ESC103F Engineering Mathematics and Computation: Lab #1

Pre-Lab

Consider the integral that will be used as the basis for Exercises 1 and 2:

$$\int_0^3 \sqrt{x+1}\, dx$$

Find the exact value for the integral using an analytical solution. This will be used as a basis for comparison with your numerical solutions in Exercise 1 and 2.

Exercise 1

Write a Matlab program that produces a **midpoint approximation** with the interval [0,3] subdivided into *n* evenly spaced subintervals. Your code should be designed to work for an arbitrary *n*.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the numerical approximations to the integral on the upper plot using the subplot command. Also plot a horizontal solid line at the exact value.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the midpoint error bound on the lower plot using the subplot command. Also plot the absolute value of the actual error using a solid line.

Label all axes and add legends to both plots as appropriate.

Given information:

$$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

where M_n is the midpoint approximation, with

$$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$$

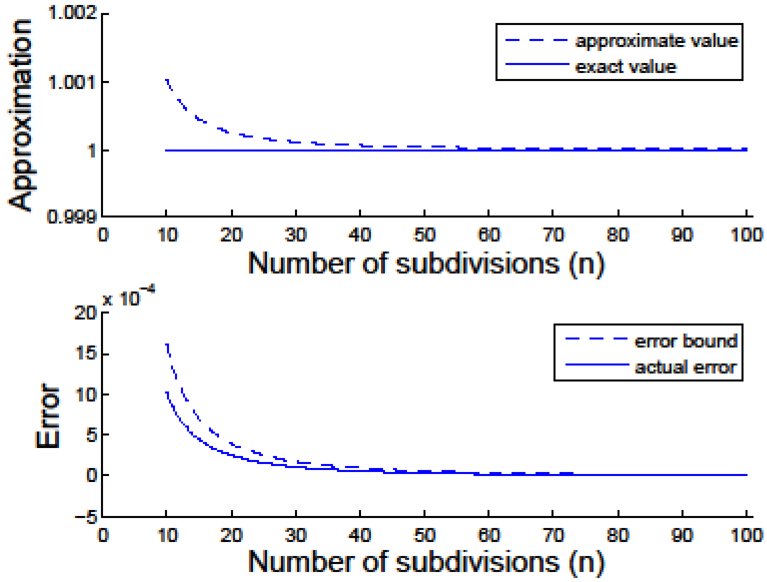
$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$|E_M| = \left| \int_0^3 f(x)dx - M_n \right| \le \frac{(3-0)^3 K}{24n^2}$$

where E_M is the actual error, and

$$|f''(x)| \le K \text{ on } [0,3]$$

To help you develop your plot for Exercise 1, here is a sample plot for illustrative purposes of the results obtained for a different integral, $\int_0^{\pi/2} \cos x\, dx$:



Exercise 2

Write a Matlab program that produces a **trapezoidal approximation** with the interval [0,3] subdivided into *n* evenly spaced subintervals. Your code should be designed to work for an arbitrary *n*.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the numerical approximations to the integral on the upper plot using the subplot command. Also plot a horizontal solid line at the exact value.

For all integer values of *n* between 10 and 100 and with *n* on the x-axis, plot using a dashed line the trapezoidal error bound on the lower plot using the subplot command. Also plot the absolute value of the actual error using a solid line.

Label all axes and add legends to both plots as appropriate.

Given information:

$$T_n = \sum_{i=1}^{n} \left(\frac{f(x_{i-1}) + f(x_i)}{2}\right)\Delta x$$

where T_n is the trapezoidal approximation, with

$$\left|E_T\right| = \left|\int_0^3 f(x)dx - T_n\right| \le \frac{(3-0)^3 K}{12n^2}$$

where E_T is the actual error, and

$\left|f''(x)\right| \le K$ on [0,3]

Testing your code for Exercise 2

In Matlab, find a built-in function for performing trapezoidal numerical integration and use this function to check your answers in Exercise 2.