1) $\gcd(332, 128)$

$$332 = 2(128) + 104$$
$$128 = 1(104) + 24$$
$$104 = 4(24) + 8$$
$$24 = 3(8) + 0$$

$\therefore \gcd(332, 128) = 8$

2) $128z \equiv 4 \pmod{332}$, $0 \leq z \leq 332$

$$4 = 128x + 332y$$

$$8 = 104 - 4(24)$$
$$8 = 104 - 4(128 - 104)$$
$$8 = 104 - 4(128) + 104$$
$$8 = 2(104) - 4(128)$$
$$8 = 2(332 - 2(128)) - 4(128)$$
$$8 = 2(332) - 4(128) - 4(128)$$
$$8 = 2(332) - 8(128)$$

$\div 2$

$$4 = 1(332) - 4(128)$$

$x = 1$, $y = -4$

$z = 1$

$4 = 332a + n$

$128z = 332b + n$

$128z = 332b - 332a + 4$

$128z = 332K + 4$

$4 = 128z - 332K$

3) $ax + by = 5$ 5 does not divide n

$x = 5k + n$

$x = 5k + n$

$ax = 5(1) - by$

multiple of x

$\gcd(x, y)$

$= 1$???

5 does not divide x $\rightarrow$ does not divide multiple of x

$\therefore \perp$

4) make $p \rightarrow \neg q \equiv F$

T F

T

$p \equiv T,\ q \equiv T$

5) $\neg(p \rightarrow q)$ $p \wedge (q \vee r)$

p	q	x	
T	T	F	x
T	F	T	/
F	T	F	/
F	F	F	/

$\therefore 3$

p	q	r	y
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

6) $\exists y \forall x\, S(x,y) \quad x, y \in \mathbb{Z}$

$S(x,y)$ is $y < (x+2)^2$

some y for all x where $S(x,y)$

$\therefore$ false

$x \in \mathbb{Z}^+$

$P(x)$: 4 divides x

$Q(x)$: x is odd

7) $\forall x P(x) \equiv F$

8) $\exists x P(x) \wedge \exists x Q(x) \equiv T$

9) $\exists x (P(x) \wedge Q(x)) \equiv F$

10) $\forall x (Q(x) \rightarrow \neg P(x)) \equiv T$

11) $\forall x (\neg P(x) \rightarrow Q(x)) \equiv F \rightarrow 10$

12) $f : \mathbb{N} \rightarrow \mathbb{N}$

$f(n) \geq (2n)!$, $n \geq 2$ can be proved by induction

(c) $f(2) = 24$, $f(k+1) \geq (2k+2) f(k)$

$f(3) \geq (2(2)+2)(24)$

$f(3) \geq 144$

$(2(3))! \Rightarrow 720$

$$\begin{array}{r} \overset{2}{24} \\ \times\ 6 \\ \hline 144 \end{array}$$

$\therefore$ c wrong

(B) $f(2) = 30$ $\qquad f(3) \geq (2k+2)(2k+1)f(k)$

$6 \times 5 \times 30$

$30 \times 30 = 900$

$\therefore$ B correct

(A) $f(2) = 24$ $\qquad f(3) \geq 6!$

$\therefore$ A correct

13) if $Q(k-1)$ then $Q(k+1)$

$t_{k+1} = t_{k-1} x \Rightarrow$ 2nd order $\therefore 2$

14) $\{1,4,5,8\} \cap \{5,8,9\} = \{5,8\}$

$9 \notin \{5,8\}$

$\{(1,2),(2,2)\} \overset{?}{\subset} \{0,2\} \times \{1,2\}$

$\hookrightarrow \{(0,1),(0,2),(2,1)(2,2)\}$

15) $A = \{1,2,3,4,5,6,7,8,9\}$

$B = \{\text{primes}\}$

$|A \cap (\mathbb{Z} - B)| = |A \cap \{\text{set of not primes}\}|$

$= |\{1,4,6,8,9\}$

16) $|S| = 3$

$P(S) = 2^3 = 8$

$|S \times P(S)| = 3 \times 8 = 24$

$f: \mathbb{N} \to P(\mathbb{N}) - \{\emptyset\}$, $f(x) = \{n : n \in \mathbb{N}, x \le n \le 2x\}$

