EXPERIMENT #5 **SIGNAL FILTERING** **ECE212H1F**

OBJECTIVES:

- To study practical applications of first-order RC filters.
- To assess performance of a filter by measuring the total harmonic distortion carried by a signal after filtering.

GENERAL COMMENT:

- Whenever assessing performance of a more complex circuit, it is good practice to test each subcircuit independently - unattached to the rest of the circuit.
- Use the **GW Function Generator Model GFG-8016G** for the 200 Hz signal in sections 4.1 and 4.4 only. Other than that, use the **GW Function Generator Model GFG-813**, which was used in Experiments #2 and #3.

TABLE OF CONTENTS:

REQUIRED READING:

Alexander and Sadiku, *Fundamentals of Electric Circuits*, 6th ed., **Chapter 5 Operational Amplifiers and Chapter 14 Frequency Response.**

INTRODUCTION:

(A) SIGNAL AND NOISE

Quite often an information-carrying signal becomes contaminated by an unwanted interference signal referred to as a "noise" signal.

In practice we may encounter a situation in which a low-frequency signal is contaminated by a high-frequency noise (induced by an electromagnetic interference), as illustrated in Fig. 1.

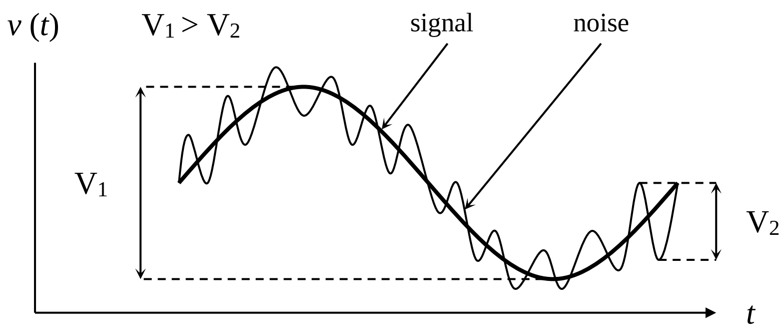


Fig. 1

Or a high-frequency signal might be contaminated by a low-frequency noise often caused by incorrect wiring, as illustrated in Fig. 2.

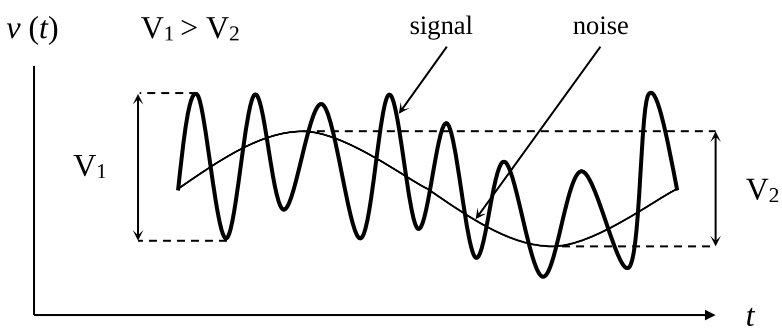


Fig. 2

In both cases described, an appropriate filter would be needed that would pass an information-carrying signal and reject, or minimize, a noise signal.

In this experiment we will study the filtering properties of a first-order low-pass filter (LPF) and a first-order high-pass filter (HPF) using a system shown in Fig. 3, where $v_1(t)$ and $v_2(t)$ are two sinusoidal signals with frequencies of 200 Hz and 22.4 kHz, respectively.

These two signals are summed up by the summing network to the signal $v_{sum}(t)$, which is fed parallel into a low-pass filter and a high-pass filter to separate them again. The low-pass filter is designed such as to pass the 200 Hz signal and reject the 22.4 kHz signal, while the high-pass filter is designed to pass the 22.4 kHz signal and to reject the 200 Hz signal.

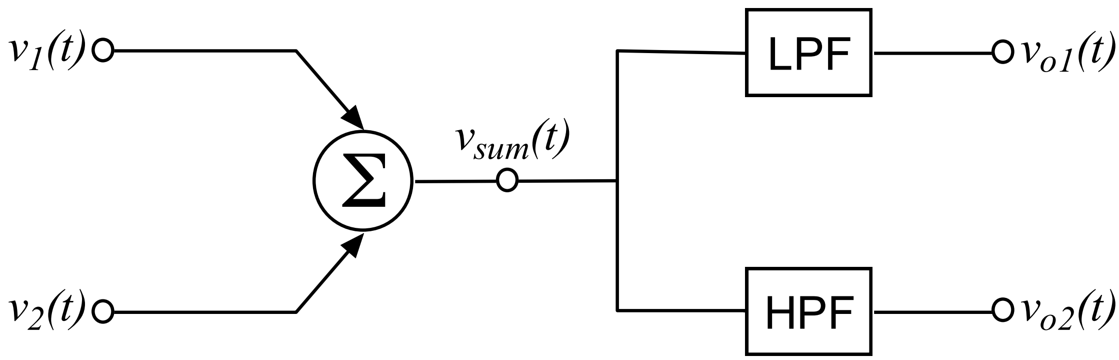


Fig. 3

(B) FIRST ORDER FILTERS

The high-pass and low-pass filters used in this experiment consist of RC circuits, i.e. first-order circuits, similar to those investigated in Experiment #4, section 4.2, which are fed by a sinusoidal input signal.

