

# Operators

## Propositional Operators

Symbol	Name	Word	Example
$\neg$	Negation	NOT	$\neg p$ means "not p"
$\wedge$	Conjunction	AND	$p \wedge q$ means "p and q"
$\vee$	Disjunction	OR	$p \vee q$ means "p or q"
$\rightarrow$	Implication	IF	$p \rightarrow q$ means "if p, then q"
$\leftrightarrow$	Bi-Implication	IIF	$p \leftrightarrow q$ means "p if and only if q"
$\oplus$	Exclusive OR	XOR	$p \oplus q$ means "either p or q, but not both"

- Operators must be applied according to precedence.
- When two operators conflict, the operator with the lower precedence value is applied first.
- Parenthesis are always applied before operators.

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\Rightarrow$	4
$\leftrightarrow$	5
$\oplus$	6

## Negation

p	$\neg p$
T	F
F	T

$p = \text{"The weather is good"}$

Example expression:

$\neg p$

i.e. "NOT  $p$ "

e.g. "The weather is not good."

## Conjunction

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p = \text{"The sky is clear"}$

$q = \text{"It is warm"}$

Example expression:

$p \wedge q$

i.e. " $p$  AND  $q$ "

e.g. "The sky is clear and it is warm."

## Disjunction

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

```
p = "The music is playing"
q = "The guests are talking"
```

Example expression:

```
p ∨ q
```

i.e. "p OR q"

e.g. "The music is playing or the guests are talking"

## Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

```
p = "It is pouring"
r = "The old man is snoring"
```

Example expression:

```
p → r
```

i.e. "If p, then r."

e.g. "If it is pouring, then the old man is snoring."

## Bi-Implication

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

```
p = "A triangle has three equal sides"
```

```
q = "The triangle is equilateral"
```

Example expression:

```
p  $\Leftrightarrow$  q
```

i.e. "p IFF q"

e.g. "A triangle has three equal sides, if and only if the triangle is equilateral."

## Exclusive OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

```
p = "The light is green"
```

```
q = "The light is red"
```

Example expression:

```
p  $\oplus$  q
```

i.e. "p XOR q"

e.g. "The light is green or the light is red, but not both."

## Set Operators

Operation	Notation	Definition	Description
Intersection	$A \cap B$	$\{x : x \in A \text{ and } x \in B\}$	The set of elements shared by A and B.

Operation	Notation	Definition	Description
Union	$A \cup B$	$\{x : x \in A \text{ or } x \in B \text{ or both}\}$	Elements of A and B are joined with no duplicates.
Difference	$A - B$	$\{x : x \in A \text{ and } x \notin B\}$	Every element shared with B is removed from A.
Symmetric difference	$A \oplus B$	$\{x : x \in A - B \text{ or } x \in B - A\}$	Union of unshared elements between A and B.
Complement	$\bar{A}$	$\{x : x \notin A\}$	Every element in the universal set not in A.

## Set Assertions

Condition	Notation	Definition	Description
Subset	$A \subseteq B$	$\forall x(x \in A \rightarrow x \in B)$	Every element of A is in B.
Proper Subset	$A \subset B$	$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$	Every element of A is in B, but B has more elements.
Not Subset	$A \not\subseteq B$	$\forall x(x \in B \rightarrow x \notin A)$	Every element of A is not in B.