Artificial Neural Network

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Outline

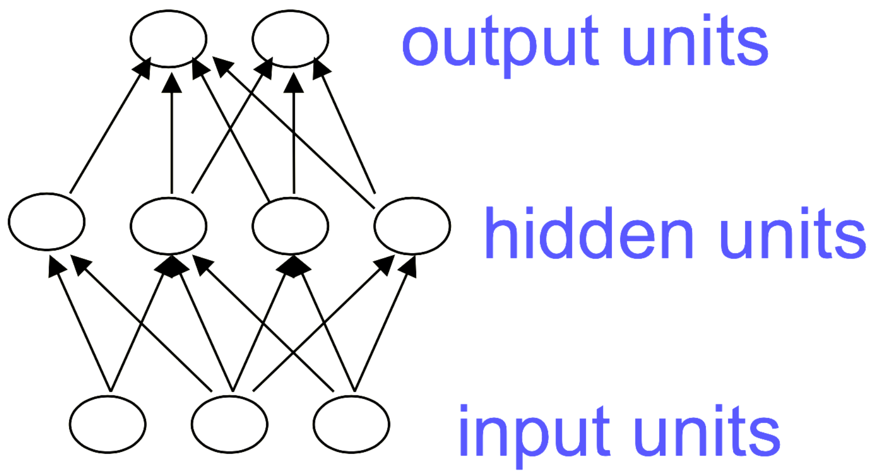
- An overview of the main types of neural network architecture
- Perceptrons: The first generation of neural networks
- A geometrical view of perceptrons
- Why the learning works
- What perceptrons can’t do

The main types of neural network architecture

- Feed-forward neural networks
- Recurrent networks
- Symmetrically connected networks

Feed-forward neural networks

- These are the commonest type of neural network in practical applications.
  - The first layer is the input and the last layer is the output.
  - If there is more than one hidden layer, we call them "deep" neural networks.
- They compute a series of transformations that change the similarities between cases.
  - The activities of the neurons in each layer are a non-linear function of the activities in the layer below.



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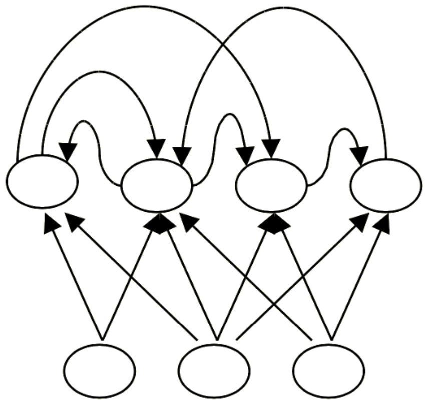
Recurrent networks

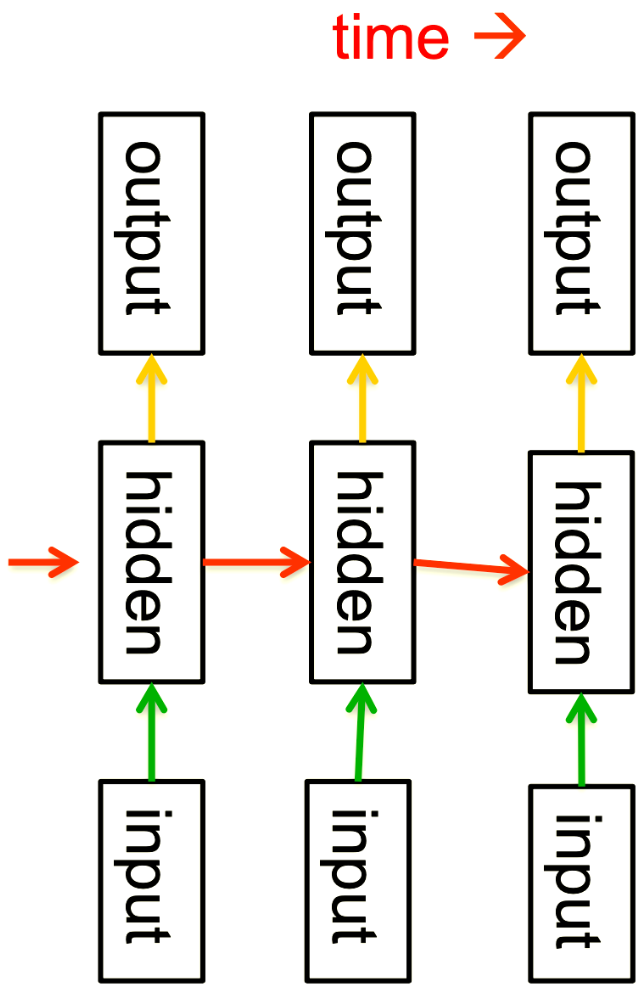
- These have directed cycles in their connection graph.
  - That means you can sometimes get back to where you started by following the arrows.
- They can have complicated dynamics and this can make them very difficult to train.
  - There is a lot of interest at present in finding efficient ways of training recurrent nets.
- They are more biologically realistic.



Recurrent nets with multiple hidden layers are just a special case that has some of the hidden→hidden connections missing.

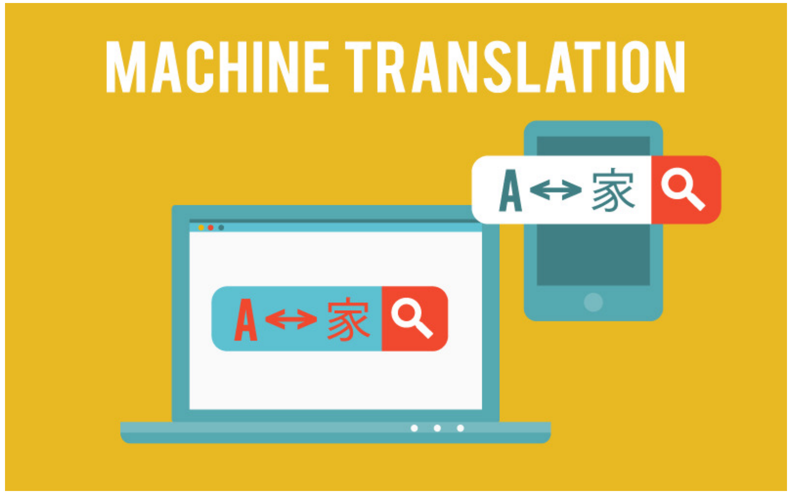
Recurrent networks

- Recurrent neural networks are a very natural way to model sequential data:
  - They are equivalent to very deep nets with one hidden layer per time slice.
  - Except that they use the same weights at every time slice and they get input at every time slice.
- They have the ability to remember information in their hidden state for a long time.
  - But its very hard to train them to use this potential



Example of what recurrent neural nets can now do

- Generate text
- Machine Translation
- Speech Recognition
- Image captioning
- Action Recognition
- …

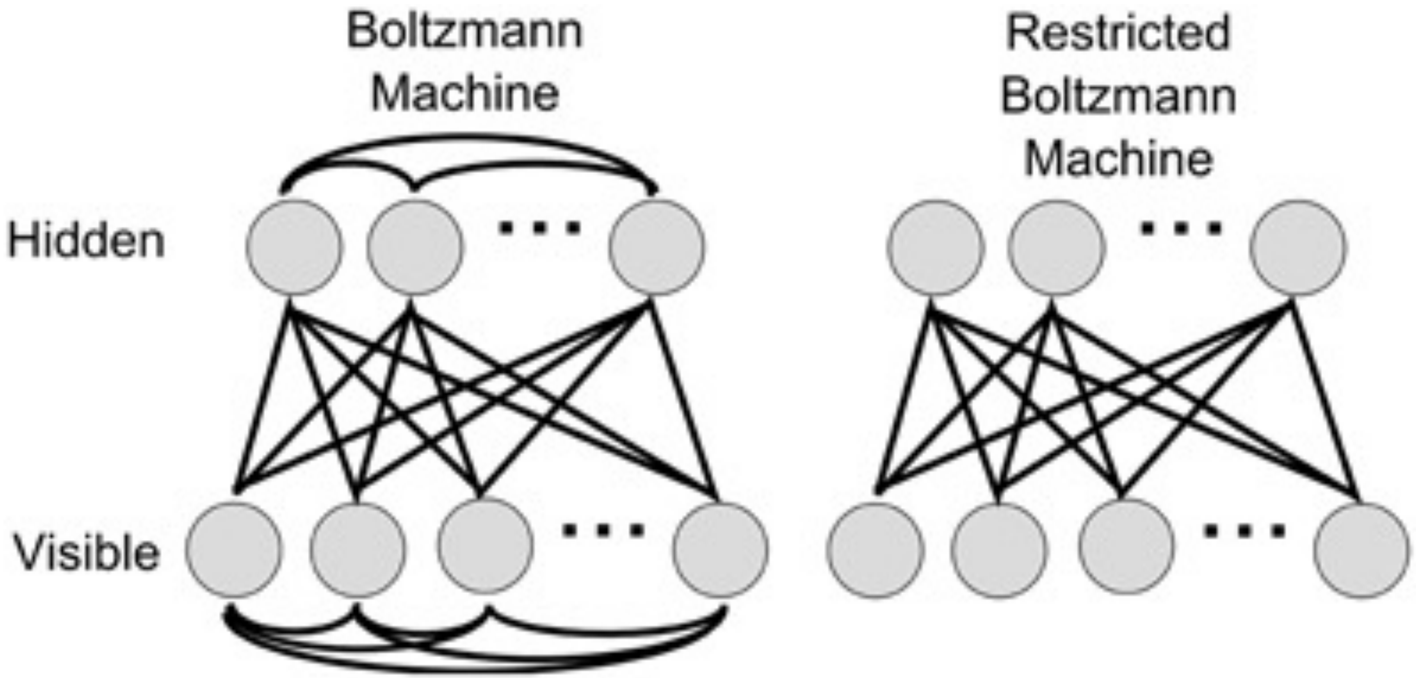


Symmetrically connected networks

- These are like recurrent networks, but the connections between units are symmetrical (they have the same weight in both directions).
  - John Hopfield (and others) realized that symmetric networks are much easier to analyze than recurrent networks.
  - They are also more restricted in what they can do because they obey an energy function.
- Symmetrically connected nets without hidden units are called "Hopfield nets".

