UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING
First Year Program – Core 8 and TrackOne

FIRST YEAR PROGRAM
ENGINEERING PROBLEM SOLVING LABS

MAT188: Laboratories #11-12
Final Project

FINAL PROJECT
Engineering Problem Solving Using MATLAB

In this series of labs you have gained a strong introduction to MATLAB and how it can be used to understand situations and solve problems. You have been introduced to many fundamental functions and commands and these have been summarized in the MATLAB Summary.

The purpose of this final project is to review the material presented in the past MATLAB labs and exercises and use your engineering problem solving skills to better understand a real-life scenario. Through this project you will work with on a real-life situation and create a script that will perform simple analysis or computation so that you can draw conclusions from your analysis. Although the suggested examples are chosen to be reasonably straightforward, they demonstrate how engineers face real-world problems and how they use numeric and symbolic computation to help them reach solutions.

For this project, five suggested problems are described below. **Choose ONE that is most interesting to you and work towards finding a solution to the problem for this project.** *You can also come up with your own situation, but each project must have the following components*:

1) ***Clearly Defined Problem Statement***: Each project must have a clear problem statement, which needs to be solved. This doesn't mean that the problem is such that there is only one "right" answer.
2) ***Real-World Scenario (Mathematical Model)***: Each project must be based on a real-world situation, which includes a mathematical model that represents the situation using fundamental scientific principles.
3) ***Design Parameter***: At least one "design" or decision parameter must be included in the problem, such that either the use of *for loops* and/or *conditional statements* (e.g., if-elseif-else-end) are a required part of the analysis.
4) ***Deliverables***: The submission for each project must include:
   a) Script: You will need to create an *m*-file that generates all the plots and carries out your analysis. It should include every calculation that you do you should include comments throughout your script describing what variables and functions you are using and why.
   b) Plots: You will need to create at least one two-dimensional plot for your project. Some projects may require multiple plots or three-dimensional plots as well. Plots must be properly labeled and include legend.
   c) Concluding Statement: You will write a short paragraph that summarizes your analysis and what your suggestion is for a "solution" to the problem.
   d) A Description of your Engineering Problem Solving Process: You will also submit short descriptions of how you have addressed each of the four key aspects of Pólya's Problem Solving Process that we have discussed in this course (see details below).

Preparation

1. Read through this lab document and the five suggested problems below.
2. Choose one of the five suggested problems OR develop your own final project problem.

George Pólya's Problem Solving Process

Understanding the Problem

What is the unknown? (GOAL)
What relevant information is provided? (KNOWNS)
What is the condition? What fundamental principles are related? (FOUNDATION)
Draw a diagram. (VISUALIZATION)

Devising a Plan

Consider the unknown. What are the possible connections between the information provided and the unknown.
Can the problem be restated in a more useful manner?
Do you know a related problem? Can you use its answer or its method?

Carrying Out the Plan

Check each step as you carry out the plan. Is it leading you in the right direction?

Looking Back

Can you check the result? How about the solution?
Is the answer complete? Have you solved the problem asked?

Submission for Lab #11

By the due date of **Wednesday, April 4th, 2018 at 11:59PM** you must submit, your brief descriptions of:

1) *Step 1: Understanding the Problem*: What is your goal? What relevant information have you been given? What are the fundamental scientific and/or mathematical concepts that you will have to use to solve this problem? Include a simple hand-sketched diagram if it would be helpful.
2) *Step 2: Devising a Plan*: **Without** writing or including in your submission ANY MATLAB code, write out your step-by-step process that you plan on using in order to address this problem. This might include:
    a. The plot or plots you will generate to support your assessment and solution. What do the *x* and *y* (or *x*, *y*, and *z*) axes represent?
    b. The "algorithm", or step-by-step process you will use in your MATLAB script. This should be written in plain English, no need to use proper or exact MATLAB syntax. If appropriate, include a graphical interpretation (flowchart) for your problem solving process. Please also include any decisions/assumptions that you have made.

