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Homework 6

Problem 1)

a)
To solve this problem we simply need to follow the provided objective function and constraints. By maximizing the distance to get to vertex 7 we are guaranteed to get the lowest value if we set up our constraints properly as discussed in the lectures. If we were to try selecting min we would get a result of 0.

Objective Function: max dt

Subject to:

ds=0

dv-du<=w(u,v) for all (u,v)$\in E$

Code:

```
max d7

ST

    d0=0

    d1-d0<=10

    d3-d0<=5

    d0-d4<=1

    d1-d4<=1

    d4-d3<=2

    d5-d3<=10

    d4-d5<=2

    d4-d6<=4

    d6-d5<=7

    d2-d1<=2

    d2-d4<=4

    d6-d2<=2

    d2-d6<=2
```

d7-d2<=8

d7-d6<=3

END

```
 LP OPTIMUM FOUND AT STEP      7

        OBJECTIVE FUNCTION VALUE

        1)      15.00000

  VARIABLE        VALUE          REDUCED COST
        D7        15.000000          0.000000
        D0         0.000000          0.000000
        D1         8.000000          0.000000
        D3         5.000000          0.000000
        D4         7.000000          0.000000
        D5         5.000000          0.000000
        D6        12.000000          0.000000
        D2        10.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         2.000000          0.000000
        4)         0.000000          1.000000
        5)         8.000000          0.000000
        6)         0.000000          1.000000
        7)         0.000000          1.000000
        8)        10.000000          0.000000
        9)         0.000000          0.000000
       10)         9.000000          0.000000
       11)         0.000000          0.000000
       12)         0.000000          1.000000
       13)         1.000000          0.000000
       14)         0.000000          1.000000
       15)         4.000000          0.000000
       16)         3.000000          0.000000
       17)         0.000000          1.000000

 NO. ITERATIONS=       7
```

The shortest distance to vertex 7 is 15.

b)

To solve this problem we simply need to follow the provided objective function and constraints. By maximizing the distance to get to the sum of all vertex we are guaranteed to get the lowest value needed to travel to each vertex if we set up our constraints properly as discussed in the lectures. If we were to try selecting min we would get results of 0's.

Objective function: maximize $\sum_{v \in V} d_v$

Subject to:

ds=0

dv-du<=w(u,v) for all (u,v)$\in E$

Code:

max d0+d1+d2+d3+d4+d5+d6+d7

ST

d0=0

d1-d0<=10

d3-d0<=5

d0-d4<=1

d1-d4<=1

d4-d3<=2

d5-d3<=10

d4-d5<=2

d4-d6<=4

d6-d5<=7

d2-d1<=2

d2-d4<=4

d6-d2<=2

d2-d6<=2

d7-d2<=8

d7-d6<=3

END

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      72.00000

  VARIABLE        VALUE          REDUCED COST
        D0         0.000000          0.000000
        D1         8.000000          0.000000
        D2        10.000000          0.000000
        D3         5.000000          0.000000
        D4         7.000000          0.000000
        D5        15.000000          0.000000
        D6        12.000000          0.000000
        D7        15.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          8.000000
        3)         2.000000          0.000000
        4)         0.000000          7.000000
        5)         8.000000          0.000000
        6)         0.000000          4.000000
        7)         0.000000          5.000000
        8)         0.000000          1.000000
        9)        10.000000          0.000000
       10)         9.000000          0.000000
       11)        10.000000          0.000000
       12)         0.000000          3.000000
       13)         1.000000          0.000000
       14)         0.000000          2.000000
       15)         4.000000          0.000000
       16)         3.000000          0.000000
       17)         0.000000          1.000000

 NO. ITERATIONS=       1
```

The shortest distance to all vertices are:

Vertex 1: 8

Vertex 2: 10

Vertex 3: 5

Vertex 4: 7

Vertex 5: 15

Vertex 6: 12

Vertex 7: 15

Problem 2)

We want to maximize profit, to do this we want to make the most possible ties while losing the least amount of money to costs. For silk its straight forward, its only used in one type of tie, and each tie nets a profit therefor we can expect to produce the maximum amount of those. For the other three kinds of ties it will be a balancing act, each tie uses some of the pooled resources. Which ties will be chosen to get made in excess will be based off of their profit per yard of fabric used. This will be done with the following objective function.

Objective Function:

Let S be amount of silk ties, P be the amount of Polyester ties, B the number of blend 1 ties and C be the number of blend 2 ties.

Max((6.7S-2.5S+.75S)+(3.55P+.48P+.75P)+(4.31B-.75B-.75B)+(4.81C+.81C+.75C))

Subject to:

Min and Max production amounts:

S>=6000

S<=7000

P>=10000

P<=14000

B>=13000

B<=16000

C>=6000

C<=8500

Amount of material:

.125S<=1000

.08P+.05B+.03C<=2000

.05B+.07C<=1250

Code:

max P

ST

ts>=6000

ts<=7000

tp>=10000

tp<=14000

tb>=13000

tb<=16000

tc>=6000

