Class Notes

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Proof of Onto and One-to-One

$$\forall x_1 \forall x_2 [f(x_1) = f(x_2) \rightarrow x_1 = x_2]$$

$$y = f(x) = e^x$$

$$ln(y) = ln(e^x) = x$$

$$e^{x_1} = e^{x_2} => ln(e^{x_1}) = ln(e^{x_2}) = x_1 = x_2$$

$$n = p \cdot b^t \Leftrightarrow \frac{n}{p} = b^t$$
$$\therefore \quad \log_b(\frac{n}{p}) = t$$

After Class Notes

1. Theorem: if a, b, and c are integers such that a^3|b and b^2|c then a^6|c

(p ∧ q) → r

a^3|b ⇒ b = ka^3 k∈z
b^2|c ⇒ c = jb^2 j :∈z

= j(ka^3)^2 = (jk^2)a^6 = la^6 ⇒ means c is divisible by a^6.

2. Theorem: xy|z → (x|z ∧ y|z)

x ∈ Z
y ∈ Z
z ∈ Z

z = k(xy) k ∈ Z

(ky)x = (l)x ⇒ z is divisible by x ⇒ x|z
(kx)y = (m)y ⇒ z is divisible by y ⇒ y|z

3. Theorem $(5n+3 = 2k) \rightarrow (n=2l+1) \equiv \neg q \rightarrow \neg p$

5n+3 is even, because of 2k.
n is odd, because of 2l + 1.

$(\neg q \rightarrow \neg p) \equiv ((n = 2l) \rightarrow (5n + 3 = 2k + 1))$

$5n+3 = 5(2l) + 3 = 10l + 3 = 10l + 2 + 1 = 2(5l+1) + 1$

$= 2m + 1$ where $m \in Z$

4. $3 \nmid xy \rightarrow 3 \nmid x$

By contrapositive:
$\neg q \rightarrow \neg p$

$(3|n) \rightarrow (3|xy)$

$x = 3k \rightarrow xy=3m$
$k \in Z$ and $m \in Z$

$xy = (3k)y = 3(ky) = 3m$