Short Project Model Based Predictive Control

MPC control applied to structure under earthquakes

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This paper is a report on a short project that aim to apply a MPC controller to prevent earthquakes effects over buildings.

OBJECTIVES

Apply a MPC controller to a more and more complex model of building to minimize the accelerations applied on the story of a building under an earthquake influence.

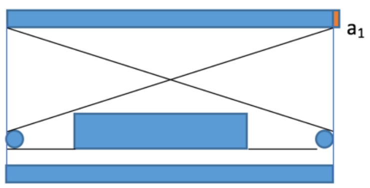
LINEAR MODELS

The model studied are in two dimensions, only shear stress is applied to these model.
The equation of the system is given by:

$$\begin{pmatrix}\dot{x}\\ \ddot{x}\end{pmatrix} = \begin{pmatrix}0 & I\\ -M^{-1}.K & -M^{-1}.C\end{pmatrix}\begin{pmatrix}x\\ \dot{x}\end{pmatrix} + \begin{pmatrix}0\\ M^{-1}L\end{pmatrix}u + \begin{pmatrix}0\\ I\end{pmatrix}\ddot{x_g} \quad (1)$$

Where $x = \begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{pmatrix}$ displacement of each story of the building, $\dot{x} = \begin{pmatrix}\dot{x_1}\\ \dot{x_2}\\ \vdots\\ \dot{x_n}\end{pmatrix}$ celerity of each story of the building, where n is the number of story of the building.

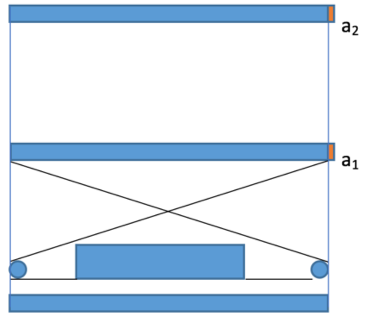
Several cases are studied, a building with one floor and a building with two floors.
Case of a building with one floor:



figure(1) *2D Building with 1 story equipped with a tendon system*

$$M = 2922.7\ kg \quad (2)$$
$$L = -4.k_c.\cos(\alpha) \quad (3)$$
$$C = 1579.4\ N.s.m^{-1} \quad (4)$$
$$K = 1.3877.10^6\ N.m^{-1} \quad (5)$$

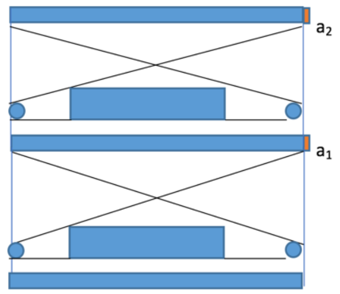
Case of a unit with two floors and a single controller:



figure(2) *2D Building with 2 story equipped with a tendon system*

$$L = -4.k_c.\cos(\alpha) \quad (6)$$
$$M = \begin{pmatrix}974 & 0\\ 0 & 974\end{pmatrix} \quad (7)$$
$$K = \begin{pmatrix}2.74 & -1.64\\ -1.64 & 3.02\end{pmatrix}.10^6 \quad (8)$$
$$C = \begin{pmatrix}382.65 & -57.27\\ -57.27 & 456.73\end{pmatrix} \quad (9)$$

Case of a unit with two floors and two controllers:



figure(3) *2D Building with 3 story equipped with 2 tendon system*

$$L = -4.k_c.\cos(\alpha) \quad (10)$$
$$M = \begin{pmatrix}974 & 0\\ 0 & 974\end{pmatrix} \quad (11)$$
$$K = \begin{pmatrix}2.74 & -1.64\\ -1.64 & 3.02\end{pmatrix}.10^6 \quad (12)$$
$$C = \begin{pmatrix}382.65 & -57.27\\ -57.27 & 456.73\end{pmatrix} \quad (13)$$

For all the cases : $k_c = 371950.8\ N/m$.

DATA

The data used in order to test the controller is the acceleration North-South, East-West, Up-Down of the earthquake El Centro (May 1940, Imperial Valley, USA).