Symmetrically connected networks with hidden units

- These are called “Boltzmann machines”.
  - They are much more powerful models than Hopfield nets.
  - They are less powerful than recurrent neural networks.
  - They have a beautifully simple learning algorithm.



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Perceptron:
The first generation of neural networks

The standard paradigm for statistical pattern recognition

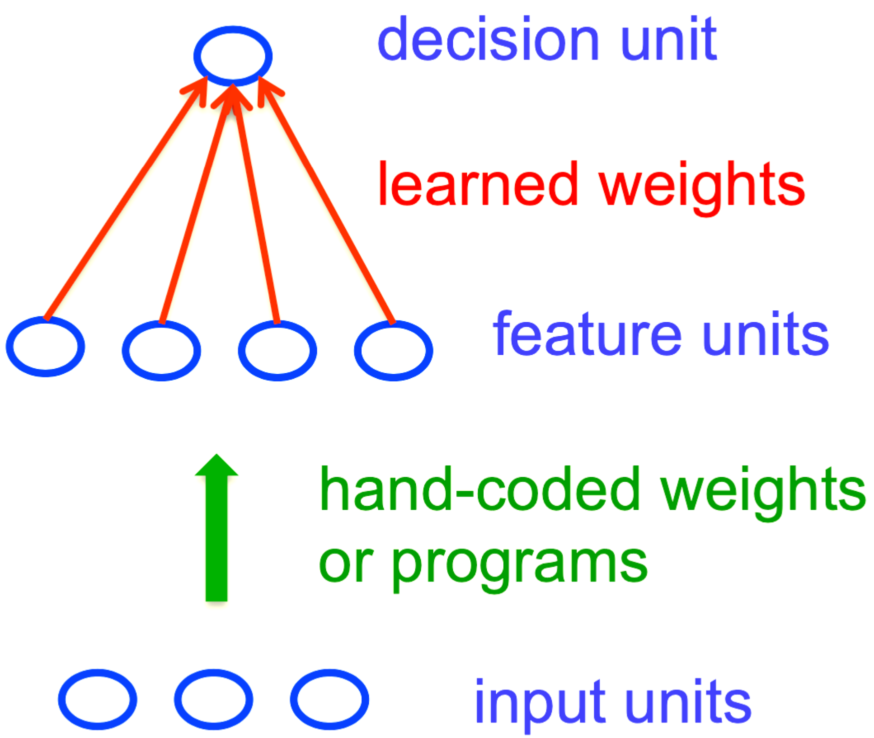
1. Convert the raw input vector into a vector of feature activations.

   Use hand-written programs based on common-sense to define the features.

2. Learn how to weight each of the feature activations to get a single scalar quantity.

3. If this quantity is above some threshold, decide that the input vector is a positive example of the target class.

The standard Perceptron architecture



The history of perceptrons

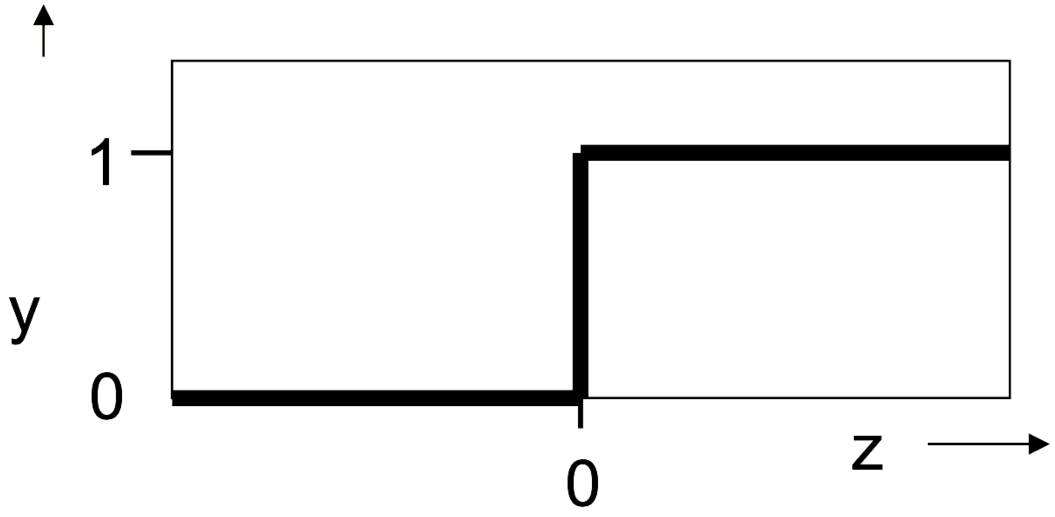
- They were popularised by Frank Rosenblatt in the early 1960's.
  - They appeared to have a very powerful learning algorithm.
  - Lots of grand claims were made for what they could learn to do.
- In 1969, Minsky and Papert published a book called "Perceptrons" that analyzed what they could do and showed their limitations.
  - Many people thought these limitations applied to all neural network models.
- The perceptron learning procedure is still widely used today for tasks with enormous feature vectors that contain many millions of features.

Binary threshold neurons (decision units)

- McCulloch-Pitts (1943)
  - First compute a weighted sum of the inputs from other neurons (plus a bias).
  - Then output a 1 if the weighted sum exceeds zero.

