Mixed Effects Models in R Report

Christianah Titilope Oyewale

November 22, 2024

Abstract

This study analyzes the effects of sleep deprivation on reaction time using the ‘sleepstudy‘ dataset. The dataset captures reaction times across multiple days of sleep deprivation for 18 subjects. A linear mixed-effects model was used to account for repeated measurements within subjects, revealing an increasing trend in reaction times with days of deprivation. Key findings show that reaction times increase by an average of 11.44 milliseconds per day, highlighting significant cognitive performance degradation over time. These results underscore the cumulative effects of sleep deprivation on psychomotor performance.

1 Introduction

Sleep is a critical component of overall health, playing a key role in cognitive functions such as attention, memory, and decision-making. Chronic sleep deprivation is increasingly recognized as a significant public health concern, with detrimental effects on individual performance and safety. Prolonged periods of inadequate sleep can impair psychomotor performance, which is the ability to respond quickly and accurately to stimuli. This is particularly relevant in occupations and activities requiring sustained attention, such as healthcare, transportation, and manufacturing, where errors caused by slowed reaction times can have dire consequences.

The ‘sleepstudy‘ dataset [1] used in this analysis provides a unique opportunity to examine the effects of sleep deprivation on psychomotor performance. The dataset captures average reaction times for 18 subjects across multiple days of sleep deprivation. By employing a linear mixed-effects modeling approach, this study accounts for individual differences in baseline reaction times, as well as the repeated nature of the measurements within subjects.

The goal of this analysis is to quantify the impact of sleep deprivation on reaction times and to identify trends in performance degradation over time. This report aims to:

- Explore the data to understand its structure and variability.
- Compute descriptive statistics to summarize reaction time trends across days and subjects.
- Build a mixed-effects model to estimate the fixed and random effects influencing reaction times.

- Perform residual analysis to validate the model.
- Interpret the results and discuss their implications for understanding the cognitive costs of sleep deprivation.

Through these analyses, this study provides insights into how reaction times progressively decline under conditions of sustained sleep deprivation. Such findings can inform policies and interventions aimed at mitigating the risks associated with sleep-related performance impairments.

2 Dataset and Data Preparation

The dataset authored by Belenky et al. [1], included as the built-in dataset sleepstudy in the lme4 package for R authored by Bates et al. [2], was extracted from eighteen participants' three hours sleep condition where they performed a ten-minute "psychomotor vigilance test" where they had to monitor a display and press a button as quickly as possible each time a stimulus appeared for 10 days. The dependent measure in the dataset is the participant's average response time (RT) on the task for that day. The data meets the definition of multilevel data due to repeated measurements on the same dependent variable (mean RT) for the same participants over ten days.

The data was filtered to exclude data for Day 0 and Day 1, leaving 144 observations across 8 days (Day 2 to Day 9). The key variables include reaction, which is the reaction time in milliseconds; days, which is the number of days of sleep deprivation; and subject, which is the identifier for each of the 18 participants.

3 Exploratory Data Analysis

After data preparation, the data exploration provided an understanding of the data. The summary statistics showed that the mean reaction times increased from 265 ms on Day 2 to 351 ms on Day 9. The standard deviation of reaction times increased with more days of deprivation. The data visualization from figure 1, the line plot shows how reaction times change over days of sleep deprivation for each subject, with separate lines for each individual. Most subjects exhibit a general upward trend in reaction times, indicating performance degradation over days. Individual trajectories vary, highlighting differences in how sleep deprivation affects subjects. The coloured lines represent individual subjects, demonstrating subject-level variability.

The scatter plot in figure 1 provides a point-by-point view of reaction times for all subjects across days. Points are scattered upward as days increase, consistent with increasing reaction times due to sleep deprivation. Variability in reaction times increases over days, showing that the impact of sleep deprivation differs among subjects. The boxplot summarizes the distribution of reaction times for each day, showing the median, interquartile range, and outliers. Median reaction time increases progressively over days, indicating worsening performance. The interquartile range (box height) widens over time, suggesting greater variability in reaction times as deprivation continues. Outliers highlight extreme responses in reaction times for certain individuals.

