Robust Optimization

Annkathrin Krämmer, Sabina Przioda, Christian Kreipl, Johannes Milz

Technische Universität München

May 23, 2015

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Robust Optimization

test

Robust Optimization

test

Model of Car

Model of Car

- Bla

Das ist eine Bemerkung.

Problem Formulation - Constraints for Parameters

$$\min_{x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u}} f_0(x, u)$$
$$s.t. f_i(x, u) \leq 0 fori = 1, ..., n_f$$
$$g_j(x, u, p) = 0 forj = 1, ..., n_x$$

with a uncertain parameter vector $p \in \mathbb{R}^{n_p}$.

$$\mathbb{P}_{box} = \{p | \, p_l \leq p \leq p_u\}$$

p_l: lower bound, p_u: upper bound
weather: $\{p | \, 0 \leq p \leq 1\}$
height profil: $\{p | \, 0 \leq p \leq 45\}$

Problem Formulation - Constraints for Parameters
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Problem Formulation - Constraints for Parameters

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Problem Formulation - Worst Case Formulation

EXAMPLE?

$$\begin{aligned} \Phi_i(u) = \max_{x \in \mathbb{R}^{n_x}, p \in \mathbb{R}^{n_p}} & f_i(x,u) \\ s.t. g(x,u,p) &= 0 \\ p &\in \mathbb{P}_{box} \end{aligned}$$

Robust counterpart

$$\begin{aligned} \min_{u \in \mathbb{R}^{n_u}} & \Phi_0(u) \\ s.t. \Phi_i(u) \leq 0 \forall i &= 1, ..., n_f \end{aligned}$$

$\Rightarrow$ bilevel structure!

Problem Formulation - Worst Case Formulation
EXAMPLE?
$\Phi_i(u) = \max_{x \in \mathbb{R}^{n_x}, p \in \mathbb{R}^{n_p}} f_i(x,u)$
$s.t. g(x,u,p) = 0$
$p \in \mathbb{P}_{box}$
Robust counterpart
$\min_{u \in \mathbb{R}^{n_u}} \Phi_0(u)$
$s.t. \Phi_i(u) \leq 0 \forall i = 1, ..., n_f$
$\Rightarrow$ bilevel structure!

Problem Formulation - Approximation Technique

Linearization

$$\begin{aligned}
\tilde{\Phi}_i(u) = \max_{(x-\bar{x})\in\mathbb{R}^{n_x},(p-\bar{p})\in\mathbb{R}^{n_p}} & f_i(\bar{x},u) + \frac{\partial f_i}{\partial x}(\bar{x},u)(x-\bar{x}) \\
s.t. \frac{\partial g}{\partial x}(\bar{x},u,\bar{p})(x-\bar{x}) & + \frac{\partial g}{\partial p}(\bar{x},u,\bar{p})(p-\bar{p}) = 0 \\
& p-\bar{p} s.th. p \in \mathbb{P}_{box}
\end{aligned}$$

$$\begin{aligned}
\min_{u\in\mathbb{R}^{n_u},\bar{x}\in\mathbb{R}^{n_x}} & \tilde{\Phi}_0(u) \\
s.t. \tilde{\Phi}_i(u) \leq 0 \forall i = 1, ..., n_f \\
g(\bar{x},u,\bar{p}) = 0
\end{aligned}$$

$\Rightarrow$ Standard Optimization Problem (SQP or else)

Problem Formulation - Approximation Technique

Linearization

$$\begin{aligned}\tilde{\Phi}_i(u) = \max_{(x-\bar{x})\in\mathbb{R}^{n_x},(p-\bar{p})\in\mathbb{R}^{n_p}} f_i(\bar{x},u) + \frac{\partial f_i}{\partial x}(\bar{x},u)(x-\bar{x})\\ s.t. \frac{\partial g}{\partial x}(\bar{x},u,\bar{p})(x-\bar{x}) + \frac{\partial g}{\partial p}(\bar{x},u,\bar{p})(p-\bar{p}) = 0\\ p-\bar{p} s.th. p \in \mathbb{P}_{box}\end{aligned}$$

$$\begin{aligned}\min_{u\in\mathbb{R}^{n_u},\bar{x}\in\mathbb{R}^{n_x}} \tilde{\Phi}_0(u)\\ s.t. \tilde{\Phi}_i(u) \leq 0 \forall i = 1,...,n_f\\ g(\bar{x},u,\bar{p}) = 0\end{aligned}$$

$\Rightarrow$ Standard Optimization Problem (SQP or else)

======= »»»> origin/master

Discretization

- Why?

