

Mathematical Specialists

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Contents

0.1	Important symbols	1
0.2	Addition principle	2
0.3	Multiplication principle	2
0.4	Factorials	2
0.4.1	Dividing factorials	2
0.4.2	Special cases	2
0.5	Permutations	3
0.5.1	Permutations in a circle	3
0.5.2	Like objects repetitions	3
0.5.3	Restrictions	4
0.5.4	Grouped items	4
0.5.5	Special cases	4
0.6	Combinations	4
0.6.1	Combinations with restrictions	5
0.7	Pascal's triangle	6
0.8	Non-right-ange-triangles	6
0.8.1	Sine rule	6
0.8.2	Cosine rule	7
0.9	Vectors	7
0.9.1	Addition of vectors	7
0.9.2	Multiplication of vectors by a scalar	7
0.9.3	Calculating the magnitude of a vector	7
0.9.4	Calculating the angle of a vector	8
0.9.5	The negative of a vector	8
0.9.6	Converting between component form and polar form	8
0.9.7	How to find the vector between points that is at the tip of other vectors	8
0.9.8	Unit vectors	8
0.9.9	Dot products	9
0.9.10	Angle between two vectors	9
0.9.11	Scalar resolutes	9
0.9.12	Vector resolutes	9
0.9.13	Applications of vectors	10
0.9.14	Relative vectors	10
0.10	Euclidean geometry	10
0.10.1	Postulates	10
0.10.2	Theorems	10
0.11	Proofs	12
0.11.1	Number systems	12
0.11.2	Propositions	13
0.11.3	Rules of equality	13
0.11.4	Proof by contraposition	13
0.11.5	Proof by contradiction	14
0.12	Circles	14
0.12.1	Terms	14
0.12.2	Theorems	15

The purpose

This document was written to be used as a summary to help revise the content covered mathematical specialists. For any inquiries, feedback, and further explanations, contact lachlanprivate@duck.com or through the discord server: <https://discord.gg/6P8rddkXFr> . I encourage you to let me know of any topic I missed, how I could explain it better, or how it could be reworded or formatted to be more helpful in it's purpose. The goal of this document is to be a comprehensive summary of everything you need to know.

0.1 Important symbols

Symbol	Mathematical definition	Simple definition
\cup	Union	<i>A and B</i> , or think as in 'add'.
\cap	Intersection	<i>A or B</i> , or think as in 'multiply'.
$n!$	$n \times (n-1) \times (n-2) \times (n-3) \dots \times 3 \times 2 \times 1$	The product of integers between the given value and 1.
nP_r	$\frac{n!}{(n-r)!}$	The number of combinations there are of length r from a group of length n where the order matters.
nC_r	$\frac{(n-r)!}{r!}$ or $\frac{n!}{r!(n-r)!}$	The number of combinations there are of length r from a group of length n where the order does not matter.
$\binom{n}{r}$	nC_r	The same as the combinations but in a different notation.

0.2 Addition principle

If there are n ways of performing operation A , and m ways of performing operation B , there are $n + m$ ways of performing operation $A \cup B$.

As in, let there be 2 ways to perform task A , and 3 ways to perform task B . If there is an option to perform A or B , there is a total of $2 + 3 = 5$ ways to perform an operation.

0.3 Multiplication principle

If there are n ways of performing operation A , and m ways of performing operation B , there are $n \times m$ ways of performing operation $A \cup B$.

As in, let there be 4 ways to perform task A , and 5 ways to perform task B . With considering 1 way to perform A , there are 5 ways to perform B , repeat the process through the number of ways to perform A .

	$B, 1$	$B, 2$	$B, 3$	$B, 4$	$B, 5$
$A, 1$	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
$A, 2$	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
$A, 3$	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
$A, 4$	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)

Where there are $4 \times 5 = 20$ ways to perform the operations.

