DELAY IN CONTROL SYSTEMS

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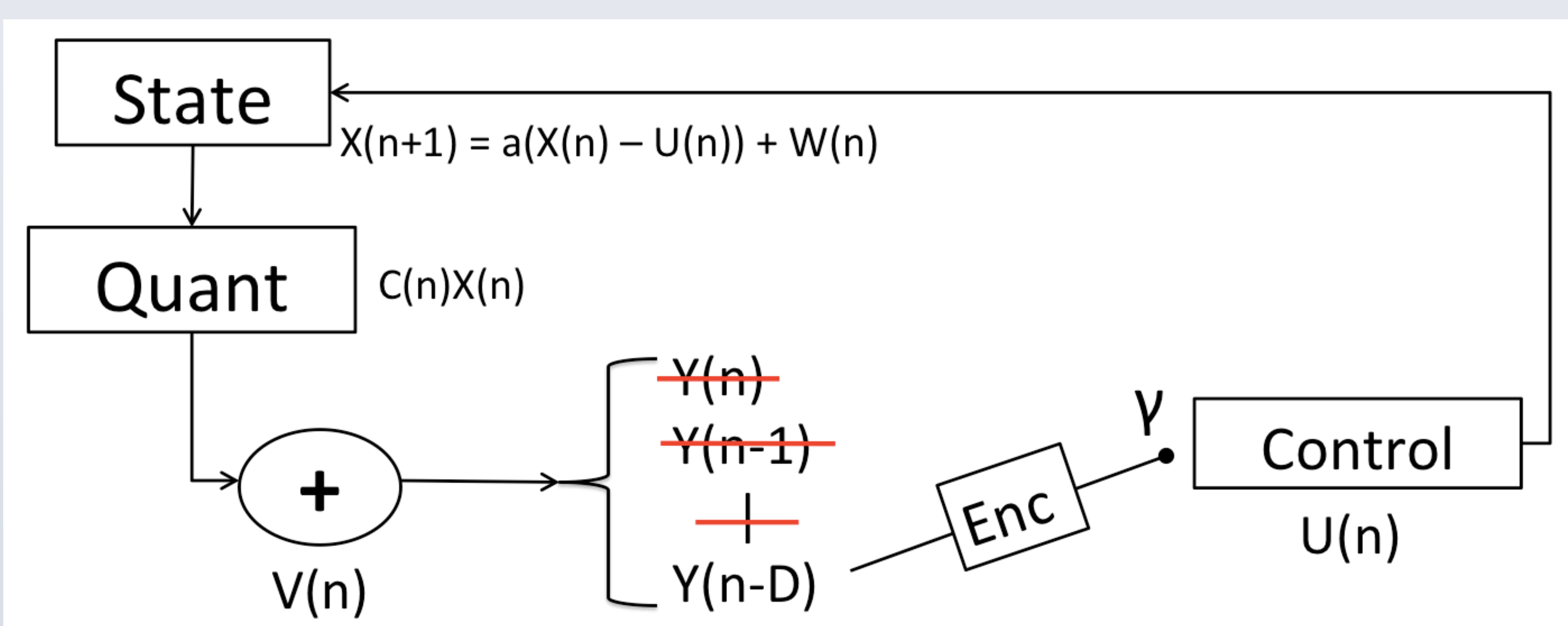
ABSTRACT

Communication channels are embedded in all technologies, and delay is present in all real-world channels. In control systems, the plant is growing increasingly unstable as time passes. The plant needs regular feedback from the controller in a short timeframe, and delay will increase the instability of the system. In contrast, delay means we are sending more bits across a channel before doing anything, which leads to higher precision in the message and higher reliability of decoding success. In this case, delay increases the stability of the system. Furthermore, the message encoding rate determines the message's precision and reliability. Our study is exploring the relationship between delay and system stability through various control and encoding strategies.

