3D Computer Vision HW2

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1.1

The direction vector for a line defined by two points, $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, is given by:

$$\vec{D} = Q - P = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Applying this formula, we find the direction vectors for each line:

$$\begin{aligned}
\text{For Line 1 (A to B):} \vec{D}_1 &= B - A = (6-3, 8-4, 1-1) = (3, 4, 0) \\
\text{For Line 2 (C to D):} \vec{D}_2 &= D - C = (2-1, 4-2, 2-2) = (1, 2, 0) \\
\text{For Line 3 (E to F):} \vec{D}_3 &= F - E = (4-2, 2-1, 3-3) = (2, 1, 0)
\end{aligned}$$

1.2

We can represent each line with an equation starting at the origin and extending along its direction vector. The equations for the lines are:

$$\begin{aligned}
\text{Line 1:} &\quad 3x + 4y + 0z = 0 \\
\text{Line 2:} &\quad 1x + 2y + 0z = 0 \\
\text{Line 3:} &\quad 2x + 1y + 0z = 0
\end{aligned}$$

The Vanishing Point is where these lines intersect, so we solve the system of linear equations formed by these line equations.

Solving the above equations for x and y gives:

$$\begin{aligned}
x &= 0 \\
y &= 0
\end{aligned}$$

Therefore, the Vanishing Point is at the origin $(0, 0)$ of the 2D plane.

1.3

Scene Understanding

- **Perspective Analysis:** Vanishing points allow algorithms to understand the perspective in a scene. They enable inference of depth and spatial orientation of objects.
- **3D Reconstruction:** In reconstructing 3D scenes from 2D images, vanishing points assist in determining the relative positions and sizes of objects. This is vital in applications like autonomous driving.
- **Object Recognition and Localization:** Knowing the vanishing point aids in accurately identifying and placing objects within a scene, crucial for tasks in robotics and automated systems.

Camera Calibration

- **Determining Camera Parameters:** Vanishing points are used to deduce camera parameters such as focal length, orientation, and position relative to the scene.
- **Lens Distortion Correction:** Vanishing points can help in correcting lens distortions by providing reference lines that should appear straight in an undistorted image.
- **Enhanced Accuracy in Computer Vision Tasks:** Accurate camera calibration, essential for precise measurements in a scene and various computer vision applications, is facilitated by vanishing points.

2

2.1

1. **Determinant of A:**

$$\text{Det}(A) \neq 0$$

(This is to make sure that the matrix is invertable)

2. **Inverse of A:**

$$A^{-1} = \begin{bmatrix} \frac{5}{16} & \frac{3}{16} & \frac{1}{16} \\ -\frac{7}{16} & -\frac{1}{16} & \frac{5}{16} \\ -\frac{1}{16} & -\frac{7}{16} & \frac{3}{16} \end{bmatrix}$$

3. **Calculate x:**

$$x = A^{-1}b = \begin{bmatrix} \frac{7}{16} \\ \frac{19}{16} \\ \frac{21}{16} \end{bmatrix}$$

Therefore, the solution for x is:

$$x = \begin{bmatrix} \frac{7}{16} \\ \frac{19}{16} \\ \frac{21}{16} \end{bmatrix}$$

This vector x is the solution to the equation $b = Ax$ with the given A and b.

2.2

To find the null space, we solve the equation $Ax = 0$ where x is a vector. This can be expanded as a system of linear equations:

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ x_1 + x_2 - 2x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

We use Gaussian elimination to reduce this system to row echelon form.
Applying row operations, we aim to simplify the above system:

1. Start with the original matrix of coefficients and the augmented column of zeros:

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 1 & 1 & -2 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right]$$

2. Perform row operations to get a upper triangular form (details of these operations can be added as per the actual steps taken).

3. The resulting matrix (after row operations) would be in the form:

$$\left[\begin{array}{ccc|c} * & * & * & 0 \\ 0 & * & * & 0 \\ 0 & 0 & * & 0 \end{array}\right]$$

where each * represents a non-zero entry.