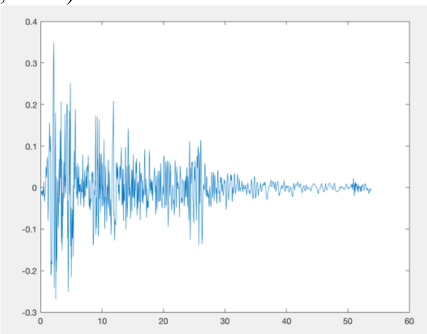


figure (4): *Acceleration in G North-Sud measured for the earthquake El Centro*

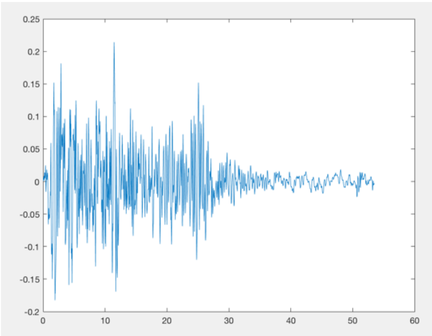


figure (5): *Acceleration in G East-West measured for the earthquake El Centro*

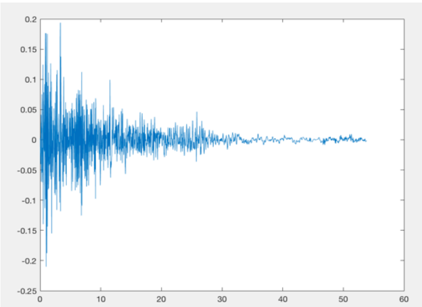


figure (6): *Acceleration in G East-West measured for the earthquake El Centro*

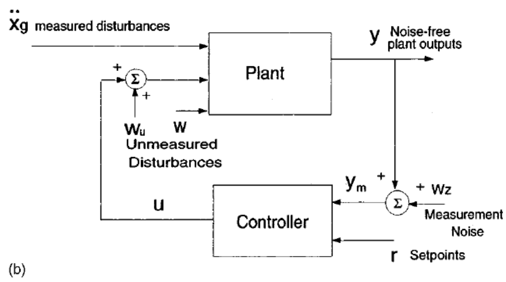
The acceleration measured is expressed in g and each measure is taken with a sampling time of 0.02 s.

Limit of the data:
Those data were measured on the floor of a building, which mean that it is not the acceleration of the ground. The rigid soil structure of the building in interaction with the soft soil of the surrounding work as a low pass filter and high frequency motion of the ground may be masked.
In the following, we only use the acceleration North-South

STRATEGY

This problem can't be processed as a simple problem because of three phenomena:

- The disturbance of the ground
- The noise of the captor
- The noise of the process



figure(7): *Model of the controller applied to the system from:*
"About tendon system control with a MPC controller and active tendon"

The input and output noise are independent; we decide to use a kalman filter in order to compute the input of the system.
We first implement a kalman filter in order to predict the future behaviour of the system from step 1 to H_p.

PREDICT STATE OF THE MODEL BY THE KALMAN FILTER

We first need to find a good kalman filter in order to create our prediction model. Indeed, the acceleration data of the earthquake El Centro doesn't inform about about the noise in the seismic measure.
The design of the kalman filter is done in case of a single floor building, the ground disturbance is considered as a process noise w (the variance of the signal) and the output measurement as the noise v.
A wrong estimation of the noise give the following result:

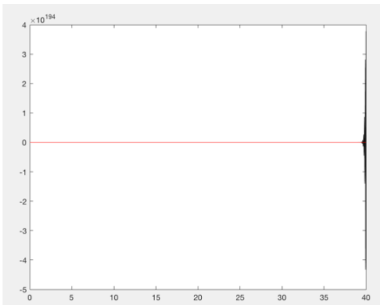


figure (8) *Estimation of the next step by a kalman filter in red the true evolution of the system without input, in black the predicted state of the system by a kalman filter*

We try different the noise of different accelerometers given their technical sheet.
The accelerometer's noise is 130 LSB RMS with 1 LSB RMS= 0.000098 m.s^{-2}.