Discretization

- Why?

Discretization

- Why?

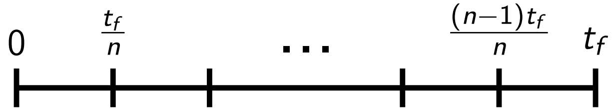
  infinte dimensional optimization problem

Discretization
- Why?
  infinte dimensional optimization problem

Discretization

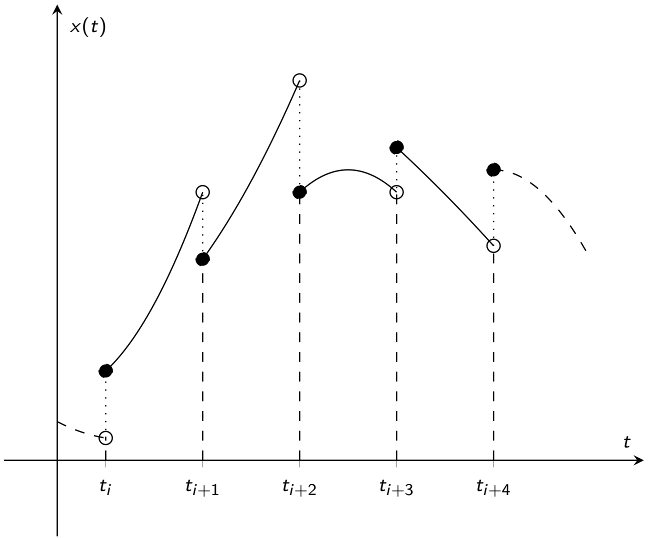
- Why?

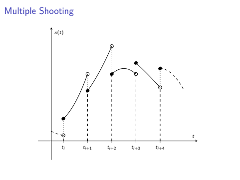
  infinte dimensional optimization problem
- How?



Discretization
- Why?
  infinte dimensional optimization problem
- How?

Multiple Shooting





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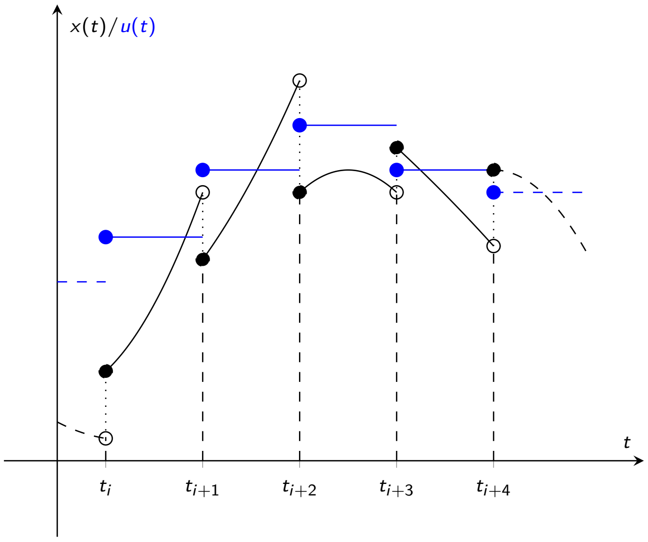

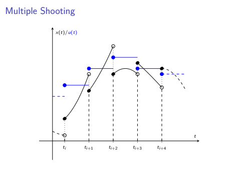

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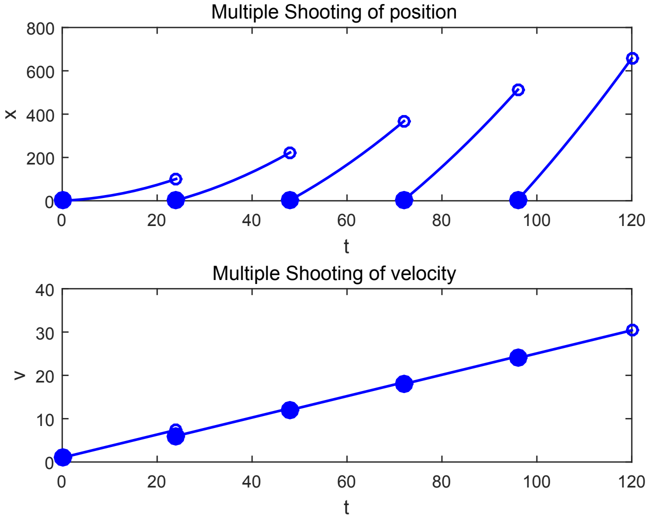