```
	tc<=8500

	.125ts<=1000
	.08tp+.05tb+.03tc<=2000
	.05tb+.07tc<=1250

	Ps-6.7ts+2.5ts+.75ts=0
	Pp-3.55tp+.48tp+.75tp=0
	Pb-4.31tb+.75tb+.75tb=0
	Pc-4.81tc+.81tc+.75tc=0

	P-Ps-Pp-Pb-Pc=0
END
```

```
 LP OPTIMUM FOUND AT STEP      7

        OBJECTIVE FUNCTION VALUE

        1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         P    120196.000000          0.000000
        TS      7000.000000          0.000000
        TP     13625.000000          0.000000
        TB     13100.000000          0.000000
        TC      8500.000000          0.000000
        PS     24150.000000          0.000000
        PP     31610.000000          0.000000
        PB     36811.000000          0.000000
        PC     27625.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)      1000.000000          0.000000
        3)         0.000000          3.450000
        4)      3625.000000          0.000000
        5)       375.000000          0.000000
        6)       100.000000          0.000000
        7)      2900.000000          0.000000
        8)      2500.000000          0.000000
        9)         0.000000          0.476000
       10)       125.000000          0.000000
       11)         0.000000         29.000000
       12)         0.000000         27.200001
       13)         0.000000          1.000000
       14)         0.000000          1.000000
       15)         0.000000          1.000000
       16)         0.000000          1.000000
       17)         0.000000          1.000000

 NO. ITERATIONS=       7
```

To produce a maximum profit of $120,196 the following amounts of ties will need to be made:

Silk Ties: 7000

Polyester Ties: 13625

Blend 1 Ties: 13100

Blend 2 Ties: 8500

Problem 3)

Part A will be achieved by selecting the combination of ingredients who have the lowest combined calories while still meeting nutritional requirements. This will be done by setting up all of the constraints such that the nutritional requirements are met, then selecting the minimum caloric option from those combinations. Part B is the same, but instead the option with the lowest price will be selected. This will be accomplished with the following objective function.

Part A:

i) Objective function:
Min $\sum_{i\in I} Ca_i$ where i is the ingredients and Ca is the calories per 100 grams.
Subject to:

$2 \leq \sum_{i\in I} G_i * F_i \leq 8$	such that F is fat and G is gram(100g units)
$\sum_{i\in I} G_i * P_i \geq 15$	Such that P is protein
$\sum_{i\in I} G_i * C_i \geq 4$	Such that C is carbohydrates
$\sum_{i\in I} Mg_i * S_i \leq 200$	such that Mg is milligrams, and S is sodium
$\sum_{i\in I} G_i * L_i \geq .4 \sum_{i\in I} G_i$	such that L is 1 when i is leafy green, 0 when not.
$\sum_{i\in I} G_i * C_i = C$	such that C is the cost.

ii)

min Ca

ST

P>=15

F<=8

F>=2

C>=4

S<=200

Al+As-.4At-.4Al-.4As-.4Ac-.4Asu-.4Ast-.4Acp-.4Ao>=0

P-.85At-1.62Al-2.86As-.93Ac-23.4Asu-16Ast-9Acp=0

F-.33At-.2Al-.39As-.24Ac-48.7Asu-5Ast-2.6Acp-100Ao=0

C-4.64At-2.37Al-3.63As-9.58Ac-15Asu-3Ast-27Acp=0

S-9At-28Al-65As-69Ac-3.8Asu-120Ast-78Acp=0

Ca-21At-16Al-40As-41Ac-585Asu-120Ast-164Acp-884Ao=0

Co-1At-.75Al-.5As-.5Ac-.45Asu-2.15Ast-.95Acp-2Ao=0

END

