***De Motu's* applicability and the underdetermination of inverse square centripetal forces**

By: Lo Min Choong Julian | 3 May 2024

Written for the partial completion of NTU's HY4130 Special Topics in Philosophy of Science, History and Philosophy of Science: From Ptolemy to Newton

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In 1684, Newton sent a copy of the first version of *De Motu Corporum in Gyrum* to Edmond Halley, a leading astronomer. *De Motu* was written in response to a question—Halley asked Newton what path a planet would trace if the only force acting on it was an inverse square force directed at some point, i.e. a centre. Newton's predecessors, Ismaël Bullialdus and Giovanni Borelli, suggested without evidence that the Sun has an attractive force which causes planets to move in elliptical orbits. Robert Hooke, Newton's contemporary, boasted that he had evidence of elliptical orbits, but failed to produce evidence even under a prize incentive. Thus, when Newton answered that it would trace an ellipse, proof or evidence in support of the non-novel claim would have been significant. In this essay, I will show that *De Motu* alone was significant as it unified key Keplerian and Galilean concepts and attributed them to a single physical phenomenon. *De Motu* demonstrated that there was an underlying structure behind Kepler's three rules and Galileo's local motion theory, giving each more weight as other competing theories were seen as "ad hoc" in comparison. Thus, regarding the problem of underdetermination, *De Motu* shifted the focus to two concerns: whether an inverse square centripetal force exists, and to what extent is *De Motu* applicable as a mathematical framework in explaining physical phenomena and observational data.

Before we can discuss the significance of the framework, we will discuss the significance of Newton's definitions and hypotheses. *De Motu* opens with three definitions and four hypotheses. The first describes "vis centripeta", i.e. centripetal force—"I call centripetal that force by which a body is impelled or drawn towards any point which is regarded as a centre [of force]." (Newton, 1974, p. 277). This is significant as this is the first description of this type of force, which is the central phenomenon of Newton's inquiry. Definition 2 and Hypothesis 2 describe Newton's conceptualisation of inertia; he states an external force will alter the motion of a body, which will otherwise "[move] uniformly in a straight line to infinity" (p. 277). Definition 3 is his conceptualisation of resistance; Hypothesis 1 states resistance in the first nine propositions is "zero" (p. 277). Hypothesis 3 is Newton's version of the parallelogram rule, which states that the distances covered by a body can be described as a consequence of two forces (p. 277). This is significant as it highlights Newton's position on forces, in contrast with the Galilean and Huygensian versions of motions in their parallelogram rule. Hypothesis 4, which is absent from version 1, but is present in the registered version at the Royal Society, states that the centripetal force acting on a body is described by the square of the time interval, but only during a body's initial motion (p. 278). This is significant as it describes uniform acceleration during a body's initial motion. This, paired with Problem 5, 6, 7, shows that Galilean motion is an approximation, which will be discussed later. Thus, these definitions and hypotheses serve as the basis and scope of Newton's work. It is worth noting what Newton proved mathematically, and what is out of scope. Newton assumed the existence of a centripetal force (along with Newtonian inertia), and then built a rich mathematical framework on it. While Newton's framework suggests that such concepts are not ad hoc, he failed to "prove" the existence of his key underlying assumptions. Thus, *De Motu* prompted a line of inquiry: do inverse square centripetal forces exist? We will see that this is not the only point of contention, as the applicability of *De Motu* can be questioned further.

