Hexacopter Gazebo Parameters

Dynamics and SDF Configuration Notes

Lahiru Cooray

1 Overview

This document records the physical and simulation parameters used to model the custom hexacopter in **Gazebo Harmonic**, updated to a *component-wise* inertial model:

- Motors are *separate links* (cylindrical bodies) with their own mass/inertia.
- Arms are *separate links* modeled as **hollow square rods**.
- Landing-gear struts are *separate links* modeled as **hollow square rods**.
- Rotors (propellers) keep their own inertia (two-blade, boxed approximation).

All units are SI unless noted.

2 Coordinate and Color Conventions

Gazebo's right-handed frame and colors:

$$\text{X (red)}, \quad \text{Y (green)}, \quad \text{Z (blue)}.$$

For a two-blade propeller (spin about Z):

$$I_{xx} \approx I_{zz} \gg I_{yy}.$$

3 Component Inventory (Inertia Model)

Components represented in the *inertia* calculation:

1. **Base assembly** (base+top+battery+electronics lumped) as a hexagonal prism.
2. **6 Arms** as hollow square rods (separate links).
3. **6 Motors** as solid cylinders (separate links).
4. **6 Rotors** (two-blade props; inertia included).
5. **6 Landing-gear struts** as hollow square rods (separate links).
6. Optional: sensors/payloads (IMU, cameras, etc.) as needed.

4 Mass & Geometry Data (current values)

Component	Mass (kg)	Key Dimensions (m)	Notes
Base assembly	$M = 1.9949$	$R = 0.15$ (circumradius), $h = 0.085$	Lumps base+top+battery+electronics
Arm (each)	0.0126	$L = 0.15885$, $a = 0.01$ (outer), $b = 0.008$ (inner)	Hollow square tube
Motor (each)	0.072	$r = 0.014$, $h = 0.03$	Solid cylinder
Rotor (each)	0.008	$D = 11\,\text{in} \approx 0.2794$, $w = 0.02$, $t = 0.001$	Two-blade boxed approx.
Landing gear (each)	0.02825	$L = 0.30315$, $a = 0.01$, $b = 0.008$	Hollow square rod (vertical)

Total mass (example). With the above numbers, the vehicle mass is

$$m_{\text{tot}} = 1.9949 + 6\big(0.0126 + 0.072 + 0.008 + 0.02825\big) = \boxed{2.72\ \text{kg}}.$$

5 Inertia Equations (per-component, about local CoM)

5.1 Hollow Square Rod (Arms & Landing Gear)

Let length L, outer width a, inner width b, mass m. Using the working convention adopted for this model:

$$I_{zz} = \frac{1}{6}\, m\,(a^2 + b^2), \tag{1}$$

$$I_{xx} = I_{yy} = \frac{1}{12}\, m \left[L^2 + \frac{(a^2 + b^2)}{2}\right]. \tag{2}$$

Orientation: for arms aligned in-plane along their local x (long) axis, I_{xx} is the smallest; for landing gear aligned with z (long) axis, swap axis labels accordingly when interpreting.

Arm numeric check (matches SDF). With $m = 0.0126$, $L = 0.15885$, $a = 0.01$, $b = 0.008$:

$$I_{xx} = I_{yy} = \boxed{2.658 \times 10^{-5}}, \qquad I_{zz} = \boxed{3.444 \times 10^{-7}}\ \text{kg}\,\text{m}^2.$$

Landing-gear numeric check. $m = 0.02825$, $L = 0.30315$, $a = 0.01$, $b = 0.008$:

$$I_{xx} = I_{yy} = \boxed{2.1654 \times 10^{-4}}, \qquad I_{zz} = \boxed{7.722 \times 10^{-7}}\ \text{kg}\,\text{m}^2.$$

5.2 Motor as Solid Cylinder

Mass m, radius r, height h:

$$I_{zz} = \frac{1}{2} m r^2, \tag{3}$$

$$I_{xx} = I_{yy} = \frac{1}{12} m (3r^2 + h^2). \tag{4}$$

Numeric (matches SDF with $h = 0.03$): $m = 0.072$, $r = 0.014$, $h = 0.03$:

$$I_{xx} = I_{yy} = \boxed{8.928 \times 10^{-6}}, \qquad I_{zz} = \boxed{7.056 \times 10^{-6}}\ \text{kg}\,\text{m}^2.$$

