EXPERIMENT #4 **FIRST AND SECOND ORDER CIRCUITS** **ECE212H1F**

OBJECTIVES:

- To study the voltage-current relationship for a capacitor.
- To study the step responses of a series RC circuit.
- To estimate the resistance and inductance of a coil, using time domain measurements.
- To study the step response of a series RLC circuit.

GENERAL COMMENTS:

- For the use of the oscilloscope refer to instructions for experiment #3, the lab guidelines and the *Laboratory Equipment Instruction Manual*.

- Instead of a step function input for first and second order circuits, a square wave will be used. The period of the square wave has to be set to a value that is large compared to the time constant of the circuit. This is taken into account by the choice of frequencies given in the instructions.

- Since currents cannot be measured directly, in sections 4.1 and 4.2 the voltage across the 100Ω resistor is measured. Make sure it is connected as shown in Fig. 4 and Fig. 5, i.e. that one of its terminals is connected to ground.

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EXPERIMENT

REQUIRED READING:

Alexander and Sadiku, *Fundamentals of Electric Circuits*, 6th ed., Chapters 6-8.

INTRODUCTION:

(A) CIRCUIT ELEMENTS

The basic elements of electric circuits are the linear time-invariant resistance, inductance, and capacitance, for which the following temporal relations hold:

$$v_R = Ri_R, \qquad v_L = L\frac{di_L}{dt}, \qquad i_C = C\frac{dv_C}{dt}.$$

For example, if an ideal capacitor were connected to a triangular voltage source, the current would be a square wave. In the case of an inductor, if the current is forced to be a triangular wave, an ideal inductance develops a square wave voltage. A real inductor, however, has a triangular wave superimposed on the square wave due to its resistance

(B) FIRST ORDER CIRCUITS

For a series RC circuit, the capacitor voltage response to a step input of V_S volts, assuming no initial charge, is:

$$v_C(t) = V_S\left(1 - e^{-t/RC}\right)$$

This is shown in Fig. 1. Two parameters define this response: the final value V_s and the time-constant (τ=RC). Another parameter shown in Fig. 1 is the 10%-90% rise time T_r (2.20τ). This is a useful alternative parameter, from which the time constant can be derived.

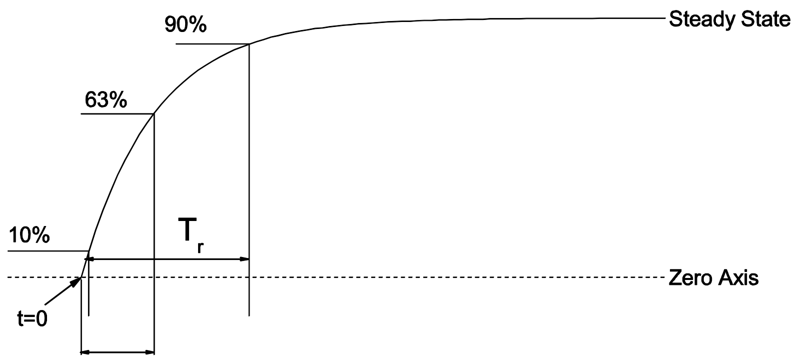

Fig. 1

(C) SECOND ORDER CIRCUITS

The techniques that are used to study second order circuits are very similar to the ones used to study first order circuits. However, the form of the responses of a second order circuit is more complicated than that of first order circuits, therefore more parameters are required to define them.

Time-domain response of a second order circuit consists of two parts – ***natural response*** and ***forced response***. The forced response for a step function input is the step function itself, while the natural response depends only on the circuit elements and decays for time $t \rightarrow \infty$.

The natural response of a second-order circuit, like the series RLC circuit in this experiment, can be ***underdamped***, ***overdamped***, or ***critically damped***. The type of natural response is described by the damping factor ζ, which for the series RLC circuit is:

$$\varsigma = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{C/L}$$

When $\zeta < 1$, the circuit is underdamped, when $\zeta > 1$ the circuit is overdamped, and when $\zeta = 1$ it is critically damped. In case of an underdamped circuit the natural response is a damped sine wave, in case of an overdamped circuit it is of the form:

$$v_C^{natural}(t) = K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t},$$

and in case of a critically damped circuit it is:

$$v_C^{natural}(t) = K_1 e^{-\alpha t} + K_2 t e^{-\alpha t}$$

In an underdamped series RLC circuit, the complete response can be described by the following expression:

$$v_C(t) = V_S - \frac{V_S}{\sqrt{1-\varsigma^2}} e^{-\alpha t} \sin(\omega_d t + \phi)$$

where:

$$\alpha = \varsigma\omega_0,\ \omega_d = \omega_0\sqrt{1-\varsigma^2},\ \omega_0 = \frac{1}{\sqrt{LC}},\ \text{and}\ \sin(\phi) = \sqrt{1-\varsigma^2}.$$

The constant ω_0 is the natural, undamped angular frequency of the circuit.

