Probability distributions

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Discrete distribution

Example: X - result of six sided dice roll

The expected value

$$\mathbb{E}[X] = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6$$

$$= \frac{21}{6} = 3.5$$

The results of a dice roll X are distributed according to a discrete distribution with the following probability density function:

$$f(x) = \begin{cases} \frac{1}{6} & : x = 1 \\ \frac{1}{6} & : x = 2 \\ \frac{1}{6} & : x = 3 \\ \frac{1}{6} & : x = 4 \\ \frac{1}{6} & : x = 5 \\ \frac{1}{6} & : x = 6 \end{cases}$$

Bernoulli (binary) distribution

Example : X - result in roulette when betting on color

$$\mathbb{E}[X] = \frac{19}{37}\cdot 0 + \frac{18}{37}\cdot 1 = \frac{18}{37} = 0.4865$$

In general for the probability density function (pdf):

$$f_X(k) = \begin{cases} p & : k = 1 \\ 1-p & : k = 0 \end{cases}$$

we have

$$\mathbb{E}[X] = 0\cdot(1-p) + 1\cdot p = p$$

Binomial distribution

Example : Y - result of n attempts in roulette

In general the pdf for the binomial distribution $B(n,p)$:

The probability of k successes

$$p(k) = f_{B(n,p)}(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

The number of k-element (k - successes) subsets of an n - element set (n trials)

The probability of $n-k$ losses

The expected value from definition :

$$\begin{aligned}\mathbb{E}[Y] = {} & 0 \cdot \binom{n}{0} p^0 (1-p)^{n-0} + 1 \cdot \binom{n}{1} p^1 (1-p)^{n-1} \\ & + 2 \cdot \binom{n}{2} p^2 (1-p)^{n-2} + \ldots + (n-1) \binom{n}{n-1} p^{n-1} (1-p)^{n-(n-1)} \\ & + n \cdot \binom{n}{n} p^n (1-p)^{n-n} \\ = {} & \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} = \sum_{k=0}^{n} k \cdot p(k)\end{aligned}$$

The expected value calculated in a clever way :

$$\mathbb{E}[Y] = \mathbb{E}[\underbrace{X + X + X + \ldots + X}_{n \text{ times}}] = \underbrace{\mathbb{E}[X] + \mathbb{E}[X] + \ldots + \mathbb{E}[X]}_{n \text{ times}}$$

$$= n\, \mathbb{E}[X] = n \cdot p$$