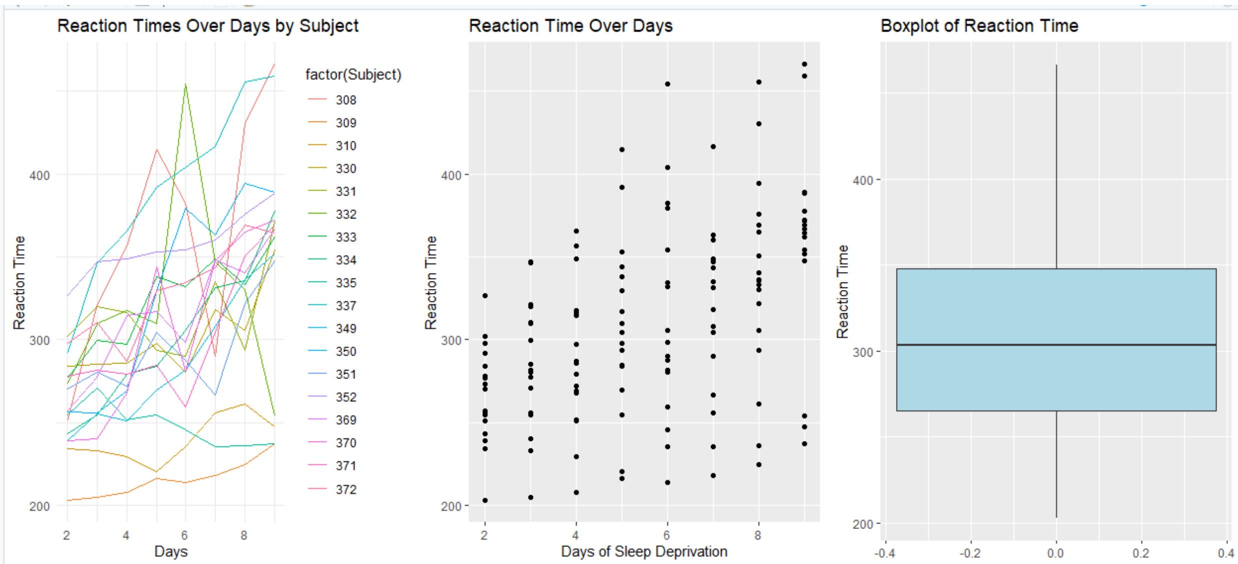

Figure 1: Line, Scatter, and Box of the data.

This density plot in figure 2 shows the distribution of reaction times for each subject, with densities shaded by subject. Subjects have unique reaction time profiles, reflected by varying shapes and locations of density curves. Some subjects consistently exhibit slower reaction times (shifted right), while others are quicker (shifted left). Overlaps between densities suggest similarities among certain subjects' performances. Similarly, the density plot visualizes the reaction time distributions for each day, with densities shaded by days. Reaction times tend to shift to the right (higher values) as days progress, consistent with increasing mean reaction times. The spread of densities increases over days, illustrating greater variability in later days of sleep deprivation. Overlaps between densities for early days are more pronounced, indicating less differentiation than in later days.

4 Descriptive Statistics

Descriptive analysis showed that the reaction times increased steadily from a mean of 265 ms on Day 2 to 351 ms on Day 9, with growing variability, suggesting worsening performance and differing individual susceptibilities as sleep deprivation continues. Subjects exhibit significant differences in baseline performance, consistency, and response to sleep deprivation, highlighting both resilience and vulnerability among participants.

Further visualizing the data from figure 3, the histogram on the left shows the distribution of reaction times across all subjects and days. The data is roughly unimodal with the majority of reaction times clustered between 250 ms and 350 ms, indicating this range as the typical response time. A smaller number of reaction times exceed 400 ms, showing some extreme delays, likely outliers or subjects particularly affected by sleep deprivation. The boxplot (right panel) displays the spread of reaction times for each day of sleep deprivation. Reaction times tend to increase as the number of days without sufficient sleep grows, indicating the detrimental effect of prolonged sleep deprivation. The variability (spread of

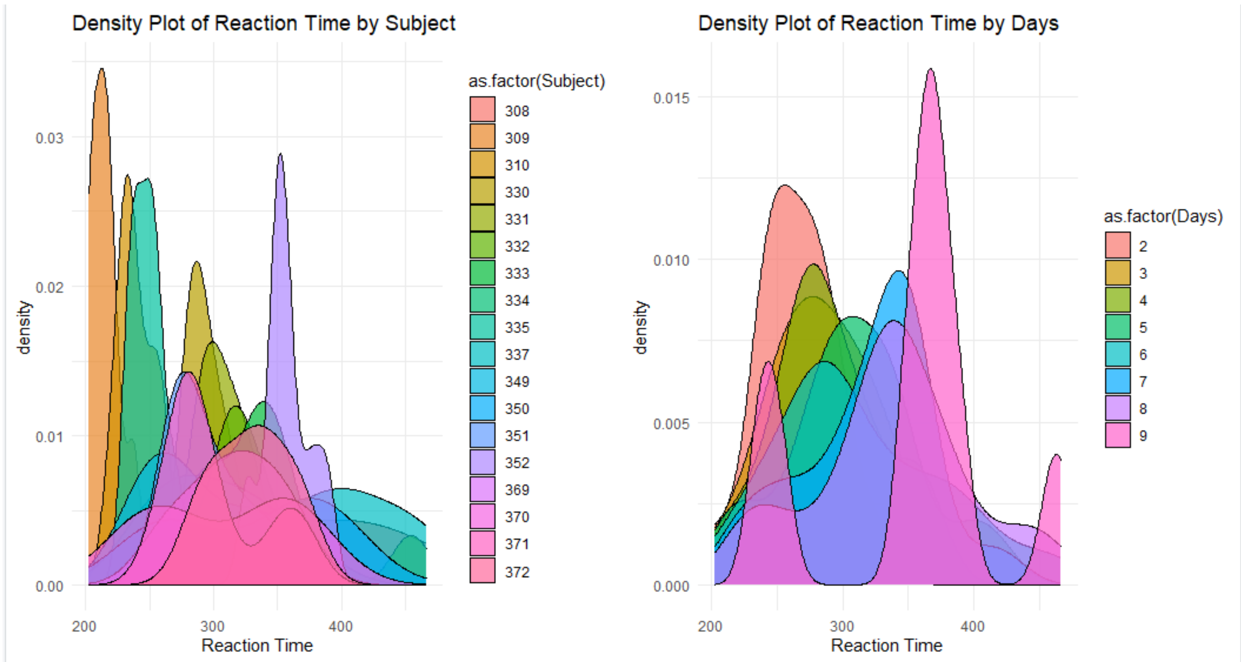

Figure 2: Density Plot of Reaction Time by Subject and by Day.

data) also increases with more days, as evidenced by longer box heights and more outliers on Days 7–9. This suggests that subjects exhibit more diverse responses to sleep deprivation over time.

5 Model Fitting and Result Interpretation

From the visualized data described, the reaction times are repeatedly measured for the same participants over several days. A linear mixed-effects model that accounts for both the fixed effect of time (days) and the random variability among participants was used in the modelling written below,

$$\text{Reaction} \sim \text{Days} + (1|\text{Subject})$$

or

$$\text{Reaction} = \beta_0 + \beta_1 \cdot \text{Days} + u_{\text{Subject}} + \epsilon$$

where

β_0 : The overall intercept, representing the baseline reaction time when the number of days (*Days*) is 0 (Day 2, as Day 0 and Day 1 were removed from the dataset).

β_1 : The fixed effect of Days, representing the average rate of change in reaction time per day of sleep deprivation.