0.4 Factorials

Factorials is the result of multiplying all of the integers between the given integer and 1. This is given the notation and formula:

$$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \dots \times 3 \times 2 \times 1$$

Example:

$$\begin{aligned} 5! &= 5 \times 4 \times 3 \times 2 \times 1 \\ &= 120 \end{aligned}$$

0.4.1 Dividing factorials

When given some factorial over another factorial in such cases as:

$$\begin{aligned} &\frac{4!}{2!} \\ &= \frac{4 \times 3 \times 2 \times 1}{2 \times 1} \\ &= \frac{4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

0.4.2 Special cases

$$0! = 1$$

0.5 Permutations

Permutations is the number of ways of choosing r things from n distinct things where *order matters*. This is given the notation and formula:

$${}_nP_r = \frac{n!}{(n-r)!}$$

Example:

$$\begin{aligned} {}^6P_4 &= \frac{6!}{(6-4)!} \\ &= \frac{6!}{2!} \\ &= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} \\ &= \frac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} \\ &= 6 \times 5 \times 4 \times 3 \\ &= 360 \end{aligned}$$

0.5.1 Permutations in a circle

In cases where the positions being concerned is a circle such as the permutations of a circular seating arrangement, if the standard permutations formula is applied, there are several over-counting of the like arrangement in a different perspective. In these cases apply the formula:

$$\frac{{}_nP_r}{r} = \frac{\frac{n!}{(n-r)!}}{r}$$

0.5.2 Like objects repetitions

The number of ways of arranging n objects made up of indistinguishable objects, n_1 in the first group, n_2 in the second group and so on, is:

$$\frac{n!}{n_1!n_2!n_3!\dots n_r!}$$

Example:

Find the number of permutations of the letters in the word WOOLLOOMOOLOO.

There are 8 'O's and 3 'L's

Permutations:

$$\begin{aligned} &\frac{13!}{8!3!} \\ &\frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1)} \\ &\frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times 3 \times 2 \times 1) \times (3 \times 2 \times 1)} \\ &\frac{13 \times 12 \times 11 \times 10 \times 9}{3 \times 2} \\ &\frac{13 \times \cancel{12} \times 11 \times 10 \times \cancel{9}}{\cancel{3} \times \cancel{2}} \\ &13 \times 6 \times 11 \times 10 \times 3 \\ &25,740 \end{aligned}$$

0.5.3 Restrictions

When considering restrictions, deal with the restrictions first.

Example:

Find the number of arrangements of the letters of the word DARWIN beginning and ending with a vowel.

Number of letters = 6

Number of positions = 6

Beginning and end must be a vowel, so the available positions are decreased by 2: number of positions - 2 = 4

Since 2 vowels must be used: number of letters - 2 = 4

Permutations:

$4P_4$

$$4!$$

$$4 \times 3 \times 2 \times 1$$

$$24$$

0.5.4 Grouped items

When items are grouped together, treat each group as a single object. Find the number of arrangements of the groups, then multiply by the number of arrangement within each group.

Example:

Find the number of arrangement of the letters of the word EQUALS if the vowels are kept together.

Number of vowels = 3

Number of letters = 6

Number of positions = 6

As the 3 vowels are grouped together, the number of positions decreases:

number of positions - 3_{for the members of the group} + 1_{for the group} = 4

Permutations within vowel group:

$${}^3P_3 = 3! = 3 \times 2 \times 1 = 6$$

Permutations of all positions:

$${}^4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$$

Multiply together:

$$6 \times 24 = 144$$

0.5.5 Special cases

$${}^nP_n = n!$$

$${}^nP_0 = 1$$

0.6 Combinations

Combinations is the number of ways of choosing or selecting r objects from n distinct objects where *order does not matter*. This is given the notation and formula:

$${}^nC_r = \frac{{}^nP_r}{r!}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Example:

$$\begin{aligned}
 {}^4C_2 &= \frac{{}^4P_2}{2!} \\
 &= \frac{\frac{4!}{(4-2)!}}{2!} \\
 &= \frac{\frac{4!}{2!}}{2!} \\
 &= \frac{\frac{4 \times 3 \times 2}{2}}{2} \\
 &= \frac{\frac{4 \times 3 \times \cancel{2}}{\cancel{2}}}{2} \\
 &= \frac{4 \times 3}{2} \\
 &= \frac{12}{2} \\
 &= 6
 \end{aligned}$$

0.6.1 Combinations with restrictions

Combinations with specific terms

When given a restriction that something must be something, consider the restriction first:

For example:

Grace belongs to a group of 8 workers. How many ways can a team of four workers be selected if Grace must be on the team?

