PHYA-UA 210 HW2

Robert Zhao

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Q1: How does the error calculated in this manner compare with a direct computation of the error as the difference between your value for the integral and the true value of 4.4? Why do the two not agree perfectly?

The error calculated through $Eq(5.28)$ is: -0.026633333333333137, while the error by direct computation is: -0.026660000000000572, which is slightly larger. The reason why the two values do not agree perfectly is that for Eq(5.28) $\epsilon_2 = ch_2^2 = \frac{1}{3}(I_2 - I_1)$, it assumes that $I = I_1 + ch_1^2$ and $I = I_2 + ch_2^2$ and ignores higher-order terms, which causes slight deviation from the real value.

Q2:

a)

We have already known that

$$E = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$$

$$\Rightarrow E - \frac{1}{2}m(\frac{dx}{dt})^2 = V(x)$$

.

$$\Rightarrow E - \frac{1}{2}m(\frac{dx}{dt})^2(x = a) = V(a)$$

.

Now since we have $V(x)$ and $V(a)$:

$$V(a) - V(x) = \frac{1}{2}m((\frac{dx}{dt})^2(x = x) - (\frac{dx}{dt})^2(x = a))$$

As we know when $t = a, E = V(a)$, we have $v_0 = \frac{dx}{dt}(x = a) = 0$ So we have

$$V(a) - V(x) = \frac{1}{2}m(\frac{dx}{dt})^2$$

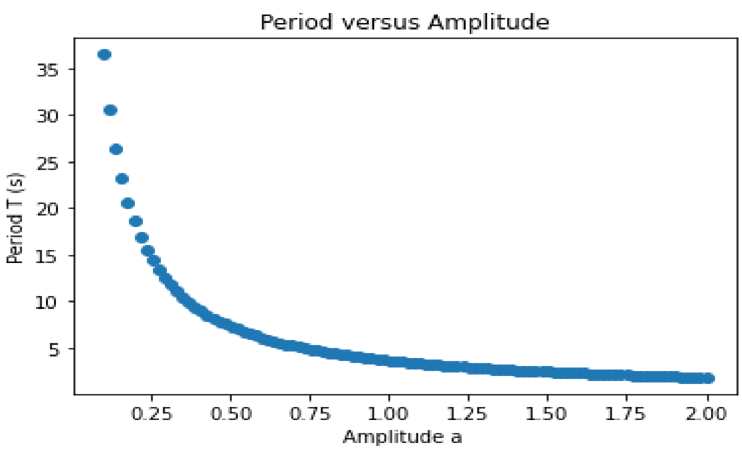
$$\Rightarrow \frac{dx}{dt} = -\sqrt{(V(a) - V(x)) \cdot \frac{2}{m}}$$

When the particle rolls from the initial position to the midpoint, it takes $\frac{1}{4}$ period.

$$T = 4\int_0^{\frac{T}{4}} dt = 4\int_a^0 \frac{dx}{\frac{dx}{dt}} = 4\int_a^0 -\frac{dx}{\sqrt{(V(a) - V(x)) \cdot \frac{2}{m}}}$$

$$\Rightarrow \; = 4\sqrt{\frac{m}{2}}\int_0^a \frac{dx}{\sqrt{V(a) - V(x)}} = \sqrt{8m}\int_0^a \frac{dx}{\sqrt{V(a) - V(x)}}$$

b)



c)

Since force is the gradient of the potential and $V(x) = x^4$. When x goes to infinity, the potential is almost vertical, and force applied to the particle is also quite large, which causes large acceleration and larger velocity. Therefore, the period for the particle gets smaller.

For the period diverges as the amplitude goes to zero: Since $V(x) = x^4$, then we have

$$E = a^4 = \frac{1}{2}m(\frac{dx}{dt})^2 + x^4$$

So we have $(\frac{dx}{dt})^2 \propto a^4 - x^4$, where $\frac{dx}{dt}$ means the velocity between 0 and a. When $a \to 0$, then the velocity $\frac{dx}{dt} \approx a^2 \cdot \sqrt{\frac{2}{m}}$, so we can have $\frac{dx}{dt} \propto a^2$ for $a \to 0$. Since we have shown $\frac{dx}{dt} \propto \sqrt{V(a) - V(x)}$ in part a), then we have $\sqrt{V(a) - V(x)} \propto a^2$

Since $T \propto \int_0^a \frac{dx}{\sqrt{V(a)-V(x)}}$ and $\sqrt{V(a) - V(x)} \propto a^2$, then we have $T \propto \int_0^a \frac{dx}{a^2}$

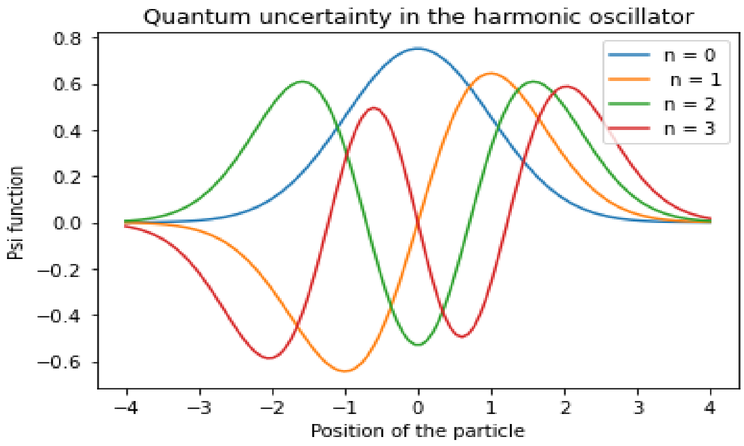
$\Rightarrow T \propto \frac{1}{a}$ for $a \to 0$

Therefore when the amplitude a goes to zero, the period diverges.

Q3:

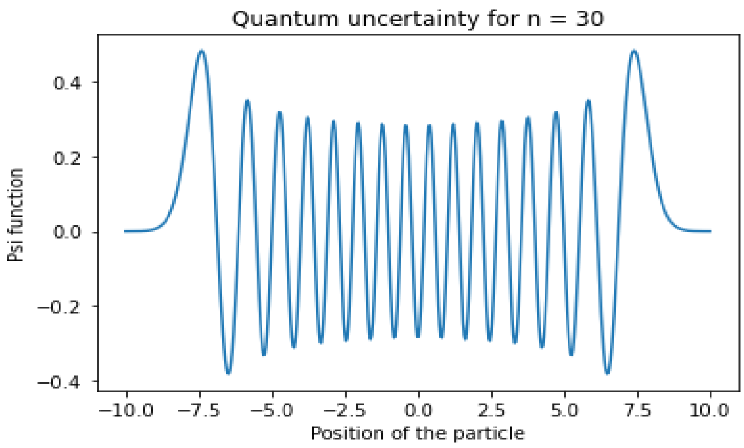
part a)

The wavefunction for n = 0 to n = 3 are alternatively even and odd functions, with $n+1$ peaks in the boundary.



part b)

The wavefunction for n = 30 has 31 peaks in the boundary.



part c)

I transferred the integral from infinite to finite through Eq(5.75). The value I get is $\sqrt{<x^2>} =$ 2.3452078797796547

part d)

The exact evaluation through Gauss-Hermite quadrature is $\sqrt{< x^2 >} = 2.345207879911714$.

Link for GitHub: https://github.com/LagrangePointL3/phys-ua210