The Survival of Zombie Firms: Evidence from the European Sovereign Debt Crisis

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Preliminary and incomplete draft: Please do not cite or circulate without permission

October 2022

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Abstract

It is generally agreed that an efficient economy should feature a productivity-enhancing reallocation where resources are shifted to high-productivity firms and inefficient firms are scrapped. However, recent studies document that an increasing number of "zombie firms" survive. The paper focuses on the effect of policies. It constructs a model to explain how poorly-designed policies can induce under-capitalized banks to roll over loans to otherwise inviable firms. Rescue policies that are frequently used during a financial crisis, such as tightened capital requirements and bank bailouts, may not boost the recovery but distort the credit allocation and hamper aggregate productivity.

The model generates implications that firms of different productivity would respond to a shock differently at the exit margin. This motivates empirical work to structurally examine whether there are zombie firms in the economy. Using European firm-level data from 2010 to 2014, the paper finds an significantly reduced exit response of low-productivity firms during the sovereign debt crisis. The reduced responsiveness and increased survival indicate the existence of zombie firms.

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1 Introduction

An efficient economy should see a productivity-enhancing reallocation where resources are shifted to high-productivity firms and inefficient firms should be scrapped. However, recent studies document an increasing prevalence of "zombie" firms in Europe. Firms that would typically exit in a competitive market now have an increased survival rate. An analysis from KPMG UK [1] argues that "underperforming 'zombie firms' are creating a major drag on the UK economy". Carillion, Britain's second biggest construction company and state contractor, is a high-profile example of a zombie firm, which was in an article in The Guardian. Prior to its collapse in January 2018, the company has been in financial difficulty for several years but it managed to keep afloat with excessive debt.

In this paper, I formalize a mechanism of how credit misallocation from banks with impaired balance sheets, induced by poorly-designed policies, could result in the promotion of zombie firms. The partial equilibrium model features three dates. Firms with idiosyncratic productivity shocks have to rely on bank financing to fund capital and operation. Banks can lend funds to firms or invest in risky assets. Banks are solvent with non-negative equity and are well-capitalized if the capital adequacy ratio is high enough. The government has four policy instruments: (1) a regulatory capital adequacy ratio, (2) tightness of the regulatory capital requirement, (3) banks bailout decisions, and (4) injected capital with an effective monitor ratio if the government chooses to bail out for bank liability loss. In my model, zombie lending and zombie firms occur only if firms have low productivity and banks are under-capitalized failing to satisfy the regulatory capital ratio.

I use the model implication to motivate empirical strategy to examine zombie firms across Europe from 2010 to 2014. The testable hypothesis is that low-productive firms are less responsive to a financial shock. I can estimate the causal effect of the interest rate on firm exit probability. I construct a firm-specific interest rate as the interest rate spread relative to a risk-free rate and interpret the firm-specific interest rate as a financial crisis proxy. To address the endogeneity concern, I instrument the interest rate spread with the average interest rate of all the other firms in the same industry, year, and country. The relevance restriction is satisfied as the leave-it-out average interest rate spread is highly correlated to firm-specific interest rates. The identifying assumptions are (1) the parallel trend assumption: the difference in exit rate between financially strong and weak firms is constant if the shock is homogeneous; (2) the exclusion restriction: conditional on a set of controls, the industry-country-year wide average interest rate affects the firm exit likelihood only through its correlation with firm-specific interest rate. I find a significantly reduced exit response of low-productivity firms during the sovereign debt crisis, consistent with the model prediction.

My paper is organized as follows. Section 2 discusses related literature on the topic. Section 3 describes the theoretical framework. Section 4 introduces model implications and how the model relates to empirical studies. Section 5 provides descriptive empirical evidence about firm dynamics in the exit margin in response to a financial crisis. Section Finally, Section 6 concludes.

[1] https://home.kpmg/uk/en/home/media/press-releases/2019/05/zombies-are-a-major-drag-on-the-uk-economy-kpmg-analysis.html

2 Literature

The paper directly contributes to the literature studying the phenomenon of a particular credit misallocation: zombie lending. The pioneer literature focuses on the Japanese crisis. and demonstrates that large Japanese banks often engaged in sham loan restructurings that kept credit flowing to otherwise insolvent borrowers (Caballero et al., 2008). Since 2008, the focus has been extended beyond Japan to Europe. Credit misallocation increased the failure rate of healthy firms and reduced the failure rate of non-viable firms (McGowan et al. 2017, Acharya et al. 2017 and Schivardi et al. 2017). This paper complements this work, supporting the notion that there exist zombie firms during the European sovereign debt crisis.

To explain the rising prevalence of zombie firms during a crisis, a strand of works (e.g., Acharya et al. 2017, Albertazzi and Marchetti 2010, and Giannetti and Simonov 2013) point out that misguided policies encourage inefficient lending behavior. Acharya et al. (2017) provide evidence that after bank rescue support (i.e., the Outright Monetary Transaction program), banks regain lending capacity but some still remain weakly capitalized. Most likely driven by risk-shifting incentives, these banks extend loans to weaker firms with pre-existing lending relationships. There are also proposed motivations for zombie lending, such as regulatory pressure to increase their capital ratios and a moral suasion motive (Blattner et al. 2019, Caballero and Hammour 1991). Overall, the literature suggests that poorly-designed policy measures aggravate the problems of loan misallocation. Motivated by this empirical evidence, my theoretical framework is constructed to highlight how government policies can cause zombie lending.

According to the literature (e.g., Caballero et al. 2008, Kwon et al. 2015), there are various forms of zombie lending: interest rate concessions, debt-equity swaps, debt forgiveness, and moratoriums on loans. Most financial assistance involves (a) reductions in interest payments, or (b) outright debt forgiveness for troubled firms. The model described in the paper is able to capture zombie lending in these two forms. Closely related, Bruche and Llobet (2013) propose a theoretical framework of zombie lending. They consider only the first type of zombie lending. Moreover, to prevent zombie lending they suggest an incentive-compatible mechanism design with a contingent transfer, which differs from my proposed solutions.

In addition, the paper is closely related to the literature on recession cleansingness and business dynamism. Since the pioneering work of Schumpeter et al. (1939), economists have been interested in how business cycles affect the allocation of resources. The conventional view of recessions, emphasized in Caballero and Hammour (1991), is based on the assumption that markets select the most productive firms and hence a recession is a time of accelerated productivity-enhancing reallocation. However, the cleansing hypothesis has been challenged by several studies highlighting potential distortions. These studies reveal "sullying" or "scarring" effects of recessions. My paper is in line with previous conclusions that credit frictions may therefore alter the productivity-enhancing effect of recessions (Barlevy, 2003). By looking at the firm dynamics, especially the exit rates in a financial crisis, I find reduced exit responses for low-productivity firms. The existence of zombie firms implies that the European sovereign debt crisis is not cleansing. This finding complements international evidence on declining firm dynamism.

3 Model

In this section, I develop a partial equilibrium firm investment model with a firm-bank lending nexus.

3.1 Setup

Consider an economy with three dates, $t = 1, 2, 3$, with banks, firms, and government. There is no discounting across dates. There is no particular role for consumers so I do not model consumer behavior explicitly.

Banks

There exists a continuum of risk-neutral banks, that operate to maximize the expected value of their equity. Bank assets include corporate loans, risky assets, and/or risk-free assets. Bank liability is exogenously given and assumed to be constant over time. Banks face two types of shock: firm-level idiosyncratic productivity shock ε, which affects firm profitability and ability of loan repayment; bank-level idiosyncratic asset return shock η. Assume η is identically distributed over time, serially independent, and orthogonal to ε.

Banks are supervised by the government. I assume that they have to satisfy two requirements:

1. Solvency: Equity ≥ 0
2. Well-capitalization: $\frac{\text{Equity}}{\text{Asset}} \geq \kappa$, where κ is the regulatory capital adequacy ratio

The requirements are held ex-ante. But ex-post, they may be violated depending on shock realizations. This feature is different from usual constraints which have to be satisfied all the time. It also reflects a degree of regulatory forbearance.

