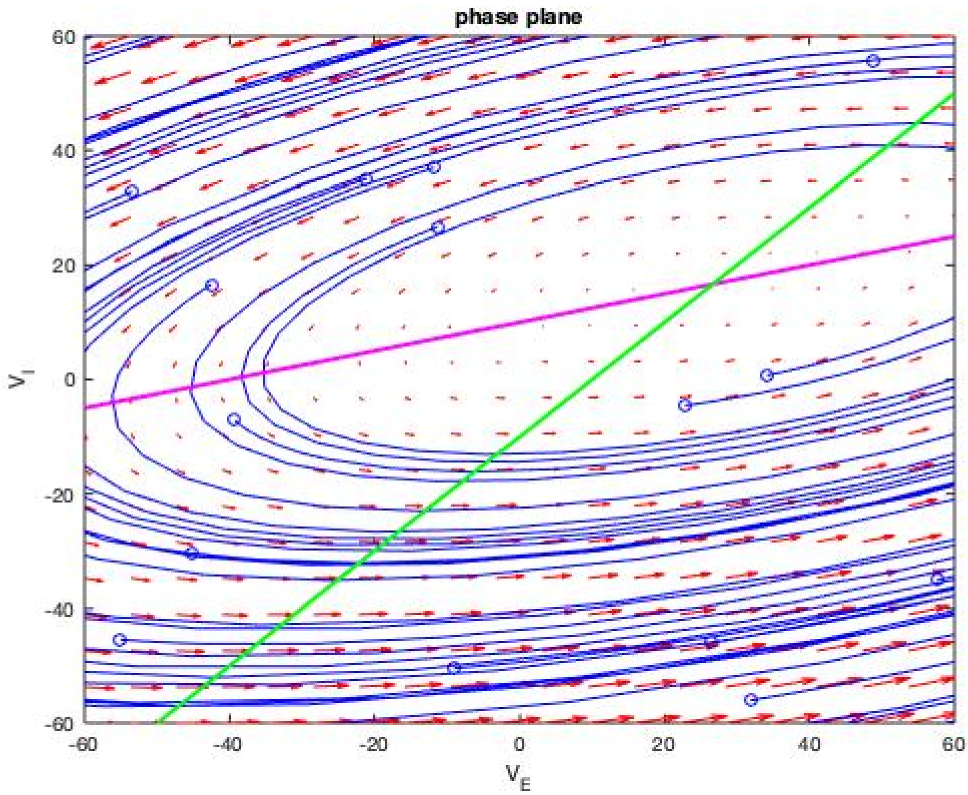
Neuroscience HW1-6 - Lachin Naghashyar

Phase plane

Consider the following code in matlab:

```
# clc
# clear
# figure
# % initial parameters noip = 10;
# interval = 60; Mee = 1.25; Mei = -1; Mie = 1;
# Mii = 0;
# Ye = -10;
# Yi = 10;
# Te = 0.01;
# Ti = 0.05;
# f = @(t,Y) [(-Y(1)+Mee*Y(1)+Mei*Y(2)-Ye)/Te;(-Y(2)+Mie*Y(1)+Mii*Y(2)-Yi)/Ti]; y1 =
# y2 = linspace(-interval,interval,20);
# % creates two matrices one for all the x-values on the grid, and one for
# % all the y-values on the grid. Note that x and y are matrices of the same
# % size and shape, in this case 20 rows and 20 columns
# [x,y] = meshgrid(y1,y2);
# u = zeros(size(x));
# v = zeros(size(x));
# % we can use a single loop over each element to compute the derivatives at % each p
# t=0; % we want the derivatives at each point at t=0, i.e. the starting time for i =
# Yprime = f(t,[x(i);
# y(i)]); u(i) = Yprime(1);
# v(i) = Yprime(2);
# end
# quiver(x,y,u,v,'r');
# xlabel('V_E')
# ylabel('V_I')
# % axis tight equal;
# hold on
# for i = 1:noip
# [ts,ys] = ode45(f,[0,50],[rand()*interval*((-1)^floor(rand()*interval)); ... rand()
# plot(ys(:,1),ys(:,2),'b') plot(ys(1,1),ys(1,2),'bo') % starting point plot(ys(end,1
# ylim([-interval interval]);
# end
# syms t fplot((t+Ye-Mee*t)/Mei,'m','LineWidth',2); fplot((Yi-Mie*t)/(Mii-1),'g','Lin
# hold('off')
```



Q1:

if we set noip=15, we will have the above phase plane.