SUBMISSION GUIDELINES

Submit a single PDF or JPG file of maximum 2 pages through Backboard. This can be handwritten as long as it is legible, or it can be a combination of typed text and hand-drawn diagrams.

Submission for Lab #12

By the due date of **Wednesday, April 11th, 2018 at 11:59PM** you must submit:

1) *Step 3: Carrying Out Your Plan*: What did you do? How did you use MATLAB to solve the problem? This should include:
    a. The final version of your MATLAB script,
    b. Any relevant plot or plots that you created to support your assessment, and
    c. A brief description of ways in which you had to adapt your plan from that which you originally envisioned in Step 2 above.
2) *Step 4: Looking Back*: How do you know that you have adequately addressed the problem? How are you sure that you have achieved your goal? Is there anything you have learned about the fundamental scientific and mathematical concepts and/or how to use MATLAB for engineering problem solving? Make sure to include a short *Concluding Statement* that summarizes your analysis and what your suggestion is for a "solution" to the problem.

SUBMISSION GUIDELINES

Please submit a single PDF file of maximum 2 pages of text, PLUS all your MATLAB files (script, plot, etc.) through Blackboard. Your MATLAB script should run without any adjustments by the user (i.e., the TA). Make sure you test and run your code before submission.

Engineering Problem #1: Electric Cars

Background

Electric cars, and plug-in hybrid-electric cars, are becoming more mainstream; the sales of cars like Tesla's Model S, Ford Fusion Energi, Nissan Leaf, Chevrolet Bolt and other 2017 models have generally increased over the past few years, and are projected to grow over the next decade as well.

This case study focuses on the economics of these vehicles as a mode of commuter transportation for those who travel to/from the University of Toronto (return-trip).

Problem Statement

- It is possible to determine the before-tax cost, in CAD, for *the base model* (*"lowest cost model"*) for each of the four vehicles described above. Some of these vehicles are eligible for a government-funded credit, which will need to be included in this price.
- It is possible to use the estimated range of each of these vehicles, as determined by the 2017 Fuel Consumption Guide (NRCAN.gc.ca), in litres/100km.
- Use the fuel cost of $112.9 cents/litre for any fuel calculations, if necessary, and variable electricity cost for a 24-hour period beginning 11/16/2017 at 12:00am and ending 11:59pm. Assume a 9am-6pm school day.
- Ignore any maintenance and/or licensing costs; only consider the principal cost of the vehicle and any fuel costs as appropriate.

The Questions

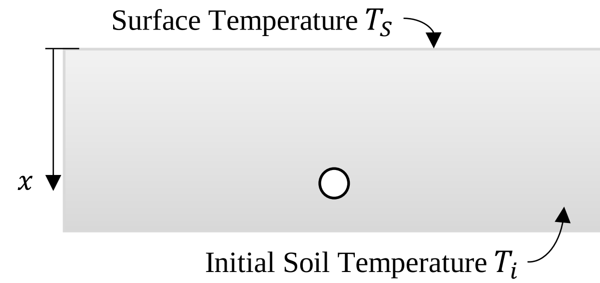
Determine what the most economical vehicle is for commuting to/from the Bahen Centre Garage from *at least* 10 different points within a 50km radius of UofT.

- Which vehicle, from those listed, is the most economical overall? Why?
- Which vehicle, from those listed, is most economical at each 10km radius away from Toronto? What impact, if any, does average traffic have on this?
- How does this compare to existing transit infrastructure (or walking or biking)? What are other concerns to take into consideration?