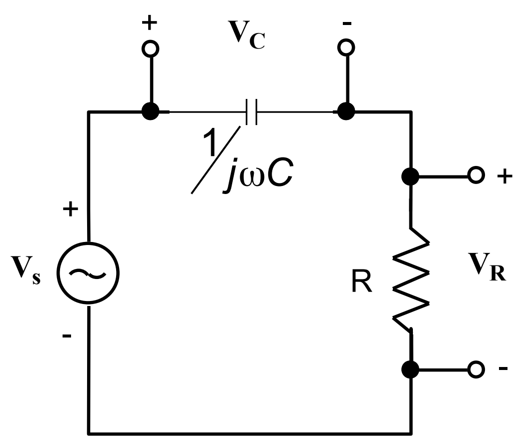


Fig. 4.

The phasor-domain circuit in Fig. 4 is in sinusoidal steady state at a frequency f with a radian frequency $\omega=2\pi f$. The phasor voltages across the resistor and the capacitor, $\mathbf{V_R}$ and $\mathbf{V_C}$, can be derived by voltage division:

$$\mathbf{V_R} = \frac{R}{R + \frac{1}{j\omega C}} \mathbf{V_S}$$
$$\mathbf{V_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \mathbf{V_S} \qquad (5.1)$$

In the context of high- and low-pass filters two special cases are of interest: in the first one the magnitude of the reactance of the equivalent impedance of the RC circuit is small compared to the value of the resistance R, i.e. $1/\omega C = R$, in the second one the value of the resistance R is small compared to the magnitude of the reactance, i.e. $R = 1/\omega C$. The former occurs at high frequencies, the latter at low frequencies. The ranges of frequencies that can be considered "high" and "low" depend on the circuit parameters.

In the case of low frequencies, the reactance is large compared to the resistance and the voltage drop across the capacitor is therefore much larger than that across the resistor. Resistor and capacitor voltages can be approximated by:

$$
\begin{aligned}
\mathbf{V_R} &= \frac{R}{R + 1/j\omega C}\mathbf{V_S} \xrightarrow[R = 1/\omega C]{} \mathbf{V_R} \approx \frac{R}{1/j\omega C}\mathbf{V_S} = j\omega CR\mathbf{V_S} \\
\mathbf{V_C} &= \frac{1/j\omega C}{R + 1/j\omega C}\mathbf{V_S} \xrightarrow[R = 1/\omega C]{} \mathbf{V_C} \approx \frac{1/j\omega C}{1/j\omega C}\mathbf{V_S} = \mathbf{V_S}
\end{aligned}
\tag{5.2}
$$

The capacitor voltage is found to be approximately equal to the source voltage while the voltage across the resistor is small and decreases with decreasing ω, i.e. the source voltage would be measured across the capacitor and virtually no signal would be detected across the resistor. This is consistent with the notion that a capacitor acts as a DC open circuit. The factor of j indicates a 90º phase shift for the resistor voltage compared to the source voltage.

In the case of high frequencies the reactance is small compared to the resistance and the voltage drop across the resistor is therefore much larger than that across the capacitor. Resistor and capacitor voltage can be approximated:

$$
\begin{aligned}
\mathbf{V_R} &= \frac{R}{R + 1/j\omega C}\mathbf{V_S} \xrightarrow[R?\ 1/\omega C]{} \mathbf{V_R} \approx \frac{R}{R}\mathbf{V_S} = \mathbf{V_S} \\
\mathbf{V_C} &= \frac{1/j\omega C}{R + 1/j\omega C}\mathbf{V_S} \xrightarrow[R?\ 1/\omega C]{} \mathbf{V_C} \approx \frac{1/j\omega C}{R}\mathbf{V_S} = \frac{1}{j\omega CR}\mathbf{V_S}
\end{aligned}
\tag{5.3}
$$

Here the resistor voltage is approximately equal to the source voltage while the capacitor voltage decreases with increasing ω, i.e. the source voltage would be measured across the resistor with virtually no signal detected across the capacitor. The $1/j$ factor indicates a phase shift of -90º for the capacitor voltage relative to the source voltage.

According to the expressions in (5.2) and (5.3), the capacitor lets signals of low frequency "pass through" while it "rejects" signals of high frequency. The resistor on the other hand "rejects" signals of low frequency and lets signals of high frequencies "pass through". Depending on if one considers the voltage across the capacitor or the resistor as the "output" voltage, the RC circuit from Fig. 1 can be utilized as a low-pass or a high-pass filter, respectively (see Figs. 6 and 7).

