

FINANCIAL FRICTIONS, DEBT TAX SHIELDS, AND THE MACROECONOMY

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Abstract

This paper evaluates the aggregate effects of financial frictions in the presence of tax shields of debt financing. Previous studies indicate that removing financial frictions will stimulate investment and reduce misallocation. However, with the tax bias of debt over equity, I show that the macroeconomic implications can be different depending on the interaction between tax shields and financial frictions. I build a dynamic quantitative general equilibrium model of investment. Heterogeneous firms face financial frictions in both debt and equity markets. I estimate the financial constraint parameter by targeting the slope of investment with respect to leverage. My results show that a small tax shield can mitigate misallocation while a large tax shield with loosening credit constraints exacerbate misallocation of capital. My counterfactual experiments demonstrate that by removing financial frictions, aggregate output increases 3% and welfare grows 2%. Aggregate gains can be 10 times larger when accounting for the tax shield.

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1 Introduction

The paper revisits a classical question in macroeconomics: what are the aggregate consequences of financial frictions? It is a commonly held view that financial frictions can reduce aggregate output and total factor productivity (TFP) ¹. However, literature in macroeconomics often ignores an important distortion of the tax shield of debt financing when modeling firm investment dynamics, which is a standard presumption in the corporate finance literature. Debt financing has a tax shield or tax benefit ² because interest on debt is deductible against corporate tax while returns to equity are not. The bias of the tax code toward debt may stimulate borrowing and contributes to higher leverage. In the absence of financial frictions, debt bias creates a deadweight loss. However, with financial frictions, the impact of debt bias is ambiguous. Therefore financial frictions can have different macroeconomic implications when interacting with debt tax shields.

The paper studies how the magnitude of tax shields interacts with the impact of financial frictions and quantifies the aggregate implications of financial frictions in the presence of the debt tax shield. I find that the bias in tax code toward debt over equity can exacerbate the aggregate effects of financial frictions. While a small increase in tax shield can mitigate misallocation, a larger tax shield with loosening financial constraint will aggravate misallocation of capital.

In this paper, I build a dynamic general equilibrium model in which heterogeneous firms finance investment with either equity or debt. Firms have two potential financing instruments. Firstly, they can issue one-period debt securities. Secondly, they can raise funds directly from shareholders in the event of a cash flow shortfall. The firm's financing decision is distorted by two types of financial frictions. The first is a borrowing constraint such that firms can only borrow up to the minimum possible cash flow and a fraction of their tangible capital. The second is the costly external equity issuance. The tax shield of debt in my model is a nonpecuniary wedge between the discount rate and the rate on debt. The debt tax shield makes firms behave impatiently and incentivizes firms to use debt to balance the tax advantage of borrowing with current and future expected financing costs. Firms are also subject to corporate income tax. I integrate my model into an otherwise standard model of firm dynamics with idiosyncratic productivity shocks, capital adjustment costs, firm entry and exogenous exit. Finally, I close the model with a representative household.

I structurally estimate the model parameters in a simulated method of moments (SMM) procedure using firm-level balance sheet data from COMPUSTAT between 1980 and 2018. Fol-

¹For example, see [Khan and Thomas \(2013\)](#), [Midrigan and Xu \(2014\)](#), [Moll \(2014\)](#)

²On this topic “tax shield”, “tax benefit”, “tax advantage”, and “debt bias” are used almost interchangeably in the literature.

lowing [Hennessy and Whited \(2007\)](#) and [Catherine et al. \(2021\)](#), I use the net equity issuance rate to pin down external equity financing costs. To identify the scope of financial constraint, I carefully choose the slope of investment with respect to leverage (defined as the debt-to-earnings ratio) as my target moment. In the data, this sensitivity is -1, which means a one-unit increase in debt-to-earnings is on average associated with a decline in investment of 1 percentage point. I estimate a linear cost of external equity issuance of about \$0.04 per \$1 new equity issued. My estimate of the borrowing constraint parameter is about 0.15, indicating a sizable degree of debt market financial frictions in the economy. Compared to the literature, the borrowing constraint is tighter in my model.

I use my structural model to explore the interaction between the tax shield and financial friction. By varying the magnitude of the tax shield and the degree of the borrowing constraint, I uncover that the tax benefit can be regarded as a double-edged sword. When firms are highly financially-constrained, a larger tax shield reduces the cost of capital. Therefore the tax shield can expand debt capacity and boost investment and firm growth. Aggregate productivity increases as well as the total credit. However, when firms are close to being unconstrained, an increase in the tax shield leads to over-borrowing and exacerbates the misallocation problem.

In my main quantitative experiment, I evaluate the importance of financial frictions on the aggregate economy. I find that by removing financial frictions in debt and/or equity markets, aggregate capital stocks grow about 10%, output gains 3%, productivity increases about 0.2%, and consumption-equivalent welfare rises about 2%. Consistent with the literature (e.g., [Catherine et al. \(2021\)](#)), the aggregate costs of financial frictions mainly come from an insufficient supply of capital input. Comparing the macroeconomic outcomes with and without debt bias, my results demonstrate that failing to account for the tax advantage of debt financing will underestimate the aggregate costs of financial frictions, which can be 10 times smaller. Moreover, in order to quantify the magnitude of the interaction between the tax shield and financial frictions, I decompose the aggregate gains into three components: (1) gains due to the tax shield alone; (2) gains due to financial frictions; and (3) gains due to the interaction term.

Lastly, I conduct a series of robustness checks. I first explore a range of alternative values of estimated structural parameters and externally calibrated parameters. I show that the implications of tax shields and the aggregate effects of financial frictions are robust in this exercise. Then I re-calibrate the model by targeting the mean leverage ratio, a moment widely used in the literature, to mainly identify the borrowing constraint parameter. I show that the model implies less severity of financial frictions: a smaller equity issuance cost estimate and a larger borrowing constraint parameter estimate. It has a flaw in matching the slope of investment to leverage. In addition, I show that there is a non-monotonic relationship between the borrow-

ing constraint and leverage. This analysis points out that using cross-sectional moments, such as leverage, to pin down structural parameters may have an identification problem.

The contribution of this paper is twofold. Firstly, to identify frictions in the debt market I propose to use the slope of investment with respect to leverage as a target moment. I show that using cross-sectional moments, such as the mean leverage ratio, will underestimate the extent of financial friction in the economy. Second, the paper demonstrates that a tax bias of debt over equity may exacerbate the aggregate effects of financial frictions of firms. While a small increase in the tax shield of debt financing can mitigate capital misallocation, a large increase in the tax shield with fewer financing frictions can aggravate the misallocation problem. This highlights the interaction between the magnitude of tax benefit and the effects of financial frictions and implies that removing financial frictions does not necessarily boost investment or lead to long-term economic growth.

This paper is organized as follows. Section 2 reviews related literature. Section 3 introduces the tax shield policy background and discusses the economic rationale for and against the debt bias in the current tax system. Section 4 formulates a dynamic investment model with financial frictions, debt tax shields, and other distortions. Section 5 structurally estimates the model. In section 6, I explore the interaction between tax shields and financial frictions and analyze the aggregate effect of financial frictions. I also do some robustness checks. Section 7 concludes.

2 Literature

My paper builds on several strands of literature. First, this paper contributes to the broad quantitative literature on the effects of financial frictions (e.g., [Hennessy and Whited \(2007\)](#), [Buera et al. \(2011\)](#), [Khan and Thomas \(2013\)](#), [Midrigan and Xu \(2014\)](#), [Moll \(2014\)](#), [Jo and Senga \(2019\)](#), and [Ottonello and Winberry \(2020\)](#)). The most closely related paper is [Catherine et al. \(2021\)](#) which quantitatively examines the impact of collateral constraints. My model shares common features with [Catherine et al. \(2021\)](#): borrowing constraints, costly external equity issuance, and a tax benefit. However, there are substantive differences. First, they focus on one source of financing friction: collateral constraints, while I also analyze the effects of other distortions (e.g., costly equity issuance and taxes) and the interactions between financial frictions and taxes. Second, in model specifications, the main differences are (a) my model has decreasing returns to scale in production; (b) goods are homogeneous and the good market is perfectly competitive, allowing for easier aggregation; (3) there is no real estate in my model. Third, in estimation, [Catherine et al. \(2021\)](#) exploit variations in real estate prices and use a reduced-form coefficient, the sensitivity of firm-level investment to collateral values, to identify the scope of financial frictions. They find that collateral constraint induces output

losses of 7.1%, and TFP (misallocation) losses of 1.4%. Instead, my estimation method uses the slope of investment with respect to the debt-to-EBITDA ratio where the computation is considerably simpler.

Second, My work relates to the extensive theoretical and empirical corporate finance literature that studies how preexisting debt affects firms' decisions to undertake new investments (e.g, [Kalemli-Özcan et al. \(2020\)](#), [Crouzet and Tourre \(2020\)](#), [Barbiero et al. \(2020\)](#), [Jordà et al. \(2020\)](#), [Albuquerque \(2021\)](#), and [Perla et al. \(2020\)](#)). The long-standing question goes back to the seminal work of [Myers \(1977\)](#). He hypothesizes that outstanding debt may distort investment downwards as profits primarily benefit existing debt holders but not potential new investors in the presence of default risk. The paper refers to it as a “debt overhang problem”. Relative to the literature (e.g., [Diamond and He \(2014\)](#)), I make two simplifications in modeling. First, my model only features one type of debt instrument, the short-term debt; second, firms exit from the markets only because of an exogenous death shock. However, with costly external equity financing, the simplicity can still capture the nexus between the firm's capital structure and its investment efficiency in the absence of endogenous and/or strategic default.

I follow [Crouzet and Tourre \(2020\)](#) and [Blickle et al. \(2022\)](#) and compute the slope of investment with respect to leverage. The slope is negative on account of debt overhang effects. Here leverage is defined as the ratio of debt and EBITDA. This leverage measure relates borrowing to a proxy for cash flow which can remove some of the endogeneity associated with the firm's financing decisions. While [Crouzet and Tourre \(2020\)](#) uses the moment to identify the adjustment cost parameter, I use it to primarily pin down parameters governing financial frictions.