If there is no restriction, it would be equal to 8C_4 . But as Grace must be on the team, n must be equal to $8 - 1$ and r must be also $4 - 1$

Therefore, the answer is:

$${}^7C_3 = 35$$

35 total combinations.

Combinations from multiple groups

When given a two combinations, multiply them together:

For example:

From 7 women and 4 men in a workplace, how many groups of 5 can be chosen containing 3 women and 2 men?

Women:

$${}^7C_3 = 35$$

Men:

$${}^4C_2 = 6$$

Total:

$$35 \times 6 = 210$$

Permutations and Combinations combined

When given two combinations, in which the output is a permutations: For example:

How many arrangement of the letters in the word "DUPLICATE" can be made that have two vowels and three consonants?

First of all, there are 4 vowels, and 5 consonants. And 2 vowels are selected, while 3 vowels are selected:

$${}^4C_2 = 6$$

$${}^5C_3 = 10$$

And there are $5!$ ways to arrange 5 objects, therefore

$$6 \times 10 \times 5! = 7200$$

0.7 Pascal's triangle

The Pascal's triangle is a pattern formed by adding the top 2 adjacent numbers and a 1 is placed on either side of the bottom row to resemble a triangle:

[illegible]

Each element in the Pascal's triangle can be used to calculate combinations, hence, the triangle can be written using Combinations notation (nC_r):

$$\begin{array}{cccccccc}
n = 0: & & & & & & & {}^0C_0 \\
n = 1: & & & & {}^1C_0 & & {}^1C_1 & \\
n = 2: & & & {}^2C_0 & & {}^2C_1 & & {}^2C_2 \\
n = 3: & & {}^3C_0 & & {}^3C_1 & & {}^3C_2 & & {}^3C_3 \\
n = 4: & & {}^4C_0 & & {}^4C_1 & & {}^4C_2 & & {}^4C_3 & & {}^4C_4 \\
n = 5: & & {}^5C_0 & & {}^5C_1 & & {}^5C_2 & & {}^5C_3 & & {}^5C_4 & & {}^5C_5
\end{array}$$

Pascal's triangle shows that the r^{th} element of the n^{th} row of Pascal's triangle is given by nC_r . It is assumed that the 1 at the beginning of each row is the 0^{th} element. This gives the *Pascal's identity*:

$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r \text{ for } 0 < r < n$$

The Pascal's triangle can be extended to the binomial theorem, where the rule for expanding an expression such as $(a + b)^n$. Where:

$$\begin{aligned}(a+b)^0 &= 1a^0b^0 \\(a+b)^1 &= 1a^1b^0 + 1a^0b^1 \\(a+b)^2 &= 1a^2b^0 + 2a^1b^1 + 1a^0b^2 \\(a+b)^3 &= 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3 \\(a+b)^4 &= 1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4 \\(a+b)^5 &= 1a^5b^0 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5\end{aligned}$$

0.8 Non-right-angle-triangles

0.8.1 Sine rule

The sine rule is:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Where a , b , and c are lengths of sides of a triangle, and A , B , and C are angles opposite the sides.

0.8.2 Cosine rule

The cosine rule is:

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

Where c is the length of side opposite θ , and a and b are the other two sides.

0.9 Vectors

Vectors have a magnitude and a direction. There are three ways for writing vectors, either the Cartesian form/component form or the polar form. Vectors are represented by \underline{a} . the Cartesian form is written as:

$$\underline{v} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The component form notation is written as:

$$\underline{v} = 2\hat{i} + 3\hat{j}$$

And polar form is written as:

$$\underline{v} = (r, \theta)$$

Where r is the radius and θ is the direction in angles.

\hat{i} is the unit vector in the x-axis, and \hat{j} is the unit vector in the y-axis. Unit vectors are the basis vectors and are multiplied by scalars to move the tip of the vector and represent different vectors.