Fig. 2 shows the step response of an underdamped circuit. Some of the parameters that can be measured to define this type of response are:

- final value V_S
- 10%-90% rise time T_r
- period of the oscillation T_d
- time constant ($1/\alpha$) of the exponential decay envelope
- % overshoot: $\frac{V_p - V_S}{V_S} \times 100$
- settling time T_S, the time required for the response to converge toward the final value within some specified percentage, for example 5% as in Fig. 2.

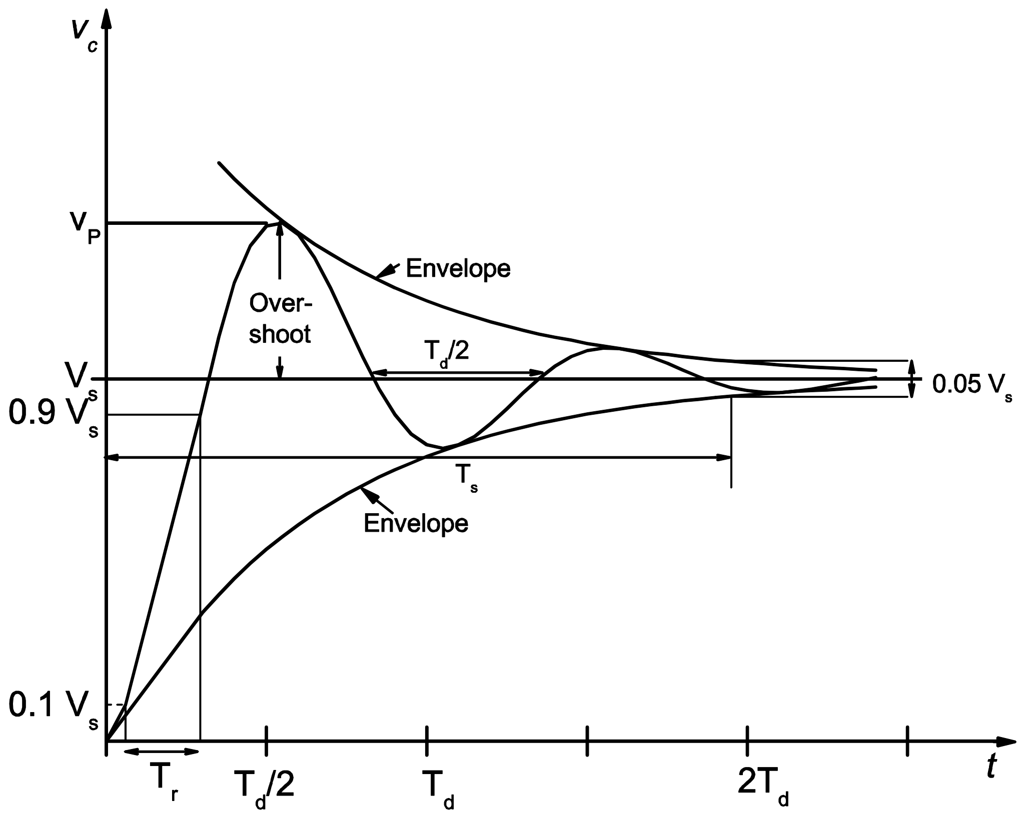

Fig. 2

Fig. 3 shows normalized step response curves of a linear second order system for different damping factor values illustrating the transition from an underdamped, through a critically damped, to an overdamped response for increasing values of ζ.