SYSTEM DYNAMICS



$$X(n+1) = a(X(n) - U(n)) + W(n)$$

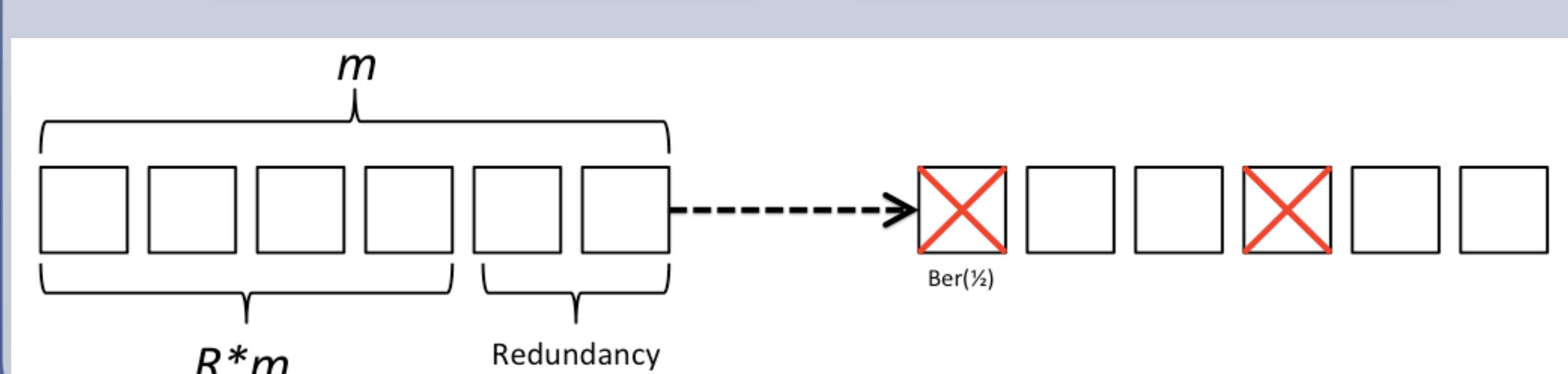
$$Y(n) = \begin{cases} Q(X(n)) + V(n) & : n \equiv 0 \pmod{D+1} \\ 0 & : \text{else} \end{cases}$$

$$Q(X(n)) = C(n)X(n)$$

$$U(n) = \begin{cases} L[X(n) \mid Y(n-D)] & : n \equiv 0 \pmod{D+1} \\ 0 & : \text{else} \end{cases}$$

$$X(0) \sim N(0,1)$$
$$W(n) \sim N(0, \sigma_w^2)$$
$$V(n) \sim N(0, \sigma_v^2)$$

$$m = D + 1$$
$$C(n) \sim N(1, 2^{-2Rm})$$
$$(\text{Success})\ \gamma = 1 - 2^{-((1-R)m+1)}$$



RESULTS

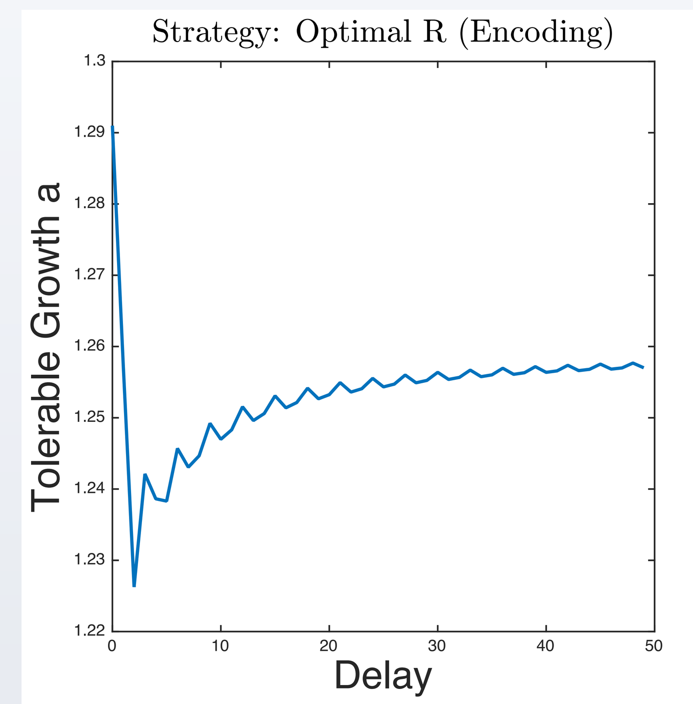


Figure 1
Instantaneous control/ communication is best, but delay is asymptotically good if D > 1