At the end of each period, a bank needs to decide whether it would like to continue to lend to the firm, especially when the firm fails to repay the loan. Bank's objective is to choose optimal asset portfolios to maximize its expected next-period equity value. The period-by-period continued lending and asset investment decisions take government regulations into account. However, both firms and banks cannot anticipate government policies. So there is no role of the forward expectation on policy measures. Facing some policy instruments, banks may have incentives to continue lending loans to unproductive firms. This would generate zombie lending and zombie firms.

Firms

There exists a continuum of risk-neutral monopolistic competitive firms (production units) in the economy. Each firm produces a variety of goods. The final consumption good is numeraire. Assume each variety uses the Cobb-Douglas production function. And the final good follows the Dixit-Stiglitz structure. The firm has pre-determined capital stock, either financed by the bank or its retained earnings. In the beginning, the firm has no equity and has to rely on bank financing to fund capital and operation. Each period firm faces idiosyncratic productivity shock

ε. After the shock is revealed, the firm chooses labor input and starts producing products. The optimal input and price decisions are based on a firm maximization problem that maximizes its current-period profits, net of wage, loan repayment, and fixed costs. If the earnings cannot cover the loan and the bank discontinues financing, the firm will exit the market. In contrast, if the firm fails to repay the loan during this period but still has the access to bank funds for the next period, then the firm can stay in the market. When the firm survives, the productivity ε will evolve stochastically, i.e., ε is serially dependent.

For simplicity's sake, I assume firms and banks are one-to-one mappings: a firm can only borrow from one bank and a bank only lends to one firm. I also assume that the pair cannot be changed. These assumptions help to capture an important characteristic that zombie lending usually occurs among weak banks with weak firms (Acharya et al. 2017). The correlation is the extreme value 1. If relaxed, the mechanism still works in general but the correlation would be smaller.

Government

The government supervises banks by looking at their balance sheets. Government can know whether a bank is solvent and well-capitalized only from the reported, or book-value balance sheet. But the government has no idea about the market-value of the balance sheet. So if a bank underreports or delays the loan loss while the balance sheet looks good, the government cannot detect the problem immediately. This forms asymmetric information between the government and banks. Government decisions will not take history into account. In other words, policies are not serially correlated.

In addition to the capital adequacy ratio κ, the government has three main policy instruments:

1. χ, tightness of regulatory capital requirement: if the government tells from the bank balance sheet that the bank is solvent but does not satisfy the minimum capital ratio, then the bank is not allowed to invest in any type of assets. In addition, it subjects to a fraction of χ punishment from its equity value.

   χ can be interpreted as the cost to raise the capital in no time. A higher χ suggests that it would be more costly for a non-compliant bank breaking the rule. This might create a distorted incentive for banks to lie on the balance sheet.

2. λ, bailout decision: when banks have a risk of bankruptcy or solvency problem, the government has to decide whether to bail out for bank liability loss and the size of the bailout if so. $\lambda = 0$ means no bailout.

3. (θ, β): if the bailout is implemented, the government needs to monitor the use of injected capital. θ is the effective monitoring ratio, where $0 \leq \theta \leq 1$. If a bank is caught by the government for inappropriate use (i.e., loan extension to zombie firms), then the bank incurs a punishment of β, where $\beta \geq 0$.

I will describe later in detail how government programs affect banks continued lending decisions.

3.2 Timing

t = 1

All banks are ex-ante identical with the liability of face value D, where $0 < D < 1$, 1 unit of corporate loan, and risky asset a. All firms have the same expected productivity ε_1. A bank balance sheet is illustrated in Table 1. I assume ex-ante banks are solvent and satisfy the government requirements for an adequate capital ratio. Appendix A provides conditions under which banks have no incentive to deviate from such asset portfolio as in Table 1.

Table 1: Bank balance sheet at t = 1

Asset	Liability
1	D
a	Equity
	1 + a - D

All firms are ex-ante identical in expected productivity and have the same technology. Every firm borrows 1 unit of corporate loan 1 from the bank to fund for capital. The corporate loan is subject to a gross interest rate R_1. All firms and banks are identical so they make the same above decisions.

t = 2

Each firm realizes its idiosyncratic productivity shock ε_2. Each firm hires labor $L_2|\varepsilon_2$ and sets price $P_2|\varepsilon_2$ to maximize its profit. The firm starts production, sells output, and pays wages and per-period operating costs. Depending on the productivity, the firm may earn a profit or incur a loss. In the latter case, the firm may have trouble repaying the loan in full amount. If the bank continues lending, then the indebted firm can survive. Otherwise, the firm will exit at the end of the date.

The bank faces idiosyncratic shock on asset returns η_2. Depending on the realized firm productivity and risky asset return, banks may become insolvent and/or under-capitalized. Banks are now heterogeneous in loan market value, solvency, and level of capitalization. If firms repay the loan, then the bank's balance sheet looks as Table 2. Alternatively, the balance sheet looks as Table 3.

Table 2: Bank balance sheet at t = 2, with firm loan repayment

Asset	Liability
R_1	D
$a\eta_2$	Equity
	$R_1 + a\eta_2 - D$

Simultaneously, the government announces its policy instruments. At the end of the period, banks respond in terms of lending and investment decisions to maximize their expected equity. A bank will lend to a firm if the bank expects the firm will be more likely to become highly productive at t = 3, i.e., the firm is more likely to repay the loan. Since firms are now

Table 3: Bank balance sheet at t = 2, without firm loan repayment

Asset	Liability
$a\eta_2$	D
	Equity
	$a\eta_2 - D$

heterogeneous, banks' expectations are also different. Suppose ε^* is the threshold productivity for firms to repay the loan at t = 2. If the bank works with a high-productivity firm (i.e., $\varepsilon_2 \geq \varepsilon^*$), the bank has the incentive to continue to invest in corporate loans. Otherwise, the bank would rather invest in risky assets and stops lending to firms. Appendix A describes the investment incentive constraints in detail. For the investment decisions to hold, I make the following assumption. Assumption 1 states that a more productive firm in this period is more likely to be profitable next period.

Assumption 1 *$\varepsilon^{**}(\varepsilon_2)$ is decreasing in ε_2.*

t = 3

Firms that receive the loan or have enough internal funds continue the operation. New idiosyncratic productivity ε_3 and risky asset return η_3 are revealed for surviving firm-bank pairs. There is no zombie lending in this period.

3.3 Bank and firm types at t = 2

By construction, the more productive a firm is, the higher revenue and profit it earns at t = 2. Thus I divide firms at t = 2 into two types by the productivity threshold ε^* where $\pi(\varepsilon^*, 1) = 0$. Similarly, I also classify banks at t = 2 into two types by the risky asset return threshold $\eta^* = \frac{D}{a(1-\kappa)}$. Table 4 shows four kinds of firm-bank pairs: HH (High-type firm and High-type bank), HL (High-type firm and Low-type bank), LH (Low-type firm and High-type bank), and LL (Low-type firm and Low-type bank). It also reports the firm exit response for each pair. The exit responses are guaranteed by the above conditions and assumptions.

For convenience, I make an assumption on the support of η. Assumption 2 says that η is bounded below at the value where the capital ratio κ is satisfied with full loan repayment. The assumption guarantees that as long as the firm repays the loan in full amount, the bank is well-capitalized.

Assumption 2 (Lower bound of risky asset return) $\min \eta \geq \frac{1}{a}(\frac{D}{1-\kappa} - R_1)$.

3.4 Firm exit at t = 2

In the LL pair, zombie lending and zombie firms might occur if these firms can still borrow from the bank. If so, low-productivity firms are able to survive. Zombie generated in the model is

Table 4: Bank and firm types

		Bank type	
		High ($\eta_2 \geq \eta^*$)	Low ($\eta_2 < \eta^*$)
Firm type	High ($\varepsilon_2 \geq \varepsilon^*$)	bank well-capitalized & continue lending; firm survive	bank well-capitalized & continue lending; firm survive
	Low ($\varepsilon_2 < \varepsilon^*$)	bank well-capitalized but stops lending; firm exits	bank under-cap. and even insolvent; firm may survive

among weak banks and weak firms. This is consistent with documented facts in Acharya et al. (2017).

