I. INTRODUCTION

This report examines the torsion constant of two identical steel wires with different diameters. When a wire, fixed on both ends, experience an external twisting force (Figure 1), the internal system of the wire will exert a restoring torque (τ) opposite in direction. For small external twisting forces, the restoring torque is proportional to the angular displacement ($\tau = -\kappa\theta$). τ is thus, a type of stress and this proportionality constant, known as the torsion constant, κ, is analogous to the Young's Modulus. κ depends on an object's material and dimensions. τ can also be calculated through $\tau = Fr$, where F is the perpendicular force applied and r is the length of the moment arm.

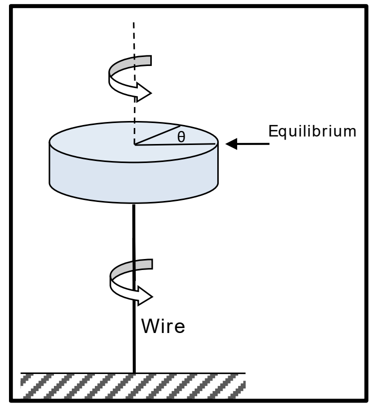

Figure 1: *Angular displacement of a wire from its equilibrium position caused by an external torque. (Pasco 2017)*

$I = \frac{1}{2}M({R_1}^2 + {R_2}^2)$ [1]

$I = \frac{1}{2}MR^2$ [2]

$I = \frac{1}{12}MR^2$ [3]

$I = MR^2$ [4]

Figure 2: *I of ring [1], disk [2], rod [3], and point mass [4], where M is the mass, R is the radius, R_1 is the inner radius, and R_2 is the outer radius.*

When a wire fixed at one end and a mass attached to the other is twisted and released, the system exhibits simple harmonic motion with period $T = 2\pi\sqrt{\frac{I}{\kappa}} = \frac{2\pi}{\omega}$, where ω is the angular frequency. Theoretical equations for rotational inertia, I, which is analogous to mass for translational motion, for a ring, disk, rod, and point mass are listed in Figure 2. When κ and ω are known, I could also be calculated by rearranging the equation: $T = 2\pi\sqrt{\frac{I}{\kappa}} \rightarrow I = \left(\frac{T}{2\pi}\right)^2 \kappa \rightarrow I = \omega^{-2}\kappa$.

The Pasco Scientific Torsional Pendulum Experiment apparatus will apply torque to wires fixed at both ends while collecting force applied and angular displacement data. With the length of the moment arm measured, the Capstone software is used to generate a torque-angular displacement graph for each wire. The objective of this experiment is to find the torsion constants of the two wires along with the rotational inertia of a disk, a hollow cylinder, and a rod with two point-masses at each end.

II. PROCEDURE

To set up the Pasco Torsional Pendulum apparatus (Figure 3), the rotary motion sensor (RMS)[1] and the force sensor[2] are connected to the Universal Interface and a computer running the Capstone software. The tare button on the force sensor is pressed before every test.

To find κ of both steel wires, the disk is removed and the collection frequency of both sensors is set to 50Hz. A string is attached and coiled around the pulley underneath the disk with the other end attached to the force sensor. During testing, the selected steel wire is secured underneath the RMS at its rotational axis. Then the force sensor is suspended parallel against the pulley at the same altitude and is pulled until the pulley has made one revolution. This is repeated for the other wire. The diameters of the two wires measure 0.78 and 1.17±0.01 mm using a caliper. The moment arm, which is the radius of the pulley, pertains a length of 14.36±0.01 mm.

