# I. INTRODUCTION

This report examines the relationship between the gravitational acceleration of an object along an inclined plane and the angle of elevation of the plane (Figure 1). A sonar sensor installed on an inclined track of varying slope will record position data as a cart slides down the track as a function of time to determine its acceleration. Given the gravitational field strength in Toronto, Canada to be 9.80678 m/s$^2$ (Wolfram Alpha 2017), the objective of this experiment is to experimentally determine the gravitational field strength in Toronto.

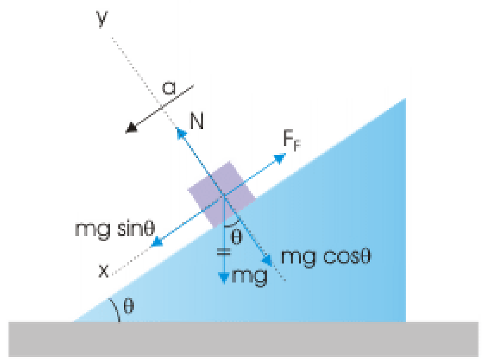


**Figure 1:** *The forces acting on an object on an inclined plane, where m is the mass, g is the gravitational field strength, θ is the angle of elevation, a is the acceleration, N is the normal force, and $F_f$ is the force of friction. (Internet Archive 2017)*

# II. PROCEDURE

The experiment apparatus consists of the Pasco Scientific Newton's Laws Experiment Set along with a computer which runs the Pasco Capstone Software. Initially, the aluminum track is leveled using a spirit level. The sonar sensor, secured to one end of the track is then connected to the Universal Interface which is connected to the computer. The angle of the track is altered by placing metal blocks of identical dimensions in increments of one on the bottom of the track supporting the sonar sensor. Due to the stacking characteristic of the blocks, the first block has a height of 23.12±0.01 mm with each additional block of 16.96±0.01 mm. Five angles were experimented using five metal blocks. Upon recording using the sonar sensor, the cart, at the base of the track, experiences an initial push force allowing it to travel up the track. The force of gravity and kinetic friction decelerates the cart as it climbs up the track. Before the cart reaches the force sensor, its velocity decelerates to zero and the force of gravity then accelerates the cart down the track with kinetic friction opposing its motion (Figure 2). Due to this change of the direction of kinetic friction in relation to the force of gravity, the position data for when the cart is accelerating down the track will be analyzed. If the initial push force caused the cart to contact the sonar sensor, the trial is redone since the force exerted by the sonar sensor on the cart will affect its acceleration down the track due to Newton's Third Law. With the Capstone Software running and the collection frequency set to 50 Hz, five trials were conducted for each angle.

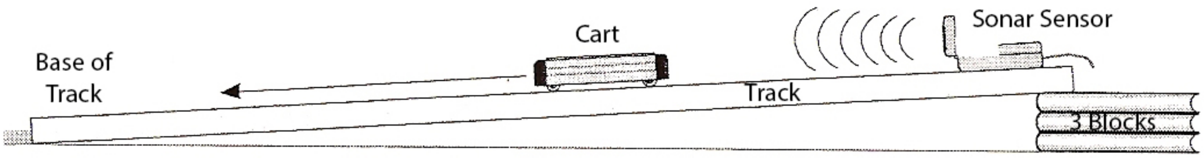


**Figure 2:** *The experiment apparatus where three metal blocks are placed under the track on the side of the sonar sensor. Only position data for when the cart is accelerating down the track will be used to calculate its acceleration.*

Using the Capstone Software, the position of the cart as it accelerates down the track is graphed as a function of time (Figure 3) and is fitted to a quadratic function. The equation of the quadratic function of $At^2 + Bt + C$ can be expressed in terms of the equation for position under constant acceleration using $X_f = \frac{1}{2}at^2 + V_0t + X_0$. As a result, the acceleration of the cart can be expressed as $2A$, with its uncertainty being twice of the uncertainty from the quadratic equation.

