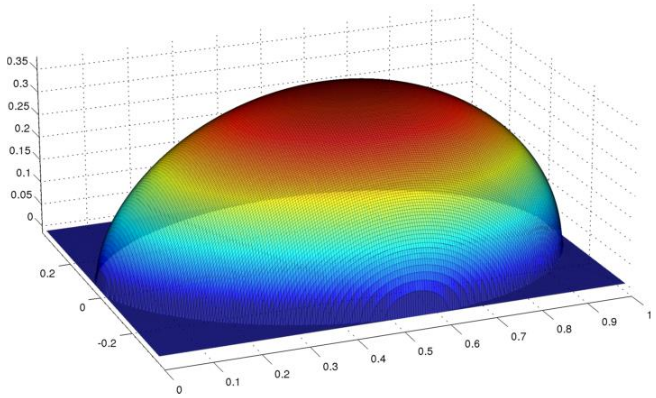

EGG MODELLNG

Semester 4 Assignment

ABSTRACT

Using mathematical forms such as exponential point modelling, to the implementation of an extended ellipse equation; the volume of an egg will be found. These answers will be compared with the displacement of the egg (found through measuring water displacement) to then contrast various errors, or limitations with each generated model.

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Contents

To be able to model equations is an art, an art form that manipulates the way numbers interact and connects their usefulness in the real world. In the following essay, various numerical and algebraic methods will be discussed and contrasted to model and find the volume of an egg. These techniques will be explored to draw conclusions as to various limitations of each model; to the opposite end of the spectrum depicting the strengths of each model.

For the following questions; the conclusions are compared to the practical in-class component of this assignment. This was where an egg was dropped into a beaker full of water, with its displacement being measured. The resultant egg was then measured, peeled, measured (in that order) and then finally sliced to make for separate slices of the egg in order to use the extended Simpsons rule. For all of the various measurements taken refer to Appendix 1.1.

Question 4 – Using Extended Simpsons Model

Assumptions

To ensure the final volume received is as accurate as it can be, several assumptions need to be made about the recordation of the egg information. Refer below for a list of these assumptions and their various knock-on effects:

- **Various slice measurements are taken from inside of the slice**
  Making measurements from inside of the circumscribed circle will ensure consistency across each slice alongside not overestimating the resultant measurements, resulting in an egg that is more likely to be underestimated than overestimated [due to the fact that the pen ink was recorded from around the egg]
- **No slice is the exact same on the sheet (that is next to one another)**
  From real life it is known that each 'arc' of the slices cut from the egg is different due to there being a large circle and then a small circle. For the process of measurement these circles could have easily been mixed up, leading to misleading results (the wrong side of the slice being recorded)
- **Uniform 5 mm slices made across cutting utensil**
  An egg slicer was used to slice the egg into 11 even pieces, this having a width of 5 mm which was implemented in the resultant Extended Simpsons rule.
- **No excess egg was stuck or discarded in utensil**
  Having chunks taken from the egg will result in a skewed volume at the end
- **Each egg slice aligned properly with the previous, the line drawn is placed properly**
  Having a slice misaligned, will mean that when calculating the final volume there will be problems aligning each slice correctly.
- **Egg has uniform density**
  While this doesn't directly affect the volume given from the Extended Simpsons rule, it does effect the volume received from displacing water. Due to the bubble made at the top of the egg, this isn't the case; although for simplicity in the model this needs to be assumed.
- **Photocopying or scanning the resultant slice page doesn't affect or skew the width of the slices**
  If this were to happen, each individual in the group would have a different 'egg' to be finding all of the volumes of; effectively making in-class measurements useless
- **The displacement volume is correct**
  Making this assumption means that the recorded displacement volume is the exact volume of the egg, so it is possible to draw conclusions based on the relevant models. Without this assumption all conclusions would be invalid.

Procedure

To calculate the volume of an egg using the Extended Simpsons Rule a series of steps must be followed to get the egg into the right form for calculation.

Step 1 (Slice Egg)

It is assumed that the egg has already been displaced and measured (using a Caliper) before performing these steps. Once peeled, save some of the shell alongside taking a pen and drawing a line going down the entire length of the egg. Place egg horizontally on egg slicer, with widths 5mm apart, this will result in 11 slices. Measure the shell width with a micrometer and record the width.

Step 2 (Inscribe each slice)

Carefully take each slice out of the egg slicer and place onto a 1mm by 1mm grid of paper. Carefully draw around each slice with pen, trying to minimize the ink leaking across the page [a pencil can be used, except when photocopying for peers it will be harder to see]). Finally, draw an external line stemming from the bottom of the slice (where the line marker is) to be able to align them all when constructing Simpsons model. Take care to record which egg slice is which, starting from 1 and ending with 11.

Step 3 (Photocopy Page)

Once each slice is inscribed onto the grid paper, photocopy the page for each team member, discarding the original (it will start to stench of egg).

Step 4 (Drawing lines across egg slices)

In order to transform the egg slice grid into something useable the egg slices must be transposed onto a grid specific to the Extended Simpsons Rule. In order to achieve this, begin measuring lines 5 millimetres apart from the centre one. Once the middle line is drawn, continue from the 5mm intervals outwards. If no extra line can fit along the 5mm interval, don't draw a line as it is assumed to be 0mm at that point [due to Extended Simpsons Rule].

Step 5 (Measuring lines across egg slices)

Once each line is drawn across the grid, measure each individual line (in mm) to the closest 0.5mm and record it on the line. This will be used to effectively measure the 'heights' of the egg at each of those points.

Step 6 (Construct the grid over the egg)

Once each measurement is completed, find the slice with the most intervals (9 on the example egg slices). Begin recording a grid (column by column) over the top of the lines starting from A and ending in I (or whatever interval the egg goes up to). A represents 0 (and so on) in the Extended Simpsons Rule grid.

Refer to Appendix 1.2 for a depiction of these steps, alongside a visual key (on top of the Appendix) which outlines each slice and their relation to the egg shape.

Step 7 (Construct Grid within Excel)

As can be recalled from the Maths C textbook, the Extended Simpsons Rule works by incorporating a grid of various heights (based on constant widths); to then calculate the overall volume (using a series of parabolas. Refer below for what the grid will look like within Excel:

	0	1	2	3	4	5	6	7	8	9	10
A	0	0	0	5.5	10	20	17	0	8.5	17.5	0
B	0	10	20	28.5	30	30	33	22.5	21.5	26.5	14.5
C	18.5	24.5	31	35	37	35.5	41	32.5	28	31	21
D	25	30.5	35.5	39	41	38.5	45.5	37.5	33.5	33.5	25
E	27.5	32.5	37.5	40.5	42.5	40.5	46.5	40	36.5	32.5	26.5
F	27	33	36.5	38.5	42	42	45	39	37.5	31.5	26
G	23	29.5	32.5	34.5	39	38.5	41.5	37	35	27.5	22
H	19	23.5	24	26	32.5	32.5	34	31	30	17.5	15
I	0	3.5	3	4.5	16	22.5	19	17.5	20	0	0
J	0	0	0	0	0	0	0	0	0	0	0

Extended Simpsons Grid Lengths [Unshelled Egg]

In the above grid, each value is in millimetres with an error of $\pm 0.5\text{mm}$. Another thing to note is the various colours, as can be recalled from Extended Simpsons Rule, the colours represent the H1, H2, H3 and H4 areas. While ideally due to this being a round shape, there shouldn't be any 'corners' (H1 and H3) although, this is only an approximation so there needs to exist corners to allow for the Extended Simpsons Model to work.

The key in the above image is:

Blue – H1 (Outside Corners)

Purple – H2 (Side Lengths)

Green – H3 (Indented Corners)

Orange – H4 (Insides)

There is a note to make at this point that these values represent the **unshelled egg** which should equate to around 54mL, although for an accurate contrast to the other models made as is asked in Question 6, the shelled egg volume should be found. This can be constructed by taking the micrometer width, multiplying by two (for each side of the egg) and then adding it to each of the lengths (resulting in a length of 0.756mm). This leading to a new generated grid, as shown below:

	0	1	2	3	4	5	6	7	8	9	10
A	0	0	0	12.512	10.756	41.512	17.756	1.512	9.256	36.512	0
B	0	43.024	41.512	117.024	61.512	123.024	67.512	93.024	44.512	109.024	30.512
C	19.256	50.512	31.756	71.512	37.756	72.512	41.756	66.512	28.756	63.512	21.756
D	51.512	125.024	72.512	159.024	83.512	157.024	92.512	153.024	68.512	137.024	51.512
E	28.256	66.512	38.256	82.512	43.256	82.512	47.256	81.512	37.256	66.512	27.256
F	55.512	135.024	74.512	157.024	85.512	171.024	91.512	159.024	76.512	129.024	53.512
G	23.756	60.512	33.256	70.512	39.756	78.512	42.256	75.512	35.756	56.512	22.756
H	39.512	97.024	49.512	107.024	66.512	133.024	69.512	127.024	61.512	73.024	31.512
I	0	8.512	3.756	10.512	16.756	46.512	19.756	36.512	20.756	0	0

Extended Simpsons Grid Lengths [Shelled Egg]

Once the data is inputted into excel in the form above (with the various colours showing which areas of the grid belongs to which section). The values in each of the grids need to be skewed to their respective Odd or Even value. In the Extended Simpsons Rule; these work by referring to each cell location and multiplying by a factor of 1, 2 or 4 depending if the cell is Even/Even, Odd/Even or Odd/Odd respectively. The cell multiplication factor is decided by the column value (**always starting at 0**) and the row value (where A=0, B=1, etc.). In this case 0 is an even number. To respectively skew all of the values in the above grid, enter the formulas shown below:

	0	1	2	3	4	5	6	7	8	9	10
A	0	0	0	=2*(R2+0.756)	=S2+0.756	=2*(T2+0.756)	=U2+0.756	=2*(V2+0.756)	=W2+0.756	=2*(X2+0.756)	0
B	0	=4*(P3+0.756)	=2*(Q3+0.756)	=4*(R3+0.756)	=2*(S3+0.756)	=4*(T3+0.756)	=2*(U3+0.756)	=4*(V3+0.756)	=2*(W3+0.756)	=4*(X3+0.756)	=2*(Y3+0.756)
C	=O4+0.756	=2*(P4+0.756)	=Q4+0.756	=2*(R4+0.756)	=S4+0.756	=2*(T4+0.756)	=U4+0.756	=2*(V4+0.756)	=W4+0.756	=2*(X4+0.756)	=1*Y4+0.756
D	=2*(O5+0.756)	=4*(P5+0.756)	=2*(Q5+0.756)	=4*(R5+0.756)	=2*(S5+0.756)	=4*(T5+0.756)	=2*(U5+0.756)	=4*(V5+0.756)	=2*(W5+0.756)	=4*(X5+0.756)	=2*(Y5+0.756)
E	=O6+0.756	=2*(P6+0.756)	=Q6+0.756	=2*(R6+0.756)	=S6+0.756	=2*(T6+0.756)	=U6+0.756	=2*(V6+0.756)	=W6+0.756	=2*(X6+0.756)	=1*Y6+0.756
F	=2*(O7+0.756)	=4*(P7+0.756)	=2*(Q7+0.756)	=4*(R7+0.756)	=2*(S7+0.756)	=4*(T7+0.756)	=2*(U7+0.756)	=4*(V7+0.756)	=2*(W7+0.756)	=4*(X7+0.756)	=2*(Y7+0.756)
G	=O8+0.756	=2*(P8+0.756)	=Q8+0.756	=2*(R8+0.756)	=S8+0.756	=2*(T8+0.756)	=U8+0.756	=2*(V8+0.756)	=W8+0.756	=2*(X8+0.756)	=1*Y8+0.756
H	=2*(O9+0.756)	=4*(P9+0.756)	=2*(Q9+0.756)	=4*(R9+0.756)	=2*(S9+0.756)	=4*(T9+0.756)	=2*(U9+0.756)	=4*(V9+0.756)	=2*(W9+0.756)	=4*(X9+0.756)	=2*(Y9+0.756)
I	=O10	=2*(P10+0.756)	=Q10+0.756	=2*(R10+0.756)	=S10+0.756	=2*(T10+0.756)	=U10+0.756	=2*(V10+0.756)	=W10+0.756	0	0

Extended Simpsons Formula Skews [Shelled Egg]

As can be noted in the above image, each respective cell is skewed by a multiplication factor depending on its cell location.

Step 8 (Calculate volume from grid [within Excel])

Once the above grid has been constructed, the Extended Simpsons Rule can be applied to calculate the final volume for the egg (based on this method). This final volume will be found by adding up each coloured grid (H1, H2, H3 and H4 [refer above for key]). Once the sum of each colour has been found it can be inputted into the Extended Simpsons rule, as shown below:

$$Volume \approx \frac{1}{9}w^2 \times (H_1 + 2H_2 + 3H_3 + 4H_4)$$
$$Where\ w = 5mm\ (constant\ width)$$

This can be depicted within Excels formula view, displayed below:

H1	=O17+P16+R15+X15+Y16+Y22+W23+P23+O22
H2	=O18+O19+O20+O21+Q23+R23+S23+T23+U23+V23+X22+Y21+Y20+Y19+Y18+Y17+W15+V15+U15+T15+S15+Q16
H3	=P22+P17+R16+X16+W22
H4	=SUM(S16:W16)+SUM(Q17:X17)+SUM(P18:X18)+SUM(P19:X19)+SUM(P20:X20)+SUM(P21:X21)+SUM(Q22:V22)
Total	=(1/9)*5^2*(P26+(2*P27)+(3*P28)+(4*P29))
	=P31/1000

Extended Simpsons Formula Volume Calculations [Shelled Egg]

These formulas resulting in the values of: (Reminder that 1mL = 1cm^3)

H1	242.108	
H2	664.96	
H3	435.096	
H4	4521.77	
Total	58234.4	mm^3
	58.2344	mL

Extended Simpsons Formula Volume Calculations [Shelled Egg]

Step 9 (Calculating Percentage Error)

Due to the final volume being found (of the shelled egg), it can be compared to the displaced volume generated from the displacement experiment (achieved in-class). To find percentage error, the formula below should be used:

$$Percentage\ Error = \frac{|displacedValue - extendedSimpsonsValue|}{displacedValue} \times 100$$

$$= \frac{|59.25 - 58.2344|}{59.25} \times 100$$

$$= 1.71\%\ error$$

Extending to Un-Shelled Egg

If the extra shell width wasn't taken into account and steps 7-9 were redone the volume of the unshelled egg (from the Extended Simpsons Rule) would be:

H1	228.5	
H2	638.5	
H3	423	
H4	4425	
Total	56873.6	mm^3
	56.8736	mL

Extended Simpsons Formula Volume Calculations [Un-Shelled Egg]

This producing a percentage error (from the unshelled displacement) of:

$$Percentage\ Error = \frac{|displacedValue - extendedSimpsonsValue|}{displacedValue} \times 100$$

$$= \frac{|54 - 56.8736|}{54} \times 100$$

$$= 5.3215\%\ error$$

Comparatively, this is quite a large difference between the calculated value and the real value (given by the displacement experiment). This could extend to the fact that the un-shelled egg was only measured once, this resulting in a displaced value that is seemingly lower than expected from these results. Other than that, it could have been the difference between the densities; resulting in less water being displaced.

Limitations and Errors

- **Measurement from Ruler ($\pm$0.5mm)**
  This is one of the larger limitations placed upon these results; this is due to the fact that it is open to a lot of parallax error by the user.
- **Bled ink across the grid paper**
  These errors stem from the driver of the utensils to mix up and write inaccuracies, these inaccuracies would have stemmed from the stencilling of the egg slices onto the grid paper; leading to the ink bleeding across the page.
- **Differences between Extended Simpsons Rule and Curves**
  Ideally (as explained above) there shouldn't be 'corners' on this shape/grid although due to this being an approximation these need to exist
- **Changing to smaller measurement intervals**
  Intrinsically, the accuracy is measured by a different of 5mm, making this smaller will result in a more accurate model.
- **Displacement value is exact**
  Having this as incorrect will result in skewed conclusions to be made about the volume of the egg.

Improvements

The obvious changes that could be made, would be to record a smaller interval (resulting in a more accurate model) to writing in a dark pencil to ensure no ink bleeding across the page. These improvements revolve around this model which resulted in an error of about 1.71% which is **very** accurate with many more improvements only bringing this percentage down by a small fraction (due to it already being quite low). Further within this report (Question 5) other models will be addressed with a much better accuracy than this model.

Conclusions

Scientifically, due to both of the respective volumes having a difference of around 5% and lower these readings are accurate depictions of the volumes. This is due to normally in a lab analysis of an experiment a percentage difference of 10% is generally acceptable as an accurate experiment. This drawing the conclusion that the measurements received from this approximation are quite accurate.

Question 5

To begin modelling the egg several assumptions need to be made in order to appropriately model of the shape.

Assumptions

- **Only making one based on shelled volume and lengths**
  Purely using the shelled volume will mean allow for simplicity in the model, as the shelled has a more cemented foundation and there are more averaged values to base measurements off.
- **No bumps or crack in egg**
  Assuming this will ensure there exists a uniform curve across the egg.
- **Each model will be constructed to revolve around y-axis**
  Doing so will allow a more accessible comparison between generated models.

Construction of model

As can be quickly noted about the shape of an egg is it looks strikingly similar to the shape of an ellipse; which of course follows the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The biggest difference of course is the fact that an ellipse is symmetrical along the y and x axis as opposed to the egg shape which is only symmetrical across the x axis. Herein lies the problem with constructing an ellipse; an ellipse needs to be constructed which allows for a different radius at each point subsequent to the y-axis [contrastingly asymmetrical against the symmetrical ellipse]. In order to reach this conclusion, an explanation for where the ellipse equation comes from will be explained.

As is known from the unit circle:

$$x^2 + y^2 = r^2$$

The above formula is essentially stating that the x and y values are dependent on **one** r value.

Dividing by r^2 leaves:

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

Which is the ellipse equation due to the fact that the x and y values depend on **two** different r values. If the first $r = a$ and the second, $r = b$ the equation turns into:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

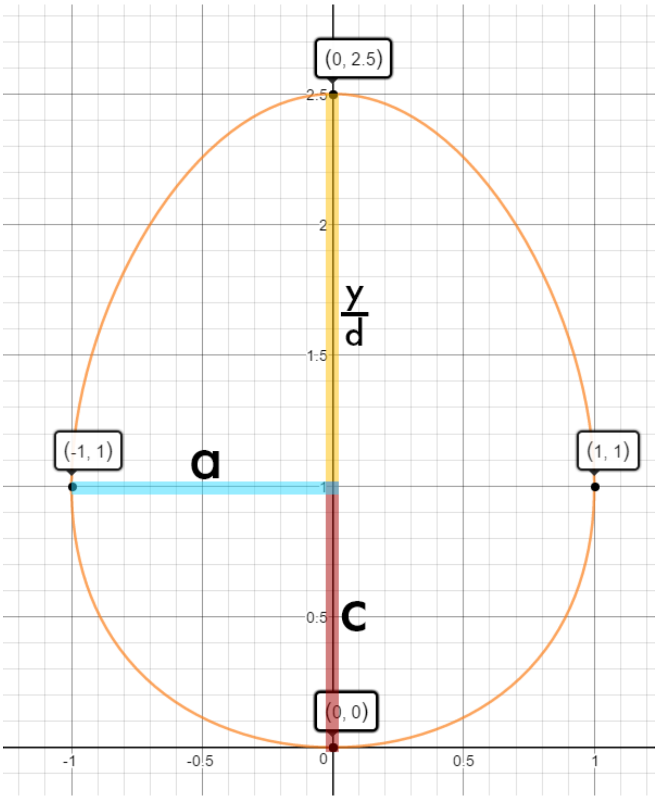
Which is exactly as depicted above. Extending from this, the 'a' value controls the minor axis (in this case the minor axis represents the 'width' of the egg) with the 'b' value controlling the major axis (the height of the egg). Although, it is known that for an egg model the 'b' value needs to change in accordance to the y value. This leading to a new formula:

$$\frac{x^2}{a^2} + \frac{y^2}{\left(c + \frac{y}{d}\right)^2} = 1$$

Where c will now represent the point from the middle of the ellipse down to the base of the ellipse with d representing the proportionality constant between y and the upper length (depicts the top height of the ellipse). The only problem with this equation, is to ensure that when revolving there is a constant positive volume, to counteract this the most negative y-value should be zero. To ensure this is the case c must be subtracted from y, moving the ellipse up by that amount. Refer to formula below:

$$\frac{x^2}{a^2} + \frac{(y-c)^2}{\left(c + \frac{y}{d}\right)^2} = 1$$

To view the relationships between the final formula, refer to the image below:



Based off:

$$\frac{x^2}{1^2} + \frac{(y-1)^2}{\left(1 + \frac{y}{5}\right)^2} = 1$$

Once this new formula was constructed it was a simple manner of inputting the relevant measurements received from the in-class experimental part of this assignment, in order to depict the hybrid-elliptical shape. Utilising the measurements shown in Appendix 1.1 the equation for the egg turns into:

$$\frac{x^2}{\left(\frac{44.61}{2}\right)^2}+\frac{(y-28)^2}{\left(28+\frac{y}{d}\right)^2}=1$$

The width of the egg was divided by two, due to the fact that the 'a' value represents just the radius at that point, so the resultant width needs to be divided by two. The number 28 comes from the length from the base to the thickest point (28mm) and has been reflected in the above equation. In order to make these values relatable to real life measurements the equation will be rewritten in terms of centimetres; refer below.

$$\frac{x^2}{(2.2305)^2}+\frac{(y-2.8)^2}{\left(2.8+\frac{y}{d}\right)^2}=1$$

Upon reaching this point there is no value for 'd', to find the proportionality constant of 'd' the resultant x and y values can be inputted to leave 'd' as the only variable left in the equation.