0.9.1 Addition of vectors

Let $\underline{a} = 3\hat{i} + -2\hat{j}$, and $\underline{b} = 7\hat{i} + 1\hat{j}$

And the addition of these vectors $\underline{a} + \underline{b} = \underline{c}$

$$\begin{aligned}\underline{c} &= \underline{a} + \underline{b} \\ \underline{c} &= 3\hat{i} + -2\hat{j} + 7\hat{i} + 1\hat{j} \\ \underline{c} &= 10\hat{i} + -1\hat{j}\end{aligned}$$

0.9.2 Multiplication of vectors by a scalar

Let $\underline{x} = 4\hat{i} + -5\hat{j}$

And $\underline{s} = 2\underline{x}$

$$\begin{aligned}\underline{s} &= 2\underline{x} \\ \underline{s} &= 2(4\hat{i} + -5\hat{j}) \\ \underline{s} &= 8\hat{i} + -10\hat{j}\end{aligned}$$

0.9.3 Calculating the magnitude of a vector

The magnitude of a vector is written as $|\underline{v}|$, and to calculate the magnitude of a vector use the pythagoras's theorem:

Let $\underline{w} = 3\hat{i} + 4\hat{j}$

$$\begin{aligned}\underline{w} &= 3\hat{i} + 4\hat{j} \\ |\underline{w}| &= \sqrt{3^2 + 4^2} \\ |\underline{w}| &= 5\end{aligned}$$

0.9.4 Calculating the angle of a vector

To calculate the angle of a vector, use the sine, cosine, and tangent ratios.

The trigonometric ratio are as follows:

Name	Ratio
Sine	$\frac{\text{Opposite}}{\text{Hypotenous}}$
Cosine	$\frac{\text{Adjacent}}{\text{Hypotenous}}$
Tangent	$\frac{\text{Opposite}}{\text{Adjacent}}$

For example:

Let $\underline{a} = 3\hat{i} + 4\hat{j}$

To calculate the angle of \underline{a} , I will use the tangent rule:

$$\begin{aligned}\underline{a} &= 3\hat{i} + 4\hat{j} \\ \tan \theta &= \frac{4}{3} \\ \tan^{-1}\left(\frac{4}{3}\right) &= \theta \\ &= 53.1301024^\circ\end{aligned}$$

0.9.5 The negative of a vector

The negative of a vector is the the vector of same length, but pointing in the opposite direction.

For example:

Let $\underline{a} = -1\hat{i} + 3\hat{j}$

The inverse or the negative vector would be $-\underline{a} = 1\hat{i} - 3\hat{j}$

As you can see, \underline{a} points off to the left and up, and $-\underline{a}$ points to the right and down.

0.9.6 Converting between component form and polar form

To convert between the component form and polar from, use the trigonometric ratios.

If given a polar vector, the component form is $\underline{v} = r \cos(\theta)\hat{i} + r \sin(\theta)\hat{j}$

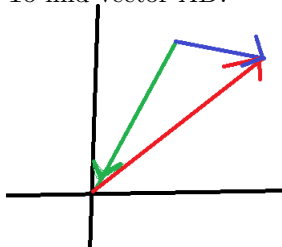
0.9.7 How to find the vector between points that is at the tip of other vectors

If given vectors \underline{a} and \underline{b} and points A and B that lie at the tips of the vectors, and told to find the vector \vec{AB} , it will be equal to $-\underline{a} + \underline{b}$.

For example:

Let vectors $\underline{a} = 1\hat{i} + 4\hat{j}$, $\underline{b} = -2\hat{i} + 0\hat{j}$, and points $A = (1, 4)$, $B = (-2, 0)$

To find vector \vec{AB} :



Where the blue vector is \vec{AB} , red is \underline{b} , and green is $-\underline{a}$

0.9.8 Unit vectors

To use a vector as the unit vector for others, use the formula:

$$\hat{u} = \frac{\underline{u}}{|\underline{u}|}$$

0.9.9 Dot products

The dot product represents one vector being projected onto the other vector, and multiplying the length of the projection by the length of the other vector. This is calculated by:

$$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos \theta$$

or

$$\underline{a} \cdot \underline{b} = a_i b_i + a_j b_j$$

NOTE: If the result is positive, the angle between the two vectors is between 0 to 90 °, if it is 0, it is at 90 °, and if it is a negative, it is between 90 to 180 °.