De Motu alone is significant because of its rich mathematical framework. Newton's results are built on analytic geometry and by taking the limits of certain variables. All proofs in *De Motu* are of the mathematical kind. For example, Theorem 1 shows that if a centripetal force (directed at a point) acts on a body, Kepler's area rule holds (p. 278). This result is significant as it enabled him to prove his other theorems. In Theorem 1, Newton provided a two-part geometric proof showing that discrete impulse forces (directed at a point) at regular intervals will cause a body to trace out triangles of equal areas. I refer to Figure 1 to explain the first part of the proof. SBA and ScB have equal areas because BA and cB are of equal lengths, and because both heights are the same, as both share the same vertex, S. ScB and SCB have equal areas as cB and CB are of equal lengths, and because both heights are the same (sharing vertex S). The same method shows that all other triangles are of equal areas. In the second part, Newton took the limit of the time interval (to zero), thereby stating that the force is no longer discrete but continuous. Thus, the path traced becomes a curve instead of a polygon, while retaining the fact that each segment is of equal area. Thus, Newton demonstrated the relationship between time and area. One could argue that this proof is problematic, as a continuous force might not be representable over non-infinitesimal arcs by the action of infinitely many discrete impulse forces. Regardless, I have shown that Newton derived results via a rich mathematical framework. This strengthens the status of Kepler's area rule, as it added a physical basis, i.e. an attribution of a physical cause; if one could demonstrate the existence of a centripetal force, one would strengthen the status of Kepler's area rule.

Figure 1

Diagram of theorem 1

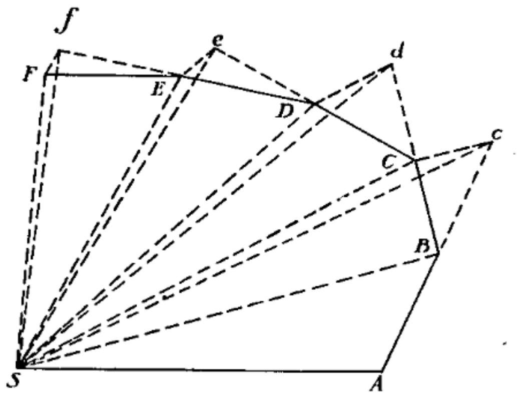

Figure 1.

Note. Taken from *De Motu*, Newton, 1974, p. 258

De Motu is significant because it unified several key concepts, i.e. Kepler's three rules and Galileo's two "laws". Newton proved key Keplerian results via his mathematical framework. The question remains: how applicable are his results? As previously discussed, Newton demonstrated that Kepler's area rule is a consequence of a centripetal force. Thus, does such a centripetal force exist? Given *De Motu's* mathematical nature, the question was never the aim of the tract. This is a recurring theme. Theorem 2 states that if and only if an inverse square centripetal force acts on a set of bodies, and the bodies move uniformly (i.e. v^2/r) in concentric circles around the force centre, then Kepler's 3/2 power rule holds (p. 278). One could question the relevance of these results, as we know that the planets do not move in concentric circles, nor do they move in uniform motion. Mars is 40% faster in Capricorn than in Cancer. In *Astronomia Nova* (1609), Kepler stated that planets trace an ellipse (Wilson, 1986, p. 9). Consider a different derivation—Theorem 4 states that if bodies move in confocal ellipses under an inverse square centripetal force directed at a focus, the 3/2 power rule holds (p. 282). This is perhaps more applicable than its counterpart, Theorem 2, since it is relevant to Kepler's observational data, as the planets move in approximate ellipses. This result, amongst others, still rests on the existence of inverse square centripetal forces. Problem 3 states if a body revolves in an ellipse with a force directed at a focus of said ellipse, the force must be inverse square (p. 281). Regarding the converse, i.e. if we began with an inverse square force, the path need not be elliptical. Problem 4 shows that depending on the initial velocity and "field strength", inverse square centripetal forces yield different (calculable) conic parabolas (p. 284). Thus, while Newton provided a procedure to determine an elliptical path, one would still need to know key variables. The underdetermination problem for *De Motu*, as of 1684, is that one could not have conclusively confirmed nor denied the existence of such forces.

