CoverPage

- Lalalala
- Наименование вуза, факультета, кафедры
- Тема работы на русском и английском языке
- Имя, фамилия, номер группы автора работы
- Должность, ученая степень, фамилия, инициалы научного руководителя ВКР
- место и год написания

тут немного текста для компилирования в командной строке и еще володя

вода

ляляляляя

Аннотация

Let me tell about the Riemann-Hilbert problem on elliptic curve. We take a meromorphic matrix function $A(z)$ over a complex manifold and solve the equation $\dot{y} = A(z)y$ depending on A we will get different solution, but the solutions has a monodromy, which can be represented as a set of linear maps. Each map correspods to a singular point of a A. This takes us to a monodromy, which is a representation of the fundamental group of punctured surface. But, given the monodromy, how can we find A such that A satisfies it? This question is a naive statement of a Riemann-Hilbert problem, or Inverse Monodromy problem.

It turns out that we can generalize our construction and treat the solutions as a trivial bundle sections and A as the matrix of a flat connection inside the bundle. In this more general setting a huge opportuity potentially slumbers, because the language of bundles and connections is more algebraic than the analytic one which was presented in the beginning. Let us confine the generalization to the case of stable bundles since their Chern classes behave better than for a non-stable one, and if the bundle has suficiently large degree. (The degree of the divisor of the arbitrary section), then the bundle has a set of global sections which form a basis at every point. The profit of this confinement will not be seen in out current inquisitions, but this property gives much in terms of algebraic structure of the bundle. Let us confine the considerations to the case of logarithmic(Fuchsian) connections which have simple poles only. However, one can try to construct the connections and solutions explicitly.

We solve (blablabla)
And we are expecting to get (blablabla) results

1 Introduction

- Постановка задачи ВКР (Понятно математику)
- Мотивировка
- История вопроса

Let us have an elliptic curve which is a holomorphic one-dimensional manifold of genus 1 The work treats flat logarighmic connections in holomorphic bundles of rank 2 and degree 0 over elliptic curve. The curve has a fixed modular parameter τ. The bundle will be called E and the base will be called B. Horizontal sections of the connection are such sections that $\nabla s = 0$ or $ds = Adz \otimes s$ holds where both sides belong to $E \otimes T^*M$. Any matrix function that changes along the bundle sections such that the horizontal section stays horizontal with respect to this modifying matrix is connection matrix for a particular bundle. [Matveeva and Poberezhny] in their article which deal with the case of a connection over a two-dimensional stable bundle. They proof that the answer always exist. They find a Fuchsian connection corresponding to a rigid representation (here: the representation induced from a sphere) with 3 branch points. They find explicitly a conection which is associated to the stable bundle $\mathcal{O}_\lambda(0) \oplus \mathcal{O}_{-\lambda}(0)$ for given λ. The logarithm of a local monodromy of any connection is conjugated to the residue of the matrix of the connection at the given monodromy point. Then, using these facts they find the solution for local monodromies and give an integral equation that should be satisfied by a conjugated (deformed) connection matrix so the global monodromy was trivial.

There is a couple of thoughts I have not yet elaborated. First, it would be marvelous to infer some general properties of those connections using Serre Duality, Riemann-Roch cohomology theorem, Chern characters and Ext groups, which are standard algebraic instruments when we are studying complex manifolds. Second, I like to think about connections as sections itself, since they are actually special elements of $F = Hom(E, E \otimes T^*M)$ which is a bundle itself. The Leibniz rule and the flattness are two conditions which I should restate in this settings so I could distinguish «good» sections from «bad» ones. The Fuchsianness of the connection means that there are no divisor points with multiplicity more than 1.

The classic analytic way to operate with the sections are Jacoby θ-functions, as far as I know they were discovered in the beginning of XIX century. Any line-bundle has sections, represented by a product of theta-functions. We will heavily use it in our constructions. Another classic object is Hypergeometric Differential Equation. The canonical form is

$$z(1-z)\frac{d^2y}{dz^2} + [c-(a+b+1)z]\frac{dy}{dz} - aby$$

This equation has regular singular points at $0, 1, \infty$, and different monodromies depending on a, b, c. This second-order linear differential equation can be represented as

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{ab}{z(z-1)} & \frac{c-(a+b+1)z}{z(z-1)} & 0 \end{pmatrix} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix} = \frac{d}{dz} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix} \tag{1}$$

this equation has a two-dimensional space of solution. The Riemann-Hilbert problem has a rich history, that will not be covered here. Riemann in 1857 has published a paper where he considered the hypergeometric equations and its monodromies. Hilbert in 1905 solved the regular Riemann-Hilbert problem for rank 2 bundles. The Riemann-Hilbert problem on the Riemann sphere was solved negatively by Alexander Bolybruch in 1988. He has provided an explicit counterexample of a punctured Riemann sphere fundamental group representation that can not be a monodromy representation of a Fuchsian system. Bolybruch has obtained other results, one of them is that for any *irreducible* representation can be obtained as a monodromy. The Riemann-Hilbert problem has connections with integrable systems and algebraic geometry. [See this and that book (FokasIts, for example)]

2 Preliminaries

Определения и теоремы, которые входят в Introduction Понятие когерентного пучка обобщает понятие локально свободного пучка конечного ранга. Локальносвободный пучок конечного ранга на многообразии в точности соответствует векторному расслоению.

2.1 Методы явных вычислений

3 Main Results

Результаты Возможно, есть связь между когомологиями пучка, соответствующего связности, и когомологиями пучка исходного расслоения.

4 Appendices