

MATH 212 PORTFOLIO

Lafe Wessel

Draft date: February 17, 2021

Changelog: *List the changes you've made since the last draft, with special attention paid to problems that have received significant revisions since the last draft (i.e., more than fixing typos). If there is any additional information you'd like me to consider as I review this submission, please say so now.*

1. Reworked Conjectures I.1-I.4

2. Added Conjecture II

Instructions: Each of the problems below is/will be presented as a conjecture. Each conjecture asks you to prove or disprove the conjecture, possibly along with some additional directions.

- If the conjecture is true, your job is to write a complete proof for the proposition. If there are multiple parts, you should consider each part in turn.
- If it is false, you should provide a counterexample plus make reasonable modifications to the stated conjecture so that a new proposition is true. Then, write a complete proof of your new proposition. You may want to run your new proposition by me before trying to write a proof—this is allowed and encouraged!

Academic Honesty Policy: The portfolio is an independent project in which no outside resources or collaboration is allowed. You may not ask other professors or discuss the problems with anyone besides me. You should not discuss even which problem you chose. Violation of this policy is grounds for failing the course. The point is that you need to be confident and competent in writing proofs for future courses.

Conjecture I. Let A and B be subsets of some universe \mathcal{U} . Then:

1. $A \setminus (A \cap \overline{B}) = A \cap B$
2. $\overline{(\overline{A} \cup B)} \cap A = A \setminus B$
3. $(A \cup B) \setminus A = B \setminus A$
4. $(A \cup B) \setminus B = A \setminus (A \cap B)$

All four statements of Conjecture I will be proved to be equal through the set equivalence method: demonstrating that the left side is a subset of the right side and that the right side is a subset of the left side.

Conjecture I.1.

Proof. Let $x \in A \setminus (A \cap \overline{B})$. Thus, x must be an element of A and not an element of $A \cap \overline{B}$, that is $x \in A$ and $x \notin A \cap \overline{B}$. If x is not in $A \cap \overline{B}$, then x must not be in \overline{B} , meaning $x \in B$. Thus, $x \in A \cap B$, and, furthermore, $A \setminus (A \cap \overline{B}) \subseteq A \cap B$.

Let $x \in A \cap B$. For this to be true, x must be in both A and B ; $x \in A$ and $x \in B$. Thus, x cannot be in the complement of B , \overline{B} . Since $x \notin \overline{B}$, $x \notin A \cap \overline{B}$. Because $x \in A$ and $x \notin A \cap \overline{B}$, $x \in A \setminus (A \cap \overline{B})$. Thus, $A \cap B \subseteq A \setminus (A \cap \overline{B})$.

Therefore, since $A \setminus (A \cap \overline{B}) \subseteq A \cap B$ and $A \cap B \subseteq A \setminus (A \cap \overline{B})$, $A \setminus (A \cap \overline{B}) = A \cap B$. □

Conjecture I.2

Proof. Let $x \in \overline{(\overline{A} \cup B)} \cap A$. Thus, $x \in A$ and $x \in \overline{(\overline{A} \cup B)}$. This means that x is not in the complement of $\overline{(\overline{A} \cup B)}$, which is $\overline{A} \cup B$. Thus, $x \notin \overline{A} \cup B$. Consequently, $x \notin \overline{A}$ and $x \notin B$. If $x \notin \overline{A}$, then $x \in A$. Since $x \in A$ and $x \notin B$, $x \in A \setminus B$. Thus, if $x \in A \setminus B$ and $x \in \overline{(\overline{A} \cup B)} \cap A$, $\overline{(\overline{A} \cup B)} \cap A \subseteq A \setminus B$.

Let $x \in A \setminus B$. If this is true, then $x \in A$ and $x \notin B$. It follows that $x \notin \overline{A}$ and, consequently, that $x \notin \overline{A} \cup B$. If $x \notin \overline{A} \cup B$, then x must be in the complement, being $\overline{(\overline{A} \cup B)}$. Since $x \in A$ and $x \in \overline{(\overline{A} \cup B)}$, then $x \in \overline{(\overline{A} \cup B)} \cap A$. Thus, $A \setminus B \subseteq \overline{(\overline{A} \cup B)} \cap A$.

Therefore, since $\overline{(\overline{A} \cup B)} \cap A \subseteq A \setminus B$ and $A \setminus B \subseteq \overline{(\overline{A} \cup B)} \cap A$, $\overline{(\overline{A} \cup B)} \cap A = A \setminus B$. □

Conjecture I.3

Proof. Let $x \in (A \cup B) \setminus A$. For x to be an element, it must be in $A \cup B$ and not in A . Thus, $x \in A \cup B$ and $x \notin A$. Since $x \notin A$, x must be a part of B for $A \cup B$ to be true: $x \in B$. If $x \notin A$ and $x \in B$, then $x \in B \setminus A$. If $x \in (A \cup B) \setminus A$ and $x \in B \setminus A$, then $(A \cup B) \setminus A \subseteq B \setminus A$.

Let $x \in B \setminus A$. Thus, x must be in B and not in A . Accordingly, $x \in B$ and $x \notin A$. It follows that $x \in A \cup B$, and, consequently, that $x \in (A \cup B) \setminus A$ since $x \notin A$. Thus, because $x \in B \setminus A$ and $x \in (A \cup B) \setminus A$, $B \setminus A \subseteq (A \cup B) \setminus A$.

Therefore, since $(A \cup B) \setminus A \subseteq B \setminus A$ and $B \setminus A \subseteq (A \cup B) \setminus A$, $(A \cup B) \setminus A = B \setminus A$. \square

Conjecture I.4

Proof. Let $x \in (A \cup B) \setminus B$. It follows that $x \in A \cup B$ and $x \notin B$. This means that $x \in A$ since $x \in A \cup B$ is true. Thus, $x \notin A \cap B$. Because $x \in A$ and $x \notin A \cap B$, $x \in A \setminus (A \cap B)$ is true. Thus, since $x \in (A \cup B) \setminus B$ and $x \in A \setminus (A \cap B)$, $(A \cup B) \setminus B \subseteq A \setminus (A \cap B)$.

Let $x \in A \setminus (A \cap B)$. Thus, $x \in A$ and $x \notin A \cap B$, meaning that $x \notin B$. It follows that $x \in A \cup B$ and, since $x \notin B$, $x \in (A \cup B) \setminus B$ are both true. Thus, if $x \in A \setminus (A \cap B)$ and $x \in (A \cup B) \setminus B$ are true, then $A \setminus (A \cap B) \subseteq (A \cup B) \setminus B$ is true.

Therefore, since $(A \cup B) \setminus B \subseteq A \setminus (A \cap B)$ and $A \setminus (A \cap B) \subseteq (A \cup B) \setminus B$ are true, it must be true that $(A \cup B) \setminus B = A \setminus (A \cap B)$. \square

Conjecture II. Define $f : \mathbb{N} \setminus \{0\} \rightarrow \mathbb{Z}$ as follows: for each $n \in \mathbb{N} \setminus \{0\}$,

$$f(n) = \frac{1 + (-1)^n(2n - 1)}{4}.$$

Then f is a bijection.

Proof.

□

Evaluation: _____

Opening: _____

Logical Correctness: _____

Reasons: _____

Use of Notation: _____

Clarity and Writing: _____

L^AT_EX Formatting: _____

Stating the Conclusion: _____

Other Comments:

Evaluation: _____

Opening: _____

Logical Correctness: _____

Reasons: _____

Use of Notation: _____

Clarity and Writing: _____

L^AT_EX Formatting: _____

Stating the Conclusion: _____

Other Comments: