Julia Sets

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Julia Set: A set of complex numbers that do not converge to any limit when a given mapping is repeatedly applied to them. In some cases the result is a connected fractal set.

First, a bit of background.

Gaston Julia, a French Mathematician, published a paper in 1918 concerning iterationsions of a rational function. Despite initial popularity, the fame quickly faded...

until Benoit Mandelbrot discovered the Mandelbrot set in 1979 with the assistance of modern computers. This allowed him to create images of the sets. Now fractals are immensely popular.

A Julia Set is an example of a set with chaotic behavior in Complex Dynamics. It is iterated multiple times over some c value to obtain the image. Any small change to c has a large effect. Hence the chaotic behavior.

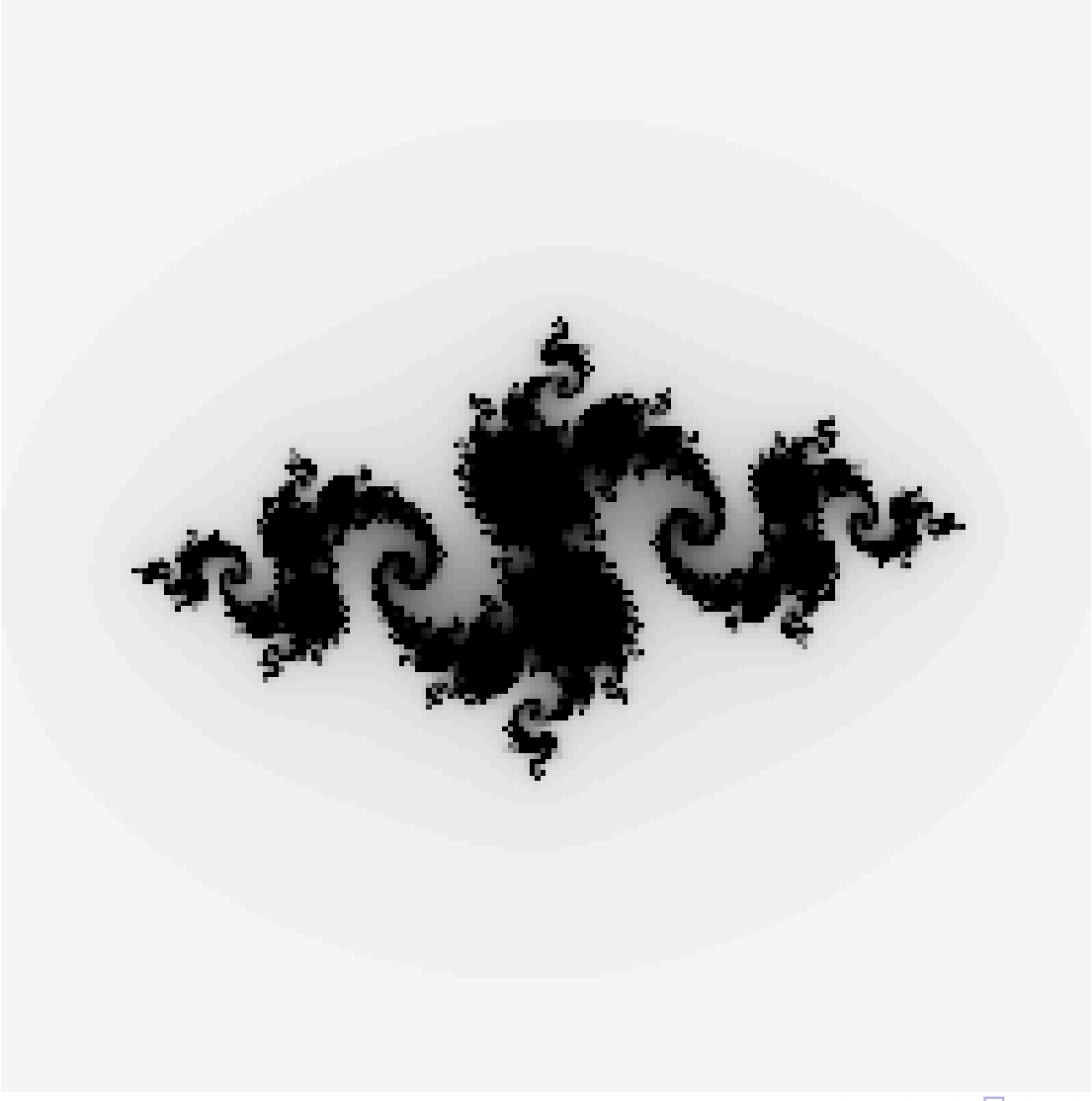
This particular Sage function uses the equation $z = z^2 + c$ to generate its Julia Set images. There are certainly other possibilities including cubic equations, exponentials, etc., but the quadratic equation is simple and quite popular.

The code is written in Python. It creates a .pgm (Portable Graymap) file, which is essentially a series of numbers corresponding to a grayscale.

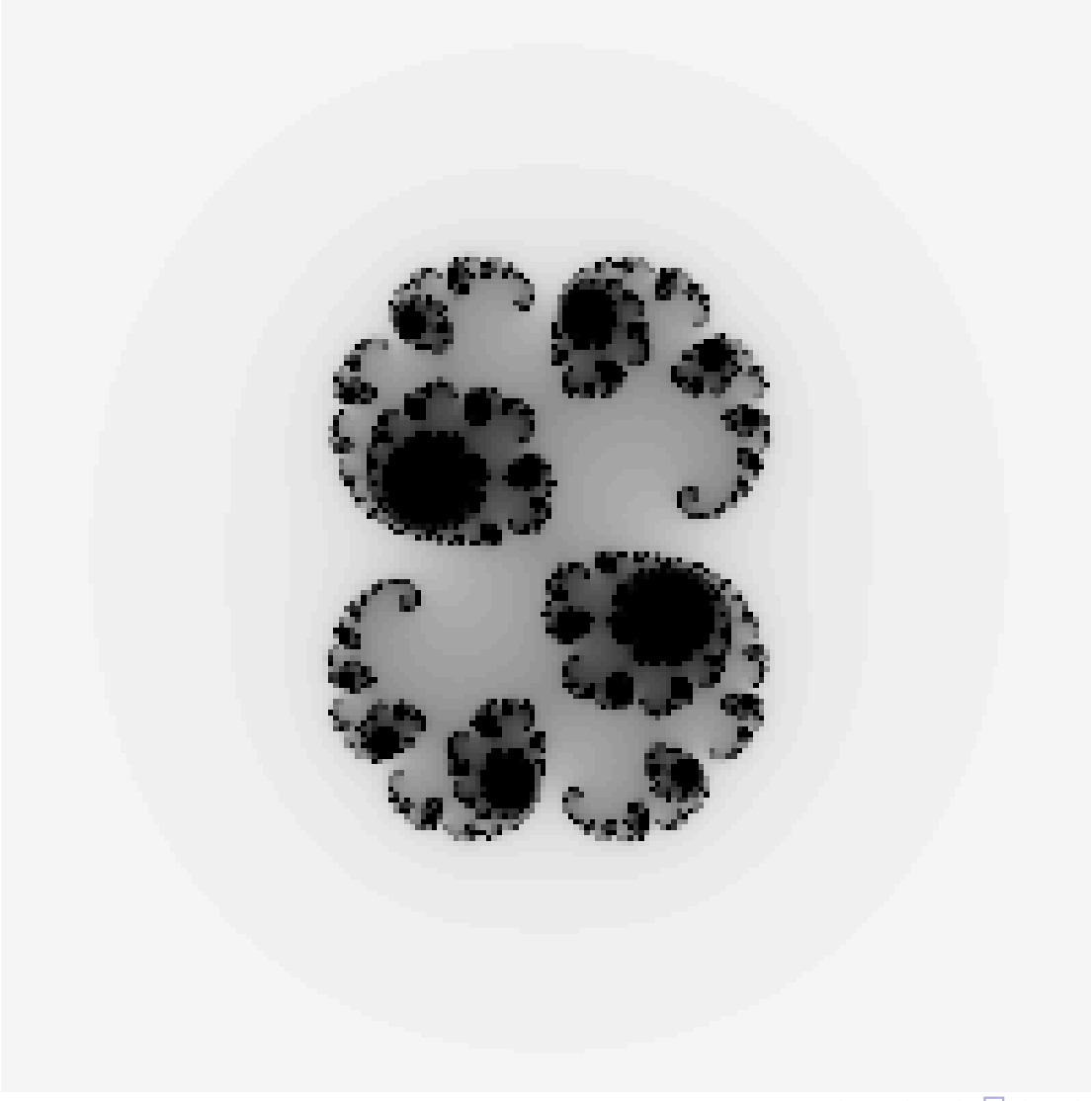
A very small sample of a .pgm file:

```
225 230 230 230 230 230 230 230 230
5 235 235 235 235 240 240 240 240 2
5 245 245
0 240 240 240 240 240 240 240 240 2
5 235 235 235 235 235 235 230 230 2
5 225 225 230 230 230 230 230 230 2
5 235 235 235 235 240 240 240 240 2
5 245
```

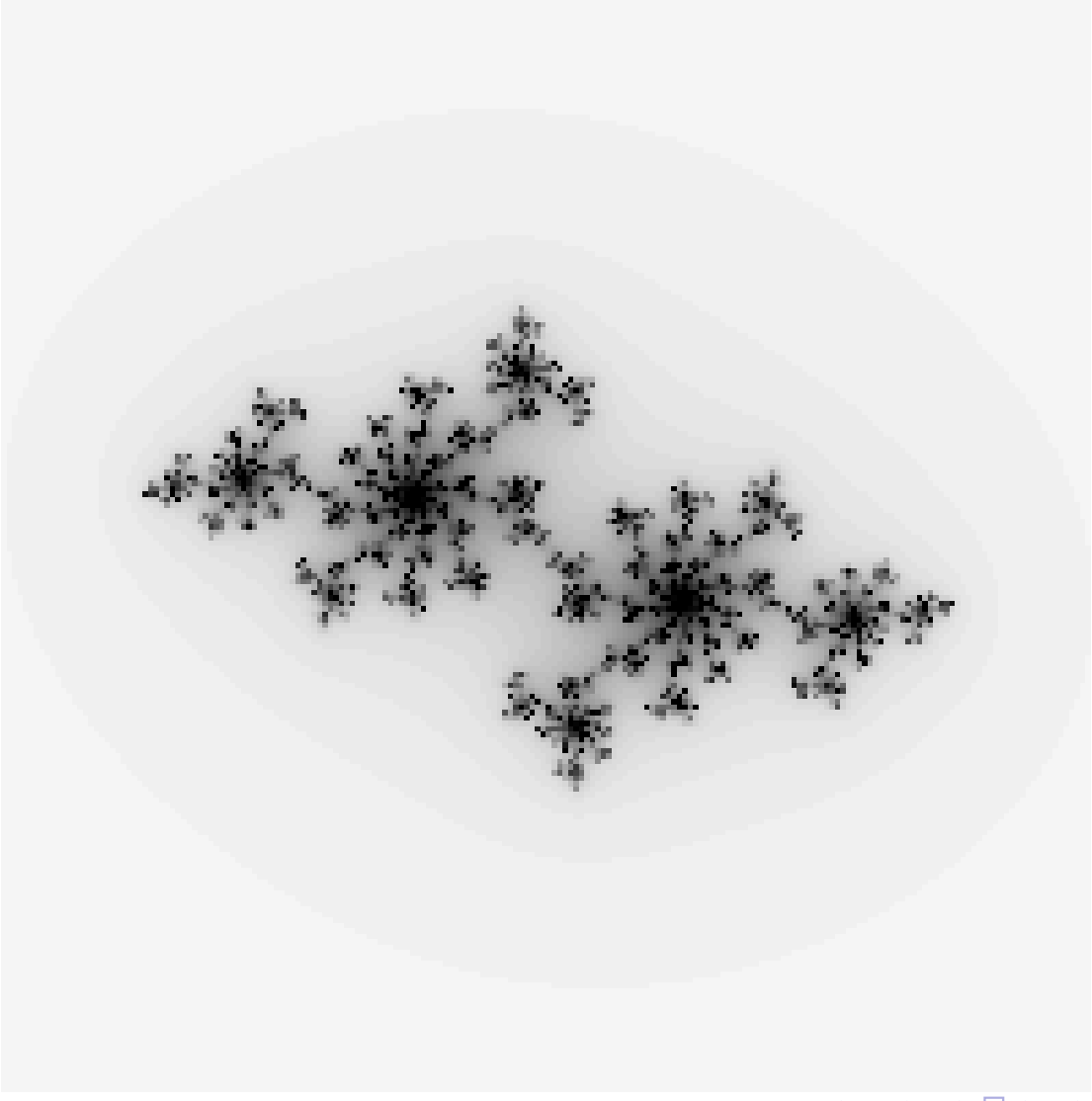
Julia Set with $c = -0.8 + 0.156i$



Julia Set with $c = 0.285 + 0.01i$