To determine which frequencies can be considered "high" and "low" with regard to the RC circuit, a third case, where resistance and reactance have the same magnitude, i.e. $R = 1/\omega C$, is considered. Resistor and Capacitor voltage are in this case:

$$\begin{aligned} \mathbf{V_R} &= \frac{R}{R + 1/j\omega C}\mathbf{V_S} = \frac{1}{1-j}\mathbf{V_S} = \left(\frac{1}{\sqrt{2}}\angle 45^\circ\right)\cdot\mathbf{V_S} \\ \mathbf{V_C} &= \frac{1/j\omega C}{R + 1/j\omega C}\mathbf{V_S} = \frac{-j}{1-j}\mathbf{V_S} = \left(\frac{1}{\sqrt{2}}\angle -45^\circ\right)\cdot\mathbf{V_S} \end{aligned} \tag{5.4}$$

The voltages across the capacitor and across the resistor both have the same magnitude – the magnitude of the source voltage divided by the square root of two. Resistor and Capacitor voltage display phase shifts of 45º and -45ºrespectively, compared to the source voltage. This frequency is called the ***cutoff frequency*** and has the value $f_c=1/2\pi RC$.

Below the cutoff frequency the magnitude of the capacitor voltage is larger than that of the resistor voltage while above this frequency the magnitude of the resistor voltage is larger than that of the capacitor voltage. Magnitude and phase shifts of both voltages, as functions of the frequency on a logarithmic frequency scale are shown in Fig. 5. At very low frequencies the phase shift of the capacitor voltage is close to zero, the same holds for the resistor voltage at very high frequencies. At very high frequencies the phase shift of the capacitor voltage is close to -90º and the phase shift of the resistor voltage is close to 90º at very low frequencies.

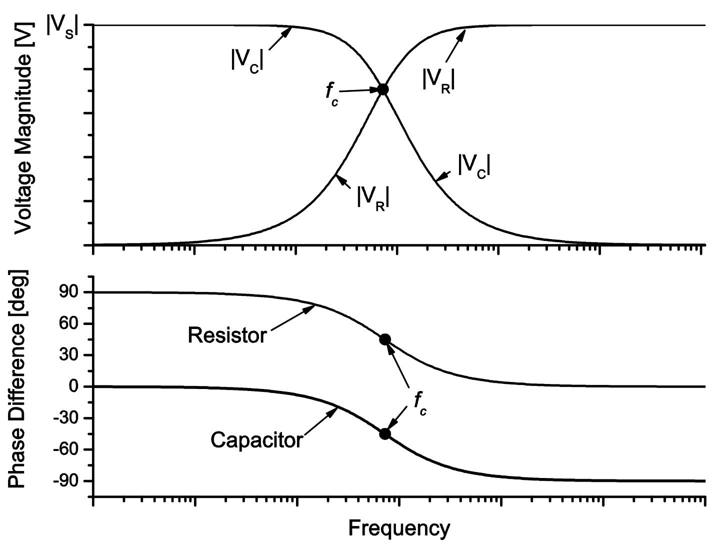


Fig. 5.

Therefore any signals of frequencies below the cutoff frequency can be considered as "passing through" the low-pass filter and being "rejected" by the high-pass-filter, and vice versa for signals of frequencies above the cutoff frequency. A frequency range for which the signal passes through is commonly referred to as a ***passband***, a frequency range for which a signal is rejected is called a ***stopband***.

In an experiment, where the output voltage is monitored on the screen of an oscilloscope, the circuit is set up as in Fig. 6 for a low-pass filter and as in Fig. 7 for a high-pass filter. The output voltage phasors in Figs. 6 and 7 can be predicted through (5.1). An alternative way to express these output voltages in terms of the input voltages is through a complex voltage gain called the ***transfer function T(jω)***:

$$T(j\omega) = \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{V_o(j\omega)}{V_i(j\omega)} \tag{5.5}$$

where $\mathbf{V_i}$ and $\mathbf{V_o}$ are the phasors of the input and output voltages. It can be directly derived from (4.1) and contains both magnitude and phase information. This is referred to as the ***frequency response*** of the filter and can alternatively be described by its magnitude and phase, i.e. the ***gain response***, which is the gain $A_v=|V_o(f)/V_i(f)|$ as a function of frequency, and the ***phase response***, which is the phase delay of the output signal compared to the input signal as a function of frequency. The gain response is often plotted on a ***decibel*** (dB), i.e. logarithmic, scale, where $A_v[\text{dB}] = 20 \log_{10}(|V_o(f)/V_i(f)|)$.

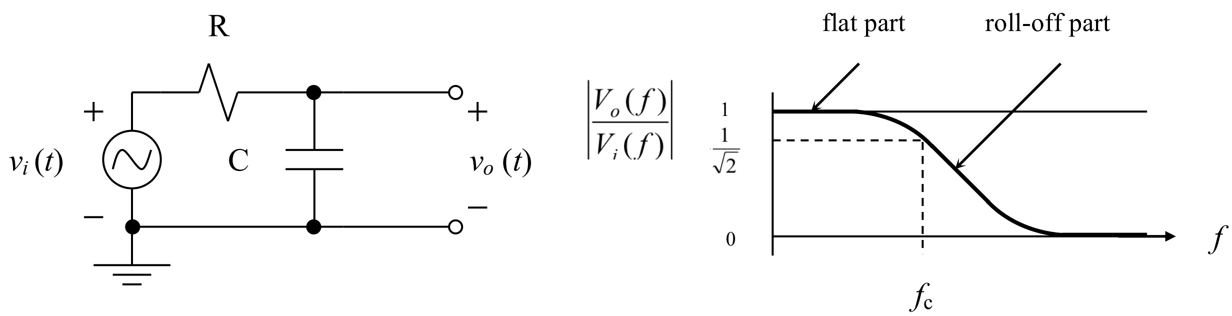


Fig. 6

Fig. 6 shows the first-order low-pass filter and its gain response. The gain response consists of a "flat" part at low frequencies, where the gain stays approximately constant ad a "roll-off" part, which is a frequency range where the gain decreases with increasing frequency.

Fig. 7 shows the same for the first-order high-pass filter. Here the roll-off part exists at a range of low frequencies, the flat part at high frequencies.

On a dB scale the cutoff frequency is the frequency where the gain is -3 dB; it is therefore also referred to as $f_{-3\text{dB}}$. In terms of the phase response, this is the frequency where the absolute value of the phase delay is 45 degrees.

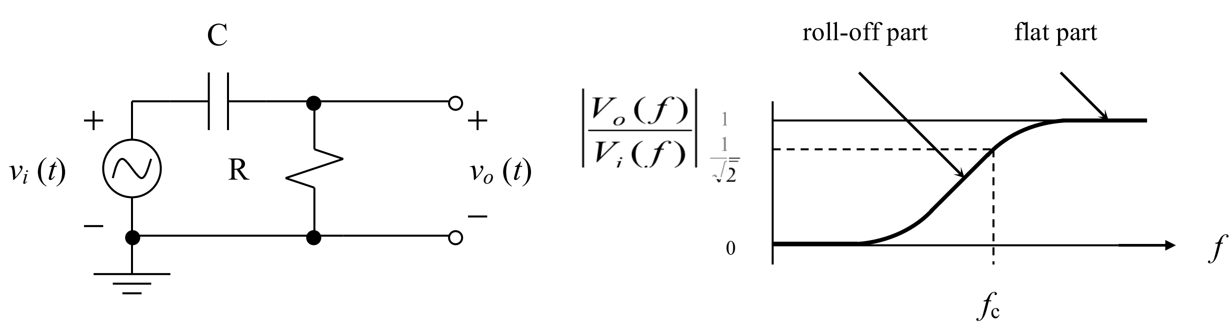


Fig. 7

Fig. 8 illustrates how to derive the phase delay between two signals $v_1(t)$ and $v_2(t)$ in degrees or radians from the time delay read off the oscilloscope.

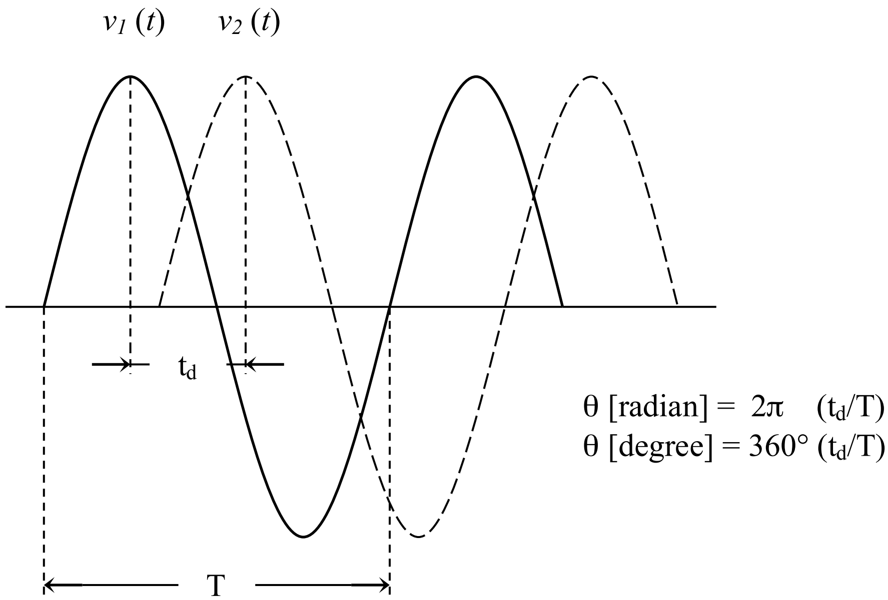


Fig. 8

The frequency response and the time-domain response (see Experiment #3) of a first-order linear circuit are interrelated via the expression $\tau = 1/\omega_c$, where τ is the time constant of the circuit and $\omega_c = 2\pi f_c$, the angular cutoff frequency.