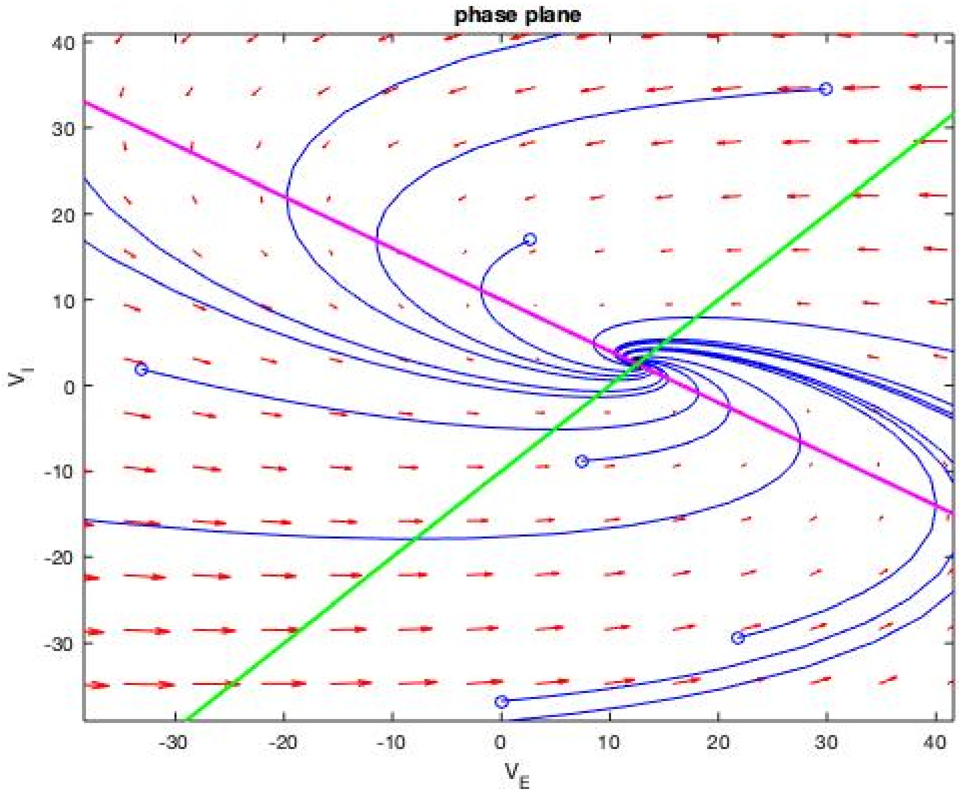
It is an unstable focus. As we can see in the figure, the phase-plane trajectory is a counter-clockwise spiral. Clearly, the fixed point is at the intersection of nullclines which are the two straight lines ($dv_E/dt = 0$ and $dv_I/dt = 0$). Also, to see the directions, we need to pay attention to the red arrows and based on them, we can see that the fixed point is an unstable one. When the fixed point is unstable, nearby configurations are pushed away from the fixed point with even a small movement away from the fixed point. Also, we can check the stability of the fixed point by looking at the real parts of the eigenvalues of its matrix (here the real parts of its eigenvalues should be positive). This way, to wrap up this part, we can see that the open circles are illustrating the initial values of v_E and v_I and since they are moving apart from their initial values, the fixed point is an unstable one.

Q2: Based on the equations, changing M_{EE} won't have any effects on V_I but it will effect V_E.

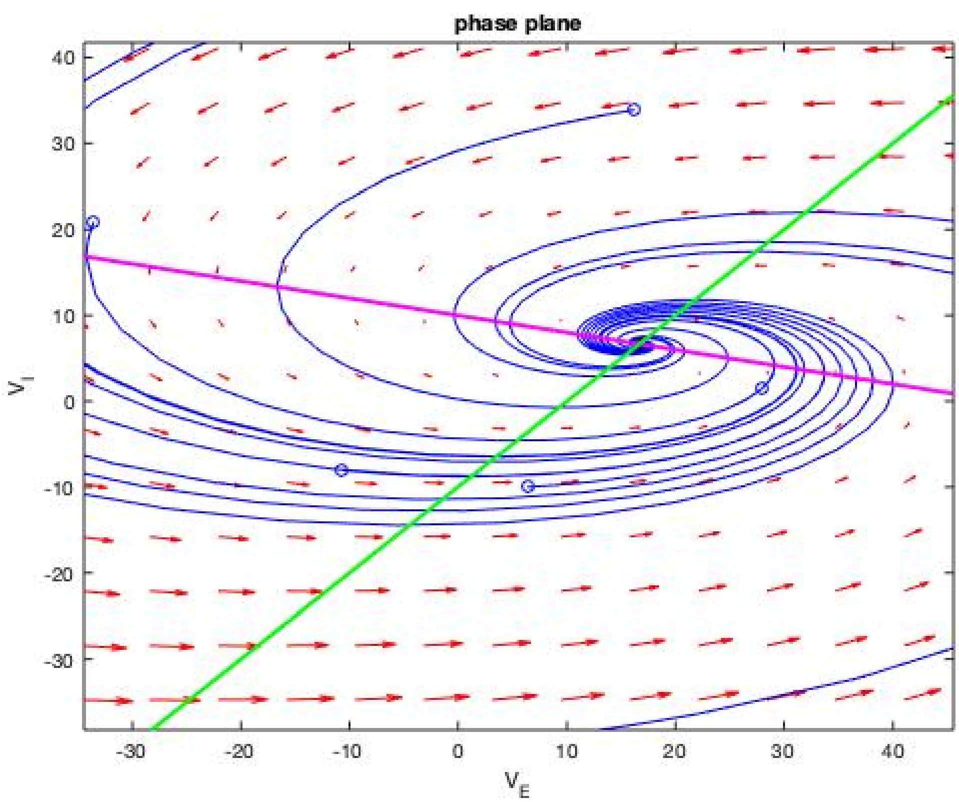
$$\begin{cases} \frac{\mathrm{V}_E}{dt} = \left(-\mathrm{V}_E + [\mathrm{M}_{EE}\ \mathrm{V}_E + \mathrm{M}_{EI}\ \mathrm{V}_I - \gamma_E]_+\right)/\tau_E \\ \frac{\mathrm{V}_I}{dt} = \left(-\mathrm{V}_E + [\mathrm{M}_{IE}\ \mathrm{V}_E + \mathrm{M}_{II}\ \mathrm{V}_I - \gamma_I]_+\right)/\tau_I \end{cases}$$

It will also change the behavior of fixed point:

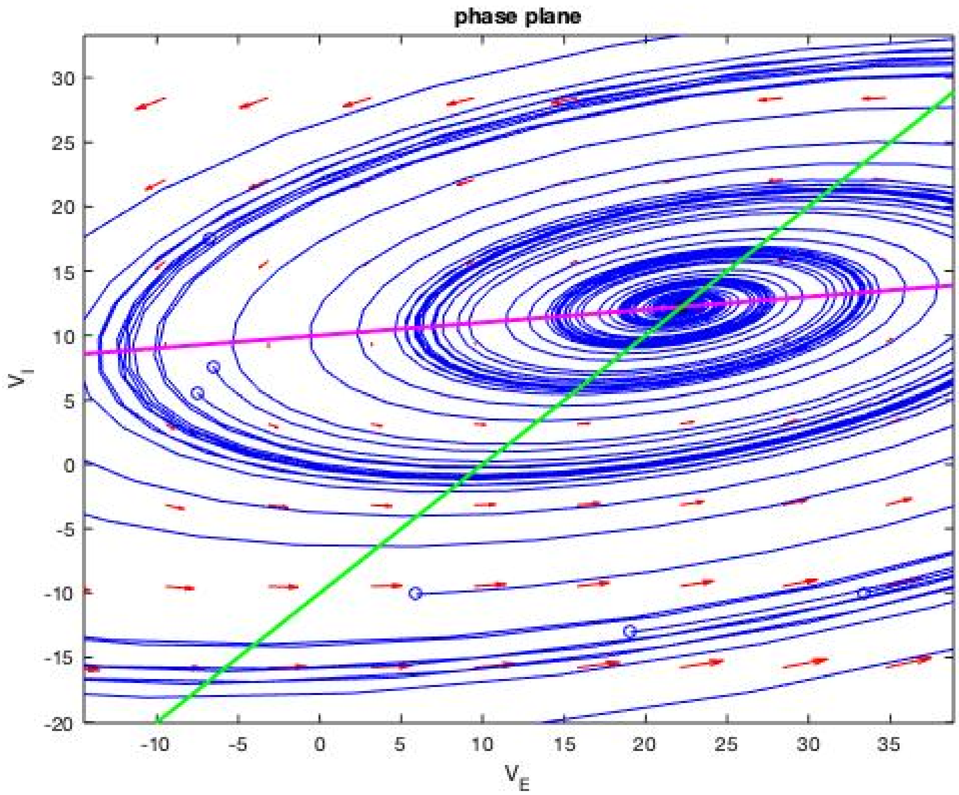
Altering M_{EE} values:



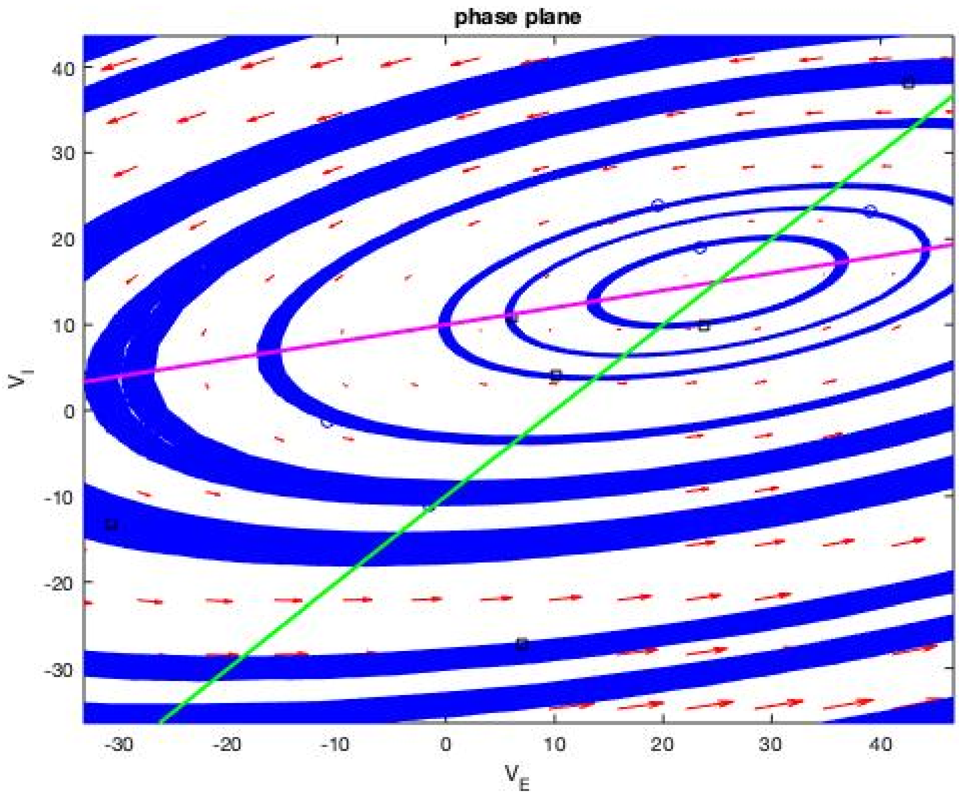
$M_{EE} = 0.4$ which is a stable focus.



$M_{EE} = 0.8$ which is also a stable focus.



$M_{EE} = 1.1$ which is a stable focus.



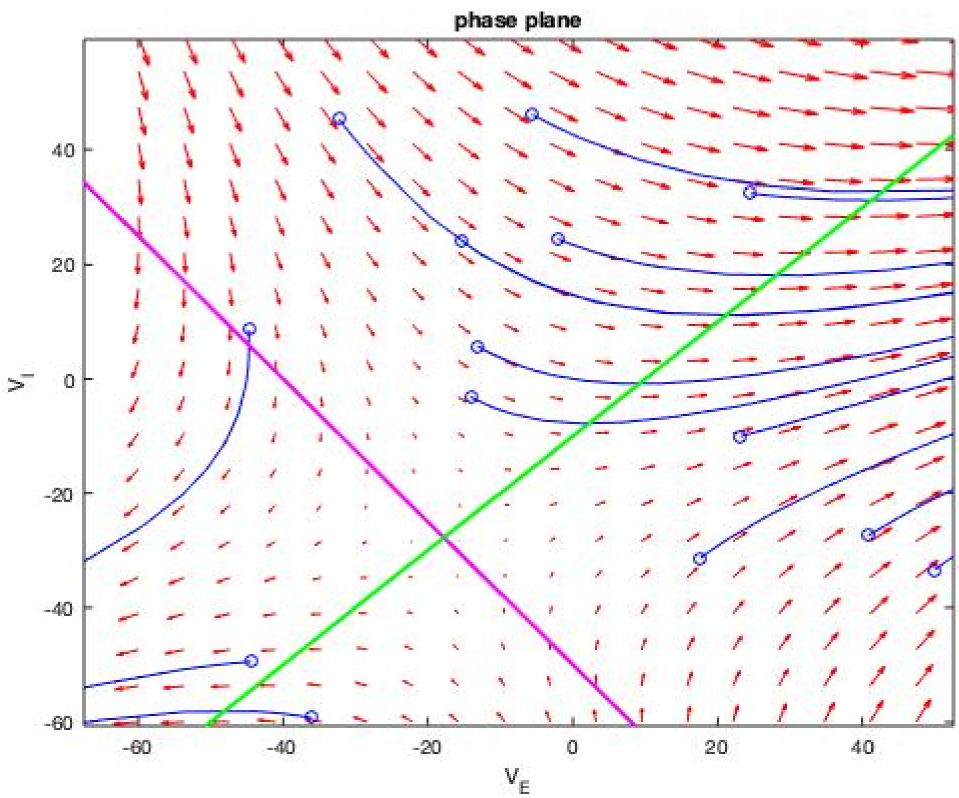
$M_{EE} = 1.2$ is a bifurcation point, it seems like it is starting to change its behavior. (because we know that in 1.25 we have an unstable focus)

$M_{EE} = 1.4$ forms an unstable focus.