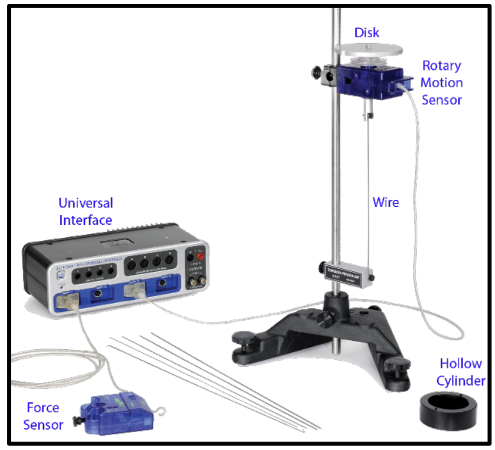

Figure 3: *Pasco Torsional Pendulum apparatus with the rod and two point-masses missing in the image. (Pasco 2017)*

To find I of the disk, it is replaced onto the pulley with the string removed and the collection frequency of both sensors set to 200Hz. Only the thin steel wire is used and is secured underneath the RMS. During testing, the disk is initially turned about 90º from its equilibrium position and released to allow oscillation for 5-7 seconds. The angular displacement is graphed over time and is fitted using a sine function (Figure 4) through the software to obtain ω, which will be used to calculate I. The mass of the disk is measured to be 123.8±0.1 g using a scale and its radius 44.62±0.01 mm using the caliper. It's theoretical I is $I_D = \frac{1}{2}MR^2 = \frac{1}{2}(0.1238\ Kg)(0.04462\ m)^2 =$ **1.232E-4 Kgm^2**. Its uncertainty is neglected assuming the uncertainty from the graph fitting is much larger.

To find I of the hollow cylinder and the rod with two point-masses at each end, a similar procedure from above is followed. Each of the two masses are placed on top of the disk in separate

[1] Uncertainty: ±0.002 rad
[2] Uncertainty: ±0.03 N

tests and ω of the disk-hollow cylinder and the disk-rod systems are obtained through sine fitting. I of the two systems will be calculated and then subtracted by I_D to obtain their individual I values. The masses of the hollow cylinder, rod, and one point-mass is 469.0, 27.6, and 158.2±0.1 g, respectively. The inner and outer radius of the hollow cylinder is 32.56 and 38.20±0.01 mm, respectively. The radius of the rod is 190.50±0.01 mm while the distance between the centre of the rod to the point mass is 155.50±0.01 mm. The theoretical I of the hollow cylinder is $I_{HC} = \frac{1}{2}M\left({R_1}^2 + {R_2}^2\right) = \frac{1}{2}(0.469\ Kg)\left((0.03256\ m)^2 + (0.03820\ m)^2\right) =$ **5.91E-4 Kgm^2**. The theoretical I of the rod with two point-masses is $I_R = \frac{1}{12}M_{rod}{R_{rod}}^2 + 2\left(M_{point\ mass}{R_{point\ mass}}^2\right) = \frac{1}{12}(0.0276\ Kg)(0.19050\ m)^2 + 2\left((0.1582\ Kg)(0.15550\ m)^2\right) =$ **7.73E-3 Kgm^2**.

