Neuroscience HW4_1 Lachin Naghashyar 98110179

Hopfield Model

- 1.1.1 : we have 3 prototypes with 4 neurons:

$$x_1 = \begin{bmatrix} +1 \\ +1 \\ +1 \\ +1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} +1 \\ +1 \\ +1 \\ -1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} +1 \\ +1 \\ -1 \\ -1 \end{bmatrix}$$

- we know that weights of contributions of M prototypes are obtained as $W_{ij} = \frac{1}{N} \sum_{\mu=1}^{M} p_i^{\mu} p_j^{\mu}$

here $M=3$ and $N=4$:

$W_{11} = \frac{1}{4}(+1+1+1+1) = \frac{3}{4}$ $\quad W_{12} = \frac{1}{4}(+1+1+1) = \frac{3}{4}$ $\quad W_{13} = \frac{1}{4}$ $\quad W_{14} = \frac{-1}{4}$

$W_{21} = \frac{1}{4}(1+1+1) = \frac{3}{4}$ ...

$$\hookrightarrow W = \begin{bmatrix} 3/4 & 3/4 & 1/4 & -1/4 \\ 3/4 & 3/4 & 1/4 & -1/4 \\ 1/4 & 1/4 & 3/4 & 1/4 \\ -1/4 & -1/4 & 1/4 & 3/4 \end{bmatrix}$$

- 1.1.2: we are looking for $\operatorname{sgn}[WS]$:

$S = [+1 \ -1 \ -1 \ +1]^T$

$WS = [-1/2 \ -1/2 \ -1/2 \ 1/2]^T \rightarrow \operatorname{sgn}[WS] = [-1 \ -1 \ -1 \ +1]^T$

by setting $S_1 = [-1 \ -1 \ -1 \ +1]^T$, we'll have $S_{i+1} = \operatorname{sgn}[[-2 \ -2 \ -1 \ 1]^T]$ which is the same as S_1, hence, we don't get any of the prototypes as the output (it looks like it converges to a local minima)

1.1.3: $W' = W + N1$, $N \sim U[-2, 2]$

$x_1 = [+1\ \ +1\ \ +1\ \ +1]^T$ ∴ $(W'x_1) = [\frac{3}{2}+4n\ \ \ \frac{3}{2}+4n\ \ \ \frac{3}{2}+4n\ \ \ \frac{1}{2}+4n]^T$

check sgn → $\frac{1}{2} + 4n > 0$ $\quad 4n > \frac{-1}{2} \rightarrow \boxed{n > \frac{-1}{8}}$

x_2: $sgn(W'x_2) = [2+2n\ \ \ 2+2n\ \ \ 1+2n\ \ \ -1+2n]^T$

sgn → $1+2n > 0$ $\quad n > \frac{-1}{2}$, $-1+2n < 0$ $\quad \boxed{n < \frac{1}{2}}$

x_3: $sgn(W'x_3) = [\frac{3}{2}\ \ \ \frac{3}{2}\ \ \ -\frac{1}{2}\ \ \ -\frac{3}{2}]^T$

sgn →

Probability of $\frac{-1}{8} < n < \frac{1}{2}$ → $\frac{5}{8}$ = Since it is a number from uniform distribution $[-2, 2]$

1.1.4: from the previous section we have: $\frac{-1}{8} < n < \frac{1}{2}$ hence, an interval that this always holds, is $[\frac{-1}{8}, \frac{1}{8}]$

→ $K = \frac{1}{8}$

yes, as for the third vector x_3, the noises cancel eachother out.

we can choose x_1, x_2 and x_3 to be vectors that contain exactly 2 +1s and 2 -1s

example: $x_1 = [+1\ \ +1\ \ -1\ \ -1]^T$, $x_2 = [+1\ \ -1\ \ +1\ \ -1]^T$
$x_3 = [-1\ \ +1\ \ +1\ \ -1]^T$

1.1.5. No not necessarily

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