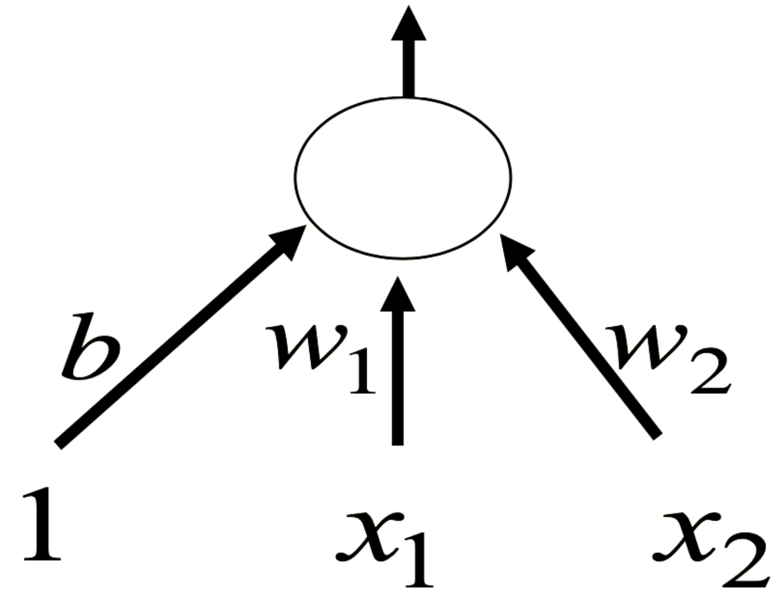
$$z = b + \sum_i x_i w_i$$

$$y = \begin{cases} 1 \text{ if } & z \geq 0 \\ 0 \text{ otherwise} & \end{cases}$$



How to learn biases using the same rule as we use for learning weights

- A threshold is equivalent to having a negative bias.
- We can avoid having to figure out a separate learning rule for the bias by using a trick:
  - A bias is exactly equivalent to a weight on an extra input line that always has an activity of 1.
  - We can now learn a bias as if it were a weight.



The perceptron convergence procedure: Training binary output neurons as classifiers

- Add an extra component with value 1 to each input vector. The "bias" weight on this component is minus the threshold. Now we can forget the threshold.
- Pick training cases using any policy that ensures that every training case will keep getting picked.
  - If the output unit is correct, leave its weights alone.
  - If the output unit incorrectly outputs a zero, add the input vector to the weight vector.
  - If the output unit incorrectly outputs a 1, subtract the input vector from the weight vector.
- This is guaranteed to find a set of weights that gets the right answer for all the training cases if any such set exists.

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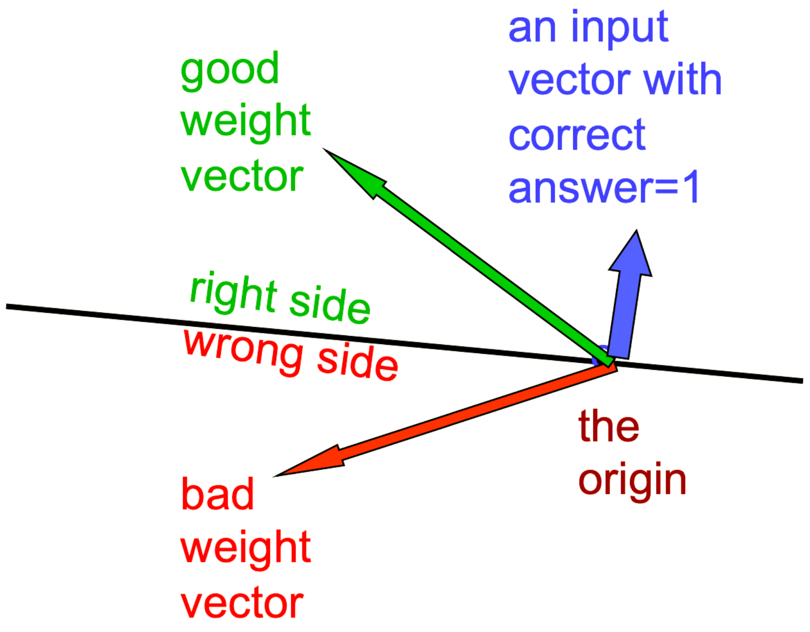
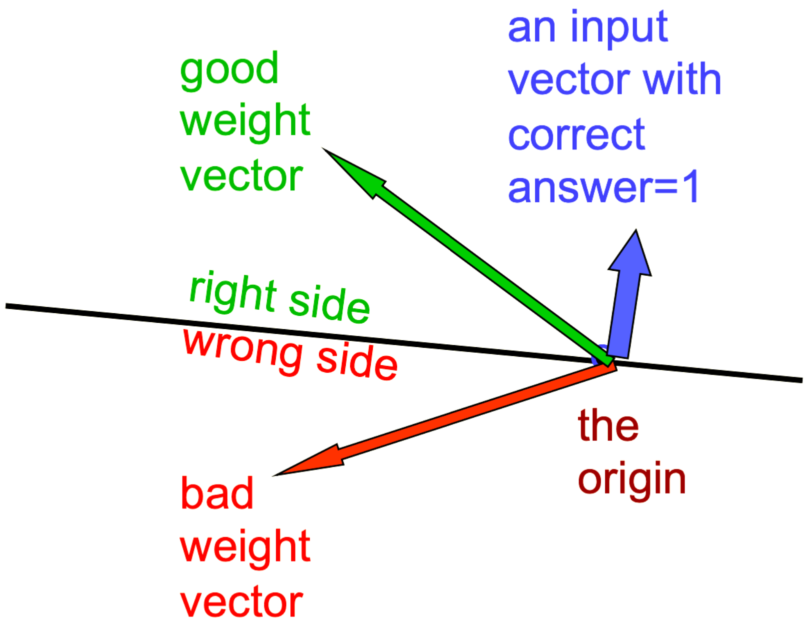
Weight-space