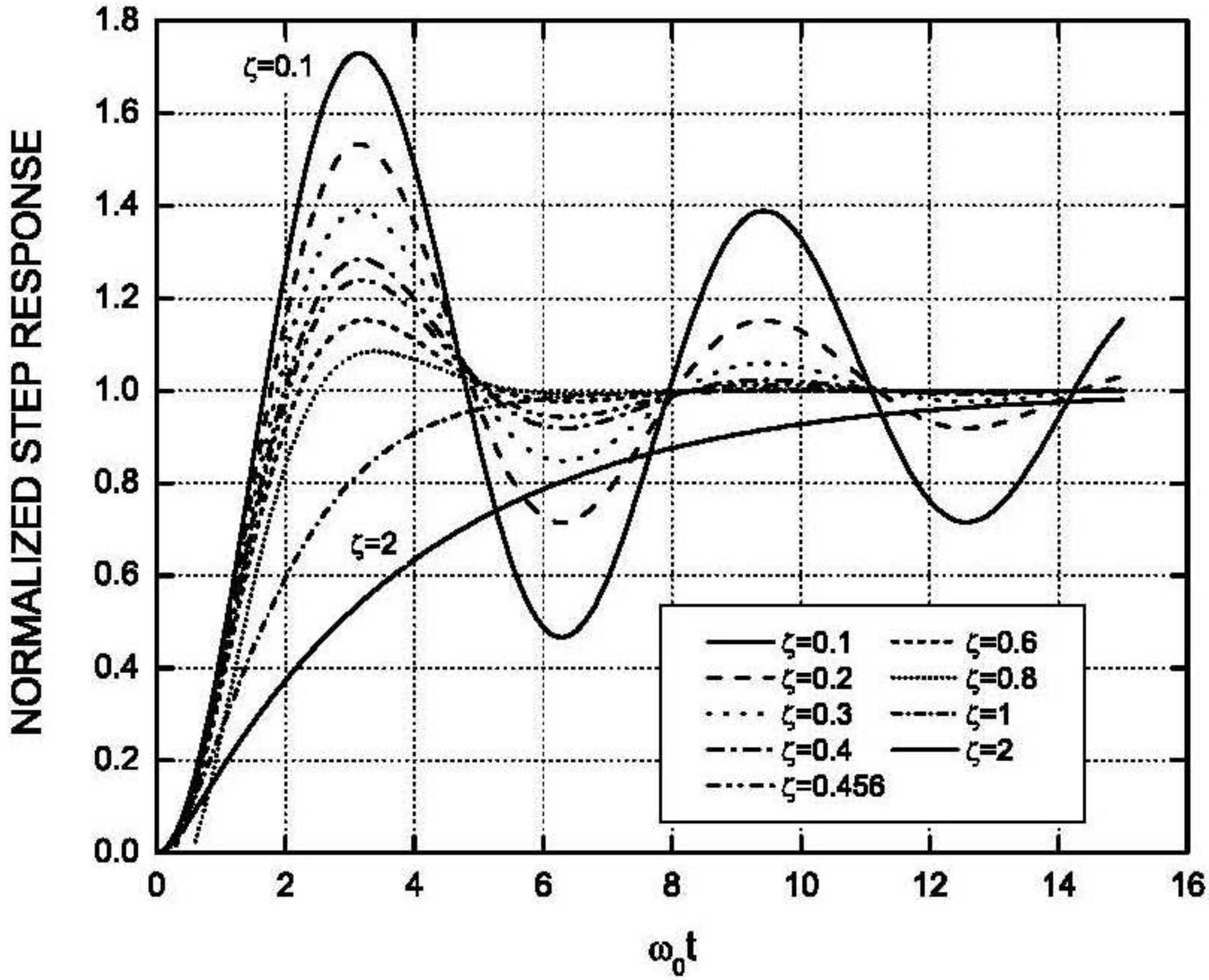

Fig. 3

PRE-LAB ASSIGNMENT (Preparation)

Except for the graph in Fig. C-2 (which can be cut and pasted from the instruction sheet), all graphs must be drawn directly into your lab book.

(A) Voltage-Current Characteristic of a Capacitor

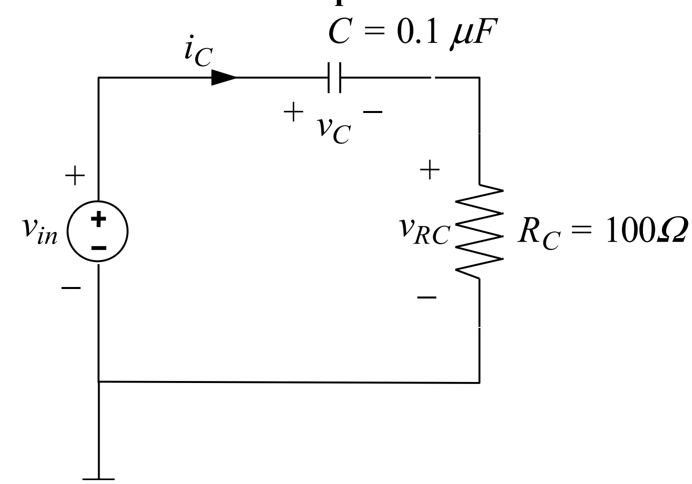

Fig. A-1 (Identical to Fig. 4)

- The voltage across a 0.1 μF capacitor is a triangular wave of 6 V peak-to-peak at a frequency of 100Hz. Derive and sketch an expression for the capacitor current $i_C(t)$. Ignore the effect of R_c on the circuit and consider the capacitor as ideal.
- Repeat the previous step for a 6 V peak-to-peak sine wave at 100 Hz.

