

Robust Optimization

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Johannes Milz

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- 6 Visualization

- Our work is divided into 4 parts
 - Model of a Car
 - Mathematical problem formulation
 - Implementation
 - Visualization

Motivation I

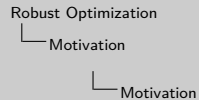
- steering a car
- minimize fuel consumption
- constraints
 - avoid crashes
 - dynamics

Problem

Dynamics change considerably, e.g. for different weather.

→ Robust Optimization

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Motivation I

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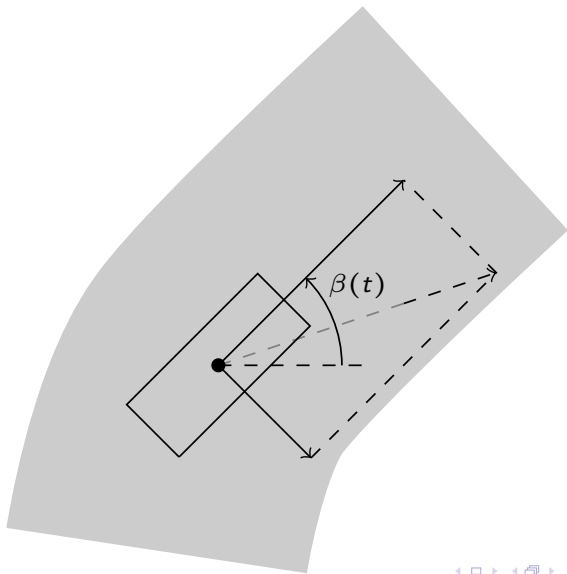
Problem

Dynamics change considerably, e.g. for different weather.

→ Robust Optimization

- Our goal is to steer a car such that the fuel consumption is minimal
- Constraints: Avoid Crashes
 - our car is not allowed to leave the road
 - hit other cars or construction sites
- Dynamics of the car
 - motion of the car
 - slipping behavior
 - acceleration behavior
- Problem: Changing weather has a major impact on the slipping behavior
 - we cannot drive fast on icy roads
 - Rain causes aquaplaning
- Way out: Robust Optimization
 - This Optimization method takes changing parameters such as weather into account.
 - Car is steered save for changing conditions such as weather, different roads...

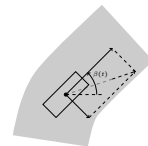
Model of Car



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Robust Optimization
└─ Model of Car
 └─ Model of Car

Model of Car



- Our car is described as a pointmass, for simplicity
- Road is visualized in gray
- Car as a rectangle
- steering angle is denoted by $\beta(t)$
 - We applied Newton's mechanic
 - The motion/ direction of motion is described with
 - The steering angle $\beta(t)$
 - The acceleration force pushing the car forward
 - The central forces
 - If the speed of the car is too high, it slides and may leave the road.

Dynamical System

$$\dot{x}(t) = G(x(t), u(t), p) \quad x(0) = x_0,$$

where

$x \in C^1([0, t_f], \mathbb{R}^n)$ is the state variable,
 $u \in C^1([0, t_f], \mathbb{R}^m)$ is the control variable,
 $p \in \mathbb{R}^{n_p}$ is the uncertain parameter vector.

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Robust Optimization

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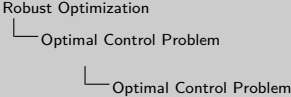
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- Motion of the car is described as a dynamical system, i.e., with a ODE and an initial value
- t_f is the final time of our ride
- our states: path, velocity, steering angle
- control: brake, motor torque (gas pedal), steering angle velocity
- parameter: weather, rolling friction coefficients, uncertain sea level due to wrong NAVI data

Optimal Control Problem

$$\begin{aligned} \min_{x(\cdot) \in \mathbb{R}^{n_x}, u(\cdot) \in \mathbb{R}^{n_u}} & f_0(x, u) \\ \text{s.t.} \quad & \dot{x}(t) = G(x(t), u(t), p) \quad \forall t \in [0, t_f], \\ & x(0) = x_0, \\ & f_i(x(t), u(t)) \leq 0 \quad \text{for } i = 1, \dots, n_f, \\ & \quad \quad \quad \forall t \in [0, t_f]. \end{aligned}$$