$g: P(\mathbb{N}) - \{0\} \to \mathbb{N}$, $g(x) = \min(x)$

17) $f(2) = \{2, 3, 4\}$

18) is f onto $\to$ no $\{1, 3, 6\}$

is f one to one $\to$ yes

19) is g onto $\to$ yes

is g one to one $\to$ no

20) $g \circ f \to g(f(x))$ $f(x) \to \{x \ldots 2x\} \to$

$g(\{x \ldots 2x\}) \to x$

onto? $\to$ yes

one to one? $\to$ yes

R, defined on $\mathbb{Z}$, xRy iff $x + y > 0$

21) Reflexive?

$xRx \equiv T$ for all x?

$-1 + -1 \not> 0$

$\therefore$ no

Symmetric?

$xRy \to yRx$

$\therefore$ yes

22) Antisym?: $\therefore$ no

Transitive?:

$aRb \wedge bRc \to aRc$

$-1R2 \wedge 2R-1 \to -1R-1$ ✗

$\therefore$ no

S defined on $P(\mathbb{N})$, XSY iff $X \subseteq Y \wedge |X| \equiv |Y| \pmod 2$

23) reflexive? $\to$ yes

symmetric? $\to$ no $\quad \{1,2,3\} \subseteq \{1,2,3,4\}$

$\{1,2,3,4\} \not\subseteq \{1,2,3\}$

24) antisym? $\to$ no

transitive? $\to$ yes

$aRb \wedge bRc \to aRc$

25) R defined on $\{-100 \ldots 100\}$, xRy iff $xy > 0 \vee x = y = 0$

eq classes

all negative num

all positive num

0

26) $\sum_{i=1}^{3} \binom{5}{i} = \binom{5}{1} + \binom{5}{2} + \binom{5}{3}$

$= 5 + 10 + 10 = 25$

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$\binom{5}{1} = \frac{5!}{1!\,4!} = 5$

$\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5 \times 4 \times \cancel{3!}}{2 \times \cancel{3!}} = \frac{20}{2} = 10$

$\binom{5}{3} = \frac{5!}{3!(2!)} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!} = 10$

27) binary strings of length 8

contain ≤ 2 1's

total $= {}^8P_8 = 8!$

2 1's, + 1 1's, + 0 1's

${}^8C_1 \times {}^8C_1 + {}^8C_1 + 1$

$64 + 8 + 1 = 69 + 4 = 73$

bin string of len 4

Y → total num of 0's in the string

Z → total num of 0's in the first 3 bits

Y	0	1	2	3	4
$Pr(Y=y)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

28) $Pr(Y=2) = \frac{6}{16}$

$= \frac{3}{8}$

Z	0	1	2	3
$P(Z=z)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

→ 0011, 0101, 1001

29) $Pr(Z \text{ is even}) = Pr(Z=2) + Pr(Z=0)$

$= \frac{3}{8} + \frac{1}{8}$

$= \frac{4}{8}$

$= \frac{1}{2}$

30) $Pr(Y=2 \wedge Z \text{ is even})$

$= \left(\frac{6}{16} \times \frac{1}{8}\right) + \left(\frac{6}{16} \times \frac{3}{8}\right)$

$= \frac{3}{64} + \frac{9}{64} = \frac{12}{64}$

$= \frac{6}{32}$

$= \frac{3}{16}$

$=$

$2\overline{)32}$ 16, 2, 12

$y \backslash z$	0	1	2	3
0				
1				
2	O		O	
3				
4				

31) yes $P(Y=2 \wedge Z \text{ is even}) \neq 0$

32) no, $P(Y=0 \wedge Z=1) = 0$

h m s

⚠ time now : 1:29:05

y	-2	4	8
$Pr(Y=y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(Y) = -\frac{2}{2} + \frac{4}{4} + \frac{8}{4}$