To begin calculating 'd' various x and y coordinates need to be inputted. Due to the model being known, the height at the very top of the egg will be the length (57.48mm or 5.748cm); at this point x will equal 0. Therefore, the above equation turns into:

$$\frac{0^2}{(2.2305)^2}+\frac{(5.748-2.8)^2}{\left(2.8+\frac{5.748}{d}\right)^2}=1$$

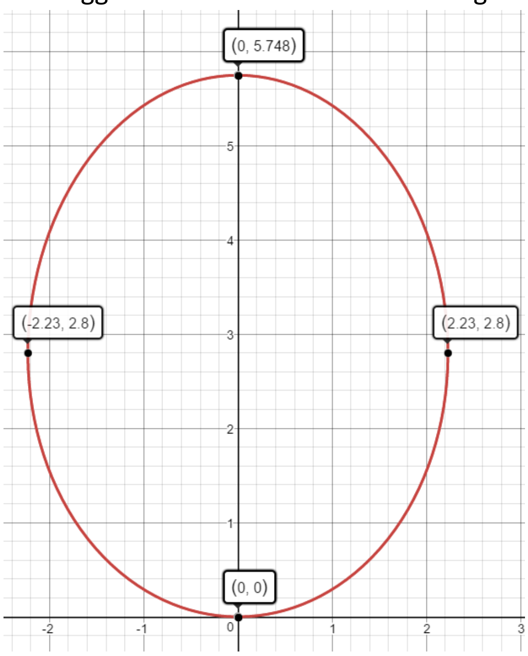
Which can be simplified to (full steps in Appendix 2.1):

$$\begin{aligned} d &= \frac{5.748}{0.148} \\ &= 38.8378 \end{aligned}$$

This means that the resultant proportionality constant for this egg curve is 38.8378. This resulting in the equation:

$$\frac{x^2}{(2.2305)^2}+\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}=1$$

Which looks like:



As can be noted in the above image, if the measurements from the in-class were the only things that were used this model is "perfect" as the curve generated follows and meets at the exact points recorded (2.8cm from the base of the egg all the way up to the extremities of $\pm$2.23cm). To then calculate the volume of this curve, the volumes of revolution will be utilised around the y axis; this follows the form of:

$$\pi \times \int_a^b x^2 \, dy$$

A note to make at this point is that for the volumes of revolution to work as long as it is in the form of x^2 it doesn't matter if there is a single or double point passing through (in the case as above) this is due to the x^2 complying for both negative and positive answers. The limits to this equation are the respective extremes of the model (along the y axis), being 0 and 5.748 respectively.

Due to this being in the form of x^2 no tedious algebra is needed to solve for it (contrasted to solving for y^2 from this equation). Refer below for solution of x^2:

$$\frac{x^2}{(2.2305)^2} + \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2} = 1$$

$$\frac{x^2}{(2.2305)^2} = 1 - \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2}$$

$$x^2 = (2.2305)^2 \left(1 - \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2} \right)$$

Once x^2 has been found it is simply a matter of finding the integral of the above equation, this integral can be found below (full steps in Appendix 2.2):

$$\int (2.2305)^2 \left(1 - \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2} \right) dy$$

$$= (2.2305)^2 \left[y - \frac{2.8 + \frac{y}{38.8378}}{2275194.264} + \frac{0.1468778228\left(\ln\left(2.8 + \frac{y}{38.8378}\right)\right)}{2275194.264} - \frac{0.2123943176}{2275194.264\left(2.8 + \frac{y}{38.8378}\right)} \right]$$

Once the integral has been found, it is simply a matter of substituting values (the upper and lower limits) into this new integral function to retrieve the definite integral. The upper and lower limits for this function is the height of the egg (in accordance to our model). [Lower limit = 0, Upper limit = 5.748]. Refer to Appendix 2.2 for the calculations of this definite integral, below is the minified version.

$$\int_0^{5.748}\left((2.2305)^2-(2.2305)^2\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}\right)dy$$
$$=(2.2305)^2\left[y-\frac{2.8+\frac{y}{38.8378}}{2275194.264}+\frac{0.1468778228\left(\ln\left(2.8+\frac{y}{38.8378}\right)\right)}{2275194.264}\right.$$
$$\left.-\frac{0.2123943176}{2275194.264\left(2.8+\frac{y}{38.8378}\right)}\right]$$
$$=19.0596$$

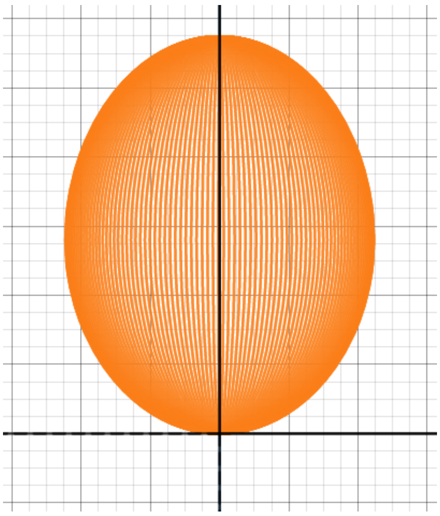
Once the definite integral was found it can be substituted back into the volumes of revolution equation to retrieve the prospective mL from this model.

$$\pi\times\int_a^b x^2\,dy$$
$$=\pi\times\int_0^{5.748}(2.2305)^2\left(1-\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}\right)dy$$
$$=\pi\times 19.0596$$
$$=59.87749934$$
$$=59.8775\text{mL}$$

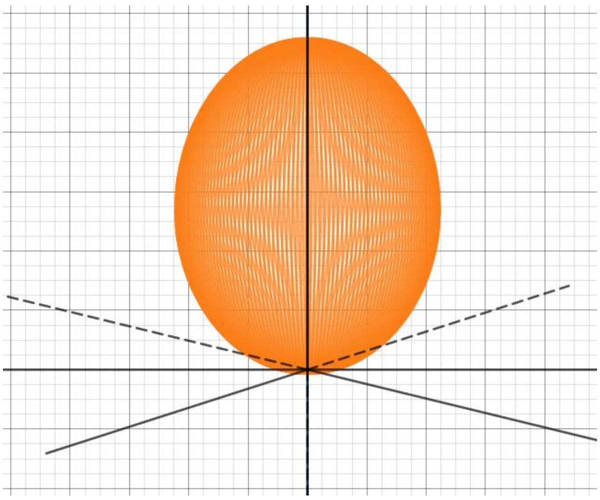
This resulting in a percentage error of:

$$Percentage\ Error=\frac{|displacedValue-displacedModelValue|}{displacedValue}\times 100$$
$$=\frac{|59.25-59.87749934|}{59.25}\times 100$$
$$=1.0591\%\ error$$

To finally depict what this model looks like through a volumes of revolution, the resultant image can be found through various online graphing tools. Refer below for a graphical representation of this curve rotated around the y-axis:



2D Representation of Volume of Revolution



3D Representation of Volume of Revolution

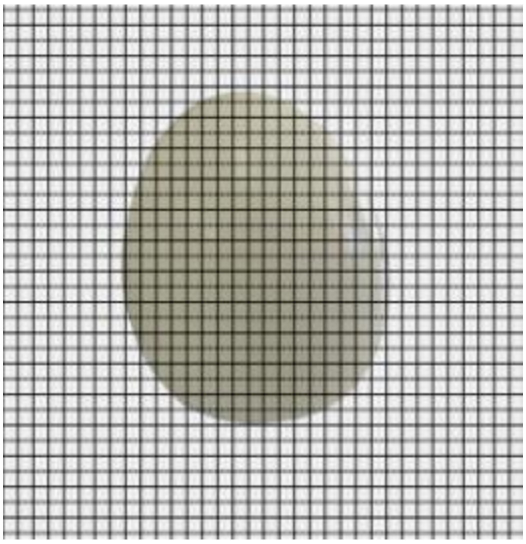
Conclusions about Ellipse Model

Comparatively, the percentage error gained through this model is quite an improvement over the Simpson's model by a difference of $(1.71 - 1.0591) = 0.6509\%$ this proving that the ellipse gives a much more accurate depiction of the overall volume of the egg. While this ellipse model is quite exact, within the improvements upon this model a polynomial will be formed which will depict a much better model of an egg.

Improvements

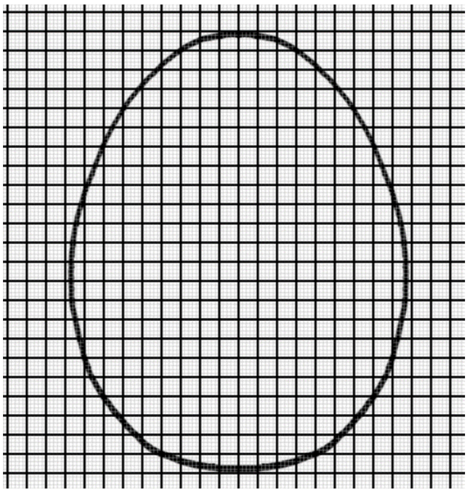
In order to offer an improvement upon this model, the actual egg curve will need to be circumscribed as a polynomial equation. This can be achieved by taking a picture taken of the egg (before it was de-shelled) and aligning it horizontally, lying the egg vertically. Refer to Appendix 3.1 for the original photo used. Refer to Appendix 3.2 for a complete to scale print out of the egg.

This photo was then transferred to the computer in order to align and scale to the appropriate size of the egg based on the real life measurements. To scale properly the egg was traced around from the image and placed on top of a metric 1cm by 1cm grid (with intervals of 0.5mm). This allowed appropriate pixel resizing of the egg (Refer below).



