

FINANCIAL FRICTIONS, DEBT TAX SHIELDS, AND THE MACROECONOMY

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Abstract

This paper evaluates the aggregate effects of financial frictions in the presence of tax shields of debt financing. Previous studies indicate that removing financial frictions will stimulate investment and reduce misallocation. However, with the tax bias towards debt over equity, I show that the macroeconomic implications of financial frictions can be different. I build a dynamic stochastic general equilibrium model of investment. Heterogeneous firms face financial frictions in both debt and equity markets: borrowing constraints and costs of external equity issuance. I estimate the borrowing constraint parameter by targeting the slope of investment with respect to leverage. My results show that a large tax shield with loose credit constraints can exacerbate the misallocation of capital. Using the U.S. firm-level data, my counterfactual experiments demonstrate that by removing financial frictions, aggregate capital increases by 10%, output by 3%, and welfare by 2%. Aggregate gains can be 10 times larger when accounting for the tax shield.

Keywords: Heterogeneous Firms, Financial Frictions, Borrowing Constraints, Tax Shields

JEL Codes: G30, H25, O40

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1 Introduction

The paper revisits a classical question in macroeconomics: what are the aggregate consequences of financial frictions? It is a commonly held view that financial frictions can reduce aggregate output and total factor productivity (TFP) ¹. However, literature in macroeconomics often ignores an important distortion of the tax shield of debt financing, a standard presumption in the corporate finance literature, when modeling firm investment dynamics. Debt financing has a tax shield or tax benefit ² because interest expenses on debt are deductible against the corporate tax. However, returns to equity do not have such a tax benefit. This creates a bias toward debt financing over equity financing and the bias of the tax code toward debt may stimulate borrowing and contributes to higher leverage.

Without financial frictions in the market, the debt tax shield only affects firms' capital structure (i.e., the composition of debt and equity financing) ³. But when the market has financial frictions, it can also affect firms' real activities. On one hand, it may expand the debt capacity of financially-constrained firms and hence incentive them to invest. On the other hand, the tax benefit may lead to overborrowing and as a result, distort resource allocation. Therefore the role of tax shield in the effect of financial frictions is ambiguous. Financial frictions can have different macroeconomic implications when interacting with the debt tax shield.

The paper quantifies the aggregate effects of financial frictions in the presence of a debt tax shield. It also studies how the magnitude of tax shields interacts with the impact of financial frictions. I find that the bias in the tax code toward debt financing over equity financing can exacerbate the aggregate effects of financial frictions of firms as well as the misallocation problem. After lifting financial frictions, aggregate productivity can instead decrease due to the interaction between the magnitude of tax shields and the effects of financial frictions.

In this paper, I build a dynamic stochastic general equilibrium model in which heterogeneous firms finance investment with either equity or debt. Firms have two potential financing instruments. Firstly, they can issue one-period risk-free debt securities. Secondly, they can raise funds directly from shareholders in the event of a cash flow shortfall. The firm's financing decision is distorted by two types of financial frictions. The first is a borrowing constraint such that firms can only borrow up to the minimum possible cash flow and a fraction of their tangible capital. The second is the costly external equity issuance. The tax shield of debt in my model is a nonpecuniary wedge between the discount rate and the rate on debt. The debt tax shield makes firms behave impatiently and incentivizes firms to use debt to balance the tax

¹For example, see [Khan and Thomas \(2013\)](#), [Midrigan and Xu \(2014\)](#), [Moll \(2014\)](#).

²On this topic, "tax shield", "tax benefit", "tax advantage", and "tax bias" are used almost interchangeably in the literature.

³For example, see [Modigliani and Miller \(1958\)](#).

advantage of borrowing with current and future expected financing costs. Firms are also subject to corporate income tax. I integrate my model into an otherwise standard model of firm dynamics with idiosyncratic productivity shocks, convex capital adjustment costs, firm entry, and exogenous firm exit. Finally, I close the model with a representative household. There is no aggregate shock in my model.

Because of various costs and frictions, my model cannot be solved analytically. I adopt the discretization approach and solve it numerically using dynamic programming. I structurally estimate the model parameters in a simulated method of moments (SMM) procedure using firm-level balance sheet data from COMPUSTAT between 1980 and 2018. I target 6 moments to pin down 5 structural parameters. Following [Hennessy and Whited \(2007\)](#) and [Catherine et al. \(2021\)](#), I use the net equity issuance rate to pin down external equity financing costs. To identify the scope of the borrowing constraint, I carefully choose the slope of investment with respect to leverage (defined as the debt-to-earnings ratio) as my target moment. In the data, this sensitivity is -1, which means a one-unit increase in debt-to-earnings is on average associated with a decline in investment of 1 percentage point. In addition, I target the volatility of debt rate, investment rate, and sales growth rates of both short-term and long-term to identify the parameters governing the underlying productivity process and the capital adjustment cost. I estimate a linear cost of external equity issuance of about \$0.04 per \$1 new equity issued. My estimate of the borrowing constraint parameter is about 0.15, indicating that firms can pledge only about 15% of their capital stock. This suggests a sizable degree of debt market financial frictions in the economy. I calibrate the baseline tax rate of 0.2 taken from [Gomes and Schmid \(2010\)](#). With a risk-free interest rate of 4%, the effective interest rate on debt after the tax deduction is 3.2%. Therefore the cost of debt financing is smaller than the cost of equity financing, encouraging firms to borrow to finance their investment projects. My parameter estimates are broadly comparable to previous estimates. My model is also able to reproduce a range of non-targeted moments including the asset-to-sales ratio and correlation of investment rate. Moreover, it can explain about 30% of misallocation in the data, making my model suitable to discuss misallocation.

I use my structural model for quantitative analysis. I first quantify the importance of financial frictions on the aggregate economy, I simulate two economies. One is the estimated Baseline economy with debt tax shields. Another is a Counterfactual economy with no tax shield. I calculate aggregate variables in log deviation relative to an “unconstrained” benchmark where equity is free and all assets can be collateralized. I find that by removing financial frictions in debt and/or equity markets, aggregate capital stocks grow about 10%, output gains 3%, productivity increases by about 0.2%, and consumption-equivalent welfare rises by about 2%. Consistent with the literature (e.g., [Catherine et al. \(2021\)](#)), the aggregate costs of financial

frictions mainly come from an insufficient supply of capital input. Comparing the macroeconomic outcomes of these two economies, my results demonstrate that failing to account for the tax advantage of debt financing will underestimate the aggregate costs of financial frictions of aggregate capital, output, and welfare, which can be 10 times smaller.

Then I explore the interaction between the tax shield and financial friction. I vary the magnitude of the tax shield and the degree of the borrowing constraint. I uncover that the tax benefit can be regarded as a double-edged sword. When firms are highly financially-constrained, a tax shield reduces the cost of capital and expands debt capacity. It boosts investment and firm growth. Both aggregate credit and productivity increase. On the contrary, when firms are close to being unconstrained, an increase in the tax shield leads to excessive leverage as firms still exploit the tax benefits. Less financial friction can compound the problem of over-borrowing. Thus, while a small increase in the tax shield can mitigate misallocation, a larger increase in the tax shield with relaxed borrowing constraints exacerbates the misallocation of input and decreases aggregate productivity. As a result, the response of aggregate productivity to financial frictions is nonlinear.

To further evaluate the magnitude of the interaction between the tax shield and financial frictions, I decompose the aggregate gains of financial frictions into three components: (1) gains due to the tax shield alone; (2) gains due to financial frictions alone; and (3) gains due to the interaction term. I find that both tax shields and a removal of financial frictions have positive effects on capital, output, productivity, and welfare. However, the interaction term counteracts these positive effects, suggesting that a larger tax shield with less financial frictions can affect the aggregate economy adversely. This is consistent with previous results. Quantitatively, the interaction term has the same importance as the financial frictions.

Lastly, I conduct a series of robustness checks. I first explore a range of alternative values of estimated structural parameters and externally calibrated parameters. I show that the implications of tax shields and the aggregate effects of financial frictions are robust in this exercise. Then I re-calibrate the model by targeting the mean leverage ratio, a moment widely used in the literature, to mainly identify the borrowing constraint parameter. I show that the model implies less severity of financial frictions: a smaller equity issuance cost estimate and a larger borrowing constraint parameter estimate. It has a flaw in matching the slope of investment to leverage. In addition, I show that there is a non-monotonic relationship between the borrowing constraint and leverage. This analysis points out that using cross-sectional moments, such as leverage, to pin down structural parameters may have an identification problem.

The contribution of this paper is twofold. Firstly, the paper demonstrates that a tax bias of debt financing can exacerbate the aggregate effects of financial frictions of firms. The quantitative difference is significant. The interaction between the magnitude of tax benefits and

the effects of financial frictions implies that removing financial frictions does not necessarily boost investment or long-term economic growth. Instead, it may lead to a problem of excessive leverage. As a result, removing financial frictions can even decrease aggregate productivity in the presence of the debt tax shield. Second, to identify financial frictions in the debt market, I propose to use the slope of investment with respect to leverage as a target moment. I show that using cross-sectional moments, such as the mean leverage ratio, will underestimate the extent of financial friction in the economy and can have an identification concern when taking the tax shield into account.

My paper has important policy implications. Recent years have witnessed tax policy reforms to neutralize the tax bias for debt over equity in the U.S. and Europe (i.e., Tax Cuts and Job Act of 2017 in the U.S.). By showing that the debt tax shield could change the macroeconomic implications of financial frictions, my results highlights that the tax bias can have a negative impact on the overall economy. Moreover, my results suggest caution in providing credit support in the presence of debt tax shields. While the purpose of the paper is not to assess a tax policy, it sheds light on the importance of the interplay between fiscal monetary policies.

This paper is organized as follows. Section 2 reviews related literature. Section 3 introduces the tax shield policy background and discusses the economic rationale for and against the debt bias in the current tax system. Section 4 formulates a dynamic investment model with financial frictions, debt tax shields, and other distortions. Section 5 structurally estimates the model. In Section 6, I evaluate the aggregate effect of financial frictions and explore the interaction between tax shields and financial frictions. In Section 7, I do some robustness checks. Finally, Section 8 concludes.

2 Literature

My paper builds on several strands of literature. First, this paper contributes to the broad quantitative literature on the effects of financial frictions (e.g., [Hennessy and Whited \(2007\)](#), [Buera et al. \(2011\)](#), [Khan and Thomas \(2013\)](#), [Midrigan and Xu \(2014\)](#), [Moll \(2014\)](#), [Jo and Senga \(2019\)](#), [Ottonello and Winberry \(2020\)](#), [Bau and Matray \(2020\)](#), and [Karabarbounis and Macnamara \(2021\)](#)). The most closely related paper is [Catherine et al. \(2021\)](#) which quantitatively examines the impact of collateral constraints. My model shares common features with [Catherine et al. \(2021\)](#): borrowing constraints, costly external equity issuance, and a tax benefit. However, there are substantive differences. First, they focus on one source of financing friction: collateral constraints, while I also analyze the effects of other distortions (e.g., costly equity issuance and taxes) and the interactions between financial frictions and taxes. Second, in model specifications, the main differences are (a) my model has decreasing-

returns-to-scale in production; (b) goods are homogeneous and the good market is perfectly competitive, allowing for easier aggregation; (3) there is no real estate in my model. Third, in estimation, [Catherine et al. \(2021\)](#) exploit variations in real estate prices and use a reduced-form coefficient, the sensitivity of firm-level investment to collateral values, to identify the scope of financial frictions. They find that collateral constraint induces output losses of 7.1%, and TFP (misallocation) losses of 1.4%. Instead, my estimation method uses the slope of investment with respect to the debt-to-EBITDA ratio where the computation is considerably simpler. Relative to the same efficiency benchmark of free equity (i.e., $\eta_1 = 0$), my results suggest that the aggregate costs of output and productivity are quantitatively smaller. And compared to my main results that are relative to a benchmark of free equity and pledgeability of 1 (i.e., $\eta_1 = 0, s = 1$), the aggregate cost of output is also smaller but it's larger for aggregate productivity. This exactly confirms my argument that relaxing financial constraints can reduce productivity.

Second, my findings echo the literature that studies the misallocation of capital and aggregate productivity (e.g., [Restuccia and Rogerson \(2008\)](#), [Hsieh and Klenow \(2009\)](#), [Midrigan and Xu \(2014\)](#), [Gilchrist et al. \(2013\)](#), and [Karabarbounis and Macnamara \(2021\)](#)). These studies support that financing frictions lead to input misallocation. A major difference with the aforementioned works is that my paper allows for tax deductibility of interest. I contribute to the literature that financing frictions may mitigate the extent of misallocation. In the presence of other distortions such as the tax benefit of debt financing, whether removing financial frictions will increase aggregate output and productivity is ambiguous. One exception is the recent paper of [Karabarbounis and Macnamara \(2021\)](#) that incorporates a similar tax structure in the interest income with a focus on risky long-term debt. But it does not discuss the nonlinear effects of debt tax shields on aggregate variables. The paper finds a TFP loss of 19% for public firms, much larger than about 8% of my model. This may be because I assume away default. I expect that an endogenous exit will amplify my results.

Third, this paper speaks to the literature on taxation, firm capital structure, and dynamic trade-off theory in corporate finance. The standard theory from [Modigliani and Miller \(1958\)](#) states that with certain assumptions in place, a firm shall be indifferent between various sources of financing for its projects. However, with tax deductibility of interest, firms raise debt to balance the value of interest tax shields against costs associated with financial distress and bankruptcy, which determines an optimal amount of debt in a firm's capital structure (e.g., [Modigliani and Miller \(1963\)](#), [Scott Jr \(1976\)](#), [Miller \(1977\)](#), [Myers \(1984\)](#), [Hennessy and Whited \(2005\)](#), and [Li et al. \(2016\)](#)). Unlike these works, my model is a standard neoclassical macroeconomic model abstract from complicated debt structure and endogenous default. Instead, I use costly equity issuance as the cost of the interest tax deductibility to form a tradeoff

of borrowing. In this respect, my paper is similar in spirit to [DeAngelo et al. \(2011\)](#). But they focus on capital structure dynamics and I study the role of the tax shield to understand the real effects of financial frictions. With that being said, my model can be easily extended to incorporate more components as in the corporate finance literature to study the optimal capital structure.

Last but not the least, my work on the structural estimation and targeted moment of the slope of investment with respect to leverage relate to the extensive theoretical and empirical corporate finance literature that studies how preexisting debt affects firms' decisions to undertake new investments (e.g., [Kalemli-Özcan et al. \(2020\)](#), [Crouzet and Tourre \(2020\)](#), [Barbiero et al. \(2020\)](#), [Jordà et al. \(2020\)](#), [Albuquerque \(2021\)](#), and [Perla et al. \(2020\)](#)). The long-standing question goes back to the seminal work of [Myers \(1977\)](#). He hypothesizes that outstanding debt may distort investment downwards as profits primarily benefit existing debt holders but not potential new investors in the presence of default risk. The paper refers to it as a "debt overhang problem". Relative to the literature (e.g., [Diamond and He \(2014\)](#)), I make two simplifications in modeling. First, my model only features one type of debt instrument, the short-term debt; second, firms exit from the markets only because of an exogenous death shock. However, with costly external equity financing, the simplicity can still capture the nexus between the firm's capital structure and its investment efficiency in the absence of endogenous and/or strategic default. I follow [Crouzet and Tourre \(2020\)](#) and [Blickle et al. \(2022\)](#) and compute the slope of investment with respect to leverage. The slope is negative on account of debt overhang effects. Here leverage is defined as the ratio of debt and EBITDA. This leverage measure relates borrowing to a proxy for cash flow which can remove some of the endogeneity associated with the firm's financing decisions. While [Crouzet and Tourre \(2020\)](#) uses the moment to identify the adjustment cost parameter, I use it to primarily pin down parameters governing financial frictions.

3 Policy Background

In this section, I provide an overview of tax policies on debt financing as well as the economic rationale for and against the debt tax shield.