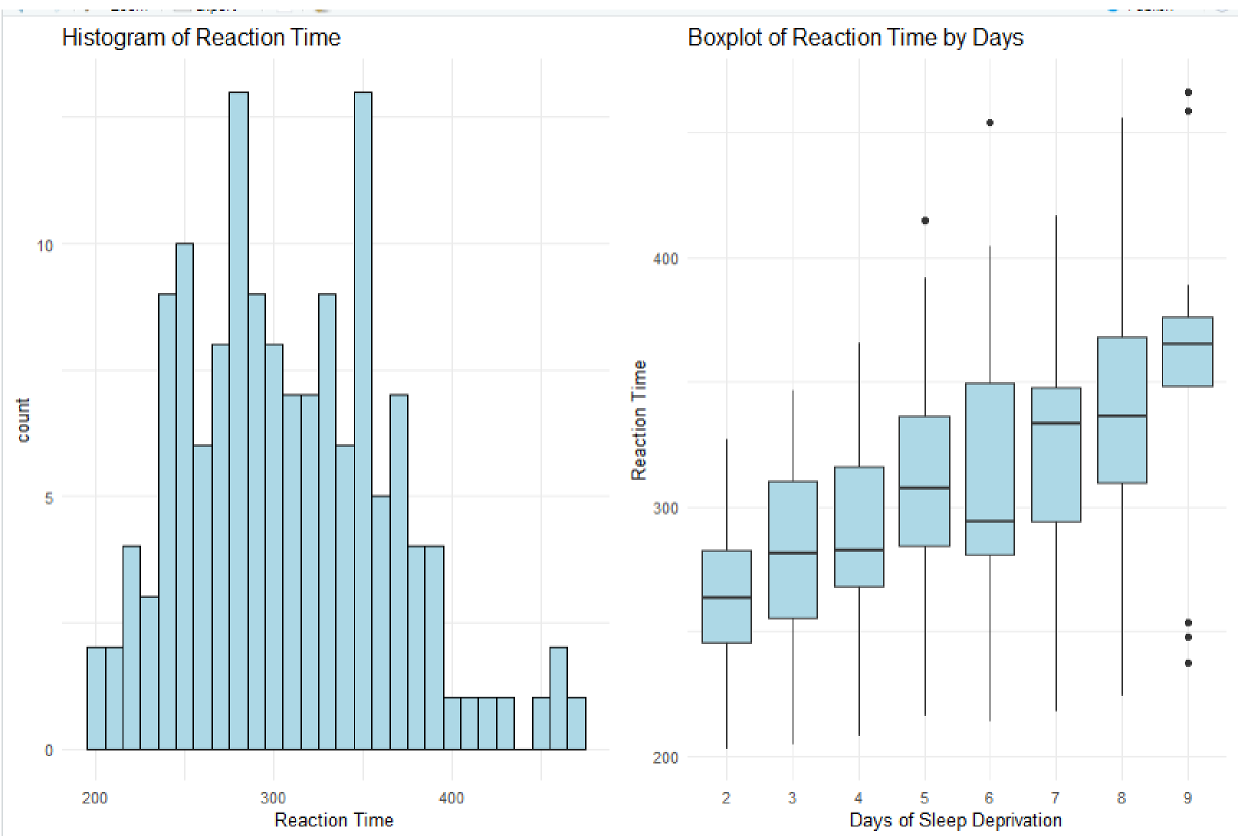

Figure 3: Histogram of Reaction Time (Left Panel) and Boxplot of Reaction Time by Days (Right Panel)

u_{Subject} : The random effect for Subject accounting for variability in baseline reaction times between participants.

ϵ : The residual error term, representing within-participant variability in reaction times.

Linear Mixed Model Output

Model Formula:

$$\text{Reaction} \sim \text{Days} + (1|\text{Subject})$$

Model Summary:

- **Data:** sleepstudy
- **REML Criterion at Convergence:** 1430.02
- **Number of Observations:** 144

- **Number of Groups (Subjects):** 18

Random Effects:

Groups	Name	Std. Dev.
Subject	(Intercept)	41.80
Residual		30.22

Fixed Effects:

Effect	Estimate
Intercept	245.10
Days	11.44

Based on the output of the model, the mixed-effects model shows that the average reaction time across all participants starts at 245.10 milliseconds (intercept) on Day 2 and increases by 11.44 milliseconds for each additional day of sleep deprivation (fixed effect for Days). The random effect indicates that baseline reaction times vary among participants with a standard deviation of 41.80 milliseconds, while the residual variability (unexplained within-subject differences) is 30.22 milliseconds.

6 Residual Analysis

The residual plots shown in figure 4 provide diagnostic tools to assess the assumptions of the linear mixed-effects model. These plots evaluate the distribution of residuals, the normality assumption, and the consistency of variance across fitted values. By examining these visualizations, we can determine the adequacy of the model and identify any potential violations of underlying assumptions.

The histogram of residuals (left panel) shows that the residuals are approximately symmetrically distributed around zero, suggesting that the residuals follow a roughly normal distribution. However, slight skewness may be observed at the tails. The normal Q-Q Plot (middle panel) compares the distribution of residuals to a theoretical normal distribution. Most points lie close to the diagonal line, indicating that the normality assumption is reasonably satisfied, though minor deviations at the extremes suggest slight non-normality. The residuals vs. fitted values (right panel) assesses the homoscedasticity (constant variance) assumption. The residuals appear to be randomly scattered around zero without a clear pattern, suggesting that the variance of residuals is consistent across fitted values.

Overall, the diagnostics suggest the model assumptions are reasonably met, but slight deviations at the extremes may warrant further investigation.

The R code used for this analysis is available on GitHub at the following link: Project Assignment on Mixed Effects Models in R.

7 Conclusion

Sleep deprivation significantly increases reaction times, with an average increase of 11.44ms per day from Day 2 onwards. This highlights the progressive degradation of psychomotor performance due to sleep loss, emphasizing the importance of adequate rest for tasks requiring sustained attention and quick responses.