5.3 Rotor (Two-Blade, Boxed Approximation)

Total mass m, span $L = 0.2794$, width $w = 0.02$, thickness $t = 0.001$; axes: span$\to Y$, width$\to X$, thickness$\to Z$:

$$I_{xx} = \frac{1}{12}m(L^2 + t^2), \quad I_{yy} = \frac{1}{12}m(w^2 + t^2), \quad I_{zz} = \frac{1}{12}m(w^2 + L^2). \tag{5}$$

Numeric (as used in SDF):

$$I_{xx} = \boxed{5.205 \times 10^{-5}}, \; I_{yy} = \boxed{2.667 \times 10^{-7}}, \; I_{zz} = \boxed{5.230 \times 10^{-5}} \; \mathrm{kg\,m^2}.$$

6 SDF Snippets (about local CoM)

6.1 Arm (hollow square tube)

```
<inertial>
  <!-- CoM pose of the arm relative to its link frame -->
  <pose>0.079425 0 -0.005 0 0 0</pose>
  <mass>0.0126</mass>
  <inertia>
    <!-- Hollow square rod: L=0.15885, a=0.01, b=0.008 (about local CoM) -->
    <ixx>2.658e-05</ixx>
    <ixy>0</ixy>
    <ixz>0</ixz>
    <iyy>2.658e-05</iyy>
    <iyz>0</iyz>
    <izz>3.444e-07</izz>
  </inertia>
</inertial>
```

6.2 Motor (solid cylinder)

```
<inertial>
  <mass>0.072</mass>
  <inertia>
    <!-- r=0.014, h=0.03 (about cylinder axis as local z) -->
    <ixx>8.928e-06</ixx>
    <ixy>0</ixy>
    <ixz>0</ixz>
    <iyy>8.928e-06</iyy>
    <iyz>0</iyz>
    <izz>7.056e-06</izz>
  </inertia>
</inertial>
```

6.3 Rotor (two-blade boxed)

```
<inertial>
  <mass>0.008</mass>
  <inertia>
    <ixx>5.205e-05</ixx>
    <ixy>0</ixy>
    <ixz>0</ixz>
    <iyy>2.667e-07</iyy>
    <iyz>0</iyz>
    <izz>5.230e-05</izz>
  </inertia>
</inertial>
```

6.4 Landing-gear (hollow square rod, vertical)

```
<inertial>
  <pose>0 0 -0.151575 0 0 0</pose>
  <mass>0.02825</mass>
  <inertia>
    <!-- L=0.30315, a=0.01, b=0.008 -->
    <ixx>2.165e-04</ixx>
    <ixy>0</ixy>
    <ixz>0</ixz>
    <iyy>2.165e-04</iyy>
    <iyz>0</iyz>
    <izz>7.722e-07</izz>
  </inertia>
</inertial>
```

7 Base Structure: Hexagonal Prism (unchanged lump)

For reference, with $M = 1.9949$, $R = 0.15$, $h = 0.085$:

$$I_{zz} = \frac{5}{12} M R^2, \qquad I_{xx} = I_{yy} = \frac{5}{24} M R^2 + \frac{M h^2}{12}, \qquad z_{\text{CoM}} = \frac{h}{2}.$$

Numerically,

$$I_{zz} = \boxed{1.8702 \times 10^{-2}}, \quad I_{xx} = I_{yy} = \boxed{1.0552 \times 10^{-2}} \text{ kg m}^2, \quad z_{\text{CoM}} = \boxed{0.0425 \text{ m}}.$$

8 Assumptions & Notes

- Each component is modeled with **uniform density**.
- Arms and landing gear are modeled as **hollow prismatic tubes** (square cross section) with constant outer and inner side lengths a and b.
- Motors are treated as **solid cylinders** with axis along Z.
- Rotors use a **conservative rectangular boxed approximation**, which is sufficient for rigid-body dynamics and produces correct total inertia magnitudes.
- All components are modeled as **geometrically symmetric about their centroidal axes**, so the principal axes coincide with the body axes.
- Under this assumption, the inertia tensor for each link becomes **diagonal**:

  $$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix},$$

  which simplifies both analytical and simulation modeling in Gazebo.
- Coupling terms (I_{xy}, I_{xz}, I_{yz}) are neglected since symmetry ensures these cross-products are zero.
- The coordinate frames for each link are defined such that their local x, y, z axes align with the principal inertia axes of the geometry.