```
 LP OPTIMUM FOUND AT STEP      15

        OBJECTIVE FUNCTION VALUE

        1)      114.7541

  VARIABLE        VALUE          REDUCED COST
        CA       114.754097          0.000000
         P        15.000000          0.000000
         F         4.508197          0.000000
         C         4.022248          0.000000
         S       121.779861          0.000000
        AL         0.585480          0.000000
        AS         0.000000         14.513662
        AT         0.000000         16.901640
        AC         0.000000         36.289616
       ASU         0.000000        408.387970
       AST         0.878220          0.000000
       ACP         0.000000         97.551910
        AO         0.000000        886.404358
        CO         2.327283          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.650273
        3)         3.491803          0.000000
        4)         2.508197          0.000000
        5)         0.022248          0.000000
        6)        78.220139          0.000000
        7)         0.000000         -6.010929
        8)         0.000000          7.650273
        9)         0.000000          0.000000
       10)         0.000000          0.000000
       11)         0.000000          0.000000
       12)         0.000000         -1.000000
       13)         0.000000          0.000000

 NO. ITERATIONS=       15
```

iii) Cost of the lowest calorie salad is \$2.33.

Part B:

i) Objective function:

Min $\sum_{i \in I} G_i * C_i$ where i is the ingredients

Subject to:

$2 \leq \sum_{i \in I} G_i * F_i \leq 8$	such that F is fat and G is gram(100g units)
$\sum_{i \in I} G_i * P_i \geq 15$	Such that P is protein
$\sum_{i \in I} G_i * C_i \geq 4$	Such that C is carbohydrates
$\sum_{i \in I} Mg_i * S_i \leq 200$	such that Mg is milligrams, and S is sodium
$\sum_{i \in I} G_i * L_i \geq .4 \sum_{i \in I} G_i$	such that L is 1 when i is leafy green, 0 when not.
$\sum_{i \in I} G_i * C_i = C$	such that C is the cost.

ii)

min Co

ST

P>=15

F<=8

F>=2

C>=4

S<=200

Al+As-.4At-.4Al-.4As-.4Ac-.4Asu-.4Ast-.4Acp-.4Ao>=0

P-.85At-1.62Al-2.86As-.93Ac-23.4Asu-16Ast-9Acp=0

F-.33At-.2Al-.39As-.24Ac-48.7Asu-5Ast-2.6Acp-100Ao=0

C-4.64At-2.37Al-3.63As-9.58Ac-15Asu-3Ast-27Acp=0

S-9At-28Al-65As-69Ac-3.8Asu-120Ast-78Acp=0

Ca-21At-16Al-40As-41Ac-585Asu-120Ast-164Acp-884Ao=0

Co-1At-.75Al-.5As-.5Ac-.45Asu-2.15Ast-.95Acp-2Ao=0

END

```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      1.554133

  VARIABLE        VALUE          REDUCED COST
        CO         1.554133          0.000000
         P        15.000000          0.000000
         F         8.000000          0.000000
         C        35.576324          0.000000
         S       144.348907          0.000000
        AL         0.000000          0.402912
        AS         0.832298          0.000000
        AT         0.000000          1.002081
        AC         0.000000          0.486914
       ASU         0.096083          0.000000
       AST         0.000000          0.405609
       ACP         1.152364          0.000000
        AO         0.000000          7.281258
        CA       278.488403          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -0.131261
        3)         0.000000          0.051847
        4)         6.000000          0.000000
        5)        31.576324          0.000000
        6)        55.651089          0.000000
        7)         0.000000         -0.241358
        8)         0.000000          0.131261
        9)         0.000000         -0.051847
       10)         0.000000          0.000000
       11)         0.000000          0.000000
       12)         0.000000          0.000000
       13)         0.000000         -1.000000