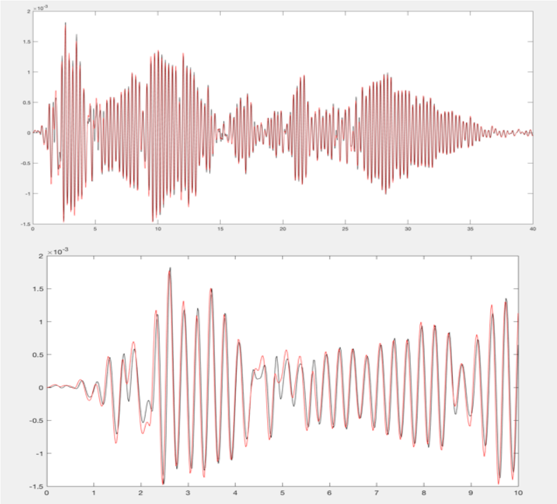


figure (9) *Performance of the Kalman filter during the whole earthquake ,* **figure (10)** *Performance of the Kalman filterfor the 2.5 first seconds of the earthquake.*

From this simulation we can see that the earthquake changes of direction after a time between 0.06 and 0.12 second which mean that the CPU time must be lower than this characteristic time (we may not be able to apply the control at each step) and the prediction horizon is pertinent for 3 or 5 steps ahead (because the direction of the acceleration may change after this number of steps).

BUILDING OF THE CONTROLLER FOR THE LINEAR MODEL

Estimation of the parameters of the cost function.

We realise several test using the first model in order to design the cost function.
We test the cost function is given by the following equation:

$$J = \frac{1}{2}.X^T.Q.X + \frac{1}{2}.U^T.R.U \qquad (14)$$

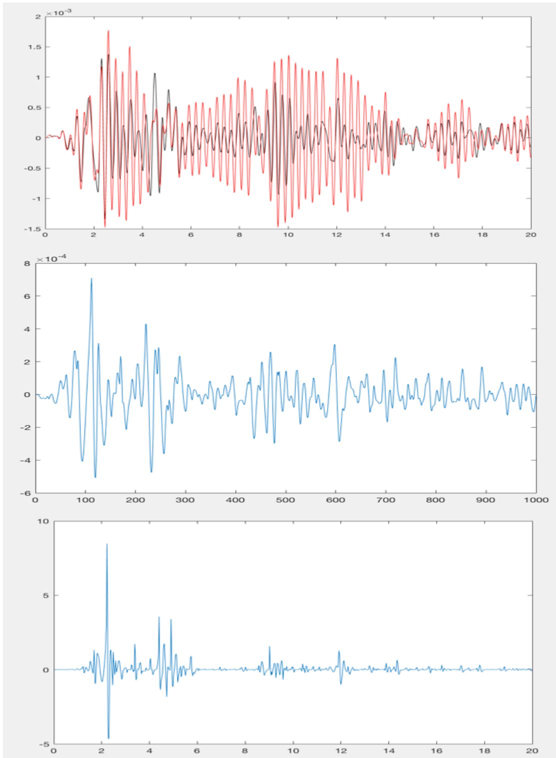
where $X = \begin{pmatrix} \ddot{x} \\ \dot{x} \end{pmatrix}$ and u the input.
We fix $Q=I_2$ (identity matrix) and modify the value of R. The control is done considering the form of the prediction,

$$X_{k+n} = \Phi.X_{k+n-1} + \Gamma_u.u_{k+n-1} + \Gamma_e.e \qquad (15)$$

where e, the error in acceleration ($e = y_k - \hat{y}_k$) considered as constant between the steps $k+1$ and $k+H_p$.
$\Phi = e^{A.t}$,$P_1 = \int_0^{dt} e^{At}dt$, $\Gamma_u = P_1.B$, Γ_e the kalman filter.
We test R=2000, R=400, R=20 for the 20 first seconds (due to computation time) without any added constraint:
R=2000:

figure (11*) evolution of the position of the floor of the building with the controller (in red) and without the controller in case of R=2000,* **figure(12)** *evolution of the input u (the displacement of the mass of the tendon system) for R=2000,* **figure(13***) evolution of the minimum power in Watt necessary to apply the input for R=2000*