The tax system in the United States and around the world has a long history that Interest expenses on debt are tax-deductible, while a similar deduction for the cost of equity (in the form of dividends or share appreciation) is rarely ever granted (e.g., [Bank \(2014\)](#)). As a result, returns to equity-financed investment are taxed at both the corporate level and the shareholder level, while debt-financed investment faces only shareholder-level tax. Some empirical works (e.g., [Feld et al. \(2013\)](#), [Heider and Ljungqvist \(2015\)](#)) document that taxes and the debt tax

shield are important drivers of firms' capital structure.

The economic rationale for tax bias for debt is related to market failures (i.e., adverse selection, agency problem, signaling) that discourage the use of external finance and lead to under-leverage, suggesting a role for tax policy that favors debt (e.g., [De Mooij \(2012\)](#), [Pozen and Goodman \(2012\)](#)). For example, corporate debt can fix an agency problem and improve managerial decisions because issuing debt constrains the use of free cash flow and protects shareholders (e.g., [Jensen \(1986\)](#)). The signaling rationale argues that debt issuance signals good health to outside investors because it shows that the company is confident to service its debt in the future (e.g., [Ross \(1977\)](#)).

However, there is a consensus that these justifications are not convincing (e.g., [De Mooij \(2012\)](#)). In contrast, many studies argue that the tax bias for debt can distort firms' decisions on financing and investment (e.g., [Weichenrieder and Klautke \(2008\)](#)). In particular, tax advantages for debt finance can disproportionately hurt young and innovative firms that invest heavily in R&D expenditures for lack of assets that can be easily used as collateral (e.g., [Pozen and Goodman \(2012\)](#)). Moreover, the tax shield can generate negative externalities as excess debt increases systemic risk and macroeconomic instability (e.g., [Bianchi \(2011\)](#), [Schularick and Taylor \(2012\)](#), [Jordà et al. \(2013\)](#)).

Given the above concerns, the debt tax shield has been the subject of analysis and discussion among lawmakers. It gains renewed interest in light of the 2008 financial and economic crisis. Many governments and international organizations have started to make tax reforms and introduce various measures (e.g., Allowance for Corporate Equity (ACE) and Comprehensive Business Income Tax (CBIT)) to reduce or eliminate the tax benefit of debt. Belgium was among the very few countries in the world that neutralized the debt bias. Since 2006, Belgium allows for a notional interest deduction on equity capital. In the United States, Congress passed the Tax Cuts and Jobs Act (TCJA) in 2017. In addition to a reduction in the corporate tax rate, the Act limits interest deductibility permanently to 30% of earnings. Previously, interest expenses are generally fully deductible. At the supranational level, European Commission proposed a debt-equity bias reduction allowance to help businesses access the financing they need and to become more resilient in May 2022. The allowance on equity is deductible for 10 consecutive tax years. The proposal also introduces a reduction of debt interest deductibility by 15%.

In light of these debates as described above, the quantitative analysis seems useful. This paper is to shed light on the role of the debt tax shield on the aggregate implications of financial frictions.

4 The Model

In this section, I present a stochastic general equilibrium model of dynamic investment by heterogeneous firms under tax distortions and financial frictions on both debt and equity financing: (1) borrowing constraints, and (2) costly external equity. The economy consists of a continuum of a unit mass of firms. Firms produce a homogeneous good consumed by a representative consumer. Time is discrete on an infinite horizon. The model builds on [Strebulaev and Whited \(2012\)](#) and it is closely related to [Hennessy and Whited \(2007\)](#), [Katagiri \(2014\)](#) and [Catherine et al. \(2021\)](#).

4.1 Firms

Production Technology

Firms are risk-neutral. Each firm owns predetermined capital stock k and hires labor n . It produces a homogeneous good with decreasing-returns-to-scale production technology:

$$y = zk^\alpha n^\nu, 0 < \alpha + \nu < 1 \quad (1)$$

z is a firm's idiosyncratic total factor productivity and follows a Markov chain. There is no aggregate uncertainty in the model. Labor is flexible and is hired in a competitive labor market at a wage of W . The capital accumulation of each firm is standard, $i = k' - (1 - \delta)k$. The investment decision takes place before the realization of the next period's productivity z' . Therefore my model is different from [Midrigan and Xu \(2014\)](#) and [Moll \(2014\)](#) where firms rent capital in a competitive capital rental market. The capital stock depreciates at the rate δ each period. Investment is reversible and subject to a standard quadratic adjustment cost $\psi(k, i)$:

$$\psi(k, i) = \psi_1 + \frac{1}{2}\psi_0 \left(\frac{i}{k}\right)^2 k \mathbb{1}_{i \neq 0} \quad (2)$$

Firms face corporate taxation. Firms' income is taxed at the rate τ . For simplicity, I assume the same income tax rate at both positive and negative cash flow (which I will define later). In general, this tax cost can represent a variety of costs from holding too much cash, such as agency costs.

Given the optimal choice of labor, the firm's earnings before an interest payment, a tax payment, and depreciation (EBITDA) are defined as follows:

$$\pi(k, z) = zk^\alpha n^\nu - Wn = (1 - \nu)y \quad (3)$$

To finance the asset, firms have three financing sources: internal funds, outside equity from the household, and debt.

Debt Market Frictions

Each firm faces a borrowing limit on a one-period riskless discount debt. Following [Strebulaev and Whited \(2012\)](#), this borrowing constraint restricts the new debt level, b' , by firms' value of their capital and future after-tax earnings in the worst state of the world. The borrowing constraint can thus be expressed as:

$$b' \leq (1 - \tau)(\underline{z}k'^\alpha \underline{n}'^\nu - W \underline{n}') + s(1 - \delta)k' \quad (4)$$

in which \underline{z} is the lowest possible value that the shock z can attain. k' is the firm's future period capital. \underline{n}' is the corresponding optimal labor input. I assume that only a fraction s of the capital stock can be liquidated. Firms can always repay the debt. The assumption of a riskless debt reflects the asymmetric information problem between creditors and firms. The state variable b takes both positive and negative values. A negative value of b denotes cash ⁴.

Consistent with corporate finance literature, I assume that debt financing has a tax shield, where debt incurs taxable interest at the after-corporate tax rate. This creates an incentive for firms to increase their leverage. Let τ^S denote the tax shield rate. Thus, the present value of debt issued in the next period is $\frac{b'}{1+r(1-\tau^S)}$. r is the risk-free interest rate. The discount rate of $1 + r(1 - \tau^S)$ implies the interest deductions on the debt. At the baseline, τ^S and τ have the same value.

Equity Market Frictions

The costly external equity injections carry a fixed and proportional cost and are thus a more expensive source of funds than internally generated cash flows. As in [Hennessy and Whited \(2005\)](#), [Hennessy and Whited \(2007\)](#), and [DeAngelo et al. \(2011\)](#), the reduced-form fashion can preserve tractability and is necessary for estimation. Suppose $e_1(k, k', b, b', z)$ is the net cash flow. The cost of external cash flow is thus given by:

$$\eta(x) = (-\eta_0 + \eta_1 e_1) \mathbb{I}_{e_1 < 0} \quad (5)$$

⁴Alternatively, I can define two variables: gross debt and cash. But as pointed out in [DeAngelo et al. \(2011\)](#), assuming no debt issuance costs and positive agency costs of holding cash, a firm never simultaneously has positive values of both gross debt and cash because using the cash to pay off debt would leave the tax bill unchanged and reduce agency costs.

where $\eta_0 > 0$ and $\eta_1 > 0$ are the fixed and linear components of the equity cost function. The external equity issuance costs can be interpreted as adverse selection costs. Without firm default, the equity market frictions can also be interpreted as distress costs that balance the tax shield of debt.

For a given desired next-period capital stock k' and debt b' , $e_1(k, k', b, b', z)$ is defined to be:

$$e_1(k, k', b, b', z) = (1 - \tau)(y - Wn) + (1 - \delta)k - b - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)}$$

where the firm's internal fund:

$$e_{IN}(k, b, z) = (1 - \tau)(y - Wn) + (1 - \delta)k - b$$

The external funding requirement is equal to the desired capital stock less the internal fund:

$$\begin{aligned} e_{EX}(k, k', b, b', z) &= k' + \psi(k, i) - e_{IN}(k, b, z) - \frac{b'}{1 + r(1 - \tau^S)} - \eta(e_1) \\ &= -e_1(k, k', b, b', z) - \eta(e_1) \end{aligned}$$

Assume that firms cannot retain earnings. If cash inflows exceed optimally chosen cash outflows, $e_1(k, k', b, b', z) > 0$, then the firm must pay the entire fund out to shareholders. On the other hand, if the distribution to shareholders is negative, $e_1(k, k', b, b', z) < 0$, then shareholders need to fill the gap e_{EX} .

Entry and Exit

In each period, incumbent firms may exit the economy. The death shock $\pi_d \in (0, 1)$ is common across firms. Since the debt is risk-free, therefore there is no endogenous exit in the model. If the firm will continue to operate for the next period, then it may invest and/or borrow. If the firm will exit, then it keeps the residual of current production after wage bill and debt repayment. Then the cash flow $e(k, k', b, b', z)$ is defined as:

$$\begin{aligned} &e(k, k', b, b', z) \\ = &\begin{cases} e_0 = (1 - \tau)(y - Wn) + (1 - \delta)k - b & \text{if the firm will exit} \\ e_1 = (1 - \tau)(y - Wn) + (1 - \delta)k - b - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)} & \text{if the firm will survive} \end{cases} \end{aligned}$$

Exiting firms are replaced by an equal mass of entrants so that the total mass of production firms is fixed in each period. Entering firms are fully equity-financed with initial capital stock k_0 . The initial productivity of an entrant, z_0 , is randomly drawn from the ergodic distribution of z . They then proceed as incumbent firms.

Timing

At the beginning of each period, an incumbent firm is identified with a state vector (k, b, z) : the predetermined capital stock k , the amount of debt carried from the previous period b , and the current period idiosyncratic productivity z . The firm makes the optimal labor choice and learns its exogenous exit status. Labor choices are static. Therefore firms with the same (k, z) will make the same labor choices, regardless of their exit shock realizations.

If the firm is assigned to exit, it simply chooses labor n to maximize its current dividend payment to shareholders. The dividends e_0 are output, less wage payment, and debt repayment, alongside the returns from capital liquidation. If the firm is continuing beyond the period, then additionally, it makes intertemporal decisions on future capital k' and borrowing b' . The current dividend payment is e_1 .

For the next period, the initial state of a continuing incumbent is (k', b', z') . It starts operating, along with entering firms with the initial state $(k_0, 0, z_0)$.

Firm Distribution

The distribution of firms over (k, b, z) is denoted by a probability measure μ , defined on the Borel algebra \mathcal{S} by the open subsets of the product space, $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$. The evolution of the firm distribution Γ is determined in part by the actions of continuing firms and in part by entry and exit:

$$\begin{aligned} \mu' &= \Gamma(\mu) \\ \mu'(z_j) &= (1 - \pi_d) \int_{\{(k, b, z_i) | (k', b') \in A\}} \pi_{ij} d\mu(k, b, z_i) + \pi_d \chi(k_0) H(z_j), \quad \forall (A, z_j) \in \mathcal{S} \end{aligned} \tag{6}$$

where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$.

Firm Problem

Let $V(k, b, z)$ be the expected discounted value of a firm that enters with (k, b) and idiosyncratic productivity z at the beginning of the current period. Then the Bellman equation for an

incumbent firm is:

$$V(k, b, z) = \max_{k', b'} \left\{ \pi_d e_0 + (1 - \pi_d) \left[e_1 + \eta(e_1) + \frac{1}{1+r} \mathbb{E}(V(k', b', z')|z) \right] \right\} \quad (7)$$

subject to

$$b' \leq (1 - \tau)(\underline{z}k'^\alpha \underline{n}'^\gamma - W\underline{n}') + s(1 - \delta)k'$$

where

$$e_0 = (1 - \tau)(y - Wn) + (1 - \delta)k - b$$

if the firm will exit

$$e_1 = (1 - \tau)(y - Wn) + (1 - \delta)k - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)} - b$$

if the firm will survive

$$\eta(e_1) = (-\eta_0 + \eta_1 e_1) \mathbb{1}_{e_1 < 0}$$

$$\psi(k, i) = \frac{1}{2} \psi_0 \left(\frac{i}{k} \right)^2 k \mathbb{1}_{i \neq 0}$$

Capital structure choices in my model are made each period and are state-contingent exhibiting (local) path dependence as in [DeAngelo et al. \(2011\)](#). There is no single optimal capital structure.

Firm Optimal Conditions

To develop the intuition behind the model, I examine the optimality conditions. Differentiating with respect to b' , the first-order condition for the optimal debt is:

$$1 + \eta_1 \mathbb{1}_{e_1 < 0} = -\frac{1 + r(1 - \tau^S)}{1 + r} \mathbb{E}(V_b(k', b', z') + \zeta'|z) \quad (8)$$

where ζ is the Lagrange multiplier on the borrowing constraint. The left side represents the marginal cost of equity finance. The right side represents the expected marginal cost of debt next period. At an optimum, the firm is indifferent between issuing equity and issuing debt. Together, debt derives value from the costly external equity and the tax benefit.

With the envelope condition, the optimality condition can be rewritten as:

$$1 + \eta_1 \mathbb{1}_{e_1 < 0} + \zeta = \frac{1 + r(1 - \tau^S)}{1 + r} \mathbb{E}(1 + \eta_1 \mathbb{1}_{e'_1 < 0} + \zeta'|z)$$

The left hand is the marginal benefit of one extra unit of debt and the right hand represents the marginal cost of debt financing. The term $\eta_1 \mathbb{1}_{e'_1 < 0}$ suggests that raising an extra unit of debt today implies a higher likelihood of external equity financing tomorrow. The shadow

value term ζ' shows that preserving debt capacity today can avoid bumping up against the constraint tomorrow, as explained in [DeAngelo et al. \(2011\)](#).

4.2 Household

An infinitely-lived representative household holds one-period noncontingent bonds B^H and owns firms. Given the real wage W and the risk-free rate r , the household determines its current consumption C^H , hours worked N^H and new bond holdings $B^{H'}$, to maximize its lifetime expected utility:

$$V^H(B^H) = \max_{C^H, N^H, B^{H'}} \{ \log C^H - \varphi N^H + \beta^H V^H(B^{H'}) \} \quad (9)$$

subject to

$$C^H + \frac{B^{H'}}{1+r} = WN^H + B^H + T^H + \Pi^H$$

where

$$\Pi^H = \int \left(\underbrace{(1 - \pi_d)[e_1 + \eta(e_1)]}_{\text{Continuing}} + \underbrace{\pi_d e_0}_{\text{Exit}} - \underbrace{\pi_d \left[k_0 - \frac{b'}{1+r(1-\tau^s)} \right]}_{\text{Entrant}} \right) d\mu(k, b, z)$$

I assume log-utility for consumption and linear disutility for labor supply as in [Hopenhayn and Rogerson \(1993\)](#) and [Gomes \(2001\)](#). Π^T is the dividend payment of the firm. T^H is the lump-sum transfer rebated to the household. β^H is the discount factor for future utility.

4.3 Equilibrium Definition

Consider a stationary general equilibrium of the model. The equilibrium is defined by a set of value functions $\{V, V^H\}$, decision rules $\{k', b', n, B^{H'}, N^H\}$, prices $\{W, r\}$, and a measure of firms μ such that:

1. All firms optimize: V solves (7) with associated policy rules $\{k', b', n\}$.
2. The household optimizes: V^H solves (9) with associated policy rules $\{C^H, B^{H'}, N^H\}$.

