3D Computer Vision HW1

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1.1

The plots for signals are are follows:

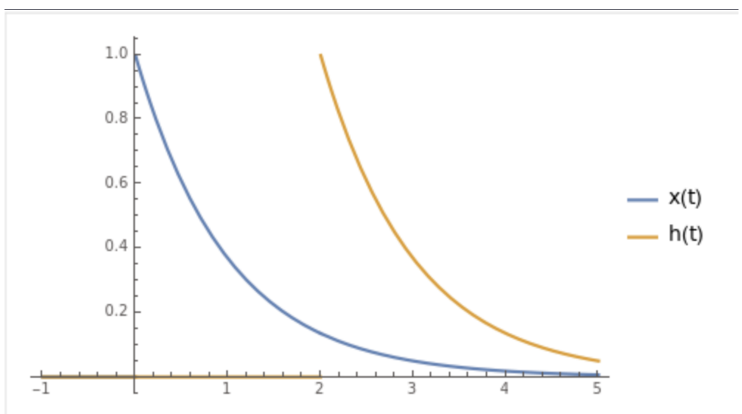

Figure 1: The plot of $x(t)$ and $h(t)$.

To find the explicit piecewise expression for the convolution of $x(t)$ and $h(t)$, we would typically break the problem into intervals based on the step functions:

1. For $t < 0$, both $x(t)$ and $h(t)$ are zero, so the convolution is zero.
2. For $0 \leq t < 2$, $h(t)$ is still zero, so the convolution is zero.
3. For $t \geq 2$, both $x(t)$ and $h(t)$ are non-zero, and the convolution integral needs to be evaluated from 2 to t.

The convolution for $t \geq 2$ is given by the integral:

$$\int_{2}^{t} e^{-\tau} e^{-(t-\tau-2)} d\tau$$

This integral simplifies to:

$$\int_{2}^{t} e^{-t} e^{2} d\tau = e^{2-t}(t-2)$$

Therefore, the piecewise function for the convolution $x(t) * h(t)$ is:

$$x(t) * h(t) = \begin{cases} 0 & \text{for } t < 2, \\ e^{2-t}(t-2) & \text{for } t \geq 2. \end{cases}$$

1.2

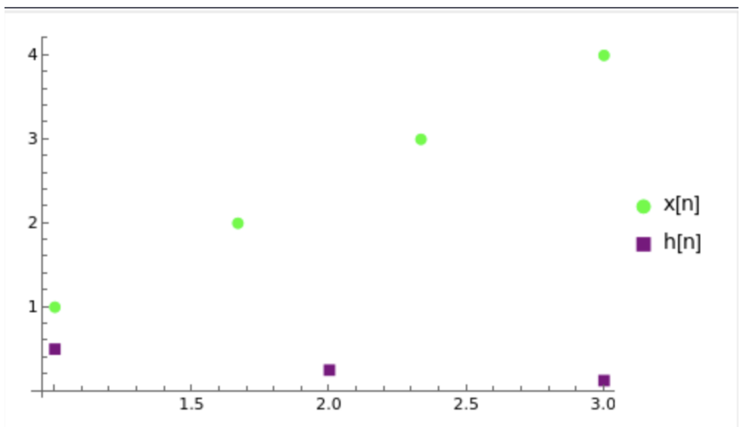

Figure 2: Plots of the discrete signals $x[n]$ and $h[n]$.

The convolution of the discrete signals $x[n] = [1, 2, 3, 4]$ and $h[n] = [0.5, 0.25, 0.125]$ is given by the sequence:

$$x[n] * h[n] = [0.5, 1.25, 2.125, 3, 1.375, 0.5]$$

This result represents the discrete convolution of the two sequences, where each term of the resulting sequence is the sum of products of the terms from $x[n]$ and $h[n]$, offset by the index of $h[n]$ and summed over the range of overlap.

1.3

Signal $X(f) = \text{sinc}(f)$:

The sinc function is defined as $\text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$. The inverse Fourier transform of the sinc function is well-known and corresponds to a rectangular pulse in the time domain.

$$x(t) = \frac{\sqrt{\pi/2}(\text{Sign}[1-t] + \text{Sign}[1+t])}{2}$$

This expression can be simplified to a rectangular function, which is typically represented as:

$$x(t) = \mathrm{rect}\left(\frac{t}{2}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

Signal $H(f)$:

The given $H(f)$ is a rectangular function in the frequency domain, which is 1 for $|f| \leq 1$ and 0 otherwise. The inverse Fourier transform of a rectangular function in the frequency domain is a sinc function in the time domain. The result obtained is:

$$h(t) = \frac{\sqrt{2/\pi}\sin(t)}{t}$$

This is the sinc function in the time domain, and it's important to note that this is not the normalized sinc function used in signal processing, which includes a factor of π. The sinc function in signal processing is defined as $\mathrm{sinc}_{\mathrm{sp}}(t) = \frac{\sin(\pi t)}{\pi t}$, but the result here is without the π factor because the rectangular function $H(f)$ is not normalized.

1.4

The convolution of the time-domain signals obtained from the inverse Fourier transforms of $X(f)$ and $H(f)$ is given by the integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

where $x(t)$ is the inverse Fourier transform of $X(f) = \mathrm{sinc}(f)$, resulting in a rectangular pulse, and $h(t)$ is the inverse Fourier transform of $H(f)$, which is a rectangular function in the frequency domain, resulting in a sinc function in the time domain. The convolution of these two functions is not trivial and does not simplify to a standard form easily. However, it is known that the convolution of a rectangular pulse with a sinc function results in a band-limited interpolation of the rectangular pulse, often referred to as a "sinc interpolation" of the pulse.