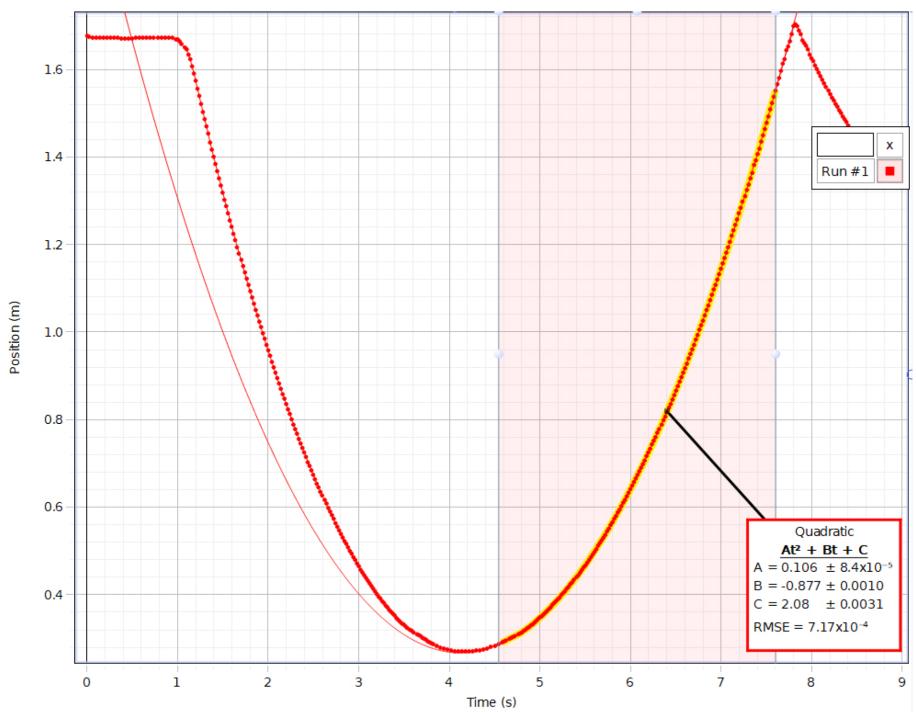


**Figure 3:** *Position versus time graph created using the Capstone Software from the first trial of the smallest angle of elevation using one metal block. Position increases as the cart moves further away from the sonar sensor. The cart's acceleration down the track is* $2A = 2(0.106) = 0.212\ m/s^2$. *And the uncertainty is* $2(8.4E-5) = 1.7E-4$.

The angle of elevation is calculated by using the height of the stack of blocks and the length of the track measured from the base, touching the ground, to the block support (923±1 mm). This effectively forms a right triangle and the following equation is used:

$$sin(\theta) = \frac{o}{h}$$

E.g. [$o = 23.12\ mm$ // $h = 923\ mm$]

$$\sin(\theta) = \left(\frac{23.12}{923}\right) \rightarrow \theta = 0.0251 \text{ RAD}$$

where $\theta$ is the angle of elevation, $o$ is the height of the stack of blocks, and $h$ is the length of the track. The uncertainty of the angle of elevation is calculated using following equation:

$$\Delta\theta = \theta\sqrt{\left(\frac{\Delta o}{o}\right)^2 + \left(\frac{\Delta h}{h}\right)^2}$$

E.g. $[\theta = 0.0251$ // $o = 23.12\ mm$ // $h = 923\ mm$ // $\Delta o = 0.005\ mm$ // $\Delta h = 0.5\ mm]$

$$\Delta\theta = 0.0251\sqrt{\left(\frac{0.005}{23.12}\right)^2 + \left(\frac{0.5}{923}\right)^2} = \pm 1.46E-5$$

where $\theta$, $o$, $h$ are the angle of elevation, height of the stack of blocks, length of the track, respectively, and $\Delta\theta$, $\Delta o$, $\Delta h$ are the uncertainty associated with $\theta$, $o$, $h$, respectively.

