MODEL OF CAR

JOHANNES

CONTENTS

NOTATION

β	steering angle
$\mu(s) = \sum_{k=0}^{2} \bar{\mu}_k s^k$	static friction coefficient
r	osculating circle
ω	velocity of steering angle
w	weather

1 ODE

If the maximum static friction $F_s(t, \bar{\mu}) = \sum_{k=0}^{2} \bar{\mu}_k w(t)^k mg$ is greater than or equal $F_{res}(t) = \sqrt{\left(\frac{mv(t)^2}{r(t)}\right)^2 + (m\dot{v})^2}$, the car does not slide. If the maximum static friction $F_s(t, \bar{\mu})$ is less than $F_{res}(t)$ the car slides. The rolling friction is denoted by $\mu_r mg$. The dynamics reads as

$$
\begin{aligned}
\dot{y} &= v(t) \cos\beta(t) - v_r(t) \sin\beta(t) \\
\dot{z} &= v(t) \sin\beta(t) + v_r(t) \cos\beta(t) \\
\dot{v} &= \frac{1}{m}\left(\frac{M(t)}{R} - F_B(t) - \frac{1}{2} c_w \rho A v(t)^2 - \left(f_{R0} + f_{R1} v(t) + f_{R4} v(t)^4\right) mg\right) \\
\dot{v}_r &= \begin{cases} \left(\frac{F_{res}^2(t) - F_s^2(t)}{c_3 m^2}\right)^5 \phi(v(t)) & \text{if } F_{res}^2(t) - F_s^2(t) \geq 0, r(t) \neq 0 \\ \phi(v(t)) & \text{if } c_3 m^2 \geq F_{res}^2(t) - F_s^2(t) \geq 0, r(t) \neq 0 \\ 0 & \text{if } r(t) \neq 0, \end{cases} \\
\dot{\beta} &= \omega_\beta,
\end{aligned}
$$

$$
G(x, u, p) = \begin{bmatrix} \text{objectfun} \\ x_4 \cos x_6 - x_5 \sin x_6 \\ x_4 \sin x_6 + x_5 \cos x_6 \\ \chi(x, u, p) \\ \begin{cases} \left(\frac{\omega(x,y,p)}{c_3}\right)^5 \phi(x_4) & \text{if } \omega(x, y, p) \geq 0, \\ \phi(x_4) & \text{if } c_3 \geq \omega(x, y, p) \geq 0, \\ 0 & \text{else} \end{cases} \\ u_3 \end{bmatrix},
$$

where

$$\chi(x,u,p) = \frac{1}{m}\left(\frac{u_1}{R} - u_2 - F_A(x_4)\right) - \tilde{F}_R(x,p)$$
$$\phi(x_4) = \frac{x_4^2}{r} - \mu_r g$$
$$\psi(x_4,x_5) = \left(\frac{x_4^2}{r}\right)^2 + x_4^2$$
$$\omega(x,y,p) = \frac{F_{res}^2(t) - F_s^2(t)}{m^2} = \left(\frac{x_4^2}{r(t)}\right)^2 + \chi(x,u,p)^2 - \left(\sum_{k=0}^{2} \bar{\mu}_k w(t)^k g\right)^2$$
$$= \left(\frac{x_4^2}{r(t)}\right)^2 + \chi(x,u,p)^2 - \left(p_4 + p_5 w(t) + p_6 w(t)^2\right)^2 g^2$$
$$F_A(x_4) = \frac{1}{2} c_w \rho A x_4^2$$
$$\tilde{F}_R(x,p) = \left(p_1 + p_2 x_4 + p_3 x_4^4\right) g$$
$$\text{objectfun}(x,u,p) = c_1 F_A(x_4) x_4 + \frac{u_1}{R} x_4 + F_R(p) x_4 - c_2 x_4$$
$$x = \left(x_1 \ \dots \ x_6\right)^T$$
$$u = \left(u_1 \ u_2 \ u_3\right)^T$$
$$p = \left(p_1 \ \dots \ p_6\right)^T$$