- This space has one dimension per weight.
- A point in the space represents a particular setting of all the weights.
- Assuming that we have eliminated the threshold, each training case can be represented as a hyperplane through the origin.
    - The weights must lie on one side of this hyper-plane to get the answer correct.



Weight-space

- Each training case defines a plane (shown as a black line)
  - The plane goes through the origin and is perpendicular to the input vector.
  - On one side of the plane the output is wrong because the scalar product of the weight vector with the input vector has the wrong sign.



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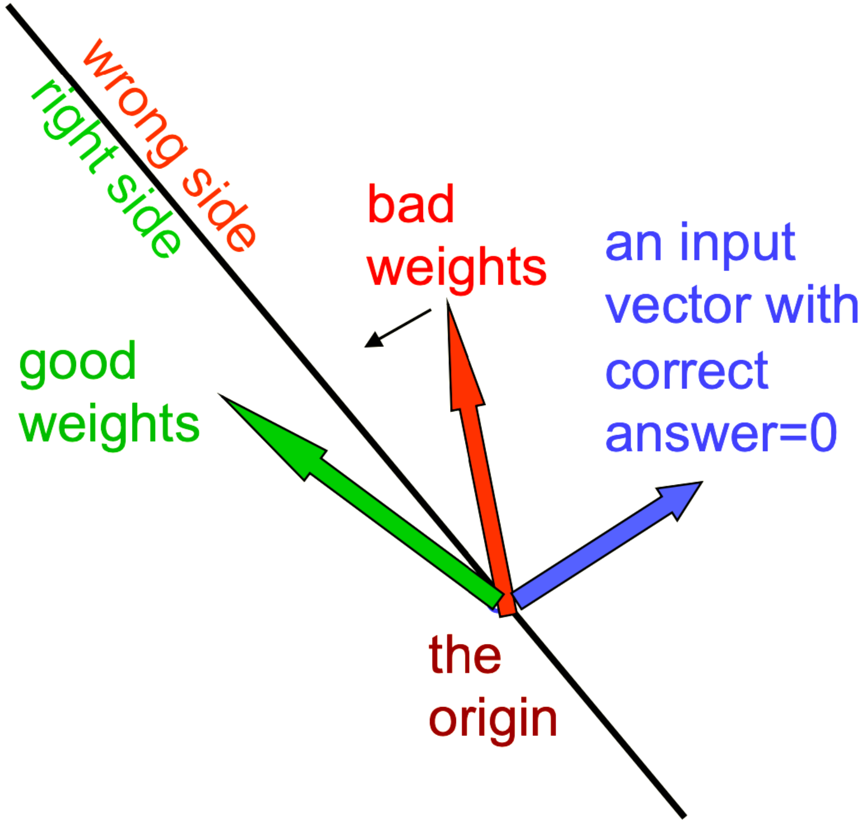
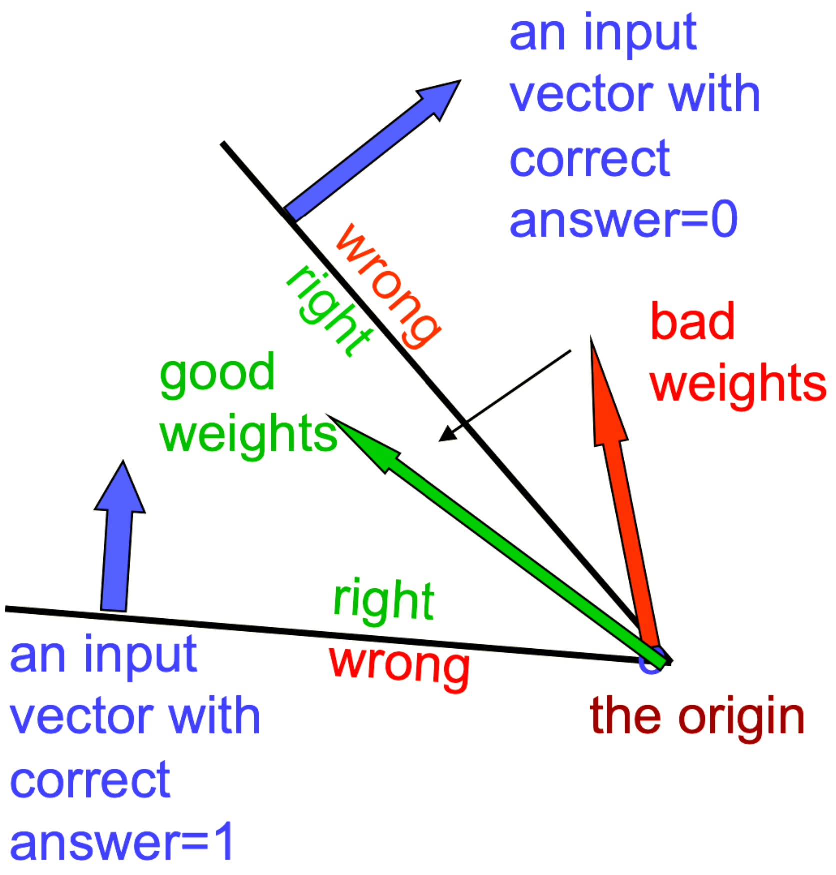


Weight-space

- To get all training cases right we need to find a point on the right side of all the planes.
  - There may not be any such point!
- If there are any weight vectors that get the right answer for all cases, they lie in a hyper-cone with its apex at the origin.
  - So the average of two good weight vectors is a good weight vector.
  - The problem is convex.



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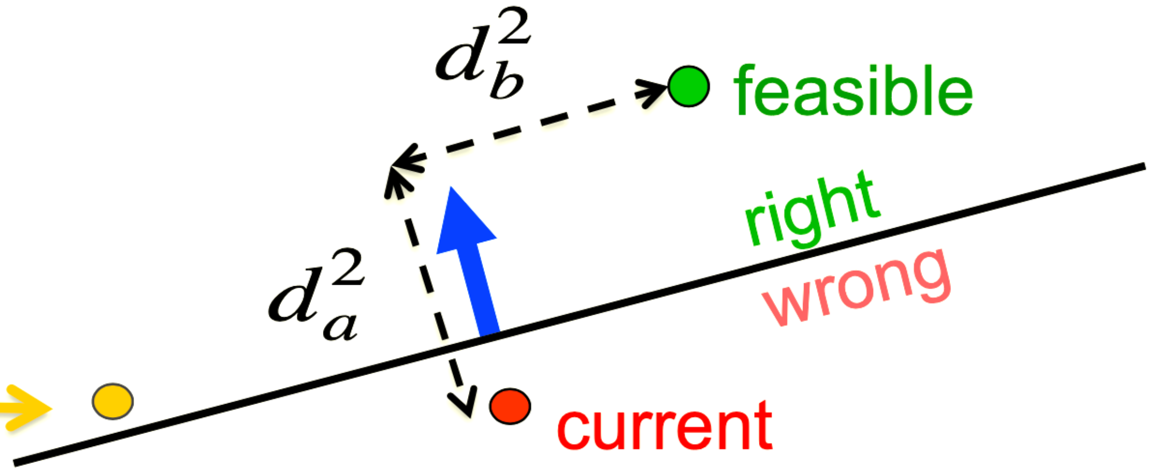
Why the learning procedure works

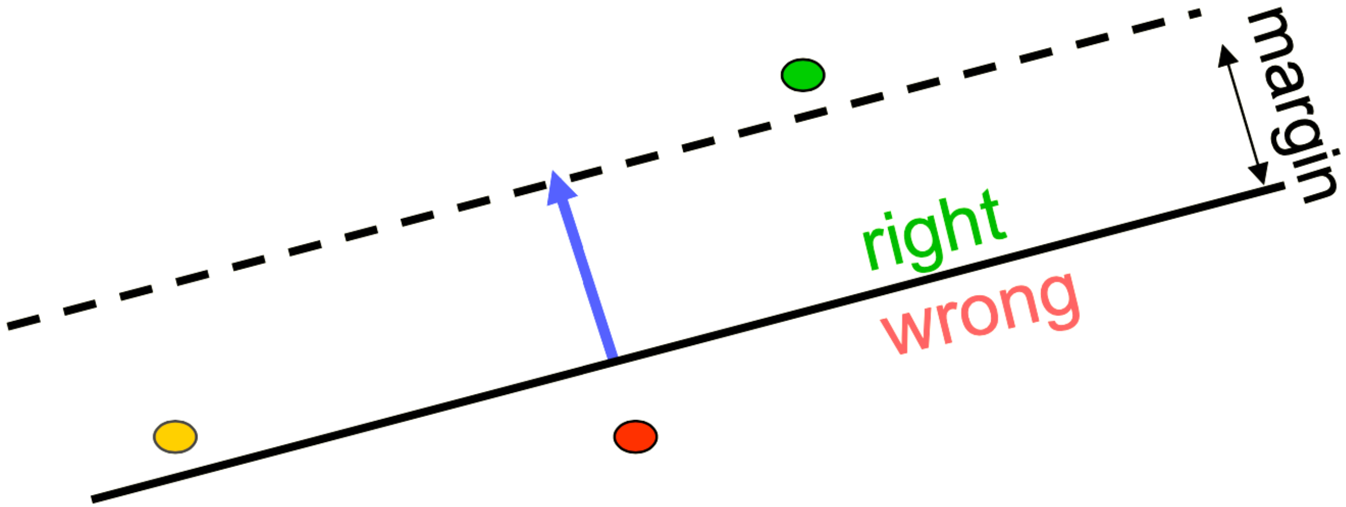
- Consider the squared distance $d_a^2 + d_b^2$ between any feasible weight vector and the current weight vector.
  - Hopeful claim: Every time the perceptron makes a mistake, the learning algorithm moves the current weight vector closer to all feasible weight vectors.

