Proof Checker Notes

Yu-Yang Lin

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1 Syntax Grammar

$$
\begin{array}{rl}
\text{(types)} & \tau ::= \mathrm{bool} \mid \tau \to \tau \mid \mathrm{nat} \mid \mathrm{list}\ \tau \\
\text{(hypotheses)} & \mathrm{A}\ ,\mathrm{B} ::= \top \mid \bot \mid \mathrm{A} \wedge \mathrm{B} \mid \mathrm{A} \vee \mathrm{B} \mid \mathrm{A} \supset \mathrm{B} \mid \forall\, x : \tau\ .\ \mathrm{A} \mid \exists\, x : \tau\ .\ \mathrm{A} \mid \mathrm{t} = \mathrm{t} : \tau \\
\text{(terms)} & \mathrm{e}\ ,\mathrm{t} ::= x \mid \mathrm{t}\ \mathrm{t} \mid \mathrm{true} \mid \mathrm{false} \mid [\ ] \mid \mathrm{t} :: \mathrm{t} \mid \mathrm{zero} \mid \mathrm{suc}(\ \mathrm{t}\ ) \\
\text{(term context)} & \psi ::= .\ \mid \psi, x : \tau
\end{array}
$$

$$
\begin{array}{ccl}
\psi & \vdash & \mathrm{t} : \tau \\
\psi & \vdash & \mathrm{A\ prop}
\end{array}
$$

2 Rules for terms and hypotheses

Natural Numbers:

$$\frac{}{\psi \vdash \mathrm{zero} : \mathrm{nat}}\ \text{(nat-zero)} \qquad \frac{\psi \vdash \mathrm{t} : \mathrm{nat}}{\psi \vdash \mathrm{suc}(\ \mathrm{t}\ ) : \mathrm{nat}}\ \text{(nat-suc-n)}$$

Booleans:

$$\frac{}{\psi \vdash \mathrm{true} : \mathrm{bool}}\ \text{(bool-true)} \qquad \frac{}{\psi \vdash \mathrm{false} : \mathrm{bool}}\ \text{(bool-false)}$$

Lists:

$$\frac{}{\psi \vdash [\ ] : \mathrm{list\ t}}\ \text{(list-empty)} \qquad \frac{\psi \vdash \mathrm{t}' : \mathrm{t} \qquad \psi \vdash \mathrm{t}'' : \mathrm{list\ t}}{\psi \vdash \mathrm{t}' :: \mathrm{t}'' : \mathrm{list\ t}}\ \text{(list-hd::tl)}$$

Variables:

$$\frac{x : \tau \in \psi}{\psi \vdash x : \tau}\ \text{(var)}$$

Application:

$$\frac{\psi \vdash \mathrm{t} : \tau \to \tau' \qquad \psi \vdash \mathrm{t}' : \tau}{\psi \vdash \mathrm{t}\ \mathrm{t}' : \tau'}\ \text{(app)}$$

Truth and Falsity Propositions:

$$\frac{}{\psi \vdash \top\ \mathrm{prop}}\ (\top\text{-prop}) \qquad \frac{}{\psi \vdash \bot\ \mathrm{prop}}\ (\bot\text{-prop})$$

Binary Relation Propositions:

$$\frac{\psi \vdash \mathrm{A\ prop} \qquad \psi \vdash \mathrm{B\ prop}}{\psi \vdash \mathrm{A} \wedge \mathrm{B\ prop}}\ (\wedge\text{-prop})$$

$$\frac{\psi \vdash \mathrm{A\ prop} \qquad \psi \vdash \mathrm{B\ prop}}{\psi \vdash \mathrm{A} \vee \mathrm{B\ prop}}\ (\vee\text{-prop})$$

$$\frac{\psi \vdash \text{A prop} \qquad \psi \vdash \text{B prop}}{\psi \vdash \text{A} \supset \text{B prop}} \quad (\supset\text{-prop})$$

$$\frac{\psi \vdash \text{t}: \tau \qquad \psi \vdash \text{t}': \tau}{\psi \vdash (\text{t} = \text{t}' : \tau) \text{ prop}} \quad (\text{eq-prop})$$