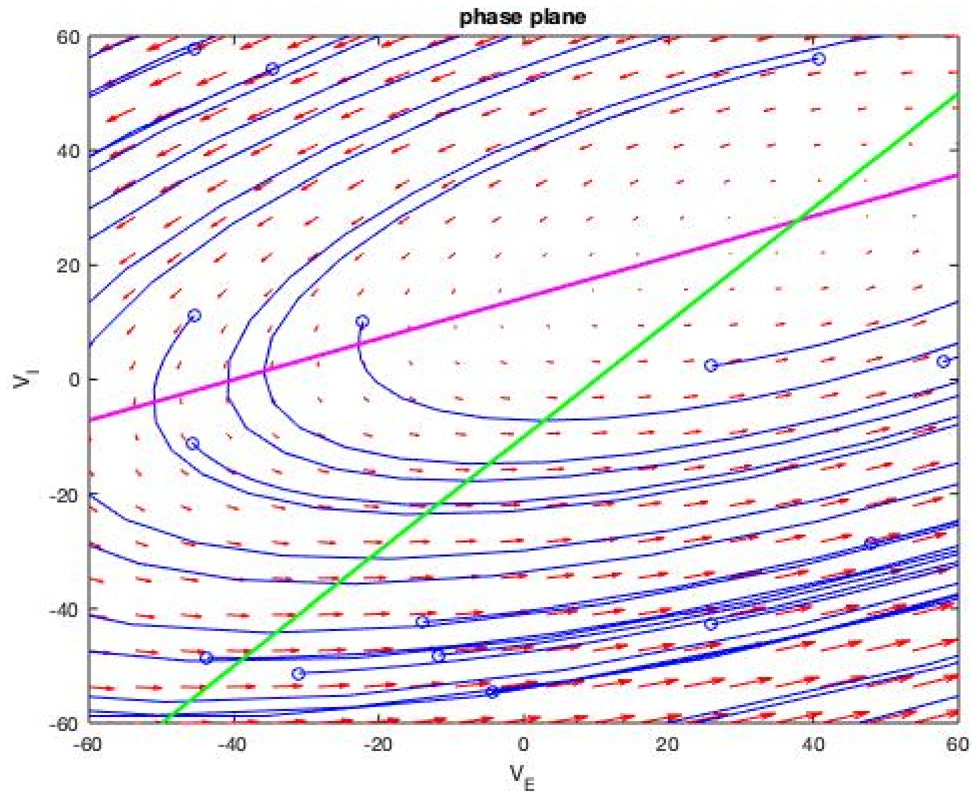
As you can see, starting from lower values of M_{EE}, the trajectory forms a stable focus and then after a bifurcation point, it turns to unstable focus.

Q3:

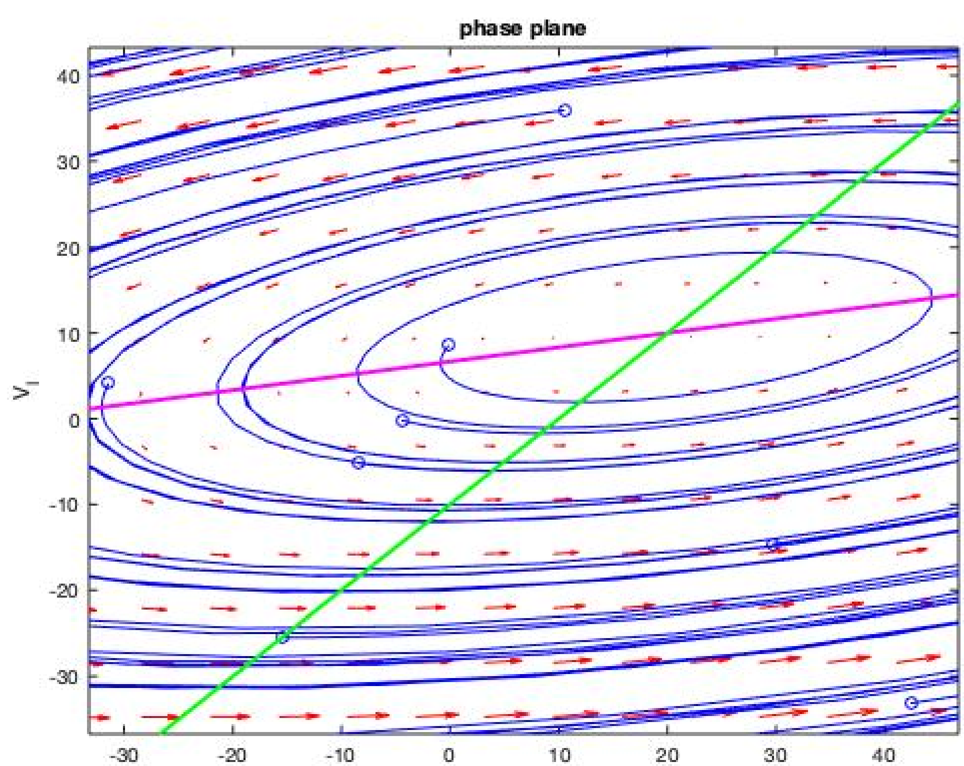
- M_{EI}:



$M_{EI} = 0.2$ which is an unstable node.