3. The bond market clears:

$$B^H = \int b d\mu = \mathbf{B}$$

4. The government budget is balanced:

$$T^H = \int \left(\underbrace{\tau(y - Wn)}_{\text{Corporate income tax}} - \underbrace{\frac{\tau r b'}{(1+r)[1+r(1-\tau^S)]}}_{\text{Tax benefit of debt}} \right) d\mu$$

5. The good market clears:

$$\begin{aligned} \mathbf{C} &= \int y d\mu - \underbrace{\pi_d k_0}_{\text{Entrant}} + \underbrace{\pi_d \int (1-\delta)k d\mu}_{\text{Exit}} - \underbrace{(1-\pi_d) \int ((i+\psi) - \eta) d\mu}_{\text{Continuing: investment, capital AC, and equity financing cost}} \\ &= \mathbf{Y} + \pi_d(1-\delta-\kappa_0)\mathbf{K} - (1-\pi_d)(\mathbf{I} + \mathbf{\Psi} - \mathbf{H}) \end{aligned}$$

where aggregate output \mathbf{Y} , capital stock \mathbf{K} , investment \mathbf{I} , adjustment costs $\mathbf{\Psi}$, equity issuance cost \mathbf{H} , consumption \mathbf{C} . κ is the fraction of the steady-state aggregate capital stock held by each entrant and $k_0 = \kappa_0 \mathbf{K}$

6. The labor market clears:

$$N^H = \int n(k, b, z) d\mu = \mathbf{N}$$

5 Estimation

Because the model has no closed-form solution, I estimate key parameters using a Simulated Method of Moments (SMM) in this section. I estimate the model in two steps. First, I exogenously fix a subset of parameters. Second, I estimate the remaining parameters to match moments in the data.

5.1 Parameterization

I assume that firm productivity is a log-normal AR(1) process:

$$\ln(z') = \rho \ln(z) + \varepsilon'$$

where $\varepsilon' \sim \mathcal{N}(0, \sigma^2)$. The parameters (ρ, σ) of the driving process are unknowns that must be estimated. The shock z takes values in the interval $[\underline{z}, \bar{z}]$ and I use the procedure of [Tauchen \(1986\)](#) to discretize the stochastic shock into a 5-state Markov chain.

5.2 Predefined parameters

The model comprises 16 parameters. I externally calibrate 11 of them. I set the capital share $\alpha = 0.25$ and labor share $\nu = 0.6$, implying a decreasing return to scale of 0.85. These values are close to the values commonly used in the investment literature (e.g., [Khan and Thomas \(2013\)](#), [Bloom et al. \(2018\)](#), [Jeenas \(2019\)](#), and [Ottonello and Winberry \(2020\)](#)). The depreciation rate δ is fixed at 0.1, in line with [Bloom et al. \(2018\)](#) and [Karabarbounis and Macnamara \(2021\)](#). I use a tax rate τ of 0.2, consistent with [Gomes and Schmid \(2010\)](#). τ^S is also set at 0.2. As employed by [Khan and Thomas \(2013\)](#), I set the exogenous exit rate π_d to 0.1. I use the relative initial capital stock of potential entrants to the average incumbent firm κ_0 as 0.2, which is based on the estimation in [Jeenas \(2019\)](#). I set both the fixed investment adjustment cost ψ_1 and the fixed equity issuance cost η_0 to 0, following [Catherine et al. \(2021\)](#). I set the risk-free interest rate $r = 0.04$, as in [Jo and Senga \(2019\)](#). The value is standard in the real business cycle literature. The subjective discount factor β^H implies the long-run real interest rate. So the value is $\frac{1}{1+r} = 0.96$. Finally, I follow [Bloom et al. \(2018\)](#) and set the labor disutility φ at 2. Table 1 summarizes these externally calibrated parameters.

5.3 Data and Target Moments

I structurally estimate the remaining 5 parameters: the productivity persistence ρ , the standard deviation of innovation to productivity σ , the convex capital adjustment cost ψ_0 , the linear equity issuance cost η_1 , and the borrowing constraint s .

To calculate data moments, I employ the COMPUSTAT industrial files. I use the fundamental annual sample of nonfinancial, unregulated publicly listed US firms from 1980 to 2018. Details on the data and sample selection are provided in Appendix A. I choose moments that are informative about parameters. The SMM estimates parameters by minimizing the distance

between model-implied moments and their empirical counterparts:

$$\hat{\theta} = \arg \min_{\theta} [m(\theta) - m(X)]' W [m(\theta) - m(X)] \quad (10)$$

where $m(X)$ and $m(\theta)$ are the vectors of moments from the data X and model with parameters θ , respectively. W is the moment weighting matrix. To obtain an asymptotically efficient SMM estimator, w is the inverse of the variance-covariance matrix of data moments. I describe my model solution algorithm and structural estimation method in detail in Appendix B.

In total, I use 6 moments. Given the model is overidentified, the identification is not a one-to-one mapping between data moments and structural parameters. All of the model parameters jointly affect all of these moments in some way. Nonetheless, some moments have a greater influence on certain parameters. I show the local identification in Appendix Figures E.1 to E.5. In Appendix A.2, I compare my data moments to the values reported in literature.

Idiosyncratic Productivity Process (ρ, σ)

I primarily use three moments to pin down parameters governing the productivity process (ρ, σ) . Following [Midrigan and Xu \(2014\)](#) and [Catherine et al. \(2021\)](#), I use 1-year and 5-year standard deviation of sales growth rate $(\sigma(\Delta y_{-1}), \sigma(\Delta y_{-5}))$ to simultaneously estimate these two parameters. The volatility of the short-run and long-run empirical growth rates is 0.35 and 0.8 respectively in the data. I also use the volatility of the debt-to-assets ratio, $\sigma(b/k)$. In the data, the ratio is 0.32. Figure E.1 and E.2 can confirm the local identification of these two parameters.

Capital Adjustment Cost ψ_0

I choose the dispersion of the investment rate, $\sigma(i/k)$, to infer the capital adjustment cost parameter ψ_0 , as in [DeAngelo et al. \(2011\)](#) and [Eisfeldt and Muir \(2016\)](#). Large adjustment costs lead the firm to a smooth investment. Adjustment cost should also have a sizeable effect on the volatility of short-term output $\sigma(\Delta y_{-1})$. Therefore, larger adjustment costs can be identified by smaller investments and short-term output volatility. Figure E.3 shows that $\sigma(i/k)$ is monotonic in ψ_0 .

Equity Market Frictions: Linear External Equity Issuance Cost η_1

In the spirit of [Hennessy and Whited \(2007\)](#), the cost of external equity issuance parameter η_1 , heavily depends on the average ratio of net positive equity issuance scaled by assets, $\mu(e/k)$, because a higher cost of external equity financing implies lower equity issuance. The

value is 0.1 in the sample. Figure E.4 shows that the net equity issuance rate is monotonically decreasing in η_1 .

Debt Market Frictions: Borrowing Constraint s

As mentioned above, I use the slope of the investment rate with respect to the ratio of debt and EBITDA, β , to primarily identify the borrowing constraint parameter. It stands in contrast to literature where the leverage ratio has been widely used⁵. The literature typically relates high leverage to a higher degree of financial friction. However, high leverage can result from overborrowing when firms are financially slack. Therefore, using leverage as a target can yield a biased estimate.

The pre-existing leverage is a debt overhang measure in empirical studies (e.g., Kalemli-Özcan et al. (2020), Blickle et al. (2022), Perla et al. (2020)). A negative investment response to pre-existing debt suggests that excessive levels of debt can reduce investment. I use EBITDA as a proxy of cash flow to scale the debt. This measure can remove some of the endogeneity associated with financing decisions. Figure 1 plots the sensitivity of moments slope of investment with respect to leverage (Left Panel) and mean leverage (Right Panel) to the borrowing constraint parameter s respectively. It shows that the slope β is increasing in s while the response of mean leverage is a U-shaped. The monotonicity demonstrates that the borrowing constraint parameter s can be identified by the slope β .

5.4 Results

Table 2 reports parameter estimates and model fits for both targeted moments and non-targeted moments.

Parameter Estimates

Panel A of Table 2 shows parameter estimates. The estimated productivity process is persistent with $\rho = 0.872$. The estimated standard deviation of the innovation to productivity σ is 0.109. The estimated value for convex capital adjustment cost ψ_0 is 0.056. I estimate that the linear equity issuance cost $\eta_1 = 0.036$ and the borrowing constraint $s = 0.147$. The estimates on financial frictions suggest that increasing \$1 of capital provides about \$0.14 of debt capacity. For per \$1 of new equity issued, firms have to pay a cost of about \$0.04.

Parameter estimates are broadly comparable to existing estimates in the literature. The productivity process parameters (ρ , σ) are close to estimates (0.909 and 0.118) in Khan and

⁵Catherine et al. (2021) provides a detailed survey of the use of the moment.

Thomas (2013). The estimated adjustment cost ψ_0 is greater than 0.004 in Catherine et al. (2021) and less than 0.1519 in DeAngelo et al. (2011). The estimate of the linear cost of equity issuance η_1 is similar to 0.059 in Hennessy and Whited (2005) and somewhat smaller than 0.091 in Hennessy and Whited (2007) and Catherine et al. (2021). The borrowing constraint s is smaller than some other estimates in the literature (e.g., 0.25 in Catherine et al. (2021)). A possible reason for a tight s is a different specification for credit constraints. In my model, borrowing constraint is not only based on assets but also the expected minimum earnings.

Model Fit

Panel B of Table 2 shows that the model matches the targeted moments reasonably well, despite being over-identified. My baseline model roughly matches the dispersion of leverage and investment rate. It somewhat overpredicts the sales growth rate volatility. But it matches perfectly the average net equity issuance rate, and importantly, the slope of investment with respect to the debt-to-EBITDA ratio β .

As for non-targeted moments, the model under-matches the mean leverage $\mu(b/k)$. The model implies -0.051, as opposed to 0.1 from the data. Other than that, the model can reproduce the average investment rate and the volatility of equity issuance. It also leads to a successful fit for the mean assets-to-sales ratio and investment correlation.

In Panel C of Table 2, I also examine the dispersion of the marginal product of capital (MPK). I measure it by calculating the standard deviation of log MPK. In my sample, the within-industry MPK dispersion is 1.128. The dispersion generated by the model is 0.347, which accounts for about one-third of the total MPK dispersion in the data. Appendix A.1 describes how I get the empirical estimate.

6 Quantitative Analysis

In this section, I conduct three counterfactual experiments. First, I quantify the aggregate effects of financial frictions. To do that, I simulate two economies: an estimated Baseline economy with a debt tax shield and a Counterfactual economy with No Tax Shield. I compare the macroeconomic variables of these two economies in log deviation from the “unconstrained” benchmark to evaluate the quantitative importance of the tax shield. Second, I explore the interaction between the magnitude of tax shields and the impact of financial frictions by varying the tax shield rate and the degree of the borrowing constraint. To estimate the contribution of the interaction term, I further decompose the aggregate costs of financial friction

6.1 The Aggregate Effects of Financial Frictions

In this subsection, I use the structural model to evaluate the aggregate effects of financial friction. Table 3 reports percentage changes in macroeconomic variables when financial frictions are removed. My Unconstrained Benchmark (B1) corresponds to a model when equity is free ($\eta_1 = 0$) and firms can pledge all value of the capital stock as collateral ($s = 1$). Since $s \neq \infty$, firms can still benefit from the tax shield of debt financing. Note that this is not the first-best allocation.

Column (1) reports results for the Baseline estimated economy relative to Benchmark B1. In contrast, Column (2) of Table 3 reports aggregate outcomes for the Counterfactual No Tax Shield economy. Benchmark B1 in this scenario has no tax shield either. When debt financing does not have a tax advantage, free equity implies the same capital allocations regardless of the value of the borrowing constraint s .

Table 3 shows that financial frictions greatly affect investment and economic growth. Lifting financial frictions increases by 10% for aggregate capital stock and 3% for aggregate output. It also improves the labor market. Both aggregate employment and wage increase. From a larger gain in capital stock than in output, the table shows that the output loss mainly comes from the loss of the input of capital stock. Removing financial frictions improves access to credit for financially-constrained firms. This boosts investment and attenuates the misallocation of input. Therefore, TFP increases.

The table demonstrates that aggregate gains of capital stock and output in Column (2) are 10 times smaller than those in Column (1). The difference in consumption-equivalent aggregate welfare is also significant. The welfare gain is 2.5% in the Baseline economy and only 0.45% in the Counterfactual economy. Appendix C.1 describes how to calculate welfare change in detail.

Altogether, Table 3 demonstrates that not accounting for the tax shield will underestimate the aggregate costs of financial frictions, capital stock, output, and welfare in particular.

In Appendix, I consider two alternative unconstrained benchmarks as robustness checks: (1) $s = 1$, and (2) $\eta_1 = 0$, which is also the unconstrained benchmark used in Catherine et al. (2021). I report the results in Appendix Table D.2.

Efficient Allocation and TFP Loss

Since Benchmark B1 still has distortions from taxes and capital adjustment costs, I derive efficient allocation and TFP loss. Table 3 also reports differences in TFP losses for both the Baseline economy and the Counterfactual No Tax Shield economy.

I follow procedures in Gilchrist et al. (2013) and Karabarbounis and Macnamara (2021)

and compute the efficient level of aggregate TFP and the size of TFP loss.

Consider a problem faced by a social planner is to maximize aggregate output, given aggregate labor and capital:

$$Y = \max_{k_i, n_i} \int (z_i k_i^\alpha n_i^\nu) di, 0 < \alpha + \nu < 1$$

$$\text{subject to } \int n_i di = N \text{ and } \int k_i di = K$$

where K and N are the aggregate capital and labor stocks. The solution to this problem implies that the marginal product of labor (MPL) and the marginal product of capital (MPK) are equated across firms. Then the optimal input choices are given by:

$$n_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{N}{\Gamma} \right)$$

$$k_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{K}{\Gamma} \right)$$

$$\text{where } \Gamma = \int z_i^{\frac{1}{1-(\alpha+\nu)}} di$$

Under the efficient allocation, the first-best TFP is:

$$\text{TFP}^{FB} = \frac{Y}{K^\alpha N^\nu} = \Gamma^{1-(\alpha+\nu)} = \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} di \right)^{1-(\alpha+\nu)}$$

TFP loss is then defined as:

$$\text{TFP Loss} = \frac{\text{TFP}^{FB}}{\text{TFP}} - 1$$

[Gilchrist et al. \(2013\)](#) and [Karabarbounis and Macnamara \(2021\)](#) assume that (z, MPK) are jointly log-normally distributed across firms. I also make the same assumption and the relative TFP loss due to resource misallocation is approximated by (See [Appendix C.2](#) for details):

$$\text{Relative TFP Loss} = \log \left(\frac{\text{TFP}^{FB}}{\text{TFP}} \right)$$

$$\approx \frac{1}{2} \alpha (1 - \alpha) \left(\frac{1 - \nu}{1 - \alpha - \nu} \right)^2 \text{Var}(\log(\text{MPK}))$$

Financial frictions will reduce TFP by increasing the dispersion in MPK across firms.

The last row of [Table 3](#) reports the difference in the TFP loss. For the Baseline economy, removing financial frictions will lower the TFP loss by 0.56%. And for the Counterfactual No

Tax Shield economy, TFP loss decreases by 0.33%. These changes are consistent with the above results that aggregate costs are larger in the Baseline economy with the debt tax shield.

Table 4 reports TFP losses for the economy with different degrees of financial frictions. Column (1) represents an economy with free equity ($\eta_1 = 0$). Column (2) represents an economy with $s = 1$. Column (3) is the frictionless Benchmark B1. And Column (4) is the Baseline economy. Panel A reports values for models with the tax shield. Panel B reports values for models without the tax shield.

The table first shows that relative to the first-best TFP, the TFP loss decreases from 8% to about 7.5% when reducing financial frictions. Second, while Column (3) has fewer financial frictions than Column (2) in Panel A, i.e., no equity market friction, the TFP loss is higher. This indicates that removing financial frictions does not necessarily increase aggregate productivity. But it is not the case in Panel B without the tax shield, which means it is the tax shield that leads to non-monotonicity in the aggregate effects of financial frictions.

6.2 Interaction between Tax Shields and Financial Frictions

To explain why the debt tax shield can lead to nonlinearity in the impact of financial frictions, in this subsection I explore the interaction between the magnitude of tax shields and the extent of financial frictions.

Figure 2 plots the effect of the tax shield rate τ^s on aggregate debt and TFP for the baseline economy. The left panel of Figure 2 shows that as the tax shield increases from 0.1 to 0.6, the total credit increases by about 3%. With a larger tax shield, the effective cost of debt is smaller. As a result, firms have a greater incentive to borrow and finance investments via debt in order to reap a greater tax benefit of interest deduction. The right panel of Figure 2 reveals that the impact on productivity is non-monotonic: the productivity increases first and then decreases. This suggests that by raising investment and stimulating borrowing, the debt bias can improve efficiency. However, a larger extent of tax relief with excessive debt finance will exacerbate preexisting distortions and hamper long-term economic growth.

