Energy Conservation Between 2D Motion

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Abstract

This report is a detailed documentation explaining and proving the phenomenon of Energy Conservation between 2D Motion. The experiment performed for this report is based on the theoretical assumption that when two objects, performing two different types of motion, translationally collide and undergo 2D motion before and after collision, the energy of the system remains conserved. With this assumption, a theoretical model for the value of projectile range 'd' for varying launch angles 'θ' is derived. The experiment is performed using a simple pendulum system, with some modifications, to allow projectile motion of the colliding metallic bobs. The final comparison of experimental and theoretically determined values of 'd' confirmed that our hypothetical data model was valid, as both sets of values showed similar behavior and linear relationship with angle 'θ'. There were some differences between experimental and theoretical results, but those were due to the lack of ideal isolated environment and the uncertainties in the measured values, and they do does not affect the validity of the hypothesis and this experiment.

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Introduction:

All objects in motion possess energy. As objects move around over time, the energy associated with them, for example, Kinetic, Gravitational Potential, Heat etc., might change forms, but if the system is isolated, i.e. there is no exchange of forces and energies in and out of the system, then the total energy of the system remains conserved.

With this experiment, we are trying to answer the question about the energy of the bodies which collide translationally but exhibit two-dimensional motion, both before and after the collision. We are trying to create and observe a fully-translational collision as well as to understand the concept of energy conservation in such a physical system.

Based on the concept of energy conservation in 2D motion, we attempt to predict the horizontal landing range of a steel bob, which undergoes projectile motion, using a simple pendulum system. After the linear collision of the pendulum bob with the resting bob (whose landing range we want to observe), we first determine the initial launch velocity 'v_o' for the projectile motion and then use the kinematics of projectile motion to determine the range 'd'.

This report is an approach to clearly explain the concept and background theory of energy conservation in 2D motion, the experiment setup and procedure, observational data, analysis of the data, discussion about the uncertainties and errors in the experiment and data collection, and the conclusion of this experiment.

Theory:

The Law of Conservation of Energy states that,

"In an isolated system, the energy of the system is always conserved, i.e., the energy can convert from one form to another, but the total energy of the system will remain constant."

For a physical system with only mechanical energies in action and no external forces being applied, there is an inter-conversion of Kinetic and Gravitational Potential energies of the system, the total energy of the system being constant throughout. This means that the change in both types of energies will be equal but opposite.

Mathematically, based on the Energy Conservation Law, this can be written as,

$$Change\ in\ K.E. = -Change\ in\ P.E.$$

$$\Delta K.E. = -\Delta P.E. \qquad - Equation(1)$$

Negative sign shows the opposite behavior (inverse relation) of both energies, i.e., with an increase in one of them, the other one decreases.

We know that Kinetic Energy possessed by an object is equal to half the product of its mass and square of its velocity.

$$K.E. = \frac{1}{2}mv^2 \qquad - Equation(2)$$

Where,

m = mass of the object, and

v = velocity of the object.

Also, Gravitational Potential Energy of an object is the product of its mass, gravitational acceleration, and vertical component of its position.

$$P.E. = mgy \qquad - Equation(3)$$

Where,

m = mass of the object,

g = gravitational acceleration = 9.81m/s^2, and

y = vertical component of position of the object.

During projectile motion, there is no force acting on the system except the gravitational force. Therefore, the energy of the system will transition between Kinetic Energy 'K.E.' and Gravitational Potential Energy 'P.E.', the total energy being conserved. As the ball swings from a certain amplitude with a certain angle towards the mean position, the Potential Energy converts into Kinetic Energy.