(B) Step-Response of a RC circuit

Consider the circuit in Fig. B-1 and disregard the current monitoring resistor R_C ($R_C << R$) in all your calculations. The signal $v_{in}(t)$ is a 6V positive step function and initially no charge is stored in the capacitor.

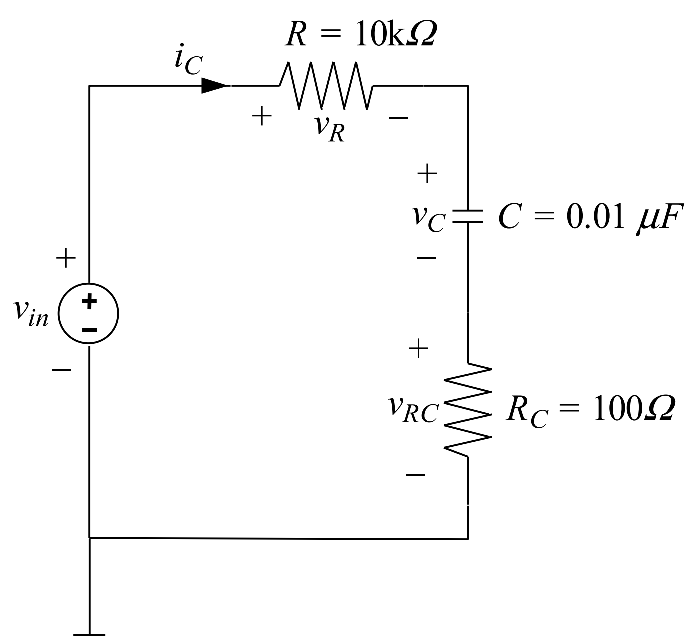

Fig. B-1 (identical to Fig. 5)

- Find the initial and final conditions and determine the time constant τ.
- Derive an expression for the capacitor voltage $v_C(t)$ and current $i_C(t)$.
- Determine the 10%-90% rise time T_r.

(C) Parameter Estimation

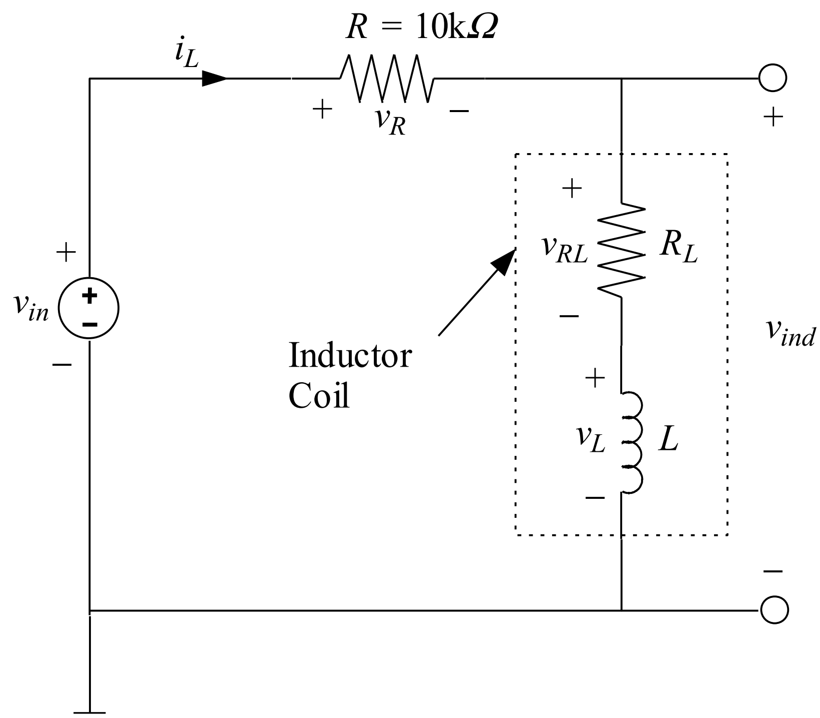

Fig. C-1 (identical to Fig. 6)