$= -1 + 1 + 2$

$= 2$

$4\overline{)64}$ 16, 4, 24

$var(Y) = E(Y^2) - E(Y)^2$

$= 22 - 4$

$= 18$

$E(Y^2) = \frac{4}{2} + \frac{16}{4} + \frac{64}{4}$

$= 2 + 4 + 16$

$= 22$

35) geo $\sim\left(\frac{1}{6}\right)$

36) $P(\text{expensive}) = \frac{2}{3} = P(\overline{\text{free}})$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$

$P(\text{free}) = \frac{1}{3}$

$P(\text{'e'} \mid \text{free}) = \frac{1}{2}$

$P(\text{'e'} \mid \text{expensive}) = \frac{3}{9} = \frac{1}{3}$

$$P(\text{free} \mid \text{output 'e'}) = \frac{\frac{1}{2}\times\frac{1}{3}}{\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{2}{3}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}{}_{\times 3}^{\times 3} + \frac{2}{9}{}_{\times 2}^{\times 2}}$$

$$= \frac{1}{6} \div \left(\frac{3}{18} + \frac{4}{18}\right)$$

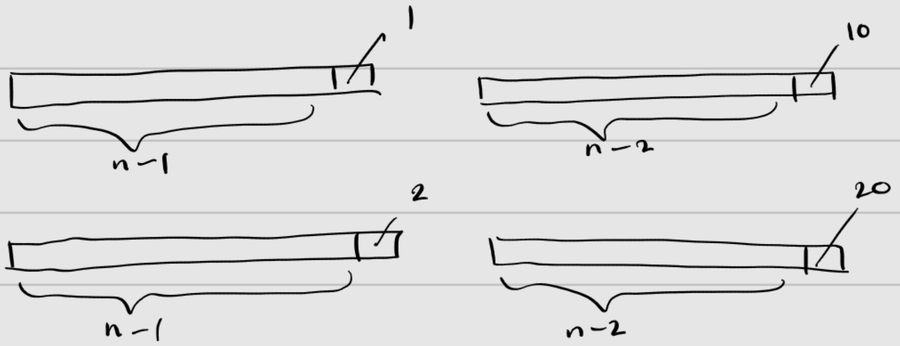
$$= \frac{1}{\cancel{6}_1} \times \frac{\cancel{18}^{3}}{7}$$

$$= \frac{3}{7}$$

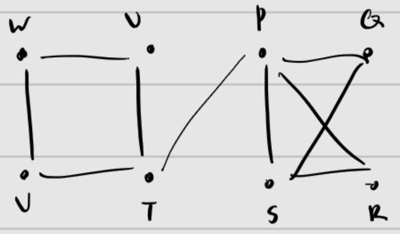
$n \geq 1$, s_n = no of ternary strings $(0, 1, 2)$ of length n w/o consecutive 0's

37) $s_2 = |\{01, 10, 11, 12, 21, 22\}|$
$= 7$

38) recurrence



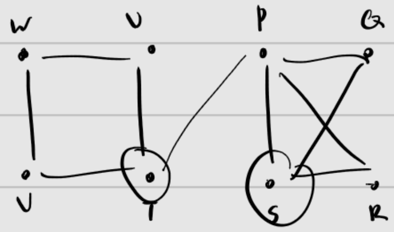
$\therefore\ s_n = 2s_{n-1} + 2s_{n-2}$



39) vertices w/ degree 2 = V, W, U, R, Q

= 5

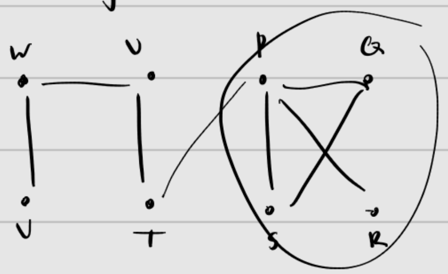
40) closed euler trail

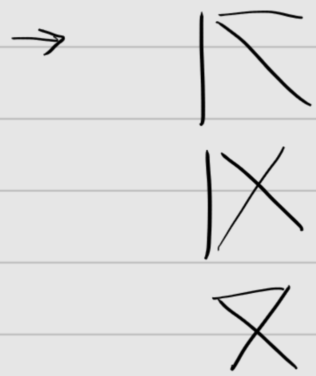


∴ add ST to make even degree

41) no of bridges = 1

42) spanning trees that dont include UT and RS





∴ 3 spanning trees

43) connected simple graph (5 vertices)