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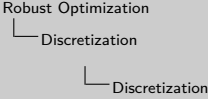
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Discretization

- infinte dimensional optimization problem

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Discretization

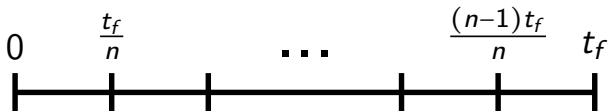
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What do we approximate?
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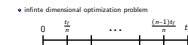
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Robust Optimization
└─ Discretization
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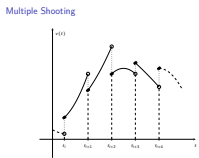
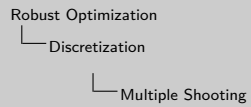
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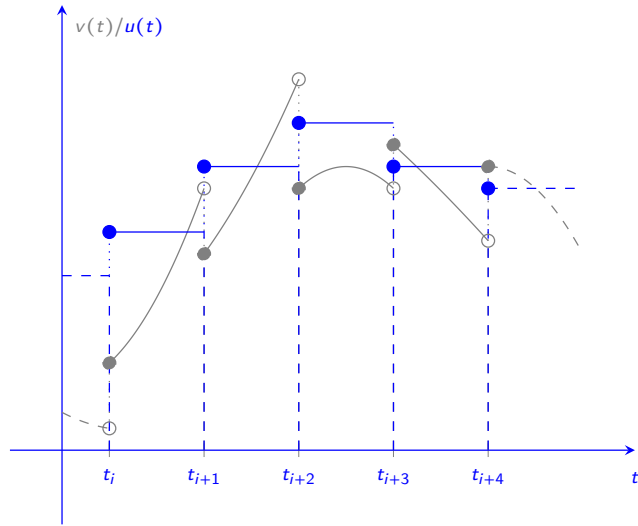
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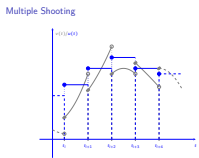
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Multiple Shooting