- In the circuit in Fig. C-1, a "real" (i.e., non-ideal) inductor coil is modeled by the series combination of an "ideal" inductor L in series with a small resistance R_L. Assume $R_L = 25\ \Omega$ and L = 25 mH. The input signal is a 6 V peak-to-peak triangular wave at a frequency of 500 Hz.
- The inductor current, i_L, can be approximated by assuming that the voltage across the inductor coil, v_{ind}, is small so that the input signal, v_{in}, appears almost entirely across the resistor R, i.e. $i_L \approx v_{in}/R$. Use this approximation to find the voltage across the "real" inductor, v_{ind}, and sketch it for a time of two periods.
- Fig. C-2 shows the input voltage $v_{in}(t)$ and the voltage $v_{ind}(t)$ across a different "real" inductor. Use the two graphs to estimate its inductance and resistance, L and R_L.

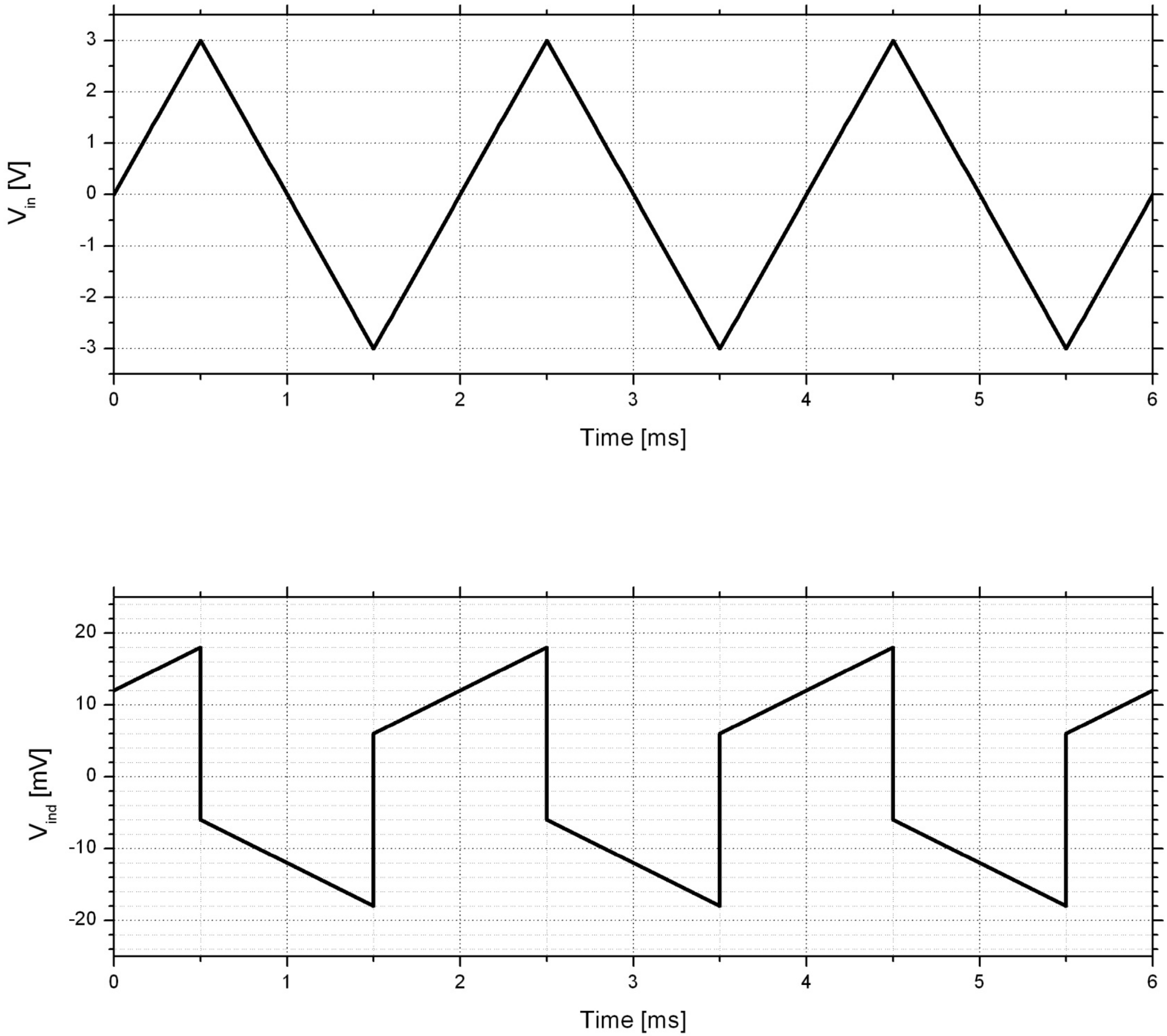

Fig. C-2

(D) Step Response of a Series RLC Circuit – Underdamped Response

For the circuit in Fig. D-1, set the variable resistor R to 1 kΩ and assume that the input voltage is a 6 V positive step function. Initially there is no current in the inductor and the capacitor is uncharged.