To inspect the interaction between tax shields and financial friction, I plot the aggregate effects of the tax shield rate τ^s for both the Baseline economy and a less-friction economy ($s = 1$) in Figure 3. When the borrowing constraint is relaxed, the magnitude of the effects of the tax shield rate is much larger. Firms that are financially constrained previously now have a larger debt capacity. The aggregate debt now increases by 25%, much greater than 3% in the Baseline economy. The negative impact of a larger tax shield on aggregate TFP is also more significant with a loosening borrowing constraint.

Similarly, I plot the aggregate effects of the borrowing constraint s at different tax shield

rates in Figure 4. For economies with tax shields ($\tau^S > 0$), the borrowing constraint has a nonlinear impact on aggregate productivity following the same economic intuition. With less debt market friction (i.e., a bigger value of s), the larger the tax shield is, the greater drag it exerts on the economy. But for an economy with No Tax Shield ($\tau^S = 0$), advantages gained from the improvement of a relaxed borrowing constraint will only advance marginally and then level off after a specific point.

Decomposing the Aggregate Effects

To estimate the contribution of the interaction term, I decompose the aggregate effects of financial frictions. In this case, I consider the Constrained Efficiency Benchmark (B2) with free equity and no tax shield ($\eta_1 = 0, \tau^S = 0$) and then decompose the difference of macroeconomic variables between the Baseline economy and Benchmark B2 into three components: (i) due to the tax shield alone, measured by the difference of aggregate variables between the Baseline economy and the economy with no tax shield ($\tau^S = 0$); (ii) due to financial frictions alone, measured by the difference of aggregate variables between the Baseline economy and the economy with free equity ($\eta_1 = 0$); and (iii) due to the interaction between the tax shield and financial frictions, measured by the reminder of the difference between the Baseline economy and Benchmark B2, after subtracting the first two components. Column (1) of Table 5 reports the total percentage changes in aggregate variables of the baseline economy relative to Benchmark B2. Columns (2) to (4) of Table 5 report the contributions of each component, respectively.

Consistent with the literature, financial frictions decrease capital, output, productivity, and welfare, as shown in Column (3). Comparing Column (2) and Column (3), the contributions of the tax shield are quantitatively smaller than those of financial frictions for most macroeconomic variables. But the tax shield is much more important in increasing aggregate welfare.

However, the interaction term counteracts the positive effects of the financial frictions and the tax shield, as shown in Column (4). This suggests that while the individual effects of a larger tax shield or reduced financial frictions can stimulate investment and firm growth respectively, a larger tax shield with less financial frictions can affect the aggregate economy adversely, consistent with the above results. Moreover, the interaction term is the same quantitatively important as the financial frictions.

7 Robustness Checks

In this section, I estimate a series of robustness checks.

7.1 Aggregate Implications with Alternative Parameter Values

Table 6 reports various parameter robustness checks. Starting at benchmark point estimates from Table 2 and externally calibrated parameter values from Table 1, I vary the magnitude of a single parameter up and down to alternative values used in literature and compare the implications for a range of macroeconomic aggregates, while keeping other parameters fixed. Each row corresponds to a different robustness check. In the last row, I modify multiple parameter values so that they are consistent with Catherine et al. (2021). Results in Table 6 show that changes in aggregate variables are not significant.

In addition, I also examine the response of aggregate TFP to the change in borrowing constraints for different parameter values. Specifically, I consider 6 externally-calibrated parameters: capital share α , labor share ν , capital depreciation rate δ , risk-free rate r , labor disutility φ , and exit rate π_d . Figure 5 shows that overall the changes are qualitatively similar to the baseline results of Figure 2 that the responses of productivity increase first and then decrease. The hump-shaped response of aggregate TFP is preserved in all 6 panels.

7.2 Targeting Mean Leverage Ratio

Estimation Results Estimation results are reported in Appendix Table D.3. Model 1 is the Baseline model targeting the slope of investment with respect to leverage β . Model 2 is estimated by targeting the mean leverage instead. The estimates suggest that overall firms are less financially-constrained in Model 2.

Compared to the Baseline model, the idiosyncratic TFP shocks are less persistent and less volatile ($\rho = 0.835, \sigma = 0.076$). I estimate a much smaller capital adjustment cost $\psi_0 = 0.008$ compared to 0.056 in Model 1, a much less expensive equity issuance cost $\eta_1 = 0.008$ compared to 0.036 in Model 1, and a much more relaxed borrowing constraint $s = 0.349$ compared to 0.147 in Model 1.

For targeted moments, Model 2 does a fairly good job. Model 2 slightly underpredicts the average net equity issuance rate. It matches perfectly the key moment of the average leverage ratio $\mu(b/k)$. On the contrary, it does a much worse job of matching the non-targeted slope moment β . The model-implied slope is -9.5, significantly smaller than -1 in the data.

Effects of Financial Frictions Figure 6 shows that the effects of financial frictions on firm-level responses are similar to the Baseline model. The average and volatility of the investment rate and leverage ratio change nonlinearly with the borrowing constraint parameter s .

Table D.4 reports macroeconomic outcomes for Model 2 in the bottom panel. The non-monotonicity holds for aggregate variables including capital stock, output, and employment.

Figure 7 shows that productivity eventually decreases when relaxing the borrowing constraint.

Parameter Identification The non-monotonicity relationship between cross-sectional moments (e.g., leverage) and the debt market financial friction parameter s may give rise to an identification problem. s can be well identified only if leverage is large enough. The top panel of Figure 6 shows that when the moment of leverage is relatively large (e.g., 0.1), then it is well above the U shape and falls in the region where the value is monotonically increasing with s . Therefore, a larger targeted moment can avoid the identification problem and help pin down the parameter s . But the failure of Model 2 to match the key moment, the slope of investment to the debt-EBITDA ratio β , indicates that it is still problematic to only target cross-sectional moments. On the contrary, there is no identification concern using the slope of investment in the Baseline model since the response is unambiguously monotonic.

8 Conclusion

Motivated by recent tax policy reforms around the world, I study how debt tax shields interact with financial frictions and affect firms' financing and investment decisions as well as the aggregate economy.

To quantify the aggregate implications of financial frictions in the presence of a tax shield, I build a dynamic general equilibrium model of investment by heterogeneous firms with a tax bias of debt over equity and financial frictions in both debt and equity markets. Specifically, I consider two types of financial frictions: borrowing constraints and costs of external equity issuance. I integrate my framework in an otherwise standard model with idiosyncratic productivity shocks, capital adjustment costs, firm entry, exogenous death shock, and a representative household. I structurally estimate the model parameters by matching micro from public firms' data. In particular, I identify the borrowing constraint parameter by targeting the slope of investment with respect to leverage. My estimate of the borrowing constraint parameter is about 0.15, indicating a sizable degree of financial frictions in the economy.

I find that in the presence of a tax benefit of debt, aggregate capital stocks grow about 10%, output gains 3%, productivity increases by about 0.2%, and consumption-equivalent welfare rises by about 2%. The aggregate impacts of financial frictions are about 10 times larger than a Counterfactual economy with no tax shield. This suggests that the debt tax shield may exacerbate the aggregate effects of financial frictions of firms.

An exploration of the interaction between the tax shield and financial friction shows that on one hand, the tax-induced debt bias can reduce the negative impacts of financial frictions for credit-constrained firms by incentivizing them to borrow and invest. On the other hand,

the resulting over-borrowing from a larger increase in the tax shield and reduced financial frictions may distort resource allocation and hence drag down aggregate productivity. Therefore, the effects of financial frictions on aggregate productivity is ambiguous, depending on the interaction with the interest tax deductibility.

Overall, these findings highlight the importance of debt bias in the tax code on misallocation and the interplay between fiscal and monetary policies. Understanding the interaction between the tax benefit of debt financing and financial frictions is critical in evaluating the aggregate costs of financial frictions and pursuing long-term economic growth.

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Table 1: Externally Calibrated Parameter Values

Parameter	Description	Value	Source
Technology			
α	Capital share	0.25	Typical in literature
ν	Labor share	0.6	Typical in literature
δ	Capital depreciation rate	0.1	Bloom et al. (2018)
τ	Corporate tax rate on profits	0.2	Gomes and Schmid (2010)
τ^S	Corporate tax rate on interest	0.2	Gomes and Schmid (2010)
π_d	Exit rate	0.1	Khan and Thomas (2013)
κ_0	Fraction of the steady-state aggregate capital stock held by each entrant	0.2	Jeenas (2019)
Financial Frictions			
ψ_1	Fixed investment adjustment cost	0	Catherine et al. (2021)
η_0	Fixed external financing/equity issuance cost	0	Catherine et al. (2021)
Preference			
φ	Labor disutility	2	Bloom et al. (2018)
β^H	Subjective discount factor	0.96	Jo and Senga (2019)
Price			
r	Risk-free interest rate	0.04	Jo and Senga (2019)

Note: The table reports the notation, description, value, and source for the set of externally calibrated parameters.

Table 2: Model Estimation Results

Panel A. Estimated Parameters

Parameter	Description	Model	SE
ρ	Productivity persistence	0.872	(0.0020)
σ	SD of innovations to productivity	0.109	(0.0005)
ψ_0	Convex investment adjustment cost	0.056	(0.0018)
η_1	Linear equity issuance cost	0.036	(0.0001)
s	Frac. of debt that can be collateralized	0.147	(0.0182)

Panel B. Model Fit: Targeted Moments

Moment	Description	Model	Data
$\sigma(b/k)$	debt rate volatility	0.365	0.32
$\sigma(i/k)$	investment rate volatility	0.478	0.53
$\sigma(\Delta y_{-1})$	1-year sales growth rate volatility	0.374	0.35
$\sigma(\Delta y_{-5})$	5-year sales growth volatility	0.938	0.8
$\mu(e/k)$	average net equity issuance rate	0.100	0.1
β	slope of i/k wrt debt/EBITDA	-0.998	-1

Panel C. Model Fit: Non-Targeted Moments

Moment	Description	Model	Data
$\mu(b/k)$	mean leverage	-0.051	0.1
$\mu(i/k)$	mean investment rate	0.187	0.40
$\mu(k/y)$	mean assets/sales	1.671	1.76
$\text{corr}(i/k, i/k_{-1})$	autocorrelation of investment rate	0.281	0.32
$\sigma(e/k)$	net equity issuance rate volatility	0.243	0.45
$\sigma(\log \text{MPK})$	dispersion in $\log(\text{sales/capital})$	0.347	1.128

Note: Panel A of the table reports point estimates and standard errors (in parentheses) for each of the parameters estimated via the SMM. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is the inverse of the moment covariance matrix. Panel B and C report model-implied moments and data moments. The empirical moments are computed from a panel of U.S. firms in Compustat annual data from 1981-2016.

Table 3: Aggregate Effects of Financial Frictions

	Distance to $\eta_1 = 0, s = 1$	
	(1) Baseline	(2) No Tax Shield
$\Delta\%$ Capital	9.88	0.96
$\Delta\%$ Labor	0.50	-0.33
$\Delta\%$ Output	2.96	0.24
$\Delta\%$ TFP	0.20	0.20
$\Delta\%$ Welfare	2.51	0.46
Δ TFP Loss	0.56%	0.33%

Note: The table compares various aggregate quantities across the estimated Baseline economy with tax shields ($\tau^S = 0.2$) and the Counterfactual No Tax Shield economy ($\tau^S = 0$) relative to their frictionless Benchmark B1 when equity is free ($\eta_1 = 0$) and firms can pledge all value of the capital stock as collateral ($s = 1$), respectively. The frictionless benchmark in Column (1) has tax shields, whereas the frictionless benchmark in Column (2) has No Tax Shield either. The aggregate quantities are computed from the stationary distributions μ of the respective economies. $\Delta\%$ Welfare represents the percentage consumption equivalent variation.

Δ TFP Loss represents the difference in the TFP Loss between the frictional and the frictionless benchmark models for both Baseline and Counterfactual economies. I first compute the TFP loss for the frictionless benchmarks (relative to the first-best TFP). Then I compute the TFP loss for the Baseline (Counterfactual) economy (relative to the first-best TFP). Last, I take the difference of values from step 1 and step 2.

Table 4: Relative TFP Loss

(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	(4) Baseline
A. With tax shield 7.659%	7.465%	7.493%	8.055%
B. Without tax shield 7.730%	7.828%	7.730%	8.064%

Note: The table reports TFP loss for the Baseline economy in Column (4) and three counterfactual economies with fewer friction from Column (1) to Column (3). Column (1) represents an economy with free equity ($\eta_1 = 0$). Column (2) represents an economy in that firms can pledge all value of the capital stock. Column (3) is the frictionless Benchmark B1 as in Table 3. Panel A reports values for models with the tax shield. Panel B reports values for models without the tax shield.

Table 5: Decomposing Aggregate Effects of Financial Frictions

	(1)	(2)	(3)	(4)
	Total	Financial frictions alone	Tax shield alone	Interaction
		$\eta_1 = 0$	$\tau^S = 0$	(1)-(2)-(3)
$\Delta\%$ Capital	1.73	4.97	0.77	-4.01
$\Delta\%$ Labor	1.29	2.52	1.62	-2.85
$\Delta\%$ Output	1.41	2.95	1.17	-2.71
$\Delta\%$ TFP	0.20	0.20	0.01	0.00
$\Delta\%$ Welfare	0.67	0.04	0.46	0.17

Note: The table reports aggregate effects of financial frictions and decomposition relative to the constrained efficiency Benchmark B2 with no free equity and no tax shield $\eta_1 = 0, \tau^S = 0$. Column (1) reports the percentage changes in aggregate variables of the Baseline economy relative to B2.

Column (2) reports the contribution of the financial friction alone, measured by the difference of aggregate variables between the Baseline economy and the economy with free equity ($\eta_1 = 0$).

Column (3) reports the contribution of the tax shield alone, measured by the difference of aggregate variables between the Baseline economy and the economy with no tax shield ($\tau^S = 0$).

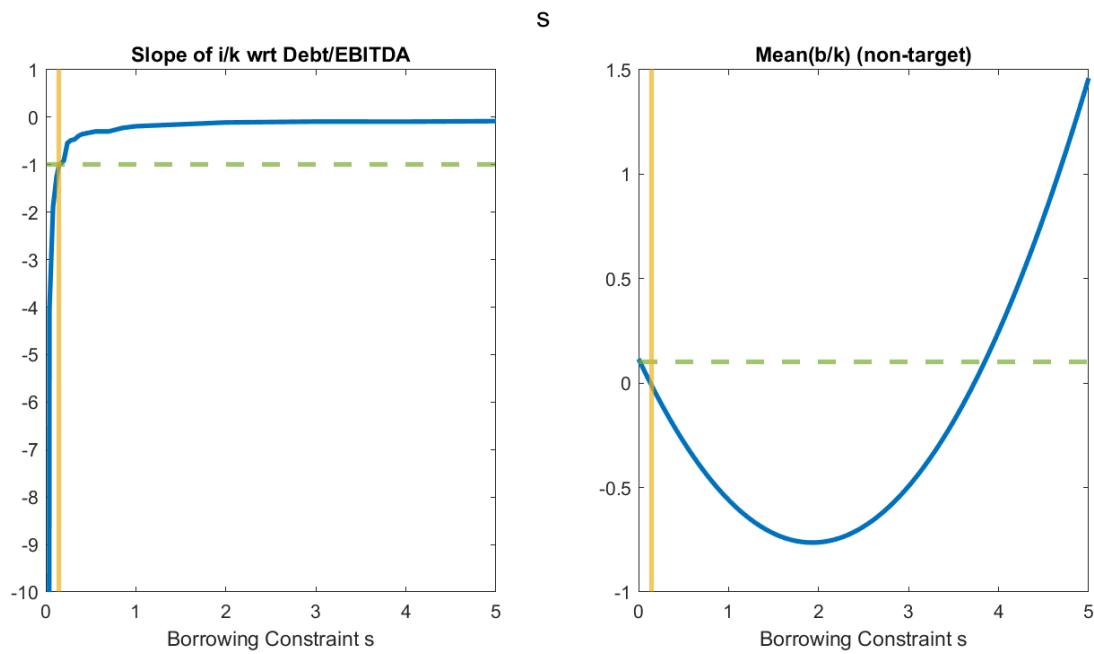
Column (4) reports the contribution of the interaction term between the tax shield and financial frictions, measured by the reminder of the difference between the Baseline economy and Benchmark B2, after subtracting the first two components.

