Mathematics in Deep Learning

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MATH199

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Mathematics in Deep Learning

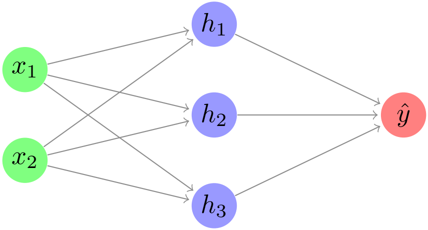


Figure: MLP network with two inputs and one hidden layer

Remark

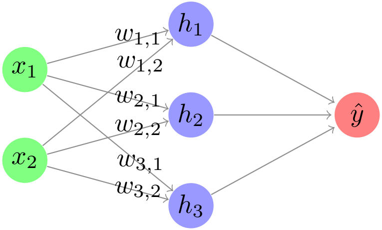
We've tidied up our diagram to not show biases and activation functions, but they're still there and being used in calculations.

Calculations:

$$h_1 = w_{1,1} \cdot x_1 + w_{1,2} \cdot x_2 + b_1$$

$$h_2 = w_{2,1} \cdot x_1 + w_{2,2} \cdot x_2 + b_2$$

$$h_3 = w_{3,1} \cdot x_1 + w_{3,2} \cdot x_2 + b_3$$



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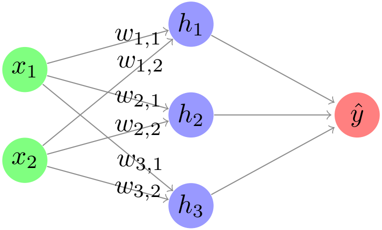

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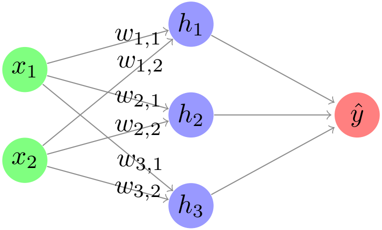

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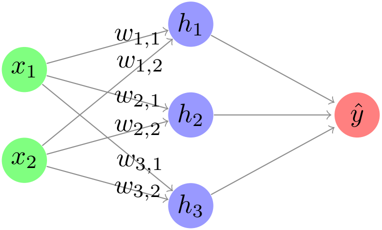

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With an activation function, the output of the hidden layer is:

$$\underline{h} = f(W \cdot \underline{x} + \underline{b})$$

Typically, architecture is much more complex, with many hidden layers. This is what deep learning is referring to.

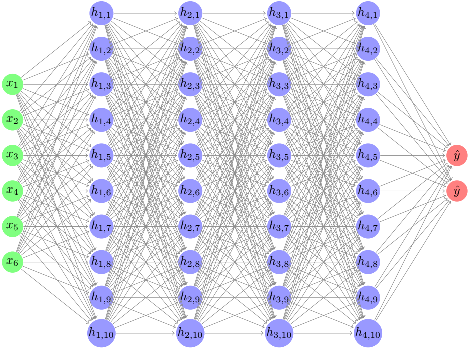

Figure: Example of what a deep NN could look like

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Suitable values for these edges need to be determined in a process called training, which uses an algorithm called gradient descent (very similar to Euler's Method, i.e. using calculus).

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Each of these use the same connections we saw before, just arranged in different ways to form different architectures.