Third, my findings echo on the literature that studies misallocation of capital and aggregate productivity (e.g., [Restuccia and Rogerson \(2008\)](#), [Hsieh and Klenow \(2009\)](#), [Midrigan and Xu \(2014\)](#), [Gilchrist et al. \(2013\)](#), and [Karabarbounis and Macnamara \(2021\)](#)). These studies support that financing frictions lead to input misallocation. A major difference with the aforementioned works is that my paper allows for tax deductibility of interest. I contribute to the literature that financing frictions may mitigate the extent of misallocation. In the presence of other distortions such as the tax benefit of debt financing, the relationship between financial frictions and aggregate productivity is non-monotonic.

Last but not the least, this paper speaks to the literature on taxation, firm capital structure, and dynamic trade-off theory in corporate finance. The standard theory from [Modigliani and Miller \(1958\)](#) states that a firm shall be indifferent between various sources of financing for its projects. However, With tax deductibility of interest, firms raise debt to balance the value of interest tax shields against costs associated with financial distress and bankruptcy, which determines an optimal amount of debt in a firm's capital structure (e.g., [Modigliani and Miller](#)

(1963), [Miller \(1977\)](#), [Myers \(1984\)](#), [Hennessey and Whited \(2005\)](#), and [Li et al. \(2016\)](#)). I show that the tax shield is a key assumption driving the non-monotonic results and it plays an important role to understand the real effects of financial frictions.

3 Policy Background

In this section, I provide an overview of the tax policy on debt financing as well as economic rationale for and against the tax benefits of debt.

The tax system in the United States and around the world has a long history that generally favors debt over equity because interest expenses on debt are tax-deductible, while a similar deduction for the cost of equity (in the form of dividends or share appreciation) is rarely ever granted (e.g., [Bank \(2014\)](#)). As a result, returns to equity-financed investment are taxed at both the corporate level and the shareholder level, while debt-financed investment faces only shareholder-level tax. The differential treatment creates a tax bias in favor of debt financing. Some empirical works (e.g., [Feld et al. \(2013\)](#), [Heider and Ljungqvist \(2015\)](#)) document that taxes and the debt tax shield are important drivers of firms' capital structure.

The economic rationale for tax bias for debt is related to market failures (i.e., adverse selection, agency problem, signaling) that discourage the use of external finance and lead to underleverage, suggesting a role for tax policy that favors debt (e.g., [De Mooij \(2012\)](#), [Pozen and Goodman \(2012\)](#)). However, there is a general consensus that these justifications are not convincing (e.g., [De Mooij \(2012\)](#)). In contrast, many studies argue that the tax bias for debt can distort firms' decisions on financing and investment. In particular, tax advantages for debt finance can disproportionately hurt young and innovative firms that invest heavily in R&D expenditures for lack of assets that can be easily used as collateral. Moreover, the tax shield can generate negative externalities as excess debt increases systemic risk and macroeconomic instability (e.g., [Schularick and Taylor \(2012\)](#), [Jordà et al. \(2013\)](#)).

Given the above concerns, the debt tax shield has been the subject of analysis and discussion among lawmakers. It gains renewed interest in light of the 2008 financial and economic crisis. Many governments and international organizations have started to make tax reforms and introduce various measures (e.g., Allowance for Corporate Equity (ACE) and Comprehensive Business Income Tax (CBIT)) to reduce or eliminate the tax benefit of debt. Belgium was among the very few countries in the world that neutralized the debt bias. Since 2006, Belgium allows for a notional interest deduction on equity capital. In the United States, Congress passed the Tax Cuts and Jobs Act (TCJA) in 2017. In addition to a reduction in the corporate tax rate, the Act limits interest deductibility permanently to 30% of earnings. Previously, interest expenses are generally fully deductible. At the supranational level, European Commission

proposed a debt-equity bias reduction allowance to help businesses access the financing they need and to become more resilient in May 2022. The allowance on equity is deductible for 10 consecutive tax years. The proposal also introduces a reduction of debt interest deductibility by 15%.

4 The Model

In this section, I present a general equilibrium model of dynamic investment by heterogeneous firms under tax distortions and financial frictions on both debt and equity financing: (1) borrowing constraints, and (2) costly external equity. The economy consists of a continuum of a unit mass of firms. Firms produce a homogeneous good consumed by a representative consumer. Time is discrete on an infinite horizon. The model builds on [Strebulaev and Whited \(2012\)](#) and it is closely related to [Hennessy and Whited \(2007\)](#), [Katagiri \(2014\)](#) and [Catherine et al. \(2021\)](#).

4.1 Firms

Production Technology

Firms are risk-neutral. Each firm owns predetermined capital stock k and hires labor n . It produces a homogeneous good with decreasing-returns-to-scale production technology:

$$y = zk^\alpha n^\nu, 0 < \alpha + \nu < 1 \quad (1)$$

z is a firm's idiosyncratic total factor productivity and follows a Markov chain. There is no aggregate uncertainty in the model. Labor is flexible and is hired in a competitive labor market at a wage of W . The capital accumulation of each firm is standard, $i = k' - (1 - \delta)k$. The investment decision takes place before the realization of the next period's productivity z' . Therefore my model is different from [Midrigan and Xu \(2014\)](#) and [Moll \(2014\)](#) where firms rent capital in a competitive capital rental market. The capital stock depreciates at the rate δ each period. Investment is reversible and subject to a standard quadratic adjustment cost $\psi(k, i)$:

$$\psi(k, i) = \psi_1 + \frac{1}{2}\psi_0 \left(\frac{i}{k}\right)^2 k \mathbb{1}_{i \neq 0} \quad (2)$$

Firms face corporate taxation. Firms' profits net of interest payments and capital depreciation are taxed at the rate τ . Given the optimal choice of labor, the firm's earnings before an interest

payment, a tax payment, and depreciation (EBITDA) is defined as follows:

$$\pi(k, z) = zk^\alpha n^\nu - Wn = (1 - \nu)y \quad (3)$$

Debt Market Frictions

Each firm faces a borrowing limit on a one-period discount debt. Following [Strebulaev and Whited \(2012\)](#), this borrowing constraint restricts the amount of new debt level, b' , by firms' value of their capital and after-tax future earnings in the worst state of the world. The borrowing constraint can thus be expressed as:

$$b' \leq (1 - \tau)(\underline{z}k'^\alpha \underline{n}'^\nu - W\underline{n}') + s(1 - \delta)k' \quad (4)$$

in which \underline{z} is the lowest possible value that the shock z can attain. k' is the firm's future period capital. \underline{n} is the corresponding optimal labor input. I assume that only a fraction of the capital stock s can be liquidated. In this specification of borrowing constraint, debt is risk-free and firms never default. Firms can always repay the debt. The state variable b takes both positive and negative values. A negative value of b denotes cash.

Consistent with corporate finance literature, I assume that debt financing has a tax shield, which creates an incentive for firms to increase their leverage. Let τ^S denote the tax shield rate. Thus, the present value of debt issued in the next period is $\frac{b'}{1+r(1-\tau^S)}$. r is the risk-free interest rate. The discount rate of $1 + r(1 - \tau^S)$ implies the interest deductions on the debt. At the baseline, τ^S and τ have the same value.

Equity Market Frictions

The costly external equity injections carry a fixed and proportional cost and are thus a more expensive source of funds than internally generated cash flows. Suppose $e_1(k, k', b, b', z)$ is the net cash flow. The cost of external cash flow is thus given by:

$$\eta(x) = (-\eta_0 + \eta_1 e_1) \mathbb{1}_{e_1 < 0} \quad (5)$$

where $\eta_0 > 0$ and $\eta_1 > 0$ are the fixed and linear components of the equity cost function.

For a given desired next-period capital stock k' and debt b' , $e_1(k, k', b, b', z)$ is defined to be:

$$e_1(k, k', b, b', z) = (1 - \tau)(y - Wn) + (1 - \delta)k - b - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)}$$

where the firm's internal fund:

$$e_{IN}(k, b, z) = (1 - \tau)(y - Wn) + (1 - \delta)k - b$$

The external funding requirement is equal to the desired capital stock less the internal fund:

$$\begin{aligned} e_{EX}(k, k', b, b', z) &= k' + \psi(k, i) - e_{IN}(k, b, z) - \frac{b'}{1 + r(1 - \tau^S)} \\ &= -e_1(k, k', b, b', z) \end{aligned}$$

Assume that firms cannot retain earnings. If cash inflows exceed optimally chosen cash outflows, $e_1(k, k', b, b', z) > 0$, then the firm must pay the entire fund out to shareholders. On the other hand, if the distribution to shareholders is negative, $e_1(k, k', b, b', z) < 0$, then shareholders need to fill the gap e_{EX} . Together, debt derives value from the costly external equity and the tax benefit.

Entry and Exit

In each period, incumbent firms may exit the economy. The death shock $\pi_d \in (0, 1)$ is common across firms. Since the debt is risk-free, therefore there is no endogenous exit in the model. If the firm will continue to operate for the next period, then it may invest and/or borrow. If the firm will exit, then it keeps the residual of current production after wage bill and debt repayment. Then the cash flow $e(k, k', b, b', z)$ is defined as:

$$\begin{aligned} &e(k, k', b, b', z) \\ = &\begin{cases} e_0 = (1 - \tau)(y - Wn) + (1 - \delta)k - b & \text{if the firm will exit} \\ e_1 = (1 - \tau)(y - Wn) + (1 - \delta)k - b - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)} & \text{if the firm will survive} \end{cases} \end{aligned}$$

Exiting firms are replaced by an equal mass of entrants so that the total mass of production firms is fixed in each period. Entering firms are fully equity-financed with initial capital stock k_0 . The initial productivity of an entrant, z_0 , is randomly drawn from the ergodic distribution of z . They then proceed as incumbent firms.