By Assumption 2, bank in the LL pair has risky asset return $a\eta_2 \in [\frac{D}{1-\kappa} - R_1, \frac{D}{1-\kappa}]$. For various values of D, there are two cases of zombie lending. The first type is when a bank has enough household deposits and all banks are solvent. Government policies on capital regulations affect the occurrence of zombie lending. The second case is when the deposits are small (which usually happens when a negative demand shock hits the economy) and some banks face the risk of bankruptcy. The policies regarding the bailout also matter. Definition 1 and 2 describe these two cases.

Definition 1 (Solvent banks) *If $R_1 \leq D\frac{\kappa}{1-\kappa}$ (D is large enough), the weakest banks are solvent. Zombie lending depends on the tightness of capital requirement χ.*

Definition 2 (Insolvent banks) *If $R_1 > D\frac{\kappa}{1-\kappa}$ (D is insufficient), the weakest banks become insolvent. Then zombie lending for insolvent banks depends on government bailout λ and supervision (θ, β).*

Conditions 1 describes the sufficient condition for the first type of zombie lending, interest rate reduction. Bank will continue lending to the otherwise default firm if the expected return (loan repayment and risky asset returns minus liability) is higher than the current equity value net of regulation punishment. Bank exploits the asymmetric information that government does not have the perfect information about the loan condition. Therefore bank delays the reporting.

Condition 2 describes another type of zombie lending, debt forgiveness or "evergreening", when some banks almost bankrupt in the economy. The government decides to bail out these banks by giving a fund of size $\lambda < D$. The strict inequality indicates a consumer welfare loss where households cannot receive the full deposit D. Banks can have the opportunity to manipulate credit use and "gamble for resurrection". The reduced-form parameter λ is structurally from taxes collected from households and firms. It would be costly to bail in the full amount so the parameter restriction on λ makes sense. The injected capital is intended to pay off the adjusted liability immediately. However, if the supervision is insufficient (small θ and small β), the bank may have the incentive to distort the use of the capital by keeping the credit flow to its firm. The incentive increases as the bank lending position are eased by the bailout capital but the bank still keeps under-capitalized.

Condition 1 (Zombie lending: interest rate concession)

$$R_2[1-\Phi(\varepsilon^{**}(\varepsilon_2))]r(\varepsilon_2)+(a\eta_2-r(\varepsilon_2))\overline{\eta}-D \geq (a\eta_2-D)(1-\chi), \textit{with } r(\varepsilon_2) \leq a\eta_2$$

where $r(\varepsilon_2)$ *is the corporate loan at* $t = 2$*, which depends on the firm productivity realization.*

Condition 2 (Zombie lending: debt forgiveness)

$$(1-\theta)\{R_2[1-\Phi(\varepsilon^{**}(\varepsilon_2))]r(\varepsilon_2)+(\lambda+a\eta_2-r(\varepsilon_2))\overline{\eta}-D\}+\theta(-\beta) \geq \lambda, \textit{with } r(\varepsilon_2) \leq \lambda+a\eta_2, 0 \leq \lambda < D \blacksquare$$

4 Model implications

In this section, I examine model implications from the following perspectives. First, I present the endogenous firm exit response and aggregate productivity. I discuss how policies affect these aggregate results. Then I derive comparative statics on firm exit likelihood across time and across interest rates. These generate predictions on the causal effect of interest rates on firm exit response. The hypothesis can be tested empirically later. In the end, I briefly talk about welfare implications.

4.1 Firm endogenous exit response

As shown above, firm idiosyncratic productivity shock is not the only reason that affects the firm exit status at t = 2. Bank continued lending may keep some low-type firms alive, which depends on bank type and government policies. Specifically, when firm productivity ε is no smaller than ε^*, the productivity threshold, firms will continue to operate. When firm productivity ε is smaller than ε^*, the firm has the default risk. In the LH pair, since banks will invest in risky assets by parametric assumptions, these low-type firms will exit for sure. For the LL pair, if zombie lending Conditions 1 and 2 are satisfied, these low-type firms will survive.

The productivity threshold ε^* is determined by the productivity distribution $\mathcal{F}$ and underlying production function. When the regulatory capital ratio κ rises, η^* rises, the risky asset return threshold $\eta^* = \frac{D}{a(1-\kappa)}$ also rises. Hence more firm-bank pairs turn out to be the LL and more zombie firms might be generated.

Other policies such as capital regulation tightness χ, effective bank debt λ or efficiency supervision (θ, β) will not change ε^* and η^* directly but affect the zombie lending incentives. When χ increases, λ decreases, θ decreases, and/or β decreases, zombie lending conditions are more likely to hold and fewer firms would exit.

4.2 Aggregate productivity efficiency measure

To illustrate the model implication on aggregate productivity efficiency, I make the following parametric assumptions. Assumption 3 states that the firm-level idiosyncratic productivity follows a normal distribution. The mean productivity is ε^*, at which the firm is break-even at t = 2. In addition, I assume the ex-ante productivity expectation at t = 1 is also ε^*. These assumptions simplify the calculation. Assumption 4 also assumes that the risky asset return follows the normal distribution. The normality assumptions of these two shocks give a closed-form solution to the average productivity of the economy.

Assumption 3 (Productivity distribution) $\varepsilon \sim \mathcal{N}(\varepsilon^*, \sigma_\varepsilon^2)$. *The ex-ante expectation at t = 1* $\varepsilon_1 = \varepsilon^*$. ϕ *is the productivity probability density function;* Φ *is the cumulative density function.*

Assumption 4 (Risky asset return distribution) $\eta \sim \mathcal{N}(\overline{\eta}, \sigma_\eta^2)$. ψ *is the productivity probability density function;* Ψ *is the cumulative density function.*

By construction, the average productivity at t = 1 is ε^*. Proposition 1 summarizes the baseline case when there is no zombie lending, while Proposition 2 discusses the aggregate productivity efficiency with zombie lending and zombie firms. These two propositions highlight the loss of productivity when weak firms are kept alive. The productivity efficiency measure at t = 3 follows a similar procedure, with additional assumptions on productivity evolution.

Proposition 1 (Baseline: no zombie lending) *In the baseline case where government policies do not generate incentives for zombie lending, all low-type firms will exit the economy. Average productivity is* $E(\varepsilon_2|\varepsilon_2 > \varepsilon^*)$. *With the normality assumptions, the aggregate productivity is*

$$A = \varepsilon^* + \sigma_\varepsilon \frac{\phi(0)}{1 - \Phi(0)} = \varepsilon^* + 2\sigma_\varepsilon \phi(0)$$

Proposition 2 (Zombie lending) *When banks have incentives for zombie lending under government policies, only low-type firms in the LH pair will exit. Low-type firms in the LL pair stay alive. With the normality assumptions, the economy-wide average productivity is*

$$A' = \varepsilon^* + 2\sigma_\varepsilon \phi(0) \frac{1 - \Psi(\eta^*)}{1 + \Psi(\eta^*)}$$

$A' < A$*, aggregate productivity in the zombie lending case is smaller compared to the baseline case.*

By shifting firm endogenous exit status, policy instruments can affect aggregate productivity. On one hand, government policies affect aggregate productivity by switching the economy between the baseline and the zombie lending scenario. Without a loss of generality, starting from the baseline scenario, low-type banks respond to policy changes in zombie lending when χ increases, λ decreases, θ decreases, and/or β decreases. Since the economy switches to the zombie lending scenario, aggregate productivity declines.

On the other hand, policies can move the average productivity within the zombie lending regime. Risky asset return threshold $\eta^* = \frac{D}{a(1-\kappa)}$ increases when capital ratio κ increases. Then $\frac{1-\Psi(\eta^*)}{1+\Psi(\eta^*)}$ decreases. As a result, the aggregate productivity of the economy declines.

The model suggests that the policy instruments affect firm dynamics and aggregate productivity efficiency. When government only monitors financial statements and summary statistics (i.e., capital ratio), it ignores the other critical aspects such as bank loan loss and liquidity coverage, which can slow down the economy's recovery from a crisis. Therefore it's important to re-orient the regulatory framework towards a macroprudential perspective. It's crucial to consummate and improve the existing banking supervision system. While the model admits that policymaker is subject to constraint from the labor market to have regulatory forbearance, more economic structural rescue policies are necessary for addition to financial aid. For example, programs that facilitate job turnover would expedite reallocating workers from unproductive firms to productive firms. This decreases the "benefit" of zombie lending and thus encourages policies to prevent the survival of zombie firms. McGowan et al. (2017) also has a similar policy discussion that reallocation-friendly policies are welcome.

