XOR operation designated with 'x'

1. If A(n) is a XOR sequence of type A(n) = 0x1x2x3x ... xn, where n>=0, then it is always in force:
if n%4=0 then A(n) = n;
if n%4=1 then A(n) = 1;
if n%4=2 then A(n) = n+1;
if n%4=3 then A(n) = 0;

2. If G(n) is a XOR sequence of type G(n) = A(0)xA(1)xA(2)xA(3)x ... A(n), where n>=0, then it is always in force:
if n%8=0 then G(n) = n;
if n%8=1 then G(n) = n;
if n%8=2 then G(n) = 2;
if n%8=3 then G(n) = 2;
if n%8=4 then G(n) = n+2;
if n%8=5 then G(n) = n+2;
if n%8=6 then G(n) = 0;
if n%8=7 then G(n) = 0;

3. The result of the XOR operation on any subsequence A(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m) is always equal to G(n-1)xG(n+m), where 1<=n, m<="number of elements in sequence".

Properties of XOR operations:
1. Commutative: AxB = BxA
2. Associative: Ax(BxC) = (AxB)xC
3. Identity Element: Ax0 = A
4. Self-inversive: AxA = 0

1. Proof for XOR sequence A(n).

1.1 Taking account of the properties of XOR, then for any integer 'n' (n>=0) the following XOR operations are always true:

sequence	**n=0, A(0) n>0, A(n-1)**	**operation**	**n**	**result**
A(n)	0	XOR	n	**n**
A(n+1)	n	XOR	(n+1)	**1**
A(n+2)	1	XOR	(n+2)	**(n+2)+1**
A(n+3)	(n+2)+1	XOR	(n+3)	**0**

1.2 If A, C, D are integers and A%C=D, then for any integer 'n' (n>=0), it is in force:
(A+n*C)%C = D

Statements 1.1 and 1.2 prove the postulated correlation of the results for XOR sequence of type A(n).

2. Proof for XOR sequence G(n).

2.1 Since G(n)=A(1)xA(2)x ... A(n) = G(n-1)xA(n) and taking account of
the properties of XOR and of type A(n) sequence, then for any integer 'n' (n>=0),
the following XOR operations are always true:

sequence	n=0, G(0) n>0, G(n-1)	operation	A(n)	result
G(n)	0	XOR	n	**n**
G(n+1)	1	XOR	n	**(n+1)**
G(n+2)	n	XOR	(n+2)	**2**
G(n+3)	2	XOR	0	**2**
G(n+4)	2	XOR	(n+4)	**(n+4)+2**
G(n+5)	(n+4)+2	XOR	1	**(n+5)+2**
G(n+6)	(n+5)+2	XOR	(n+6)+1	**0**
G(n+7)	0	XOR	0	**0**

2.2 If A, C, D are integers and A%C=D, then for any integer 'n' (n>=0), it is in force:
(A+n*C)%C = D

Statements 2.1 and 2.2 prove the postulated correlation of the results for XOR sequence of type G(n).

3. Proof for A(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m) = G(n-1)xG(n+m)

Taking account of the XOR properties, the following operations could be made:

A(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m)
=
[A(1)xA(2)...A(n-1)] x [A(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m)] x [A(1)xA(2)x...A(n-1)]
=
[A(1)xA(2)...A(n-1)] x [A(1)xA(2)...A(n-1)xA(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m)]
=
G(n-1)xG(n+m)

Thus A(n)xA(n+1)xA(n+2)x...A(n+(m-1))xA(n+m) = G(n-1)xG(n+m)