Homework Set 1
”Galactic Dynamics” course at SJTU

Due Date: 2pm, Oct. 8 (Sunday), 2023

1. (10 points) There is a supermassive compact object at the center of our Milky Way galaxy. Observations show that $M = 4.2 \times 10^6 M_\odot$ is packed within 10^{-3}pc. Most astronomers believe that it must be a supermassive black hole. If the central “supermassive compact object” is a cluster of small black holes, estimate the upper mass limit of a constituent black hole. Hint: use the relaxation timescale of the system as a constraint, which cannot be too short or the black hole cluster has already evaporated.

2. (10 points) For most luminous ellipticals stellar densities are low and stellar collisions are extremely rare. The possible exceptions are low-luminosity ellipticals like M32. M32 can be approximated as a spherical mass distribution with the density $\rho(r) = 3 \times 10^5 (\frac{r}{1\ \mathrm{pc}})^{-2.4} M_\odot/\mathrm{pc}^3$ (e.g. Lauer et al. 1998, AJ, 116, 2263). Assume that most of this mass is in stars similar to the Sun.

   - What is M32's relaxation time (in Gyr) at $r = 0.1\,\mathrm{pc}$ and $\sigma = 240\,\mathrm{km\ s}^{-1}$?
   - Assume that a typical red giant star has radius $10R_\odot$ and lives for 10^8 years. Estimate inside what distance from the center of M32 do most red giants collide with other stars (not necessarily another red giant) during its life time?

3. (5 points) [Problem 1.5 in BT08]

   (a) Why is the estimated mass-to-light ratio of clusters of galaxies, equation (1.25), proportional to the assumed value of the Hubble constant h_7?

   (b) Dark matter was discovered by Zwicky (1933), who compared the mass-to-light ratio of the Coma cluster of galaxies (as measured by the virial theorem, §4.8.3) with the mass-to-light ratios of the luminous parts of spiral galaxies as measured by circular-speed curves, and concluded that there was 400 times as much dark matter as luminous matter in the Coma cluster. However, Zwicky's conclusion was based on a Hubble constant $H_0 = 558\,\mathrm{km\ s}^{-1}\,\mathrm{Mpc}^{-1}$. How would his conclusion about the ratio of dark to luminous matter have been affected had he used the correct value of the Hubble constant, which is smaller by a factor of eight?

4. (10 points) [Problem 2.2 in BM98]

   I_B is the B-band surface brightness of a galactic disk measured in $L_\odot\,\mathrm{pc}^{-2}$. Show that, when viewed face-on, the B-band surface brightness of the disk will be measured to be μ_B magnitudes per square arcsecond, where

$$\mu_B = 27.05 - 2.5\log(I_B). \tag{1}$$

   You will need to use the fact that the B-band absolute luminosity of the Sun is $M_B(\odot) = 5.48$. Note that the surface brightness is independent of distance.

5. (15 points) [Slightly revised from BT08, Problem 1.4]

   An axisymmetric transparent galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ having axis ratio q. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes (iso-density contours) and central surface brightness I_n. A second distant observer, observing the galaxy from a line of sight that is inclined by an angle i to the symmetry axis, sees an image with elliptical isophotes with axis ratio $Q < 1$ and central surface brightness I_0.

(a) What is the relation between q, Q, and i? *Hint: the answers are different for oblate ($q < 1$) and prolate ($q > 1$) galaxies.*

(b) What is the relation between I_0, I_n, and Q? *Hint: 1. the answer depends on if the galaxy is oblate or prolate. 2. Projected surface brightness is the integration of luminosity density along the line of sight.*

(c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10° of the symmetry axis? From within 10° of the equatorial plane? In other words, what fractions of a large sample of randomly oriented axisymmetric galaxies are observed to have an inclination of $i \leq 10°$ (nearly face-on) and $80° \leq i \leq 90°$ (nearly edge-on), respectively?

(d) *Optional question.* (5 bonus points) In astronomy, we often need to generate a uniform distribution of stars (or points) on the surface of a sphere of radius R. If you are given a random number generator, describe how you can achieve this. This is related to question (c).