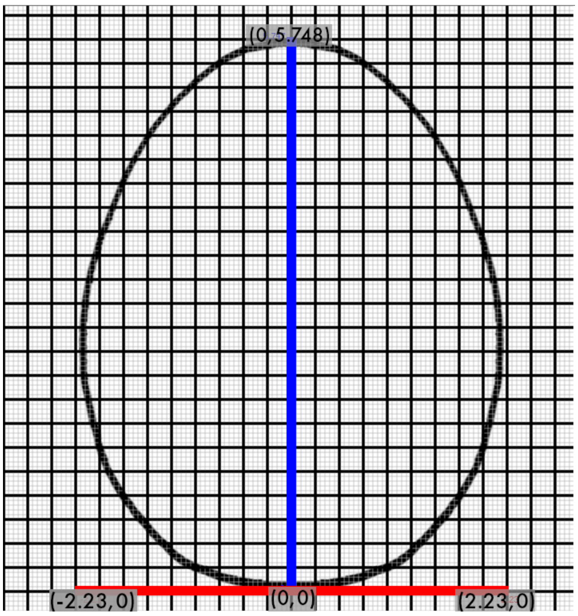
Cut out egg on top of metric grid

From this point the egg was digitally traced, to allow for a SVG Vector to be constructed (essentially allowing for points to be mapped onto the egg). An outline of the traced egg can be found below. For computational code generation refer to Appendix 3.3.



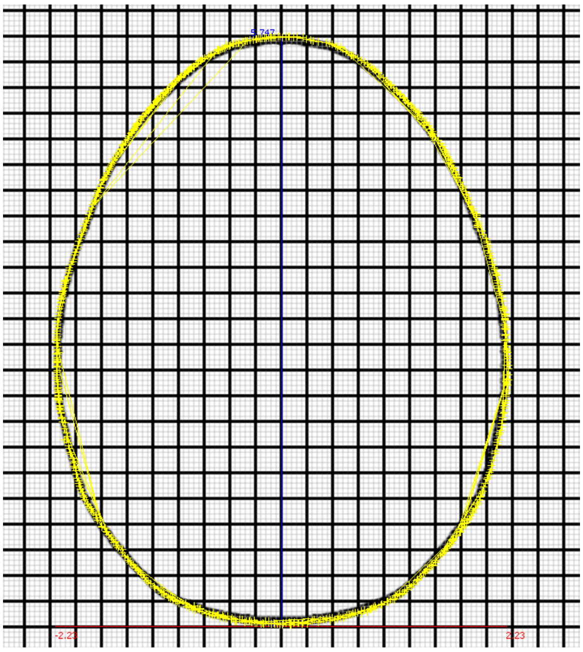
Outlined SVG vector of the egg

After the SVG computer file was constructed of the egg, the various coordinates could be found by outlining the position of each part of the curve as its place on a predetermined coordinate system. This coordinate system would take into the knowns (base -> height length, height and width) of the egg and then trace the curve plotting each of these points.



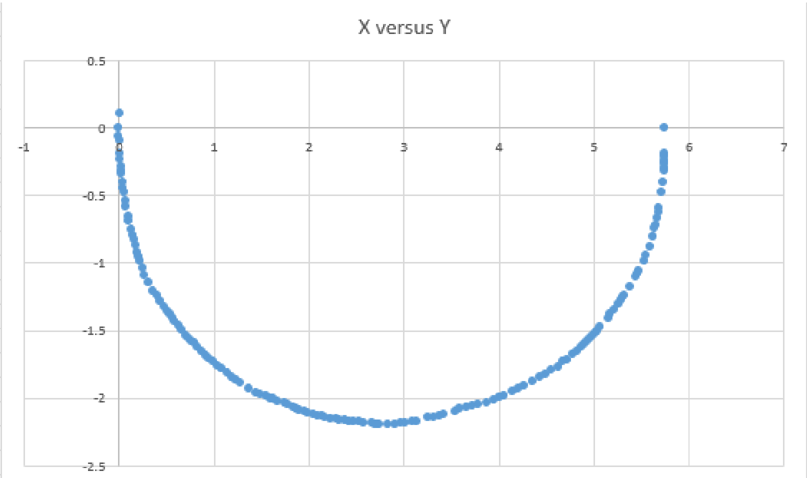
Inscribed Coordinate System

Once this had been set up some more code was utilised to trace the curve (Appendix 3.4), resulting in the image as noted below:



Traced Paths

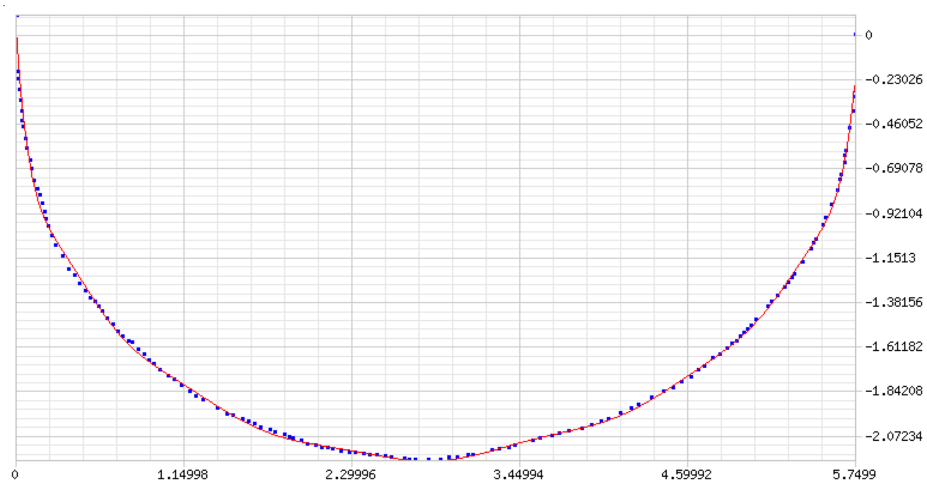
This resulted in about a couple of thousand unique points along the entire curve, although due to a polynomial fit needing to be constructed; only half of the points will need to be used. Lying within the domain of $\{-2.23 < x < 0\}$. In order to keep processing times down every second point was taken, with any close duplicates removed. This resulting in a grand total of 156 unique points for half of the egg. The resultant x and y coordinates were then flipped, to the generate a scatter plot (refer below) that looked like a parabola (in order to best fit a curve). Refer to Appendix 3.5 outlining each generated point.



Depiction of scatter plot

To then extrapolate and fit a curve to these points, a polynomial order needed to be determined. It was quickly realised that for each order that is incremented the resultant amount of coefficients increases. To find an averaged calculation time an order of 14 was used as after that point processing times begin to take longer than 30 minutes. Refer to Appendix 3.6 for the code used to generate this fit.

Effectively, the fitting algorithm works by implementing a Gaussian matrix to then solve for the unknown 15 coefficients. Although, the thing that separates this algorithm from others used within early methods in Math C is its iterative function. This is where it sorts through each combination of 15 points and solves the matrix, resulting in a comparison between heaps of different alignments of the curves. These curves are then tested among each point; to eventually find the curve which enlists the most points having been successfully met. Once this code was run on the 156 unique points the graph below was generated through the code.



Depiction of Model against points

This model being:

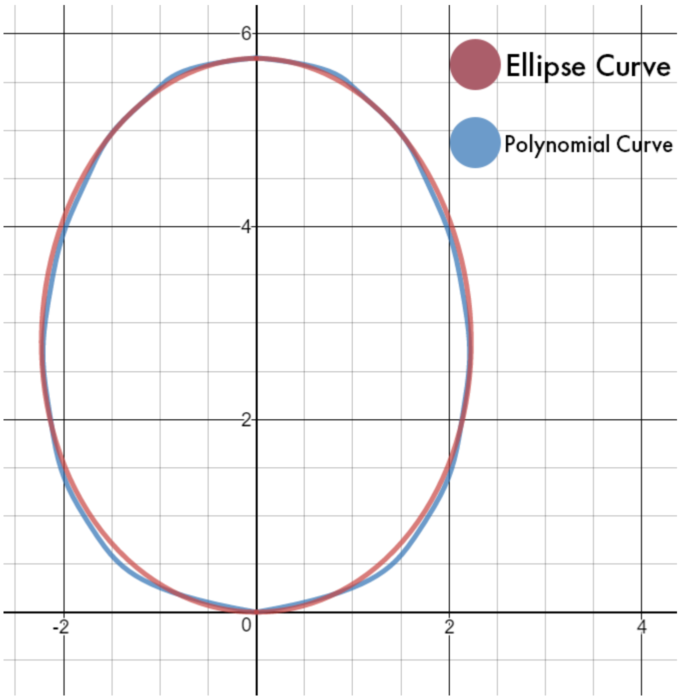
$$y = 0.00002476312803365381 \times x^{14} - 0.0009178590891438972 \times x^{13} + 0.015046116894752253 \times x^{12} - 0.14345656989993 \times x^{11} + 0.8797832124328367 \times x^{10} - 3.6216063790911295 \times x^{9} + 10.127583595192807 \times x^{8} - 19.03740932937551 \times x^{7} + 23.355759574131245 \times x^{6} - 18.277771148930487 \times x^{5} + 10.902563464899021 \times x^{4} - 9.31249077161999 \times x^{3} + 9.085271066255547 \times x^{2} - 5.760935396945824x + 0$$

This constructing an r^2 value of 0.99986

Although, in order for this to be useful in relation to the egg; the x and y variables need to be inverted back. This making the generated curve:

$$x = 0.00002476312803365381 \times y^{14} - 0.0009178590891438972 \times y^{13} + 0.015046116894752253 \times y^{12} - 0.14345656989993 \times y^{11} + 0.8797832124328367 \times y^{10} - 3.6216063790911295 \times y^{9} + 10.127583595192807 \times y^{8} - 19.03740932937551 \times y^{7} + 23.355759574131245 \times y^{6} - 18.277771148930487 \times y^{5} + 10.902563464899021 \times y^{4} - 9.31249077161999 \times y^{3} + 9.085271066255547 \times y^{2} - 5.760935396945824y + 0$$

This curve when placed upon the previous Ellipse model looks as depicted below:



Visual comparison between models

Note: in the above image, the polynomial was only shown within the relevant bounds, and inverted to show either side to depict a fair comparison.