EQUIPMENT

- GW Function Generator Model GFG-813
- GW Function Generator Model GFG-8016G
- TEKTRONIX TDS 210 oscilloscope
- Protoboard
- 2 DC power supplies
- Digital Multimeter (DMN)
- Components: μA741 OP-Amp, 22 kΩ (2), 18 kΩ (3), 15 kΩ (2), 12 kΩ (2), and 10 kΩ (3) resistors, 1 nF (2) and 10 nF (2) capacitors

EXPERIMENT

5.1 Summing Amplifier

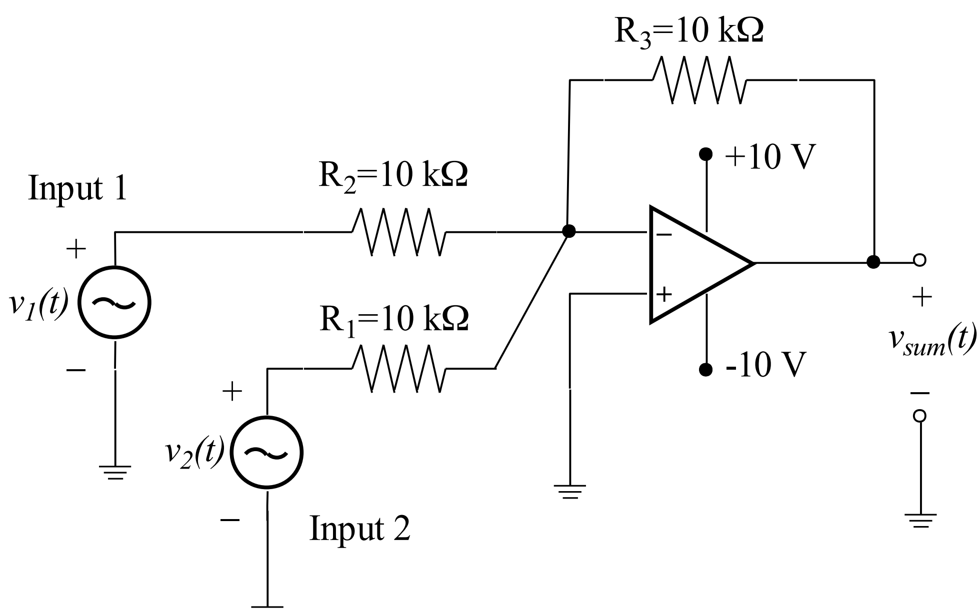


Fig. 9

Preparation:

- Derive the general expression for $v_{sum}(t)$ in the circuit of Fig. 9. For the case of two 2 $V_{p\text{-}p}$ sinusoidal inputs of different frequencies, what are the highest and lowest possible values for $v_{sum}(t)$?

Experiment:

Confirm by measurement the principle of superposition using the summing amplifier of Fig. 9.

- Build the summing amplifier circuit shown in Fig. 9. Follow the steps taken in Experiment #3, section 3.1.1 for the inverting amplifier.
- Use the **GW Function Generator Model GFG-8016G** (output labeled "50 Ω") to apply voltage $v_1(t)$, adjusted to a sine wave of amplitude 2 $V_{p\text{-}p}$ at a frequency of 200 Hz, to "input 1" of the amplifier. Set the other input of the amplifier to 0 V by grounding the "input 2" terminal. Do not short-circuit the output terminals of the function generator. Measure the input $v_1(t)$ and the output $v_{sum}(t)$, and confirm the closed-loop gain of the amplifier for the input signal $v_1(t)$. Enter the measured data in Table 1.
- Reverse the procedure by grounding the "input 1" terminal and by applying voltage $v_2(t)$ using the **GW Function Generator Model GFG-813** adjusted to a sine wave of amplitude 2 $V_{p\text{-}p}$ and a frequency of 22.4 kHz to "input 2". Confirm the closed-loop gain for the input signal $v_2(t)$. Enter the measured data in Table 1.
- Apply both inputs, voltage $v_1(t)$, set to a sine wave of amplitude 2 $V_{p\text{-}p}$ at a frequency of 200 Hz, and voltage $v_2(t)$, set to a sine wave of amplitude 2 $V_{p\text{-}p}$ at a frequency of 22.4 kHz, to the input of the summing amplifier. Observe and sketch the output voltage $v_{sum}(t)$ (as seen on the oscilloscope) in your lab-book. Label both axes and comment on the output waveform.