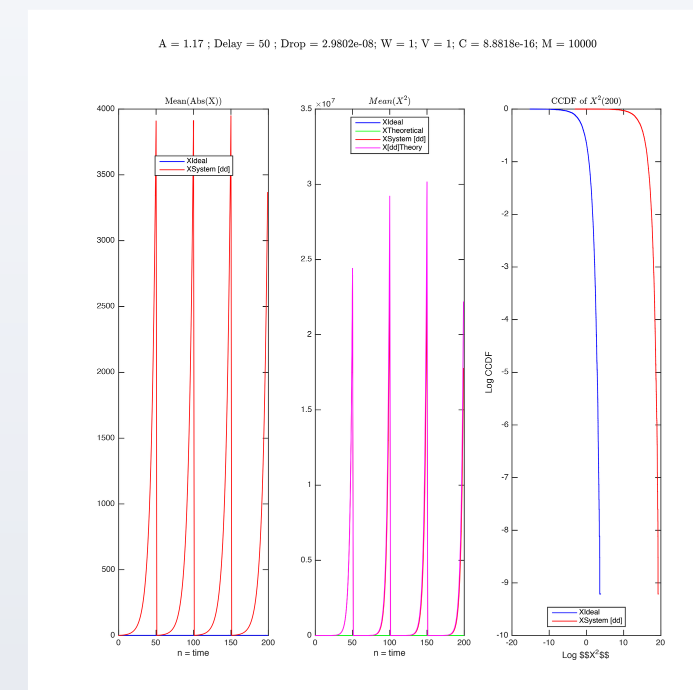


Figure 2
Simulation

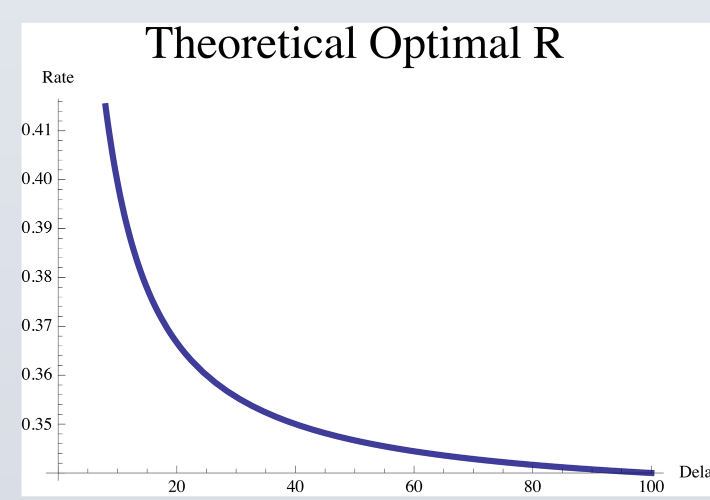


Figure 3
Optimal encoding rate decays and converges to ~0.34

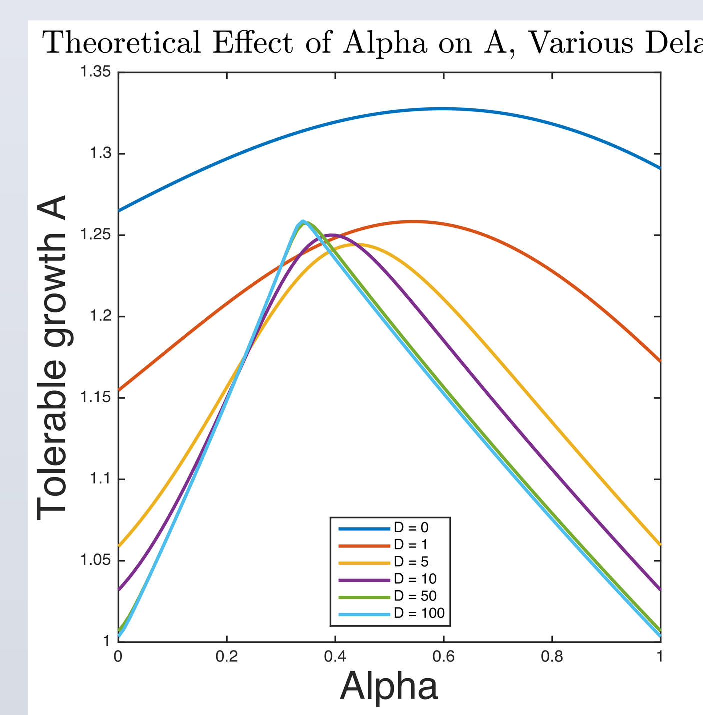


Figure 4
Encoding rate changes as delay increases

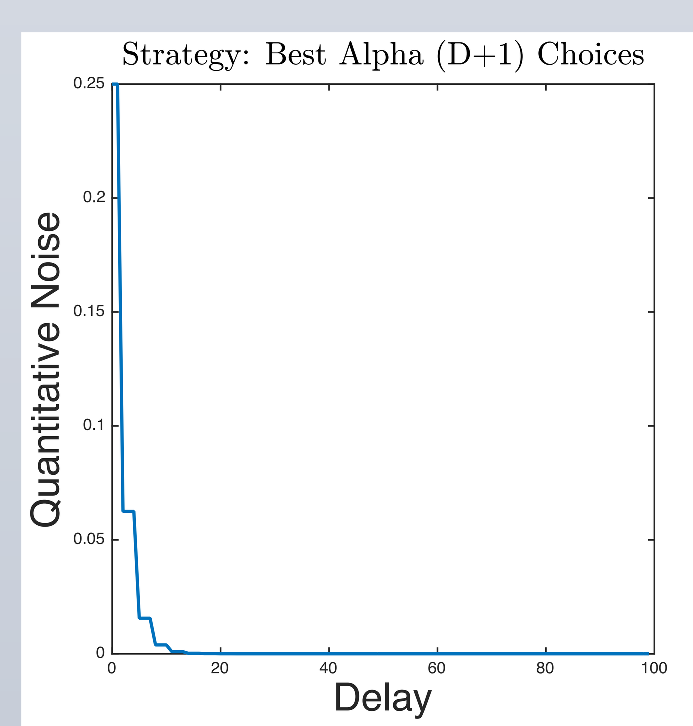


Figure 5
Graphical representation of the decay of the variance of C(n), $E[C^2(n)] - E[C(n)]^2$

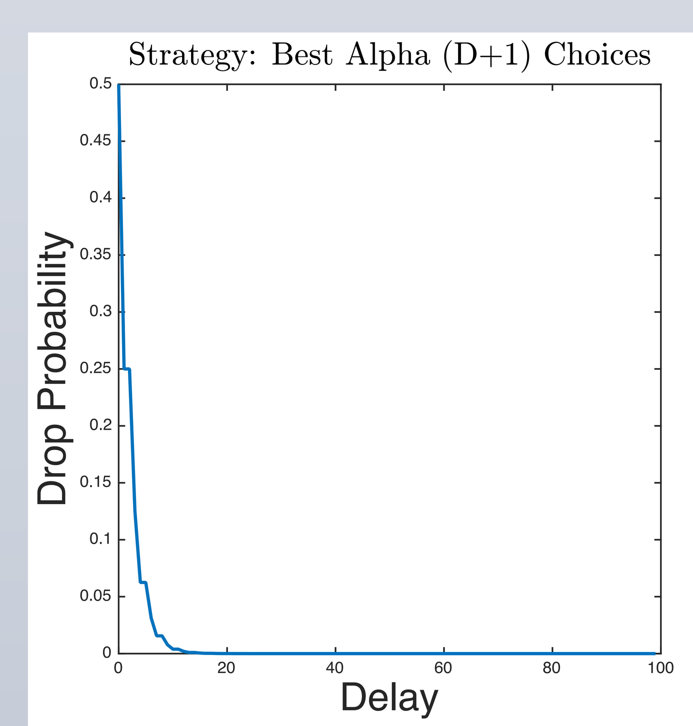


Figure 6
Graphical representation of the decay of drop probability, $2^{-((1-R)^{*}\text{code} + 1)}$

CONCLUSIONS

- Instantaneous control is best
- Real-world applications D >= 1
- Performance of the system improves asymptotically:
  - We don't need to achieve infinite delay to reap all possible benefits
- Jaggedness of the plot stems from the discrete nature of the packet drop model
