Robust Optimization of a Car

Annkathrin Krämmer, Sabina Przioda, Christian Kreipl, Johannes Milz

Technische Universität München

May 27, 2015

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- Our work is divided into 4 parts
  - Model of a Car
  - Mathematical problem formulation
  - Implementation
  - Visualization

Motivation

- steering a car
- minimize fuel consumption
- constraints
  - dynamics
  - avoid crashes

Problem

Dynamics change considerably, e.g., for different weather.

→ Robust Optimization

Motivation

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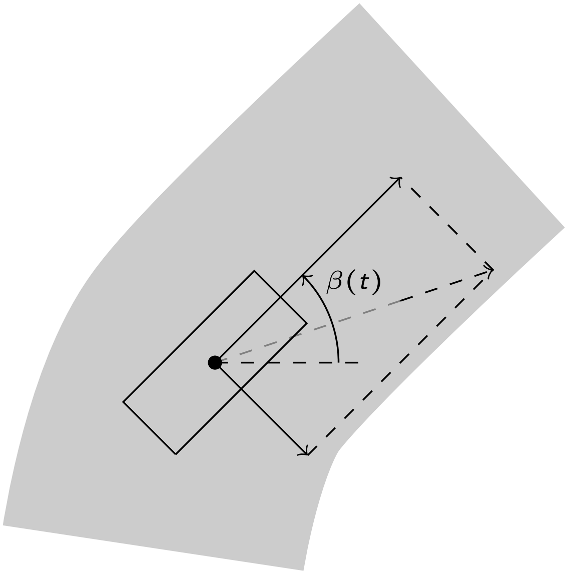
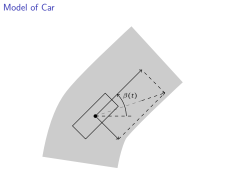
Problem
Dynamics change considerably, e.g., for different weather.

→ Robust Optimization

- Our goal is to steer a car such that the fuel consumption is minimal
- Contraints: Avoid Crashes
  - our car is not allowed to leave the road
  - hit other cars or construction sites
- Dynamics of the car
  - motion of the car
    - slipping behavior
    - acceleration behavior
- Problem: Changing weather has a major impact on the slipping behavior
  - we cannot drive fast on icy roads
  - Rain causes aquaplaning
- Way out: Robust Optimization
  - This Optimization method takes changing parameters such as

Model of Car





- Our car is described as a pointmass, for simplicity
- Road is visualized in gray
- Car as a rectangle
- steering angle is denoted by $\beta(t)$
  - We applied Newton's mechanic
  - The motion/ direction of motion is described with
    - The steering angle $\beta(t)$
    - The acceleration force pushing the car forward
    - The central forces
    - If the speed of the car is to high, it slides and may leave the road.

Dynamical System

$$\dot{x}(t) = G(x(t), u(t), p) \qquad x(0) = x_0,$$

where

$x \in C^1([0, t_f], \mathbb{R}^n)$ is the state variable,
$u \in C^1([0, t_f], \mathbb{R}^m)$ is the control variable,
$p \in \mathbb{R}^{n_p}$ is the uncertain parameter vector.

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- Motion of the car is described as a dynamical system, i.e., with a ODE and an initial value
- t_f is the final time of our ride
- our states: path, velocity, steering angle
- control: brake, motor torque (gas petal), steering angle velocity
- parameter: weather, rolling friction coefficients, uncertain sea level due to wrong NAVI data

Optimal Control Problem

$$
\begin{aligned}
&\min_{x(\cdot)\in\mathbb{R}^{n_x},\,u(\cdot)\in\mathbb{R}^{n_u}} f_0(x,u) && \\
\text{s.t.}\quad &\dot{x}(t) = G(x(t),u(t),p) && \forall t \in [0,t_f],\\
&x(0) = x_0, && \\
&f_i(x(t),u(t)) \le 0 && \text{for } \forall t \in [0,t_f],\ i = 1,\dots,n_f
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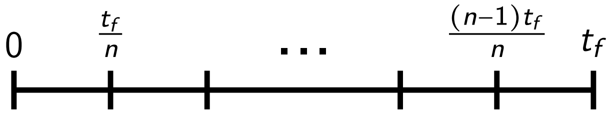
Discretization

