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Mini Project Report

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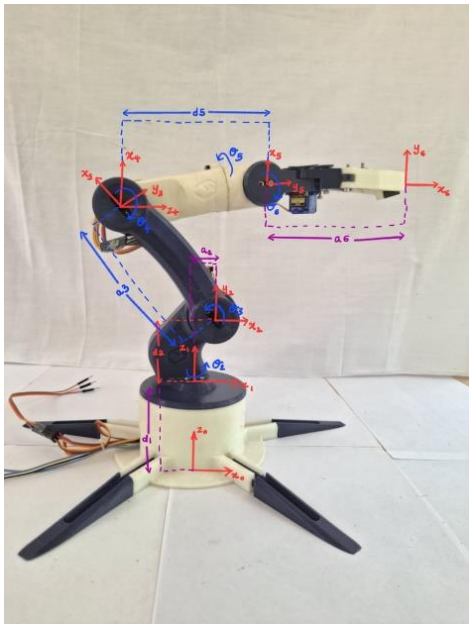
The robot arm we selected is a 5 DOF arm with all the joints revolute. It has 6 links.



DH Table

Link	a_i	α_i	d_i	θ_i
1	0	0	6.1	0
2	1.3	π/2	7	θ_2^*
3	12	π	0	$\theta_3^* + \pi/4$
4	0	-π/2	0	$\theta_4^* - \pi/4$
5	0	π/2	12.2	θ_5^*
6	13	π	0	$\theta_6^* + \pi/2$

Forward Kinematics Equations

We derived the forward kinematics equations through a MATLAB code. Those are as below.

$$H = \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_{11} \\ R_{21} & R_{22} & R_{23} & t_{21} \\ R_{31} & R_{32} & R_{33} & t_{31} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_{11} = cos(th6 + 1.57079)*(cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*cos(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2)) + 1.0*sin(th2)*sin(th5)) - sin(th6 + 1.5707)*(1.0*cos(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*cos(th2))

R_{12} = 1.0*cos(th6 + 1.5707)*(1.0*cos(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*cos(th2)) + sin(th6 + 1.5707)*(cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*cos(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2)) + 1.0*sin(th2)*sin(th5))

R_{13} = 1.0*cos(th5)*sin(th2) - 1.0*sin(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*cos(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2)

t_{11} = 1.3*cos(th2) + 12.021*cos(th3 + 0.7853)*cos(th2) + 13.0*cos(th6 + 1.5707)*(cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*cos(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2)) + 1.0*sin(th2)*sin(th5)) - 13.0*sin(th6 + 1.5707)*(1.0*cos(th3

+ 0.7853)*sin(th4 - 0.7853)*cos(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*cos(th2)) - 12.171*cos(th3 + 0.7853)*sin(th4 - 0.7853)*cos(th2) + 12.171*cos(th4 - 0.7853)*sin(th3 + 0.7853)*cos(th2)

R_{21} =- sin(th6 + 1.5707)*(1.0*cos(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*sin(th2)) - cos(th6 + 1.5707)*(1.0*cos(th2)*sin(th5) - cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*sin(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2)))

R_{22} = 1.0*cos(th6 + 1.5707)*(1.0*cos(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*sin(th2)) - sin(th6 + 1.5707)*(1.0*cos(th2)*sin(th5) - cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*sin(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2)))

R_{23} =- 1.0*cos(th2)*cos(th5) - 1.0*sin(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*sin(th2) + 1.0*sin(th3 + 0.78539816339744830961566084581988)*sin(th4 - 0.78539816339744830961566084581988)*sin(th2))

t_{21} = 1.3*sin(th2) - 13.0*sin(th6 + 1.5707)*(1.0*cos(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2) - 1.0*cos(th4 - 0.7853)*sin(th3 + 0.7853)*sin(th2)) + 12.021*cos(th3 + 0.7853)*sin(th2) - 13.0*cos(th6 + 1.5707)*(1.0*cos(th2)*sin(th5) - cos(th5)*(1.0*cos(th3 + 0.7853)*cos(th4 - 0.7853)*sin(th2) + 1.0*sin(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2))) - 12.171*cos(th3 + 0.7853)*sin(th4 - 0.7853)*sin(th2) + 12.171*cos(th4 - 0.7853)*sin(th3 + 0.7853)*sin(th2)

