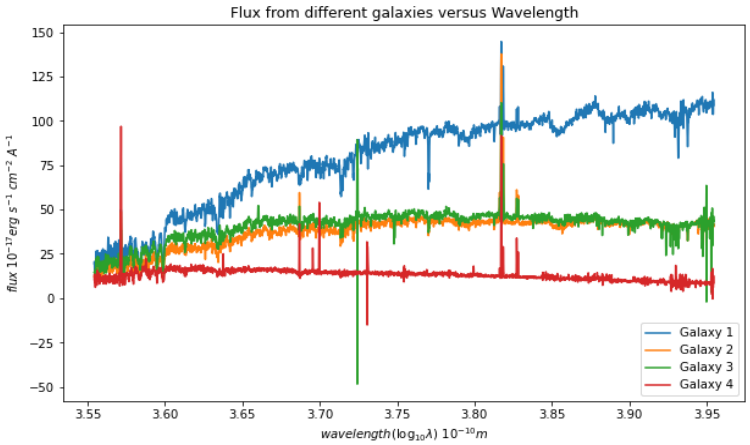
PHYA-UA 210 HW6

Robert Zhao

November 5, 2023

PS6

1 Part a)



We can find that the flux from different galaxies have peaks when the wavelength is at 3.82. So we have

$$10^{3.82} = \lambda \Rightarrow \lambda = 6606 \times 10^{-10} m = 660nm$$

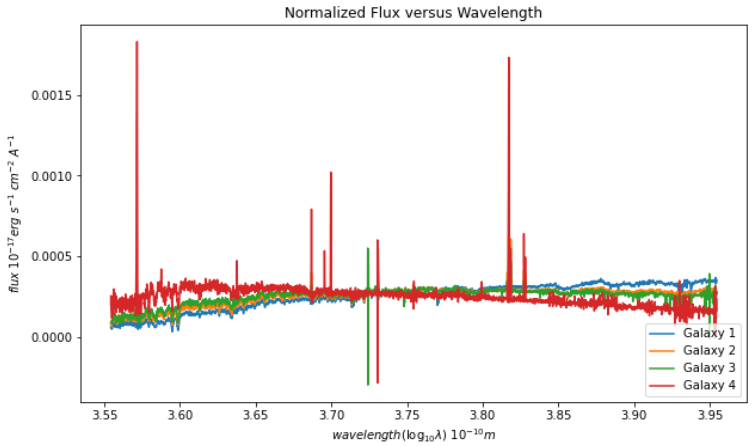
According to the formula for the energy of a photon, $E = \frac{hc}{\lambda}$, and we know that the formula defining the energy level transition in the Hydrogen atom is $E = -2.178 \times 10^{-18} J(\frac{1}{(n_1)^2-(n_2)^2})$. Therefore we have:

$$\frac{hc}{\lambda} = -2.178 \times 10^{-18} J(\frac{1}{(n_1)^2} - \frac{1}{(n_2)^2})$$

$$\Rightarrow n_1 = 3, n_2 = 2$$

Therefore, when the Hydrogen atom emits the photon, the electron of the atom is transmitted from energy level 3 to energy level 2.

2 Part b)

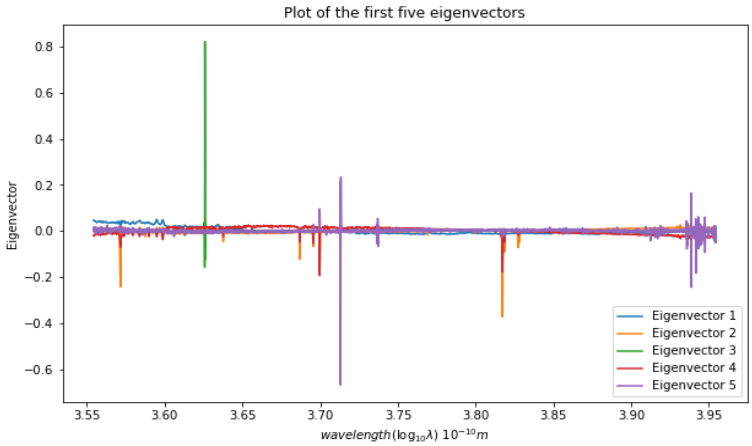


Firstly, I made a summation of all the flux of each galaxy to get the normalization constant for each galaxy, and then each element in this array of flux was divided by the normalization constant.

3 Part c)

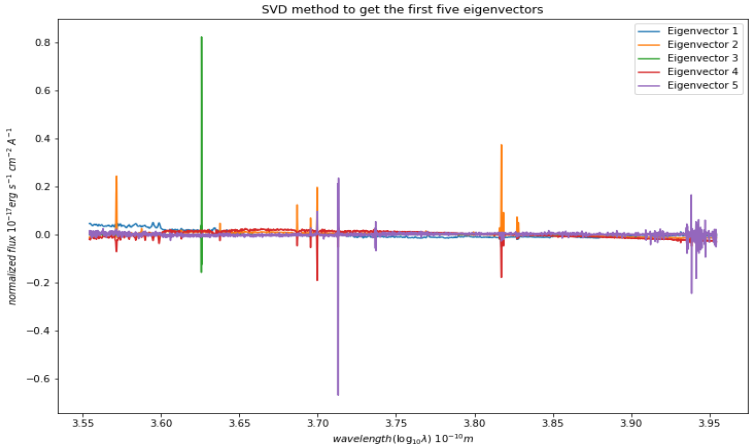
The matrix of the residuals is calculated through the formula $r_i = f_i - f_m$, with f_m is the mean of the normalized spectra.

4 Part d)



I got the covariance matrix through the formula $\mathbf{C} = \mathbf{R} \cdot \mathbf{R^T}$, where $\mathbf{R}$ is the transpose of the matrix of residuals (so that the dimension of the matrix is $N_{wave} \times N_{wave}$). Then I got the eigenvectors of the covariance matrix. The plot is for the first five eigenvectors in the matrix of eigenvectors.

5 Part e)



Let $\mathbf{X}$ be the matrix of eigenvectors of the covariance matrix $\mathbf{C} = \mathbf{R} \cdot \mathbf{R^T}$. Now we have $\mathbf{R} \cdot \mathbf{R^T} \cdot \mathbf{X} = \lambda \mathbf{X}$. As the matrix $\mathbf{V}$ is composed of the right eigenvectors of $\mathbf{R^T} \cdot \mathbf{R}$, then we have

$$\mathbf{R^T} \cdot \mathbf{R} \cdot \mathbf{V} = \lambda \mathbf{V}$$

$$\Rightarrow \mathbf{R R^T} \cdot \mathbf{R V} = \lambda \mathbf{R V}$$

$$\Rightarrow \mathbf{X} = \mathbf{R} \cdot \mathbf{V}$$

We know that $\mathbf{R}$ is the transpose matrix of the residual matrix, and we can get $\mathbf{V}$ through the SVD method. After getting the eigenvecotr $\mathbf{X}$, I normalized it and then we got the eigenvectros for the covariance matrix.
The computation time for the method in part d) is 26.9251389503479s, while the computation time for the method in part e) is 34.93965196609497s.

6 Part f)

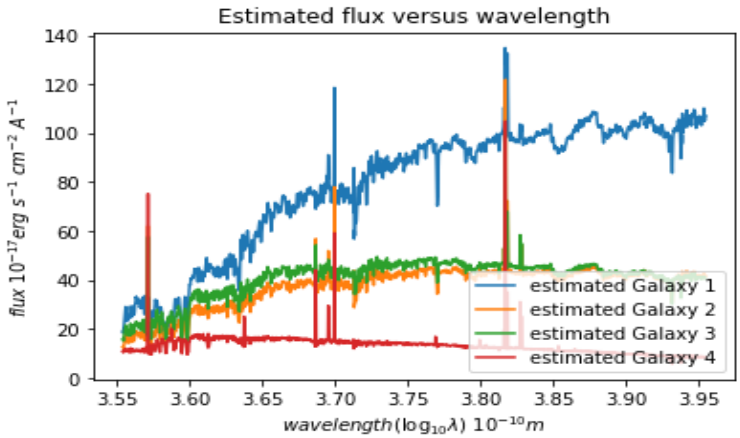
It is true that the SVD method spends more time than constructing the covariance matrix and then finding its eigenvectors. However, the condition number of the covariance matrix $\mathbf{C}$ is larger than the matrix $\mathbf{R}$. The condition number for matrix $\mathbf{C}$ is $3.8433522822351785 \times 10^{18}$, while the condition number for matrix $\mathbf{R}$ is: 4649219523235597.0. Since the condition number is large for matrix $\mathbf{C}$, the matrix can be unstable. It is close to be singular and may not be able to be solved through SVD if a small change occurs to the matrix.

