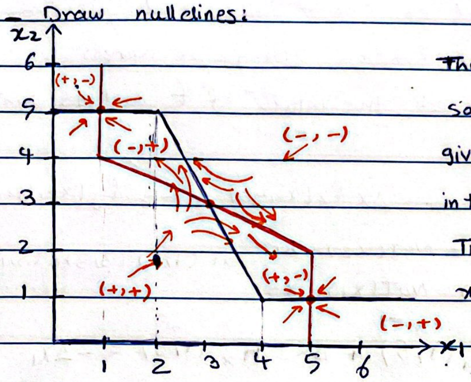
Neuroscience HW2-1 Lachin Naghashyar

Dynamic model of a SR Latch system:

$x_1' = NOT(x_2) - x_1 = h(x_1, x_2)$

$x_2' = NOT(x_1) - x_2 = l(x_1, x_2)$

- Draw nullelines:



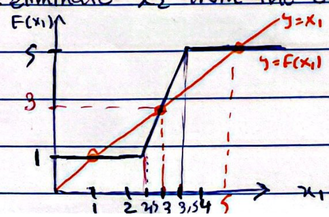
The first nullcline is obtained by solving $h(x_1,x_2)=0$ or $x_1'=0$ which gives $NOT(x_2)=x_1$, so it is represented in this graph with blue.
The other one is simalarly obtained for $x_2'=0$ or $NOT(x_1)=x_2$ and is shown in red. The fixed points which appear in their intersections, are the orange ones.

- Show the signs of x_1' and x_2' on different parts of the graph with pairs such as $(+,-)$.

- $(0.5, 5.5) \rightarrow x_1' = NOT(5.5) - 0.5 = 1 - 0.5 = 0.5 \rightarrow +$
  $\hookrightarrow x_2' = NOT(0.5) - 5.5 = 5 - 5.5 = -0.5 \rightarrow -$ $\Big\}$ $(+,-)$
- $(2,2) \rightarrow x_1' = 5-2 = 3 \rightarrow +,\ x_2' = 3 \rightarrow + \rightarrow (+,+)$
- $(5,5) \rightarrow x_1' = x_2' = -4 \rightarrow - \rightarrow (-,-)$
- $(2,4) \rightarrow x_1' = -1,\ x_2' = 1 \rightarrow (-,+)$
- $(4,2) \rightarrow (+,-)$ • $(5.5, 0.5) \rightarrow (-,+)$

$\rightarrow$ As you can see, $(3,3)$ is stable whereas $(1,5)$ & $(5,1)$ are unstable

- eliminate x_2 from the equation: $\rightarrow x_1 = NOT(x_2) = \underbrace{NOT(NOT(x_1))}_{F(x_1)}$



$[0,2] \rightarrow NOT(x_1) = 5 \rightarrow NOT(5) = 1$

$2.5 \rightarrow NOT(2.5) = 4 \rightarrow NOT(4) = 1$

$[4,6] \rightarrow NOT(NOT(x_1)) = NOT(1) = 5$

$3.5 \rightarrow 5$

$\underline{NOT(3) = 3}$ $NOT(NOT(1)) = 1$
$NOT(NOT(5)) = 5$

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So we know that the x_1 value in fixed points, is 1, 3 and 5
the corresponding x_2 value can be found by using $x_2 = NOT(x_1)$
which gives us the three fixed Points (1,5), (3,3) and (5,1) as before.
- use linear approximation to check stability.

One method to check their stability, is as follows:
Note that h and l, depend on two variables and we can
write linear approximation for function $f(x,y)$ to express
$f(x_0+\Delta x, y_0+\Delta y)$ in terms of the values of f and its partial
derivatives at (x_0, y_0):

$$f(x_0+\Delta x, y_0+\Delta y) \approx f(x_0,y_0) + \Delta x f_x(x_0,y_0) + \Delta y f_y(x_0,y_0)$$

write this for $h(x_1,x_2) = NOT(x_2) - x_1$ at (1,5) & (3,3) & (5,1)
and $l(x_1,x_2) = NOT(x_1) - x_2$

$$h(x_1,x_2)\begin{cases}(1,5): 0 + \Delta x_1 \, \overset{-1}{h_{x_1}(1,5)} + \cancel{\Delta x_2 \, \overset{0}{h_{x_2}(1,5)}} = -\Delta x_1 \\ (3,3): 0 + \; -\Delta x_1 - 2\Delta x_2 = -\Delta x_1 - 2\Delta x_2 \\ (5,1): \; -\Delta x_1\end{cases}$$

So for $\Delta x > 0 \rightarrow$ the point gets smaller, meaning its stable
(as for $\Delta x < 0 \rightarrow$ gets bigger and closer to the fixed point)
it's the same for $l(x_1,x_2)$

$$l(x_1,x_2)\begin{cases}(1,5): -\Delta x_2 \rightarrow \text{stable} \\ (3,3): -\Delta x_2 - 2\Delta x_1 \\ (5,1): -\Delta x_2 \rightarrow \text{Stable}\end{cases}$$

but it is not the same for (3,3) because if x_1 and x_2 change
in different directions, it becomes unstabe, to be more precise:

Use the linearazation in slides of chapter 1 set III;

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$$h(x_1, x_2) = a(\overbrace{x_1 - x_{1_0}}^{u}) + b(\overbrace{x_2 - x_{2_0}}^{w}) = au + bw$$

$$l(x_1, x_2) = c(x_1 - x_{10}) + d(x_2 - x_{20}) = cu + dw$$

$a = \frac{\partial h}{\partial x_1}\Big|_{(x_{10}, x_{20})}$ and similar for others (slide num 56)

$$\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{\text{Jacobian Matrix} \to L} \begin{bmatrix} u \\ w \end{bmatrix}$$

eigenvalues of L are yeild by solving $\det(L - \lambda I) = 0$

$$\to \lambda^2 - \underbrace{(a+d)}_{T}\lambda + \underbrace{ad - bc}_{\Delta} = 0$$

eigen values: $\lambda_1 = \frac{T + \sqrt{T^2 - 4\Delta}}{2}$, $\lambda_2 = \frac{T - \sqrt{T^2 - 4\Delta}}{2}$

if both eigenvalues have negative real parts $\to$ stable
else $\to$ unstable

In our case: $h(x_1, x_2) = NOT(x_2) - x_1$ & $l(x_1, x_2) = NOT(x_1) - x_2$

- $(1,5)$: $a = -1$ $b = 0$ $c = 0$ $d = -1$
- $(3,3)$: $a = -1$ $b = -2$ $c = -2$ $d = -1$
- $(5,1)$: $a = -1$ $b = 0$ $c = 0$ $d = -1$

$\to (1,5)$: $T = -2$, $\Delta = 1 \to \lambda_1 = \lambda_2 = -1 \to$ stable

$\to (5,1)$: $T = -2$, $\Delta = 1 \to$ stable

$\to (3,3)$: $T = -2$, $\Delta = 1 - 4 = -3 \to \lambda_1 = \frac{-2 + \sqrt{4 + 12}}{2} = 1$

$\lambda_2 = -3$

$\to$ it has one eigenvalue less than zero $\to$ unstable

Also we can see that for $\Delta < 0 \to$ where $\lambda_2 < 0 < \lambda_1$

the fixed point is a saddle