- infinte dimensional optimization problem
  ➔ approximate the optimal control problem



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infinte dimensional optimization problem
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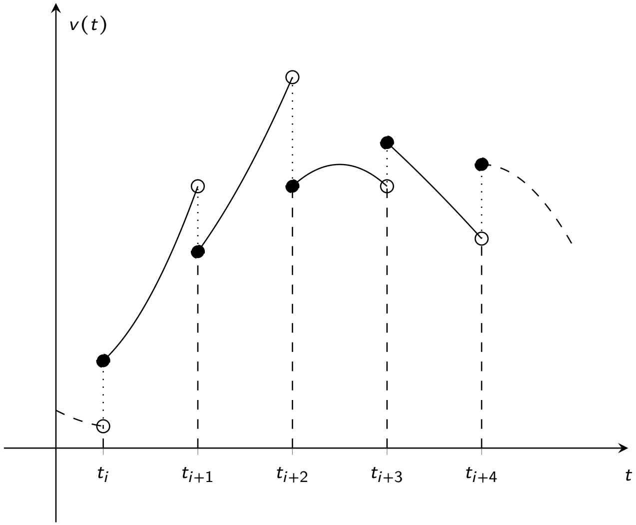
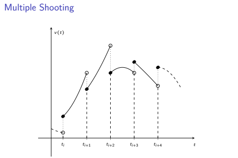
Multiple Shooting





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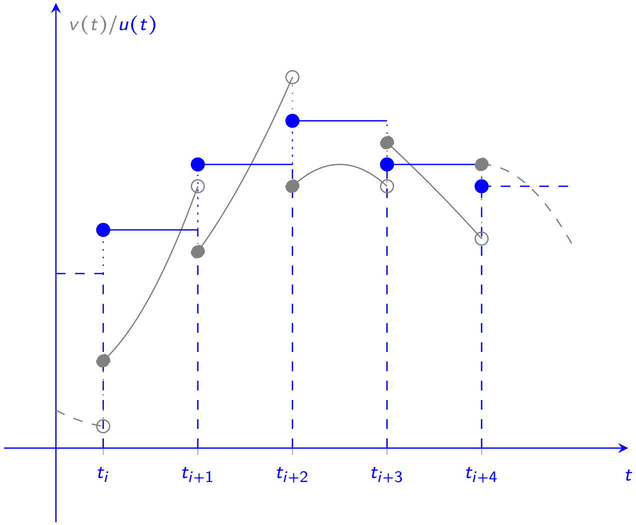

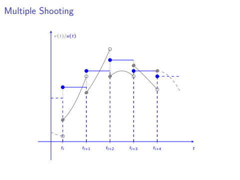

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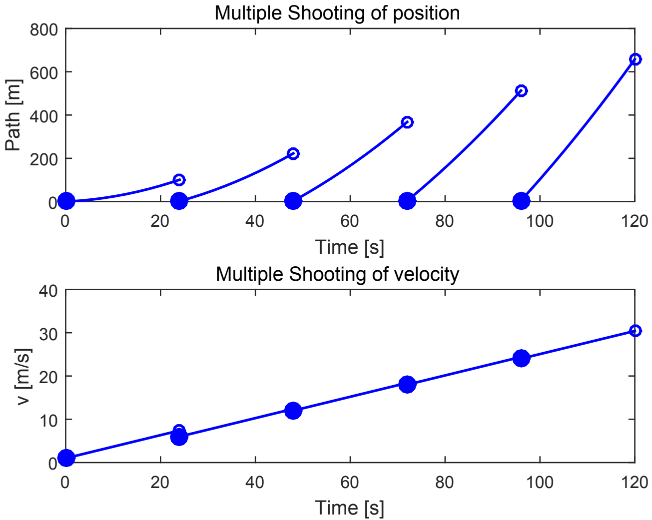
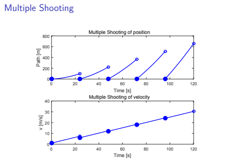