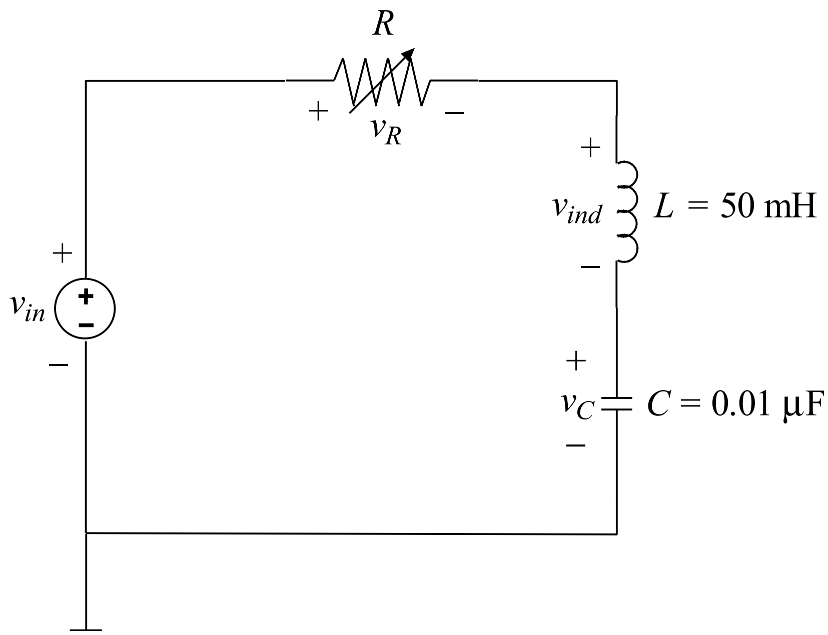

Fig. D-1 (identical to Fig. 7)

- Determine the initial and final values of v_C.
- Find the solution for the capacitor voltage $v_C(t)$ and plot it versus time. Make use of the normalized step response curves shown in Fig. 3.
- Determine the damping factor ζ, the natural angular frequency ω_0, the period of the oscillation T_d, and estimate the percentage overshoot from the normalized step response in Fig. 3.
- Determine the value of the resistor for a critically damped response.
- Determine the value of the resistor for an overdamped response, where $\zeta = 2$.

<u>EQUIPMENT</u>

- GW Function generator Model GFG-813
- TEKTRONIX TDS 210 oscilloscope
- Protoboard
- Components: 100 Ω, 1 kΩ, and 10 kΩ resistors, 0.01 µF and 0.1 µF capacitors, 50 mH inductor
- Decade resistor box
- GenRad RLC Digibridge Model 1657/Instek RLC Meter Model 815

EXPERIMENT

4.1 Voltage-Current Characteristic of a Capacitor

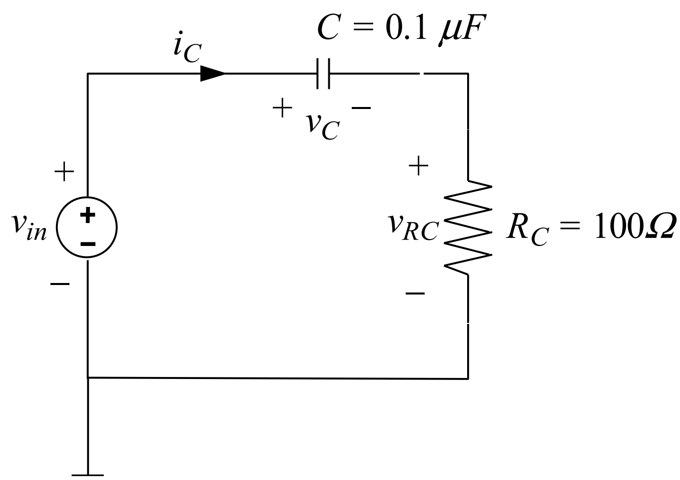

Fig. 4

Preparation:

- Part (A) of your pre-lab assignment

Experiment:

- Connect the output of the function generator (labeled MAIN) to one of the BNC connectors on the protoboard. Set the waveform to a triangular wave form at 100Hz.
- Complete the circuit in Fig. 4. Make sure one of the resistor terminals is connected to ground
- Connect Channel 1 of the oscilloscope to measure the input voltage and adjust it to 6 V peak-to-peak.
- Connect Channel 2 of the oscilloscope to measure v_{RC} across R_C and display both wave forms simultaneously.
- Copy both wave forms into your lab book and label all axes.
- Repeat your measurements for a 6 V peak-to-peak sine wave at the same frequency.
- Compare your results with the preparation.

4.2 Step Response of a Series RC Circuit

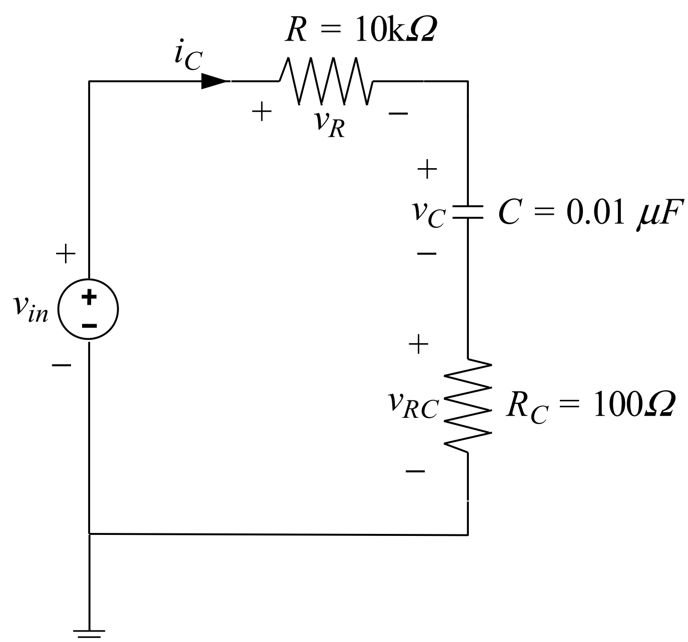

Fig. 5

Preparation:

- Part (B) of your pre-lab assignment

Experiment:

- Convert the RC circuit on your protoboard to that in Fig. 5.
- Set the function generator to a 6 V positive voltage square wave at 500 Hz. To achieve this, first set the function generator to a 6 V peak-to-peak square wave at 500 Hz. Then pull out the OFFSET knob on the function generator and adjust the DC level until the oscilloscope displays a positive square wave from 0 to 6 V.
- Connect the oscilloscope probes to simultaneously display v_{RC} and $v_C + v_{RC}$ ($v_{RC} << v_C$, so $v_C+v_{RC}\approx v_C$).
- Copy both wave forms into your lab book and label all axes.
- Measure the time constant τ and the rise time T_r using the cursors on the oscilloscope. What is the ratio T_r/τ?
- Compare your results to the preparation.

4.3 Parameter Estimation

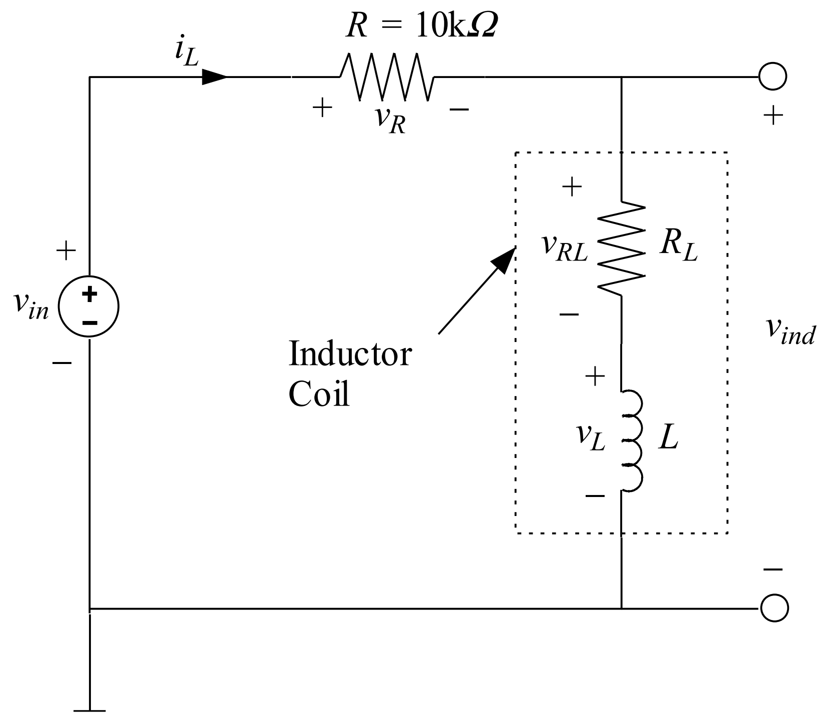

Fig. 6

Preparation:

- Part (C) of your pre-lab assignment.

Experiment:

- Set up the circuit in Fig. 6 and set the function generator to a 6 V peak-to-peak triangular wave at a frequency of 500 Hz. Make sure the DC offset is reset and the OFFSET knob on the function generator is pushed in.
- Connect the oscilloscope probes such as to measure the input voltage and the inductor voltage simultaneously.
- In your lab book, plot the <u>measured</u> waveforms $v_{in}(t)$ and $v_{ind}(t)$ From these plots, determine the actual values of the resistance R_L and the inductance L, using the same method as in part (C) of your pre-lab assignment.

4.4 Step-Response for the Series RLC circuit

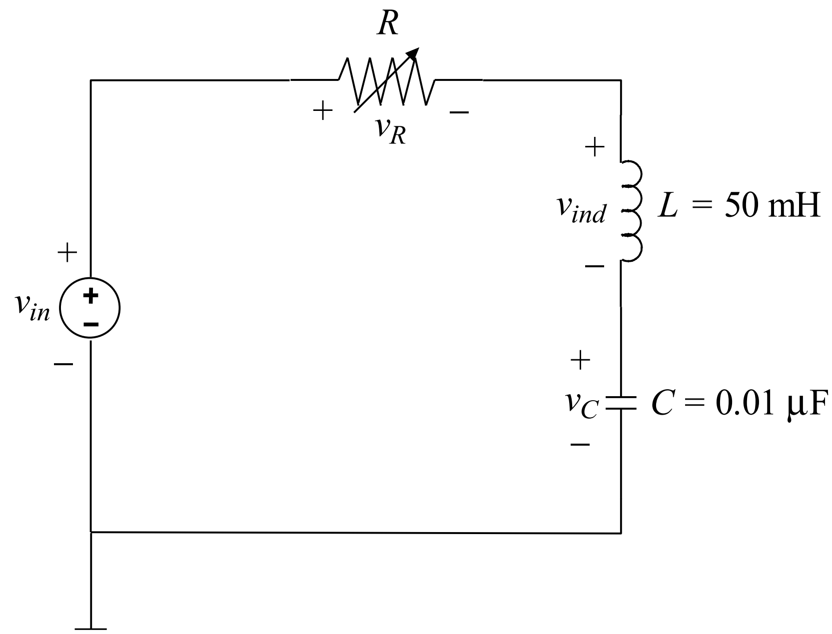

Fig. 7

4.4.1 Underdamped Response

Preparation:

- The first three steps of part (D) of your pre-lab assignment.

Experiment:

- Set up the circuit in Fig. 7, with R = 1kΩ.
- Set the function generator to a 6 V positive square wave of frequency 500 Hz by (see section 3.2). Display the capacitor voltage $v_C(t)$ on the oscilloscope. Use the oscilloscope cursors to measure the initial and final value of v_C, the rise time T_r, the percentage overshoot, the period of the oscillation T_d, and the settling time T_S, based on the 5% of final value criterion.
- Copy the wave form from the oscilloscope into your lab book and label all axes.
- Compare your results to the preparation.

4.4.2 Critically Damped and Overdamped Responses

Preparation:

- The last two steps of part (D) of your pre-lab assignment.

Experiment:

- Continue using the circuit in Fig. 7 and display the capacitor voltage $v_C(t)$ on the oscilloscope.
- Slowly increase the resistance from 1 kΩ while observing the waveform on the oscilloscope until it becomes that of a critically damped response.
- Record this value of R and compare it to the one obtained in the preparation. Is there is a discrepancy, and if so, what could be a reason for it?
- Copy the wave form from the oscilloscope into your lab book and label all axes.
- Further increase the resistance to that with $\zeta=2$ obtained in the preparation and confirm that the waveform on the oscilloscope is overdamped. Copy the wave form into your lab book and label all axes.