Problem case: The weight vector may not get closer to this feasible vector!



Why the learning procedure works

- So consider "generously feasible" weight vectors that lie within the feasible region by a margin at least as great as the length of the input vector that defines each constraint plane.
  - Every time the perceptron makes a mistake, the squared distance to all of these generously feasible weight vectors is always decreased by at least the squared length of the update vector.



Informal sketch of proof of convergence

- Each time the perceptron makes a mistake, the current weight vector moves to decrease its squared distance from every weight vector in the “generously feasible” region.
- The squared distance decreases by at least the squared length of the input vector.
- So after a finite number of mistakes, the weight vector must lie in the feasible region if this region exists.

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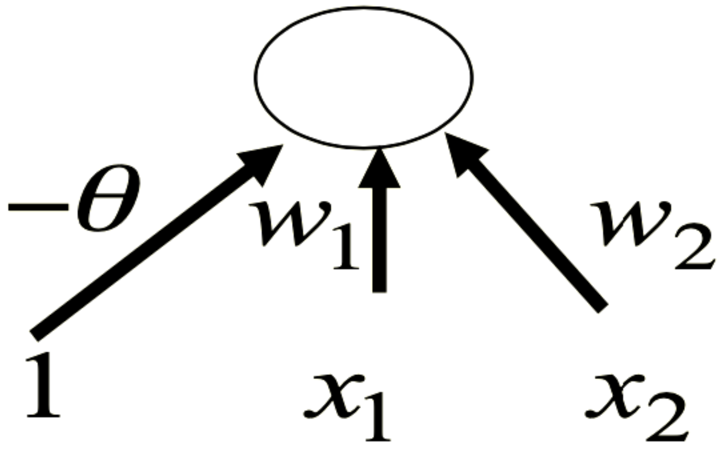
The limitations of Perceptrons

- If you are allowed to choose the features by hand and if you use enough features, you can do almost anything.
  - For binary input vectors, we can have a separate feature unit for each of the exponentially many binary vectors and so we can make any possible discrimination on binary input vectors.
  - This type of table look-up won’t generalize.
- But once the hand-coded features have been determined, there are very strong limitations on what a perceptron can learn.

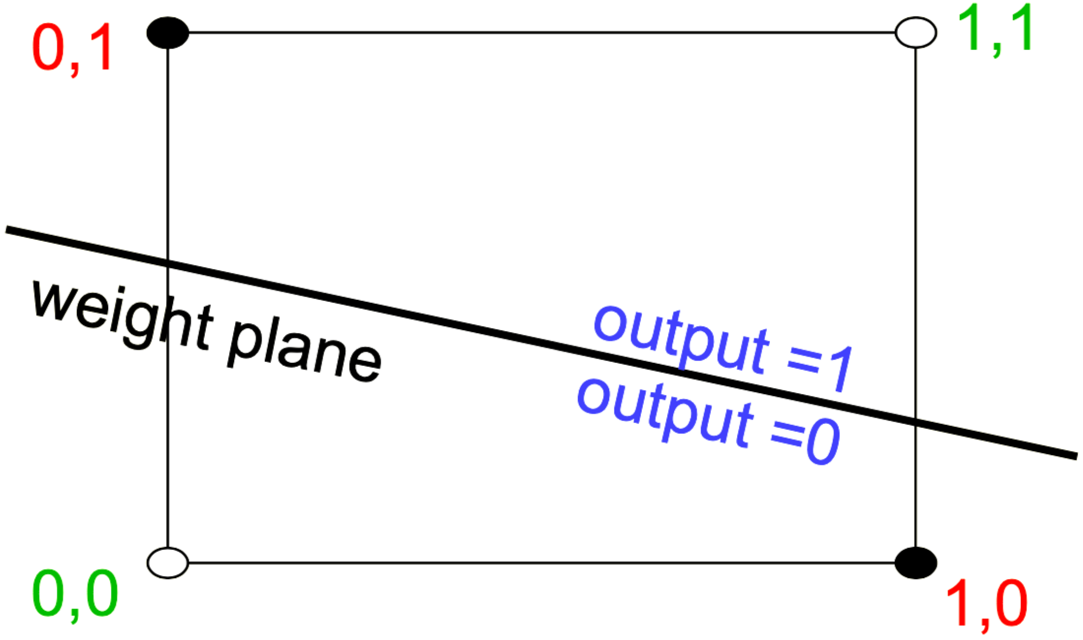
What binary threshold neurons cannot do

- A binary threshold output unit cannot even tell if two single bit features are the same!
  - Positive cases (same): (1,1)→1; (0,0)→1
  - Negative cases (different): (1,0)→0; (0,1)→0
- The four input-output pairs give four inequalities that are impossible to satisfy:
  - $w_1 + w_2 \geq \theta, 0 \geq \theta$
  - $w_1 < \theta, w_2 < \theta$