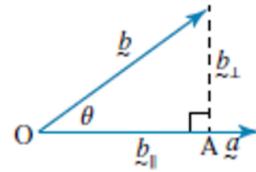
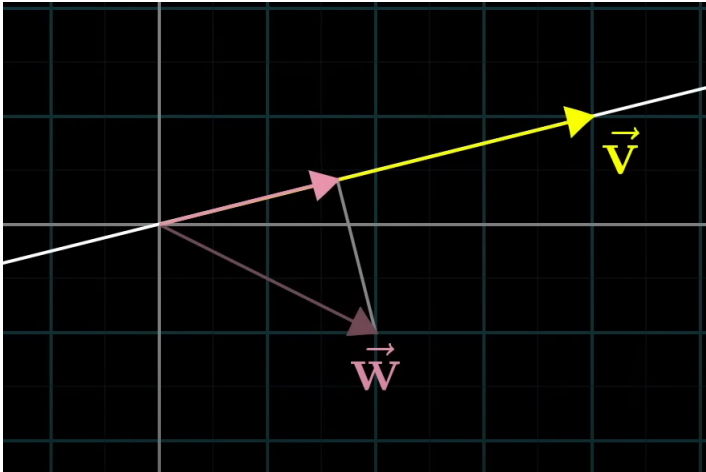
0.9.10 Angle between two vectors

Using the two formulas for calculating the dot product, the equation for cosine can be derived:

$$\cos \theta = \frac{a_i b_i + a_j b_j}{|\underline{a}||\underline{b}|}$$

0.9.11 Scalar resolutives

The scalar resolute is the length of the resultant vector if it were to be projected onto the other vector where a line from the tip of the vector projected to the tip of the projection is perpendicular to the vector being projected onto:



This can be calculated using the formula:

$$\vec{OA} = |\underline{b}| \cos \theta$$

Where \vec{OA} is the line between the origin and the tip of the projected vector, \underline{b} is the vector being projected, and θ is the angle between the two vectors. Or:

$$\vec{OA} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|}$$

And also:

$$\vec{OA} = \hat{\underline{a}} \cdot \underline{b}$$

0.9.12 Vector resolutives

The vector resolute is the same concept as the scalar resolute, but only that that we are concerned with the vector instead of the magnitude of the vector. As we are projecting a vector onto another vector, we can simply multiply the result of the scalar resolute with the unit vector of the vector we are projecting onto:

$$\underline{b}_{\parallel} = (\hat{\underline{a}} \cdot \underline{b}) \hat{\underline{a}}$$

Additionally, the vector perpendicular to the vector being projected to is given by:

$$\underline{b}_{\perp} = \underline{b} - (\hat{\underline{a}} \cdot \underline{b}) \hat{\underline{a}}$$

0.9.13 Applications of vectors

Forces

Use the non-right-angle-triangles rules for the triangles of forces. And the resultant force (the vector of the actual force experienced) is the the sum of all forces (vectors) using vector addition.

0.9.14 Relative vectors

The general relationship between relative velocities is:

$$\underline{v}_a = \underline{v}_{(a \text{ relative to } b)} + \underline{v}_b$$

Where \underline{v}_a is the actual vector and $\underline{v}_{(a \text{ relative to } b)}$ is the vector relative to b .