To calculate the volume of this generated curve, it will again be revolved around the y-axis through the formula:

$$\pi \times \int_a^b x^2 \, dy$$

Where ‘a’ and ‘b’ represent the upper and lower bounds of the height (0 and 5.748 respectively). To solve for x^2 all that needs to be done is square the generated polynomial as noted below:

$$\begin{aligned} x^2 = (&0.000024763128033653 81 \times y^{14} - 0.0009178590891438972 \times y^{13} + 0.015046116894752253 \times y^{12} \\ &- 0.14345656989993 \times y^{11} + 0.8797832124328367 \times y^{10} - 3.6216063790911295 \times y^{9} \\ &+ 10.127583595192807 \times y^{8} - 19.03740932937551 \times y^{7} + 23.355759574131245 \times y^{6} \\ &- 18.277771148930487 \times \mathrm{y}^{5} + 10.902563464899021 \times y^{4} - 9.31249077161999 \times y^{3} \\ &+ 9.085271066255547 \times \mathrm{y}^{2} - 5.760935396945824\mathrm{y} + 0)^2 \end{aligned}$$

This makes for a problem as ‘u’ substitution won’t work, leaving the only real option of calculating the integral for this curve being to expand the square. While at face value, this will seem like a tedious task; when manipulated correctly excel can be used to generate each coefficient and the respective order of polynomial.

To begin with constructing an Excel table to perform this task, an understanding of what is going on when squaring with multiple entities. See below for a walk through of this process:

Let a, b and c represent an arbitrary (n length) polynomial

$$\begin{aligned} &(ax^4 + bx^3 + cx^2)^2 \\ &= (ax^4 + bx^3 + cx^2)(ax^4 + bx^3 + cx^2) \end{aligned}$$

Instead of then expanding out as normal, construct a grid of each of these factors:

	ax^4	bx^3	cx^2
ax^4	a^2x^{4+4}	abx^{3+4}	abx^{2+4}
bx^3	abx^{4+3}	b^2x^{3+3}	bcx^{2+3}
cx^2	acx^{4+2}	bcx^{3+2}	a^2x^{2+2}

So then, if from this grid, once each cell is multiplied within the opposing cell, a pattern starts emerging (as can be noted in the above grid).

This will result in each of the resultant co factors to be found simply by combining like terms, as can be noted below. This when replicated in Excel, provides a quick and easy way to expand the equation.

	$\boldsymbol{ax^4}$	$\boldsymbol{bx^3}$	$\boldsymbol{cx^2}$
$\boldsymbol{ax^4}$	a^2x^8	abx^7	abx^6
$\boldsymbol{bx^3}$	abx^7	b^2x^6	bcx^5
$\boldsymbol{cx^2}$	acx^6	bcx^5	a^2x^4

Refer below for excel version:

	14	13	12	11	10	9	8	7	6	5	4	3	2	1
14	28	27	26	25	24	23	22	21	20	19	18	17	16	15
13	27	26	25	24	23	22	21	20	19	18	17	16	15	14
12	26	25	24	23	22	21	20	19	18	17	16	15	14	13
11	25	24	23	22	21	20	19	18	17	16	15	14	13	12
10	24	23	22	21	20	19	18	17	16	15	14	13	12	11
9	23	22	21	20	19	18	17	16	15	14	13	12	11	10
8	22	21	20	19	18	17	16	15	14	13	12	11	10	9
7	21	20	19	18	17	16	15	14	13	12	11	10	9	8
6	20	19	18	17	16	15	14	13	12	11	10	9	8	7
5	19	18	17	16	15	14	13	12	11	10	9	8	7	6
4	18	17	16	15	14	13	12	11	10	9	8	7	6	5
3	17	16	15	14	13	12	11	10	9	8	7	6	5	4
2	16	15	14	13	12	11	10	9	8	7	6	5	4	3
1	15	14	13	12	11	10	9	8	7	6	5	4	3	2

Key Linking colour with exponent

	a -x^14	b x^13	c x^12	d x^11	e x^10	f x^9	g x^8	h x^7	I x^6	j x^5	k x^4	l x^3	m x^2	n x^1
	2.48E-05	**-0.00092**	**0.015046**	**-0.14346**	**0.879783**	**-3.62161**	**10.12758**	**-19.0374**	**23.35576**	**-18.2778**	**10.90256**	**-9.31249**	**9.085271**	**-5.76094**
a -x^14	6.13E-10	-2.3E-08	3.73E-07	-3.6E-06	2.18E-05	-9E-05	0.000251	-0.00047	0.000578	-0.00045	0.00027	-0.00023	0.000225	-0.00014
b x^13	-2.3E-08	8.42E-07	-1.4E-05	0.000132	-0.00081	0.003324	-0.0093	0.017474	-0.02144	0.016776	-0.01001	0.008548	-0.00834	0.005288
c x^12	3.73E-07	-1.4E-05	0.000226	-0.00216	0.013237	-0.05449	0.152381	-0.28644	0.351413	-0.27501	0.164041	-0.14012	0.136698	-0.08668
d x^11	-3.6E-06	0.000132	-0.00216	0.02058	-0.12621	0.519543	-1.45287	2.731041	-3.35054	2.622066	-1.56404	1.335938	-1.30334	0.826444
e x^10	2.18E-05	-0.00081	0.013237	-0.12621	0.774019	-3.18623	8.910078	-16.7488	20.54801	-16.0805	9.591892	-8.19297	7.993069	-5.06837
f x^9	-9E-05	0.003324	-0.05449	0.519543	-3.18623	13.11603	-36.6781	68.946	-84.5854	66.19489	-39.4848	33.72618	-32.9033	20.86384
g x^8	0.000251	-0.0093	0.152381	-1.45287	8.910078	-36.6781	102.5679	-192.803	236.5374	-185.11	110.4166	-94.313	92.01184	-58.3444
h x^7	-0.00047	0.017474	-0.28644	2.731041	-16.7488	68.946	-192.803	362.423	-444.633	347.9614	-207.557	177.2857	-172.96	109.6733
I x^6	0.000578	-0.02144	0.351413	-3.35054	20.54801	-84.5854	236.5374	-444.633	545.4915	-426.891	254.6377	-217.5	212.1934	-134.551
j x^5	-0.00045	0.016776	-0.27501	2.622066	-16.0805	66.19489	-185.11	347.9614	-426.891	334.0769	-199.275	170.2116	-166.059	105.2971
k x^4	0.00027	-0.01001	0.164041	-1.56404	9.591892	-39.4848	110.4166	-207.557	254.6377	-199.275	118.8659	-101.53	99.05274	-62.809
l x^3	-0.00023	0.008548	-0.14012	1.335938	-8.19297	33.72618	-94.313	177.2857	-217.5	170.2116	-101.53	86.72248	-84.6065	53.64866
m x^2	0.000225	-0.00834	0.136698	-1.30334	7.993069	-32.9033	92.01184	-172.96	212.1934	-166.059	99.05274	-84.6065	82.54215	-52.3397
n x^1	-0.00014	0.005288	-0.08668	0.826444	-5.06837	20.86384	-58.3444	109.6733	-134.551	105.2971	-62.809	53.64866	-52.3397	33.18838

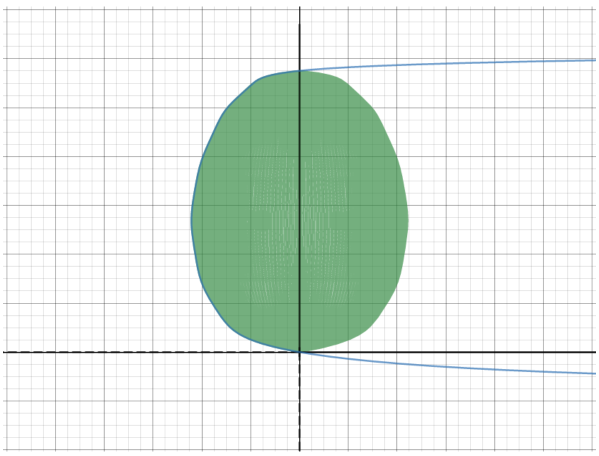
Generated graph with each coefficient

		Y VALUE:	5.748		
	Summation of each colour		Integrated	Exponent to raise to	Resultant Value
x^28	6.13213E-10		2.11453E-11	29	2.24491E+11
x^27	-4.54581E-08		-1.6235E-09	28	-2.99863E+12
x^26	1.58764E-06		5.88016E-08	27	1.88948E+13
x^25	-3.47253E-05		-1.33559E-06	26	-7.46635E+13
x^24	0.000533304		2.13322E-05	25	2.07469E+14
x^23	-0.006111327		-0.000254639	24	-4.3085E+14
x^22	0.05420426		0.002356707	23	6.93731E+14
x^21	-0.380937831		-0.017315356	22	-8.86748E+14
x^20	2.153970613		0.102570029	21	9.13845E+14
x^19	-9.894851791		-0.49474259	20	-7.66857E+14
x^18	37.13519149		1.954483762	19	5.27048E+14
x^17	-114.1253975		-6.340299861	18	-2.97448E+14
x^16	287.1457263		16.89092507	17	1.3786E+14
x^15	-590.3628827		-36.89768017	16	-5.23923E+13
x^14	990.0271863		66.00181242	15	1.63045E+13
x^13	-1357.621197		-96.97294263	14	-4.16759E+12
x^12	1547.338951		119.0260732	13	8.89938E+11
x^11	-1533.464941		-127.7887451	12	-1.66224E+11
x^10	1423.674983		129.4249984	11	29288863667
x^9	-1296.158469		-129.6158469	10	-5103001505
x^8	1103.022424		122.5580471	9	839445813.3
x^7	-804.2790982		-100.5348873	8	-119798359.5
x^6	495.4220907		70.77458439	7	14672179.92
x^5	-294.8309335		-49.13848891	6	-1772239.263
x^4	189.8394658		37.96789316	5	238232.1093
x^3	-104.6793194		-26.16982984	4	-28567.21719
x^2	33.18837665		11.06279222	3	2100.946696
				Total:	**18.80786171**

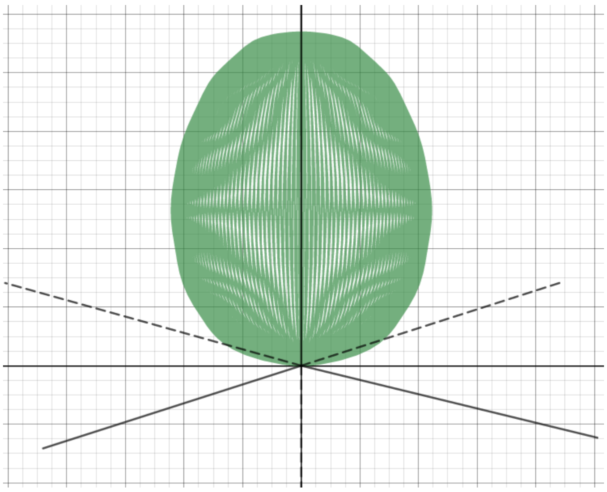
Calculated Integral

Refer to Appendix 3.7 for full extrapolation and working of this integral. Note, the zero didn't need to be taken into account as due to each coefficient relying on y; inputting zero as a value will result in just cancelling out to equal zero.

