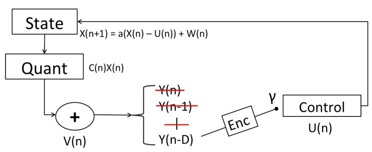
CSoI Mid-Year Report

Leah Dickstein
Mentored by: Gireeja Ranade, Anant Sahai

What I've been exploring is how time, delay and quantization affects information flows in a control system.

Problem Setup



$$X(n+1) = a(X(n) - U(n)) + W(n)$$

$$Y(n) = \begin{cases} Q(X(n)) + V(n) & : n \equiv 0 \text{ (mod D + 1)} \\ 0 & : \text{else} \end{cases}$$

$$Q(X(n)) = C(n)X(n)$$

$$U(n) = \begin{cases} L[X(n) \mid Y(n-D)] & : n \equiv 0 \text{ (mod D + 1)} \\ 0 & : \text{else} \end{cases}$$

The state is a random variable with some unstable system gain, which is observed with some AWGN and encoded and sent across a BEC to the controller. We choose to model the number of bits in the observation encoding as a linear function of delay, which is then represented as quantization noise in the observation. We use an adaptive quantizer, thus quantization noise is modeled as multiplicative noise. Encoding is done via a Reed Solomon scheme.

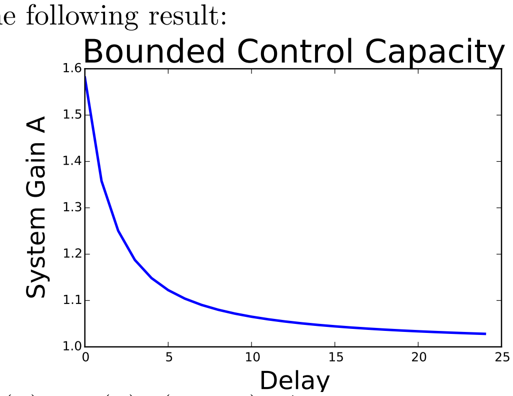
Exploring Bounded Control Capacity

I explored this problem framework under various cost functions. The question of interest was: is delay always bad? The first cost function minimizes the second moment of the state, while the second cost function also bounds the control capacity by penalizing control power. For simplicity, we chose an observation that had quantization noise but no AWGN. I obtained the following result:

$$\min \mathbb{E}[x^2(n+1)] + \sum_{k=1}^{n} a^2 u^2(k)$$

$$\alpha(n) = \frac{a^D \mu_c}{2(\mu_c^2 + \sigma_c^2)}$$

$$a^{2(D+1)} \left(\frac{\mu_c^2 + 4\sigma_c^2}{4(\mu_c^2 + \sigma_c^2)} \right) < 1$$



The control is the optimal linear memoryless controller $u(n) = \alpha(n)\, y(n-D)$. A is the maximum system gain where the state remains mean square stabilizable. The result makes sense, as the control "sacrifices" resolution of the state in exchange for reducing the power of the control. The result confirms that increasing delay is monotonically bad, even with different cost functions.

Future Work: Spring 2016

- I am currently working on a memory-based controller, and may pursue nonlinear controllers.
- I am also looking at how wireless protocols interact with control.