To stabilize the display of the output voltage $v_{sum}(t)$ when both inputs are applied to the amplifier, you might have to use the external triggering. Apply the 200 Hz signal to the **EXT TRIG** input of the oscilloscope. Press the **TRIGGER MENU** button and push the third menu box button to change the Source to EXT. For more details, refer to the oscilloscope manual.

Table 1

Conditions	$V_{o1p\text{-}p}$ [V]
$v_1(t) = 2\ V_{p\text{-}p}$ at 200 Hz $v_2(t) = 0$ V (grounded)	
$v_1(t) = 0$ V (grounded) $v_2(t) = 2\ V_{p\text{-}p}$ at 22.4 kHz	

KEEP THE SUMMING AMPLIFIER ON THE PROTOBOARD AS IT IS.
DO NOT DISASSEMBLE IT!

5.2 First-Order Low-Pass Filter

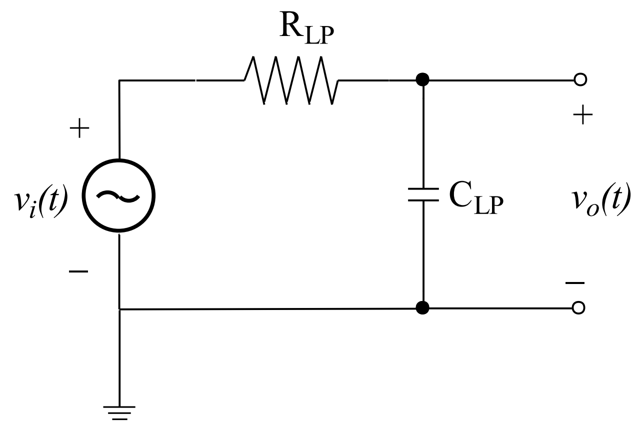


Fig. 10

Preparation:

- Pick a capacitor C_{LP} and a resistor R_{LP} from the list of available components such that the low-pass filter in Fig. 10 has a cutoff frequency of around 1 kHz.
- Derive the circuit's transfer function $T(j\omega)$ and calculate A_v[dB] and θ for the frequencies in Table 2. Plot both in Graph 1 and 2, respectively, and use a logarithmic scale for the frequency. If plotting by hand, use semi-log paper (available at the end of this handout) and use it as shown. Indicate the cutoff frequency in both graphs.

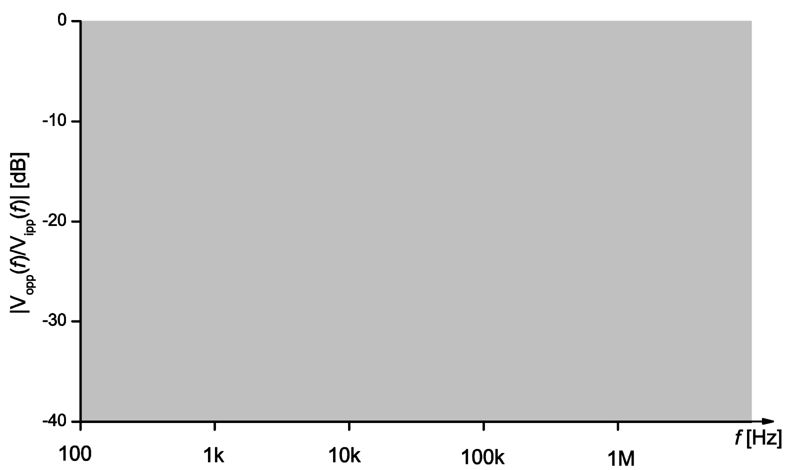
Experiment:

- Build the RC low-pass filter circuit in Fig. 10. Set the input $v_i(t)$ to a 2 V peak-to-peak sine wave at 100 Hz and display both input and output signal on the oscilloscope.
- Align the input and output signals horizontally at the center, i.e. horizontal axes of both signals are identical, as in Fig. 8. Set the oscilloscope sensitivities on both channels such that the vertical deflections of both signals are approximately equal.
- Measure the peak-to-peak output voltage, $V_{op\text{-}p}$, at this frequency and enter it in the appropriate row of Table 2.
- Use a time at which the input voltage reaches the peak value as a reference and use the cursors to measure the time delay t_d. Convert the time delay into a phase delay in degrees (see Fig. 8). Enter the phase delay in Table 2.
- Repeat the previous three steps for all frequencies listed in Table 2.
- Complete Table 2 and add a plot of the measured absolute value of the gain, $|V_{opp}/V_{ipp}|$, in dB versus frequency on a logarithmic frequency scale to Graph 1
- Add a plot of the measured phase delay, θ, in degrees, versus frequency on a logarithmic frequency scale to Graph 2.
- Adjust the frequency such that $|V_{opp}/V_{ipp}| = 1/\sqrt{2}$ and $\theta=45^o$. Record this frequency and mark it in Graph 1 and Graph 2. Compare this frequency with your predicted cutoff frequency from the preparation.