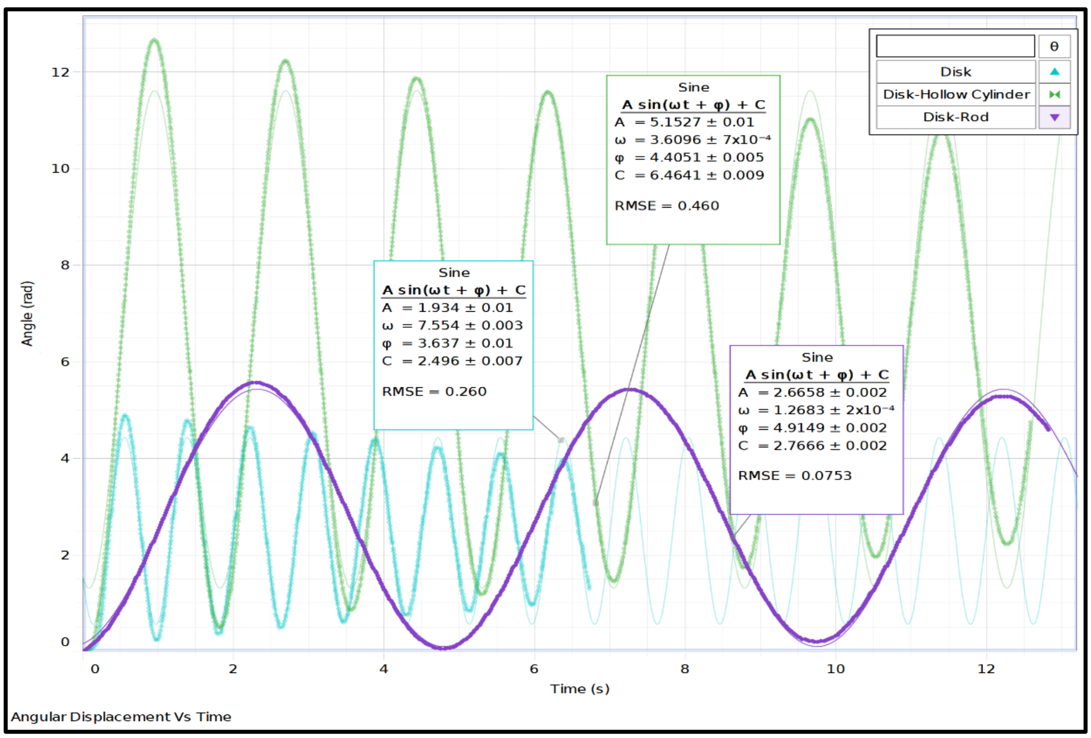

Figure 4: *Angular displacement versus time graph showcasing the curves for the disk (cyan), disk-hollow cylinder system (green), and the disk-rod system (purple). The angular frequency, ω, obtained from the sine fitting, is shown for each curve. ω is used to calculate I.*

III. RESULTS AND DISCUSSION

τ is calculated using the force applied and the length of the moment arm and is graphed over the angular displacement and linear fitted (Figure 5). Thus, uncertainty for τ is irrelevant due to further processing of this value. For example, for a 5 N applied force: $\tau = Fr = (5\ N)(0.01436\ m) = 0.07$

Nm. Since the slope of the linear function is $\frac{\Delta y}{\Delta x}, \frac{\tau}{\theta}$ is κ, which is **9.369±0.008E-3** and **4.403±0.005E-2 Nm** for the thin and thick wires, respectively.