For a visual depiction of this curve being rotated refer to the images below:



2D Representation of Volume of Revolution



3D Representation of Volume of Revolution

Therefore, due to the definite integral value being found it can be substituted into the original volumes of revolution formula to find the total volume of this curve:

$$\pi \times \int_a^b x^2\,dy$$
$$= \pi \times \int_0^{5.748} (0.00002476312803365381 \times y^{14} - 0.0009178590891438972 \times y^{13} + 0.015046116894752253 \times y^{12}$$
$$- 0.14345656989993 \times y^{11} + 0.8797832124328367 \times y^{10} - 3.6216063790911295 \times y^{9}$$
$$+ 10.127583595192807 \times y^{8} - 19.03740932937551 \times y^{7} + 23.355759574131245 \times y^{6}$$
$$- 18.277771148930487 \times y^{5} + 10.902563464899021 \times y^{4} - 9.31249077161999 \times y^{3}$$
$$+ 9.085271066255547 \times y^{2} - 5.760935396945824y + 0)^2\,dy$$
$$= \pi \times 18.80786171$$
$$= 59.08664018$$
$$= 59.0866\text{mL}$$

This leading to a percentage error of:

$$Percentage\ Error = \frac{|displacedValue - displacedModelValue|}{displacedValue} \times 100$$
$$= \frac{|59.25 - 59.08664018|}{59.25} \times 100$$
$$= 0.27571278$$
$$= 0.2757\% error$$

Assumptions to Polynomial Model

- **Egg is perfectly symmetrical across y-axis**
  Not assuming this will result in having an egg that is skewed either way, which would mean that two different polynomials would each have to be rotated halfway around the egg. For sake of simplicity this is a valid assumption
- **Lens on phone didn't skew the curves of the egg**
  Doing so will remove the ability to properly generate points from the respective curves.
- **Curves meet exactly at the top of the egg (5.748 centimetres)**
  Having this assumption wrong will result in a skewed value given by the Volumes of Revolution, as it **depends** on the resultant height of the egg

Final Conclusions

This model has resulted in the lowest percentage error by a difference of $(1.0.591 - 0.2757) = 0.7834\%$ this being a **massive** difference in the overall accuracy of the model. Although, it must be noted that this model is quite easily extensible as it is only restricted by the degree of polynomial and can theoretically be applied up to any nth degree (although a severe limiting factor is processing time/power). Further analysis will be provided within Question 6.

Question 6

Within question 4 the resultant percentage error was 1.71% for the Extended Simpsons Model. This was superseded by the Elliptical model having a percentage error of 1.0591% to then ultimately be overthrown by the Polynomial model having a percentage error of 0.2757%. Within the following question; various strengths and limitations of each model will be addressed to draw conclusions as to why each model is sufficiently better at modelling the egg.

Extended Simpsons Model

This model was the first conducted and had the overall highest (least accurate) percentage error. A strength of this model is its ability to provide an accurate volume given only the shell width and the individual slices of the egg. There exist several large reasons for such a large inaccuracy across this model, the first extending to the fact that these were 5mm slices. If accuracy was the priority over time constraints this could have easily been reduced to 1mm slices, if a blade was used instead of a slicing machine.

Pieces could have fallen out of the egg, which would have been left discarded; literally taking chunks out of the calculated volume of the egg. Another thing which let this model down was its reliance on human measuring and recordation. This is a fault due to humans being **very** prone to errors, especially mechanical ones such as recording and measuring various lengths. Finally, this model had a lot of human elements and steps to it; each step increasing the likely hood of an error occurring. By minimizing the amount of steps to be taken to reach the final volume, the more accurate the answer. This model is a good **estimate** to the volume of the egg; although its big downsides are that once this method is completed the egg must be destroy, and there are a lot of differing parts to the process of generating the conclusion.

Elliptical Model

This model was good as it purely relied on measurements given by the digital Caliper; this removing errors such as parallax error which can be crucial to maintaining accuracy in such a fine tuned experiment. Another strength of this model is its simplicity, contrastingly, this model required very few steps and just required some inputting into the final formula until the resultant equation was made.

The major error made in this elliptical model was that it didn't take into account the real life discrepancies of generating a curve and assumed a perfect curve along that shape. Ideally, this elliptical model would represent the 'perfect' egg based on the readings gathered; with the polynomial model constructed depicting the 'real life' version of the perfect egg. This offering a good strength due to the fact that this elliptical model would be what is to be expected from an egg of these lengths. Intrinsically, this model is the most **mathematically based**, this being that the curves are predicted based on an idealist model or construct of an egg.

Polynomial Model

A major factor in ensuring the accuracy of this model was the time constraints. It took a lot of processing power and time to generate the curve depicted above and took even more time to manually integrate and check all of the answers. While this is the most accurate model (a strength) the length of time taken to fully apply this model was a weakness.

The only real way to improve upon this model would be to keep constructing a higher order polynomial which will get to a point where a perfect digital representation of the egg will be constructed through mathematics. Overall, this is the most **real life** applicable model designed to mimic the actual curve of the egg, as opposed to merely estimate it based on the lengths given.

Conclusion

As can be found in the preceding report the polynomial was the most exact to the real life reading, purely due to it directly mimicking the actual curve of the egg (as depicted within a photograph). Overall, this report has provided a good analysis and comparison between three different egg models and clearly highlights the strengths and weaknesses between each prospective model.

Bibliography

Anon, 2016. *Labs - Error Analysis | Department of Physics and Astronomy | Appalachian State University*. [online] Physics.appstate.edu. Available at: <http://physics.appstate.edu/undergraduate-programs/laboratory/resources/error-analysis> [Accessed 23 Aug. 2016].

Appendix

Appendix 1.1 (Measurements from in-class component)

Egg Dimensions

Measurement:	Length(mm)	Width(mm)	Dist. Base to Thickest(mm)	Shell Width(mm)
Measurement 1	57.51	44.60	28	0.37
Measurement 2	57.46	44.63	-	0.386
Measurement 3	57.46	44.58	-	-
Average	57.48	44.61	28	0.378

Egg Volume

Measurement:	With Shell (mL)	Without Shell (mL)
Measurement 1	59.5	54
Measurement 2	59.0	-
Measurement 3	-	-
Average	59.25	54

Appendix 1.2 (Egg lines)

Refer to Attached Page.

Appendix 2.1 (Full algebraic calculation of 'd')

$$\frac{0^2}{(2.2305)^2}+\frac{(5.748-2.8)^2}{\left(2.8+\frac{5.748}{d}\right)^2}=1$$

$$0+\frac{(2.948)^2}{\left(2.8+\frac{5.748}{d}\right)^2}=1$$

$$(2.948)^2=\left(2.8+\frac{5.748}{d}\right)^2$$

$$2.8+\frac{5.748}{d}=2.948$$

$$\frac{5.748}{d}=0.148$$

$$d=\frac{5.748}{0.148}$$

$$=38\frac{31}{37}$$

$$=38.8378$$

Appendix 2.2 (Finding integral of elliptical model)

$$\int_0^{5.748}(2.2305)^2\left(1-\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}\right)dy$$

Find the indefinite integral:

$$\int(2.2305)^2\left(1-\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}\right)dy$$

$$\int\left((2.2305)^2-(2.2305)^2\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}\right)dy$$

Separate:

$$\int(2.2305)^2\,dy$$
$$=(2.2305)^2\int 1\,dy$$

$$-(2.2305)^2\int\frac{(y-2.8)^2}{\left(2.8+\frac{y}{38.8378}\right)^2}dy$$

$$Let\ u=2.8+\frac{y}{38.8378}$$
$$\frac{du}{dy}=\frac{1}{38.8378}$$
$$\therefore dy=38.8378du$$

$$=-\frac{(2.2305)^2}{38.8378}\int\frac{38.8378(y-2.8)^2}{(u)^2}du$$
$$=-\frac{(2.2305)^2}{38.8378}\int\frac{38.8378(y^2-5.6y+2.8^2)}{(u)^2}du$$

Although from u, y can be found:

$$u=2.8+\frac{y}{38.8378}$$
$$\frac{y}{38.8378}=(u-2.8)$$
$$y=38.8378(u-2.8)$$

Substitute into integral

$$=-\frac{(2.2305)^2}{38.8378}\int\frac{38.8378\left(\left(38.8378(u-2.8)\right)^2-5.6\left(38.8378(u-2.8)\right)+2.8^2\right)}{(u)^2}du$$
$$=-\frac{(2.2305)^2}{38.8378}\int\frac{38.8378(1508.3747u^2-8664.39005u+12442.47442)}{(u)^2}du$$
$$=-\frac{(2.2305)^2}{38.8378}\int\frac{58581.95531(u^2-0.1468778228u+0.2123943176)}{(u)^2}du$$
$$=-\frac{(2.2305)^2}{38.8378\times58581.95531}\int\frac{(u^2-0.1468778228u+0.2123943176)}{(u)^2}du$$
$$=-\frac{(2.2305)^2}{2275194.264}\int\frac{(u^2-0.1468778228u+0.2123943176)}{(u)^2}du$$

Separate into constituents

$$= -\frac{(2.2305)^2}{2275194.264}\int \frac{u^2}{u^2}du - \frac{(2.2305)^2}{2275194.264}\int \frac{-0.1468778228u}{u^2}du$$
$$- \frac{(2.2305)^2}{2275194.264}\int \frac{0.2123943176}{u^2}du$$
$$= -\frac{(2.2305)^2}{2275194.264}\int 1\,du + \frac{(2.2305)^2 \times 0.1468778228}{2275194.264}\int \frac{1}{u}du$$
$$- \frac{(2.2305)^2 \times 0.2123943176}{2275194.264}\int \frac{1}{u^2}du$$

Calculate simplified integrals

$$= -\frac{(2.2305)^2}{2275194.264}[u] + \frac{(2.2305)^2 \times 0.1468778228}{2275194.264}[\ln u] - \frac{(2.2305)^2 \times 0.2123943176}{2275194.264}\left[\frac{1}{u}\right]$$
$$= -\frac{(2.2305)^2}{2275194.264}\left[2.8 + \frac{y}{38.8378}\right] + \frac{(2.2305)^2 \times 0.1468778228}{2275194.264}\left[\ln\left(2.8 + \frac{y}{38.8378}\right)\right]$$
$$- \frac{(2.2305)^2 \times 0.2123943176}{2275194.264}\left[\frac{1}{\left(2.8 + \frac{y}{38.8378}\right)}\right]$$