R=200:



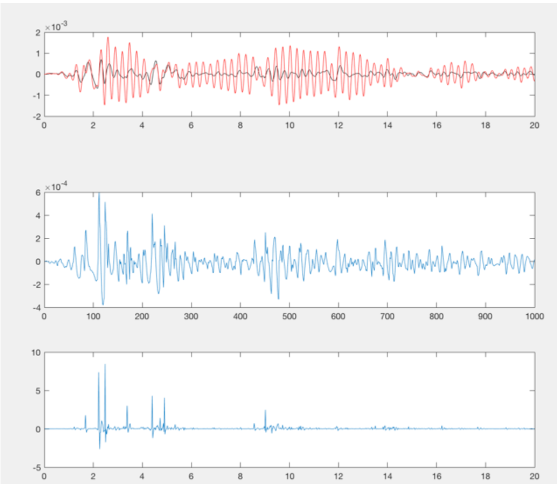
figure(14*) evolution of the position of the floor of the building with the controller (in red) and without the controller in case of R=200,* **figure(15)** *evolution of the input u (the displacement of the mass of the tendon system) for R=200,* **figure(16)** *evolution of the minimum power in Watt necessary to apply the input for R=200*

R=20:

figure(17) *evolution of the position of the floor of the building with the controller (in red) and without the controller in case of R=20,* **figure(18)** *evolution of the input u (the displacement of the mass of the tendon system) for R=20,* **figure(19)** *evolution of the minimum power in Watt necessary to apply the input for R=20*

We can see that the higher the value of R, the weaker the power necessary to apply the control and the lower the amortization on the vibration of the building. A too strong value of R makes the controller useless and a too low value of R ask for a lot of energy to activate the actuator. For now, we decide to take R=200.

We add an end point constraint equal to zero to the case R=200:

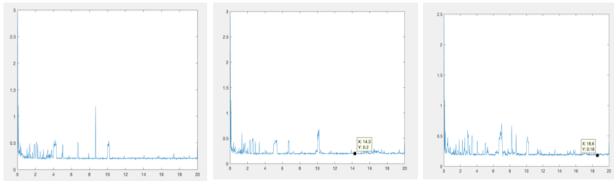


figure(20) *First figure, in red, the displacement of the building without any control, in black, the displacement of the building with the control and the terminal point constraint, second figure, input generated by the controller, third figure energy used to generate the input*

We obtain a better performance in case of cost of energy, and attenuation of the displacement of the first story.

Remark: for R=200, the CPU time taken to compute the the input without any constraint is already high:

N=3 N=4 N=5



Figure(21), figure(22), figure(23) *CPU time at each iteration for the controller without constrain and an horizon, respectively, N=3, N=4, N=5*

For N=5 and a point terminal constraint,

We can observe that the CPU time is between 0.16 and 0.2 second when there is no onset in the earthquake signal and without constraint. For a big change in the vibration of the ground, the CPU time taken to compute the input can rise till 1 second. Which really high compared to the fast evolution of the ground.

For the cases 2 and 3:

We know apply a single tendon system to the model 2:

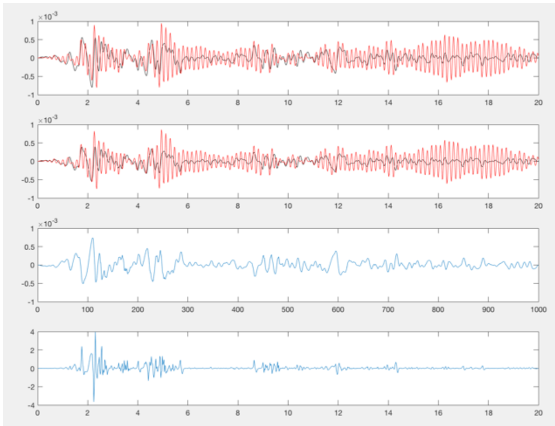
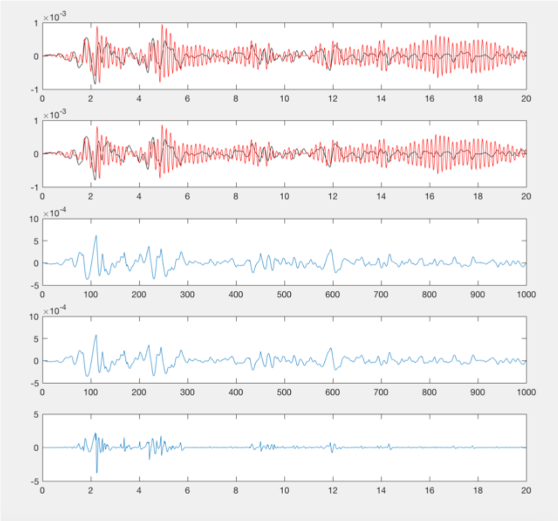


figure (24) *First figure, in red, the displacement of the building's first story without any control, in black, the displacement of the building's first story with two stories with one, second figure , in red, the displacement of the building's first story without any control, in black, the displacement of the building's first story with two stories with one, third figure input generated by the controller, fourth figure energy used to generate the input*

If we try to apply two controllers, one at each floor



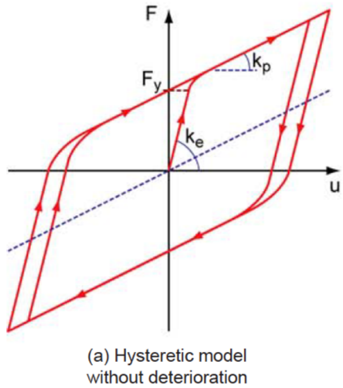
figure(25) *First figure, in red, the displacement of the building's first story without any control, in black, the displacement of the building's first story with two stories with one, second figure , in red, the displacement of the building's first story without any control, in black, the displacement of the building's first story with two stories with one, third figure input generated by the first controller, fourth figure input generated by the second controller, fifth figure energy used to generate the input.*

We can observe that the oscillations of the first compared to the second story is stronger for both model and for the controlled and uncontrolled system. The performances of the model 2 and 3 is equivalent in terms of displacement but it asks for less energy in the second case, the displacement of the mass of the tendon is lower than in the 2 model. The performances of the model 3 are slightly better but may not justify the installation of a second system on the second story.

NON LINEAR MODEL

We complicate the model by considering the non linearity of the structure. Indeed, steel reinforced concrete used for building has an elasticity limit between R_e= 60MPa and 90MPa for a shear stress and a Young's modulus of E=35GPa. That means that, in the worst case, the elasticity limit is obtained for a deformation of 1mm. This non linearity is applied to the term K of the equations of the model.
Taking into account the non linearity is used to determine the acceptance of a building. Here it would be a mean to enhance robustness of the MPC controller even if the aim of the MPC is to stay in the linear part.

We can first model the plasticity with a perfect plasticity (limit of elasticity equal to 60 MPa) and then complete with the Prager model with the same limit of elasticity and $k_p = 96\% k_e$ (beginning of plasticity), $k_e = 90GPa$ in both case.



figure(26) *Hysteresis model taken from "Non linear structural analysis for seismic design"*

The force applied to the system:

- First elasticity: $F = k_e . x$ (16)
- Deformation extension: $F = F_y + k_p . x$ (17)
  $= (k_e - k_p). x_{limit} + k_p . x$ (18)
- Return to elasticity:
  $F = F_y + k_p . x_{high} - k_e (x_{high} - x)$ (19)
- Deformation compression:
  $F = F_y + k_p . x_{high} - 2. Re + kp. (x - x_{low\_limit})$ (20)

For case 1:

- Without disturbance:

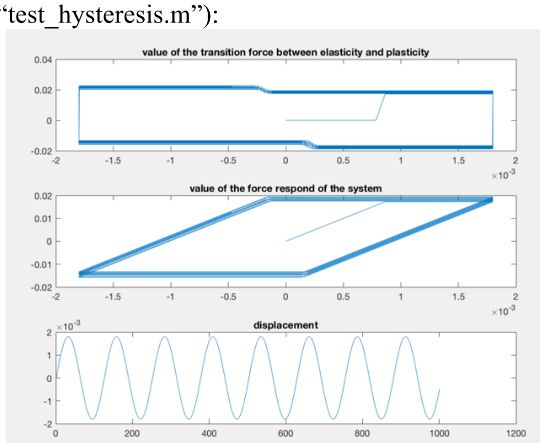
$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -M^{-1}.K_e & -M^{-1}.C \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}L \end{pmatrix} u + \begin{pmatrix} 0 \\ I \end{pmatrix} \ddot{x}_g \quad (21)$$

- With a previous plastification the strength applied by the building is
  $F = -k_p .(x - x_{in}) - k_e . x_{in}$ (22)
  The system become:

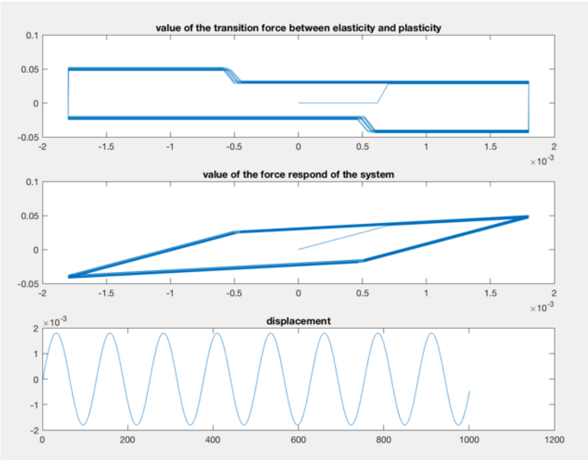
$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & I \\ -M^{-1}.K(x) & -M^{-1}.C \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}L \end{pmatrix} u + \begin{pmatrix} 0 \\ I \end{pmatrix} \ddot{x}_g + \begin{pmatrix} 0 \\ F_{\lim elastic} \end{pmatrix} \quad (23)$$

and $F_{\lim elastic} = -k_p . x_{limit} - k_e . x_{limit}$ (24)

We create a hysteresis function that change the value of K according to the displacement of the building (we create the function "test_hysteresis.m"):



figure(27): *Result of the function hysteresis for perfect plasticity*



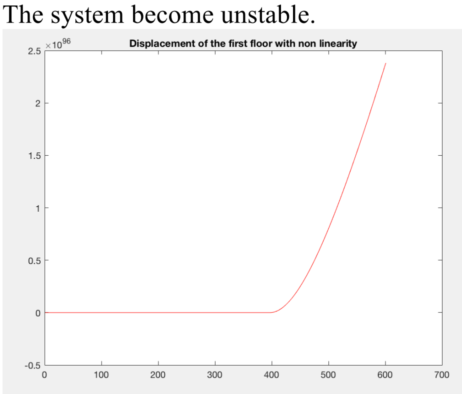
figure(28): *Result of the function hysteresis for Prager model*

For those examples, the value of k_e and k_p are chosen in order to have a clear image of the hysteresis and aren't the values mentioned before.

We apply the non linearity to see the behaviour of the building in the first case:

- Perfect plasticity:

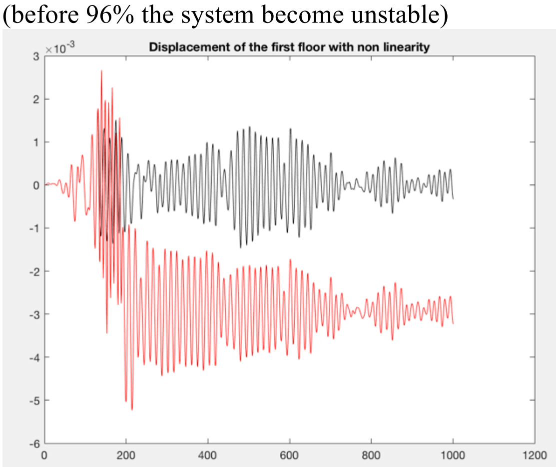
The system become unstable.