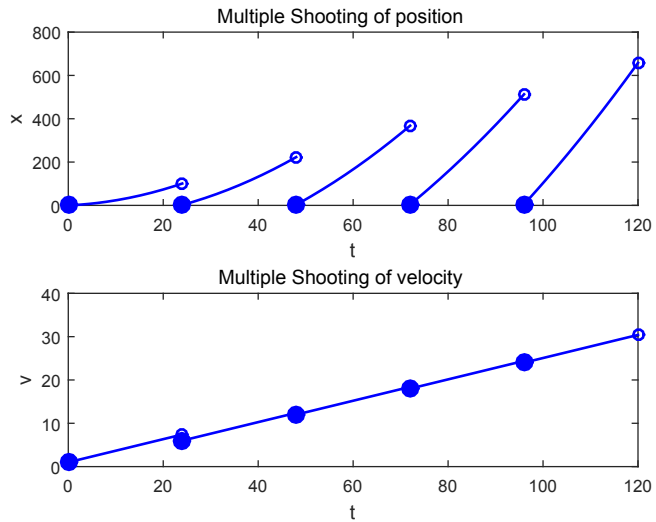
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Robust Optimization
└─ Discretization
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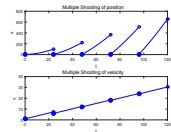
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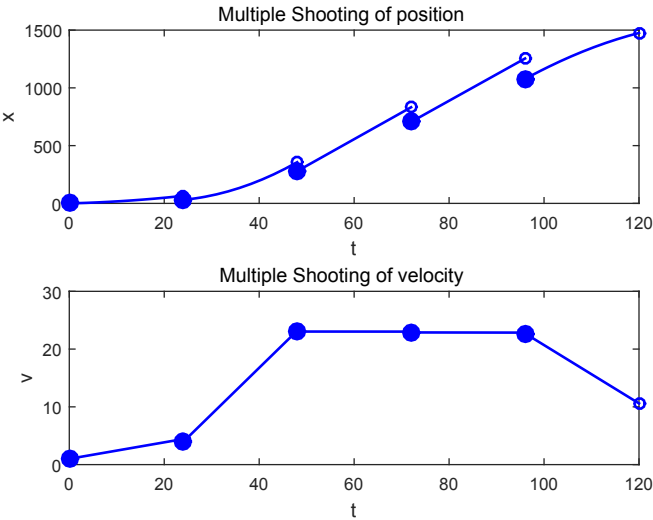
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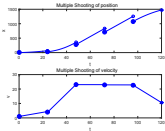
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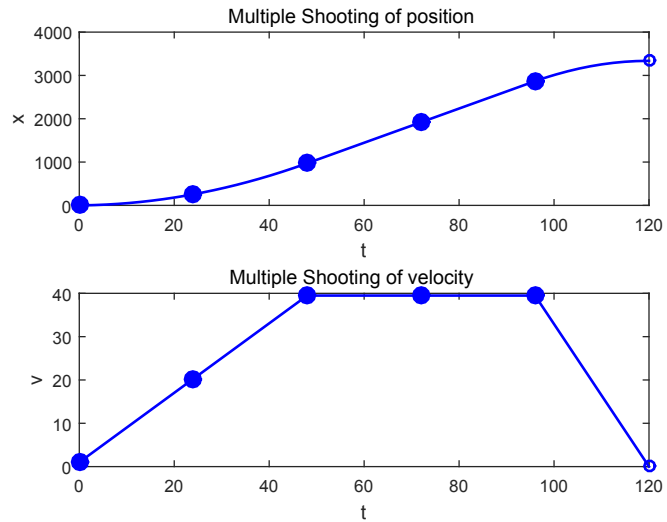
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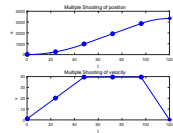
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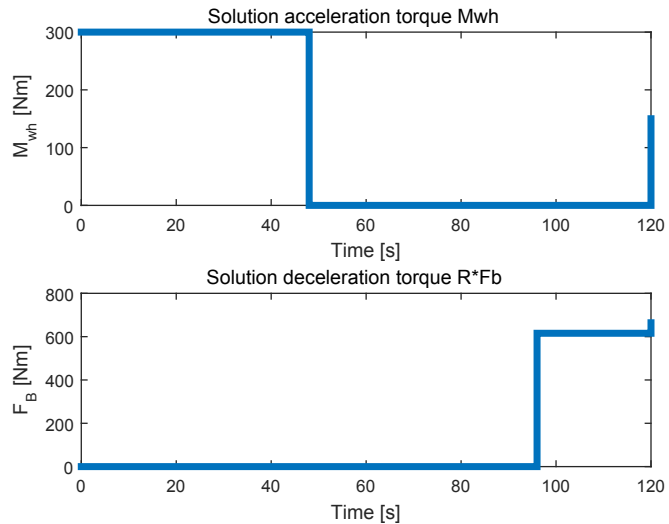
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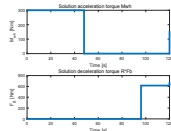
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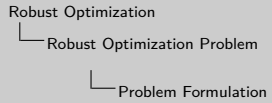
Problem Formulation

Discrete Optimization Problem

$$\begin{aligned} & \min_{x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u}} f_0(x, u) \\ \text{s.t.} \quad & f_i(x, u) \leq 0 \quad \text{for } i = 1, \dots, n_f \\ & g_j(x, u, p) = 0 \quad \text{for } j = 1, \dots, n_g \end{aligned}$$

with $p \in \mathbb{R}^{n_p}$ uncertain parameter vector

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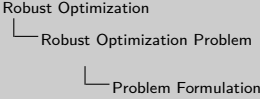
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$$\text{with } p \in \mathbb{P}_{\text{box}} = \{ p \in \mathbb{R}^{n_p} \mid p_{\text{lower}} \leq p \leq p_{\text{upper}} \}$$

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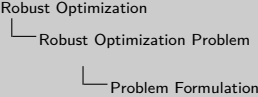
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Worst Case Formulation

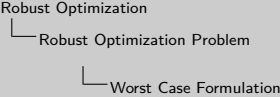
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Robust Counterpart

$$\begin{aligned} &\min_{u \in \mathbb{R}^{n_u}} \Phi_0(u) \\ \text{s.t.} \quad &\Phi_i(u) \leq 0 \quad \text{for } i = 1, \dots, n_f. \end{aligned}$$

⇒ bilevel structure!