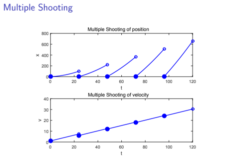

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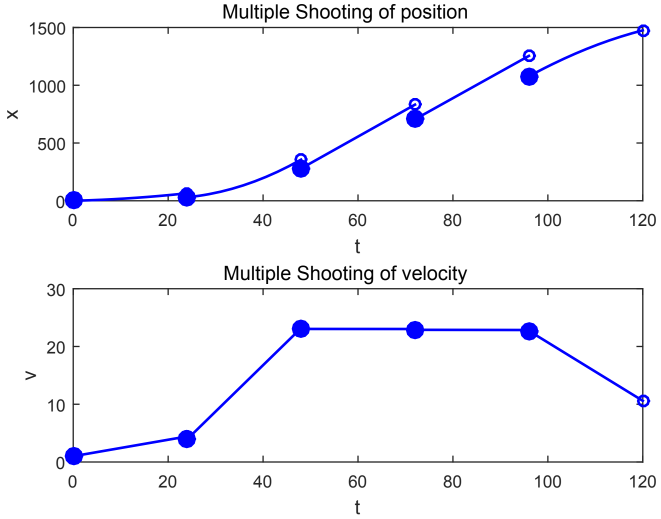

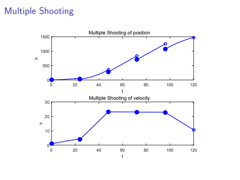

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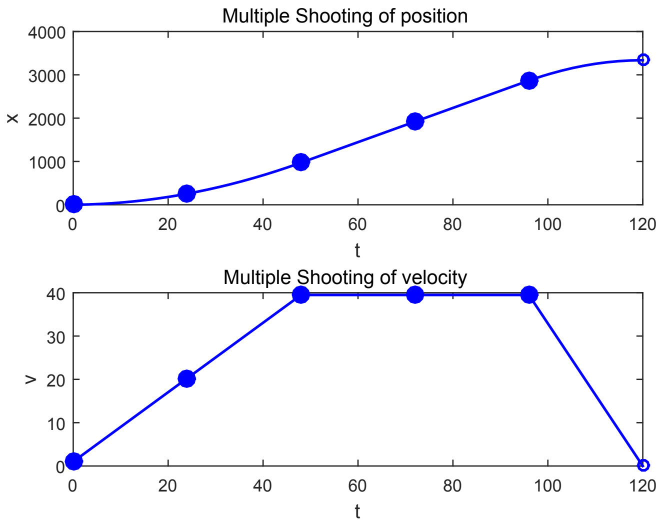

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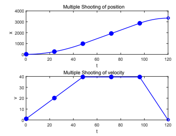

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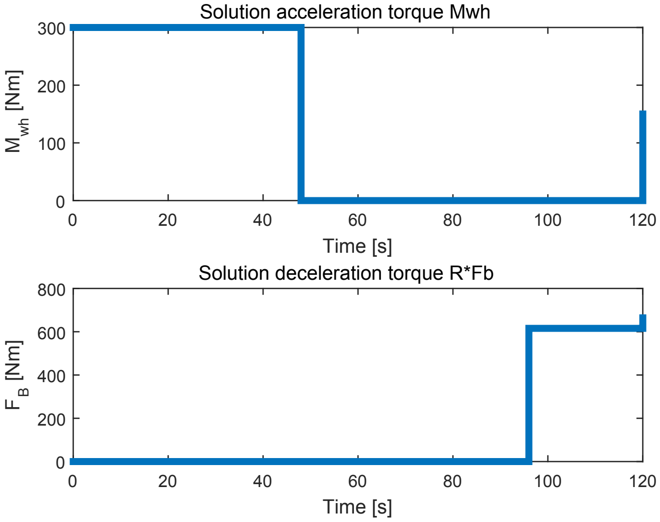

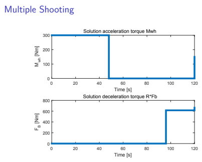

Visualization