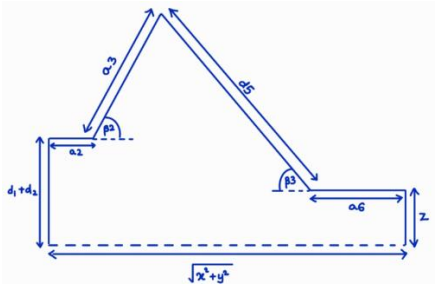
R_{31} =cos(th6 + 1.5707)*sin(th3 - 1.0*th4 + 1.5707)*cos(th5) - 1.0*sin(th6 + 1.5707)*cos(th3 - 1.0*th4 + 1.5707)

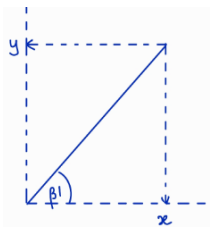
R_{32} =cos(th6 + 1.5707)*cos(th3 - 1.0*th4 + 1.5707) + sin(th6 + 1.5707)*sin(th3 - 1.0*th4 + 1.5707)*cos(th5)

R_{33} =-1.0*sin(th3 - th4 + 1.5707)*sin(th5)

t_{31} =12.021*sin(th3 + 0.7853) - 12.171*cos(th3 - th4 + 1.5707) + 6.5*cos(th6 - th5 + 1.5707)*sin(th3 - th4 + 1.5707) - 13.0*cos(th3 - th4 + 1.5707)*sin(th6 + 1.5707) + 6.5*cos(th5 + th6 + 1.5707)*sin(th3 - th4 + 1.5707) + 13.101

Inverse Kinematic Equations





$$y = x \tan \beta_1$$

$$\sqrt{x^2 + y^2} = 1.3 + 12.0 \cos \beta_2 + 12.171 \cos \beta_3 + 13$$

$$Z = 6.1 + 7.0 + 12.0 \sin \beta_2 - 12.1 \sin \beta_3$$

$$\theta_2 = \beta_1 \qquad \theta_4 = \beta_2 + \beta_3 - \frac{\pi}{2}$$

$$\theta_3 = \beta_2 \qquad \theta_5 = 0$$

$$\theta_6 = \frac{\pi}{2} - \beta_3$$

Manipulator Jacobian

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ 0 & 0 & J_{33} & J_{34} & J_{35} & J_{36} \\ 0 & 0 & \sin(th2) & -\sin(th2) & \sin(th3 - th4 + 1.57) \cdot \cos(th2) & -\sin(th2) \\ 0 & 0 & -\cos(th2) & \cos(th2) & \sin(th3 - th4 + 1.57) \cdot \sin(th2) & \cos(th2) \\ 1 & 1 & 0 & 0 & -\cos(th3 - th4 + 1.57) & 0 \end{bmatrix}$$

J_{11} = -(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000

J_{12} = -0.001*sin(th2)*(13000.0*cos(th3-th4-th6+)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0)

J_{13} = 1.0*cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79))

J_{14} = -cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{15} = 1.0*cos(th3-th4+1.57)*((sin(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-

th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th3-th4+1.57)*sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{16} = -cos(th2)*(12.171*cos(th3-th4+1.57)-12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{21} = (cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000

J_{22} = 0.001*cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.5)+12021.0*cos(th3+0.79)+1300.0)

J_{23} = sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79))

J_{24} = -1.0*sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{25} = 1.0*cos(th3-th4+1.57)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-

th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)+sin(th3-th4+1.57)*cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{26} = -1.0*sin(th2)*(12.171*cos(th3-th4+1.57)-12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{33} = -1.0*cos(th2)*(1.3*cos(th2)-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th2)*(1.3*sin(th2)-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{34} = cos(th2)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)+1.0*sin(th2)*((sin(th2)*(12021.0*cos(th3+0.79)+13

00.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{35} = sin(th3-th4+1.57)*sin(th2)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-

th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th3-th4+1.57)*cos(th2)*((sin(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{36} = -cos(th2)*((cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+ 1300.0))/1000-0.001*cos(th2)*(12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+ 0.79)+1300.0))- 1.0*sin(th2)*((sin(th2)*(13000.0*cos(th3-th4-

th6)+12171.0*sin(th3-th4+ 1.57)+12021.0*cos(th3+ 0.79)+1300.0))/1000-0.001*sin(th2)*(12171.0*sin(th3

th4+1.57)+12021.0*cos(th3+0.79)+1300.0))