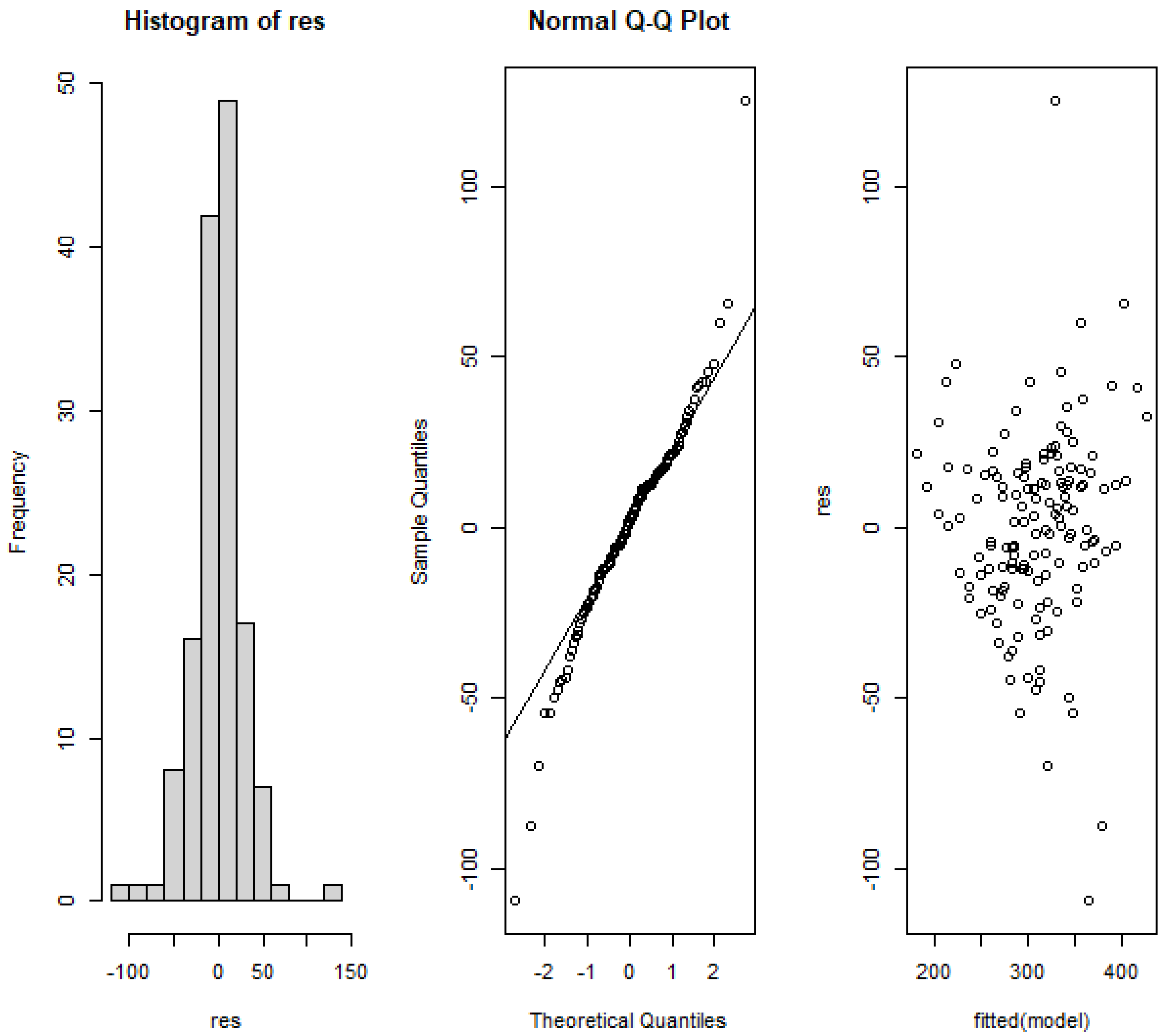

Figure 4: Residual Plot

References

[1] B. G. Wesensten, N. J. Thorne, D. R. Thomas, M. L. Sing, H. C Redmond, D. P Russo, M. B Balkin and J. Thomas, *Patterns of performance degradation and restoration during sleep restriction and subsequent recovery: A sleep dose-response study*, (Journal of sleep research, 12, 1, 1–12, 2003, Wiley Online Library).

[2] D. Bates, M. Mächler, B. M. Bolker, S. C. Walker *Fitting linear mixed-effects models using lme4*, (published online in the Journal of Statistical Software, on Oct. 2015, with DOI 10.18637/jss.v067.i01, see https://www.jstatsoft.org/ article/view/v067i01/.)