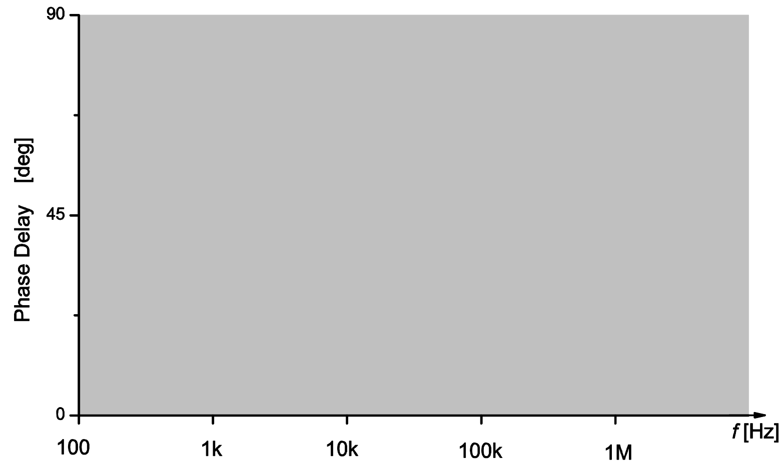
Table 2

v_i const at 2 $V_{p\text{-}p}$	Frequency f [Hz]	$V_{op\text{-}p}$ [V]	$A_v = V_{op\text{-}p} / V_{ip\text{-}p}$	A_v [dB]	θ [degrees]
	100				
	500				
	700				
	1k				
	2k				
	4k				
	7k				
	10k				
	30k				

Graph 1



Graph 2



5.3 First-Order High-Pass Filter

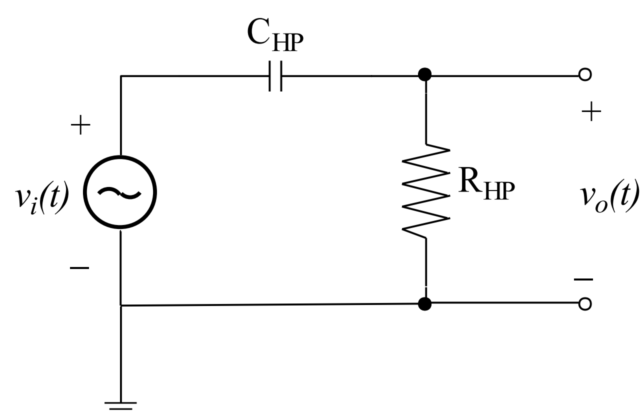


Fig. 11

Preparation:

- Pick a capacitor C_{HP} and a resistor R_{HP} from the list of available components such that the low-pass filter in Fig. 10 has a cutoff frequency of around 10 kHz.
- Derive the circuit's transfer function $T(j\omega)$ and calculate A_v[dB] and θ for the frequencies in Table 3. Plot both in graphs similar to Graph 1 and 2, respectively, i.e. use a logarithmic scale for the frequency, as in 4.2. Indicate the cutoff frequency in both graphs/

Experiment:

- Build the RC high-pass filter circuit in Fig. 11. Set the input $v_i(t)$ to a 2 V peak-to-peak sine wave, display both input and output signal on the oscilloscope, and follow the same steps that were taken in section 5.2 for all frequencies given in Table 3. Complete the table and add plots of your results to the graphs from the preparation.

- Determine the cutoff frequency by measurement the same way as in section 5.2 and indicate it in your graphs. Compare this value for the cutoff frequency with your predicted value from the preparation.

Table 3

v_i const at 2 $V_{p\text{-}p}$	Frequency f [Hz]	$V_{op\text{-}p}$ [V]	$A_v = V_{op\text{-}p} / V_{ip\text{-}p}$	A_v [dB]	θ [degrees]
	500				
	1k				
	5k				
	7k				
	10k				
	12k				
	15k				
	20k				
	100k				