4.3 Welfare and government objectives

The economy has a gain in productivity efficiency when all low-type firms exit. Output per worker grows. The government would announce policies to prevent zombie lending incentives if it only cares about efficiency.

However, it might not be the case. The government's forbearance of zombies is supported by several concerns. Firstly, the government does not lead to implementing costly bank monitoring, which is related to a higher tax. Second, the firm exit would lead to a decrease in product variety and consumer welfare. In addition, firm and bank exit would lead to an increase in unemployment if the expansion of high-type firms in terms of job openings is relatively smaller. As workers become unemployed, consumption decreases, prices drop, and the production scale shrinks, which further lowers social welfare. More importantly, if banks are insolvent, then households lose their deposits. It might cause public panic to withdraw deposits from not only low-type banks but also high-type banks. As a consequence, negative idiosyncratic shocks would potentially spill over to the entire banking sector. If in general policies are stringent, i.e., a higher κ, average productivity rises. But banks would restrain from lending to firms. Without enough bank financing, the production scale decreases, and firms would hire less labor.

Loosely speaking, government objective function $G()$ can be summarized as below:

$$G(\Phi(\varepsilon), \Psi(\eta), D, N)$$

where the distribution of realized firm productivity $\{\varepsilon_2\}$ and bank asset returns $\{\eta_2\}$ matter for the number of hires and separations. A more left-skewed productivity and asset return distribution would have more high-type firms and banks than low-type firms and banks. The concern for unemployment would be smaller. $G()$ returns policies against zombie lending. If bank liability D is smaller, more banks become insolvent so policies would show more leniency. N is the number of differentiated products in the economy. Implicitly, $G()$ also depends on the shape of the supervision function (i.e, how effective the government expenses transform into supervision), labor wage, and unemployment.

4.4 Comparative statics of firm exit response

I derive comparative statics on the aggregate exit rates for the baseline economy and the zombie lending scenario across time and across interest rates. I consider the heterogeneous response for high- and low-productivity firms respectively. Appendix A provides detailed derivations for the comparative statics results.

Firstly, I compare the exit response across time t = 2 and t = 3. Summarized in Proposition 3, it says that high-productive firms will survive at t=2 and they will continue to earn profits unless they turn out to be low-productive unfortunately at t = 3, i.e., the new productivity is below $\varepsilon^{**}(\varepsilon_2)$. Conditional on high-type firms, the exit rate will increase by a fraction of firms whose productivity is smaller than the threshold. If firms are low-productive at t = 2, all of them will exit if the economy operates efficiently. So there is no change in exit rate for low-type firms across time. However, if the economy is inefficient as zombie lending arises, the exit probability for low-type firms will increase if these firms still draw low productivity.

Proposition 3 (Zombie lending effects across time) *The change in the exit response for high- and low-type firms between $t = 2$ and $t = 3$ is:*

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 \geq \varepsilon^*) = \int_{\{\varepsilon_2 \geq \varepsilon^*\}} \Phi(\varepsilon^{**}(\varepsilon_2))d\varepsilon_2$$

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 < \varepsilon^*) = \begin{cases} 0, \ baseline \\ \int_{\{\varepsilon_2 < \varepsilon^*\}} \Phi(\varepsilon^{**}(\varepsilon_2))\Psi(\eta^*)d\varepsilon_2, \ zombie \end{cases}$$

Proposition 4 (Zombie effects when interest rate R_1 varies) *When corporate loan gross interest rate R_1 increases, $t = 2$ productivity threshold ε^* also increases to $\varepsilon^{*\prime}$, the new productivity threshold. Then the change in exit response for high- and low-type firms before and after the interest rate R_1 increases is:*

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 \geq \varepsilon^{*\prime}) = 0$$

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 < \varepsilon^{*\prime}) = \begin{cases} \int_{\{\varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}\}} \Phi(\varepsilon^{*\prime}) - \Phi(\varepsilon^*)d\varepsilon_2, \ baseline \\ \int_{\{\varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}\}} [\Phi(\varepsilon^{*\prime}) - \Phi(\varepsilon^*)] \times [1 - \Psi(\eta^*)]d\varepsilon_2, \ zombie \end{cases}$$

Secondly, Proposition 4 states the zombie lending effects on exit rates when interest rate fluctuates. When corporate loan interest rate R_1 builds up, firms have to be more productive to earn profits. The productivity threshold thus shifts upwards. In this framework, firms in the productivity threshold margins before and after the change will be sensitive. Precisely, high-type firms that are just slightly above the old threshold will exit in the baseline case. But in the zombie lending scenario, if these firms borrow from high-type banks, then they are able to continue to live. As for low-type firms, a rise in interest rates will not induce changes in bank behaviors. So exit rate remains the same. Proposition 4 shows that low-productivity firms (i.e, $\varepsilon_2 < \varepsilon^{*\prime}$) have reduced exit rate when inefficient zombie lending is allowed, compared to the exit change in the efficient economy. When the risk-free interest rate R_f differs, productivity threshold ε^* and $\varepsilon^{**}(\varepsilon_2)$ are the same. There is no change in the exit rate as long as bank investment incentive conditions still hold.

I also compare the exit rates when both time and interest rates vary. Now the interest rate R_2 moves. In the baseline model, the economy ends at t = 3 and zombie lending only occurs at t = 2. However, it's easy to extend the timing such that banks again face an option to determine whether to continue financing firms for all surviving firms from the end of t = 2. Then I can classify new high-type and low-type firms and banks by the new threshold $\varepsilon^{**}(\varepsilon_2)$ and η^{**}, which now would be a function of firm-level productivity ε_2 and bank's last-period asset return η_2. Suppose the economy has more than three periods, then Proposition 5 also describes the corresponding zombie lending effects across time and across interest rates R_2. When there is no zombie lending at t = 3, Σ is the impact of interest rates across different product groups. The corresponding effect is Σ' with zombie lending at t = 3. Σ' is smaller than Σ and under appropriate parameter values, Σ' can be even negative. It highlights that the causal effect of the interest rate on the exit rates in the zombie lending scenario is smaller.

Proposition 5 (Zombie lending effects across time and across interest rates R_2) *When corporate loan gross interest rate R_2 increases, t = 3 productivity threshold $\varepsilon^{**}|\varepsilon_2$ also increases. Suppose the new productivity threshold is $\varepsilon^{**\prime}$.*

Conditional on surviving firms at the end of t = 2, the difference in changed exit response across different productivity groups is:

$$\Sigma = \int_{\{\varepsilon_2<\varepsilon^*,\eta_2<\eta^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 - \int_{\{\varepsilon_2\geq\varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \quad (1)$$

Under regular conditions, we would expect $\Sigma > 0$.

With an extended time horizon, the bank regains the continued lending decisions to maximize its expected next-period equity facing policy instruments at t = 3. Without loss of generality, assume at t=3 zombie lending would occur between low-type banks (i.e, $\eta < \eta^{**}(\eta_2, \varepsilon_2)$) and low-type firms (i.e., $\varepsilon_3 < \varepsilon^{**}(\varepsilon_2)$), if and only if new zombie lending conditions (not stated explicitly) are satisfied.

Conditional on surviving firms at the end of t = 2, and if there is zombie lending at t = 3, then the difference in changed exit response across different productivity groups is:

$$\begin{aligned}\Sigma' = &\int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3\geq\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \qquad (2)\\ &+ \int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3<\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))] \times [1 - \Psi(\eta^{**}(\eta_2, \varepsilon_2))]d\varepsilon_2\\ &- \int_{\{\varepsilon_2\geq\varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2\end{aligned}$$

With a factor of $[1-\Psi(\eta^{**}(\eta_2, \varepsilon_2))]$, the changed exit rate for low productivity firms is smaller. As a result, $\Sigma' < \Sigma$.