<u>Engineering Problem #2</u>: Soil Temperature Modeling[1]

Background

In the construction of new urban systems, one of the important considerations is the depth at which water pipes must be buried in order to ensure they do not freeze. It is possible to model this situation by considering how the temperature within the soil at a depth x relates to the surface temperature, T_S, and the initial soil temperature, T_i, over time t (in seconds). It has been found that this can be approximately represented by the equation:



$$\frac{T(x,t) - T_S}{T_i - T_S} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) = \frac{2}{\pi}\int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-u^2}du$$

Here, erf, refers to a famous (and very useful) function in mathematics call the *Gauss Error Function*, or simply the *error function*. It has the integral form shown above. The parameter α represents the thermal diffusivity of the soil.

Problem Statement

Prepare a report that summarizes how the soil temperature within Toronto would change with time and depth over the course of a six-month period (December 1 – May 31). Most of the soil in Toronto can be characterized as clay, and typically the initial soil temperature for all depths at the start of December is $T_i = 14$°C. You have been asked to consider the following specific cases:

a) Demonstrate how the temperature changes with time at a 0.75 m depth over the course of the first 31 days (use increments of 1 day).
b) Show how the temperature changes with time (using 1 day increments) over the six-month period for the range of depths of 0.05 m to 5 m.
c) Determine:
    a. How deep the water lines need to be buried to ensure that the temperature does not drop below 5°C.
    b. How deep the water lines need to be buried in order for the surrounding soil temperature to change by less than 1% of the initial soil temperature (i.e., 14°C).

[1] Adapted from A. Gilat and V. Subramaniam, *Numerical Methods for Engineers and Scientists*, John Wiley & Sons, 2008, P7.27, pg. 306.

Engineering Problem #3: Atmospheric Absorption of Electromagnetic Radiation

Background

The way in which the electromagnetic radiation interacts with the Earth's atmosphere is an important aspect of many engineering and scientific innovations and discoveries. Satellite communications, radio astronomy, remote sensing[3], solar energy, and understanding the process of climate change all depend on our ability to describe how electromagnetic waves, from radio to light, are transmitted and absorbed by various layers of the Earth's atmosphere.

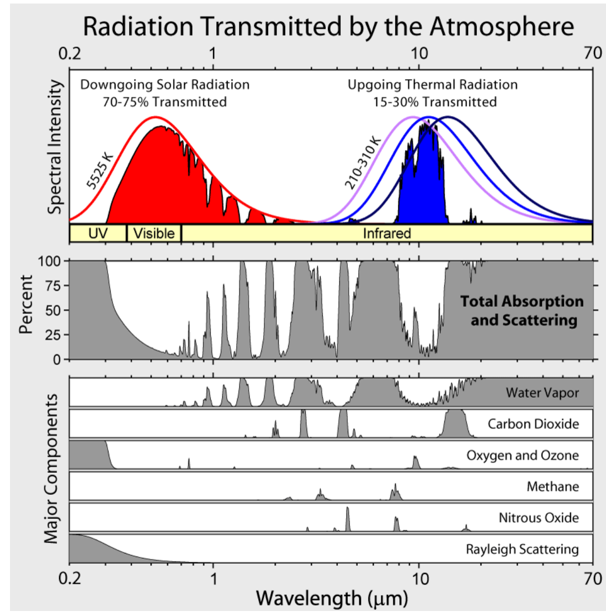


Figure 1: Radiation Transmitted by the Atmosphere[3]

For example, consider how UV, Visible, and Infrared radiation (IR) are absorbed by the various constituents of the Earth's atmosphere (Figure 1). In particular, you can see that carbon dioxide molecules have three regions of absorption in the IR spectrum. They way in which the molecule absorbs the energy contained in the IR photon, is that it transfers this energy into kinetic energy through vibrational motion[4]. Much like a person pushing someone on a swing, if the energy in the IR photon "pushes" the oxygen and carbon atoms according to their natural rhythm then the movement of these atoms can grow and be sustained. However, if the energy is associated with a different frequency of "pushing" then the impulse is out of sync with the vibrational movement of the atoms and thus the energy of the photon is not absorbed or converted into kinetic or another form of energy. This natural rhythm or movement of CO_2 depends on the atomic structure of that molecule.