Table 6: Robustness to Alternative Parameters

	Source	K	B	N	Y	TFP	U
Benchmark		0.775	0.094	0.309	0.571	1.231	-1.213
Low capital revenue elasticity	$\alpha = .21$	0.602	0.071	0.308	0.528	1.189	-1.283
High capital revenue elasticity	$\alpha = .28$	0.952	0.113	0.317	0.633	1.279	-1.133
Low labor revenue elasticity	$\gamma = .5$	0.754	0.105	0.258	0.541	1.143	-1.162
High labor revenue elasticity	$\gamma = .64$	0.814	0.081	0.337	0.617	1.304	-1.194
Low depreciation	$\delta = .06$	1.128	0.137	0.292	0.605	1.229	-1.058
High depreciation	$\delta = .15$	0.538	0.044	0.325	0.538	1.234	-1.349
Low initial capital	$\kappa_0 = .1$	0.772	0.093	0.306	0.568	1.231	-1.200
High initial capital	$\kappa_0 = .23$	0.784	0.095	0.312	0.576	1.232	-1.215
Firm exit rate	$\pi_d = .08$	0.794	0.096	0.318	0.584	1.232	-1.231
Low risk-free interest rate	$r = .03$	0.853	0.104	0.313	0.589	1.232	-1.192
High risk-free interest rate	$r = .06$	0.657	0.076	0.310	0.550	1.233	-1.250
Low labor disutility	$\varphi = 1.28$	1.088	0.134	0.486	0.816	1.232	-0.860
High labor disutility	$\varphi = 2.48$	0.649	0.073	0.250	0.481	1.231	-1.387
Catherine et al. (2021)		1.108	0.203	0.301	0.638	1.279	-1.053

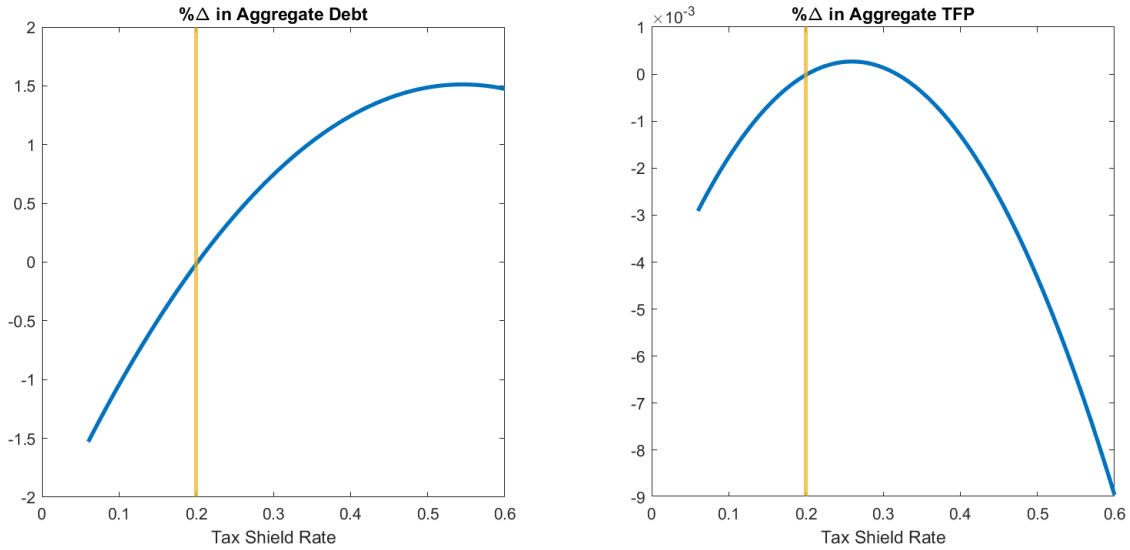
Note: The table reports aggregate capital, debt, employment, output, consumption, productivity, and utility. Different rows report these values for robustness checks varying the value of the indicated parameter, holding all other parameters fixed at their benchmark values from Table 1 and Table 2. The last row set parameter values consistent with Catherine et al. (2021) where $\rho = 0.851, \sigma = 0.131, \psi_0 = 0.004, \eta_1 = 0.091, s = 0.25, \delta = 0.06, r = 0.03, \pi_d = 0.08, \tau = 0.33$.

Figure 1: Identification of Borrowing Constraint Parameter s



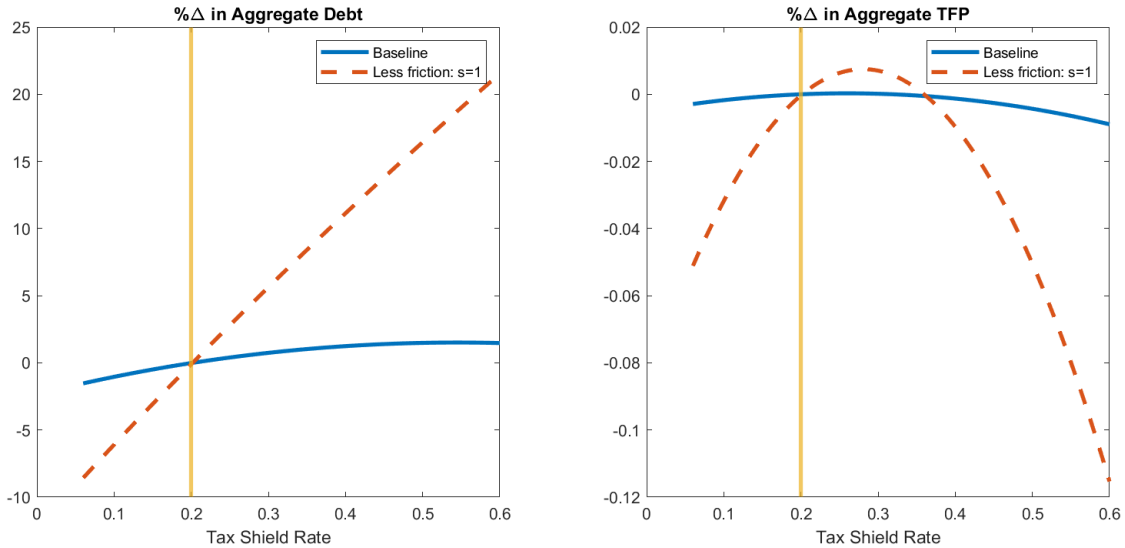
Note: The figure plots the sensitivity of moments slope of investment with respect to leverage (Left Panel) and mean leverage (Right Panel) to the borrowing constraint parameter s respectively. Yellow vertical line. The yellow vertical line corresponds to the SMM estimate of s . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.

Figure 2: Aggregate Impact of the Tax Shield Rate: Baseline Economy



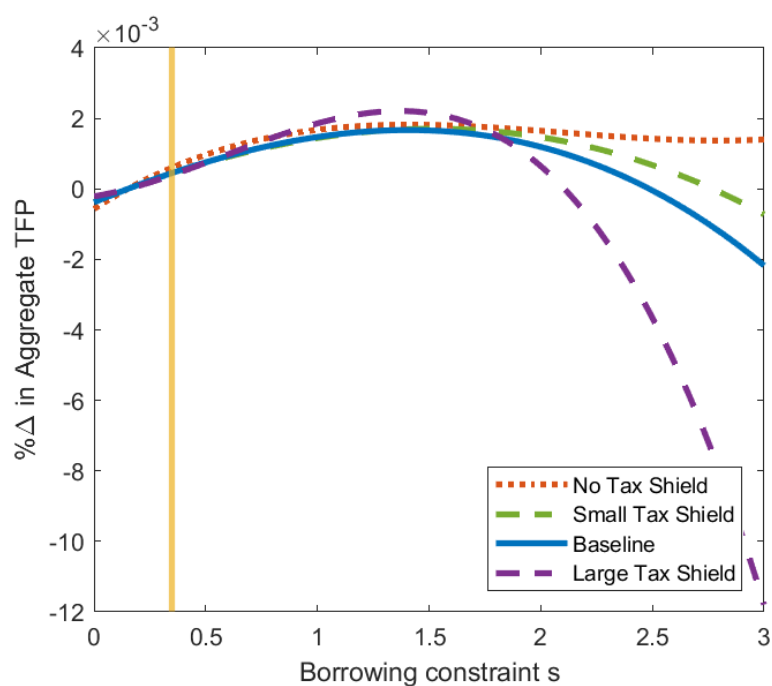
Note: The figure plots the percentage change in aggregate debt and productivity with different tax shield rates τ^S for the Baseline economy. The yellow vertical line corresponds to the Baseline tax shield rate $\tau^S = \tau = 0.2$. Numerical comparative statics are smoothed using a polynomial approximation.

Figure 3: Aggregate Impact of the Tax Shield Rate at Different Borrowing Friction



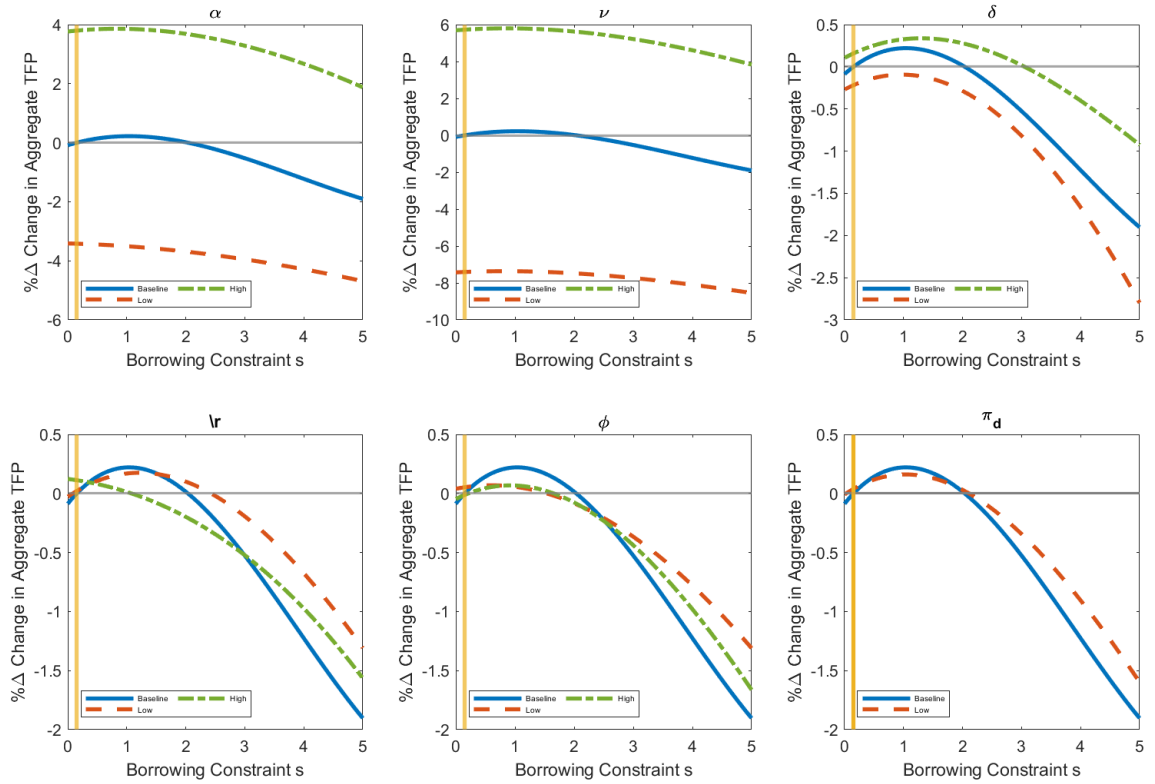
Note: The figure plots the percentage change in aggregate debt and productivity with different tax shield rates τ^S for the Baseline economy (blue solid line) and counterfactual economy with $s = 1$ (red dotted line). The yellow vertical line corresponds to the Baseline tax shield rate $\tau^S = \tau = 0.2$. Numerical comparative statics are smoothed using a polynomial approximation.

Figure 4: Impact of the Borrowing Constraint on Aggregate productivity at Different Tax Shield



Note: The figure plots the percentage change in aggregate productivity with different borrowing constraints s for the Baseline economy (blue solid line) and counterfactual economies with different tax shield rates τ^S . The yellow vertical line corresponds to the SMM estimate of borrowing constraint s . Numerical comparative statics are smoothed using a polynomial approximation.

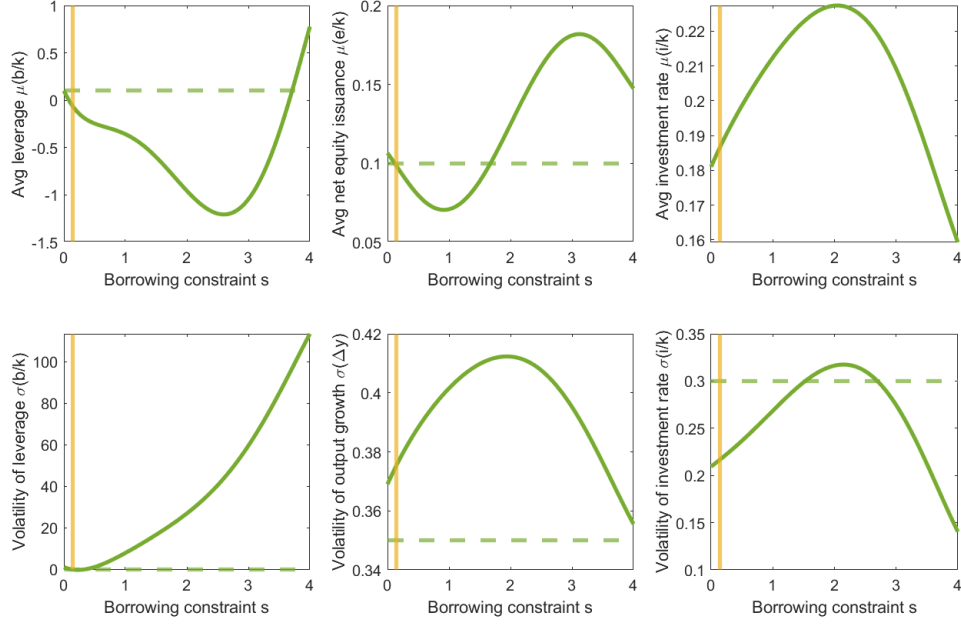
Figure 5: Aggregate TFP of the Borrowing Constraint s for Alternative Parameter Values



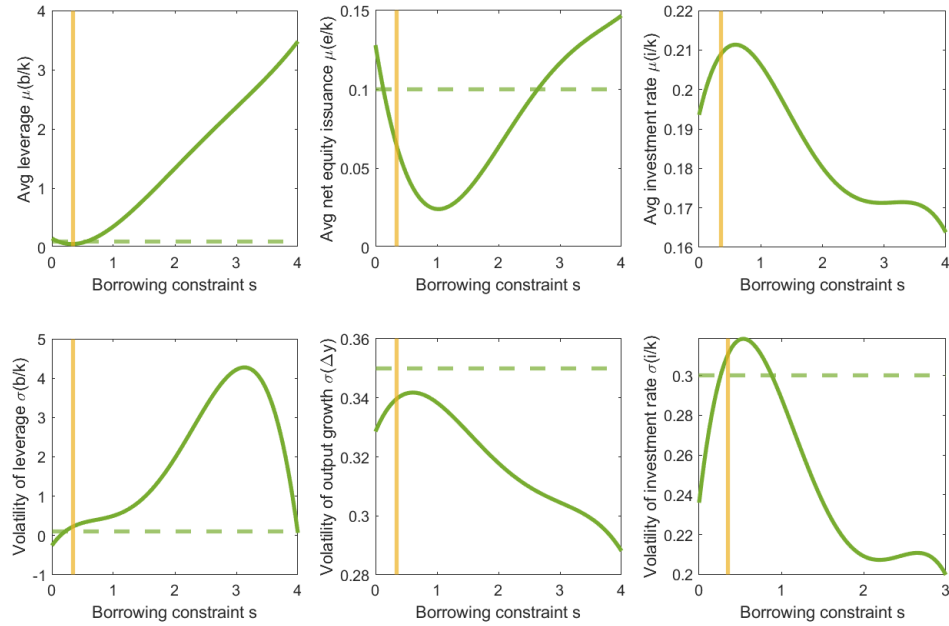
Note: This figure plots the percentage change in aggregate productivity as a function of the borrowing constraint s for different alternative parameter values. The blue line refers to my Baseline economy with baseline parameter values. The red dashed line reflects a lower parameter value. And the green dash-dotted line reflects a higher parameter value. I consider 6 externally-calibrated parameters: capital share α , labor share ν , capital depreciation rate δ , risk-free rate r , labor disutility φ , and exit rate π_d . Numerical comparative statics are smoothed using a polynomial approximation.