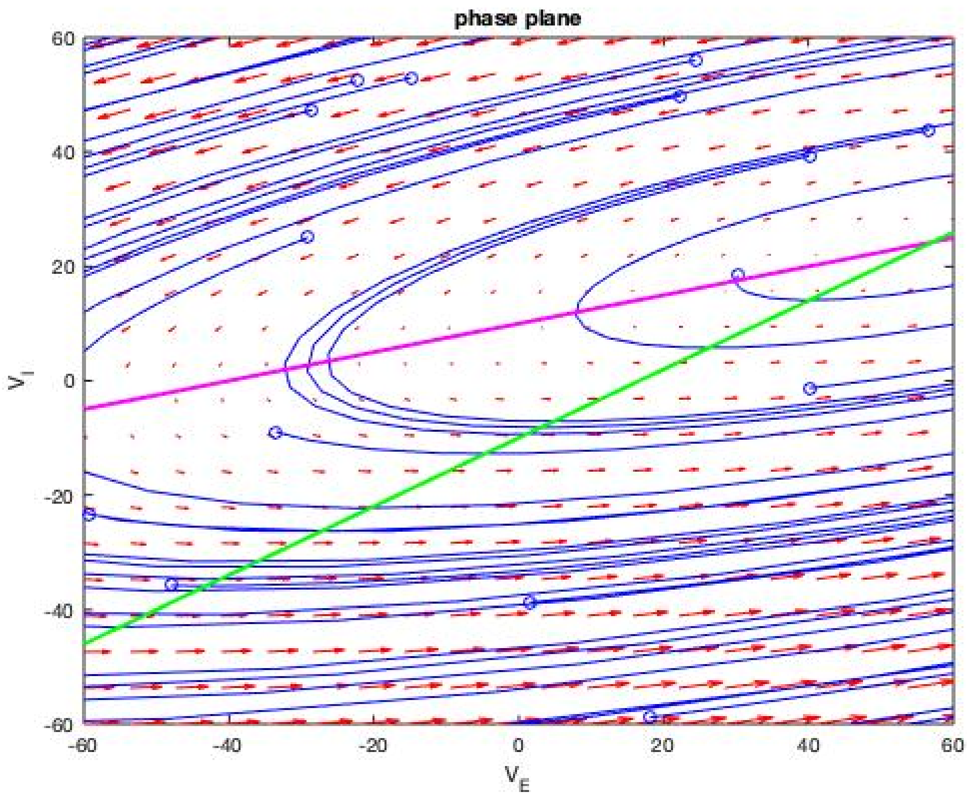
$M_{EI} = -0.7$ which is an unstable focus.



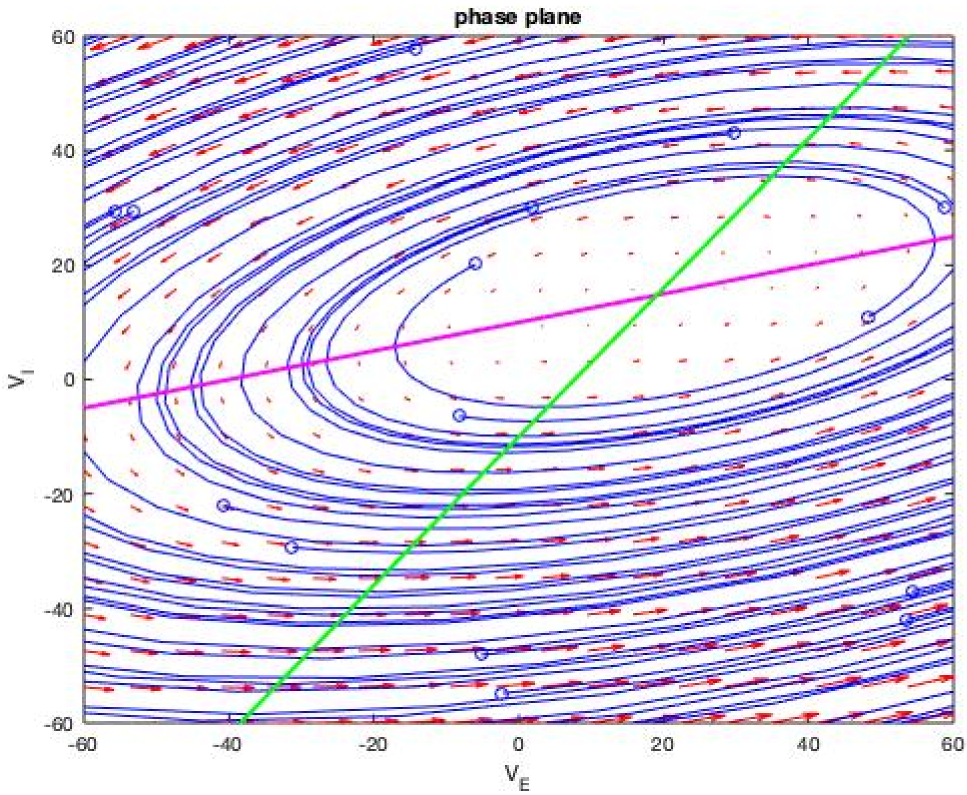
$M_{EI} = -1.5$ which is also an unstable focus.

So, starting from $M_{EI} = 0.2$ to $M_{EI} = -1.5$, the fixed point changes from an unstable node to an unstable focus.