Specifically, if one looks carefully at the molecular structure of CO_2 it is possible to determine that the absorption at $\lambda = 4.3$ μm is due to the translational movement of the atoms along the axis of the molecule. As a result, this allows us to approximate, or model the CO_2 molecule as a three masses connected by two springs (Figure 2). Through an analysis of the dynamics of this situation using the fundamental laws of physics, one can derive the following relationships between the masses, the spring constant k, the frequency of vibration $\omega = 2\pi f$, and the amplitudes of translation or movement along the x axis for each atom, D_1, D_2, and D_3:

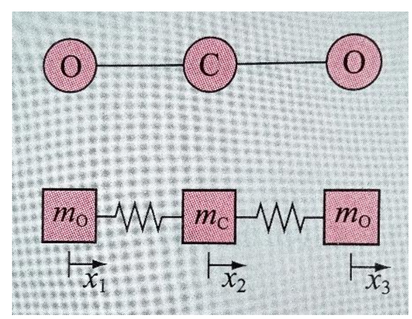


Figure 2: Spring/Mass Model of the CO_2 Molecule[6]

$$-\omega^2 D_1 = -\frac{k}{m_O} D_1 + \frac{k}{m_O} D_2$$

$$-\omega^2 D_2 = \frac{k}{m_C} D_1 - \frac{2k}{m_C} D_2 + \frac{k}{m_C} D_3$$

$$-\omega^2 D_3 = \frac{k}{m_O} D_2 - \frac{k}{m_O} D_3$$

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[2] Downloaded from https://en.wikipedia.org/wiki/Absorption_band, used through Creative Commons license: CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=2623190

[3] https://www.earthobservatory.nasa.gov/Features/RemoteSensing/remote_04.php

[4] Recall that this energy is described by $E = \frac{hc}{\lambda} = hf$, where f is the frequency in Hertz.

[5] Taken from A. Gilat and V. Subramaniam, *Numerical Methods for Engineers and Scientists*, John Wiley & Sons, 2008, pg. 172.

Problem Statement

Complete the spring/mass mathematical model of the CO_2 molecule by using MATLAB to determine the correct spring constant, k, so that the model properly predicts the absorption at $\lambda = 4.3$ μm. Use the model to understand how the atoms in the CO_2 molecules move when it interacts with a photon that has a wavelength of $\lambda = 4.3$ μm. You should consider the problem from the following perspective:

a) Observe that this can be considered to be an eigenvalue problem. Rewrite this system in an eigenvalue format, i.e., $\mathbf{Av} = L\mathbf{v}$ (here we are using L to represent the eigenvalue vector rather than λ, since in this problem λ represents the photon wavelength).
b) What are the eigenvalues and eigenvectors of this system? Use MATLAB to visualize how they depend on the spring constant k? What do they mean and what conclusions can you draw from them? *Hint*: Not all of the eigenvalues may be relevant to the actual situation. For example, some may lead to complex results for the associated wavelength. We suggest you only focus on the two eigenpairs that lead to real-valued results.
c) Knowing that measurements have shown that CO_2 molecules demonstrate an absorption at $\lambda = 4.3$ μm, what would be the correct value of the spring constant, k, for this model.
d) Briefly describe the limitations of this model. Why doesn't it predict the clear absorption around $\lambda = 15$ μm?

Engineering Problem #4: Saturn V Rocket Launch Analysis

Background

This problem involves using the ideal rocket equation to model the speed of the Saturn V rocket throughout its takeoff and burn time. Rockets are propelled by the ejection of fuel from the aft of the rocket, where Newton's third law dictates that a forward force will act upon the rocket because of this ejected mass. For a typical rocket, the fuel may make up approximately 90% of the total mass of the rocket, thus it is important to consider the changing mass of the rocket as it accelerates.