Combine both integrals

$$\int \left((2.2305)^2 - (2.2305)^2 \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2}\right)dy$$

$$= (2.2305)^2 \int 1\,dy - (2.2305)^2 \int \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2}dy$$
$$= (2.2305)^2 \left[y - \frac{2.8 + \frac{y}{38.8378}}{2275194.264} + \frac{0.1468778228\left(\ln\left(2.8 + \frac{y}{38.8378}\right)\right)}{2275194.264}\right.$$
$$\left. - \frac{0.2123943176}{2275194.264\left(2.8 + \frac{y}{38.8378}\right)}\right]$$

Calculate definite integral

$$\int_0^{5.748} \left((2.2305)^2 - (2.2305)^2 \frac{(y-2.8)^2}{\left(2.8 + \frac{y}{38.8378}\right)^2}\right)dy$$
$$= (2.2305)^2 \left[y - \frac{2.8 + \frac{y}{38.8378}}{2275194.264} + \frac{0.1468778228\left(\ln\left(2.8 + \frac{y}{38.8378}\right)\right)}{2275194.264}\right.$$
$$\left. - \frac{0.2123943176}{2275194.264\left(2.8 + \frac{y}{38.8378}\right)}\right]$$

$$= \left((2.2305)^2 \left[5.748 - \frac{2.8 + \frac{5.748}{38.8378}}{2275194.264} + \frac{0.1468778228\left(\ln\left(2.8 + \frac{5.748}{38.8378}\right)\right)}{2275194.264} - \frac{0.2123943176}{2275194.264\left(2.8 + \frac{5.748}{38.8378}\right)} \right] \right) - \left((2.2305)^2 \left[0 - \frac{2.8 + \frac{0}{38.8378}}{2275194.264} + \frac{0.1468778228\left(\ln\left(2.8 + \frac{0}{38.8378}\right)\right)}{2275194.264} - \frac{0.2123943176}{2275194.264\left(2.8 + \frac{0}{38.8378}\right)} \right] \right)$$

$$= 19.05964247679739$$

$$= 19.0596$$

Appendix 3.1 (Original Photo of Egg)

Appendix 3.2 (Full Scaled Diagram of Egg)

Refer to Attached Page.

Appendix 3.3 (MATLAB code [line tracing])

```
%# collect image from computer
I = imread(eggInscribe.png');

%# Convert to a binary (black and white) image
BW = im2bw(I, graythresh(I));

%# Find extremities of egg
BW = imfill(BW,'holes');
B = bwboundaries(BW,'noholes');

%# Plot and outline the outsides of the egg
imshow(I), hold on
for i=1:numel(B)
    plot(B{i}(:,2), B{i}(:,1), 'Color','g',
'LineWidth',2)
end
hold off
```

Appendix 3.4 (MATLAB code [point recordation])

```
%# collect image from computer
I = imread(eggInscribe.png');

%// ensure image is vectorized
im = bwmorph(~im2bw(im), 'skel', 'inf');

%// Mark colour so points that have already been visited can be known
%// yellow
im_colour = 255*uint8(cat(3,im,im,im));

%// Retrieve the starting point [from user]
imshow(im);
[col,row] = ginput(1);
close all;

%// Find closest point based on what user selected to start from
[rows,cols] = find(im);
[~,ind] = min((row-rows).^2 + (col-cols).^2);
queue = [rows(ind), cols(ind)];

%// Store all access coordinates
mask = false(size(im));

%// Counts how many coordinates tracked
n = 1;

%// Loops over each point
while ~isempty(queue)

    %// Dequeue
    pt = queue(1,:);
    queue(1,:) = [];

    %// Check for validity of point
    if im(pt(1),pt(2)) == 0
        mask(pt(1),pt(2)) = true;

    %// If visited before, skip
    elseif mask(pt(1),pt(2))
        continue;
    else
        %// If not visited, mark as visited
        mask(pt(1),pt(2)) = true;
```

```
        %// Colour image at that point
        im_colour(pt(1),pt(2),:) = [255;0;0];

        %// Update Counter
        n = n + 1;

        %// Search for neighbouring points that haven't already been found
        %// only go to the ones that are new
        [c,r] = meshgrid(pt(2)-1:pt(2)+1,pt(1)-1:pt(1)+1);
        ind = sub2ind(size(im), r, c);
        locs = im(ind);
        r = r(locs);
        c = c(locs);

        %// Restart path loop counter
        queue = [queue; r(:) c(:)];
    end
end
```

Appendix 3.5 (Received points from egg)

X	Y	X (Cont.)	Y (Cont.)	X (Cont.)	Y (Cont.)	X (Cont.)	Y (Cont.)	X (Cont.)	Y (Cont.)	X (Cont.)	Y (Cont.)
5.747	0	5.0633	-1.4688	3.4128	-2.1199	1.8685	-2.078	0.5659	-1.4045	0.0029	-0.0911
5.7411	-0.2588	5.0308	-1.5023	3.3745	-2.1311	1.8361	-2.0668	0.5364	-1.3766	-0.0029	-0.0576
5.7441	-0.1889	5.0102	-1.5191	3.3067	-2.1423	1.7683	-2.05	0.5099	-1.357	0.0029	0.1045
5.7441	-0.2141	4.9837	-1.5359	3.2566	-2.1451	1.7388	-2.0389	0.4774	-1.3207	0	0
5.7441	-0.2364	4.9572	-1.5554	3.1299	-2.1702	1.6711	-2.0277	0.4362	-1.2844		
5.7441	-0.2895	4.9306	-1.5806	3.0916	-2.173	1.6298	-2.0081	0.4008	-1.2424		
5.7411	-0.3202	4.8982	-1.5973	3.0179	-2.1814	1.5915	-1.9997	0.3566	-1.2089		
5.7293	-0.3957	4.8687	-1.6197	2.9619	-2.1814	1.5502	-1.9858	0.3153	-1.1418		
5.7087	-0.4795	4.8157	-1.6476	2.9089	-2.1926	1.4854	-1.969	0.2711	-1.0887		
5.6704	-0.6612	4.7715	-1.67	2.8264	-2.1926	1.4412	-1.9578	0.2446	-1.0384		
5.6822	-0.5969	4.7096	-1.7119	2.735	-2.1926	1.282	-1.888	0.221	-0.9853		
5.6763	-0.6221	4.6742	-1.7315	2.7291	-2.1981	1.3734	-1.9299	0.2004	-0.949		
5.6497	-0.7199	4.6212	-1.765	2.6878	-2.1926	1.226	-1.8684	0.1945	-0.9155		
5.6409	-0.745	4.5563	-1.7957	2.6584	-2.187	1.1848	-1.8405	0.1739	-0.868		
5.6203	-0.8009	4.4945	-1.8237	2.5699	-2.1842	1.1347	-1.8125	0.165	-0.8288		
5.5849	-0.8764	4.4296	-1.8432	2.5287	-2.1758	1.0816	-1.7818	0.1474	-0.7981		
5.5436	-0.9462	4.353	-1.8712	2.4727	-2.173	0.9844	-1.7287	0.1238	-0.7506		
5.5289	-0.9797	4.2587	-1.9075	2.4196	-2.1702	1.0374	-1.7622	0.1061	-0.6891		
5.4759	-1.0552	4.2086	-1.9299	2.3784	-2.1646	0.9402	-1.7007	0.0973	-0.65		
5.4582	-1.0776	4.1378	-1.955	2.3194	-2.159	0.9107	-1.6812	0.0707	-0.5857		
5.4405	-1.1055	4.0553	-1.983	2.2811	-2.1562	0.8724	-1.6476	0.0472	-0.4767		
5.3845	-1.1754	4.0082	-1.9969	2.2251	-2.1506	0.8311	-1.6253	0.0354	-0.4432		
5.3256	-1.2341	3.9404	-2.0137	2.1691	-2.1423	0.7898	-1.5889	0.0648	-0.5354		
5.3079	-1.2536	3.8726	-2.0333	2.1308	-2.1311	0.7663	-1.5806	0.0413	-0.3957		
5.2872	-1.276	3.7842	-2.0472	2.0954	-2.1311	0.7309	-1.5554	0.0265	-0.337		
5.2607	-1.3039	3.7193	-2.0612	2.0483	-2.1227	0.6955	-1.5331	0.0206	-0.3119		
5.2106	-1.3486	3.6663	-2.0696	1.9894	-2.1115	0.6602	-1.4939	0.0206	-0.2839		
5.1723	-1.3794	3.5897	-2.0808	1.9481	-2.0948	0.6219	-1.4632	0.0147	-0.228		
5.1458	-1.4045	3.5396	-2.0975	1.895	-2.0836	0.5894	-1.4269	0.0118	-0.1917		

Appendix 3.6 (MATLAB Curve fitting code)

```
fid = fopen('retrievedPoints','r');
Z = textscan(fid, '%f %f %f %f %f');
fclose(fid);

X = log(Z{1});
Y = log(Z{2});

p = polyfit(X, Y, 14); %// the 14 depicts polynomial number
Y2 = polyval(p, X);

plot(exp(X), exp(Y));
hold on
plot(exp(X), exp(Y2), 'r')
legend('Original data','Fitted curve')

print('-dpng','generatedCurve.png')
```

Appendix 3.7 (Finding integral of polynomial model)

$$x = 0.00002476312803365381 \times y^{14} - 0.0009178590891438972 \times y^{13} + 0.015046116894752253 \times y^{12} - 0.14345656989993 \times y^{11} + 0.8797832124328367 \times y^{10} - 3.6216063790911295 \times y^{9} + 10.127583595192807 \times y^{8} - 19.03740932937551 \times y^{7} + 23.355759574131245 \times y^{6} - 18.277771148930487 \times y^{5} + 10.902563464899021 \times y^{4} - 9.31249077161999 \times y^{3} + 9.085271066255547 \times y^{2} - 5.760935396945824y + 0$$

$$x^2 = (0.00002476312803365381 \times y^{14} - 0.0009178590891438972 \times y^{13} + 0.015046116894752253 \times y^{12} - 0.14345656989993 \times y^{11} + 0.8797832124328367 \times y^{10} - 3.6216063790911295 \times y^{9} + 10.127583595192807 \times y^{8} - 19.03740932937551 \times y^{7} + 23.355759574131245 \times y^{6} - 18.277771148930487 \times y^{5} + 10.902563464899021 \times y^{4} - 9.31249077161999 \times y^{3} + 9.085271066255547 \times y^{2} - 5.760935396945824y + 0)^2$$