Figure 6: Effect of Borrowing Constraints s on Firm Characteristics

(a) Model 1: targeting β

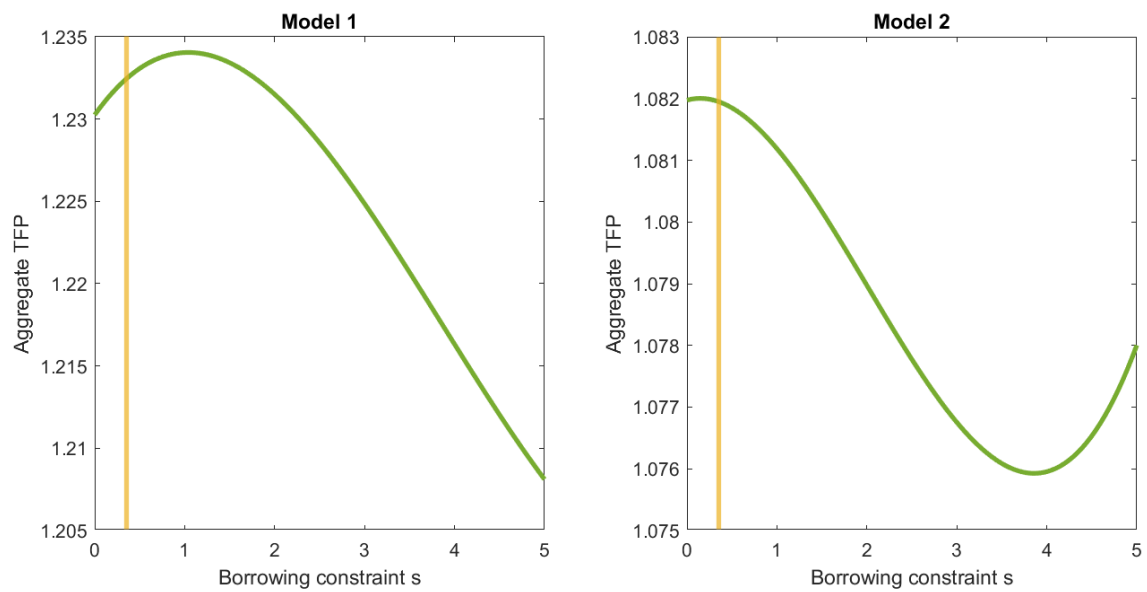


(b) Model 2: targeting $\mu(b/k)$



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

Figure 7: Effect of Borrowing Constraints s on Aggregate TFP



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

A Data

I obtained data on U.S. nonfinancial firms from the Standard and Poor’s CRSP/Compustat industrial fundamental annual files through Wharton Research Data Services (WRDS) in 2021. The data is an unbalanced panel that covers from 1980 to 2018.

A.1 Variable Definition

The variable definition and construction follow standard practices in the literature. Table D.1 lists model notation, definition and Compustat data item for each variable.

I use the beginning-of-the-period capital (PPENT) as the firms’ capital stocks. Investment $i = k' - (1 - \delta)k$ is defined as capital expenditures on property, plant, and equipment (CAPXV). Investment rate i/k

Debt b in my model is the net debt. The empirical counterpart is the sum of debt in current liabilities (DLC) and long-term debt (DLTT) minus cash (CHE). I consider two definitions of the leverage ratio. The first is defined as the ratio of net debt to lagged assets, $(DLC+DLTT-CHE)/L.A.T$. This definition is used for a targeted moment of debt rate volatility $\sigma(b/k)$ and a non-targeted moment of mean leverage ratio $\mu(b/k)$. The second is the debt-to-EBITDA ratio, $(DLC+DLTT-CHE)/EBITDA$, which is used to compute the slope of investment with respect to pre-existing debt. I follow [Crouzet and Tourre \(2020\)](#) to compute the slope of investment with respect to the debt-to-EBITDA ratio as follows:

$$\beta = \frac{\text{Cov}(i/k, b/EBITDA)}{\text{Var}(b/EBITDA)} \times 100$$

where $b/EBITDA$ is the beginning-of-the-period value and i/k is the current-period value. The slope is computed with non-negative EBITDA.

To compute the net equity issuance rate, I first compute the firm-level net equity issuance e as the stock sales (SSTK) minus cash dividends (DV) and share buybacks (PRSTKC). Then I take the maximum of e and zero and normalize it by total assets (AT). The construction is similar to [Catherine et al. \(2021\)](#), which normalizes equity issuance by value-added. They approximate value added by 60% of total sales, assuming a 40% gross margin ratio.

I measure firm revenue y using sales (SALE). I construct sales growth rate using the “DHS growth rates” defined following [Davis et al. \(1996\)](#): $\Delta y_{-t} = (y - y_{-t})/(0.5y + 0.5y_{-t})$. This measure bounds growth rates between -2 and $+2$, addressing any concerns over outliers. The empirical equivalent of model variable capital-to-sales ratio k/y is assets divided by sales.

I use the BEA nonresidential fixed investment implicit price deflator (from FRED) to deflate capital stock and investment. I use the gross GDP deflator to deflate other variables.

MPK dispersion calculation

I measure the firm marginal product of capital in logs (up to an additive constant) as the difference between log revenue and capital, $mpk = y - k$. Then I take the standard deviation of the value. The original empirical result is 1.453. Since the focus in the misallocation literature is generally on within-industry variation in the MPK, I compute the within-industry results, defined at the 4-digit SIC level. Following [David et al. \(2022\)](#), I obtain the residualized value of mpk from a regression with industry-by-year fixed effects and then compute the standard deviation. The adjusted result is 1.128. This composition-adjusted measure of the cross-sectional dispersion can ensure that the variation is due to changes in firm MPK, rather than additions or deletions from the dataset.

A.2 Sample Selection

I apply the following sample selection criteria:

1. Drop firm-year observation that is incorporated in the United States ($FIC = \text{"USA"}$)
2. Drop firm-year observations if two-digit SIC code (SIC) is in the financial industry (SIC code between 6000 and 6999), utilities (SIC code between 4900 and 4999), or public administration (SIC code between 9000 and 9999)
3. Keep observations for fiscal year (FYEAR) between 1980 and 2018
4. Drop firm-year observations with missing assets (AT), sales (SALE), cash holdings (CHE), long-term debt (DLTT), short-term debt (DLC), capital expenditure (CAPXV), earnings before interest (EBITDA), and capital stock (PPENT)
5. Drop firm-year observations with non-positive SALE or AT
6. Drop firms that are in the data for smaller than 5 years

I obtain a sample of 157,683 firm-year observations and 11,148 firms. I trim all moments at the top 99% and bottom 1%.

Comparison of key data moments to literature

The volatility of the leverage ratio (i.e., net debt-to-lagged asset) is 0.32. This is in the range of the moments in the literature. [DeAngelo et al. \(2011\)](#) uses gross debt, which reports that the standard deviation of gross leverage is 0.1086 (variance of leverage is 0.0118). [Karabarbounis and Macnamara \(2021\)](#) report a standard deviation of the gross leverage ratio of 0.42.

The investment rate volatility is 0.53, a little higher than existing estimates. Using PPEGT, the gross value property, plant, and equipment, as capital stock, [DeAngelo et al. \(2011\)](#) report that the standard deviation of investment rate is 0.1962 (variance of investment rate is 0.0385). [Karabarbounis and Macnamara \(2021\)](#) also use PPEGT and report that the standard deviation of investment rate is 0.22 and [Ottonello and Winberry \(2020\)](#) report a value of 0.33. Weighting and sample selection might explain why my data moment is more elevated.

My data moment of short-run sales growth rate volatility is 0.35 and the long-run volatility is 0.8. The estimates are similar to [Catherine et al. \(2021\)](#), which document that the volatility of 1-year and 5-year sales growth rates is 0.327 and 0.912 respectively.

My sample's average net equity issuance rate is 0.1, which is broadly consistent with the literature. [Hennessy and Whited \(2005\)](#) and [Hennessy and Whited \(2007\)](#) report a gross equity issuance rate of 4.2% and 8.9%. [Belo et al. \(2019\)](#) report that the value is 0.04. [Catherine et al. \(2021\)](#) normalize the net equity issuance by value-added and the net equity issuance rate is 0.026.

The average leverage ratio, defined as net debt to lagged assets, in my sample is 0.1. My value is similar to 0.098 in [Catherine et al. \(2021\)](#). [Crouzet and Tourre \(2020\)](#) document the ratio is 25.58%. In [Hennessy and Whited \(2007\)](#), the leverage ratio is 12.04% for their baseline estimation and 14.52% in their restricted large firm sample. [Karabarbounis and Macnamara \(2021\)](#) report the mean leverage ratio of 0.29. [Belo et al. \(2019\)](#) report a value of 0.25.

The slope of investment with respect to the ratio of debt to EBITDA in my sample is -1. My data estimate is close to -1.04 in [Crouzet and Tourre \(2020\)](#).

The non-targeted moment of the average investment rate is 0.4 in my sample. Using PPENT, my estimate is somewhat larger than previous research. For example, the average investment is 0.1868 in [DeAngelo et al. \(2011\)](#), 0.16 in [Karabarbounis and Macnamara \(2021\)](#), 0.11 in [Crouzet and Tourre \(2020\)](#), and 0.07 in [Eisfeldt and Muir \(2016\)](#). [Hennessy and Whited \(2005\)](#) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample.

In my sample, the autocorrelation of investment rate is 0.34. The value is in the range of 0.165 in [Catherine et al. \(2021\)](#), 0.17 in [Karabarbounis and Macnamara \(2021\)](#), 0.40 in [Eisfeldt and Muir \(2016\)](#), and 0.41 in [Belo et al. \(2019\)](#).

Finally, my estimate of the standard deviation of the net equity issuance rate is 0.35. My estimate is close to the moment in [Hennessy and Whited \(2007\)](#), 0.3018.

B Model

In this section, I describe the structural estimation procedure. First, for a given set of parameters, I solve the model numerically by iterating on the firm's Bellman equation, which produces the value function $V(k, b, z)$ and the policy function (k', b') . Then, I simulate the economy and search for parameters that model-generated moments could match data moments.

B.1 Numerical Solution Method

I use policy iteration to solve for the firm's problem by iterating on the Bellman equation defined in Eq. (13) until convergence.

Grid definition

I transform (32) into a discrete-state Markov chain using the method in [Tauchen \(1986\)](#). I let productivity z (in logs) have 5 points of support on the interval $[\log(\underline{z}), \log(\bar{z})] = [-3\sigma, 3\sigma]$. I let capital stock k have 100 equally-spaced (in logs) on the interval $[\log(\underline{k}), \log(\bar{k})] = [0.001, 100]$.

Since debt b is bounded above by the borrowing constraints, I set the maximal value of b with the maximal value of k , \bar{k} . Therefore $\bar{b} = (1 - \tau)(\underline{z}\bar{k}^{\alpha}\underline{n}^{\nu} - W\underline{n}') + s(1 - \delta)\bar{k}'$. The minimum of b is chosen so that the optimal choice of debt never hits the lower endpoint. I verify ex-post and set $\underline{b} = -0.01 \times \bar{b}$. I let debt b have 40 geometrically-spaced points in the interval $[\underline{b}, \bar{b}]$.

The state space for the firm's problem is $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$.

Policy function iteration

I compute the return matrix $R(k', b', k, b, z)$ for all possible values of (k, b, z) :

$$R(k', b', k, b, z) = \pi_d e_0(k, b, z) + (1 - \pi_d) [e_1(k', b', k, b, z) + \eta(e_1(k', b', k, b, z))]$$

I set $R(\cdot)$ to “missing” when (k, b, z) are such that the borrowing constraint is violated. Given a value function $V(k, b, z)$, the policy function $(k', b') = P(k, b, z)$ solves:

$$P^*(k, b, z) = \arg \max_P \left\{ R(P(k, b, z), k, b, z) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [V(P(k, b, z), z') | z] \right\}, \forall (k, b, z)$$

I assume that firms can only choose values of (k', b') on a discrete grid, where $k' \in \mathcal{K} = \{k_1, \dots, k_{N_k}\}$ and $b' \in \mathcal{B} = \{b_1, \dots, b_{N_b}\}$. (N_k, N_b) are the number of grid points for capital and debt, respectively, and N_z is the number of grid points for productivity. Therefore the number

of states is $N_k \times N_b \times N_z = 100 \times 40 \times 5 = 20,000$. The number of choices is $N_k \times N_b = 100 \times 40 = 4,000$.

I initiate with the process with a guess V_0, P_0 and specify a solution tolerance $\varepsilon_{\text{tolerance}} > 0$. To speed up the computation, I apply the Howard improvement algorithm and iterate the policy function instead of the value function iteration.

To solve for the steady state equilibrium given a set of parameters, the algorithm proceeds as follows:

1. **Outer Loop:** Suppose the real wage is in a range of $[W_a, W_c]$. Guess the value of the real wage $W_b = (W_a + W_c)/2$ using the bisection algorithm
2. Solve the firm's problem $V(k, b, z)$ and compute the stationary distribution $\Gamma(k, b, z)$ with firm policies $(k', b') = P(k, b, z)$, for $n = 1, 2, \dots$, given real wage W_b

- (a) **Inner Loop:** Starting from the policy function $(k'_{n-1}, b'_{n-1}) = P_{n-1}(k, b, z)$ and value function $V_{n-1}(k, b, z)$ from the previous round, solve for the optimal policy P_n :

$$P_n(k, b, z) = \arg \max_p \left\{ R(P(k, b), k, b, z) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [V_{n-1}(P(k, b), z') | z] \right\}$$

- (b) Set $\tilde{V}_{n-1}^1 = V_{n-1}$. For each Howard improvement step $h = 1, \dots, H - 1$, iterate the Bellman equation without optimization:

$$\tilde{V}_{n-1}^{(h+1)}(k, b, z) = \left\{ R(P_n(k, b, z), k, b) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [\tilde{V}_{n-1}^{(h)}(P_n(k, b, z), z') | z] \right\}$$

- (c) Set $V_n = \tilde{V}_{n-1}^{(H)}$

- (d) Compute the error $\|P_n - P_{n-1}\| = \max_{k, b, z} |P_n(k, b, z) - P_{n-1}(k, b, z)|$.

- If $\|P_n - P_{n-1}\| < \varepsilon_{\text{tolerance}}$, exit.
- If $\|P_n - P_{n-1}\| \geq \varepsilon_{\text{tolerance}}$, go back to Step 2(a) with $n = n + 1$

3. Calculate the implied value of aggregate consumption $C(W_b)$
4. If W and φC are within some tolerance of each other, $|W_b - \varphi C(W_b)| < \varepsilon_{\text{tol}}$, then I solve the model and set $W^* = W_b$. If not, then update my guess for W as follows and return to Step 1:
 - If $W_b < \varphi C(W_b)$, then wage is underestimated. I set $W_a = W_b$
 - If $W_b > \varphi C(W_b)$, then wage is overestimated. I set $W_c = W_b$

The contracting mapping theorem guarantees that there is a fixed point where the policy function converges under some regularity conditions. I set the step $H = 10$ for the Howard improvement algorithm.

