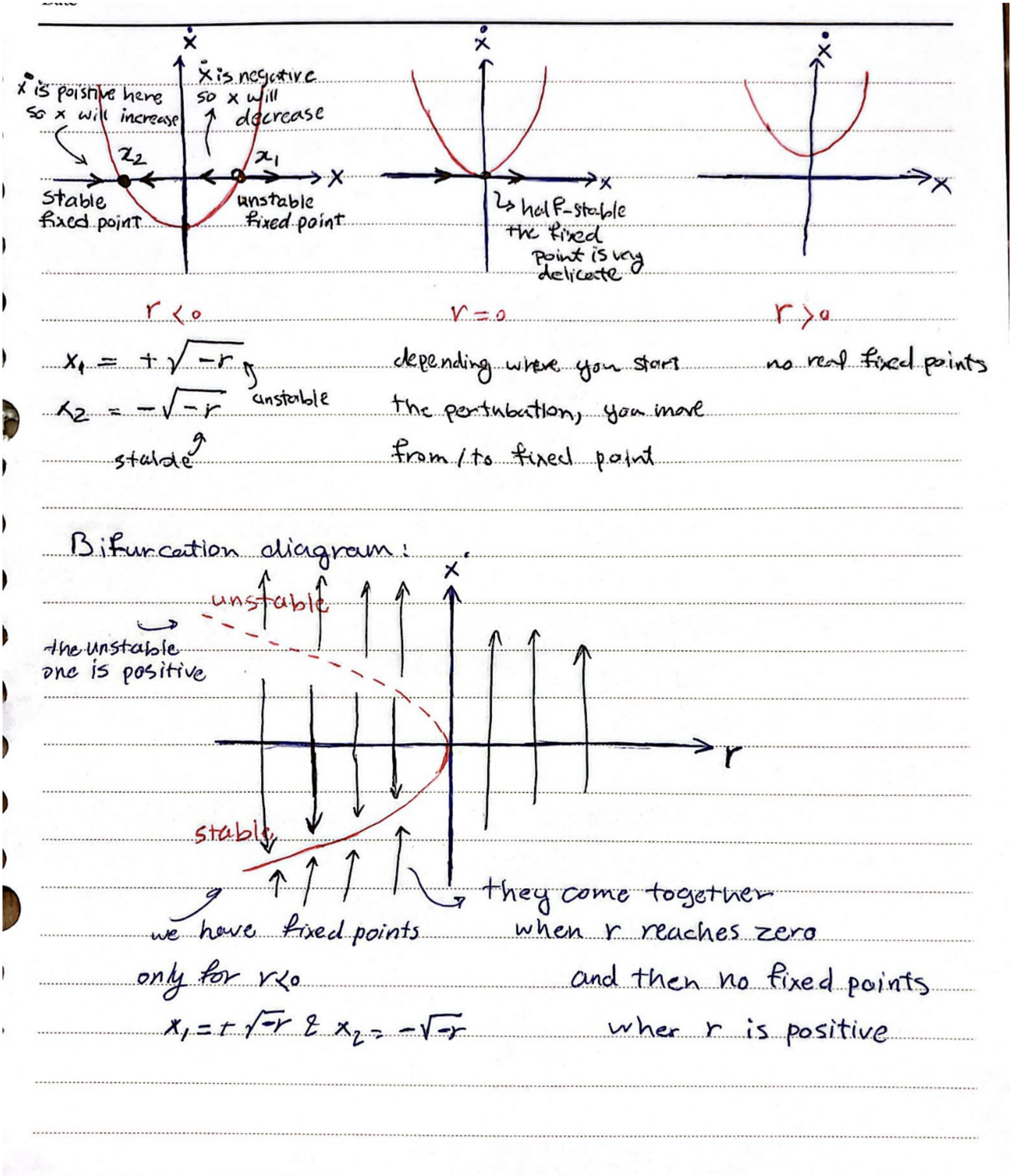
Neuroscience HW1-5 - Lachin Naghashyar

Bifurcations

First we find the fixed points: we know that fixed points satisfy the equation $r + x^2 = 0$. or $x_1 = +\sqrt{-r}$ and $x_2 = -\sqrt{-r}$. Since we are looking for real fixed points, we will have them for $r \leq 0$. Then we will have two real fixed points.

To look at the stability of fixed points, we know that the stability depends on the derivative of $r + x^2$ which is $2x$. so if x is negative, then the fixed point is stable. Meaning x_2 is stable and x_1 is unstable.

Also note that bifurcation occurs at $r = 0$:



For the second part of the question, note that the bifurcation point happens where we have stable fixed point on one side and unstable one on the other side. Also, we know that the stability of a point is defined based on $f(x)'$'s sign. If it is negative then we have a stable point and if it is positive, it is unstable. Hence, when we have $f(x)' = 0$, the stability changes (a change in the direction) and we have the bifurcation point. We can also see this in the bifurcation diagram above. At bifurcation points $r = 0$ and we have $f(x)' = 0$.