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$$\textbf{Q1)} \quad \begin{aligned} x(n) &= ax(n-1) \\ y(n) &= cx(n) + v(n) \end{aligned}$$

$$\begin{aligned} x(0) &\sim N(0,1) \\ v(n) &\sim N(0, \sigma_v^2) \end{aligned}$$

$$\textbf{Distribution}: \quad \frac{N(cx[n], \sigma_v^2) N(0, a^{2n})}{N(0, c^2 a^{2n} + \sigma_v^2)}$$

$$\textbf{Q2)} \quad \begin{aligned} x(n) &= ax(n-1) + w(n-1) \\ y(n) &= cx(n) + v(n) \end{aligned}$$

$$w(n-1) \sim N(0, \sigma_w^2)$$

$$\textbf{Distribution}: \quad \frac{N(cx[n], \sigma_v^2) N(0, a^{2n} + \sigma_w^2 \sum_{i=0}^{n-1} a^{2i})}{N(0, c^2 a^{2n} + c^2 \sigma_w^2 \sum_{i=0}^{n-1} a^{2i} + \sigma_v^2)}$$

In the previous Gaussian problem and in both these cases

$$\hat{x}[n] = \frac{y\sigma_x^2}{\sigma_y^2}$$

Gaussian: $\hat{x}[n] = \frac{y\sigma^2}{\sigma^2+1}$

No Noise: $\hat{x}[n] = \frac{1}{c} y[n]$

Noisy State, Noiseless Observation: $\hat{x}[n] = \frac{1}{c} y[n]$

Noiseless State, Noisy Observation: $\hat{x}[n] = \frac{yca^{2n}}{c^2 a^{2n} + \sigma_v^2} = \frac{yc\sigma_x^2}{\sigma_y^2}$

Noisy State, Noisy Observation: $\hat{x}[n] = \frac{yc(a^{2n} + \sigma_w^2 \sum_{i=0}^{n-1} a^{2i})}{c^2(a^{2n} + \sigma_w^2 \sum_{i=0}^{n-1} a^{2i}) + \sigma_v^2}$