With $N_m \geq N_p$, the model is over-identified. Then the test of the overidentifying restrictions of the model is:

$$J = \frac{NS}{1+S} \min_{\Theta} \hat{g}'_N \hat{W}_N \hat{g}_N \sim \chi^2(N_m - N_p)$$

Simulation and model-generated moments

Once we have solved the model for a given set of parameters, I simulate data in order to compute the simulated moments.

I simulate a balanced panel of 5,000 firms over 5,500 years, and only keep the last 50 years to ensure each firm has reached the steady-state. For each firm, I take a random draw from the distribution of productivity z and simulate a path of log.

B.2 Structural Estimation Method

I use the simulated method of moments (SMM) to estimate a vector of unknown structural parameters, $\Theta^* = (\rho, \sigma, \psi_0, \eta_1, s)$. This procedure chooses parameters to minimize the distance between model-generated moments and the corresponding data moments.

SMM Estimation

Let M be the actual data moments and $m^s(\Theta)$ is a vector of moments computed from the s th simulated sample using parameters Θ , where $s = 1, \dots, S$. S is the number of simulations. N is the number of observations in actual data. The number of targeted moments $N_m = 6$. The number of parameters of interest $N_p = 5$.

The SMM estimator of Θ^* solves:

$$\begin{aligned} \hat{\Theta} &= \arg \min_{\Theta} \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S m^s(\Theta) \right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S m^s(\Theta) \right] \\ &= \arg \min_{\Theta} \hat{g}'_N \hat{W}_N \hat{g}_N \end{aligned}$$

where \hat{W}_N is an $N_m \times N_m$ arbitrary positive definite matrix that converges in probability to a deterministic positive definite matrix W .

The simulated moment estimator is asymptotically normal for fixed S . The asymptotic

distribution of Θ is given by:

$$\sqrt{N}(\hat{\Theta} - \Theta^*) \xrightarrow{d} \mathcal{N}(0, \text{avar}(\hat{\Theta}))$$

Let

$$G = \frac{\partial m(\Theta)}{\partial \Theta}, \text{ the } N_m \times N_p \text{ gradient matrix where } G_{ij} = \frac{\partial m_i(\Theta)}{\partial \Theta_j}$$

$$\Omega = \lim_{N \rightarrow \infty} \text{Var}(\sqrt{N}\hat{M}_N), \text{ the } N_m \times N_m \text{ asymptotic variance-covariance matrix of the data moments}$$

Then

$$\begin{aligned} \text{avar}(\hat{\Theta}) &= \left(1 + \frac{1}{S}\right) (G'WG)^{-1} G'W\Omega WG (G'WG)^{-1} \\ &= \left(1 + \frac{1}{S}\right) \left[\frac{\partial \hat{m}_n(\Theta)'}{\partial \Theta} W \frac{\partial \hat{m}_n(\Theta)}{\partial \Theta} \right]^{-1} \end{aligned}$$

The optimal weighting matrix is equal to the inverse of a covariance matrix that is calculated using the influence function approach of Erickson and Whited (2002): $W = \Omega^{-1}$

$$\text{avar}(\hat{\Theta}) = \left(1 + \frac{1}{S}\right) (G'WG)^{-1}$$

The weighting matrix W is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrapping with replacement on the actual data. The estimates of variance-covariance matrix is qualitatively similar to the ones computed from the Delta Method.

C Quantitative Analysis

C.1 Welfare Change Formula

The total consumption equivalent welfare gains from the removal of financial frictions, i.e. moving from the Benchmark B1 ($\eta_1 = 0, s = 1$) to the Baseline economy, can be written as where Δ satisfies the following equation:

$$\sum_{t=0}^{\infty} (\beta^H)^t [\log(C_{t,Baseline}(1 + \Delta)) - \varphi N_{t,Baseline}] = \sum_{t=0}^{\infty} (\beta^H)^t [\log(C_{t,B1}) - \varphi N_{t,B1}]$$

This yields the following formula:

$$\begin{aligned} \sum_{t=0}^{\infty} (\beta^H)^t \log(1 + \Delta) + \sum_{t=0}^{\infty} (\beta^H)^t [\log(C_{Baseline}^*) - \varphi N_{Baseline}^*] &= \sum_{t=0}^{\infty} (\beta^H)^t [\log(C_{B1}^*) - \varphi N_{B1}^*] \\ \sum_{t=0}^{\infty} (\beta^H)^t \log(1 + \Delta) + \sum_{t=0}^{\infty} (\beta^H)^t U_{Baseline}^* &= \sum_{t=0}^{\infty} (\beta^H)^t U_{B1}^* \\ \log(1 + \Delta) &= U_{B1}^* - U_{Baseline}^* \\ \Delta &= (e^{U_{B1}^* - U_{Baseline}^*}) - 1 \end{aligned}$$

where $*$ denotes the value at the stationary distributions of the respective economies. $\Delta \times 100$ represents the percentage consumption equivalent variation of the Benchmark B1 relative to the Baseline economy. U is the stationary utility level of the household.

C.2 Efficient Allocation and TFP Loss

Firms choose capital and labor optimally where the marginal product of capital and labor are equal to their respective costs. Since labor is static, the marginal product of labor is equal to wage W .

$$\begin{aligned} MPK_i &= \alpha \frac{y_i}{k_i} = X_i \\ MPL_i &= \nu \frac{y_i}{n_i} = W \end{aligned}$$

Then the optimal capital-labor ratio is given by

$$\frac{k_i}{n_i} = \frac{\alpha}{\nu} \frac{W}{X_i}$$

Solving for the labor input yields

$$n_i = \underbrace{z_i^{\frac{1}{1-(\alpha+\nu)}} X_i^{-\frac{\alpha}{1-(\alpha+\nu)}}}_{w_i^N} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{1-\alpha}{1-(\alpha+\nu)}} \alpha^{\frac{\alpha}{1-(\alpha+\nu)}}}_{c_N}$$

Then the optimal capital input is

$$k_i = \underbrace{z_i^{\frac{1}{1-(\alpha+\nu)}} X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}}}_{w_i^K} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{\nu}{1-(\alpha+\nu)}} \alpha^{\frac{1-\nu}{1-(\alpha+\nu)}}}_{c_K}$$

where w_i^N and w_i^K denote labor and capital wedges relative to an efficient allocation of inputs. As discussed above, MPL_i and w_i^N are the same across firms. At the efficient allocation, the marginal product of capital is also equated across firms.

Aggregate labor and capital can be expressed as

$$\begin{aligned} N &= \int n_i di = c_N \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di \\ K &= \int k_i di = c_K \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di \end{aligned}$$

The aggregate output is

$$Y = \int y_i di = (c_K^\alpha c_N^\nu) \int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di$$

Then the aggregate productivity is given by

$$\text{TFP} = \frac{Y}{K^\alpha N^\nu} = \frac{\int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di}{\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di \right)^\alpha \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di \right)^\nu} \quad (11)$$

Expressing Equation (11) in logs yields

$$\log(\text{TFP}) = \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di\right) - \alpha \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di\right) - \nu \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di\right) \quad (12)$$

Define $A_i = z_i^{\frac{1}{1-(\alpha+\nu)}}$. Assume that (A_i, w_i^K, w_i^N) are jointly log-normal distributed

$$\begin{bmatrix} \log(A_i) \\ \log(w_i^K) \\ \log(w_i^N) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_a \\ \mu_K \\ \mu_N \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{a,K} & \sigma_{a,N} \\ \sigma_{a,K} & \sigma_K^2 & \sigma_{K,N} \\ \sigma_{a,N} & \sigma_{K,N} & \sigma_N^2 \end{bmatrix}\right)$$

The second-order approximations of Equation (12) are given by

$$\begin{aligned} \log\left(\int A_i (w_i^K)^\alpha (w_i^N)^\nu di\right) &= \mu_a + (\alpha\mu_K + \nu\mu_N) + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha^2\sigma_K^2 + \frac{1}{2}\nu^2\sigma_N^2 + \alpha\nu\sigma_{K,N} + \alpha\sigma_{a,K} + \nu\sigma_{a,N} \\ \log\left(\int A_i w_i^K di\right) &= \mu_a + \mu_K + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\sigma_K^2 + \sigma_{a,K} \\ \log\left(\int A_i w_i^N di\right) &= \mu_a + \mu_N + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\sigma_N^2 + \sigma_{a,N} \end{aligned}$$

Rearrange the above expressions. Then Equation (12) is given by

$$\log(\text{TFP}) = (1 - \alpha - \nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) - \frac{1}{2}\alpha(1 - \alpha)\sigma_K^2 - \frac{1}{2}\nu(1 - \nu)\sigma_N^2 + \alpha\nu\sigma_{K,N}$$

Given w_i^N is equalized across firms, then $\sigma_N^2 = 0$ and $\sigma_{K,N} = 0$. Therefore,

$$\log(\text{TFP}) = (1 - \alpha - \nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) - \frac{1}{2}\alpha(1 - \alpha)\sigma_K^2$$

The efficient allocation implies that $\sigma_K^2 = 0$. Then I can approximate the first-best TFP as

$$\log(\text{TFP}^{\text{FB}}) = (1 - \alpha - \nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) \quad (13)$$

The TFP loss is defined to be

$$\text{Relative TFP Loss} = \log\left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}}\right) = \frac{1}{2}\alpha(1 - \alpha)\sigma_K^2 \quad (14)$$

To solve for σ_K^2 . Recall that

$$\text{MPK}_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = w_i^K$$

Then MPK is also log-normally distributed

$$\begin{aligned} -\frac{1-\nu}{1-(\alpha+\nu)} \log(X_i) &= \log(w_i^K) \sim \mathcal{N}(\mu_K, \sigma_K^2) \\ \log(X_i) &= -\frac{1-(\alpha+\nu)}{1-\nu} \log(w_i^K) \sim \mathcal{N}\left(\mu_K \left[-\frac{1-(\alpha+\nu)}{1-\nu}\right], \sigma_K^2 \left[\frac{1-(\alpha+\nu)}{1-\nu}\right]^2\right) \end{aligned}$$

This solves σ_K^2

$$\begin{aligned} \text{Var}(\log(\text{MPK}_i)) &= \text{Var}(\log(X_i)) = \sigma_K^2 \left[\frac{1-(\alpha+\nu)}{1-\nu}\right]^2 \\ \sigma_K^2 &= \left[\frac{1-\nu}{1-(\alpha+\nu)}\right]^2 \text{Var}(\log(\text{MPK}_i)) \end{aligned}$$

Plug σ_K^2 into Equation (14) and TFP loss is given by

$$\log\left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}}\right) = \frac{1}{2}\alpha(1-\alpha) \left[\frac{1-\nu}{1-(\alpha+\nu)}\right]^2 \text{Var}(\log(\text{MPK}_i))$$

D Additional Tables

Table D.1: Variable Definitions

Variable	Notation	Definition	Compustat data item
Capital	k	Total net value of property, plant, and equipment	PPENT
Investment	$i = k' - (1 - \delta)k$	Capital expenditures on property, plant, and equipment	CAPXV
Debt	b	Net debt computed as the sum of short-term and long-term debt minus cash	DLC+DLTT-CHE
Sale	y	Sales	SALE
EBITDA	π	Earnings before interest, taxes, depreciation and amortization	EBITDA
Net equity issuance	e	Stock sales minus cash dividends and share buybacks	SSTK - PRSTKC - DV
Investment rate	i/k		CAPXV/L.PPENT
Leverage	-	See Appendix A.1	
Net equity issuance rate	e/k	See Appendix A.1	
Sales growth rate	Δy_{-t}	See Appendix A.1	
Capital-to-sales ratio	k/y		AT/SALE

Note: The table describes the empirical counterpart of model variables. It includes the model notation, definition, and data item from Compustat for each variable.

Table D.2: Aggregate Effects of Financial Frictions

	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	(4) Benchmark value
Panel A. Without Tax Shield				
K	0.7888	0.7916	0.7888	0.7812
B	-0.1067	0.2664	-0.6320	-0.0572
N	0.3132	0.3137	0.3132	0.3143
Y	0.5795	0.5804	0.5795	0.5781
TFP	1.2339	1.2337	1.2339	1.2315
$\Delta\%$ (K)	0.9605	1.3162	0.9605	
$\Delta\%$ (N)	-0.3315	-0.1656	-0.3315	
$\Delta\%$ (Y)	0.2384	0.4040	0.2384	
$\Delta\%$ (TFP)	0.1963	0.1736	0.1963	
Panel B. With Tax Shield				
K	0.8147	0.8516	0.8557	0.7752
B	0.1009	0.6929	0.6943	0.0939
N	0.3171	0.3099	0.3108	0.3092
Y	0.5885	0.5869	0.5886	0.5714
TFP	1.2339	1.2339	1.2338	1.2314
$\Delta\%$ (K)	1.5409	9.3966	9.8804	
$\Delta\%$ (N)	0.4018	0.2118	0.4982	
$\Delta\%$ (Y)	0.8369	2.6782	2.9646	
$\Delta\%$ (TFP)	0.2105	0.2020	0.1956	

Note: The table compares various aggregate quantities across the estimated Baseline economy with tax shields ($\tau^S = 0.2$) and the Counterfactual No Tax Shield economy ($\tau^S = 0$) relative to different frictionless benchmarks. The frictionless benchmark in Column (1) has free equity ($\eta_1 = 0$). Column (2) has no tax shields, whereas the frictionless benchmark in Column (3) has No Tax Shield either. The aggregate quantities are computed from the stationary distributions of the respective economies. $\Delta\%$ represents the percentage consumption equivalent variation. TFP Loss represents the difference in the TFP Loss between the frictional and the frictionless benchmark models for both Baseline and Counterfactual economies. I first compute the TFP loss for the frictionless benchmarks (relative to the first-best TFP). Then I compute the TFP loss for the Baseline (Counterfactual) economy (relative to the first-best TFP). Last, I take the difference of values from step 1 and step 2. 31

Table D.3: Model Estimation Results

Panel A. Estimated Parameters

Parameter	Description	Model 1	SE	Model 2	SE
ρ	Productivity persistence	0.872	(0.0020)	0.835	(0.0011)
σ	St. Dev. of innovations to productivity	0.109	(0.0005)	0.076	(0.0008)
ψ_0	Convex investment adjustment cost	0.056	(0.0018)	0.008	(0.0046)
η_1	Linear equity issuance cost	0.036	(0.0001)	0.008	(0.0000)
s	Frac. of debt that can be collateralized	0.147	(0.0182)	0.349	(0.0031)

Panel B. Model Fit: Targeted Moments

Moment	Description	Model 1	Model 2	Data
$\sigma(b/k)$	debt rate volatility	0.365	0.300	0.32
$\sigma(i/k)$	investment rate volatility	0.478	0.556	0.53
$\sigma(\Delta y_{-1})$	1-year sales growth rate volatility	0.374	0.340	0.35
$\sigma(\Delta y_{-5})$	5-year sales growth volatility	0.938	0.806	0.8
$\mu(e/k)$	average net equity issuance rate	0.100	0.067	0.1
β	slope of i/k wrt debt/EBITDA	-0.998		-1
$\mu(b/k)$	mean leverage		0.091	0.1

Panel C. Model Fit: Non-Targeted Moments

β	slope of i/k wrt debt/EBITDA		-9.500	-1
$\mu(b/k)$	mean leverage	-0.051		0.1
$\mu(i/k)$	average investment rate	0.187	0.309	0.40
$\text{corr}(i/k, i/k_{-1})$	autocorrelation of investment rate	0.281	0.015	0.32
$\sigma(e/k)$	net equity issuance rate volatility	0.243	0.219	0.45

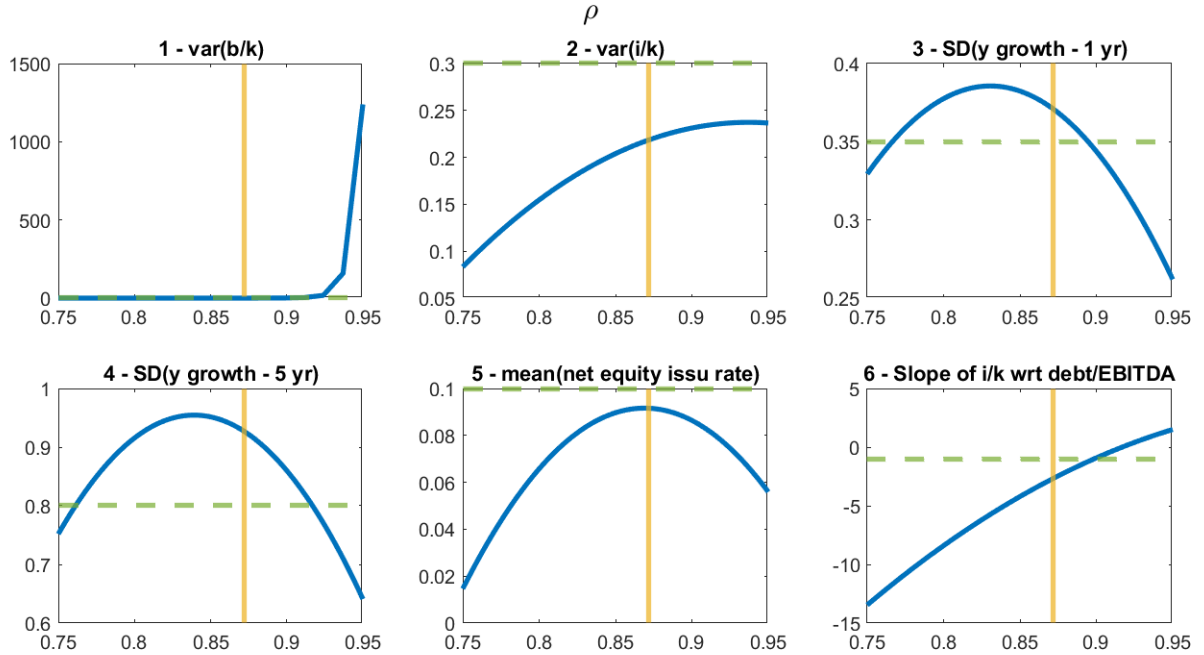
Notes: The table reports point estimates and standard errors (in parentheses) for each of the parameters estimated via SMM. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is the inverse of the moment covariance matrix. Model 1 is the Baseline model. Model 2 is the model targeting the mean leverage ratio instead of the slope of investment to leverage.