Proposition 5 proposes a difference-in-difference estimation framework to estimate the impact of interest rates on firm exit margin across firms of different productivity. It also generates a test to check whether an economy has zombie lending and zombie firms or not. If the estimate is negative, it might point to the existence of zombie lending and zombie firms.

5 Empirical work

In this section, I provide empirical evidence to support that the recent European sovereign debt crisis is a period of increasing survival of zombie firms. Low-productive firms demonstrate reduced exit responses compared to their high-productive counterparts. The estimation is in line with Proposition 2 of a reduced effect in a zombie lending scenario.

5.1 Data

I combine a rich dataset of firm-level financial information and country-level aggregate performance from 2010 to 2014. This is a period of the European sovereign debt crisis. My base dataset is the AMADEUS. The dataset contains financial statement data for public and private companies in Europe. I can also explore firm entry and exit. Earlier works that also use the AMADEUS find a reasonably close match (e.g. Bloom et al. (2016a)) by comparing aggregate employment in the dataset to those from census data. But there are still some issues in the AMADEUS. I describe the datasets and related concerns in more detail in Appendix B. I adopt a cleaning procedure similar to Arellano et al. (2012). But different from them, I did not impute employment for the firms. I describe the data cleaning procedure in detail in Appendix C.

5.2 Regression specification

Motivated by model prediction on comparative statics on the exit rate, I can examine whether the economy has zombie firms following the essence of Proposition 1 and 2. That is, I can estimate the causal effect of the interest rate on firm exit probability. If the estimated effect is negative, then it tells that there might exist zombie firms.

Firm-specific interest rate is constructed as the ratio of interest payment and total debt. I use German short-term interest rates as risk-free rates, to normalize computed firm-level interest rates. On the other hand, I can interpret the firm-specific interest rate as a financial crisis proxy. If a firm has a small financial shock, the firm would typically find it not hard for external financing with respect to loan approval, cost of financing, loan term, and so on. So a smaller interest rate can somewhat embody a smaller financial shock that a particular firm confronts.

The OLS estimation using the firm-level interest rate spread suffers the reverse causality bias. Financially weak firms with a bad reputation would be more likely to be charged a higher loan rate. To address the endogeneity concern, I instrument the interest rate spread with the average interest rate of all the other firms in the same industry, year, and country. The relevance restriction is satisfied as the leave-it-out average interest rate spread is highly correlated to firm-specific interest rates. For example, if a country is severely hit by a crisis, the interest rates of all firms in the country would have a similar magnitude. The identifying assumptions are (1) the parallel trend assumption: the difference in exit rate between financially strong and weak firms is constant if the shock is homogeneous; (2) the exclusion restriction: conditional on a set of controls, the industry-country-year wide average interest rate affects the firm exit likelihood only through its correlation with firm-specific interest rate.

Based on firm TFPR value, I divide firms into three productivity groups: low, middle, and high according to the 25% and 75% threshold. Since firm exit response is expected to be different across productivity, I include an interaction term between the interest rate spread and the TFPR dummy variable. The parameter of interest is the coefficient for the interaction term. If the estimate is negative, it suggests that as the interest rate spread increases, the rise in exit probability is higher for productive firms. The reduced responsiveness of unproductive firms indicates that the "cleansing" effects of a crisis are smaller.

In addition to the exit response, I also estimate the impact on the firm new debt issuance and investment. Specifically, they are constructed as follows:

$$\text{Investment rate}_{jt+1} = \frac{\text{Investment}_{jt+1}}{\text{Capital}_{jt+1}}$$

$$\Delta\text{Leverage}_{t} = \frac{\text{Debt}_{t} - \text{Debt}_{t-1}}{K_{t-1}}$$

The empirical models are Equation (**??**), (**??**), and (**??**). For all specifications, lagged firm controls include total fixed assets (in logarithm) and sales growth rates. They can help to control the role of firm size and growth potential. Variables of TFPR, ΔLeverage, investment rates, and firm controls are all standardized. For regression (**??**) and (**??**), I trim out observations of $\Delta\text{Leverage}_{jt+1}$ below 1 and above 99 percentiles, due to much noise. Variables change in leverage and investment rate are 1-period forward as firms need to take time to adjust. Fixed effect controls include industry-year fixed effects, country fixed effects, and firms' age group fixed effects. Standard errors are clustered at the firm level.

$$\begin{aligned} Y_{jt} = {} & \beta_1 \text{Productivity}_{j,t-1} + \beta_2 \left(\text{Productivity}_{j,t-1} \times \text{interest rate}_{jct}\right) \\ & + \beta_3 \text{interest rate}_{jct} + \text{Firm controls} + \text{Time} \times \text{Industry FE} + \varepsilon_{jt} \end{aligned} \tag{3}$$

5.3 Estimation results

Table 5 reports the 2SLS regression estimation results. As expected, a higher interest rate significantly increases the exit probability and decreases the new debt issuance and firm investment. Higher productivity is associated with lower exits and more investment. The interaction between productivity and interest rate spread returns a coefficient of positive value, significant and increasing in magnitude when productivity rises. As interest rate spread changes, change in exit rate can be expressed as:

$$\begin{aligned} \Delta\text{Pr(Exit)} &= \beta_2(\text{TFPR} \times \Delta\text{interest rate}) + \beta_3 \Delta\text{interest rate} \\ &= \Delta\text{interest rate} \times (\beta_3 + \beta_2 \text{TFPR}) \end{aligned}$$

The positive value of β_2 in Equation (**??**) shows that the financial crisis, proxied by interest rate spread has a different impact on firm exit by productivity level. And unproductive firms respond less as they are less likely to exit.

For the response to new debt issuance and investment rates, the estimates for responsiveness are similar. They both show reduced responsiveness. To sum up, the decline in firm response to a financial downturn is not only in the dynamics (i.e., entry and exit margin) but also in the investment decisions.

Recall that Proposition 8 and Proposition 9. The model predicts that the causal effect of interest rate on the exit rate Σ' in the zombie lending scenario is smaller than Σ, the causal effect in the baseline scenario. Appropriate parameterization would result in a non-negative Σ and negative Σ'. The coefficient β_3 of the interaction term corresponds to the causal effect in the model. The significant negative estimates are thus consistent with an economy if there are zombie firms.

However, there may be other factors that would generate zombie firms. For example, some government programs support weak industries in the country. On the other hand, high bankruptcy costs may also delay the exit of a firm. If these forces dominate, then we could also observe a negative estimate of the interaction term, observationally equivalent. But it has little to do with the misallocation of credits.

5.4 Robustness

The baseline estimates from Equation (**??**), (**??**), and (**??**) are consistent with the zombie lending model predictions. Here I consider several robustness checks of the estimation.

Continuous productivity measure

The baseline specification uses the discrete TFPR measure, a dummy variable of different values for each tertile group. It's helpful to echo the model predictions. However, the discrete measure fails to capture the economic magnitude. More importantly, the classification of firms is ad-hoc. To solve the issue, I use the estimated TFPR directly.

Table 6 reports the regression results. They are qualitatively similar to the estimates with the discrete measure. With the estimates in hand, I can compute marginal effects. When the interest rate spread increases from 25 to 75 percentile, the exit probability for a typical high-productivity (at 75%-tile) firm will increase by 0.079; the exit rate for a typical low-productivity (at 25%-tile) firm will increase by 0.056. When the interest rate spread increases from 1 standard deviation below the mean to 1 standard deviation above the mean, the exit probability for a typical high-productivity (1 standard deviation above the mean) firm will increase by 0.057; the exit rate for a typical low-productivity (1 standard deviation below the mean) firm will increase 0.139. The economic magnitude is considerable.

Table 5: Firm-level response to interest rate spreads, with discrete productivity measure

	Exit	ΔLeverage	Investment rate
Interest rate	0.00502***	-0.368***	-0.141***
	(0.00103)	(0.0351)	(0.0303)
Low TFPR × interest rate	0	0	0
	(.)	(.)	(.)
Medium TFPR × interest rate	0.0230***	-0.0395	0.104
	(0.000936)	(0.0307)	(0.0709)
High TFPR × interest rate	0.0315***	-0.131***	-0.0980***
	(0.00100)	(0.0369)	(0.0290)
Low TFPR	0	0	0
	(.)	(.)	(.)
Medium TFPR	-0.0751***	0.185**	0.110
	(0.00186)	(0.0650)	(0.118)
High TFPR	-0.0912***	0.365***	1.095***
	(0.00196)	(0.0767)	(0.0620)
FE	✓	✓	✓
Firm controls	✓	✓	✓
R^2	0.506	.	0.005
N	2987507	1999750	1961710

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Fixed effects include country, and year x industry group effects.