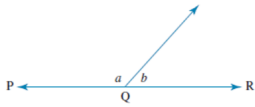

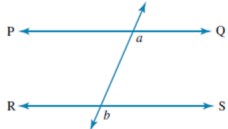

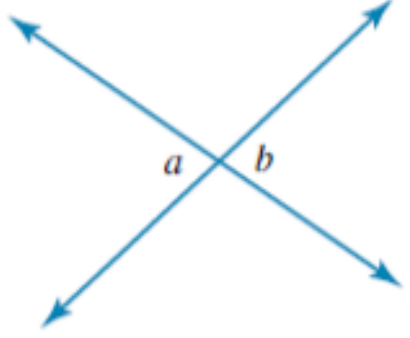

0.10 Euclidean geometry

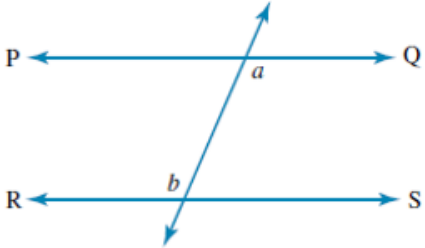

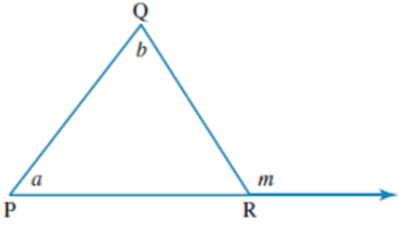

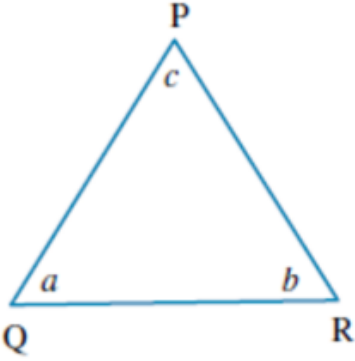

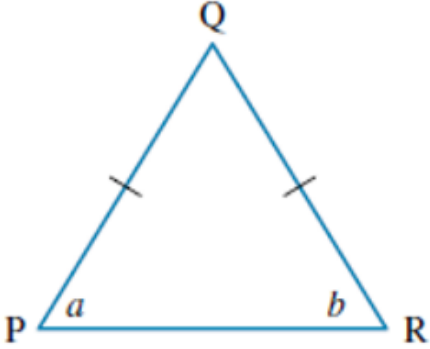

There are some basic statements that are accepted as true without proving them. In general, when these statements refer to working with numbers, the statements are called axioms. When the statements are about geometry, they are called postulates.

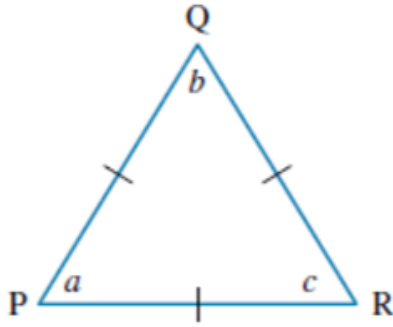

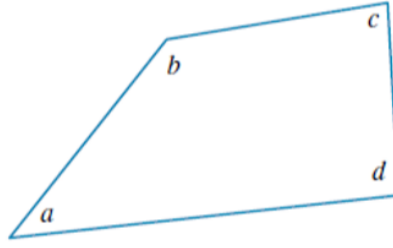

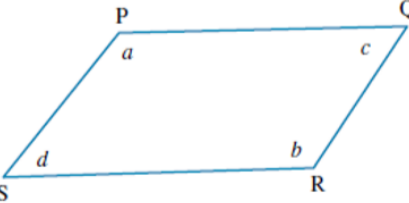
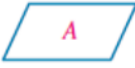
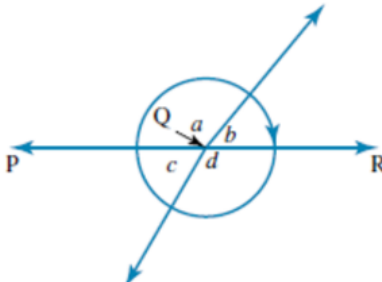

0.10.1 Postulates

- A straight line can be drawn from one point to another (that is, any two points will form a straight line)
- It is possible to produce a finite straight line
- a circle can be described with any centre and radius
- All right angles are equal to eachother
- If there is a line and a point on the line, then one line can be drawn through the point that is parallel to the original line (does not work in non-euclidean geometry)

0.10.2 Theorems

Theorem	Diagram	Symbol
Adjacent angles on a straight line are supplementary.	$\widehat{PQR} \Leftrightarrow a + b = 180^\circ$ 	
If two lines are parallel, the corresponding angles are congruent.	$PQ \parallel RS \Leftrightarrow a = b$ 	
Vertically opposite angles are congruent.		