Robust Optimization of a Car
└─Discretization
└─Multiple Shooting



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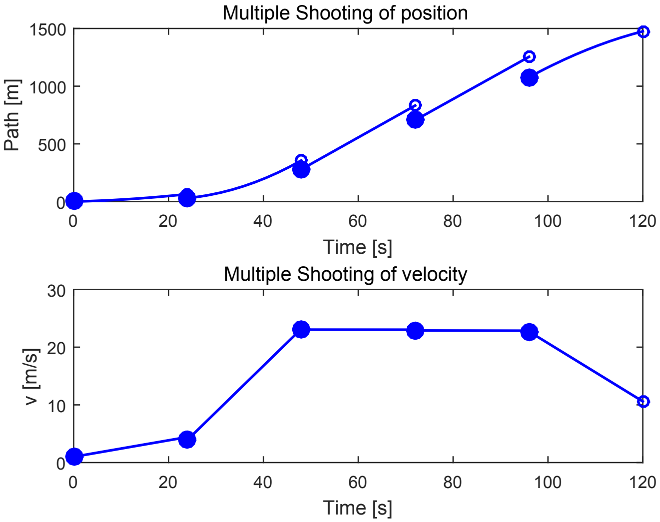

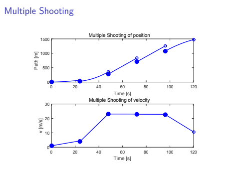

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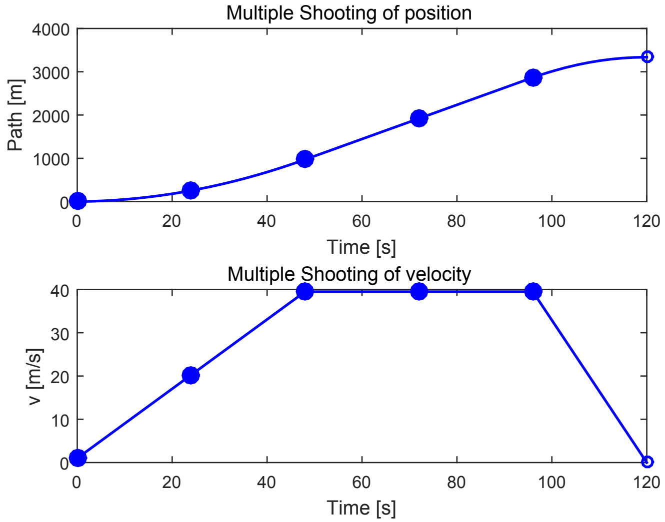

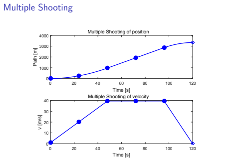

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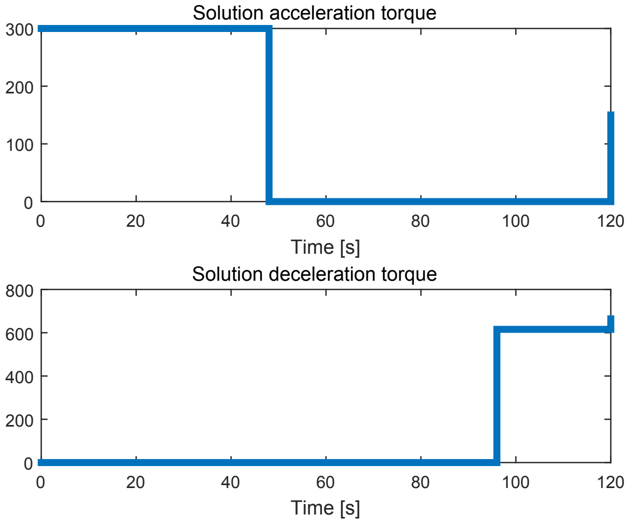