Errors are clustered at firm level.

Alternative financial crisis measure

The baseline specification exploits the computed firm-specific interest rate. In a robustness check, I consider other proxies: government bond spreads and a crisis dummy.

Government bond yields are retrieved from OECD (2015) and International Monetary Fund (IMF) International Financial Statistics (IFS) database. The OECD provides 10-year government bond yields percent per annum. The long-term government bond yield in IFS refers to one or more series representing yields to maturity of government bonds or other bonds that would indicate longer term rates. The crisis dummy equals 1 if (i) the year is between 2010 and 2012, and (ii) the country is Portugal, Italy, Ireland, Greece, or Spain (PIIGS). The table is not shown here but the results are robust with these aggregate measures.

Table 6: Firm-level response to interest rate spread, with continuous productivity measure

	Exit	ΔLeverage	Investment rate
Interest rate	0.0245***	-0.424***	-0.112***
	(0.000670)	(0.0244)	(0.0332)
TFP × interest rate	0.0104***	-0.0328	-0.0988***
	(0.000449)	(0.0190)	(0.0220)
TFP	-0.0291***	0.0904*	0.525***
	(0.000872)	(0.0400)	(0.0432)
FE	✓	✓	✓
Firm controls	✓	✓	✓
R^2	0.505	.	0.005
N	2987507	1999750	1961710

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Fixed effects include country, and year x industry group effects.

Errors are clustered at firm level.

Selected countries

Since different countries have different reporting requirements of firm information, I conduct a robustness check for a selected of economies. Following Bloom et al. (2016a), I restrict my sample to four countries: France, Italy, Spain, and Sweden. I repeat the above regressions for the reduced sample. I find that in general, the results do not change significantly.

6 Conclusion

In this paper, I construct a firm investment model to investigate the firm dynamics, in particular the exit decision, in the firm-bank lending nexus. I show sometimes government policies can lead weak banks to not disclose the true market value of assets on the balance sheet. Instead, weak banks continue lending to otherwise insolvent firms. The continued financing keeps weak firms surviving in the economy, dragging down aggregate productivity. The model predictions suggest that I can detect whether there are zombie firms using a difference-in-difference estimation framework. If there are zombie firms due to zombie lending, then the causal effect of interest rate on the exit likelihood is smaller and can be negative. I estimate the regression to test the model hypothesis. The estimates are consistent with the theory and support the view that there are zombie firms in Europe during the sovereign debt crisis.

Although the empirical analysis is not a direct test of zombie lending, it does provide a new method to examine the misallocation from the perspective of firm dynamism. The reduced exit rates revealed that the declining firms' dynamics are not a phenomenon in the US. In Europe, unproductive and indebted firms demonstrate reduced response to a crisis. It implies that the reallocation during the debt crisis is not productivity-enhancing.

A potential explanation is the distorted bank lending decision. It's relevant in a financial crisis since many banks have a negative balance-sheet shock and even face a default risk. Conventional policy instruments would tighten capital adequacy requirements and inject capital into the economy. But the paper indicates that these policies might be ineffective and even counteractive. These policy measures ease banking lending positions but do not fully recover the bank from the shock. So it would be still optimal for banks to conduct zombie lending and under-report loan loss.

Last, the paper calls for well-designed policies which could prevent banks' high risk-taking behaviors. It's also important to have related labor market assistance programs during a crisis. With additional data work, further studies could return a more accurate estimated degree of credit misallocation in the economy.

To fully quantify the effect of zombie lending on the welfare of the economy, a general equilibrium model is necessary where the interest rate will be endogenously determined. In this framework, bank liability is made up of household deposits. Therefore aggregate shocks that negatively affect household wealth might exacerbate zombie lending as available bank assets drop and more banks will become under-capitalized. In terms of empirical work, the current version only focuses on firm financial variables. There is no direct evidence for zombie lending. It would be interesting to match the firm to its bank and investigate how bank financial performance affects firm-side dynamics.

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A Model

Condition 3 states the bank investment incentive constraints at t = 1.

Condition 3 (Investment incentive constraints at t = 1) *At $t = 1$, banks have incentives to invest in corporate loans and risky assets, rather than risk-free assets, if*

$$\begin{aligned} R_1[1-\Phi(\varepsilon^*)] + a\overline{\eta} &\geq (1+a)R_f \\ R_1[1-\Phi(\varepsilon^*)] &\geq \overline{\eta} \\ R_1 \gg R_f &\geq 1 \\ \overline{\eta} \geq R_f &\geq 1 \end{aligned}$$

where ε^* is the threshold productivity for firms to repay the loan; $\Phi(\varepsilon)$ is the productivity CDF; $\overline{\eta}$ is the expected return of risky asset investment; R_f is the risk-free interest rate.

Condition 4 and 5 describe the heterogeneous investment incentive conditions at t=2 when firm idiosyncratic productivity is realized.

Condition 4 (Investment incentive constraints at t = 2, with high-productivity firms) *At $t = 2$, banks with $\varepsilon_2 \geq \varepsilon^*$ have incentives to invest in corporate loan if:*

$$\begin{aligned} R_2[1-\Phi(\varepsilon^{**}(\varepsilon_2))] &\geq R_f \\ R_2[1-\Phi(\varepsilon^{**}(\varepsilon_2))] &\geq \overline{\eta} \end{aligned}$$

where $\varepsilon^{**}(\varepsilon_2)$ is the productivity threshold for loan repayment at t=3 when the realized productivity at $t = 2$ is ε_2; R_2 is the gross interest rate for corporate loan at $t = 2$.

Condition 5 (Investment incentive constraints at t = 2, with low-productivity firms) *At $t = 2$, banks with $\varepsilon_2 < \varepsilon^*$ bank has an incentive to invest in risky assets:*

$$\overline{\eta} \geq R_2[1-\Phi(\varepsilon^{**}(\varepsilon_2))]$$

where $\varepsilon^{**}(\varepsilon_2)$ is the productivity threshold for loan repayment at t=3 when the realized productivity at $t = 2$ is ε_2; R_2 is the gross interest rate for corporate loan at $t = 2$.

Proposition 6 (Zombie lending effects across time) *The (cumulative) exit rates for high- and low-type firms at different times are as follows:*

$$\textit{Pr(Exit at } t = 2 \mid \varepsilon_2 \geq \varepsilon^*\textit{)} = 0$$

$$\textit{Pr(Exit at } t = 2 \mid \varepsilon_2 < \varepsilon^*\textit{)} = \begin{cases} 1, & \textit{baseline} \\ 1-\Psi(\eta^*), & \textit{zombie} \end{cases}$$

$$\textit{Pr(Cumulative exit at } t = 3 \mid \varepsilon_2 \geq \varepsilon^*\textit{)} = \Phi(\varepsilon^{**}(\varepsilon_2))$$

$$\textit{Pr(Cumulative exit at } t = 3 \mid \varepsilon_2 < \varepsilon^*\textit{)} = \begin{cases} 1, & \textit{baseline: no additional exit, same as before} \\ 1-\Psi(\eta^*)+\Psi(\eta^*)\Phi(\varepsilon^{**}(\varepsilon_2)), & \textit{zombie} \end{cases}$$

Then the change in the exit response for high- and low-type firms are:

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 \geq \varepsilon^*) = \int_{\{\varepsilon_2 \geq \varepsilon^*\}} \Phi(\varepsilon^{**}(\varepsilon_2))d\varepsilon_2$$

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 < \varepsilon^*) = \begin{cases} 0, \text{ baseline} \\ \int_{\{\varepsilon_2 < \varepsilon^*\}} \Phi(\varepsilon^{**}(\varepsilon_2))\Psi(\eta^*)d\varepsilon_2, \text{ zombie} \end{cases}$$