figure(29): *displacement of the building under perfect plasticity*

- Prager model with $k_p = 96\% * K$:

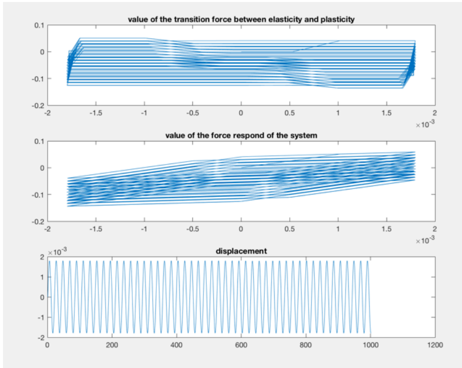
(before 96% the system become unstable)

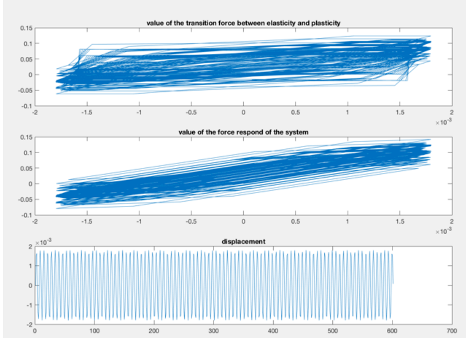


figure(30): *in black the linear evolution of the displacement of the floor, in red the non linear evolution of the displacement of the floor.*

We can see that a slight entry in the plastic domain first generate a higher amplitude of the displacement when we have big accelerations applied and then, when we return to small acceleration applied, an offset appear in the position of the building because the elasticity domain has been displaced.

This simulation has been done with $k_p = 96\% * k_e$ because under this value, the system become unstable. Also rise the problem of resolution (sampling time) of the acceleration signal, if there isn't enough point between two peaks of the variation we obtain this kind of result:



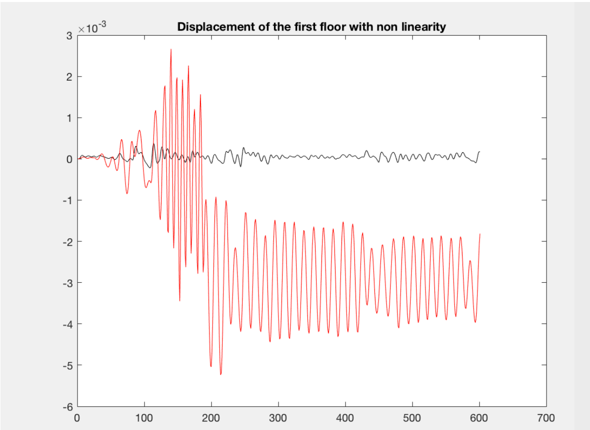


figure(31*) result of our hysteresis function for a cos variation of x with a 5 time higher frequency,* **figure(32)** *result of our hysteresis function for a cos variation of x with a 30 time higher frequency*

We can see that it begins to be chaotic for for the test with a 30 higher frequency. This is a problem that might appear with our controller (with a "bad" combination of noise).

BEHAVIOUR OF THE CONTROLLER APPLIED TO THE NON LINEAR MODEL

Now we apply the model predictive controller to the first case with $k_p = 96\% * K$ with an horizon N=5