Theorem	Diagram	Symbol
If two lines are parallel, the alternate angles are congruent.		
The exterior angle of a triangle is equal to the sum of the two interior opposite angles.		
The sum of the angles of a triangle is 180° .		
In an isosceles triangle, the opposite angles are congruent.		

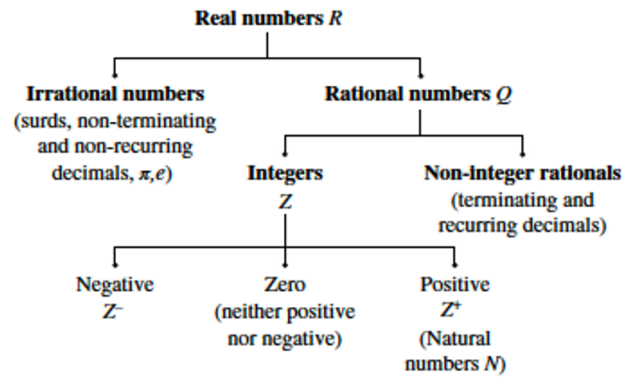
Theorem	Diagram	Symbol
In an equilateral triangle, all sides and angles are congruent.		
The sum of the angles of a quadrilateral is 360° .		
The opposite angles of a parallelogram are congruent.		
Angles at a point add to 360° .		

0.11 Proofs

0.11.1 Number systems

The notation for referring to different sets of numbers are:

Symbol	Meaning
\mathbb{R}	Real numbers
\mathbb{N}	Natural numbers
\mathbb{Z}	Integers
\mathbb{Z}^+	Positive integers
\mathbb{Z}^-	Negative integers
\mathbb{Q}	Rational numbers



0.11.2 Propositions

A proposition or mathematical statement is a sentence that is either true or false.

Symbol	Meaning	Explanation
$P \implies Q$	Implication	P implies Q
$\neg A$	Negation	The opposite of A
$\forall x$	Universal quantifier	For all values of x
\exists	Existential quantifier	There exists for some values of x
$P \Leftrightarrow Q$	Equivalence	The converse implication of $P \implies Q$ and $Q \implies P$ is also true
$A \cap B$	Union	A or B
$A \cup B$	Intersection	A and B
$A > B$	Greater than	A is larger than B
$A < B$	Less than	A is smaller than B
$A \leq B$	Less than and Equal to	A is smaller than or equal to B
$A \geq B$	Greater than and Equal to	A is larger than or equal to B
$A = B$	Equal to	A is the same value as B
$A \in \mathbb{N}$	Contained in	A is contained in the set of natural numbers

0.11.3 Rules of equality

Equality must follow these rules

Transitive property:

$$\left. \begin{matrix} a + b \\ b + c \end{matrix} \right\} \implies a = c$$

Substitution:

$$\left. \begin{matrix} a + b = c \\ b = d \end{matrix} \right\} \implies a + d = c$$

Matching:

$$\left. \begin{matrix} a + b = c \\ a + d = c \end{matrix} \right\} \implies b = d$$

Adding equals:

$$\left. \begin{matrix} a = b \\ c = d \end{matrix} \right\} \implies a + c = b + d$$

0.11.4 Proof by contraposition

Proof by contraposition is done by proving the contrapositive. The contrapositive of a proposition is: If Q , then P becomes: If not Q , then not P

0.11.5 Proof by contradiction

Proof by contradiction is done by assuming that it is false initially.

1. Assume (incorrectly) that what we want to prove is true, is actually false
2. Show that the assumption eventually leads to mathematical nonsense
3. Conclude that we are wrong to assume it is false
4. realize that if it is not false, then it must be true

Example:

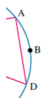
x satisfies $5^x = 2$, show that x is irrational.

1. Assume that x is rational.
2. $x = \frac{p}{q}, \in \mathbb{Z}$ (with no common factors) as all rational numbers can be represented by a fraction of integers
3. $5^{\frac{p}{q}} = 2$
4. $(5^{\frac{p}{q}})^q = (2)^q$
5. $5^p = 2^q$
6. odd = even
7. \therefore Contradiction, assumption was wrong.
8. $\therefore x$ is irrational.