Timing

At the beginning of each period, an incumbent firm is identified with a state vector (k, b, z) : the predetermined capital stock k , the amount of debt carried from the previous period b , and

the current period idiosyncratic productivity z . The firm makes the optimal labor choice and learns its exogenous exit status. Labor choices are static. Therefore firms with the same (k, z) will make the same labor choices, regardless of their exit shock realizations.

If the firm is assigned to exit, it simply chooses labor n to maximize its current dividend payment to shareholders. The dividends e_0 are output, less wage payment, and debt repayment, alongside the returns from capital liquidation. If the firm is continuing beyond the period, then additionally, it makes intertemporal decisions on future capital k' and borrowing b' . The current dividend payment is e_1 .

For the next period, the initial state of a continuing incumbent is (k', b', z') . It starts operating, along with entering firms with initial state $(k_0, 0, z_0)$.

Firm Distribution

The distribution of firms over (k, b, z) is denoted by a probability measure μ , defined on the Borel algebra \mathcal{S} by the open subsets of the product space, $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$. The evolution of the firm distribution Γ is determined in part by the actions of continuing firms and in part by entry and exit:

$$\begin{aligned} \mu' &= \Gamma(\mu) \\ \mu'(z_j) &= (1 - \pi_d) \int_{\{(k, b, z_i) | (k', b') \in A\}} \pi_{ij} d\mu(k, b, z_i) + \pi_d \chi(k_0) H(z_j), \quad \forall (A, z_j) \in \mathcal{S} \end{aligned} \tag{6}$$

where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$.

Firm Problem

Let $V(k, b, z)$ be the expected discounted value of a firm that enters with (k, b) and idiosyncratic productivity z at the beginning of the current period. Then the Bellman equation for an incumbent firm is:

$$\begin{aligned} V(k, b, z) &= \max_{k', b'} \left\{ \pi_d e_0 + (1 - \pi_d) \left[e_1 + \eta(e_1) + \frac{1}{1+r} \mathbb{E} \left(V(k', b', z') | z \right) \right] \right\} \\ &\text{subject to} \\ b' &\leq (1 - \tau)(\underline{z} k'^\alpha \underline{n}'^\gamma - W \underline{n}') + s(1 - \delta)k' \end{aligned} \tag{7}$$

where

$$\begin{aligned}
e_0 &= (1 - \tau)(y - Wn) + (1 - \delta)k - b && \text{if the firm will exit} \\
e_1 &= (1 - \tau)(y - Wn) + (1 - \delta)k - k' - \psi(k, i) + \frac{b'}{1 + r(1 - \tau^S)} - b && \text{if the firm will survive} \\
\eta(e_1) &= (-\eta_0 + \eta_1 e_1) \mathbb{1}_{e_1 < 0} \\
\psi(k, i) &= \frac{1}{2} \psi_0 \left(\frac{i}{k} \right)^2 k \mathbb{1}_{i \neq 0}
\end{aligned}$$

4.2 Household

An infinitely-lived representative household holds one-period noncontingent bonds B^H and owns firms. Given the real wage W and the risk-free rate r , the household determines its current consumption C^H , hours worked N^H and new bond holdings $B^{H'}$, to maximize its lifetime expected utility:

$$V^H(B^H) = \max_{C^H, N^H, B^{H'}} \{ \log C^H - \varphi N^H + \beta^H V^H(B^{H'}) \} \quad (8)$$

subject to

$$C^H + \frac{B^{H'}}{1 + r} = WN^H + B^H + T^H + \Pi^H$$

where

$$\Pi^H = \int \left(\underbrace{(1 - \pi_d)[e_1 + \eta(e_1)]}_{\text{Continuing}} + \underbrace{\pi_d e_0}_{\text{Exit}} - \underbrace{\pi_d \left[k_0 - \frac{b'}{1 + r(1 - \tau^S)} \right]}_{\text{Entrant}} \right) d\mu(k, b, z)$$

Π^T is the dividend payment of the firm. T^H is the lump-sum transfer rebated to the household. β^H is the discount factor for future utility.

4.3 Equilibrium Definition

Consider a stationary industrial equilibrium of the model. The equilibrium is defined by a set of value functions $\{V, V^H\}$, decision rules $\{k', b', n, B^{H'}, N^H\}$, prices $\{W, r\}$, and a measure of firms μ such that:

1. All firms optimize: V solves (7) with associated policy rules $\{k', b', n\}$.

2. The household optimizes: V^H solves (8) with associated policy rules $\{C^H, B^{H'}, N^H\}$.
3. The bond market clears:

$$B^H = \int b d\mu = \mathbf{B}$$

4. Government budget is balanced:

$$T^H = \int \left(\underbrace{\tau(y - Wn)}_{\text{Corporate income tax}} - \underbrace{\frac{\tau r b'}{(1+r)[1+r(1-\tau^S)]}}_{\text{Tax benefit of debt}} \right) d\mu$$

5. The good market clears:

$$\begin{aligned} \mathbf{C} &= \int y d\mu - \underbrace{\pi_d k_0}_{\text{Entrant}} + \underbrace{\pi_d \int (1-\delta)k d\mu}_{\text{Exit}} - \underbrace{(1-\pi_d) \int ((i+\psi) - \eta) d\mu}_{\text{Continuing: investment, capital AC, and equity financing cost}} \\ &= \mathbf{Y} + \pi_d(1-\delta-\kappa_0)\mathbf{K} - (1-\pi_d)(\mathbf{I} + \mathbf{\Psi} - \mathbf{H}) \end{aligned}$$

where aggregate output \mathbf{Y} , capital stock \mathbf{K} , investment \mathbf{I} , adjustment costs $\mathbf{\Psi}$, equity issuance cost \mathbf{H} , consumption \mathbf{C} . κ is the fraction of the steady-state aggregate capital stock held by each entrant and $k_0 = \kappa_0 \mathbf{K}$

6. The labor market clears:

$$N^H = \int n(k, b, z) d\mu = \mathbf{N}$$

5 Estimation

Because the model has no closed-form solution, I estimate key parameters using a Simulated Method of Moments (SMM) in this section. I estimate the model in two steps. First, I exogenously fix a subset of parameters. Second, I estimate the remaining parameters to match moments in the data.

5.1 Parameterization

I assume that firm productivity is a log-normal AR(1) process:

$$\ln(z') = \rho \ln(z) + \varepsilon'$$

where $\varepsilon' \sim \mathcal{N}(0, \sigma^2)$. The parameters (ρ, σ) of the driving process are unknowns that must be estimated. I use the procedure of [Tauchen \(1986\)](#) to discretize the stochastic shock into a 5-state Markov chain.

5.2 Predefined parameters

The model comprises 16 parameters. I externally calibrate 11 of them. I set the capital share $\alpha = 0.25$ and labor share $\nu = 0.6$, implying a decreasing return to scale of 0.85. These values are close to the values commonly used in the investment literature (e.g., [Khan and Thomas \(2013\)](#), [Bloom et al. \(2018\)](#), [Jeenas \(2019\)](#), and [Ottonello and Winberry \(2020\)](#)). The depreciation rate δ is fixed at 0.1, in line with [Bloom et al. \(2018\)](#) and [Karabarbounis and Macnamara \(2021\)](#). I use a tax rate τ of 0.2, consistent with [Gomes and Schmid \(2010\)](#). τ^S is also set at 0.2. As employed by [Khan and Thomas \(2013\)](#), I set the exogenous exit rate π_d to 0.1. I use the relative initial capital stock of potential entrants to the average incumbent firm κ_0 as 0.2, which is based on estimation in [Jeenas \(2019\)](#). I set both the fixed investment adjustment cost ψ_1 and the fixed equity issuance cost η_0 to 0, following [Catherine et al. \(2021\)](#). I set the risk-free interest rate $r = 0.04$, as in [Jo and Senga \(2019\)](#). The value is standard in the real business cycle literature. The subjective discount factor β^H implies the long-run real interest rate. So the value is $\frac{1}{1+r} = 0.96$. Finally, I follow [Bloom et al. \(2018\)](#) and set the labor disutility φ at 2. Table 1 summarizes these externally calibrated parameters.

5.3 Data and Target Moments

I structurally estimate the remaining 5 parameters: the productivity persistence ρ , the standard deviation of innovation to productivity σ , the convex capital adjustment cost ψ_0 , the linear equity issuance cost η_1 , and the borrowing constraint s .

To calculate data moments, I employ the COMPUSTAT industrial files. I use the fundamental annual sample of nonfinancial, unregulated publicly listed US firms from 1980 to 2018. Details on the data and sample selection are provided in Appendix A. I choose moments that are informative about parameters. The SMM estimates parameters by minimizing the distance

between model-implied moments and their empirical counterparts:

$$\hat{\theta} = \arg \min_{\theta} [m(\theta) - m(X)]' W [m(\theta) - m(X)] \quad (9)$$

where $m(X)$ and $m(\theta)$ are the vector of moments from the data X and model with parameters θ , respectively. W is the moment weighting matrix. To obtain an asymptotically efficient SMM estimator, w is the inverse of the variance-covariance matrix of data moments. I describe my model solution algorithm and structural estimation method in detail in Appendix B.

In total, I use 6 moments. Given the model is overidentified, the identification is not a one-to-one mapping between data moments and structural parameters. All of the model parameters jointly affect all of these moments in some way. Nonetheless, some moments have a greater influence on certain parameters.

Idiosyncratic Productivity Process (ρ, σ)

I primarily use three moments to pin down parameters governing the productivity process (ρ, σ) . Following [Midrigan and Xu \(2014\)](#) and [Catherine et al. \(2021\)](#), I use 1-year and 5-year standard deviation of sales growth rate $(\sigma(\Delta y_{-1}), \sigma(\Delta y_{-5}))$ to simultaneously estimate these two parameters. The volatility of the short-run and long-run empirical growth rates is 0.35 and 0.8 respectively in the data. I also use the volatility of the debt-to-assets ratio, $\sigma(b/k)$. In the data, the ratio is 0.32.