Newton proved key Galilean results via his mathematical framework, with the main point of contention being the applicability of these results. Problem 5 states that bodies in vertical fall are governed by inverse square centripetal forces (p. 286). This is a conceptual shift from Galilean's uniform acceleration. Problem 5's scholium suggests that Problem 4 and 5 give solutions to projectile motion and vertical fall, thus showing that Galilean motion is merely an approximation (p. 286). Notably, Galilean motion states if a ball is dropped from a height, i.e. a tower, it will land at the foot of the tower. Tyconians disagree, stating the ball will land slightly west. Newton disagrees, stating the ball will land slightly to the east since the top of the tower is further away from the Earth's centre, thus it will have a greater linear velocity at the top of the tower (Newton & Flamsteed, 1960, p. 301). Thus, the problem is showing that bodies in local motion experience inverse square forces. Outside *De Motu*, Newton was puzzled over variations of the "Moon test", in which he aimed to demonstrate how terrestrial gravity acts on the Moon. This suggests that Newton was concerned about this problem, but left it out of the scope of *De Motu*. This highlights Newton's priorities: he wanted to provide a mathematical framework first, as providing evidence for his key assumptions was a more challenging task. Problems 6 and 7 discussed the effects of resistance on Galilean motion, where air resistance is proportional to a body's velocity (p. 287). Regarding this, a challenge for Newton was deriving a geometric relationship between two variables where one progresses geometrically while the other progresses arithmetically.

Thus, Newton showed that Galilean motion approximates motion governed by inverse square gravity, via his rich mathematical framework.

De Motu unites key Keplerian and Galilean concepts under a single physical phenomenon. Specifically, said concepts are consequences of inverse square centripetal forces under certain conditions. Thus, Newton's results rest on the existence of such a force, which is not proven or demonstrated in *De Motu*. Regardless, the status of Kepler's three rules and Galileo's theory of local motion was strengthened because of the underlying structure in which both theories inhabit. Unlike competing theories which were seen as ad hoc in comparison, Newton attributed his results to a physical cause, and was able to build a rich mathematical framework from it. One could argue such a rich framework would not be possible if inverse square centripetal forces did not exist, thus giving weight to *De Motu's* results. Newton also showed that Galilean motion holds as approximations of motion governed by inverse square centripetal forces. Thus, this changed the state of the problem of underdetermination. If one could provide evidence that the planets and bodies in local motion experience inverse square centripetal forces, *De Motu* would be directly applicable. But, as of 1684, *De Motu* is only applicable if one accepts all of Newton's definitions and hypotheses.

De Motu's significance went beyond inverse square centripetal forces. It contains many concepts Newton was grappling with. Regarding gravity, Problem 5's scholium states "gravity is one species of centripetal force." (p. 286). This, with Newton replacing instances of "gravitas" with "vis centripeta" shows that Newton did not want to commit himself to any notion of gravity, and instead wanted to focus on the consequences of (inverse square) centripetal forces. Regarding mass, *De Motu* is absent of any mention of it, as he has yet to make the leap to universal gravity, where every two particles attract each other with an inverse square centripetal force. What Newton was comfortable with is the abstraction of forces (and action at a distance), given his use of the parallelogram rule (in contrast to

Galilean and Huygensian motion). Thus, while *De Motu* is significant as it linked key Keplerian and Galilean results, giving greater justification for their respective underdetermination problems, Newton did not have conclusive evidence for the existence of inverse square centripetal forces. This is in spite of his rich mathematical framework which suggests his results are not ad hoc. Thus, *De Motu* is only applicable under certain conditions, as of 1684.

References

Newton, I., Flamsteed, J. (1960). *The Correspondence of Isaac Newton*, vol. 2 (H. W. Turnbull, Ed.). Cambridge University Press. https://doi.org/10.1017/9781108627375 (Original work published 1680-81)

Newton, I. (1974). *The Mathematical Papers of Isaac Newton*, vol. 6 (D. T. Whiteside, Ed.). Cambridge University Press. https://api.semanticscholar.org/CorpusID:161467966 (Original work published 1684, December)

Wilson, C. (1968, March). *Kepler's Derivation of the Elliptical Path*. The University of Chicago Press 59(1), 4-25. https://www.jstor.org/stable/227848.