As *Figure 1* shows,

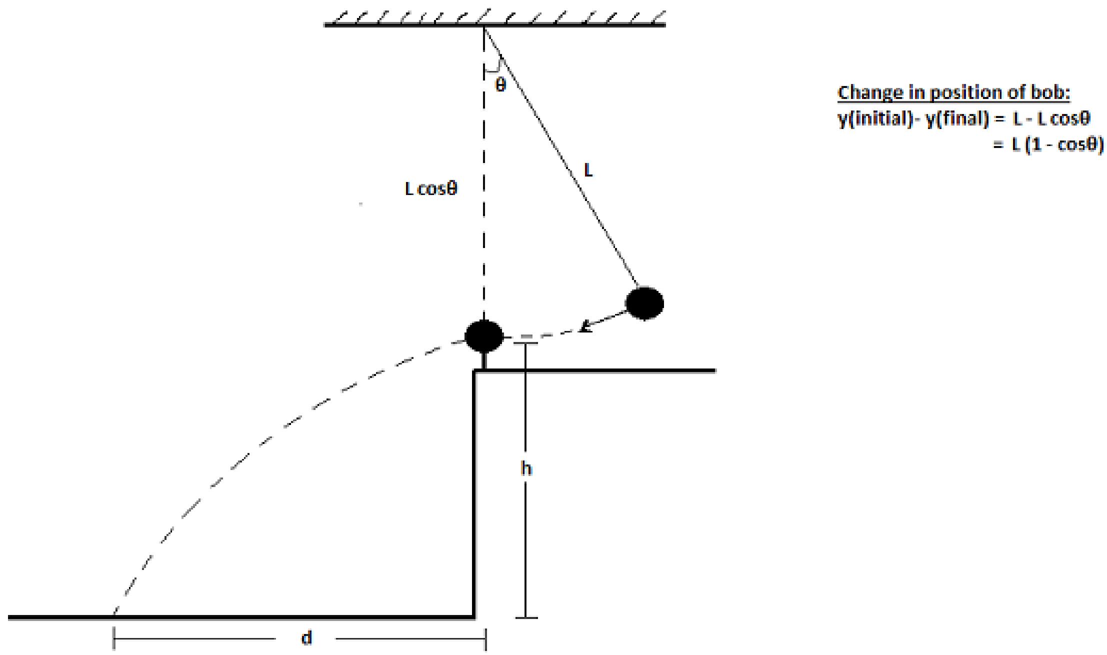

Figure 1- Sketch of the Experiment System

Equation (1) implies,

$$\Delta K.E. = -\Delta P.E.$$

$$\Rightarrow \quad K.E._{final} - K.E._{initial} = -[P.E._{final} - P.E._{initial}]$$

Using Equations (2) and (2)

$$\Rightarrow \quad \frac{1}{2}m(v_{\ final})^2 - \frac{1}{2}m(v_{\ initial})^2 = -[mgy_{final} - mgy_{initial}]$$

$$\Rightarrow \quad \frac{1}{2}m[(v_{\ final})^2 - (v_{\ initial})^2] = mg[y_{initial} - y_{final}]$$

Here,

$v_{initial} = 0$, $v_{final} = v$, and $y_{initial} - y_{final} = L(1 - \cos\theta)$, {as shown in Figure1}

Hence, the above equation becomes,

$$\Rightarrow \quad \frac{1}{2}mv^2 = mgL(1 - cos\theta)$$

$$\Rightarrow \quad v^2 = 2gL(1 - cos\theta)$$

$$\Rightarrow \quad v = \sqrt{2gL(1 - cos\theta)}$$

Now, considering the projectile motion of the steel ball,

$$\Rightarrow \quad v_{ox} = v_o = \sqrt{2gL(1 - cos\theta)} \quad \_Equation(4)$$

$$\Rightarrow \quad v_{oy} = v_o sin\theta$$

Since, the bob is at mean initially and there $\theta = 0^o \Rightarrow sin\theta = 0$,

