MODEL OF CAR

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CONTENTS

NOTATION

β	steering angle
$\mu(s) = \sum_{k=0}^{2} \bar{\mu}_k s^k$	static friction coefficient
r	osculating circle
ω	velocity of steering angle
w	weather

1 ODE

If the maximum static friction $F_s(t, \bar{\mu}) = \sum_{k=0}^{2} \bar{\mu}_k w(t)^k mg$ is greater than or equal $F_{res}(t) = \sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car does not slide. If the maximum static friction $F_s(t, \bar{\mu})$ is less than $F_{res}(t)$ the car slides. The rolling friction is approximately half of the static friction. The dynamics reads as

$$
\begin{aligned}
\dot{y} &= v(t)\cos\beta(t) - v_r(t)\sin\beta(t) \\
\dot{z} &= v(t)\sin\beta(t) + v_r(t)\cos\beta(t) \\
\dot{v} &= \frac{1}{m}\left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2}c_w\rho A v(t)^2 - \left(f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4\right) mg\right) \\
\dot{v}_r &= \begin{cases} \left(\frac{v(t)^2}{r(t)} - \frac{1}{2}\sum_{k=0}^{2} \bar{\mu}_k w(t)^k g\right) & \text{if } F_{res}(t) - F_s(t) > 0, r(t) \neq 0, \\ 0 & \text{else} \end{cases} \\
\dot{\beta} &= \omega_\beta.
\end{aligned}
$$

Smoothing approach:

$$
\dot{v}_r = \begin{cases} \left(\frac{F_{res}(t) - F_s(t)}{5m}\right)^3 \left(\frac{v(t)^2}{r(t)} - \frac{1}{2}\sum_{k=0}^{2} \bar{\mu}_k w(t)^k g\right) & \text{if } F_{res}(t) - F_s(t) > 0, r(t) \neq 0, \\ 0 & \text{else} \end{cases}
$$

Since $F_{res} - F_s \leq 0 \Leftrightarrow F_{res}^2 - F_s^2 \leq 0$, I would implement (in view of nicer derivatives)

$$
\dot{v}_r = \begin{cases} \left(\frac{F_{res}^2(t) - F_s^2(t)}{120m}\right)^3 \left(\frac{v(t)^2}{r(t)} - \frac{1}{2}\sum_{k=0}^{2} \bar{\mu}_k w(t)^k g\right) & \text{if } F_{res}^2(t) - F_s^2(t) > 0, r(t) \neq 0 \\ 0 & \text{else} \end{cases}
$$