Capital Adjustment Cost ψ_0

I choose the dispersion of the investment rate, $\sigma(i/k)$, to infer the capital adjustment cost parameter ψ_0 , as in [DeAngelo et al. \(2011\)](#) and [Eisfeldt and Muir \(2016\)](#). Large adjustment costs lead the firm to a smooth investment. Adjustment cost should also have a sizeable effect on the volatility of short-term output $\sigma(\Delta y_{-1})$. Therefore, larger adjustment costs can be identified by smaller investments and short-term output volatility.

Equity Market Frictions: Linear External Equity issuance Cost η_1

In the spirit of [Hennessy and Whited \(2007\)](#), the cost of external equity issuance parameter η_1 , heavily depends on the average ratio of net positive equity issuance scaled by assets, $\mu(e/k)$, because a higher cost of external equity financing implies lower equity issuance. The value is 0.1 in the sample.

Debt Market Frictions: Borrowing Constraint s

As mentioned above, I use the slope of the investment rate with respect to the ratio of debt and EBITDA, β , to primarily identify the borrowing constraint parameter. It stands in contrast to literature where the leverage ratio has been widely used ³. The literature typically relates high leverage to a higher degree of financial friction. However, high leverage can result from overborrowing when firms are financially slack. Therefore, using leverage as a target can yield a biased estimate.

The pre-existing leverage is a debt overhang measure in empirical studies (e.g., [Kalemli-Özcan et al. \(2020\)](#), [Blickle et al. \(2022\)](#), [Perla et al. \(2020\)](#)). A negative investment response to pre-existing debt suggests that excessive levels of debt can reduce investment. I use EBITDA as a proxy of cash flow to scale the debt. This measure can remove some of the endogeneity associated with financing decisions.

In Appendix [A.2](#), I compare my data moments to literature. As robustness, I estimate a version of the model (Model 2) using the average leverage, $\mu(b/k)$, where the leverage is defined as the ratio of debt to assets.

5.4 Results

Table [2](#) reports parameter estimates and model fits for both targeted moments and non-targeted moments.

Parameter Estimates

The estimated productivity process is persistent with $\rho = 0.872$. The estimated standard deviation of the innovation to productivity σ is 0.109. The estimated value for convex capital adjustment cost ψ_0 is 0.056. I estimate that the linear equity issuance cost $\eta_1 = 0.036$ and the borrowing constraint $s = 0.147$.

Parameter estimates are broadly comparable to existing estimates in the literature. The productivity process parameters (ρ , σ) are close to estimates (0.909 and 0.118) in [Khan and Thomas \(2013\)](#). The estimated adjustment cost ψ_0 is greater than 0.004 in [Catherine et al. \(2021\)](#) and less than 0.1519 in [DeAngelo et al. \(2011\)](#). The estimate of the linear cost of equity issuance η_1 is similar to 0.059 in [Hennessy and Whited \(2005\)](#) and somewhat smaller than 0.091 in [Hennessy and Whited \(2007\)](#) and [Catherine et al. \(2021\)](#). The borrowing constraint s is smaller than some other estimates in the literature (e.g., 0.25 in [Catherine et al. \(2021\)](#)).

³[Catherine et al. \(2021\)](#) provides a detailed survey of the use of the moment.

A possible reason for a tight s is a different specification for credit constraints. In my model, borrowing constraint is not only based on assets but also on the expected minimum earnings.

Model Fit

Panel B of Table 2 shows that the model matches the targeted moments reasonably well, despite being over-identified. My baseline model roughly matches the dispersion of leverage and investment rate. It somewhat overpredicts the sales growth rate volatility. But it matches perfectly the average net equity issuance rate, and importantly, the slope of investment with respect to the debt-to-EBITDA ratio β .

As for non-targeted moments, the model under-matches the mean leverage $\mu(b/k)$. The model implies -0.051, as opposed to 0.1 from the data. Other than that, the model is able to reproduce the average investment rate and the volatility of equity issuance. It also leads to a successful fit for the mean assets-to-sales ratio and investment correlation.

In Panel C of Table 2, I also examine the dispersion of the marginal product of capital (MPK). Following David et al. (2022), I measure it by calculating the standard deviation of log MPK. In my sample, the within-industry MPK dispersion is 0.814. The dispersion generated by the model is 0.347, which accounts for about 50% of total MPK dispersion in the data. Appendix A.1 describes how I get the empirical estimate.

6 Quantitative Analysis

In this section, I conduct three counterfactual experiments. First, I explore the interaction between tax shields and financial frictions. Then, I describe the quantitative aggregate implications of financial frictions. I compare the macroeconomic variables of the baseline economy with tax shield to the No Tax Shield counterfactual. Lastly, I evaluate the results across a range of parameterizations of my model.

6.1 Interaction between Tax Shields and Financial Frictions

Figure 1 plots the effect of the tax shield rate τ^s on aggregate debt and TFP for the baseline economy. The left panel of Figure 1 shows that as the tax shield increases from 0.1 to 0.6, the total credit increases by about 3%. With a larger tax shield, the effective cost of debt is smaller. As a result, firms have a greater incentive to borrow and finance investments via debt in order to reap a greater tax benefit of interest deduction. The right panel of Figure 1 reveals that the impact on productivity is non-monotonic: the productivity increases first and then decreases. This suggests that by raising investment and stimulating borrowing, the debt bias can improve

efficiency. However, a larger extent of tax relief with excessive debt finance will exacerbate preexisting distortions and hamper long-term economic growth.

To inspect the interaction between tax shields and financial friction, I plot the aggregate effects of the tax shield rate τ^s for both the Baseline economy and a less-friction economy ($s = 1$) in Figure 2. When the borrowing constraint is relaxed, the magnitude of the effects of the tax shield rate is much larger. Firms that are financially constrained previously now have a larger debt capacity. The aggregate debt now increases by 25%, much greater than 3% in the Baseline economy. The negative impact of a larger tax shield on aggregate TFP is also more significant with a loosening borrowing constraint.

Similarly, I plot the aggregate effects of borrowing constraint s at different tax shield rates in Figure 3. For economies with tax shields ($\tau^s > 0$), the borrowing constraint has a nonlinear impact on aggregate productivity following the same economic intuition. With less debt market friction (i.e., a bigger value of s), the larger the tax shield is, the greater drag it exerts on the economy. But for an economy with No Tax Shield ($\tau^s = 0$), advantages gained from the improvement of a relaxed borrowing constraint will only advance marginally and then level off after a specific point.

6.2 The Aggregate Effects of Financial Frictions

In this subsection, I use the structural model to evaluate the aggregate effects of financial friction. Table 3 reports percentage changes of macroeconomic variables when financial frictions are removed. My Unconstrained Benchmark (B1) corresponds to a model when equity is free ($\eta_1 = 0$) and firms can pledge all value of the capital stock as collateral ($s = 1$). Firms can still benefit from the tax shield of debt financing. Note that this is not the first-best allocation since the economy still has distortions from taxes and capital adjustment costs. Therefore, I derive efficient allocation and TFP loss. Table 3 reports TFP losses for both the Baseline economy and the No Tax Shield economy as well.

Column (1) reports results for the Baseline estimated economy relative to Benchmark B1. It shows that financial frictions greatly affect investment and economic growth. Lifting financial frictions increases 10% for aggregate capital stock and 3% for aggregate output. It also improves the labor market. Both aggregate employment and wage increase. From a larger gain in capital stock than in output, the table shows that the output loss mainly comes from the loss of the input of capital stock. Removing financial frictions improves access to credit for financially-constrained firms. This boosts investment and attenuates the mislocation of input. Therefore, TFP increases.

In contrast, Column (2) of Table 3 reports aggregate outcomes for the No Tax Shield coun-

terfactual economy. Benchmark B1 in this scenario has no tax shield either. When debt financing does not have a tax advantage, free equity implies the same capital allocations regardless of the value of borrowing constraint s . The table demonstrates that aggregate gains of capital stock and output in Column (2) are 10 times smaller than those in Column (1). This means that not accounting for the tax shield will underestimate the aggregate costs of financial frictions.

In Appendix, I consider two alternative unconstrained benchmarks: (1) $s = 1$, and (2) $\eta_1 = 0$, which is also the unconstrained benchmark used in [Catherine et al. \(2021\)](#). I report results in Appendix Table C2.

Efficient Allocation and TFP Loss

I follow procedures in [Gilchrist et al. \(2013\)](#) and [Karabarbounis and Macnamara \(2021\)](#) and compute the efficient level of aggregate TFP and the size of TFP loss.

Consider a problem faced by a social planner is to maximize aggregate output, given aggregate labor and capital:

$$Y = \max_{k_i, n_i} \int (z_i k_i^\alpha n_i^\nu) di, 0 < \alpha + \nu < 1$$

$$\text{subject to } \int n_i di = N \text{ and } \int k_i di = K$$

where K and N are the aggregate capital and labor stocks. The solution to this problem implies that the marginal product of labor (MPL) and the marginal product of capital (MPK) are equated across firms. Then the optimal input choices are given by:

$$n_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{N}{\Gamma} \right)$$

$$k_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{K}{\Gamma} \right)$$

$$\text{where } \Gamma = \int z_i^{\frac{1}{1-(\alpha+\nu)}} di$$

Under the efficient allocation, the first-best TFP is:

$$\text{TFP}^{FB} = \frac{Y}{K^\alpha N^\nu} = \Gamma^{1-(\alpha+\nu)} = \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} di \right)^{1-(\alpha+\nu)}$$

TFP loss is then defined as:

$$\text{TFP Loss} = \frac{\text{TFP}^{FB}}{\text{TFP}} - 1$$

[Gilchrist et al. \(2013\)](#) and [Karabarbounis and Macnamara \(2021\)](#) assume that (z, MPK) are jointly log-normally distributed across firms. I also make the same assumption and the relative TFP loss due to resource misallocation is approximated by (See Appendix C for details):

$$\begin{aligned} \text{Relative TFP Loss} &= \log\left(\frac{\text{TFP}^{FB}}{\text{TFP}}\right) \\ &\approx \frac{1}{2}\alpha(1-\alpha)\left(\frac{1-\nu}{1-\alpha-\nu}\right)^2 \text{Var}(\log(\text{MPK})) \end{aligned}$$

Financial frictions will reduce TFP by increasing the dispersion in MPK across firms.