- Optimal encoding rate R asymptotically decays as delay increases, converging to 0.34

FUTURE WORK

- Refining our model further
- Nonlinear or memory-based control strategies
- Generalizing beyond Reed-Solomon codes
- "Buffering" vs. "Streaming" delay
- Further exploration of 'finite blocklength' effects

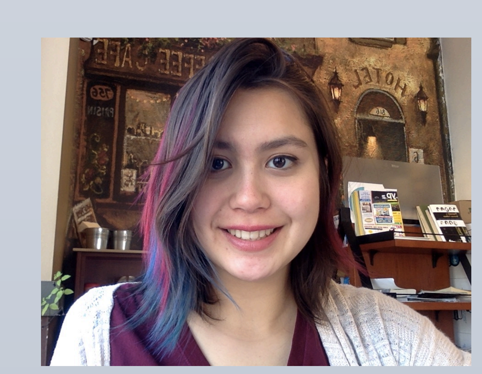
REFERENCES

[1] Shannon "A Mathematical Theory of Communication" 1948.

[2] Gireeja Ranade and Anant Sahai. Non-Coherence in Estimation and Control. Allerton 2013.

[3] S.Dey, A. Chiuso, L. Schenato. Remote estimation with noisy measurements subject to packet loss and quantization noise. \textit{Automatica} 2013

CONTACT



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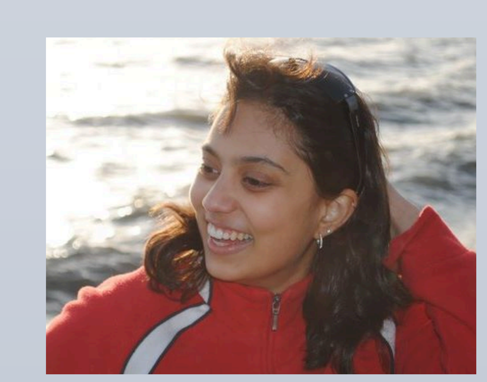

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