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Contents

1 Nonlinear Memoryless Control: Tangent Line Approximation

$$
\begin{aligned}
x(n) &= ax(n-1) + w(n-1) \\
y(n) &= cx^2(n) + v1(n) \\
y'(n) &= \alpha(n)x(n) + \beta(n) \\
\hat{x}(n) &= fy'(n) + g
\end{aligned}
$$

$$min \mathbb{E}[||X - \hat{X}||^2] = \mathbb{E}[||x(n) - fy'(n) - g||^2]$$

$$
\begin{aligned}
\frac{d}{df}\mathbb{E}[||x(n) - fy'(n) - g||^2] &= \mathbb{E}[2(x(n) - fy'(n) - g)(-y'(n))] = 0 \\
&= \mathbb{E}[-x(n)y'(n) + fy'^2(n) + gy'(n)] = 0 \\
f\mathbb{E}[y'^2(n)] &= \mathbb{E}[x(n)y'(n)] - g\mathbb{E}[y'(n)]
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dg}\mathbb{E}[||x(n) - fy'(n) - g||^2] &= \mathbb{E}[2(x(n) - fy'(n) - g)(-1)] = 0 \\
\mathbb{E}[-x(n) + fy'(n) + g] &= 0 \\
g &= \mathbb{E}[x(n) - fy'(n)]
\end{aligned}
$$

$$
\begin{aligned}
f\mathbb{E}[y'^2(n)] &= \mathbb{E}[x(n)y'(n)] - \mathbb{E}[y'(n)]\mathbb{E}[x(n) - fy'(n)] \\
f(\mathbb{E}[y'^2(n)] - (\mathbb{E}[y'(n)])^2) &= \mathbb{E}[x(n)y'(n)] - \mathbb{E}[x(n)]\mathbb{E}[y'(n)] \\
f &= \frac{cov(x(n), y(n))}{var(y'(n))}
\end{aligned}
$$

$$\hat{X}(n) = \frac{cov(x(n), y(n))}{var(y'(n))}(y'(n) - \mathbb{E}[y'(n)] + \mathbb{E}[x(n)]$$

Assuming $\alpha(n)$ and $\beta(n)$ are constants for a given n:

$$\begin{aligned}
\mathbb{E}[x(n)(\alpha x(n)+\beta)] - \mathbb{E}[x(n)]\mathbb{E}[\alpha x(n)+\beta] &= \alpha\mathbb{E}[x^2(n)] + \beta\mathbb{E}[x(n)] - \alpha(\mathbb{E}[x(n)])^2 - \beta\mathbb{E}[x(n)] \\
&= \alpha\mathbb{E}[x^2(n)] - \alpha(\mathbb{E}[x(n)])^2 \\
&= \alpha Var(x(n))
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}[(\alpha x(n)+\beta)^2] - (\mathbb{E}[\alpha x(n)+\beta])^2 &= \mathbb{E}[\alpha^2 x^2(n) + 2\alpha\beta x(n) + \beta^2] - \mathbb{E}[\alpha x(n)+\beta]\mathbb{E}[\alpha x(n)+\beta] \\
&= \alpha^2\mathbb{E}[x^2(n)] + 2\alpha\beta\mathbb{E}[x(n)] + \beta^2 - \alpha^2(\mathbb{E}[x(n)])^2 - 2\alpha\beta\mathbb{E}[x(n)] - \beta^2 \\
&= \alpha^2\mathbb{E}[x^2(n)] - \alpha^2(\mathbb{E}[x(n)])^2 \\
&= \alpha^2 Var(x(n))
\end{aligned}$$

$$\begin{aligned}
L[X(n)|Y(n)] &= \frac{1}{\alpha}(Y(n) - \mathbb{E}[Y(n)]) + \mathbb{E}[X(n)] \\
&= \frac{Y(n) - \mathbb{E}[\alpha X(n)+\beta]}{\alpha} + \mathbb{E}[X(n)] \\
&= \frac{Y(n)-\beta}{\alpha}
\end{aligned}$$

The calculation for $\alpha(n)$ and $\beta(n)$, if y' is the function and $y'(n)$ is the realization of the function at timestep n:

$$\begin{aligned}
\frac{dy}{dx} &= 2cx(n) \\
y' - y'(n) &= \frac{dy}{dx}(x - x(n)) \\
y' &= \frac{dy}{dx}x + \left(y'(n) - \frac{dy}{dx}x(n)\right) \\
&= \alpha x + \beta
\end{aligned}$$

$$\begin{aligned}
\alpha &= 2cx(n) \\
\beta &= y'(n) - \alpha x(n)
\end{aligned}$$

If I attempt to calculate the linear estimator after plugging in what α and β are, I'll just get the linear estimator of X and Y (where Y is nonlinear, the estimator gave unbounded error.)