Julia Set with $c = 0.70176 - 0.3842i$



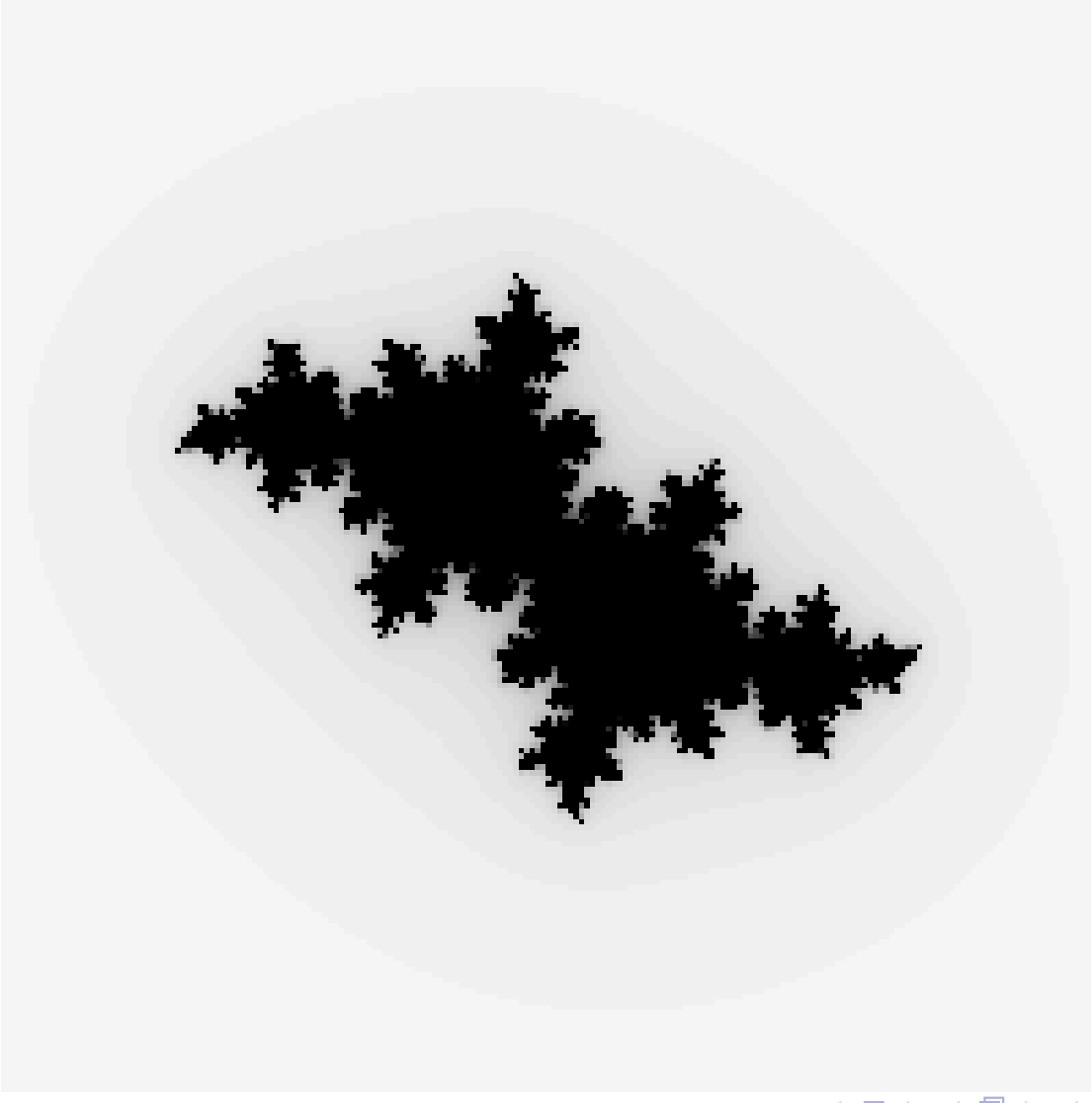
*I picked random numbers from here on out.