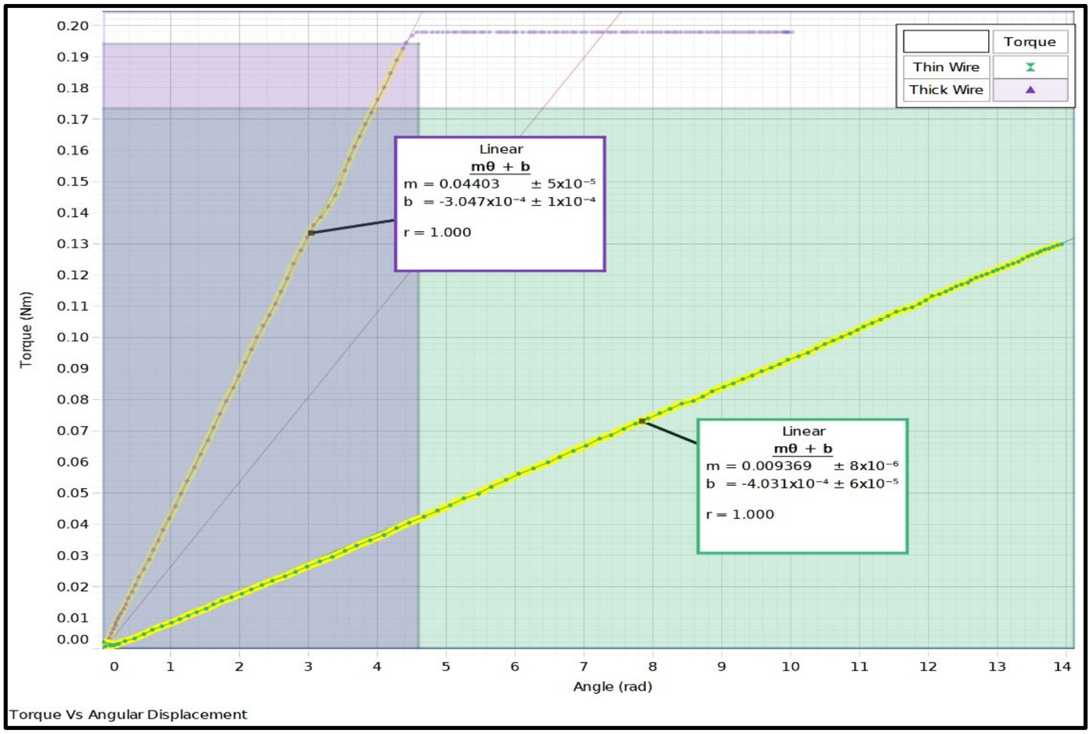

Figure 5: *Torque versus angular displacement graph showcasing the curves for the think wire (green) and the thick wire (purple) with their linear fits. The slope of the curves, m, identifies the torsion constant, τ. The thick wire is seen to have a larger τ than the thin wire.*

From Figure 4, the value of ω for the disk, disk-hollow cylinder, and the disk-rod system is 7.554±0.003, 3.6096±0.0007, 1.2683±0.0002 s^{-1}, respectively. By using $I = w^{-2}\kappa$, the I value for the three systems listed above can be calculated, which corresponds to 1.642E-4, 7.191E-4, and 5.824E-3 Kgm^2, respectively. The value of κ used is that of the thin wire. A sample calculation for the disk is: $I_D = \omega^{-2}\kappa = (7.554)^{-2}(0.009369) =$ **1.642E-4 Kgm^2**. The I of the hollow cylinder and the rod with two point-masses are obtained by subtracting I_D from the system. Thus, they correspond to 7.191E-4 $-$ 1.642E-4 $=$ **5.549E-4 Kgm^2**, and 5.824E-3 $-$ 1.642E-4 $=$ **5.660E-3 Kgm^2**, respectively.

The uncertainties of this experiment consist of the measurement uncertainties associated with the length (±0.01 mm) and weight (±0.1 g) measurements of the various masses along with the force (±0.03 N) and angle (±0.002 rad) data from the two sensors. Statistical uncertainties originate from the linear and sine fits which are used to identify κ and ω. The dominant uncertainty is likely the latter type since I is obtained through two function fits where the range of the linear and sine fits is set by inspection each time. This is an example of random error. In consequence, this assumption justifies why the percentage uncertainties are neglected in the above calculations.

IV. CONCLUSION

The experiment has succeeded in examining the torque-angular displacement relationship of the two wires. The objective of the experiment has been met by experimentally determining κ of the thin and thick wires along with I of the disk, hollow cylinder, and the rod with two point-masses at each end, which corresponds to 9.369±0.008E-3, 4.403±0.005E-2 Nm, and 1.642E-4, 5.549E-4, 5.660E-3 Kgm^2, respectively. The theoretical I of the disk, hollow cylinder, and the rod with two point-masses are 1.232E-4, 5.91E-4, and 7.73E-3 Kgm^2, respectively. Although within the same order of magnitude, the experimental I deviates moderately compared to their theoretical. This signifies that the error of the two κ values could be larger than the given uncertainties. These inconsistencies are primarily blamed on the error of the linear and sine fits performed during data processing to find κ and I.

The method to experimentally determine κ involved collecting force and angular displacement data when applying torque to the secured wire. κ is determined from the slope of the linearly fitted torque-angular displacement graph. To obtain I of the various masses, the system is allowed to oscillate caused by an initial angular displacement while collecting angular displacement data. An angle-time graph is obtained and sine fitted to find ω. I is calculated by using $I = \omega^{-2}\kappa$.

While more accurate calipers and sensors can be used to improve the experiment's accuracy, the execution of more trials per experiment exercise can likely produce significantly more accurate κ and more consistent I values. For reference, only one trial was conducted per exercise for this experiment.

The experiment can be extended by allowing the apparatus, with various combinations of masses on top of the pulley, to oscillate from an initial displacement for a period of time while collecting angular displacement data. An angle-time graph can be created and fitted to a damp sine function to determine the damping constant, α, which can be compared with the result of other masses.

References

Pasco. (2017). Torsional Pendulum Experiment - EX-5521A. Retrieved December 03, 2017, from https://www.pasco.com/prodCatalog/EX/EX-5521_torsional-pendulum-experiment/index.cfmPasco.