Hence,

$$\Rightarrow \quad v_{oy} = 0$$

In projectile motion,

$$y = v_{oy}\, t + \frac{1}{2}\, gt^2$$

Initially, y = h (Fig.1), and $v_{oy} = 0$,

$$\Rightarrow \quad h = 0 \times t + \frac{1}{2}\, gt^2$$

$$\Rightarrow \quad t^2 = 2\frac{h}{g}$$

$$\Rightarrow \quad t = \sqrt{2\frac{h}{g}} \quad \_Equation(5)$$

The range of a projectile is given by,

$$d = v_{ox}t \quad \_\_Equation(a)$$

Putting the values of v_{ox} and t from Equations (4) and (5) in Equation (a),

$$\Rightarrow \quad d = \sqrt{2gL(1-cos\theta)} \times \sqrt{2\frac{h}{g}}$$

$$\Rightarrow \quad d = 2\sqrt{L(1-cos\theta)} \times \sqrt{h}$$

$$\Rightarrow \quad d = 2\sqrt{hL(1-cos\theta)} \quad \_Equation(A)$$

In this equation,

h = height of the pendulum system mean position above the ground,

L = Length of metal rod with which the bob is attached to the fixed edge of pendulum,

$\theta =$ angle at which the bob is lifted up, and

d = the range (horizontal distance on ground) covered by the bob.

Also,

L and h are constants,

θ is the independent variable, and

d depends on θ.

Method/Procedure:

The setup consists of,

- Pre-Constructed fixed pendulum base
- Two metal bobs of equal mass,
- Thin metal rod (of considerably negligible mass) to hang the bob with fixed edge of pendulum,
- Meter rule to measure the distances,
- Graph paper and carbon paper to mark the landing point of the bob,
- Protractor to measure the angle for each observation.

As shown in the figure below,

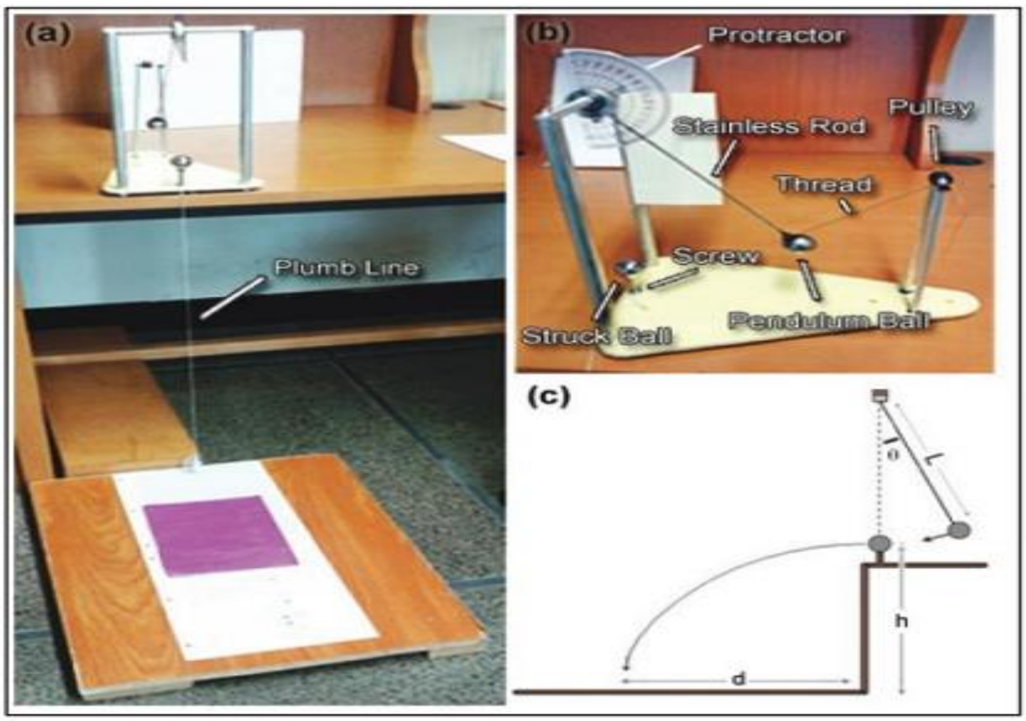

Figure 2- Experiment Setup *[1]*

For each observation, the bob is pulled up to a certain angle (measured with the protractor at the fixed edge of pendulum). We choose eleven evenly spaced data points with increments of 5 degrees between 10 to 60 degrees. We cannot go beyond this range because of no collision of the bobs when angle is less than 10 degrees, and the pole coming in between the swing path if we try to take an angle greater than 60 degrees.