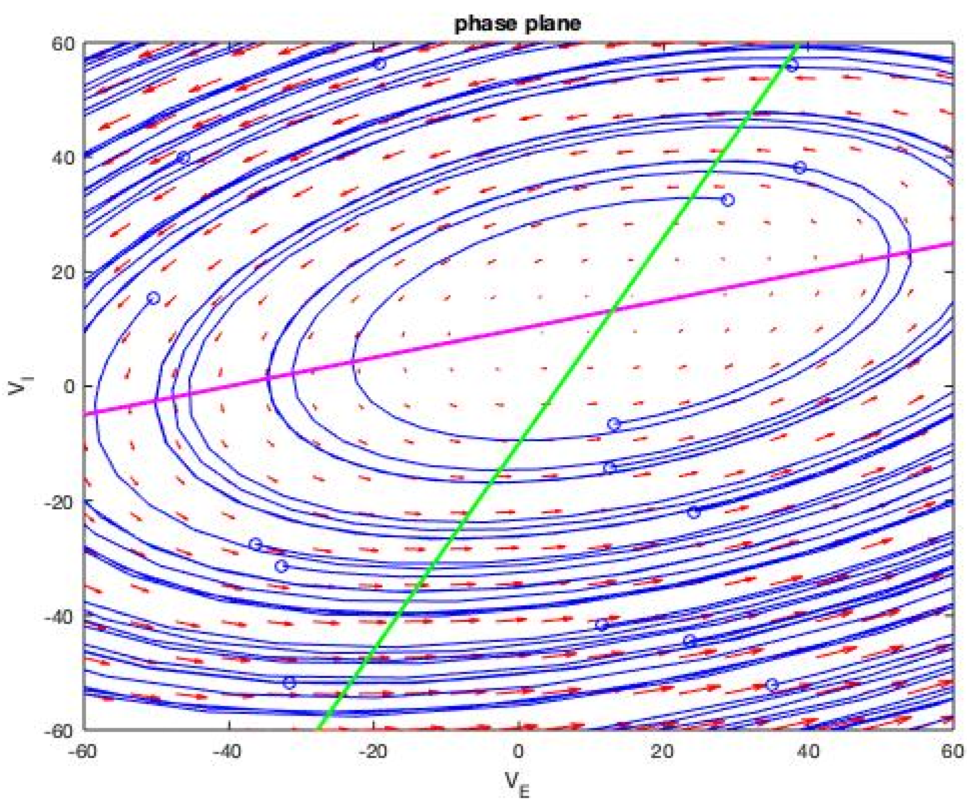
- M_{IE}:



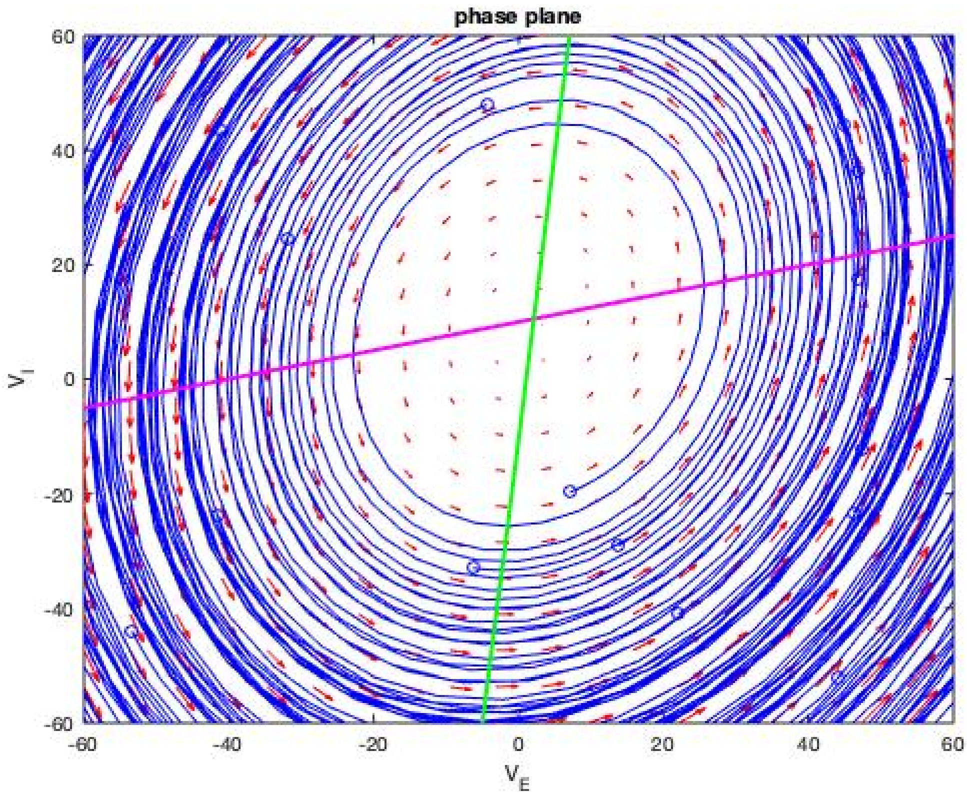
$M_{IE} = 0.6$ which is an unstable focus.



$M_{IE} = 1.3$ which is an unstable focus.



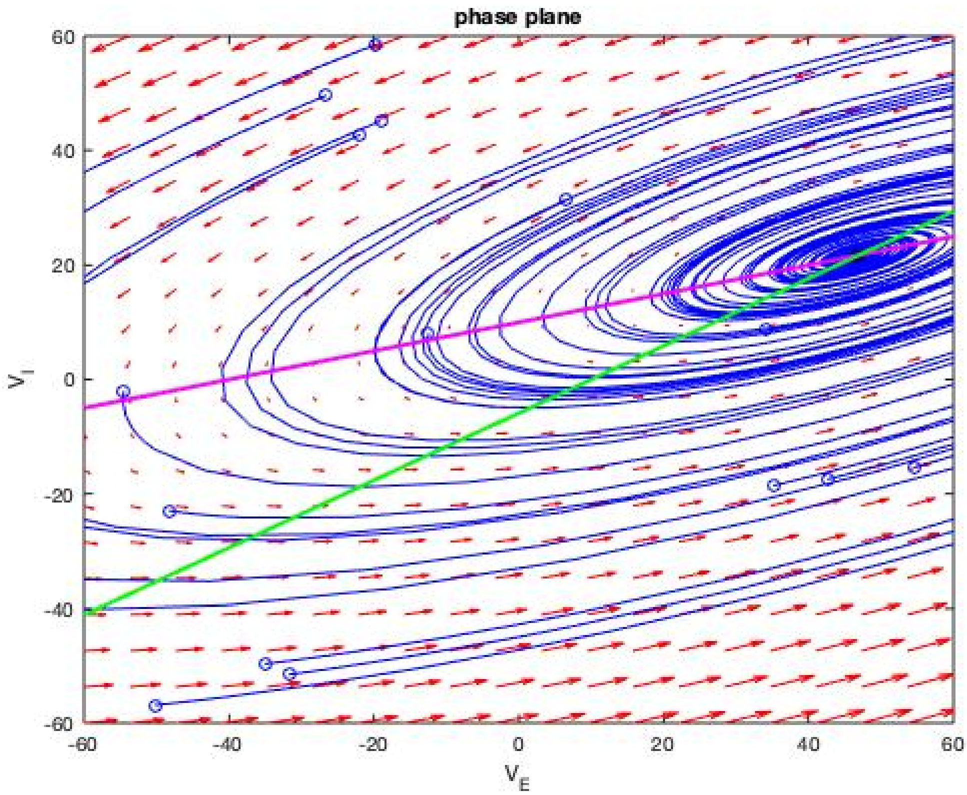
$M_{IE} = 1.8$ that forms an unstable focus.