A geometric view of what binary threshold neurons cannot do

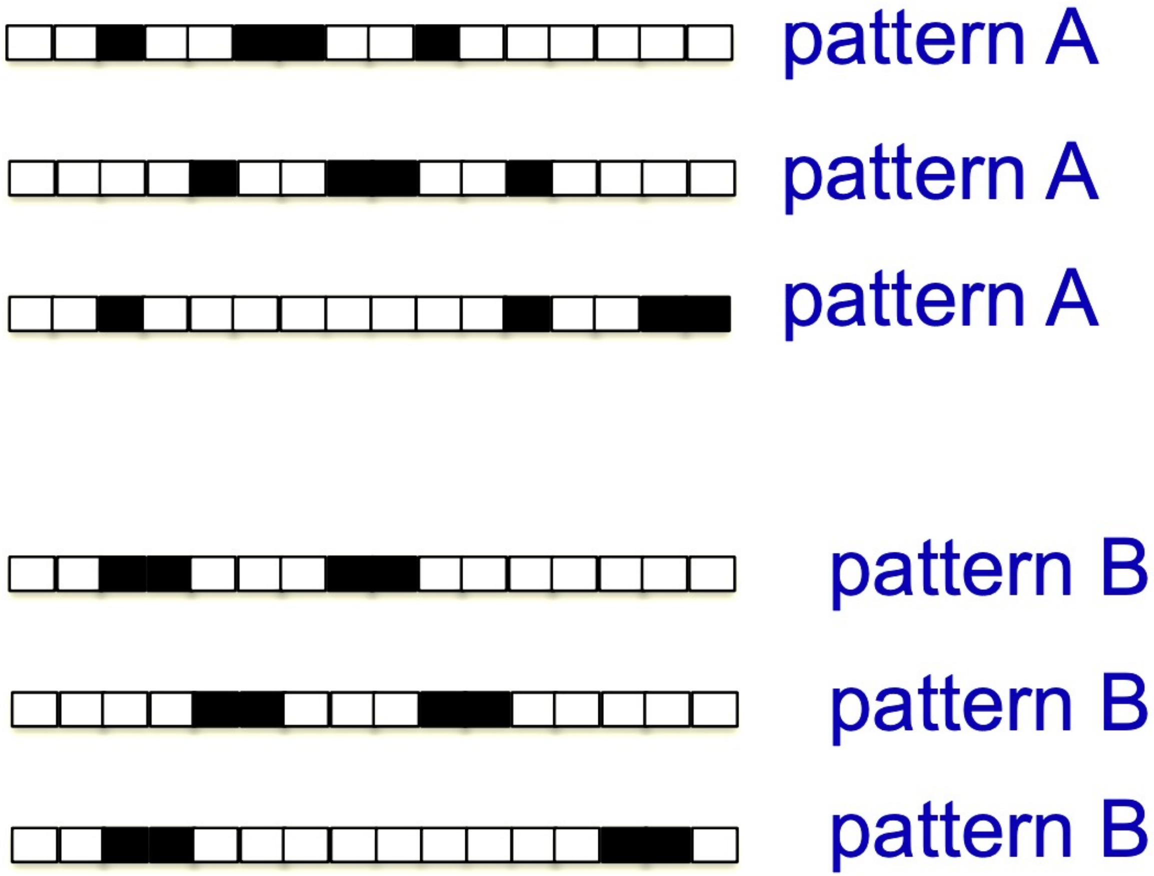
- Imagine "data-space" in which the axes correspond to components of an input vector.
  - Each input vector is a point in this space.
  - A weight vector defines a plane in data-space.
  - The weight plane is perpendicular to the weight vector and misses the origin by a distance equal to the threshold.



The positive and negative cases cannot be separated by a plane

Discriminating simple patterns under translation with wrap-around

- Suppose we just use pixels as the features.
- Can a binary threshold unit discriminate between different patterns that have the same number of on pixels?
  - Not if the patterns can translate with wrap-around!



Sketch of a proof that a binary decision unit cannot discriminate patterns with the same number of on pixels

- For pattern A, use training cases in all possible translations.
  - Each pixel will be activated by 4 different translations of pattern A.
  - So the total input received by the decision unit over all these patterns will be four times the sum of all the weights.
- For pattern B, use training cases in all possible translations.
  - Each pixel will be activated by 4 different translations of pattern B.
  - So the total input received by the decision unit over all these patterns will be four times the sum of all the weights.
- But to discriminate correctly, every single case of pattern A must provide more input to the decision unit than every single case of pattern B.
  - This is impossible if the sums over cases are the same.

Why this result is devastating for Perceptrons

- The whole point of pattern recognition is to recognize patterns despite transformations like translation.
- Minsky and Papert’s “Group Invariance Theorem” says that the part of a Perceptron that learns cannot learn to do this if the transformations form a group.
  - Translations with wrap-around form a group.
- To deal with such transformations, a Perceptron needs to use multiple feature units to recognize transformations of informative sub-patterns.
  - So the tricky part of pattern recognition must be solved by the hand-coded feature detectors, not the learning procedure.

Learning with hidden units

- Networks without hidden units are very limited in the input-output mappings they can learn to model.
  - More layers of linear units do not help. Its still linear.
  - Fixed output non-linearities are not enough.
- We need multiple layers of adaptive, non-linear hidden units. But how can we train such nets?
  - We need an efficient way of adapting all the weights, not just the last layer. This is hard.
  - Learning the weights going into hidden units is equivalent to learning features.
  - This is difficult because nobody is telling us directly what the hidden units should do.