(a, b)-tree experiment

Test program

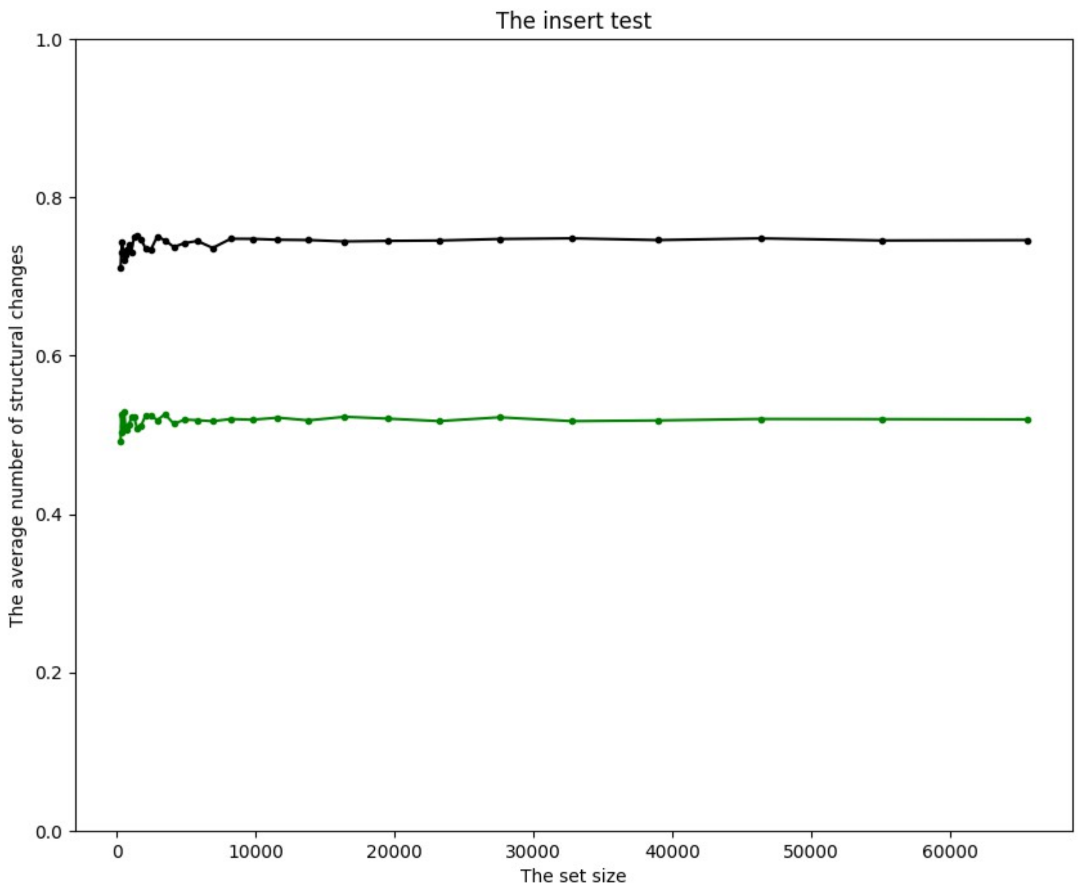
The test program evaluates the implementation of (2, 3)-tree and (2, 4)-tree,
It performs three types of tests: Insert test, Min test and Random test.
Performance is measured in terms of the average number of structural changes (ASC).
Structural change is either a node split (in insert) or merging of two nodes (in delete).

Plots

Each plot shows the dependence of the average number of structural changes on the set size n.
The black curve represents the (2, 3)-tree implementation, the green one represents the (2, 4)-tree implementation.

Insert test:

Insert n elements in random order.



(2, 3)-tree

The average number of structural changes ASC lies between ½ and 1:

- lower bound estimation:
  - ASC >= ½:
    - each insert starts in some node on the lowest internal level
    - each such node:
      - was created with one key
      - after each insertion without a split, the next insertion is performed with at least one split: the node itself has to be splitted and some nodes in higher levels can be splitted too:

        #SPLIT   >= 2 * #INSERT

        ASC   >= (2 * #INSERT) / #INSERT   = ½

- upper bound estimation:
  - ASC <= 1:
  - the number of splits #SPLIT is bounded by the number of nodes #NODES
      - each split creates one new node
  - in worst case (we insert ordered numbers):
      - there is only one branch, to that the keys are inserted, and only nodes on this branch can contain two keys
      - HEIGHT   <= 1 + $\log_2$ ((#KEYS + 1) / 2)
      - #NODES   <= #KEYS   <= #NODES + HEIGHT

             <= #NODES + 1 + $\log_2$((#KEYS + 1) / 2)
      - 1   >= ASC

        ASC   >= #NODES / (#NODES + 1 + $\log_2$((#KEYS + 1) / 2))

Min test:
Insert n elements sequentially and then n times repeat: remove the minimal element in the tree and then insert it back.

Random test:
Insert n elements sequentially and then n times repeat: remove random element from the tree and then insert random element into the tree.
Removed element is always present in the tree and inserted element is always not present in the tree.