2, 1, 1, 1, 1 ×

2, 2, 2, 1, 1 √

3, 3, 1, 1, 1 √

4, 3, 2, 1, 0 ×

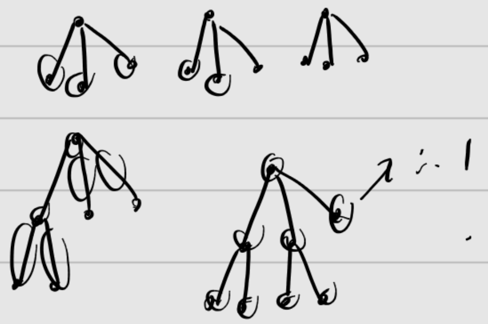
4, 4, 2, 1, 1 √

44) tree → 81 vertices — 50 degree 1

30 degree 3

1

9v → 3 deg 3 & 5 deg 1



45) flip fair coin 82 times

E(tails → heads → heads)

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$ per 3 flips

max = 27, min = 0

$27\times\frac{1}{8} = \frac{27}{8}$

??

46) (x_1, x_2, x_3) $x_1, x_2, x_3 \in \mathbb{Z}$, $x_1 + x_2 + x_3 = 38$

include 0

ordered

0 + 0 + 38

38 = 0 + 38 + 0

2 = 0 + 0 + 2,
0 + 2 + 0
2 + 0 + 0
1 + 1 + 0
1 + 0 + 1
0 + 1 + 1

6 ways

2 × 3

3 = 0 + 0 + 3.
0 + 3 + 0.
3 + 0 + 0.
1 + 2 + 0.
1 + 1 + 1.
1 + 0 + 2.
2 + 0 + 1.
2 + 1 + 0.

8 ways

4 = 0 + 0 + 0
0 + 0 + 4
0 + 4 + 0
4 + 0 + 0
1 + 1 + 2
2 + 1 + 1
1 + 2 + 1
2 + 2 + 0
2 + 0 + 2
0 + 2 + 2
3 + 1 + 0
1 + 3 + 1

38 × 3

1st num = 0 ... 38
2nd num = 0 ... (38 − 1st)
3rd num = 0 ... (38 − (1st + 2nd))

∴ total = 38 × 38 × 38

$$\begin{array}{r} 38 \\ \times\ 38 \\ \hline 304 \\ +\ 1140 \\ \hline 1444 \end{array} \qquad \begin{array}{r} 1444 \\ \times\ 38 \\ \hline 11552 \\ +\ 43320 \\ \hline 54872 \end{array}$$

??

final time 2:28:20, 3 questions dont know

Correction

1) $\gcd(332, 128)$

$332 = 2(128) + 76$

$128 = 1(76) + 52$

$76 = 1(52) + 24$

$52 = 2(24) + 4$

$24 = 6(24) + 0$

$\therefore \gcd(332, 128) = 4$

2) $128z \equiv 4 \pmod{332}$

$4 = 128z + 332y$

$4 = 52 - 2(24)$

$4 = 52 - 2(76 - 52)$

$4 = 52 - 2(76) + 2(52)$

$4 = 3(52) - 2(76)$

$4 = 3(128 - 76) - 2(76)$

$4 = 3(128) - 3(76) - 2(76)$

$4 = 3(128) - 5(76)$

$4 = 3(128) - 5(332 - 2(128))$

$4 = 13(128) - 5(332)$

$\therefore z = 13$

5) let $x \equiv \neg(p \to q) \to r$ ↪ did not see this.

$y \equiv p \wedge (q \vee r)$

p	q	r	$q \vee r$	y	$\neg(p \to q)$	x
T	T	T	T	T	F	F
T	T	F	T	T	F	T
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	F	F	T
F	T	F	T	F	F	F
F	F	T	T	F	F	T
F	F	F	F	F	F	T

$\therefore 4$

6) $\exists y \forall x\, S(x, y)$, $S(x, y)$ is "$y < (x+2)^2$" ↪ did not see square!