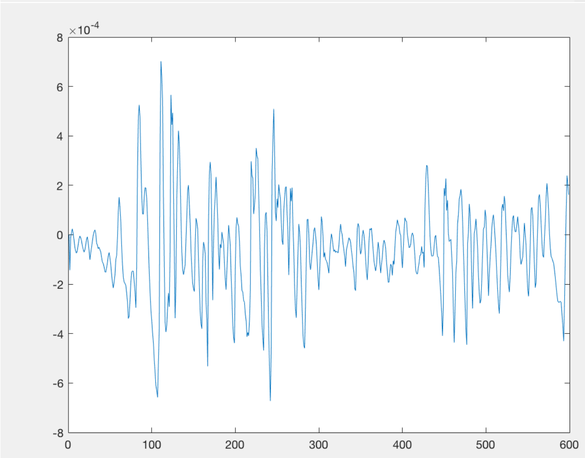
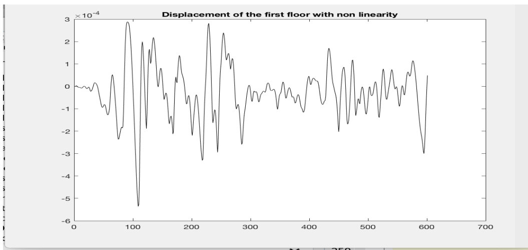
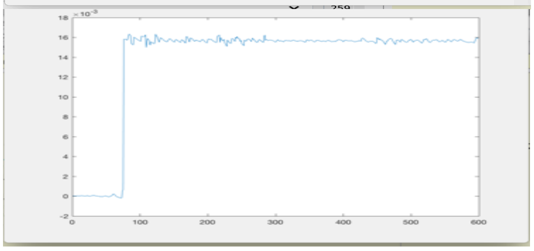


figure (33): *in red the comportment of the system without any controller, in black comportment of the system with our controller and the matrices of our cost function* $Q = I_2$ *and R=200.* **figure (34):** *evolution of the input of the system.*

We can see that the system is maintained in the elastic domain without crossing the plastic (deformation) domain once which mean that the input control doesn't change in comparison to the linear case.

We move down the limit of elasticity to 20MPa and a Young modulus of 90GPa to see the behaviour of the system in case of a deformation of the building (the system without controller become unstable and its displacement evolution isn't shown because we wouldn't be able to see the evolution of the displacement of the story with the controller)

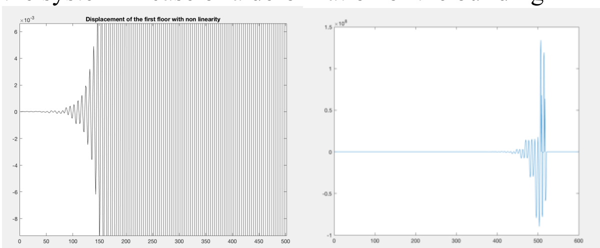




figure(35) *evolution of the displacement of the first story for* $R_e = 20MPa$ *and* $k_e = 90MPa$ **figure(36)** *evolution of the input applied* $R_e = 20MPa$ *and* $k_e = 90MPa$.

From the previous example, we can see that the entry in the plastic domain is compensated by the mass of the tendon system with an offset in order to maintain the displacement of the building around 0.

We move down the limit of elasticity to 18MPa and a Young modulus of 90GPa to see the behaviour of the system in case of a deformation of the building



figure(37) *evolution of the displacement of the first story for* $R_e = 18MPa$ *and* $k_e = 90MPa$ *figure(33) evolution of the input applied* $R_e = 18MPa$ *and* $k_e = 90MPa$.

We can observe that the system become unstable with strong oscillations in order to make the building returns to 0. We can see that the input suggested by the simulation isn't possible due to physical constraints on the displacement of the mass of the tendon, moreover, a constraint on the displacement of the building in this case has resulted in an infeasibility.

CONCLUSION

The implementation of a MPC controller is able to reduce the oscillations of story by 2 or 3 times in the elastic domain, reduce the risks to cross the plasticity domain and is able to compensate the damages done to the building by small deformation of it. The limit of this MPC is related to the physical limit of the problem, we can't indefinitely change the position of the mass of the tendon system to support the deformation of the building (and the building can't support it either, it will go first to its maximum shear stress in plasticity and then will attain rupture). Moreover, in case of non linearity, the controller is too dependant of the sampling of the signal and can't prevent strong deformation. One future work would be to see the influence of the others stress on the building generated by the up down and east west acceleration of the earthquake, on a 3D model, which combined by Von Mises equation might reveal that the elasticity limit is attained sooner than what we have with only the shear stress and see how implementing a 3D control of the tendon system in order to prevent the building collapsing.

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