Math 1ZB3 Formula Sheet

Integrals:

$\int_1^\infty \frac{1}{x^p}\mathrm{d}x$ only converges if $p > 1$

Sequences & Series:

$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} a_{n\pm 1}$

Geometric series

$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ convergent if $|r| < 1$

P-series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ only converges if $p > 1$

Remainder estimate for integral test

$R_n = s - s_n$

$\int_{n+1}^{\infty} f(x)\mathrm{d}x \le R_n \le \int_n^{\infty} f(x)\mathrm{d}x$

Alternating Series Test

$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ converges if

(i) $b_{n+1} \le b_n$ for all n

(ii) $\lim_{n\to\infty} b_n = 0$

Alternating series estimate

$|R_n| = |s - s_n| \le b_{n+1}$

Ratio test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$

$L < 1$ convergent

$L > 1$ divergent

$L = 1$ no conclusion

Root test

$\lim_{n\to\infty} \sqrt[n]{|a_n|}$ same conditions as above

Strats for series testing:

1. Test for Divergence
2. p-Series
3. Geometric Series
4. Comparison Test
5. Alternating Series Test
6. Ratio Test
7. Root Test
8. Integral Test

Power series:

$\sum_{n=0}^{\infty} c_n x^n$

Taylor series of f at a

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

Taylor's Inequality

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$\left|f^{(n+1)}(x)\right| \le M \quad$ for $|x-a| \le d$

$\lim_{n\to\infty} \frac{x^n}{n!} = 0$

Important Maclaurin Series:

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \qquad R = 1$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \qquad R = \infty$

$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$

$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$

$\arctan x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \qquad R = 1$

$\ln 1 + x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$

$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$

Binomial Coefficient

$\binom{k}{n} = \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!}$

Area of a Surface of Revolution around:

x-axis: $S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$

y-axis: $S = \int_a^b 2\pi x\sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$

Hydrostatic Pressure and Force:

$F = mg = \rho g A d$

$P = \frac{F}{A} = \rho g d$

Differential Equations:

Seperable

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{g(x)}{h(y)} \to h(y)\mathrm{d}y = g(x)\mathrm{d}x$

Integrate $\uparrow$

Orthogonal trajectories

1. Get a single differential equation
2. Get negative reciprocal of slope
3. Solve differential equation

Mixing Problems

$\frac{\mathrm{d}y}{\mathrm{d}t} = (\text{rate in}) - (\text{rate out})$

Solve differential then use initial value to find c

Newton's Law of Cooling

$\frac{\mathrm{d}T}{\mathrm{d}t} = k(T - T_s)$

Initial-value problem

$\frac{\mathrm{d}P}{\mathrm{d}t} = kP \quad P(0) = P_0 \quad$ solution: $P(t) = P_0 e^{kt}$

Linear Differential Equations

$\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x) \qquad I(x) = e^{\int P(x)\mathrm{d}x}$

1. Multiply both sides by integration factor I(x)
2. Integrate both sides

Parametrics:

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ if $\frac{\mathrm{d}x}{\mathrm{d}t} \ne 0$

Arc Length

$L = \int_\alpha^\beta \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$

Surface Area

$S = \int_\alpha^\beta 2\pi y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$

Polar and Cartesian:

$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$

Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ and x is near a and

$\lim_{x\to a} f(x) \le \lim_{x\to a} h(x) = L$

then $\quad \lim_{x\to a} g(x) = L$

Multi Variable:

For partial derivatives, consider all variables but one constant and differentiate

Tangent plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Linearization

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

Linear approximation

$f(x,y) \approx L(x,y)$

Differentials

$dz = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$

Chain Rule Case 1

$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}$

Gradient

$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y)\rangle$

Directional directive

$D_u f(x,y) = \nabla f(x,y) \cdot \boldsymbol{u}$

Double integral over rectangle

$\iint_R f(x,y)\mathrm{d}A = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$

Volume of solid

$V = \iint_R f(x,y)\mathrm{d}A$