$y = 0$ //

S defined on $P(\mathbb{N})$, XSY iff $X \subseteq Y \wedge |X| \equiv |Y| \pmod 2$

X subset of Y — no of elements of set both even / odd

23) is S reflexive?

ARA for all A

→ all sets are subsets of themselves

→ all sets have the same size as themselves → congruent (mod 2)

∴ yes

is S symmetric?

$ARB \to BRA$ for all A, B?

$\{1,2\} \, R \, \{1,2,3\}$ but $\{1,2,3\} \not R \{1,2\}$

∴ no

24) is S antisymmetric?

$ARB \to B \not R A$ for all A, B

∴ yes

is S transitive?

$ARB \wedge BRC \to ARC$ for all A, B, C.

∴ yes

27) bin string of length 8 that contain ≤ 2 1s.

contain 0 1s = 1

contain 1 1s = 8

8 spots choose 2 to place 1s.

contain 2 1s $= {}^{8}C_{2} = \frac{8!}{2!\,6!} = \frac{8\times 7}{2} = \frac{56}{2} = 28$

$\therefore$ total $= 1+8+28 = 37$ //

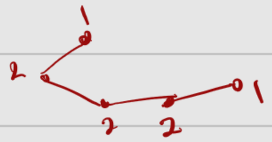
$n \geq 1$, S_n = num of ternary strings of length n that do not contain two consecutive 0s.

37) 01, 10, 11, 12, 21, 22, 02, 20

$S_2 = 8$ //

43) on 5 vertices

2, 1, 1, 1, 1 → 

2, 2, 2, 1, 1 → 

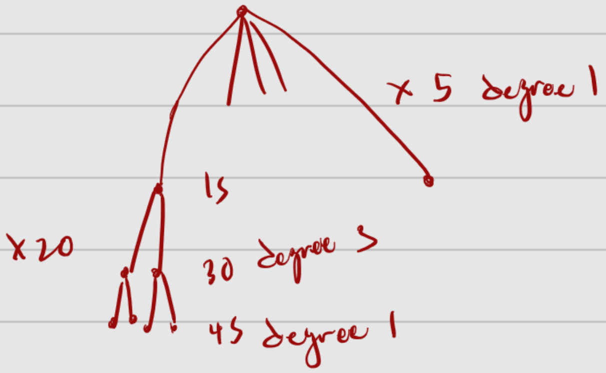
3, 3, 1, 1, 1 → odd degree → impossible

4, 3, 2, 1, 0 → impossible for simple graph

4, 4, 2, 1, 1 → impossible for simple graph

$\therefore$ 1

44)



$\therefore 15 + 5 = 20$ degree

45) fair coin flipped 82 times

Tails → Heads → Heads

last possible start

sum up all the $X_n p_n$

$$E\left(\sum_{n=1}^{80} X_n\right) = \sum_{n=1}^{80} X_n p_n$$

$X \sim \text{Bernoulli}\left(\frac{1}{2}\right)$

success chance

Since $P(X_n = 1) = \frac{1}{8}$,

$$X_n p_n = \frac{1}{8} \times 1 = \frac{1}{8}$$

$$\therefore \sum_{n=1}^{80} \frac{1}{8} = \frac{80}{8} = 10$$

46) x_1, x_2, x_3 are non negative and $x_1 + x_2 + x_3 = 38$

38 spots, 2 spacers

→ ways to split 38 items into 3 groups

$r = 38, \ n-1 = 2$

$$\begin{array}{r} 40 \\ \times 39 \\ \hline 360 \\ +1200 \\ \hline 1560 \end{array} \qquad \begin{array}{r} 780 \\ 2\overline{)1560} \\ 14 \\ \hline 16 \end{array}$$

$38 + 2 = 40$ compartments

$${}^{40}C_{38} = \frac{40!}{38!\ 2!} = \frac{40 \times 39}{2} = 780$$