Quantifier Propositions:

$$\frac{\psi, x : \tau \vdash \text{A prop}}{\psi \vdash \forall x : \tau.\ \text{A prop}} \quad (\forall\text{-prop})$$

$$\frac{\psi, x : \tau \vdash \text{A prop}}{\psi \vdash \exists x : \tau.\ \text{A prop}} \quad (\exists\text{-prop})$$

```
check_term  :  ctx -> term -> tp option
check_prop  :  ctx -> prop -> unit option

i.e.
      check_term  :  ψ → t → τ option
      check_prop  :  ψ → A → unit option
```

3 Rules for well-formedness of proofs

(proofs)	p , q	::=	by H
		|	(p , q)
		|	`let` (H′,H″) = H `in` p
		|	(p , q) `either`
		|	`match [`H`] : (`A ∨ B`) with (`
			| A `[`H′`]` : p -> C
			| B `[`H″`]` : q -> C `)`
(hypotheses context)	Γ	::=	·
		|	Γ , H : A
		|	Assume A [H] , p

$$\psi;\Gamma \ \vdash \text{p} : \text{A}$$
$$\psi \ \vdash \Gamma$$

check_proof : $\psi \to \Gamma \to$ P $\to$ A $\to$ unit option

Conjunction:

$$\frac{\psi;\Gamma, \text{H} : \text{A} \wedge \text{B} , \text{H}' : \text{A} , \text{H}'' : \text{B} \vdash \text{p} : \text{C}}{\psi;\Gamma, \text{H} : \text{A} \wedge \text{B} \vdash \texttt{let}\ (\text{H}', \text{H}'')\ =\ \text{H}\ \texttt{in}\ \text{p}} \quad (\wedge\text{L})$$

$$\frac{\psi;\Gamma \vdash \text{p} : \text{A} \qquad \psi;\Gamma \vdash \text{q} : \text{B}}{\psi;\Gamma \vdash (\text{p} , \text{q}) : \text{A} \wedge \text{B}} \quad (\wedge\text{R})$$

Disjunction:

$$\frac{\psi;\Gamma , \text{H} : \text{A} \vee \text{B} , \text{H}' : \text{A} \vdash \text{p} : \text{C} \qquad \psi;\Gamma , \text{H} : \text{A} \vee \text{B} , \text{H}'' : \text{B} \vdash \text{q} : \text{C}}{\psi;\Gamma, \text{H} : \text{A} \vee \text{B} \vdash \texttt{match}\ [\text{H}]\ \texttt{with}(\ \text{A}\ [\text{H}'] : \text{p} \mid \text{B}\ [\text{H}''] : \text{q}\ ) : \text{C}} \quad (\vee\text{L})$$

$$\frac{\psi;\Gamma \vdash \text{A}}{\psi;\Gamma \vdash \text{A} \vee \text{B}} \quad (\vee R_1)$$

$$\frac{\psi; \Gamma \vdash \mathrm{B}}{\psi; \Gamma \vdash \mathrm{A} \vee \mathrm{B}} \quad (\vee R_2)$$

Implication:

$$\frac{\psi; \Gamma, \mathrm{H} : \mathrm{A} \supset \mathrm{B} \vdash \mathrm{p} : \mathrm{A} \qquad \psi; \Gamma, \mathrm{H} : \mathrm{A} \supset \mathrm{B}\ , \mathrm{H'} : \mathrm{B} \vdash \mathrm{q} : \mathrm{C}}{\psi; \Gamma, \mathrm{H} : \mathrm{A} \supset \mathrm{B} \vdash (\mathrm{p}, \mathrm{B}\ [\ \mathrm{H'}\ ]\ \text{via H}, \mathrm{q}) : \mathrm{C}} \quad (\supset\mathrm{L})$$

$$\frac{\psi; \Gamma\ , \mathrm{H} : \mathrm{A} \vdash \mathrm{p} : \mathrm{B}}{\psi; \Gamma \vdash (\ \text{Assume A}\ [\ \mathrm{H}\ ]\ , \mathrm{p}\ ) : \mathrm{A} \supset \mathrm{B}} \quad (\supset\mathrm{R})$$

Using hypotheses:

$$\frac{}{\psi; \Gamma, [\ \mathrm{H}\ ] : \mathrm{A} \vdash \text{by H} : \mathrm{A}} \quad (\text{by})$$

$$\frac{\psi; \Gamma \vdash \mathrm{p} : \mathrm{A}}{\psi; \Gamma \vdash \text{p Therefore A} : \mathrm{A}} \quad (\text{therefore})$$