Table 3 reports TFP losses for the economy with and without financial frictions. Lifting financial frictions, TFP loss decreases from 8% to about 7.5%. Compared to the economy without both debt and equity market frictions (Column (3) $\eta_1 = 0, s = 1$), the economy with equity market frictions (Column (2) $s = 1$) has a smaller TFP loss.

Decomposing the Aggregate Effects

To quantify the magnitude of the interaction between tax shields and financial frictions, I consider the Constrained Efficiency Benchmark (B2) with free equity and no tax shield ($\eta_1 = 0, \tau^S = 0$) and then decompose the difference of macroeconomic variables between the Baseline economy and Benchmark B2 into three components: due to tax shield alone, due to financial frictions alone, and due to the interaction between the tax shield and financial frictions.

Column 1 of Table 4 reports the percentage changes in aggregate variables of the baseline economy relative to Benchmark B2.

Columns 2 to 4 of Table 4 report the fraction of three components, respectively. Column 2 computes the difference of aggregate variables between the economy with free equity ($\eta_1 = 0$) and Benchmark B2. This represents the gains due to the tax shield alone. Column 3 computes the difference of aggregate variables between the economy with no tax shield ($\tau^S = 0$) and Benchmark B2. This represents the gains due to financial frictions alone. Column 4 computes the remainder of the difference between the Baseline economy and Benchmark B2, after subtracting Columns 2 and 3, which represents the interaction between tax shield and financial frictions.

6.3 Robustness Checks

6.3.1 Aggregate Implications with Alternative Parameter Values

Table 5 reports various parameter robustness checks. Starting at benchmark point estimates from Table 2 and externally calibrated parameter values from Table 1, I vary the magnitude of a single parameter up and down to alternative values used in literature and compare the implications for a range of macroeconomic aggregates, while keeping other parameters fixed. Each row corresponds to a different robustness check. In the last row, I modify multiple parameter values so that they are consistent with Catherine et al. (2021).

I also examine the response of aggregate TFP to the change of borrowing constraints. Figure 4 shows that overall the changes are qualitatively similar to the baseline results that curves are hump-shaped.

6.3.2 Targeting Mean Leverage Ratio

Estimation Results Estimation results are reported in Appendix Table C3. Model 1 is the Baseline model. Model 2 is calibrated by targeting the mean leverage. The estimates suggest that firms are less financially-constrained in this economy.

Compared to the Baseline model, the idiosyncratic TFP shocks are less persistent and less volatile ($\rho = 0.835, \sigma = 0.076$). I estimate a smaller capital adjustment cost $\psi_0 = 0.008$, a less expensive equity issuance cost $\eta_1 = 0.008$, and a more relaxed borrowing constraint $s = 0.349$.

Model 2 slightly underpredicts the average net equity issuance rate. It matches perfectly the key moment of the average leverage ratio $\mu(b/k)$. However, it does a much worse job of matching the slope moment β . The model-implied slope is -9.5, significantly smaller than -1 in the data.

Effects of Financial Frictions Figure 5 shows that the effects of financial frictions on firm-level responses are similar to the baseline model. The average and volatility of investment rate and leverage ratio change nonlinearly with the borrowing constraint parameter s .

Table C4 reports macroeconomic outcomes for Model 2 in the bottom panel. The non-monotonicity holds for aggregate variables including capital stock, output, and employment. Figure 6 shows that productivity eventually decreases when relaxing the borrowing constraint.

Parameter Identification The non-monotonicity relationship between cross-sectional moments (e.g., leverage) and debt market financial friction parameter s may give rise to an identification problem. s can be well identified only if leverage is large enough. The top panel of

Figure 5 shows that when the moment of leverage is relatively large (e.g., 0.1), then it is well above the U shape and falls in the region where the value is monotonically increasing with s . Therefore, a larger targeted moment can avoid the identification problem and help pin down the parameter s . But the failure of Model 2 to match the key moment, the slope of investment to the debt-EBITDA ratio β , indicates that it is still problematic to only target cross-sectional moments. On the contrary, there is no identification concern using the slope of investment in the baseline model since the response is unambiguously monotonic.

7 Conclusion

Motivated by recent tax policy reforms around the world, this paper studies how debt tax shields interact with financial frictions and affect firms' financing and investment decisions as well as the aggregate economy.

To quantify the aggregate implications of financial frictions in the presence of a tax shield, I build a dynamic general equilibrium model of investment by heterogeneous firms with a tax bias of debt over equity and financial frictions in both debt and equity markets. I integrate our framework in an otherwise standard model with idiosyncratic productivity shocks, capital adjustment costs, firm entry, and exogenous death shock, and a representative household. I structurally estimate the model parameters by matching micro from public firms' data. In particular, I identify financial constraint parameters by targeting the slope of investment with respect to leverage. My estimate of the borrowing constraint parameter is about 0.15, indicating a sizable degree of financial frictions in the economy.

An exploration of the interaction between the tax shield and financial friction shows that on one hand, the tax-induced debt bias can reduce the negative impacts of financial frictions for credit-constrained firms by incentivizing them to borrow and invest. On the other hand, the resulting over-borrowing from a larger increase in the tax shield and reduced financial frictions may distort resource allocation and hence drag down aggregate productivity.

I find that in the presence of a tax benefit of debt, the aggregate impacts of financial frictions are about 10 times larger. Aggregate capital stocks grow about 10%, output gains 3%, productivity increases about 0.2%, and consumption-equivalent welfare rises about 2%. This suggests that the debt tax shield may exacerbate the aggregate effects of the financial constraints of firms.

Overall, these findings highlight the importance of debt bias in the tax code on corporate capital structure. Understanding the interaction between tax benefit of debt financing and financial frictions is critical in evaluating the aggregate costs of financial frictions.

References

- Albuquerque, B. (2021). Corporate credit booms, financial constraints, and the investment nexus. *Available at SSRN 3827924*.
- Bank, S. A. (2014). Historical perspective on the corporate interest deduction. *Chap. L. Rev.*, 18:29.
- Barbiero, F., Popov, A., and Wolski, M. (2020). Debt overhang, global growth opportunities, and investment. *Journal of Banking & Finance*, 120:105950.
- Belo, F., Lin, X., and Yang, F. (2019). External equity financing shocks, financial flows, and asset prices. *The Review of Financial Studies*, 32(9):3500–3543.
- Blickle, K. S., Santos, J. A., et al. (2022). The costs of corporate debt overhang following the covid-19 outbreak. Technical report, Federal Reserve Bank of New York.
- Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, 86(3):1031–1065.
- Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and development: A tale of two sectors. *American economic review*, 101(5):1964–2002.
- Catherine, S., Chaney, T., Huang, Z., Sraer, D. A., and Thesmar, D. (2021). Quantifying reduced-form evidence on collateral constraints. *Available at SSRN 2631055*.
- Crouzet, N. and Tourre, F. (2020). Can the cure kill the patient? corporate credit interventions and debt overhang. *Unpublished Manuscript, Northwestern University*.
- David, J. M., Schmid, L., and Zeke, D. (2022). Risk-adjusted capital allocation and misallocation. *Journal of Financial Economics*, 145(3):684–705.
- Davis, S. J., Haltiwanger, J., and Schuh, S. (1996). Small business and job creation: Dissecting the myth and reassessing the facts. *Small business economics*, 8(4):297–315.
- De Mooij, R. A. (2012). Tax biases to debt finance: Assessing the problem, finding solutions. *Fiscal studies*, 33(4):489–512.
- DeAngelo, H., DeAngelo, L., and Whited, T. M. (2011). Capital structure dynamics and transitory debt. *Journal of financial economics*, 99(2):235–261.
- Diamond, D. W. and He, Z. (2014). A theory of debt maturity: the long and short of debt overhang. *The Journal of Finance*, 69(2):719–762.

- Eisfeldt, A. L. and Muir, T. (2016). Aggregate external financing and savings waves. *Journal of Monetary Economics*, 84:116–133.
- Feld, L. P., Heckemeyer, J. H., and Overesch, M. (2013). Capital structure choice and company taxation: A meta-study. *Journal of Banking & Finance*, 37(8):2850–2866.
- Gilchrist, S., Sim, J. W., and Zakrajšek, E. (2013). Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics*, 16(1):159–176.
- Gomes, J. F. and Schmid, L. (2010). Levered returns. *The Journal of Finance*, 65(2):467–494.
- Heider, F. and Ljungqvist, A. (2015). As certain as debt and taxes: Estimating the tax sensitivity of leverage from state tax changes. *Journal of financial economics*, 118(3):684–712.
- Hennessy, C. A. and Whited, T. M. (2005). Debt dynamics. *The journal of finance*, 60(3):1129–1165.
- Hennessy, C. A. and Whited, T. M. (2007). How costly is external financing? evidence from a structural estimation. *The Journal of Finance*, 62(4):1705–1745.
- Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics*, 124(4):1403–1448.
- Jeenas, P. (2019). Firm balance sheet liquidity, monetary policy shocks, and investment dynamics. *Work*, 5.
- Jo, I. H. and Senga, T. (2019). Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation. *Review of Economic Dynamics*, 32:68–93.
- Jordà, Ò., Kornejew, M., Schularick, M., and Taylor, A. M. (2020). Zombies at large? corporate debt overhang and the macroeconomy. Technical report, National Bureau of Economic Research.
- Jordà, Ò., Schularick, M., and Taylor, A. M. (2013). When credit bites back. *Journal of money, credit and banking*, 45(s2):3–28.
- Kalemli-Özcan, Laeven, L., and Moreno, D. (2020). Debt overhang, rollover risk, and corporate investment: Evidence from the european crisis. Technical report, Working Paper.
- Karabarbounis, M. and Macnamara, P. (2021). Misallocation and financial frictions: the role of long-term financing. *Review of Economic Dynamics*, 40:44–63.