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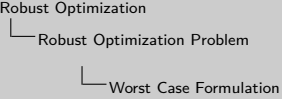
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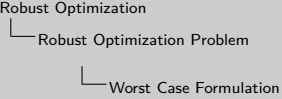
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Approximation Technique

Linearization

at a point (\bar{x}, u, \bar{p}) with $g(\bar{x}, u, \bar{p}) = 0$ and $\bar{p} = \frac{p_{lower} + p_{upper}}{2}$:

$$\begin{aligned} \tilde{\Phi}_i(u) := & \max_{(x-\bar{x}) \in \mathbb{R}^{n_x}, (p-\bar{p}) \in \mathbb{R}^{n_p}} f_i(\bar{x}, u) + \frac{\partial f_i}{\partial x}(\bar{x}, u)(x - \bar{x}) \\ \text{s.t.} \quad & \frac{\partial g}{\partial x}(\bar{x}, u, \bar{p})(x - \bar{x}) + \frac{\partial g}{\partial p}(\bar{x}, u, \bar{p})(p - \bar{p}) = 0 \\ & p \in \mathbb{P}_{box} \end{aligned}$$

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Robust Optimization

└ Robust Optimization Problem

└ Approximation Technique

Approximation Technique

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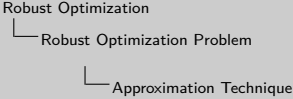
With a unique optimal solution $\tilde{\Phi}_i(u)$:

Standard Optimization Problem

$$\begin{aligned} \min_{u \in \mathbb{R}^{n_u}, \bar{x} \in \mathbb{R}^{n_x}} \quad & \tilde{\Phi}_0(u) \\ \text{s.t.} \quad & \tilde{\Phi}_i(u) \leq 0 \quad \text{for } i = 1, \dots, n_f \\ & g(\bar{x}, u, \bar{p}) = 0 \end{aligned}$$



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Approximation Technique

Linearization

at a point (\bar{x}, u, \bar{p}) with $g(\bar{x}, u, \bar{p}) = 0$ and $\bar{p} = \frac{p_{lower} + p_{upper}}{2}$:

$$\begin{aligned} \tilde{\Phi}_i(u) := & \max_{(x-\bar{x}) \in \mathbb{R}^{n_x}, (p-\bar{p}) \in \mathbb{R}^{n_p}} f_i(\bar{x}, u) + \frac{\partial f_i}{\partial x}(\bar{x}, u)(x - \bar{x}) \\ \text{s.t.} \quad & \frac{\partial g}{\partial x}(\bar{x}, u, \bar{p})(x - \bar{x}) + \frac{\partial g}{\partial p}(\bar{x}, u, \bar{p})(p - \bar{p}) = 0 \\ & p \in \mathbb{P}_{box} \end{aligned}$$

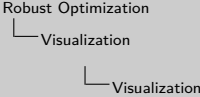
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What do we want?

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Visualization

What do we want?

- graphs contain much information
- videos catch people where they are
- Grass tiles
- Make car shiny, windows glassy
- Curves are nice to see
- this is just an arbitrary acceleration
- This 10 seconds clip already rendered 30 min
- min 18FPS needed

Visualization

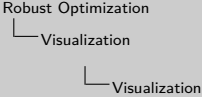
- sell our results to you
- nice graphs
- rendered video



- graphs contain much information
- videos catch people where they are
- Grass tiles
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What can we improve?

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Visualization

What can we improve?

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Visualization

- textures
- nonlinear street design
- fit acceleration to optimization results
- fiit resolution to output device
- follow the car with the camera

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Robust Optimization

- Visualization

- Visualization

Visualization

- ▀ textures
- ▀ nonlinear street design
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Robust Optimization
└ Visualization
└ References

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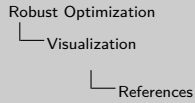
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