Term Project (II) – Spherical Jeans Equation

Due Date: 2pm, Jan. 18, 2024. 50 points

Please submit both your solution and codes. Using Jupyter Notebook is highly recommended.

The spherical Jeans Equation can be applied to the Galactic halo stars to estimate the Milky Way's mass profile. From LAMOST Data Release 5, we selected K-giant stars through the surface gravity ($\log g$) and the effective temperature ($T_{\rm eff}$) following Liu et al. (2014, ApJ, 790, 110), calculated their distances using the method presented in Xue et al. (2014, ApJ, 784, 170), and crossed match with Gaia DR2 to get their proper motions. After the substructures were identified and removed, we get a smooth halo sample of about 9000 K giants with $|z| > 5$ kpc or (2 kpc $< |z| < 5$ kpc and $[\mathrm{Fe/H}] < -1$).

The catalog 'lamost_kg_dr5_smoothhalo.fits' contains the position and kinematic information of the K-giant stars, which can be downloaded here. Useful columns include Column 2 for x (kpc), Column 4 for y (kpc), Column 6 for z (kpc), Columns 30 and 31 for V_r and $V_r^{\rm error}$ (km/s), Columns 32 and 33 for V_ϕ and $V_\phi^{\rm error}$ (km/s), Columns 34 and 35 for V_θ and $V_\theta^{\rm error}$ (km/s), Column 40 for E (estimated total energy, $(\mathrm{km/s})^2$). The position and velocity are in the Galactocentric coordinate frame. You can find more information about the data in the header of the FITS-format data file. Special thanks to Professor Xiangxiang Xue at NAOC for the generous help with this project.

To read a FITS-format file, you may use `astropy.io` package in python. The `fits.open` function in that package reads a FITS table.

(a) [30 points] Assuming that the number density of K giant stars follows $n_* \sim r^{-3.6}$, derive the enclosed mass profile $M(r)$ and the corresponding circular speed curve $V_{\rm c}(r)$ of the Milky Way with the spherical Jeans equation. Also plot $M(r)$ and $V_{\rm c}(r)$ curves with the error bars derived in (b).

Hint: You may divide the sample into different radial bins (boxes) in order to derive the radial and tangential velocity dispersions at different radius (σ_r and $\sigma_t = (\sigma_\phi^2 + \sigma_\theta^2)^{1/2}$). $\beta(r) = 1 - \sigma_t^2/(2\sigma_r^2)$. The radial bin size at different radius can vary to ensure a large number of stars in each bin to give good statistics. We recommend the boundaries of the radial bins as R_i = [6, 9, 12, 15, 20, 25, 30, 35, 40, 50, 65, 100] kpc, and ignore stars inside 6 kpc. Also, it is recommended to reject the stars with velocity uncertainty greater than 100 km/s and positive energy ($E > 0$) before the calculation, since these stars may have wrong classification or very big errors. Gravitational constant $G = 4.302 \times 10^{-6}\,\mathrm{km\ s^{-2}\,kpc\,M_\odot^{-1}}$.

(b) [10 points] Estimate the errors of the mass and circular velocity profiles.

Hint: At each radial bin, use bootstrap method in astropy.stats with large number of resamples (e.g., ~ 30000). Since each resample leads to an estimation of the mass and circular velocity, the corresponding errors of the mass and circular velocity at each radial bin can be considered as the standard deviations of the mass and circular velocity distributions from all the resamples at that radius.

(c) [10 points] Once the Galactic mass profile is determined, find the best-fit NFW dark matter halo density profile with the virial mass $M_{\rm vir}$ and concentration index c as free parameters. Since the enclosed mass profile derived in (a) includes masses of the disk and bulge, it is recommended to subtract $0.8 \times 10^{11} M_\odot$ from the derived $M(r)$ to remove the disk and bulge contribution before fitting for a NFW profile.

Hint: The NFW mass profile can be characterized by the virial mass $M_{\rm vir}$ and concentration index c as follows:

$$M(r) = M_{\rm vir} \frac{f(cx)}{f(c)}, \tag{1}$$

where $x = r/r_{\rm vir}$ and $f(x) = \ln(1+x) - x/(1+x)$. Adopting $h = 0.7$, $\Delta_{\rm vir} = 200$ and at $z = 0$, the virial radius $r_{\rm vir}$ is related to the virial mass $M_{\rm vir}$ as follows:

$$r_{\rm vir} \simeq 206\,{\rm kpc} \left[\frac{M_{\rm vir}}{10^{12} M_\odot} \right]^{1/3}. \tag{2}$$