- Katagiri, M. (2014). A macroeconomic approach to corporate capital structure. *Journal of Monetary Economics*, 66:79–94.
- Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.
- Li, S., Whited, T. M., and Wu, Y. (2016). Collateral, taxes, and leverage. *The Review of Financial Studies*, 29(6):1453–1500.
- Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American economic review*, 104(2):422–58.
- Miller, M. H. (1977). Debt and taxes. *the Journal of Finance*, 32(2):261–275.
- Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American economic review*, 48(3):261–297.
- Modigliani, F. and Miller, M. H. (1963). Corporate income taxes and the cost of capital: a correction. *The American economic review*, 53(3):433–443.
- Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221.
- Myers, S. C. (1977). Determinants of corporate borrowing. *Journal of financial economics*, 5(2):147–175.
- Myers, S. C. (1984). Capital structure puzzle.
- Ottonello, P. and Winberry, T. (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica*, 88(6):2473–2502.
- Perla, J., Pflueger, C., and Szkup, M. (2020). Doubling down on debt: Limited liability as a financial friction. Technical report, National Bureau of Economic Research.
- Pozen, B. and Goodman, L. (2012). Capping the deductibility of corporate interest expense. *Tax notes*, 137.
- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics*, 11(4):707–720.
- Schularick, M. and Taylor, A. M. (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, 102(2):1029–61.

- Strebulaev, I. A. and Whited, T. (2012). Dynamic models and structural estimation in corporate finance. *Foundations and Trends (R) in Finance*, 6(1–2):1–163.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics letters*, 20(2):177–181.

Table 1: Externally Calibrated Parameter Values

Parameter	Description	Value	Source
Technology			
α	Capital share	0.25	Typical in literature
ν	Labor share	0.6	Typical in literature
δ	Capital depreciation rate	0.1	Bloom et al. (2018)
τ	Corporate tax rate on profits	0.2	Gomes and Schmid (2010)
τ^S	Corporate tax rate on interest	0.2	Gomes and Schmid (2010)
π_d	Exit rate	0.1	Khan and Thomas (2013)
κ_0	Fraction of the steady-state aggregate capital stock held by each entrant	0.2	Jeenas (2019)
Financial Frictions			
ψ_1	Fixed investment adjustment cost	0	Catherine et al. (2021)
η_0	Fixed external financing/equity issuance cost	0	Catherine et al. (2021)
Preference			
φ	Labor disutility	2	Bloom et al. (2018)
β^H	Subjective discount factor	0.96	Jo and Senga (2019)
Price			
r	Risk-free interest rate	0.04	Jo and Senga (2019)

Note: The table reports the notation, description, value, and source for the set of externally calibrated parameters.

Table 2: Model Estimation Results

Panel A. Estimated Parameters

Parameter	Description	Model	SE
ρ	Productivity persistence	0.872	(0.0020)
σ	SD of innovations to productivity	0.109	(0.0005)
ψ_0	Convex investment adjustment cost	0.056	(0.0018)
η_1	Linear equity issuance cost	0.036	(0.0001)
s	Frac. of debt that can be collateralized	0.147	(0.0182)

Panel B. Model Fit: Targeted Moments

Moment	Description	Model	Data
$\sigma(b/k)$	debt rate volatility	0.365	0.32
$\sigma(i/k)$	investment rate volatility	0.478	0.53
$\sigma(\Delta y_{-1})$	1-year sales growth rate volatility	0.374	0.35
$\sigma(\Delta y_{-5})$	5-year sales growth volatility	0.938	0.8
$\mu(e/k)$	average net equity issuance rate	0.100	0.1
β	slope of i/k wrt debt/EBITDA	-0.998	-1

Panel C. Model Fit: Non-Targeted Moments

Moment	Description	Model	Data
$\mu(b/k)$	mean leverage	-0.051	0.1
$\mu(i/k)$	mean investment rate	0.187	0.40
$\mu(k/y)$	mean assets/sales	1.671	1.76
$\text{corr}(i/k, i/k_{-1})$	autocorrelation of investment rate	0.281	0.32
$\sigma(e/k)$	net equity issuance rate volatility	0.243	0.45
$\sigma(\log \text{MPK})$	dispersion in $\log(\text{sales/capital})$	0.347	0.814

Notes: Panel A of the table reports point estimates and standard errors (in parentheses) for each of the parameters estimated via the SMM. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is the inverse of the moment covariance matrix. Panel B and C report model-implied moments and data moments. The empirical moments are computed from a panel of U.S. firms in Compustat annual data from 1981-2016.

Table 3: Aggregate Effects of Financial Frictions

	Distance to $\eta_1 = 0, s = 1$	
	(1) Baseline	(2) No Tax Shield
$\Delta\%$ Capital	9.88	0.96
$\Delta\%$ Labor	0.50	-0.33
$\Delta\%$ Output	2.96	0.24
$\Delta\%$ TFP	0.20	0.20
$\Delta\%$ Welfare	2.51	0.46

Notes: The table compares various aggregate quantities across the estimated baseline economy with tax shields ($\tau^S = 0.2$) and the No Tax Shield counterfactual economy ($\tau^S = 0$) relative to the frictionless benchmark $\eta_1 = 0, s = 1$. The moments are computed from the stationary distributions μ of the respective economies. $\Delta\%$ Welfare represents the percentage consumption equivalent variation.

Table 4: Decomposing Aggregate Effects of Financial Frictions

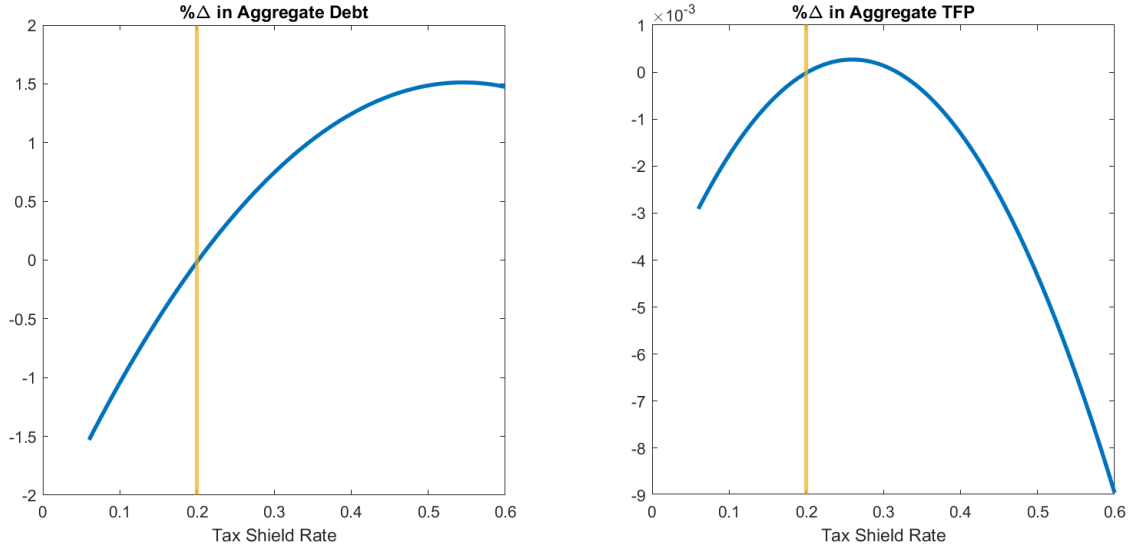
	(1)	(2)	(3)	(4)
	$\Delta\%$		Δ	
		Tax shield alone (%) $\eta_1 = 0$	Constraint alone (%) $\tau^S = 0$	Interaction (%)
Capital	1.73	−192.3	55.8	236.6
Labor	1.29	−96.5	−25.8	222.3
Output	1.41	−111.1	17.0	194.1
TFP	0.20	2.6	97.1	0.3
Welfare	0.67	−105.4	68.1	137.3

Table 5: Robustness to Alternative Parameters

	Source	K	B	N	Y	TFP	U
Benchmark		0.775	0.094	0.309	0.571	1.231	-1.213
Low capital revenue elasticity	$\alpha = .21$ Ottonello and Winberry (2020)	0.602	0.071	0.308	0.528	1.189	-1.283
High capital revenue elasticity	$\alpha = .28$ Jo and Senga (2019)	0.952	0.113	0.317	0.633	1.279	-1.133
Low labor revenue elasticity	$\gamma = .5$ Bloom et al. (2018)	0.754	0.105	0.258	0.541	1.143	-1.162
High labor revenue elasticity	$\gamma = .64$ Ottonello and Winberry (2020)	0.814	0.081	0.337	0.617	1.304	-1.194
Low depreciation	$\delta = .06$ Midrigan and Xu (2014)	1.128	0.137	0.292	0.605	1.229	-1.058
High depreciation	$\delta = .15$ Hennessy and Whited (2007)	0.538	0.044	0.325	0.538	1.234	-1.349
Low initial capital	$\kappa_0 = .1$ Khan and Thomas (2013)	0.772	0.093	0.306	0.568	1.231	-1.200
High initial capital	$\kappa_0 = .23$	0.784	0.095	0.312	0.576	1.232	-1.215
Firm exit rate	$\pi_d = .08$	0.794	0.096	0.318	0.584	1.232	-1.231
Low risk-free interest rate	$r = .03$	0.853	0.104	0.313	0.589	1.232	-1.192
High risk-free interest rate	$r = .06$	0.657	0.076	0.310	0.550	1.233	-1.250
Low labor disutility	$\varphi = 1.28$ Katagiri (2014)	1.088	0.134	0.486	0.816	1.232	-0.860
High labor disutility	$\varphi = 2.48$ Jo and Senga (2019)	0.649	0.073	0.250	0.481	1.231	-1.387
Catherine et al. (2021)		1.108	0.203	0.301	0.638	1.279	-1.053