Julia Set with $c = -0.045 + 0.764i$

*It doesn't always result in a nice image.

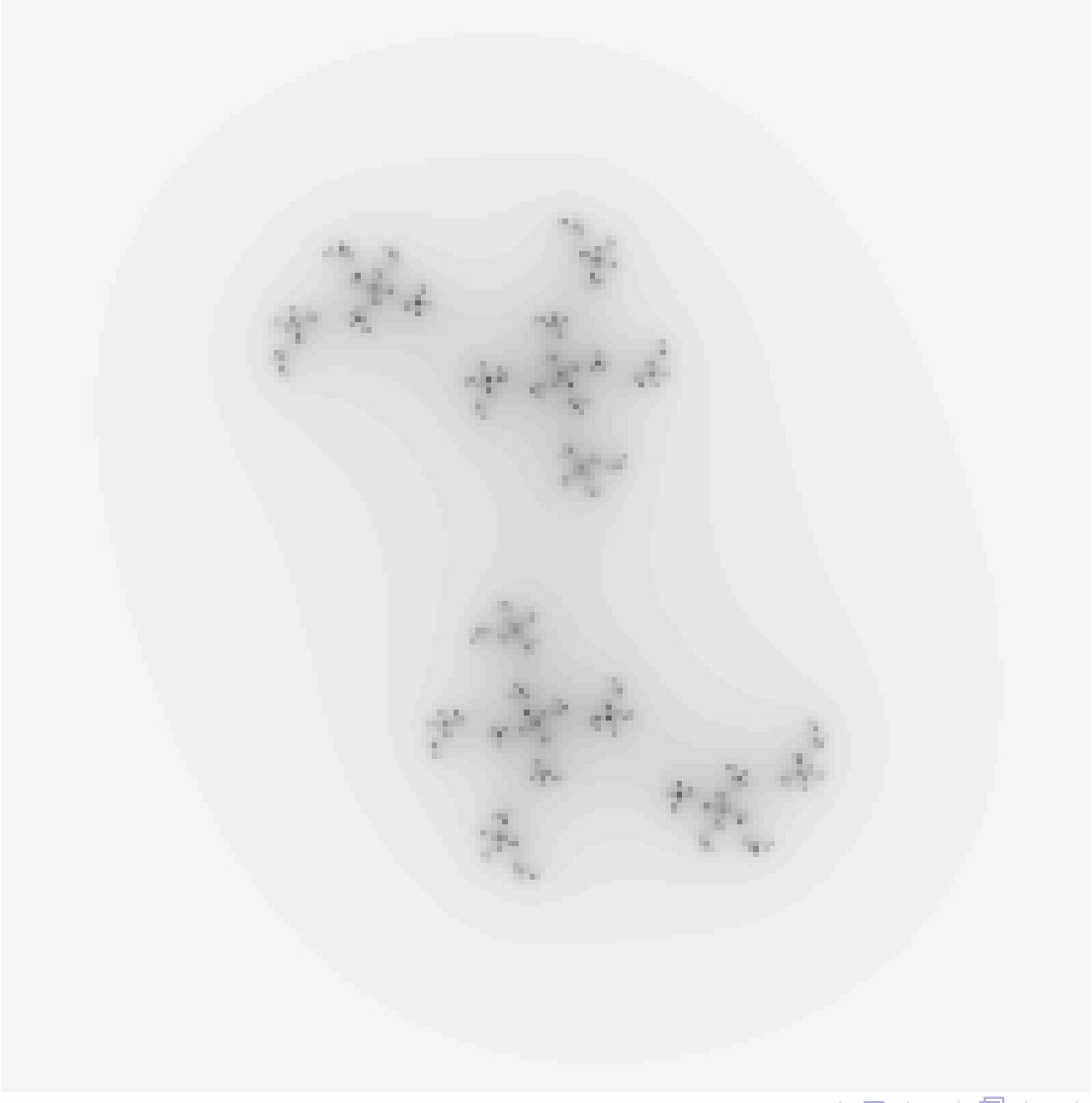
Julia Set with $c = 0.876 + 0.259i$

Julia Set with $c = -0.378 - 0.594i$

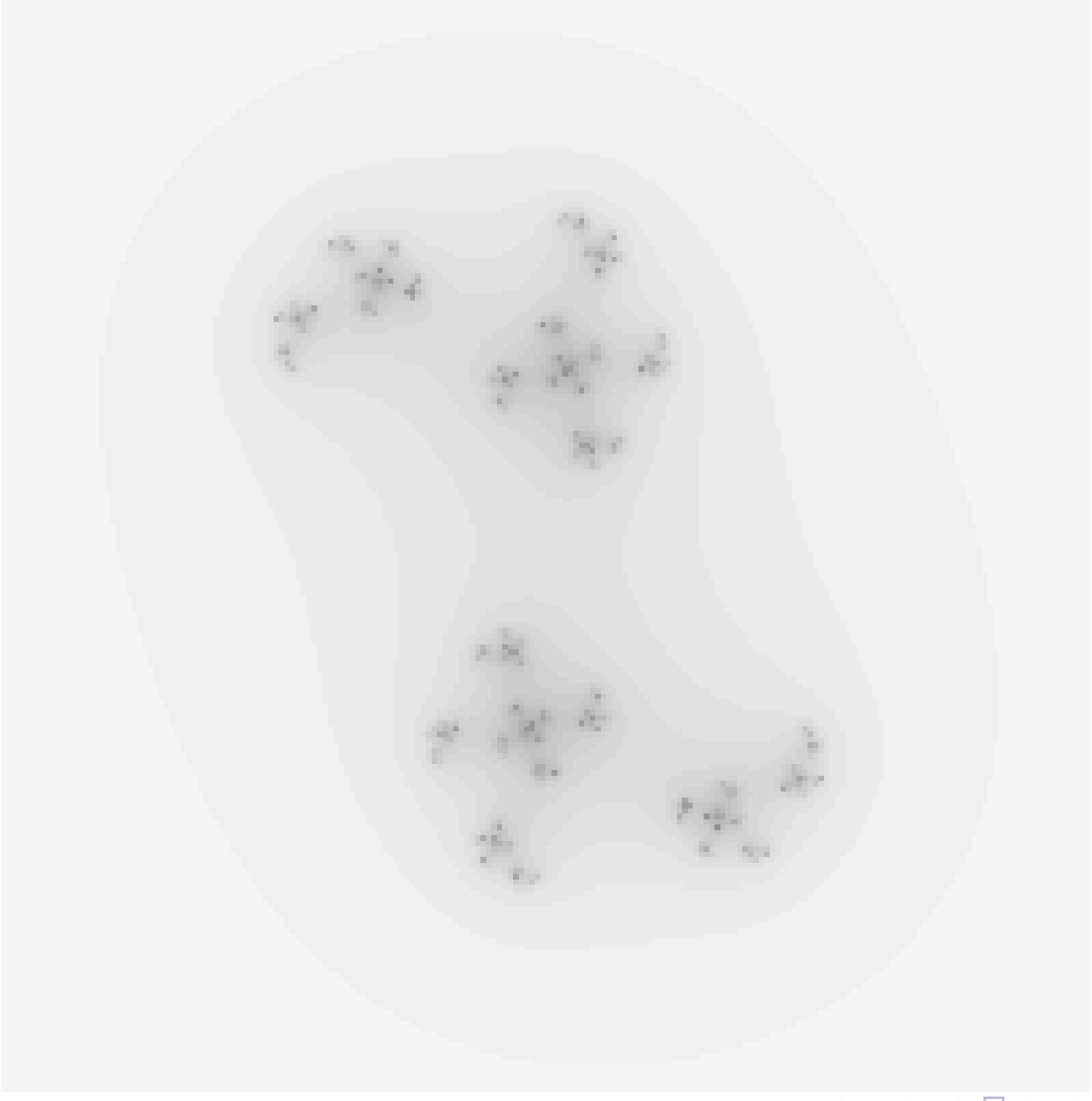


These next two have very similar c values. I changed the real component by 0.05.

Julia Set with $c = 0.45 - 0.57i$



Julia Set with $c = 0.50 - 0.57i$



There are many different iteration methods available to plot fractals. Fractals have become quite popular as a focus of jewelry, photography, and other art.

Thank You