

MODEL OF CAR

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CONTENTS

1 ODE

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NOTATION

| | |
|-----------------------------------------|----------------------------|
| β | steering angle |
| $\mu(s) = \sum_{k=0}^2 \bar{\mu}_k s^k$ | static friction |
| r | osculating circle |
| ω | velocity of steering angel |
| w | weather |

1 ODE

If the maximum static friction $\mu(w(t))mg$ is greater than or equal $\sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car does not slide. The dynamics reads as

$$\begin{aligned}\dot{y} &= v(t) \cos \beta(t) \\ \dot{z} &= v(t) \sin \beta(t) \\ \dot{v} &= \frac{1}{m} \left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2} c_w \rho A v(t)^2 - (f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4) mg \right) \\ \dot{\beta} &= \omega_\beta\end{aligned}$$

If the maximum static friction is less than $F_{res}(t) \sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car slides. The rolling friction $(f_{R0} + f_{R1}v(t) + f_{R4}v(t)^4) mg$ "turns into" dynamic friction $\mu_d mg$. For simplicity, we could assume that the coefficient μ_d is constant. However, $\mu_d \approx 0.5 \cdot \mu(w(t))$, in general. The dynamics reads as

$$\begin{aligned}\dot{y} &= v(t) \cos \beta(t) + v_r(t) \sin \alpha(t) \\ \dot{z} &= v(t) \sin \beta(t) + v_r(t) \cos \alpha(t) \\ \dot{v} &= \frac{1}{m} \left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2} c_w \rho A v(t)^2 - \mu_d mg \right) \\ \dot{\beta} &= \omega_\beta,\end{aligned}$$

where $\tan \alpha(t) = \frac{v(t)^2}{\dot{v}r(t)}$ and $\sin \alpha(t)(F_{res}(t) - \mu(w(t))mg) = \frac{mv_r(t)^2}{r(t)}$.