Exponential Distribution

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Simulation and exploration of the Central Limit Theorem in regards to Exponential Distribution

First we will simulate 1000 40 element exponential distributions, and calculate their means

```
nosim <- 1000

lambda <- .2
n <- 40
cfunc <- function(x, n) sqrt(n) * (x - (1/lambda)) / (1/lambda)
AllMeans= NULL
NormalizedMean= NULL
AllSD = NULL
for (i in seq(1,nosim,1)) {
    x <- rexp(n, lambda)
    sample_mean <- mean(x)
    normalized_mean <- cfunc(sample_mean,n)
    standard_deviation <- sd(x)
    AllMeans=rbind(AllMeans,sample_mean)
    NormalizedMean=rbind(NormalizedMean, normalized_mean)

}
rownames(NormalizedMean) <- NULL
data = data.frame(NormalizedMean)
```

The theoretical sample mean is:

```
## [1] 5
```

The actual population sample mean is:

```
## [1] 4.972
```

The theoretical standard deviation of the sample mean is:

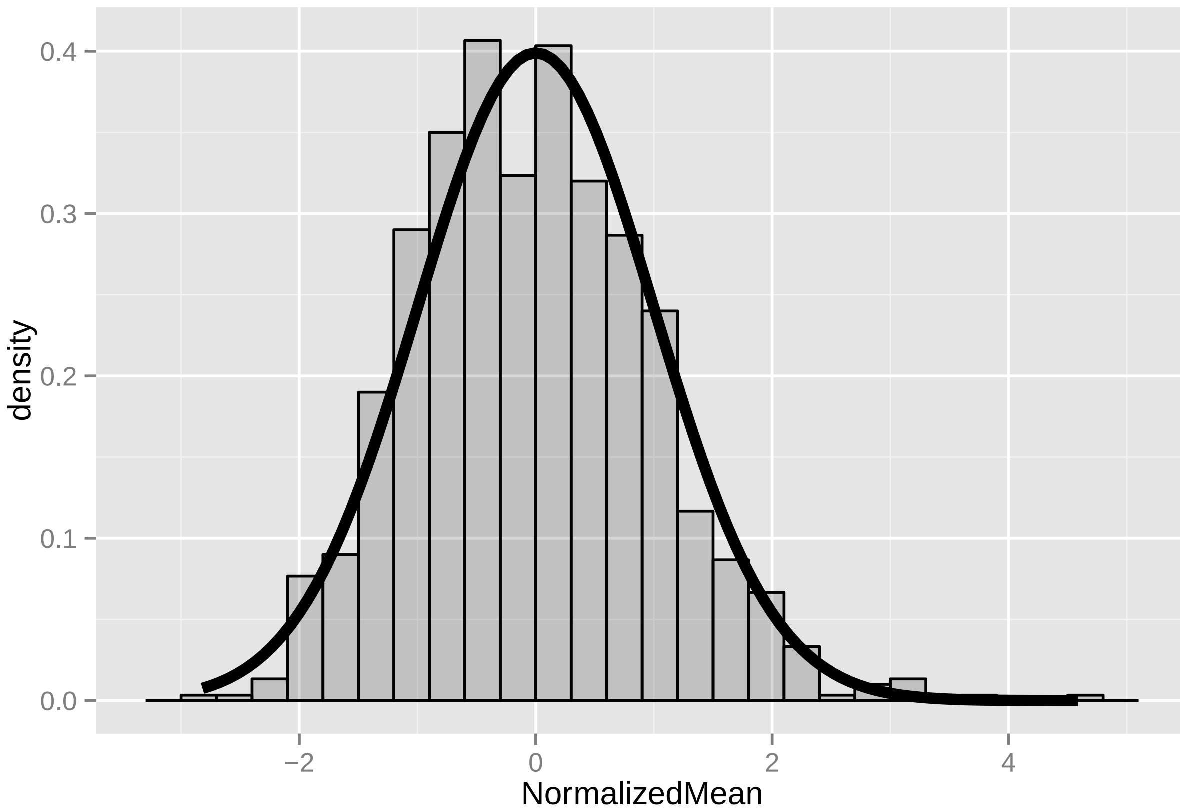
```
## [1] 0.7906
```

The actual standard deviation of the sample mean is:

```
## [1] 0.7878
```

All of the actual values are reasonably close to the theoretical values, suggesting a sufficiently large number of simulations with a large enough number of samples per simulation The normalized mean has a distribution with approximately the same density as a normal distribution:

```r
g <- ggplot(data, aes(x = NormalizedMean)) + geom_histogram(alpha = .20, binwidth=.3, colour = "black",
g <- g + stat_function(fun = dnorm, size = 2)
print(g)
```



The percentage of sample means covered by the confidence interval for 1/lambda: $\bar{X} \pm 1.96$ * (S/sqrt(n) is

```r
ul <- mean(AllMeans) +  1.96 * real_sample_mean_sd
ll <- mean(AllMeans) -  1.96 * real_sample_mean_sd
inside <- 0
for (i in AllMeans) {
    if (i > ll & i < ul){
        inside <- inside + 1
    }
}
coverage <- inside/length(AllMeans)
coverage
```

```
## [1] 0.96
```

This is the expected result, given the number of samples