Proposition 7 (Zombie effects when interest rate R_1 varies) *When corporate loan gross interest rate R_1 increases, $t = 2$ productivity threshold ε^* also increases to $\varepsilon^{*\prime}$, the new productivity threshold. Before R_1 increases,*

$$Pr(Exit \mid \varepsilon_2 \geq \varepsilon^{*\prime}) = 0$$

$$Pr(Exit \mid \varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}) = \begin{cases} 0, \text{ baseline} \\ 0, \text{ zombie} \end{cases}$$

$$Pr(Exit \mid \varepsilon_2 < \varepsilon^*) = \begin{cases} 1, \text{ baseline} \\ 1 - \Psi(\eta^*), \text{ zombie} \end{cases}$$

After R_1 increases,

$$Pr(Exit \mid \varepsilon_2 \geq \varepsilon^{*\prime}) = 0$$

$$Pr(Exit \mid \varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}) = \begin{cases} 1, \text{ baseline} \\ 1 - \Psi(\eta^*), \text{ zombie} \end{cases}$$

$$Pr(Exit \mid \varepsilon_2 < \varepsilon^*) = \begin{cases} 1, \text{ baseline} \\ 1 - \Psi(\eta^*), \text{ zombie} \end{cases}$$

Then the change in exit response for high- and low-type firms are:

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 \geq \varepsilon^{*\prime}) = 0$$

$$Aggregate\Delta Pr(Exit \mid \varepsilon_2 < \varepsilon^{*\prime}) = \begin{cases} \int_{\{\varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}\}} \Phi(\varepsilon^{*\prime}) - \Phi(\varepsilon^*)d\varepsilon_2, \text{ baseline} \\ \int_{\{\varepsilon^* \leq \varepsilon_2 < \varepsilon^{*\prime}\}} [\Phi(\varepsilon^{*\prime}) - \Phi(\varepsilon^*)] \times [1 - \Psi(\eta^*)]d\varepsilon_2, \text{ zombie} \end{cases}$$

Proposition 8 (Baseline lending effects across time and across interest rates R_2) *When corporate loan gross interest rate R_2 increases, $t = 3$ productivity threshold $\varepsilon^{**}|\varepsilon_2$ also increases. Suppose the new productivity threshold is $\varepsilon^{**\prime}$. Then changes in the exit rate before and after the interest rate shifts are:*

$$Aggregate\Delta Pr(Exit \text{ at } t = 3 \mid \varepsilon_2 \geq \varepsilon^*) = \int_{\{\varepsilon_2 \geq \varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \tag{4}$$

$$Aggregate\Delta Pr(Exit \text{ at } t = 3 \mid \varepsilon_2 < \varepsilon^*) = \begin{cases} 0, \text{baseline} \\ \int_{\{\varepsilon_2 < \varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]\Psi(\eta^*)d\varepsilon_2, \text{zombie} \end{cases}$$

Conditional on surviving firms at the end of $t = 2$, then in the zombie scenario (the only case where low-type firms have a chance of continued operation):

$$Aggregate\Delta Pr(\text{Exit at } t = 3 \mid \varepsilon_2 < \varepsilon^*, \eta_2 < \eta^*) = \int_{\{\varepsilon_2<\varepsilon^*,\eta_2<\eta^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \tag{5}$$

Then the difference in changed exit response across different productivity groups is:

$$\Sigma = \int_{\{\varepsilon_2<\varepsilon^*,\eta_2<\eta^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 - \int_{\{\varepsilon_2\geq\varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \tag{6}$$

Under regular conditions, we would expect $\Sigma > 0$.

Proposition 9 (Zombie lending effects across time and across interest rates R_2) *With an extended time horizon, the bank regains the continued lending decisions to maximize it's expected next-period equity facing policy instruments at $t = 3$.*

Without loss of generality, assume at t=3 zombie lending would occur between low-type banks (i.e, $\eta < \eta^{**}(\eta_2, \varepsilon_2)$) and low-type firms (i.e., $\varepsilon_3 < \varepsilon^{**}(\varepsilon_2)$), if and only if new zombie lending conditions (not stated explicitly) are satisfied.

Conditional on surviving firms at the end of $t = 2$, the increase in the exit rate before and after the interest rate shifts with zombie lending at $t = 3$ is :

$$\begin{aligned} &Aggregate\Delta Pr(\text{Exit at } t = 3 \mid \varepsilon_2 < \varepsilon^*, \eta_2 < \eta^*) \qquad (7)\\ &= \int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3\geq\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \\ &+ \int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3<\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))] \times [1 - \Psi(\eta^{**}(\eta_2, \varepsilon_2))]d\varepsilon_2 \end{aligned}$$

If there is zombie lending at $t = 3$, then the difference in changed exit response across different productivity groups is:

$$\begin{aligned} \Sigma' = &\int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3\geq\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \qquad (8)\\ &+ \int_{\{\varepsilon_2<\varepsilon^*,,\eta_2<\eta^*,\eta_3<\eta^{**}(\eta_2,\varepsilon_2)\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))] \times [1 - \Psi(\eta^{**}(\eta_2, \varepsilon_2))]d\varepsilon_2 \\ &- \int_{\{\varepsilon_2\geq\varepsilon^*\}} [\Phi(\varepsilon^{**\prime}(\varepsilon_2)) - \Phi(\varepsilon^{**}(\varepsilon_2))]d\varepsilon_2 \end{aligned}$$

With a factor of $[1-\Psi(\eta^{**}(\eta_2, \varepsilon_2))]$, the changed exit rate for low productivity firm is smaller. As a result, $\Sigma' < \Sigma$.

B Data Sources

The basic data sources are described in the text, but I give some more details here.

B.1 Firm-level data

The firm-level data I use is the AMADEUS. It's compiled by the Bureau van Dijk Electronic Publishing (BvD), a private-sector organization that seeks to compile accounting information on companies from all over the world. AMADEUS is one of the BvD products that contains financials, stock prices, and ownership information for approximately 19 million public and private companies in 44 European countries. It also provides information such as region, date of the corporation, last year available, industry, and an indicator of whether a firm is quoted. Amadeus data comes entirely from regulatory filings of local governments. In addition, to including large listed firms, like the Compustat, it also contains private small companies. The data set has names, industries, addresses, and ïňĄnancial accounting information from detailed harmonized balance sheets, income statements, and proïňĄt and loss accounts of ïňĄrms. It includes variables such as industry codes, employment, sales, capital, profits, and value-added.

AMADEUS classifies firms into different size categories: very large, large, medium-sized, and small. Companies on Amadeus are considered to be very large when they match at least one of the following conditions:

(a) Operating Revenue $\geq$ 100 million EUR (130 million USD)

(b) Total assets $\geq$ 200 million EUR (260 million USD)

(c) Employees $\geq$ 1,000

(d) Listed

Companies on Amadeus are considered to be large when they match at least one of the following conditions:

(a) Operating Revenue $\geq$ 10 million EUR (13 million USD)

(b) Total assets $\geq$ 20 million EUR (26 million USD)

(c) Employees $\geq$ 150

(d) Not Very Large

Companies on Amadeus are considered to be medium-sized when they match at least one of the following conditions:

(a) Operating Revenue $\geq$ 1 million EUR (1.3 million USD)

(b) Total assets $\geq$ 2 million EUR (2.6 million USD)

(c) Employees $\geq$ 15

(d) Not Very Large or Large

Companies on Amadeus are considered to be small when they are not included in another category.

Since registration of some form of company accounts is a legal requirement of all incorporated firms under EU law, the list of names should be the population. Bloom et al. (2016b) the data is reasonably comprehensive. For example, when comparing aggregate employment in the ORBIS populations to those from census data, we usually ïňĄnd a reasonably close match (e.g. Kalemli-Ozcan et al, 2015; Bloom et al. (2016a)).

However, there are some concerns about the AMADEUS. A well-known problem in the dataset is that BvD has a policy by which ïňĄrms that do not report during a certain period are automatically deleted from their later vintage products, creating an artiïňĄcial survival bias in the sample. Moreover, employment is sometimes a voluntary item for smaller firms. Kalemli-Ozcan et al. (2015) proposes one way to address these problems and maximize the coverage of firms and variables. They suggest accessing data in historical disks and using the AMADEUS and ORBIS (another BvD product) together. But due to the limited access to historical disks, the dataset I use here is still attained from the Wharton Research Data Service WRDS website. Considering the above concerns, I adopt a data-cleaning procedure and examine the validity of the sampling frame.

