Laws

Law	Formula	Law of
Idempotent	$p \wedge p = p$ $p \vee p = p$	Conjunction Disjunction
Commutative	$p \wedge q = q \wedge p$ $p \vee q = q \vee p$	Conjunction Disjunction
Associative	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$ $(p \vee q) \vee r = p \vee (q \vee r)$	Conjunction Disjunction
Distributive	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$	And over Or Or over And
De Morgans	$\neg (p \wedge q) = \neg p \vee \neg q$ $\neg (p \vee q) = \neg p \wedge \neg q$	*First* *Second*
De Morgans (II)	$\neg\forall x\, P(x) \equiv \exists x\, \neg P(x)$ $\neg\exists x\, P(x) \equiv \forall x\, \neg P(x)$	
Identity	$p \wedge T = p$ $p \vee F = p$	Conjunction Disjunction
Domination	$p \wedge F = F$ $p \vee T = T$	Conjunction Disjunction
Negation	$p \wedge \neg p = F$ $p \vee \neg p = T$	*Contradiction* *Tautology*
Double Negation	$\neg (\neg p) = p$	*Involution Law*
Contrapositive	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	
Absorption	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	*Left Absorption Law* *Right Absorption Law*
Implication	$p \rightarrow q \equiv \neg p \vee q$ $\neg(p \rightarrow q) \equiv p \wedge \neg q$	*Conditional Law* Negation of *Conditional Law*
Bi-Implication	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	*Bi-Conditional Law*

Non Equivalances

Law	Conditional Non-Equivalence	Description
Converse	p → q ≠ q → p	Order of conditional is reversed
Inverse	p → q ≠ ¬q → ¬p	Negation of both sides of inverse

De Morgans Law

- When a negation "passes" over a quantifier, it causes the quantifier to "flip".
- Existential becomes Universal, and Universal becomes Existential

$$\neg\exists x\forall y(P(x)+Q(y)=0)\equiv\forall x\exists y\neg(P(x)+Q(y)=0)$$

- Note: the negation can be applied to the expression, converting the "=" operator to "≠".

$$\forall x\exists y\neg(P(x)+Q(y)=0)\equiv\forall x\exists y(P(x)+Q(y)\neq 0)$$

Tautology and Contradiction

- A compound proposition is a tautology if the proposition is always true.
- A compound proposition is a contradiction if the proposition is always false

Tautology truth table:

p	¬p	p ∨ ¬p
T	F	T
F	T	T

Contradiction truth table:

p	¬p	p ∧ ¬p
T	F	F
F	T	F

Conditional Statements

Proposition:	p → q	Ex: If it is raining today, the game will be cancelled.
Converse:	q → p	Ex: If the game is cancelled, it is raining today.

Proposition:	**p → q**	**Ex: If it is raining today, the game will be cancelled.**
Contrapositive:	¬q → ¬p	Ex: If the game is not cancelled, then it is not raining today.
Inverse:	¬p → ¬q	Ex: If it is not raining today, the game will not be cancelled.