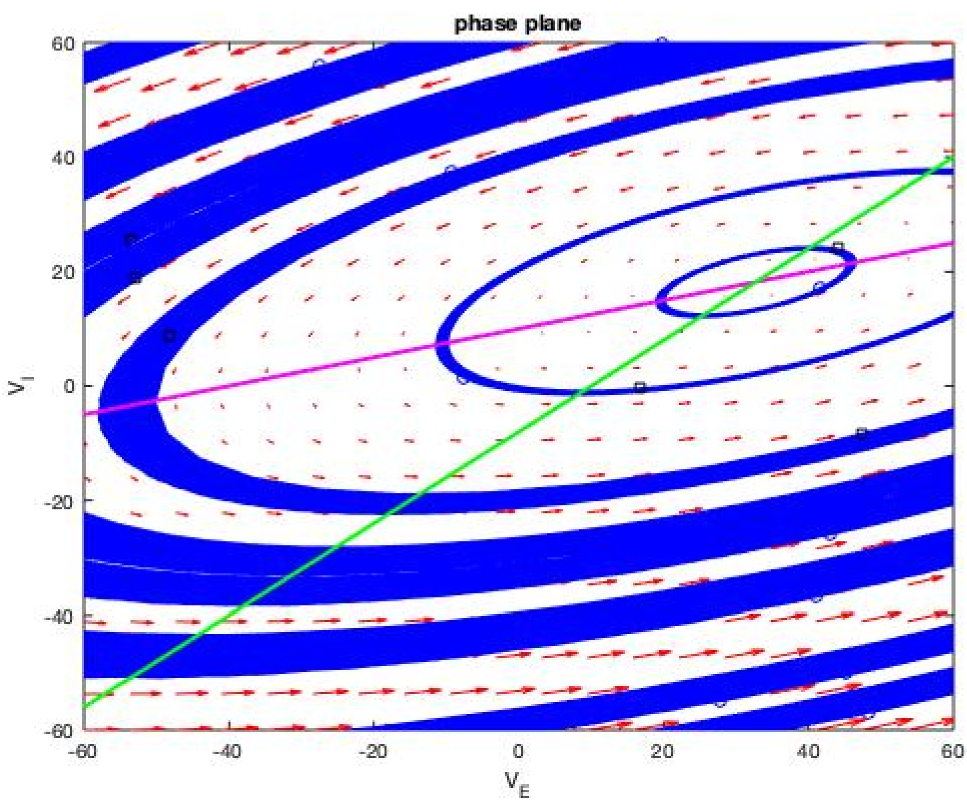
$M_{IE} = 10$ it remains to be an unstable focus.

Starting from $M_{IE} = 0.6$ to $M_{IE} = 10$, the fixed point remains an unstable focus.