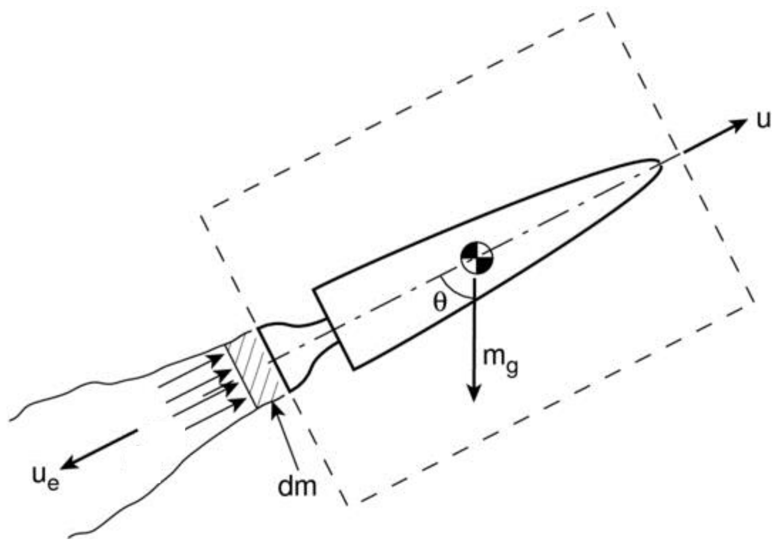

Figure 1: Rocket schematic used for ideal rocket equation derivation[1].

To derive the ideal rocket equation, we note that the momentum of the rocket, $\rho(t)$, will increase due to its decreasing mass, $m(t)$, increasing speed, $u(t)$, and the conservation of the fuel ejected in the opposite direction, an infinitesimal change in a rocket's momentum can therefore be given by:

$$\begin{aligned}\Delta\rho &= \rho_f - \rho_i \\ &= (m_r - dm)(u + du) + dm(u - u_e) - mu \\ &= m_r u + m_r du - udm - dmdu + udm - u_e dm - mu \\ &\approx m_r du - u_e dm.\end{aligned}$$

Where, m_r is the mass of the rocket, dm is some small mass ejected from the rocket, and u_e is the velocity of the ejected mass. We have ignored the very small term, $dmdu$ (i.e., very small squared equals very very very small, relative to the other larger terms). From Newton's second law, we know that the total impulse, $\sum F\,dt$, must equal the change in momentum for our rocket, $\Delta\rho$. Where for simplicity we will consider only the force of gravity acting at an angle of $\theta = 0$, which makes sense for the trajectory of a rocket . We therefore arrive at:

$$\begin{aligned}m_r du - u_e dm &= F_g dt = -m_r g dt \\ du &= -\frac{u_e}{m_r} dm_r - gdt.\end{aligned}$$

Where we have made use of the fact that $dm = -\frac{dm_r}{dt}dt$. Integrating the above result gives us the ideal rocket equation,

$$\begin{aligned} u(t) &= -u_e \ln\left(\frac{m_r}{m_{initial}}\right) - gt \\ &= g\left[\frac{u_e}{g}\ln\left(\frac{m_{initial}}{m_r}\right) - t\right] \\ &= g\left[I_{sp}\ln\left(\frac{m_{initial}}{m_r}\right) - t\right] \end{aligned}$$

Where we have introduced the specific impulse I_{sp}, which is a common measure of the efficiency of a rocket engine. The last equation gives the speed of the rocket in meters per second (m/s), it is the one we will use throughout this analysis.

Problem Statement

You are asked to address three key issues with this situation by using the detailed information below:

a) Use appropriate plots to visualize how the position, velocity and acceleration of the rocket change with time.
b) Estimate the final velocity of the Saturn V rocket once all its fuel is used.
c) Determine if the rocket achieves the exit velocity required to leave the Earth's orbit.