Then using Excel [Refer to Question 5 - Improvements] to expand this out, this turns into:

$$x^2 = 6.13213 \times 10^{-10} \times y^{28} - 4.54581 \times 10^{-8} \times y^{27} + 1.58764 \times 10^{-6} \times y^{26} - 3.47253 \times 10^{-6} \times y^{25} + 0.000533304 \times y^{24} - 0.006111327 \times y^{23} + 0.05420426 \times y^{22} - 0.380937831 \times y^{21} + 2.153970613 \times y^{20} - 9.894851791 \times y^{19} + 37.13519149 \times y^{18} - 114.1253975 \times y^{17} + 287.1457263 \times y^{16} - 590.3628827 \times y^{15} + 990.0271863 \times y^{14} - 1357.621197 \times y^{13} + 1547.338951 \times y^{12} - 1533.464941 \times y^{11} + 1423.674983 \times y^{10} - 1296.158469 \times y^{9} + 1103.022424 \times y^{8} - 804.2790982 \times y^{7} + 495.4220907 \times y^{6} - 294.8309335 \times y^{5} + 189.8394658 \times y^{4} - 104.6793194 \times y^{3} + 33.18837665 \times y^{2} + 0$$

Substitute into definite integral

$$\int_0^{5.748} (6.13213 \times 10^{-10} \times y^{28} - 4.54581 \times 10^{-8} \times y^{27} + 1.58764 \times 10^{-6} \times y^{26} - 3.47253 \times 10^{-6} \times y^{25} + 0.000533304 \times y^{24} - 0.006111327 \times y^{23} + 0.05420426 \times y^{22} - 0.380937831 \times y^{21} + 2.153970613 \times y^{20} - 9.894851791 \times y^{19} + 37.13519149 \times y^{18} - 114.1253975 \times y^{17} + 287.1457263 \times y^{16} - 590.3628827 \times y^{15} + 990.0271863 \times y^{14} - 1357.621197 \times y^{13} + 1547.338951 \times y^{12} - 1533.464941 \times y^{11} + 1423.674983 \times y^{10} - 1296.158469 \times y^{9} + 1103.022424 \times y^{8} - 804.2790982 \times y^{7} + 495.4220907 \times y^{6} - 294.8309335 \times y^{5} + 189.8394658 \times y^{4} - 104.6793194 \times y^{3} + 33.18837665 \times y^{2} + 0)\, dy$$

Calculate indefinite integral

$$\int (6.13213 \times 10^{-10} \times y^{28} - 4.54581 \times 10^{-8} \times y^{27} + 1.58764 \times 10^{-6} \times y^{26} - 3.47253 \times 10^{-6} \times y^{25} + 0.000533304 \times y^{24} - 0.006111327 \times y^{23} + 0.05420426 \times y^{22} - 0.380937831 \times y^{21} + 2.153970613 \times y^{20} - 9.894851791 \times y^{19} + 37.13519149 \times y^{18} - 114.1253975 \times y^{17} + 287.1457263 \times y^{16} - 590.3628827 \times y^{15} + 990.0271863 \times y^{14} - 1357.621197 \times y^{13} + 1547.338951 \times y^{12} - 1533.464941 \times y^{11} + 1423.674983 \times y^{10} - 1296.158469 \times y^{9} + 1103.022424 \times y^{8} - 804.2790982 \times y^{7} + 495.4220907 \times y^{6} - 294.8309335 \times y^{5} + 189.8394658 \times y^{4} - 104.6793194 \times y^{3} + 33.18837665 \times y^{2} + 0)\, dy$$

Separate into each constituent

$$= 6.13213 \times 10^{-10} \int y^{28} dy - 4.54581 \times 10^{-8} \int y^{27} dy + 1.58764 \times 10^{-6} \int y^{26} dy - 3.47253 \times 10^{-6} \int y^{25} dy$$
$$+ 0.000533304 \int y^{24} dy - 0.006111327 \int y^{23} dy + 0.05420426 \int y^{22} dy - 0.380937831 \int y^{21} dy$$
$$+ 2.153970613 \int y^{20} dy - 9.894851791 \int y^{19} dy + 37.13519149 \int y^{18} dy - 114.1253975 \int y^{17} dy$$
$$+ 287.1457263 \int y^{16} dy - 590.3628827 \int y^{15} dy + 990.0271863 \int y^{14} dy - 1357.621197 \int y^{13} dy$$
$$+ 1547.338951 \int y^{12} dy - 1533.464941 \int y^{11} dy + 1423.674983 \int y^{10} dy - 1296.158469 \int y^{9} dy$$
$$+ 1103.022424 \int y^{8} dy - 804.2790982 \int y^{7} dy + 495.4220907 \int y^{6} dy - 294.8309335 \int y^{5} dy$$
$$+ 189.8394658 \int y^{4} dy - 104.6793194 \int y^{3} dy + 33.18837665 \int y^{2} dy$$

$$= 6.13213 \times 10^{-10} \left[\frac{y^{29}}{29}\right] - 4.54581 \times 10^{-8} \left[\frac{y^{28}}{28}\right] + 1.58764 \times 10^{-6} \left[\frac{y^{27}}{27}\right] - 3.47253 \times 10^{-6} \left[\frac{y^{26}}{26}\right] + 0.000533304 \left[\frac{y^{25}}{25}\right]$$
$$- 0.006111327 \left[\frac{y^{24}}{24}\right] + 0.05420426 \left[\frac{y^{23}}{23}\right] - 0.380937831 \left[\frac{y^{22}}{22}\right] + 2.153970613 \left[\frac{y^{21}}{21}\right] - 9.894851791 \left[\frac{y^{20}}{20}\right]$$
$$+ 37.13519149 \left[\frac{y^{19}}{19}\right] - 114.1253975 \left[\frac{y^{18}}{18}\right] + 287.1457263 \left[\frac{y^{17}}{17}\right] - 590.3628827 \left[\frac{y^{16}}{16}\right] + 990.0271863 \left[\frac{y^{15}}{15}\right]$$
$$- 1357.621197 \left[\frac{y^{14}}{14}\right] + 1547.338951 \left[\frac{y^{13}}{13}\right] - 1533.464941 \left[\frac{y^{12}}{12}\right] + 1423.674983 \left[\frac{y^{11}}{11}\right] - 1296.158469 \left[\frac{y^{10}}{10}\right]$$
$$+ 1103.022424 \left[\frac{y^{9}}{9}\right] - 804.2790982 \left[\frac{y^{8}}{8}\right] + 495.4220907 \left[\frac{y^{7}}{7}\right] - 294.8309335 \left[\frac{y^{6}}{6}\right] + 189.8394658 \left[\frac{y^{5}}{5}\right]$$
$$- 104.6793194 \left[\frac{y^{4}}{4}\right] + 33.18837665 \left[\frac{y^{3}}{3}\right]$$

$$\int_0^{5.748} \Bigg(6.13213 \times 10^{-10} \left[\frac{5.748^{29}}{29}\right] - 4.54581 \times 10^{-8} \left[\frac{5.748^{28}}{28}\right] + 1.58764 \times 10^{-6} \left[\frac{5.748^{27}}{27}\right] - 3.47253 \times 10^{-6} \left[\frac{5.748^{26}}{26}\right]$$
$$+ 0.000533304 \left[\frac{5.748^{25}}{25}\right] - 0.006111327 \left[\frac{5.748^{24}}{24}\right] + 0.05420426 \left[\frac{5.748^{23}}{23}\right] - 0.380937831 \left[\frac{5.748^{22}}{22}\right]$$
$$+ 2.153970613 \left[\frac{5.748^{21}}{21}\right] - 9.894851791 \left[\frac{5.748^{20}}{20}\right] + 37.13519149 \left[\frac{5.748^{19}}{19}\right] - 114.1253975 \left[\frac{5.748^{18}}{18}\right]$$
$$+ 287.1457263 \left[\frac{5.748^{17}}{17}\right] - 590.3628827 \left[\frac{5.748^{16}}{16}\right] + 990.0271863 \left[\frac{5.748^{15}}{15}\right] - 1357.621197 \left[\frac{5.748^{14}}{14}\right]$$
$$+ 1547.338951 \left[\frac{5.748^{13}}{13}\right] - 1533.464941 \left[\frac{5.748^{12}}{12}\right] + 1423.674983 \left[\frac{5.748^{11}}{11}\right] - 1296.158469 \left[\frac{5.748^{10}}{10}\right]$$
$$+ 1103.022424 \left[\frac{5.748^{9}}{9}\right] - 804.2790982 \left[\frac{5.748^{8}}{8}\right] + 495.4220907 \left[\frac{5.748^{7}}{7}\right] - 294.8309335 \left[\frac{5.748^{6}}{6}\right]$$
$$+ 189.8394658 \left[\frac{5.748^{5}}{5}\right] - 104.6793194 \left[\frac{5.748^{4}}{4}\right] + 33.18837665 \left[\frac{5.748^{3}}{3}\right] \Bigg)$$
$$- \Bigg(6.13213 \times 10^{-10} \left[\frac{0^{29}}{29}\right] - 4.54581 \times 10^{-8} \left[\frac{0^{28}}{28}\right] + 1.58764 \times 10^{-6} \left[\frac{0^{27}}{27}\right] - 3.47253 \times 10^{-6} \left[\frac{0^{26}}{26}\right]$$
$$+ 0.000533304 \left[\frac{0^{25}}{25}\right] - 0.006111327 \left[\frac{0^{24}}{24}\right] + 0.05420426 \left[\frac{0^{23}}{23}\right] - 0.380937831 \left[\frac{0^{22}}{22}\right] + 2.153970613 \left[\frac{0^{21}}{21}\right]$$
$$- 9.894851791 \left[\frac{0^{20}}{20}\right] + 37.13519149 \left[\frac{0^{19}}{19}\right] - 114.1253975 \left[\frac{0^{18}}{18}\right] + 287.1457263 \left[\frac{0^{17}}{17}\right] - 590.3628827 \left[\frac{0^{16}}{16}\right]$$
$$+ 990.0271863 \left[\frac{0^{15}}{15}\right] - 1357.621197 \left[\frac{0^{14}}{14}\right] + 1547.338951 \left[\frac{0^{13}}{13}\right] - 1533.464941 \left[\frac{0^{12}}{12}\right] + 1423.674983 \left[\frac{0^{11}}{11}\right]$$
$$- 1296.158469 \left[\frac{0^{10}}{10}\right] + 1103.022424 \left[\frac{0^{9}}{9}\right] - 804.2790982 \left[\frac{0^{8}}{8}\right] + 495.4220907 \left[\frac{0^{7}}{7}\right] - 294.8309335 \left[\frac{0^{6}}{6}\right]$$
$$+ 189.8394658 \left[\frac{0^{5}}{5}\right] - 104.6793194 \left[\frac{0^{4}}{4}\right] + 33.18837665 \left[\frac{0^{3}}{3}\right] \Bigg)$$

$$= 18.80786171$$
$$= 18.8079$$