7 Part g)

From last section, we have got $\mathbf{X} = \mathbf{R} \cdot \mathbf{V}$. Since we know that $\mathbf{V}$ is unitary($\mathbf{V}\mathbf{V}^\mathbf{T} = \mathbf{I}$), then we have

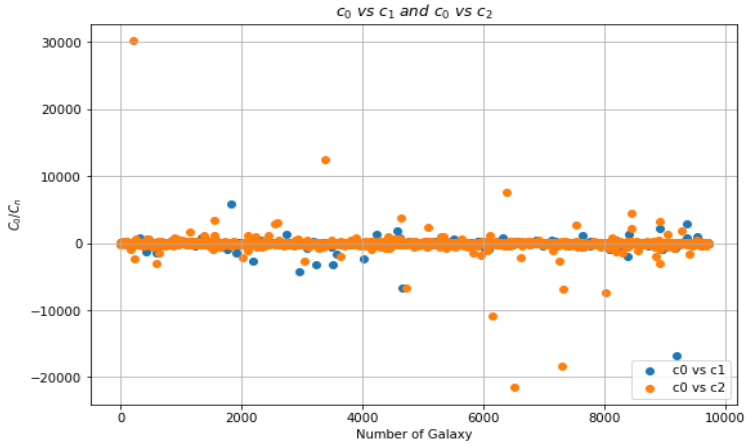
$$\mathbf{X}\mathbf{V}^\mathbf{T} = \mathbf{R}\mathbf{V}\mathbf{V}^\mathbf{T}$$
$$\Rightarrow \mathbf{X}\mathbf{V}^\mathbf{T} = \mathbf{R}$$
$$\Rightarrow (\mathbf{X}\mathbf{V}^\mathbf{T})^\mathbf{T} = \mathbf{R}^\mathbf{T}$$
$$\Rightarrow \mathbf{V}\mathbf{X}^\mathbf{T} = \mathbf{R}^\mathbf{T}$$

In this way, we get the estimated residual matrix. Then, through the formula $r = f_i - f_m \Rightarrow f_i = f_m + r$, we get the normalized flux matrix, then we multiply each element with normalization constant and get the estimated original flux.



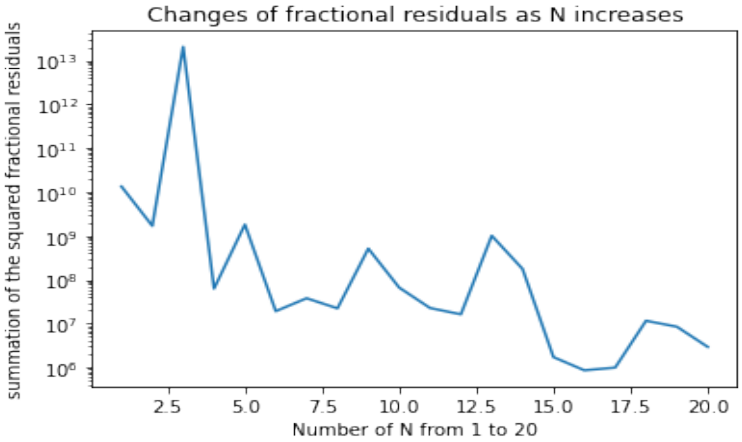
We can find that the estimated flux vs wavelength is approximated the same as the original one.

8 Part h)



The x-axis is numbers from 1 to 9713, and the y-axis is for $\frac{c_0}{c_1}$ and $\frac{c_0}{c_2}$.

9 Part i)



When calculating the squared fractional residuals, I let the matrix of original flux minus the matrix of estimated flux and then divided it by the estimated flux and squared it. After that I made a summation over all the elements of the residual matrix. From the plot, we can find that, generally speaking, though the summation of residuals fluctuates, the squared fractional residuals between the spectra and the estimated spectra decrease as the N number increases.
The fractional error for $N_c = 20$ is 2936378.1816461193

Link for GitHub: https://github.com/LagrangePointL3/phys-ua210