The Saturn V rocket consists of three stages. It is reasonable to assume that each stage fires continuously, each stage lasts as long as its burn time, and the time taken between stages is negligible. It is also known that the rocket doesn't start leaving the launch tower until 6.5 s into the Stage 1 burn time, i.e., $h(t = 6.5\text{ s}) = 0$. Some relevant parameters for each stage of the Saturn V rocket are summarized in Table 1 below.

Table 1. Selected Saturn V rocket parameters[2]

Component	Specific Impulse [s]	Gross Mass [kg]	Remaining Mass [kg]	Burn Time [s]
1	263	2,290,000	130,000	165
2	421	496,200	40,100	360
3	421	123,000	13,500	500

Since fuel is consumed as the rocket fires, the mass of the rocket will decrease. Thus, we must describe the variable m_r as a function of time. It is reasonable to assume that the fuel is consumed linearly, meaning

$$m_r(t) = m_{initial} - (m_{initial} - m_{final})\frac{t}{t_{burn}}$$

Engineering Problem #5: Robot Movement and Control

Background

In this project, we will be considering the movement (and control) of a robotic vehicle moving around a 2D space. For instance, you can think of an autonomous vacuum cleaner moving around a room.

Here is the crux of the problem: in order to understand its working, and ultimately optimize its performance, we would like to know a path that the robotic vehicle took during a certain interval of time. However, any automated vehicle will only save its position (or share it with the central computer) at regular time intervals. Of course, those intervals may be short, but we still only have access to finitely many data points. From those points we want to reconstruct, as well as possible, the entire motion of the vehicle[6].

Problem Statement

Consider the data saved in the posted *robpos.mat* data file. This file contains a matrix with 61 rows and 3 columns, with positions of the robot measured every 1 second during an interval of 1 minute. In each row, the first column represents the timestamp of the measurements (in seconds), the second column represents the robot's x-coordinate in meters at the corresponding time, and the third column represents its y-coordinate in meters.

You are asked to:

a) Use this raw data to visualize a *piecewise linear approximation* of the position of this robot as it moves through this 2D space, i.e., plot $y(t)$ versus $x(t)$. Determine the values of the robot's velocity and acceleration from this raw data by considering this approach:

If the vehicle's position at time t is given by $[x(t), y(t)]$, an approximation of its speed at time t seconds, $t = 0,1, \dots, 59$, is

$$v(t) = \sqrt{v_x^2 + v_y^2} = \sqrt{[x(t+1) - x(t)]^2 + [y(t+1) - y(t)]^2},$$

and an approximation of its acceleration at time t, $t = 1,2, \dots, 59$, is

$$a(t) = v(t) - v(t-1).$$

This is because the time increment is one second, i.e., $\Delta t = 1\text{s}$.

b) Use *polynomial curve fitting* techniques to create a smoother picture of how the robot has moved through this 2D space and how its velocity and acceleration behave. Explain how this approximate model compares to the original data. Remember that if you know how the velocity and acceleration behave with respect to the x and y directions, then the total speed and acceleration is given as the square root of the sum of the squares, i.e., $a(t) = \sqrt{a_x^2 + a_y^2}$.

c) *Path control*: You have now been asked to be part of a team that has to create a control system that keeps the robot on a circular path. This path starts and ends at $(x, y) = (0,0)$, has a diameter of 2.4 m, and takes 1 minute to complete one revolution. In order for your team to know which signals to send to the motors that are controlling the movement in the x and y directions, you have been asked to determine the velocity and acceleration values in each direction that would be required for this kind of movement over one revolution.

[6] *As an interesting (and, perhaps, motivational) side note: problem of reconstruction of an object trajectory is a very "real" and difficult problem in engineering and applied mathematics. This story was not invented purely for educational purposes of learning Matlab. In fact, the author of this project, Melkior Ornik a former ECE PhD is currently working on research in a particular setting of this problem, and the method outlined near the end of this project is an actual strategy for reconstruction proposed by his colleagues in at least one publication.*