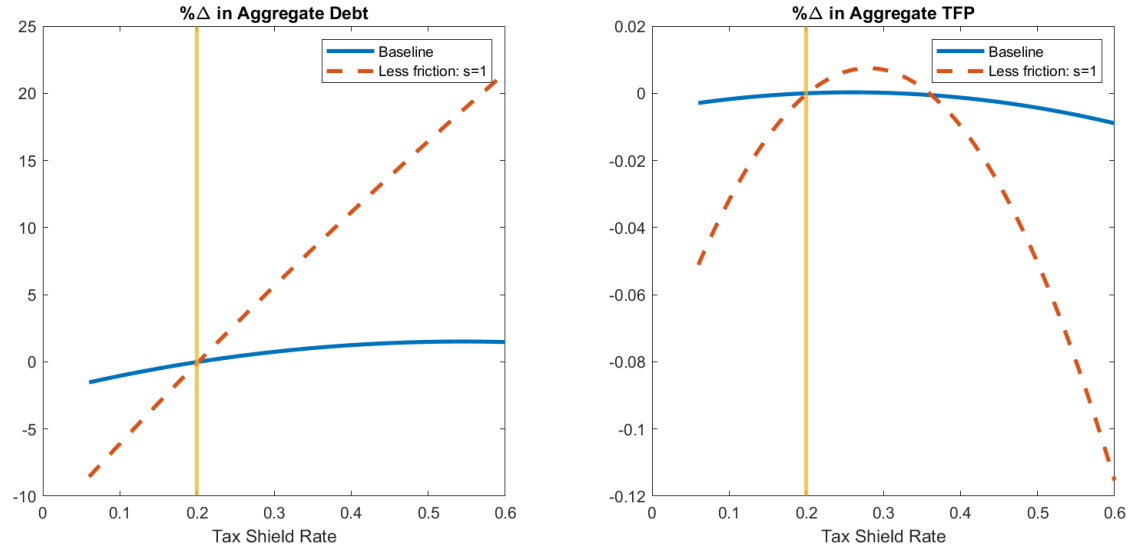
Note: The table reports aggregate capital, debt, employment, output, consumption, productivity, and utility. Different rows report these values for robustness checks varying the value of the indicated parameter, holding all other parameters fixed at their benchmark values from Table 1 and Table 2. The last row set parameter values consistent with [Catherine et al. \(2021\)](#) where $\rho = 0.851, \sigma = 0.131, \psi_0 = 0.004, \eta_1 = 0.091, s = 0.25, \delta = 0.06, r = 0.03, \pi_d = 0.08, \tau = 0.33$.

Figure 1: Aggregate Impact of the Tax Shield Rate: Baseline Economy



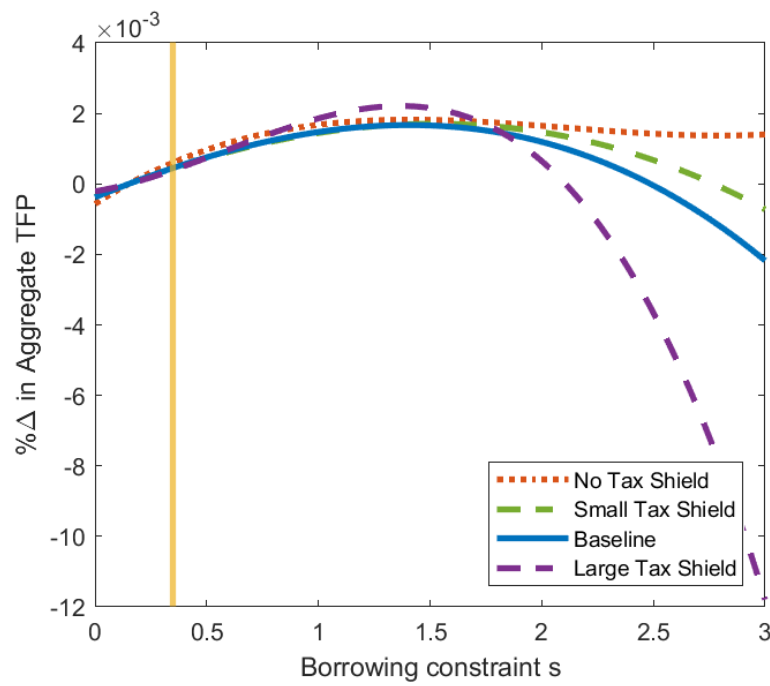
Notes: The figure plots the percentage change in aggregate debt and productivity with different tax shield rates τ^S for the Baseline economy. The yellow vertical line corresponds to the Baseline tax shield rate $\tau^S = \tau = 0.2$. Numerical comparative statics are smoothed using a polynomial approximation.

Figure 2: Aggregate Impact of the Tax Shield Rate at Different Borrowing Friction



Notes: The figure plots the percentage change in aggregate debt and productivity with different tax shield rates τ^S for the Baseline economy (blue solid line) and counterfactual economy with $s = 1$ (red dotted line). The yellow vertical line corresponds to the Baseline tax shield rate $\tau^S = \tau = 0.2$. Numerical comparative statics are smoothed using a polynomial approximation.

Figure 3: Impact of the Borrowing Constraint on Aggregate productivity at Different Tax Shield



Notes: The figure plots the percentage change in aggregate productivity with different borrowing constraints s for the Baseline economy (blue solid line) and counterfactual economies with different tax shield rates τ^S . The yellow vertical line corresponds to the SMM estimate of borrowing constraint s . Numerical comparative statics are smoothed using a polynomial approximation.

Figure 4: Aggregate TFP of the Borrowing Constraint s for Alternative Parameter Values

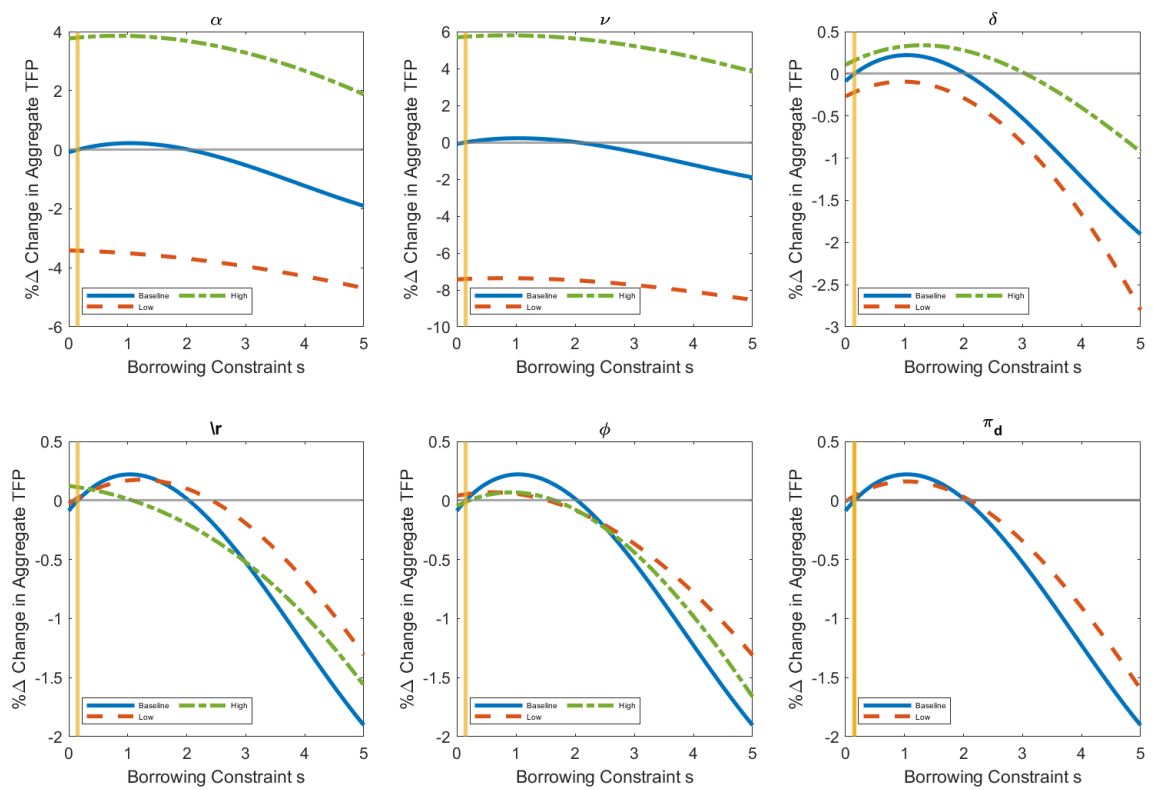
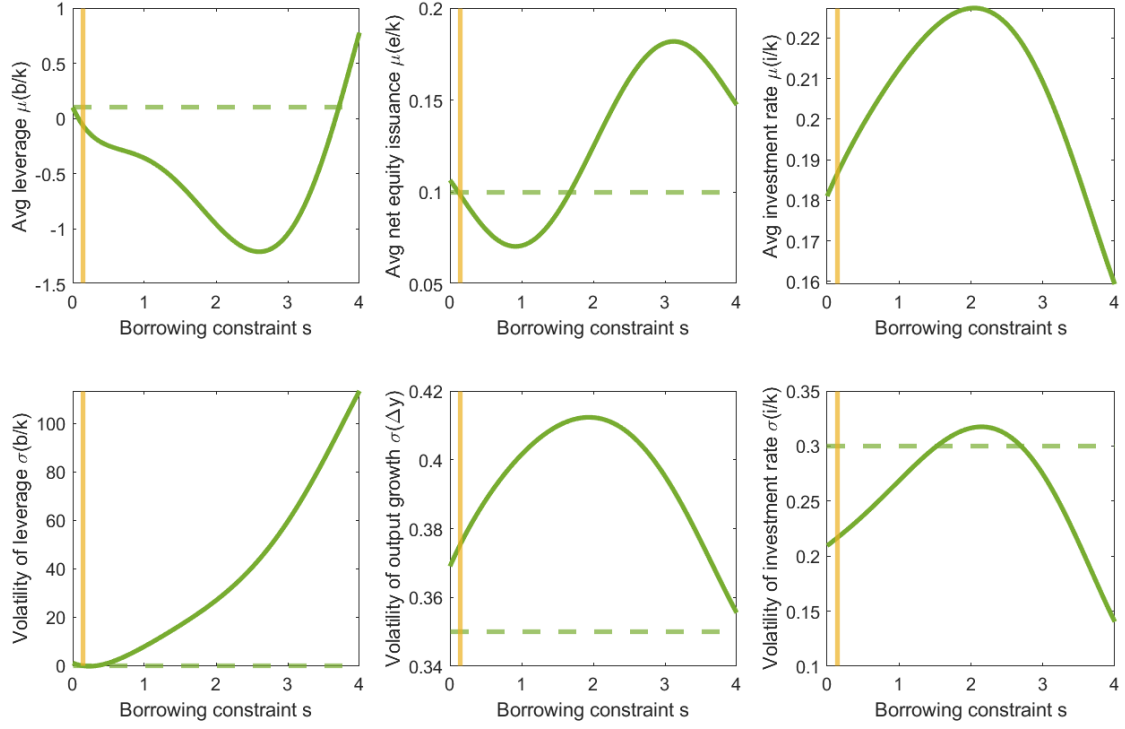