The mean acceleration of the cart for each angle from the five trials is calculated with their associated uncertainty in terms of standard error. The standard error is deduced from the following equation:

$$SE = \frac{s}{\sqrt{n}} = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{\bar{x}})^2}}{\sqrt{n}} = \frac{\sqrt{\frac{1}{4}\sum_{i=1}^{5}(x_i - \bar{\bar{x}})^2}}{\sqrt{5}}$$

where $s$, is the standard deviation, $n$ is the number of trials (5), $x_i$ is the acceleration of the cart for each trial, and $\bar{x}$ is the mean acceleration of the cart. The standard deviation is calculated using a spreadsheet using the corresponding equation for $s$.

# III. RESULTS AND DISCUSSION

Because an object accelerating down a plane has a force of kinetic friction opposing its motion (Figure 1), the following equation is used to calculate the acceleration due to the gravitational field strength. The small angles of elevation during experimentation tolerates the use small angle approximations, where $sin(\theta) = \theta$ and $cos(\theta) = 1$. In the equation,

$$a = g\ sin(\theta) - \mu_k\ g\ cos(\theta) = g\theta - \mu_k\ g$$

where $a$ is the acceleration of the object, $g$ is the gravitational field strength, $\theta$ is the angle of elevation of the incline, and $\mu_k$ is the coefficient of static friction.

The linear fit equation of $mx + b$ from the mean acceleration as a function of the angle of elevation graph (Figure 4) can be expressed in terms of $a = g\theta - \mu_k\ g$. As a result, the gravitational field strength is 9.8±0.3 m/s² while the coefficient of kinetic friction is $\mu_k\, g = 0.0208 \rightarrow \mu_k = \frac{0.0208}{9.79} = 0.00212$. The uncertainty of the coefficient of kinetic friction is calculated by $\Delta\mu_k = \mu_k\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta b}{b}\right)^2} = 0.00196$. The coefficient becomes 0.002±0.002.