C Data cleaning procedure

I delete firms when the consolidation variable `CONSOL` equals to C2, which refers to the consolidated statement with an unconsolidated companion. I delete all firms in the financial, government sectors, and non-classifiable which correspond to three-digit US SIC industry codes [600, 700) and [900, 999]. Checking the variable `LSTATUS` in the data, I drop firms that are classified as “Inactive”, “Dissolved”, “In liquidation”, and “Bankruptcy”. Then I restrict the sample to firms to have positive employment, positive total assets, non-negative liabilities, and non-negative sales.

To conduct firm-level analysis, I construct the following variables:

(a) firm startup rate and exit rate: using the AMADEUS variables `DATEINC_year` and `LASTYEAR`

(b) job creation and job destruction rate: using Davis et al. (1998) formula

(c) labor productivity: defined as the ratio of sales and the number of employees, converted to common currency Euro

(d) leverage: defined as the ratio of total debt and total assets

(e) sales growth rate:

(f) employment growth rate: using Davis et al. (1998) formula

Other works using AMADEUS may use a firm's legal status to construct the exit variable. They label a firm exits if its status changes from "active" to "inactive". However, in my sample, firms' legal status does not change over time so I cannot exploit exit information from the variable LSTATUS. I also construct dummy variables for age groups and industry groups, as shown in Table **??** and A1.

Table A1: Classification of US SIC industry code groups

Group	SIC
1 - agriculture, forestry and fishing	<100
2 - mining	[100, 150)
3 - construction	[150, 200)
4 - manufacturing	[200, 400)
5 - transportation and public utilities	[400, 500)
6 - wholesale trade	[500, 520)
7 - retail trade	[520, 600)
8 - FIRE	[600, 700)
9 - service	[700, 900)
10 - public administration	≥900

C.1 Data summary

Table A2 provides information about the coverage of AMADEUS. The dataset is rich enough to have about 11 million observations each year. Most are private companies. In terms of firm size, over half of the firms are small in terms of employees, total assets, and operating revenue. And less than 1% firms are very large [2]. From 2010 to 2014, the share of small firms increases slightly while the percentage of very large firms are almost constant. The skewness of firm size suggests that the sample is likely to be representative of underlying firm distribution. Table A3 presents simple summary statistics from the cleaned sampling panel. Most variation of the panel is due to the cross-country heterogeneity.

Table A2: Firm coverage of AMADEUS

Year	Number of observations	Quoted firm	Small companies (%)	Very large companies (%)
2010	11,193,472	5,659	61.4	0.336
2011	11,735,884	5,839	63.5	0.332
2012	12,228,851	5,924	71.5	0.333
2013	12,860,547	5,885	75.0	0.322
2014	12,071,159	5,937	78.6	0.336

[2] I use the AMADEUS-defined company size variable `compcate`. See Appendix B for details.

Table A3: Firm level variables: summary statistics

	Mean	Median	Std.	Top 5%	Botton 5%
Leverage	0.747	0.615	1.158	1.593	0.009
R&D intensity	7.836	0.020	315.750	0.283	0
Sales growth	-0.165	0.014	1.293	0.692	-1.212
Employment growth rate	0.033	0	0.367	0.667	-0.5

D Data validation tests

I check the validation of the sampling frames as follows: first, I regress industry-level variables on the median value of labor productivity of an industry. The industry-level variables contain start-up rate, exit rate, job creation, and job destruction rate. All these variables are constructed using the dataset. This step is to examine the validation using the internal information. Second, I examine the validation at the country level using external variables such as GDP growth rate and unemployment.

E Production function estimation

I estimate firm-level productivity (TFPR), assuming that firm's output is given by a Cobb-Douglas production function:

$$Y_{it} = A_{it} L_{it}^{\beta_l} K_{it}^{\beta_k} \tag{9}$$

where Y_{it} is firm value added, A_{it} is physical productivity, L_{it} is labor input and K_{it} is capital input. β_l and β_k are the output elasticity of labor and capital respectively. Labor input is measured as the costs of employees or the number of employees. Capital input is measured as the book value of fixed tangible assets. I deflate these nominal values by the industrial producer price index from the Eurostat and the IMF International Financial Statistics database. The estimation method I use is primarily based on Olley and Pakes (1992) (OP), Levinsohn and Petrin (2003) (LP), Wooldridge (2009) (WRDG), and Ackerberg et al. (2015) (ACF). Essentially, TFPR is constructed as the residuals from a Cobb-Douglas production function with capital and labor.

Specifically, I estimate the production function in logarithm:

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \varepsilon_{it} \tag{10}$$

where y_{it}, l_{it}, k_{it}, ω_{it} are the corresponding logarithm of Y_{it}, L_{it}, K_{it} and A_{it} , described in the text. ε_{it} is a production shock that is not observable by the firm before making their input decisions at time t.

The value-added variable uses `VA` from the AMADEUS. Assume that the capital stock is predetermined. So current period capital stock is the lagged value of tangible fixed assets `TFAS`. For labor input, I consider both the costs of employees `STAF`, and the headcount `EMPL`. The former may help adjust for differences in the quality of workers across firms. Following the literature of Olley and Pakes (1992) and Levinsohn and Petrin (2003), I construct proxy variables for productivity. The proxy variable is an investment in OP and intermediate inputs in LP. Investment is computed as the gross investment level, which equals the difference of capital stocks, adjusted with depreciation DEPR. I use material costs MATE as the intermediate inputs variable.

Since Olley and Pakes (1992) uses the control function approach to address the endogeneity concern, follow-up works such as Levinsohn and Petrin (2003), Wooldridge (2009), Ackerberg et al. (2015), De Loecker (2011) and Petrin et al. (2011) improve the estimation procedure. I use the Stata command `prodest` developed by Mollisi and Rovigatti (2017) to estimate the productivity using Wooldridge (2009). I also provide estimation results using the ordinary least square (OLS), fixed effects (FE), LP and ACF in Table A4. There are also different Stata programs, in addition to `prodest`. For example, Petrin et al. (2004) develop `levpet` Yasar et al. (2008) write a command `opreg`. But compared to these available user-written commands, `prodest`

Estimation results of production function output elasticities are reported in Table A4. Panel 1 uses the number of employees as labor inputs and Panel 2 uses the costs of employees instead. Column (1) reports the results of an OLS linear regression of log output - value added - on free and state variables. In Column (2) I add country, industry, and year fixed effects. Column (3) to (5) uses Stata's command `prodest` using the approach of LP, WRDG, and ACF respectively. I use a third-degree polynomial approximation and also account for the firm exit. Given the endogeneity concern in the original OP method, I do not report estimates using the method.

From Table A4 we can see that the output elasticities estimates are similar across different approaches. The estimates of β_l are higher than 2/3 on average. The sum of β_l and β_k is smaller than 1, implying a decreasing return to scale production function at the aggregate level. I cross-check the estimation results with Fons-Rosen et al. (2017). Using a similar dataset, they instead estimate the production function by country and industry. But their mean and median value of the estimates is similar to my results.

Table A4: Production function output elasticities

	OLS	FE	LP	WRDG	ACF
Panel 1: Number of employees					
Labor elasticity (β_l)	0.852	0.873	0.684	0.708	0.815
	(0.0004)	(0.0004)	(0.0003)	(0.0004)	
Capital elasticity (β_k)	0.165	0.142	0.085	0.082	0.160
	(0.0003)	(0.0002)	(0.0007)	(0.0008)	
Panel 2: Costs of employees					
Labor elasticity (β_l)	0.751	0.715	0.688	0.724	0.783
	(0.0002)	(0.0005)	(0.0002)	(0.0001)	(0.0006)
Capital elasticity (β_k)	0.136	0.140	0.090	0.084	0.079
	(0.0004)	(0.0002)	(0.0003)	(0.0004)	(0.0007)

Standard errors in parentheses

Pooled over all countries and industries