The bob swings back towards the mean position. There is another bob, already present at the mean position, which collides with this swinging bob and falls down the table, undergoing projectile motion and hits the ground. The point where the bob hits the ground is marked with the help of carbon paper and graph paper, the horizontal distance ‘d’ from table to the point is measured.

Alongside the experiment, we also worked on our theoretical assumption. For each observation, the range ‘d’ is also calculated using our theoretical model, Equation (A), i.e., $d = 2\sqrt{hL(1 - cos\theta)}$, to have a comparison between the experiment and our hypothesis.

Analysis of Data:

height of the pendulum system above the ground = h = 92.5 cm

= 0.925m

Length of metal rod = L = 25cm

= 0.25m

$d_{theoretical} = 2\sqrt{hL(1 - cos\theta)}$

Observations Table:

Observation Number	Angle ‘θ’ (degrees)	Theoretical Range ‘$d_{theoretical}$’ (meters)	Experimental Range ‘$d_{experimental}$’ (meters)
1	10	0.1185	0.108
2	15	0.1775	0.187
3	20	0.2362	0.226
4	25	0.2944	0.262
5	30	0.3520	0.328
6	35	0.4090	0.397
7	40	0.4652	0.448
8	45	0.5205	0.474
9	50	0.5748	0.543
10	55	0.6280	0.585
11	60	0.6800	0.618

Graphical Representation of Data:

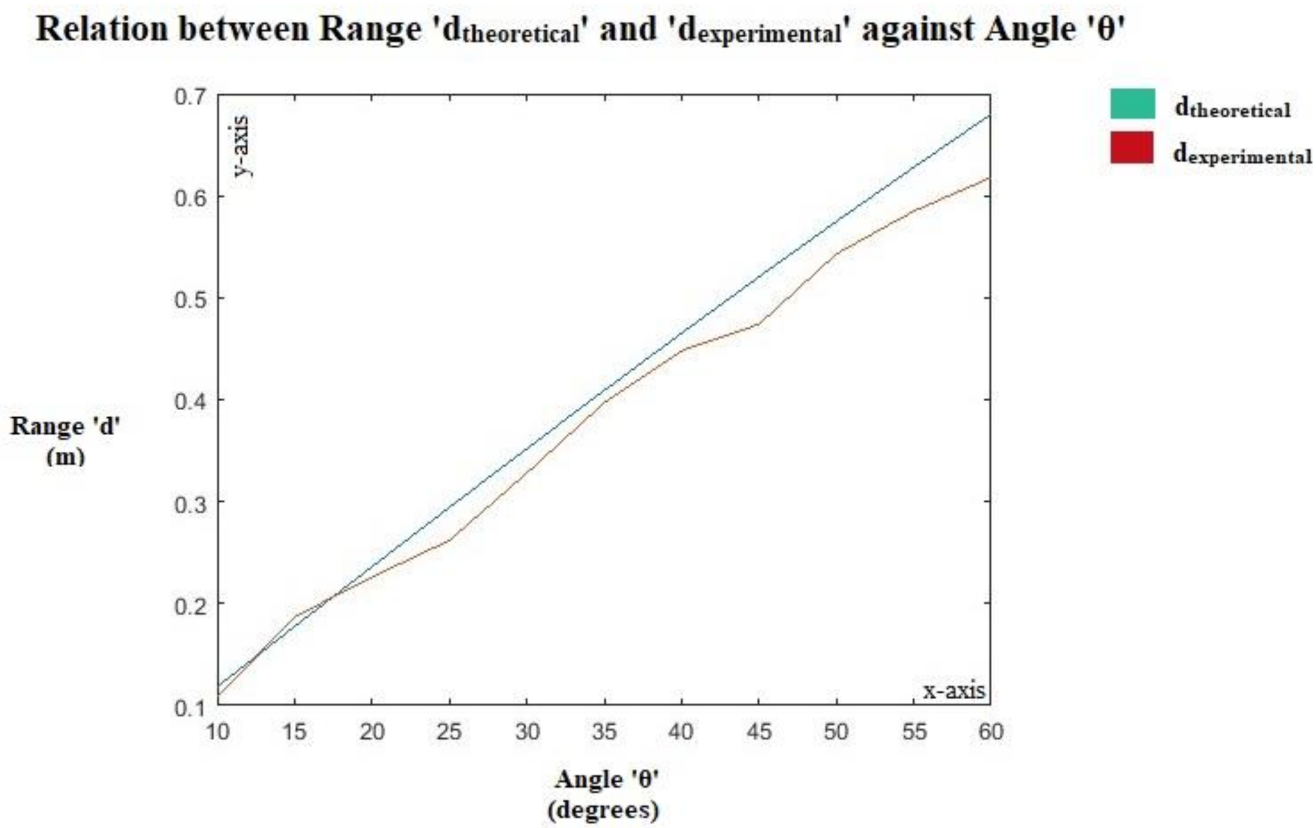

Figure 3-Relation between d(theoretical) and d(experimental) plotted against θ

Discussion:

We plotted both estimated and observed values of projectile range 'd' using MATLAB. Both graphs showed similar symmetry and behavior. Both theoretical and experimental values of 'd' showed a linear relationship with angle 'θ'. This confirmed that our hypothesis about the Conservation of Energy between 2D Motion and approach for theoretical data model was valid.

There were some differences in the theoretical and experimentally measured values. This happened because,

- For our hypothesis we considered the metal rod to be massless (of negligible mass) which was not really the case.
- The experiment was not performed in ideal controlled environment, so the system was not Isolated which is a requirement for energy conservation.
- The collision between the two bobs was not elastic as assumed, so there might be energy losses.

Along with that, the experimental value of range was measured with ruler which can have uncertainties.

The uncertainty for the ruler can be given by,

$$U_{ruler} = \frac{\frac{1}{2}(Length\ of\ \ Minimum\ Division)}{\sqrt{6}}$$

For ruler, the length of minimum division is 0.1cm = 0.001m,

So,

$$U_{ruler} = \frac{(0.5)(0.001m)}{\sqrt{6}} = 0.000204m$$

Also, the value of ‘d’ depends on ‘θ’ which we are calculating using protractor. The protractor can also have uncertainties given as,

$$U_{protractor} = \frac{\frac{1}{2}(Minimum\ Division\ Angle)}{\sqrt{6}}$$

For protractor, the minimum division is 1° , So,

$$U_{protractor} = \frac{(0.5)(1°)}{\sqrt{6}} = 0.2041°$$

We only calculated type B uncertainties because type A uncertainties are negligible in this case.

Conclusion:

The purpose of this experiment was to understand, prove and analyze the phenomenon of Energy Conservation between 2D Motion.

The experiment started with the theoretical assumption that in a physical system with two objects, performing two different types of motion, colliding linearly and undergoing 2D motion before and after collision, the energy of the system remains conserved. Based this assumption, a hypothetical model for the values of projectile range 'd' for varying launch angles 'θ' was derived.

On the comparison of experimental and theoretically determined values of 'd', it is confirmed that our hypothetical data model was valid, as both sets of values, when plotted on graph against 'θ', showed similar nature and linear relationship with angle 'θ'. The little inaccuracies in the values were because of the lack of ideal isolated system and the uncertainties in the measured values, but the hypothesis and this experiment are logically reasonable and valid approach to explain the concept of Energy Conservation in 2D motion.

References:

[1] M.Warda, A.Shahbaz, "Physics 101L- Energy Conservation between 2D Motion," [Online]. Available: https://lms.habib.edu.pk/access/content/group/dd4e06d1-61b7-4a00-bb47-f917e40dc506/Lab%20Manuals/Energy%20conservation%20between%202D%20motions.pdf.