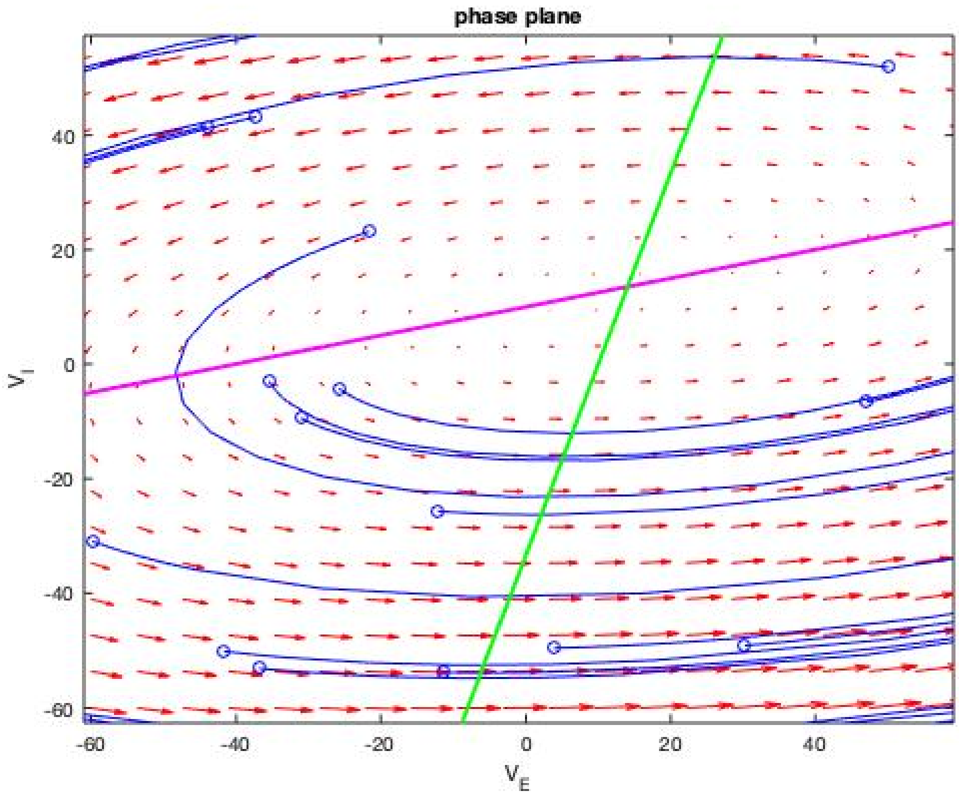
- M_{II}



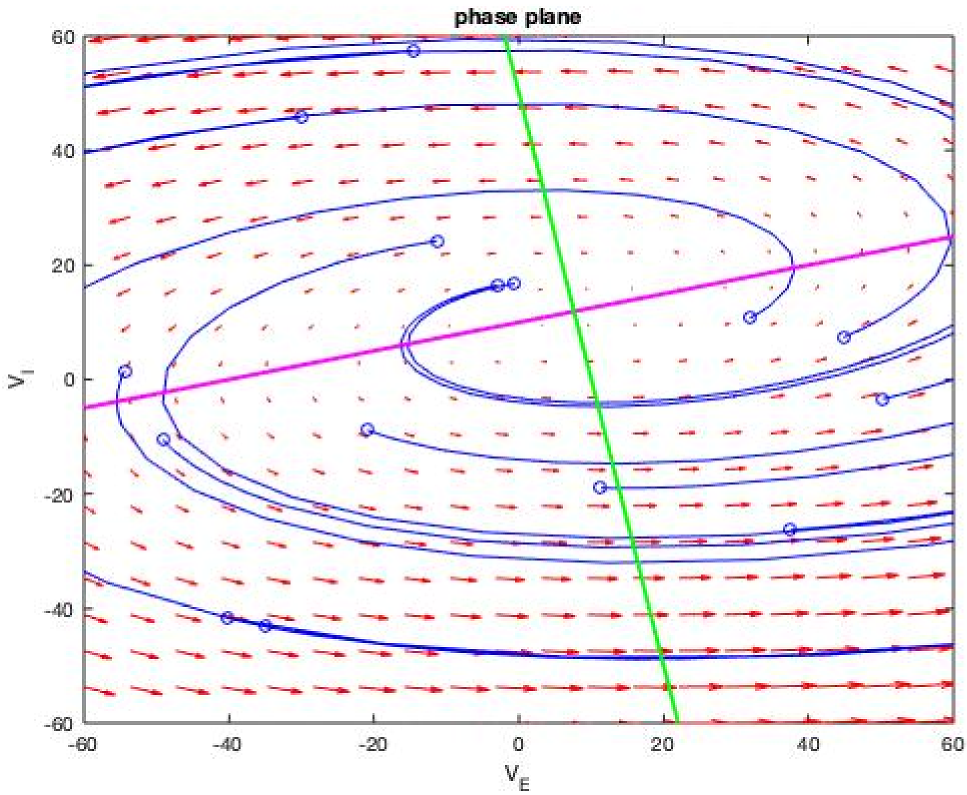
$M_{II} = -0.7$ which is a stable focus.



$M_{II} = -0.25$ which is the bifurcation point.



$M_{II} = 0.7$ that is an unstable focus.

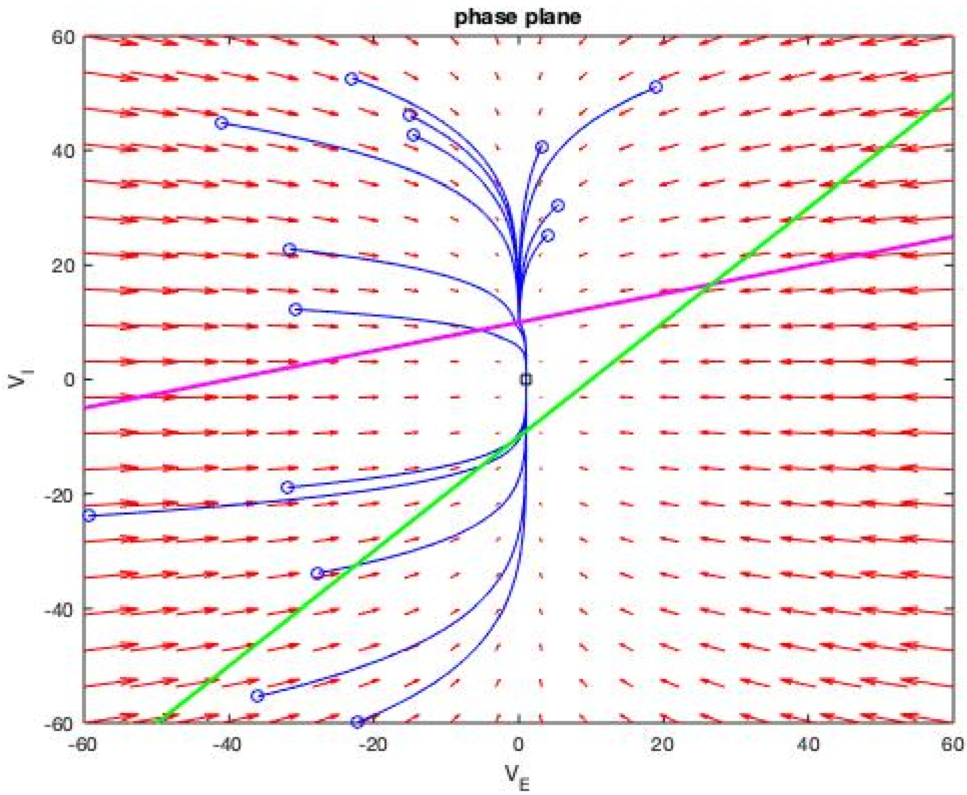


$M_{II} = 1.2$ that is again, an unstable focus.

Also for $M_{II} = -0.7$ to $M_{IE} = 1.2$, the fixed point changes from a stable focus to an unstable focus and experiences a bifurcation at $M_{IE} = -0.25$

Q4: Note that if the initial values (open blue circles) fall in to the intersection of the nullclines (the fixed point) then we will have $dv_E/dt = 0$ and $dv_I/dt = 0$ and this way, it won't move and enter the limit cycle.

Q5: I added the step function in the end of the matlab code (q6.m). Then used it in the two places it was needed



comparing to the initial phase plane, they both have the same nullclines but different vector fields. And here, the fixed point is a stable node. Vector fields are different because every time when calculating dv, the part that is added to V_t is 0 or 1, meaning it doesn't have different values of different signs and this makes the vector fields to be mostly in one direction.