MODEL OF CAR

JOHANNES MILZ

CONTENTS

NOTATION

β	steering angle
$\mu(s) = \sum_{k=0}^{2} \bar{\mu}_k s^k$	static friction
r	osculating circle
ω	velocity of steering angel
w	weather

1 ODE

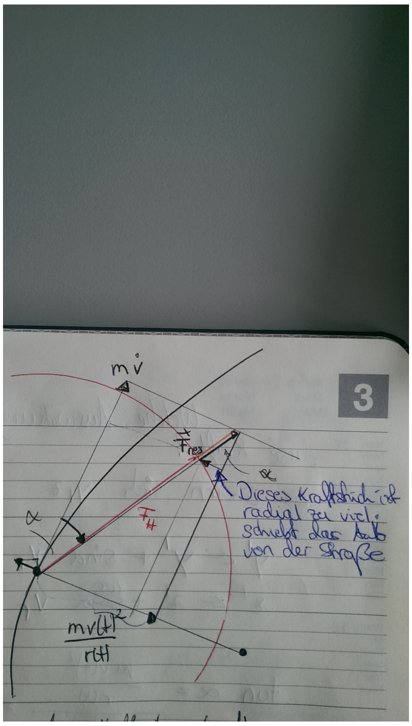
If the maximum static friction $\mu(w(t))mg$ is greater than or equal $\sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car does not slide. The dynamics reads as

$$\begin{aligned}
\dot{y} &= v(t)\cos\beta(t) \\
\dot{z} &= v(t)\sin\beta(t) \\
\dot{v} &= \frac{1}{m}\left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2}c_w\rho A v(t)^2 - \left(f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4\right) mg\right) \\
\dot{\beta} &= \omega_\beta
\end{aligned}$$

If the maximum static friction is less than $F_{res}(t) = \sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car slides. The rolling friction $\left(f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4\right) mg$ "turns into" dynamic friction $\mu_d mg$. For simplicity, we could assume that the coefficient μ_d is constant. However, $\mu_d \approx 0.5 \cdot \mu(w(t))$, in general. The dynamics reads as

$$\begin{aligned}
\dot{y} &= v(t)\cos\beta(t) + v_r(t)\sin(\frac{\pi}{2} + \alpha(t) + \beta(t)) \\
\dot{z} &= v(t)\sin\beta(t) + v_r(t)\cos(\frac{\pi}{2} + \alpha(t) + \beta(t)) \\
\dot{v} &= \frac{1}{m}\left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2}c_w\rho A v(t)^2 - \mu_d mg\right) \\
\dot{\beta} &= \omega_\beta,
\end{aligned}$$

where $\tan\alpha(t) = \frac{v(t)^2}{\dot{v}r(t)}$ and $\sin\alpha(t)(F_{res}(t) - \mu(w(t))mg) = \frac{mv_r(t)^2}{r(t)}$.


$m\dot{v}$
3
$\vec{F}_{res}$
α
F_H
α
Dieses Kraftstück ist
radial zu viel:
schiebt das Auto
von der Straße
$\frac{mv(t)^2}{r(t)}$