MODEL OF CAR

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CONTENTS

NOTATION

β	steering angle
$\mu(s) = \sum_{k=0}^{2} \bar{\mu}_k s^k$	static friction coefficient
r	osculating circle
ω	velocity of steering angle
w	weather

1 ODE

If the maximum static friction $F_s(t, \bar{\mu}) = \sum_{k=0}^{2} \bar{\mu}_k w(t)^k mg$ is greater than or equal $F_{res}(t) = \sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car does not slide. If the maximum static friction $F_s(t, \bar{\mu})$ is less than $F_{res}(t)$ the car slides. The dynamics reads as

$$
\begin{aligned}
\dot{y} &= v(t)\cos\beta(t) - v_r(t)\sin\beta(t)\\
\dot{z} &= v(t)\sin\beta(t) + v_r(t)\cos\beta(t)\\
\dot{v} &= \frac{1}{m}\left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2}c_w\rho A v(t)^2 - \left(f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4\right)mg\right)\\
\dot{v}_r &= \begin{cases} \frac{1}{m}\left(\frac{v(t)^2}{r(t)}\frac{1}{\dot{v}}\left(F_{res}(t) - F_s(t)\right)\right) & \text{if } F_{res}(t) - F_s(t) > 0, r(t) \neq 0, \dot{v} \neq 0\\ 0 & \text{else} \end{cases}\\
\dot{\beta} &= \omega_\beta.
\end{aligned}
$$

Since $\frac{v(t)^2}{r(t)} \ll \dot{v}$, I assumed that $\sin\alpha(t) \approx \tan\alpha(t)$, where $\tan\alpha(t) = \frac{v(t)^2}{\dot{v}r(t)}$ and $\sin\alpha(t) = \frac{F_r}{F_{res}(t) - F_s(t)}$. Therefore, $F_r = m\dot{v}_r = \left(\frac{v(t)^2}{r(t)}\frac{1}{\dot{v}}\left(F_{res}(t) - F_s(t)\right)\right)$.

$\sin(\arctan(\alpha)) = \frac{\alpha}{\sqrt{\alpha^2+1}}$