Table D.4: Aggregate Implications of Financial Frictions: (1) free equity (2) no collateral constraint (3) free equity and no collateral constraint

	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	Benchmark value
Model 1: targeting slope β				
$100 \times \Delta \log (K)$	1.541	9.397	9.880	0.775
$100 \times \Delta \log (N)$	0.402	0.212	0.498	0.310
$100 \times \Delta \log (\text{Output})$	0.837	2.678	2.965	0.576
$100 \times \Delta \log (\text{Wage})$	0.435	2.466	2.466	1.114
$100 \times \Delta \log (\text{TFP})$	0.211	0.202	0.196	1.234
Relative TFP loss	7.659%	7.465%	7.493%	8.055%
Model 2: targeting mean leverage				
$100 \times \Delta \log (K)$	-0.799	1.969	-0.156	0.706
$100 \times \Delta \log (N)$	-0.874	-0.101	-0.879	0.312
$100 \times \Delta \log (\text{Output})$	-0.699	0.408	-0.580	0.493
$100 \times \Delta \log (\text{Wage})$	0.175	0.509	0.299	0.949
$100 \times \Delta \log (\text{TFP})$	0.025	-0.024	-0.014	1.082
Relative TFP loss	3.402%	3.442%	3.430%	3.431%

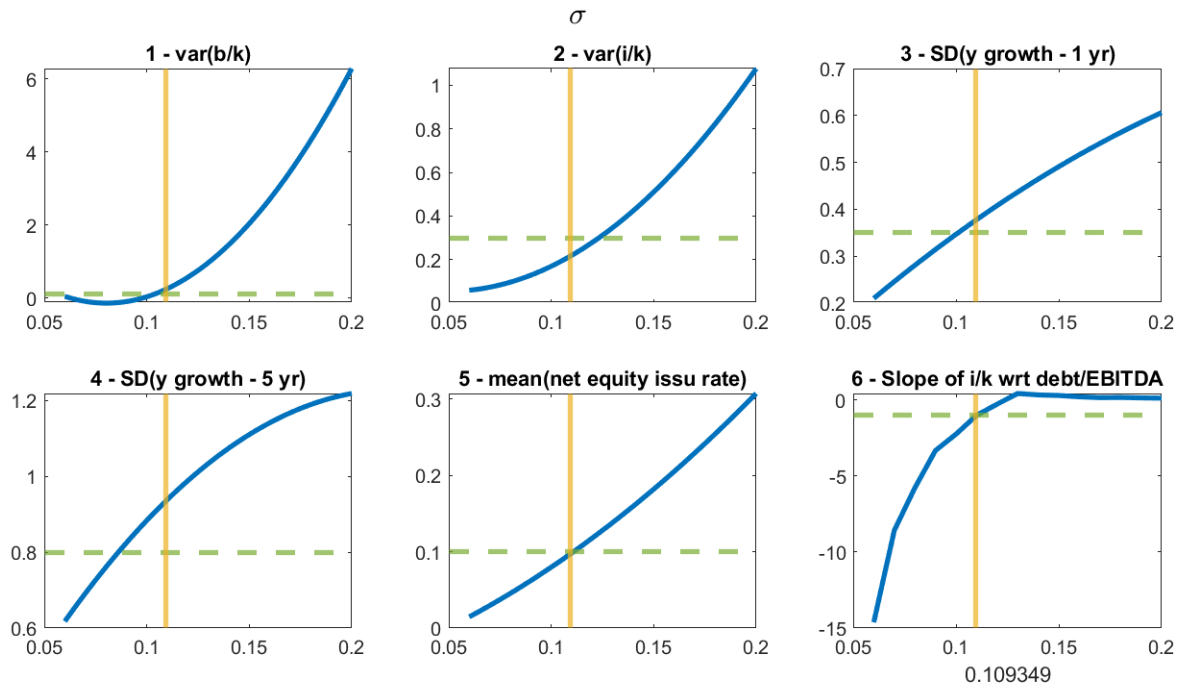
E Additional Figures

Figure E.1: Sensitivity of moments to ρ



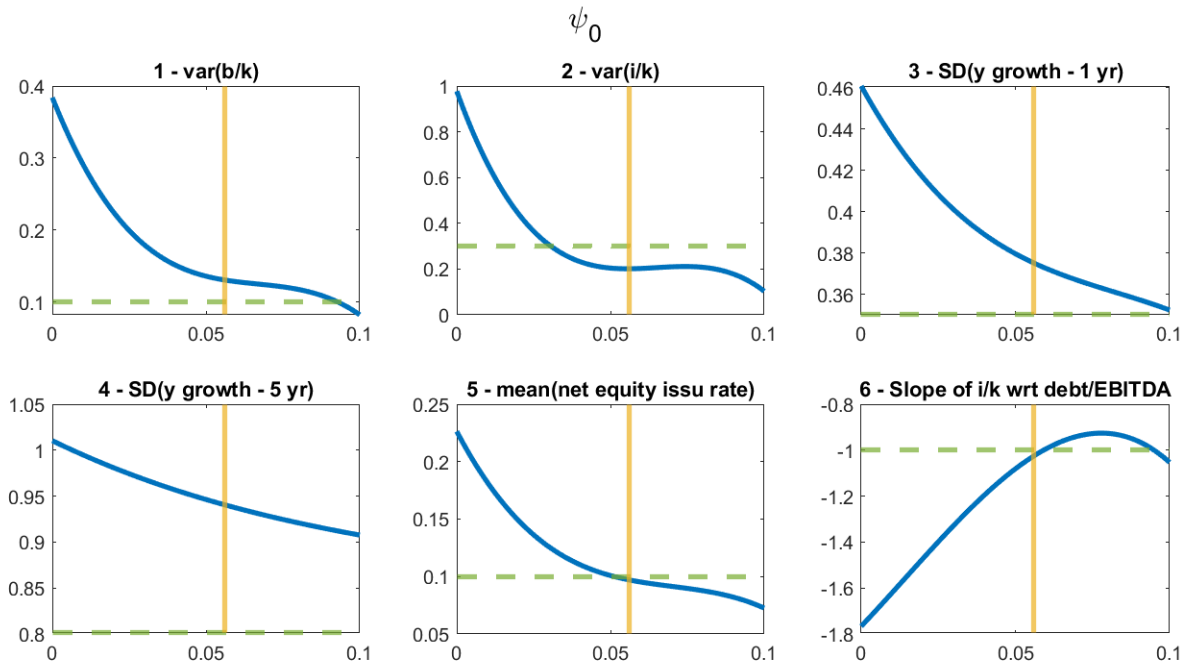
Note: In this figure, I set all estimated parameters ($\rho, \sigma, \psi_0, \eta_1, s$) at their SMM estimates in Table 2. Then I vary ρ from 0.75 to 0.95. For each value of ρ that I choose, I solve the model, simulate the data, and compute six target moments. Each panel corresponds to one moment. The yellow vertical line corresponds to the SMM estimate of ρ . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.

Figure E.2: Sensitivity of moments to σ



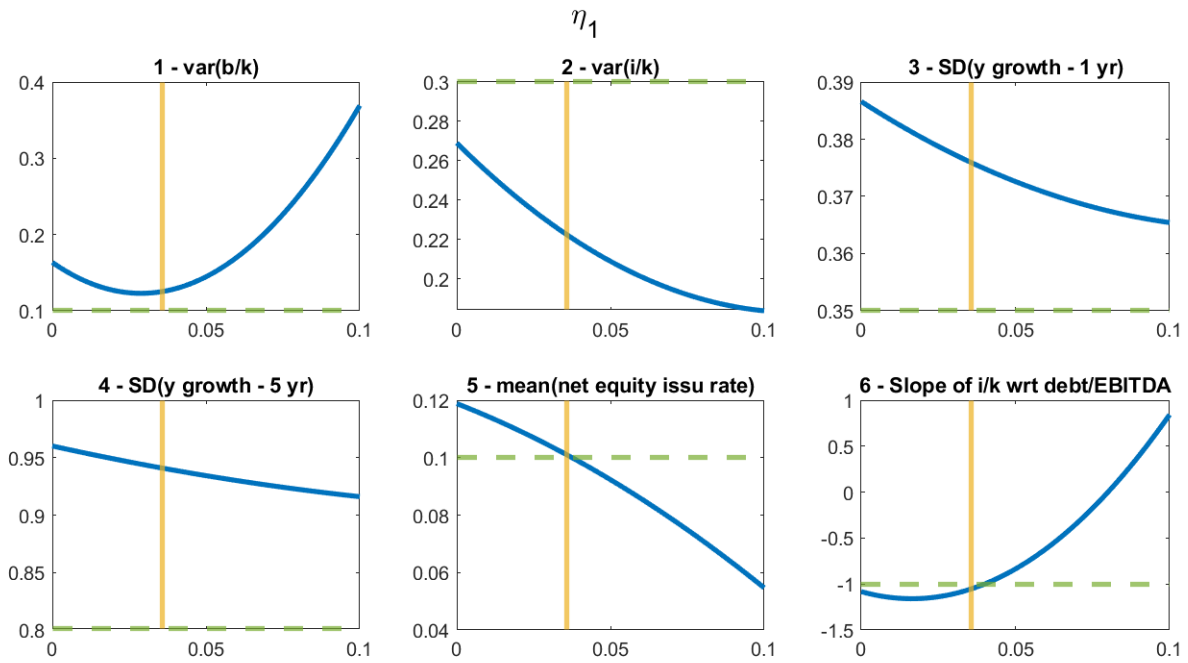
Note: In this figure, I set all estimated parameters ($\rho, \sigma, \psi_0, \eta_1, s$) at their SMM estimates in Table 2. Then I vary σ from 0.05 to 0.2. For each value of σ that I choose, I solve the model, simulate the data, and compute six target moments. Each panel corresponds to one moment. The yellow vertical line corresponds to the SMM estimate of σ . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.

Figure E.3: Sensitivity of moments to ψ_0



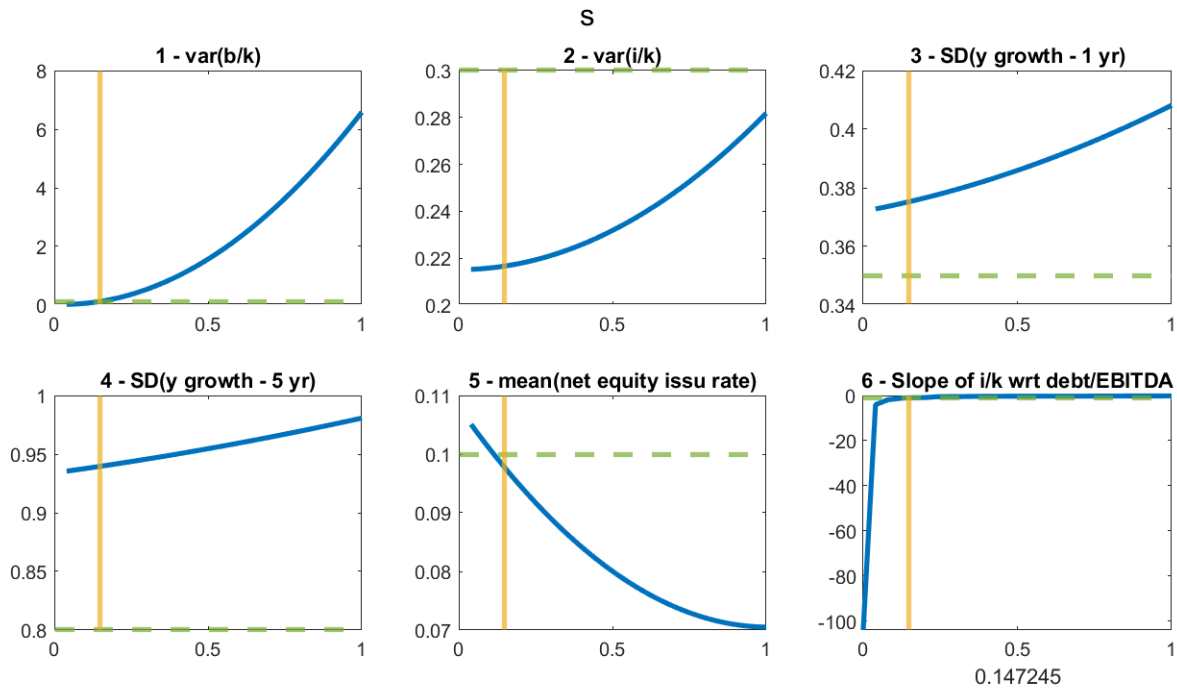
Note: In this figure, I set all estimated parameters ($\rho, \sigma, \psi_0, \eta_1, s$) at their SMM estimates in Table 2. Then I vary ψ_0 from 0 to 0.1. For each value of ψ_0 that I choose, I solve the model, simulate the data, and compute six target moments. Each panel corresponds to one moment. The yellow vertical line corresponds to the SMM estimate of ψ_0 . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.

Figure E.4: Sensitivity of moments to η_1



Note: In this figure, I set all estimated parameters ($\rho, \sigma, \psi_0, \eta_1, s$) at their SMM estimates in Table 2. Then I vary η_1 from 0 to 0.1. For each value of η_1 that I choose, I solve the model, simulate the data, and compute six target moments. Each panel corresponds to one moment. The yellow vertical line corresponds to the SMM estimate of η_1 . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.

Figure E.5: Sensitivity of moments to s



Note: In this figure, I set all estimated parameters ($\rho, \sigma, \psi_0, \eta_1, s$) at their SMM estimates in Table 2. Then I vary s from 0 to 1. For each value of s that I choose, I solve the model, simulate the data, and compute six target moments. Each panel corresponds to one moment. The yellow vertical line corresponds to the SMM estimate of s . The green dashed horizontal line corresponds to the value of each target moment. Numerical comparative statics are smoothed using a polynomial approximation.