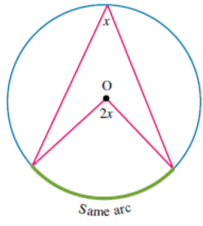

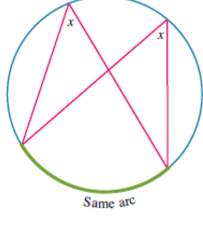

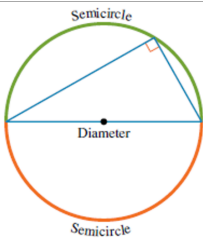

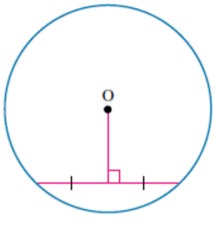

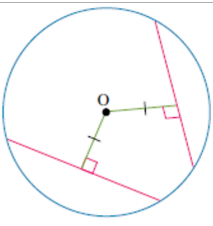

0.12 Circles

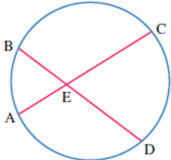

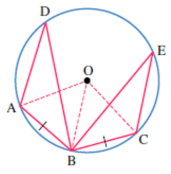

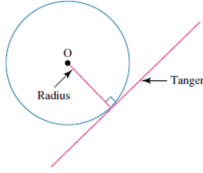

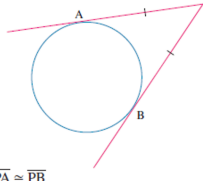

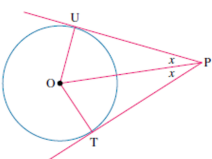

0.12.1 Terms

Some important terms used with circles are:

Term	Meaning
Circumference	The distance around the circle, or it's perimeter
Centre	The point denoted O which is the middle point of the circle
Chord	A line between two points on the circumference
Diameter	The length of a chord that intersects the centre
Radius	The length from the centre to the circumference
Arc	The part of a circle created by a chord \widehat{abd} is 
Angle at the centre	The angle made by two radii
Secant	A line the cuts the circumference of the circle at two points
Tangent	A line that touches the circle at a point
Segment	A region of a circle cut by a chord
Cyclic quadrilateral	A quadrilateral (shape with four sides) whose vertices all lie on a circle

0.12.2 Theorems

Theorem	Diagram	Symbol
The angle at the centre of the circle is twice the angle at the circumference subtended on the same arc.		
Angles at the circumference subtended by the same arc are congruent.		
The angle in a semicircle is a right angle.		
If a line from the center bisects a chord, it is perpendicular to the chord. The converse implication is also true.		
If the chords are congruent, then they are equidistant from the centre of the circle.		

Theorem	Diagram	Symbol
<p>If two chords intersect inside the circle, then the point of intersection divides each chord into two segments so that the product of the lengths of the segments for both chords is the same.</p>	 <p>$BE \cdot DE = AE \cdot CE$</p>	
<p>If chords are congruent, they subtend equal angles at the centre.</p>	 <p>Congruent chords subtend equal angles at the centre. If $AB \cong BC$, $\angle AOB \cong \angle BOC$.</p>	
<p>A tangent and a radius intersect at right angles.</p>		
<p>Two tangents drawn from an external point to a circle are congruent.</p>	 <p>$\overline{PA} \cong \overline{PB}$</p>	
<p>The angle formed by two tangents meeting at an external point is bisected by a straight line joining the centre of the circle to that external point.</p>		

Theorem	Diagram	Symbol
The angle formed by a tangent and a chord is congruent to the angle in the alternate segment.		
If a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn from an external point P, the square of the length of the tangent equals the product of the length of the secant and its external segment.	$PT^2 = PA \cdot PB$	
If two secants intersect outside the circle, then the product of one external secant segment length and its external length is equal to the product of the other secant lengths.	$PA \cdot PB = PC \cdot PD$	
The opposite angles of a cyclic quadrilateral are supplementary.	$\angle ABC + \angle CDA = 180^\circ \text{ and } \angle BCD + \angle DAB = 180^\circ$	
The exterior angle of a cyclic quadrilateral is congruent to the inner opposite angle.	$\angle CDE = \angle ABC$	