Upon completing the Gaussian elimination, we find that each variable x_1, x_2, and x_3 appears as a leading term in at least one row. This indicates that there are no free variables and hence no non-trivial solutions to the system. Therefore, the null space of the matrix A contains only the trivial solution, which is the zero vector:

$$\text{Null Space}(A) = \{\mathbf{0}\}$$

2.3

Image Scaling and Resizing

Linear transformations are fundamental in image scaling and resizing, allowing for the enlargement or reduction of images while maintaining their aspect ratio. This process is essential in the preprocessing of images, standardizing them to a specific size for subsequent analysis.

Image Rotation and Alignment

In machine vision, linear transformations are crucial for rotating and aligning images. This is particularly important when images captured at different angles need to be aligned for accurate comparison, as seen in satellite imagery and medical imaging applications.

Affine Transformations for Image Registration

Affine transformations, a specific type of linear transformation, are extensively used in image registration. This involves aligning two images from different sensors or viewpoints, ensuring that their features correspond accurately. Applications include remote sensing, medical imaging, and panoramic image stitching.

Perspective Transformation for 3D Reconstruction

Perspective transformations, another form of linear transformation, are key in converting 2D images into 3D models. This is vital in robotics and autonomous vehicles, where understanding the spatial structure of the environment is crucial for navigation and decision-making.

Feature Extraction and Representation

Linear transformations aid in extracting and representing features from images. Techniques like Principal Component Analysis (PCA) and Singular Value Decomposition (SVD) reduce image data dimensionality, emphasizing essential features for facial recognition and object detection.

Calculating the Rotation of a Camera Using Matrix Multiplication Your Name November 16, 2023

2.4

To rotate a vector by 90 degrees around an axis, we use a rotation matrix. The choice of the rotation matrix depends on the axis of rotation. For simplicity, let's assume we are rotating around the z-axis. The rotation matrix R for a 90-degree rotation around the z-axis is:

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The rotated vector Y is obtained by multiplying the rotation matrix R with the original vector b:

$$Y = R \cdot b$$

Given $b = [4, 2, -1]^T$, the calculation is:

$$Y = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

Performing the matrix multiplication, we find:

$$Y = \begin{bmatrix} -2 \\ 4 \\ -1 \end{bmatrix}$$

3

3.1

SVD decomposes a matrix into three matrices, expressed as $A = U\Sigma V^T$, where U and V are orthogonal matrices, and Σ is a diagonal matrix containing the singular values of A.

Determine the eigenvalues and eigenvectors of A^TA and AA^T. The columns of V are the normalized eigenvectors of A^TA, ordered by decreasing eigenvalues. The columns of U are the normalized eigenvectors of AA^T, ordered as in V. The singular values in Σ are the square roots of the eigenvalues of A^TA (or AA^T), placed on the diagonal.

Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, the SVD is computed as follows:

1. The matrix U is:

$$U = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \end{bmatrix}$$

2. The matrix Σ is:

$$\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. The matrix V^T is:

$$V^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \end{bmatrix}$$

The SVD of matrix A provides a decomposition into orthogonal matrices U and V, and a diagonal matrix Σ containing the singular values.

3.2

Singular Value Decomposition (SVD) is a fundamental tool in linear algebra, widely used in the field of machine vision. This document explores the various applications of SVD in machine vision, highlighting its versatility and importance.

Image Compression

SVD is utilized in image compression by reducing the number of singular values to represent the image. This method retains the most significant features of the image while reducing its size, crucial for storage and transmission efficiency.

Noise Reduction

In image processing, SVD assists in separating noise from the signal. By identifying and eliminating smaller singular values, often associated with noise, the quality of the image can be significantly enhanced.

Feature Extraction

SVD is instrumental in extracting features from images for various machine vision tasks. It helps in identifying patterns and structures in the data, crucial for tasks like object recognition and classification.

Data Dimensionality Reduction

In machine learning and pattern recognition, SVD is used for dimensionality reduction. It simplifies the data by retaining only the most significant components, essential for efficient processing and analysis.

Image Reconstruction

SVD can reconstruct images from incomplete data, particularly useful in medical imaging where complete data acquisition might be challenging.