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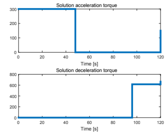

Problem Formulation

Discrete Optimization Problem

$$
\begin{aligned}
&\min_{x \in \mathbb{R}^{n_x},\, u \in \mathbb{R}^{n_u}} \; f_0(x, u) && \\
\text{s.t.} \quad & f_i(x, u) \leq 0 && \text{for } i = 1, \ldots, n_f \\
& g_j(x, u, p) = 0 && \text{for } j = 1, \ldots, n_x,
\end{aligned}
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where $p \in \mathbb{R}^{n_p}$ is an uncertain parameter vector

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where $p \in \mathbb{P}_{box} = \left\{ p \in \mathbb{R}^{n_p} \middle| \, p_{lower} \le p \le p_{upper} \right\}$

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\rightsquigarrow \Phi_i(u) := \quad & \max_{x \in \mathbb{R}^{n_x}, p \in \mathbb{R}^{n_p}} f_i(x,u) & \text{for } i = 0, \ldots, n_f \\
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Robust Counterpart

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\begin{aligned}
& \min_{u \in \mathbb{R}^{n_u}} \Phi_0(u) \\
\text{s.t.} \quad & \Phi_i(u) \leq 0 && \text{for } i = 1, \dots, n_f.
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➔ bilevel structure!

Worst Case Formulation
$\Phi_i(u) := \max_{x \in \mathbb{R}^{n_x}, p \in \mathbb{R}^{n_p}} f_i(x,u)$ for $i = 0, \dots, n_f$
s.t. $g(x,u,p) = 0$
$p \in \mathbb{P}_{box}$
Robust Counterpart
$\min_{u \in \mathbb{R}^{n_u}} \Phi_0(u)$
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\begin{aligned}
\Phi_i(u) := \quad & \max_{x\in\mathbb{R}^{n_x},\, p\in\mathbb{R}^{n_p}} f_i(x,u) && \text{for every } i = 0,\dots,n_f \\
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Approximation Technique

Linearization

at a point $(\bar{x}, u, \bar{p})$ with $\bar{p} = \frac{p_{lower} + p_{upper}}{2}$ and $g(\bar{x}, u, \bar{p}) = 0$:

$$\begin{aligned}
\tilde{\Phi}_i(u) := \quad & \max_{(x-\bar{x}) \in \mathbb{R}^{n_x}, (p-\bar{p}) \in \mathbb{R}^{n_p}} f_i(\bar{x}, u) + \frac{\partial f_i}{\partial x}(\bar{x}, u)(x - \bar{x}) \\
\text{s.t.} \quad & \frac{\partial g}{\partial x}(\bar{x}, u, \bar{p})(x - \bar{x}) + \frac{\partial g}{\partial p}(\bar{x}, u, \bar{p})(p - \bar{p}) = 0 \\
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With a unique optimal solution $\tilde{\Phi}_i(u)$:

Standard Optimization Problem

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Standard Optimization Problem

Visualization

Goals

- nice graphs
- rendered video

Visualization
Goals
- nice graphs
- rendered video

- graphs contain much information
- videos catch people where they are

- Graphs are graphs, nothing to do here
- Grass tiles
- Make car shiny, windows glassy
- Curves are nice to see
- this is just an arbitrary acceleration
- This 10 seconds clip allready rendered 30 min
- min 18FPS needed

Visualization

Improvements

- textures
- nonlinear street design
- fit acceleration to optimization results
- fit resolution to output device
- follow the car with the camera

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