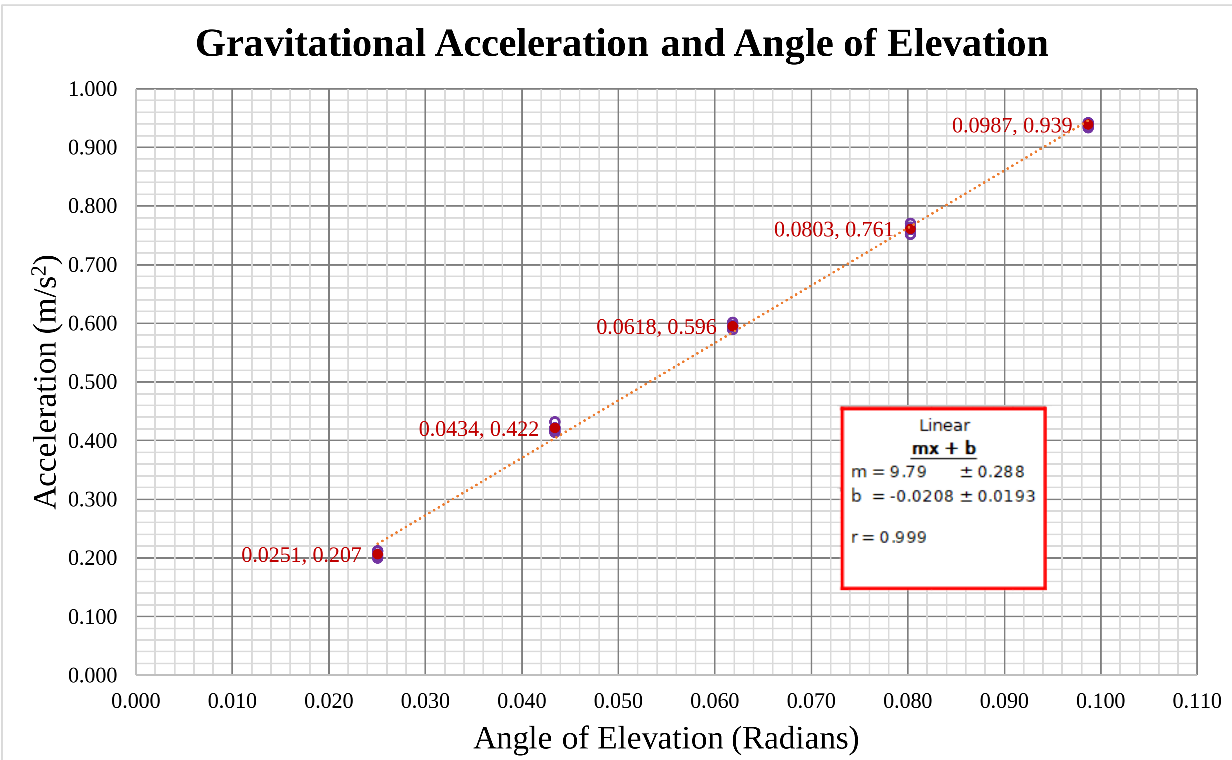


**Figure 4:** *Gravitational acceleration versus angle of elevation graph created using a spreadsheet. The mean acceleration for each angle is plotted using red data points with a linear trendline fit. The error bars for both axis is too insignificant to be identified. The maximum error for the angle of elevation is ±6.00E-5 while for acceleration is ±2.99E-3 m/s². The acceleration of each trial for each angle is plotted using purple data circles.*

The uncertainties involved in this experimentation consists of the measurement uncertainty associated with the sonar sensor (±1 mm), the length of the track (±1 mm), and the height of each block (±0.01 mm). The two statistical uncertainties consist of the quadratic fitting function for each trial to obtain the cart's acceleration (Figure 3) and the linear fitting function to obtain the strength of the gravitational field and the coefficient of kinetic friction (Figure 4). Among the measurement uncertainties and the linear fitting function, the sonar sensor likely provides the dominant uncertainty since its percentage uncertainty increases with decreasing distance between the cart and the sensor. Systematic error caused my imperfect calibration of the sensor also contributes to its uncertainty.

Nevertheless, the dominant uncertainty among all is associated with the linear fitting function because of the high percentage uncertainty for both $m$ and $b$ values (Figure 4). The trendline from the linear fit also deviates significantly more than the unnoticeable error bars from the data points.

## IV. CONCLUSION

The experiment has succeeded in identifying a linear relationship between the gravitational acceleration of an object along an inclined plane and the angle of elevation of the plane. The objective of this experiment has been met by experimentally determining the gravitational field strength in Toronto to be $9.8 \pm 0.3$ m/s$^2$ which agrees with 9.80678 m/s$^2$ (Wolfram Alpha 2017) to within the uncertainty. The coefficient of kinetic friction of the ramp was also determined to be $0.002 \pm 0.002$.

To determine the gravitational field strength in Toronto, the position data as a function of time between the cart accelerating down the track and the sonar sensor were first fitted to a quadratic function. The acceleration of the cart is then obtained for each trial and the mean was calculated for each angle of elevation. The mean acceleration as a function of the angle of elevation was then graphed and fitted to a linear function to obtain the gravitational field strength and the coefficient of kinetic friction of the track.

The experimental value of the gravitational field strength is predominantly limited by the statistical error associated with the linear fitting of the mean acceleration as a function of the angle of elevation. While more trials and more angle of elevations can be explored to improve the accuracy in addition to using a more sensitive sonar sensor, other methods that exclude linear fitting can be used to determine the gravitational field strength.

The use of a pendulum and dropping ball in free-fall are two other methods which this experiment can be extended to calculate the gravitational field strength in Toronto. The results between the different methods can be compared to determine which method can more effectively provide a greater accuracy given equipment that is easily accessible to the general public.

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