 NO. ITERATIONS=       3
```

iii) The lowest cost salad contains 278 calories.

Problem 4)

This is an extension of the transport problem and was discussed in the lectures. I will be assuming warehouse don't store anything, the number of fridges in will equal the amount out. Also plants don't need to ship their entire inventory, just enough so the demand is met. The goal then is to minimize the cost of shipping from plants to warehouse and from warehouse to retailers while still meeting the demand. This will be done with the following objective function.

Objective function:

$Min\ \sum_{i\in I}\sum_{j\in J} S_{ij} * cp_{ij} + \sum_{k\in K}\sum_{j\in J} S_{jk} * cp_{jk}$ Where Sij is the quantity shipped from plant Pi to warehouse Pj and Sjk is the quantity shipped from warehouse Wj to retailer Rk.

Subject to:

$P_{ij}, S_{jk} \geq 0$

$\sum_{j\in J} P_{1j} \leq 150$ Supply from plant 1

$\sum_{j\in J} P_{2j} \leq 450$

$\sum_{j\in J} P_{3j} \leq 250$

$\sum_{j\in J} P_{4j} \leq 150$

$\sum_{j\in J} S_{j1} \geq 100$ Demand at Retailer 1

$\sum_{j\in J} S_{j2} \geq 150$

$\sum_{j\in J} S_{j3} \geq 100$

$\sum_{j\in J} S_{j4} \geq 200$

$\sum_{j\in J} S_{j5} \geq 200$

$\sum_{j\in J} S_{j6} \geq 150$

$\sum_{j\in J} S_{j7} \geq 100$

$\sum_{k\in K} S_{1k} - \sum_{i\in I} P_{i1} = 0$ In/out at warehouse 1

$\sum_{k\in K} S_{2k} - \sum_{i\in I} P_{i2} = 0$

$\sum_{k\in K} S_{3k} - \sum_{i\in I} P_{i3} = 0$

min c

ST

cp11>=0

cp12>=0

cp21>=0

cp22>=0

cp31>=0

cp32>=0

cp33>=0

cp42>=0

cp43>=0

cw11>=0

cw12>=0

cw13>=0

cw14>=0

cw23>=0

cw24>=0

cw25>=0

cw26>=0

cw34>=0

cw35>=0

cw36>=0

cw37>=0

cp11+cp12<=150

cp21+cp22<=450

cp31+cp32+cp33<=250

cp42+cp43<=150

cw11>=100

cw12>=150

cw13+cw23>=100

cw14+cw24+cw34>=200

cw25+cw35>=200

cw26+cw36>=150

cw37>=100

cp11+cp21+cp31-cw11-cw12-cw13-cw14=0

cp12+cp22+cp32+cp42-cw23-cw24-cw25-cw26=0

cp33+cp43-cw34-cw35-cw36-cw37=0

c-10cp11-15cp12-11cp21-8cp22-13cp31-8cp32-9cp33-14cp42-8cp43-5cw11-6cw12-7cw13-10cw14-12cw23-8cw24-10cw25-14cw26-14cw34-12cw35-12cw36-6cw37=0

END

LP OPTIMUM FOUND AT STEP 13

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE	VALUE	REDUCED COST
C	17100.000000	0.000000
CP11	150.000000	0.000000
CP12	0.000000	8.000000
CP21	200.000000	0.000000
CP22	250.000000	0.000000
CP31	0.000000	2.000000
CP32	150.000000	0.000000
CP33	100.000000	0.000000
CP42	0.000000	7.000000
CP43	150.000000	0.000000
CW11	100.000000	0.000000
CW12	150.000000	0.000000
CW13	100.000000	0.000000
CW14	0.000000	5.000000
CW23	0.000000	2.000000
CW24	200.000000	0.000000
CW25	200.000000	0.000000
CW26	0.000000	1.000000
CW34	0.000000	7.000000
CW35	0.000000	3.000000
CW36	150.000000	0.000000
CW37	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	150.000000	0.000000
3)	0.000000	0.000000
4)	200.000000	0.000000
5)	250.000000	0.000000
6)	0.000000	0.000000
7)	150.000000	0.000000
8)	100.000000	0.000000
9)	0.000000	0.000000
10)	150.000000	0.000000
11)	100.000000	0.000000
12)	150.000000	0.000000
13)	100.000000	0.000000
14)	0.000000	0.000000
15)	0.000000	0.000000
16)	200.000000	0.000000
17)	200.000000	0.000000
18)	0.000000	0.000000
19)	0.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	1.000000
24)	0.000000	0.000000
25)	0.000000	0.000000
26)	0.000000	1.000000
27)	0.000000	-16.000000
28)	0.000000	-17.000000
29)	0.000000	-18.000000
30)	0.000000	-16.000000
31)	0.000000	-18.000000
32)	0.000000	-21.000000
33)	0.000000	-15.000000
34)	0.000000	-11.000000
35)	0.000000	-8.000000
36)	0.000000	-9.000000
37)	0.000000	-1.000000

NO. ITERATIONS= 13

The minimum cost is $17,100.

The optimal shipping routes are:

Plant 1 to Warehouse 1: 150

Plant 2 to Warehouse 1: 200

Plant 2 to Warehouse 2: 250

Plant 3 to Warehouse 2: 150

Plant 3 to Warehouse 3: 100

Plant 4 to Warehouse 3: 150

Warehouse 1 to Retailer 1: 100

Warehouse 1 to Retailer 2: 150

Warehouse 1 to Retailer 3: 100

Warehouse 2 to Retailer 4: 200

Warehouse 2to Retailer 5: 200

Warehouse 3 to Retailer 6: 150

Warehouse 3 to Retailer 7: 100