5.4 Complete Circuit

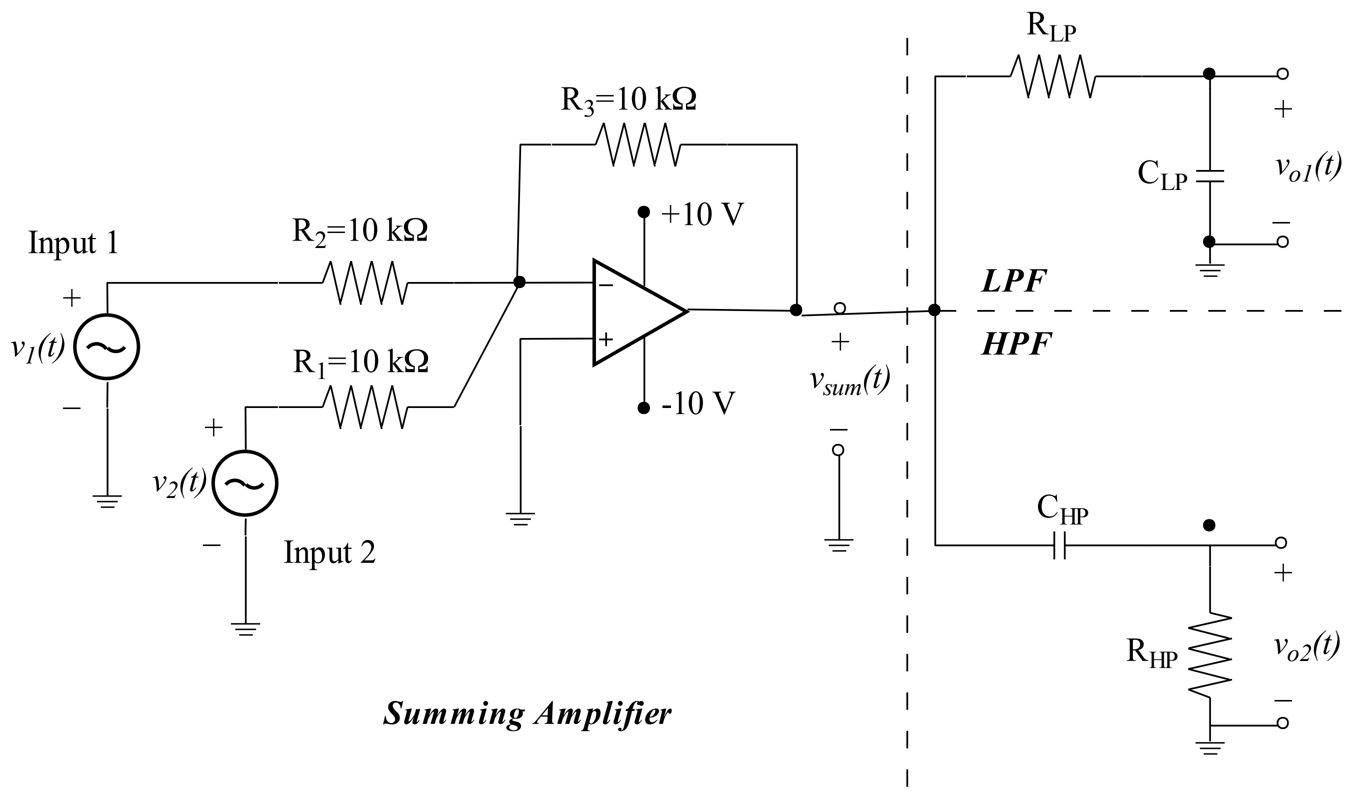


Fig. 12

Preparation:

- Find the transfer functions $T_{LPF}(j\omega)$ and $T_{HPF}(j\omega)$ for the high-pass and low-pass filters in Fig 12 and determine their magnitudes at 200 Hz and 20 kHz. Determine the peak-to-peak voltages for $v_{o1}(t)$ and $v_{o2}(t)$ for the following two cases:

(1) $v_1(t)$ = 2 V_{p-p} at 200 Hz, $v_2(t)$ = 0 V (grounded).
(2) $v_1(t)$ = 0 V (grounded), $v_2(t)$ = 2 V_{p-p} at 20 kHz.

Experiment:

- Build the complete circuit as shown in Fig. 12.
- Use the **GW Function Generator Model GFG-8016G** to apply voltage $v_1(t)$, adjusted to a sine wave of amplitude 2 V_{p-p} and a frequency of 200 Hz, to "input 1" of the amplifier. Ground the "input 2" terminal. Measure the voltage $v_{o1}(t)$ at the output of the LPF, as well as the voltage $v_{o2}(t)$ at the output of the HPF. Enter measured values in Table 4.
- Use the **GW Function Generator Model GFG-813** to apply voltage $v_2(t)$, set to a sine wave of amplitude 2 V_{p-p} and a frequency of 20 kHz to "input 2" of the amplifier. Ground the "input 1" terminal. Again measure the voltage $v_{o1}(t)$ at the output of the LPF, as well as the voltage $v_{o2}(t)$ at the output of the HPF. Enter measured values in Table 4.
- Apply both inputs (200 Hz and 20 kHz) to the input terminals of the summing amplifier. Monitor output voltages $v_{o1}(t)$ and $v_{o2}(t)$ of the filters. To stabilize the oscilloscope display, you can use external triggering as in section 5.1. Sketch both output waveforms as seen on the oscilloscope.
- Use your visual observation and comment on the filtering effectiveness of the filters. Can you detect a presence of the interfering signals? Comment.

Table 4

Conditions	V_{o1p-p} [V] at output of LPF	V_{o2p-p} [V] at output of HPF
v_1 (t) = 2 V_{p-p} at 200 Hz v_2 (t) = 0 V (grounded)		
v_1 (t) = 0 V (grounded) v_2 (t) = 2 V_{p-p} at 22.4 kHz		

Blank semilog graph paper (Rotate, crop, and size as required)