Figure 5: Effect of Borrowing Constraints s on Firm Characteristics

(a) Model 1: targeting β



(b) Model 2: targeting $\mu(b/k)$

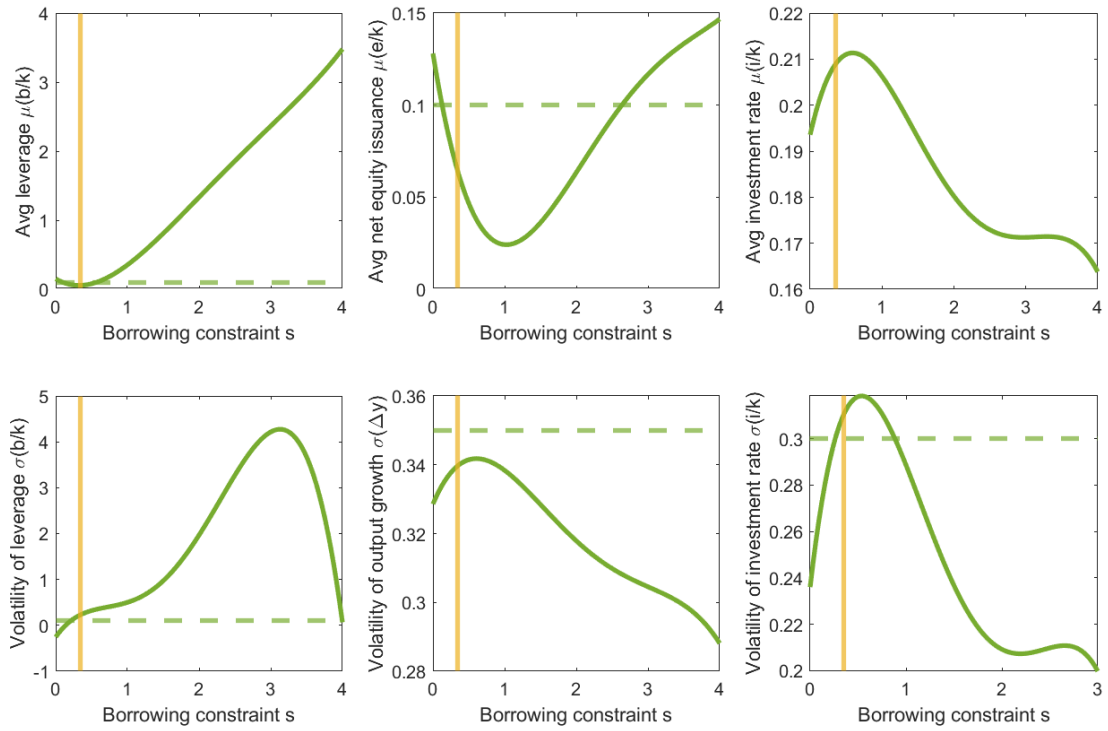
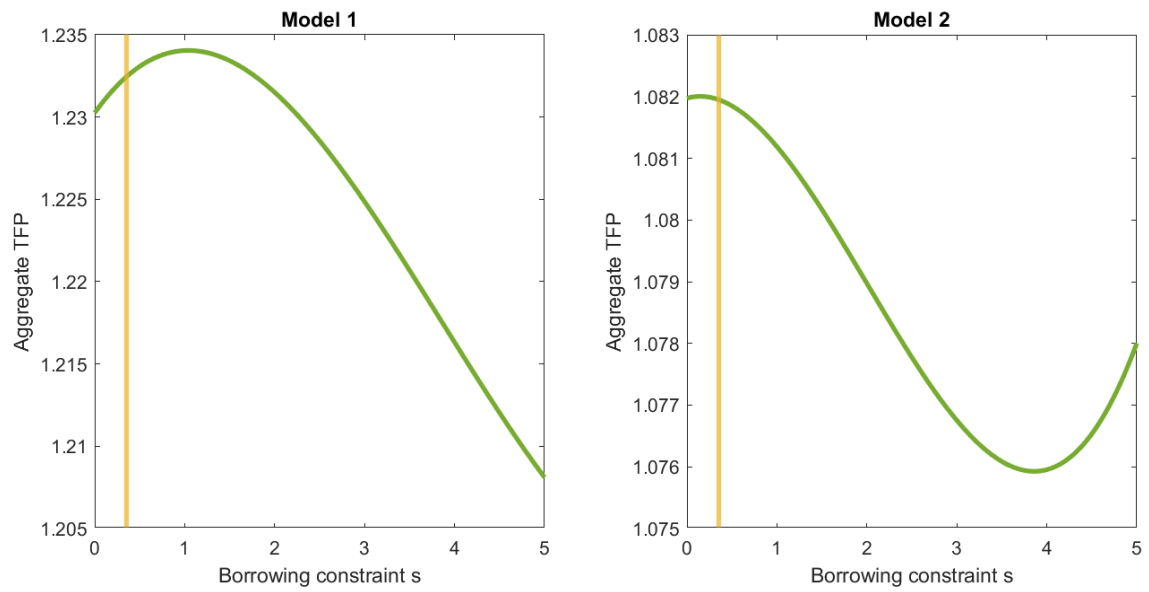


Figure 6: Effect of Borrowing Constraints s on Aggregate TF



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

A Data

I obtained data on U.S. nonfinancial firms from the Standard and Poor’s CRSP/Compustat industrial fundamental annual files through Wharton Research Data Services (WRDS) in 2021. The data is an unbalanced panel that covers from 1980 to 2018.

A.1 Variable Definition

The variable definition and construction follow standard practices in the literature. Table C1 lists model notation, definition and Compustat data item for each variable.

I use the beginning-of-the-period capital (PPENT) as the firms’ capital stocks. Investment $i = k' - (1 - \delta)k$ is defined as capital expenditures on property, plant, and equipment (CAPXV). Investment rate i/k

Debt b in my model is the net debt. The empirical counterpart is the sum of debt in current liabilities (DLC) and long-term debt (DLTT) minus cash (CHE). I consider two definitions of the leverage ratio. The first is defined as the ratio of net debt to lagged assets, $(DLC+DLTT-CHE)/L.AT$. This definition is used for a targeted moment of debt rate volatility $\sigma(b/k)$ and a non-targeted moment of mean leverage ratio $\mu(b/k)$. The second is the debt-to-EBITDA ratio, $(DLC+DLTT-CHE)/EBITDA$, which is used to compute the slope of investment with respect to pre-existing debt. I follow [Crouzet and Tourre \(2020\)](#) to compute the slope of investment with respect to the debt-to-EBITDA ratio as follows:

$$\beta = \frac{\text{Cov}(i/k, b/EBITDA)}{\text{Var}(b/EBITDA)} \times 100$$

where $b/EBITDA$ is the beginning-of-the-period value and i/k is the current-period value. The slope is computed with non-negative EBITDA.

To compute the net equity issuance rate, I first compute the firm-level net equity issuance e as the stock sales (SSTK) minus cash dividends (DV) and share buybacks (PRSTKC). Then I take the maximum of e and zero and normalize it by total assets (AT). The construction is similar to [Catherine et al. \(2021\)](#), which normalizes equity issuance by value-added. They approximate value added by 60% of total sales, assuming a 40% gross margin ratio.

I measure firm revenue y using sales (SALE). I construct sales growth rate using the “DHS growth rates” defined following [Davis et al. \(1996\)](#): $\Delta y_{-t} = (y - y_{-t})/(0.5y + 0.5y_{-t})$. This measure bounds growth rates between -2 and $+2$, addressing any concerns over outliers. The empirical equivalent of model variable capital-to-sales ratio k/y is assets divided by sales.

measure firm marginal product of capital in logs (up to an additive constant) as the difference between log revenue and capital, $mpk = y - k$.

I use the BEA nonresidential fixed investment implicit price deflator (from FRED) to deflate capital stock and investment. I use the gross GDP deflator to deflate other variables.

A.2 Sample Selection

I apply the following sample selection criteria:

1. Drop firm-year observation that is incorporated in the United States ($FIC = \text{"USA"}$)
2. Drop firm-year observations if two-digit SIC code (SIC) is in the financial industry (SIC code between 6000 and 6999), utilities (SIC code between 4900 and 4999), or public administration (SIC code between 9000 and 9999)
3. Keep observations for fiscal year (FYEAR) between 1980 and 2018
4. Drop firm-year observations with missing assets (AT), sales (SALE), cash holdings (CHE), long-term debt (DLTT), short-term debt (DLC), capital expenditure (CAPXV), earnings before interest (EBITDA), and capital stock (PPENT)
5. Drop firm-year observations with non-positive SALE or AT
6. Drop firms that are in the data for smaller than 5 years
7. Keep observations with non-missing
8. Trim variables of interest at the top 99% and bottom 1%

157,683 firm-year observations and 11,148 firms.

Comparison of key data moments to literature

The volatility of the leverage ratio (i.e., net debt-to-lagged asset) is 0.32. This is in the range of the moments in the literature. [DeAngelo et al. \(2011\)](#) uses gross debt, who report that the standard deviation of gross leverage is 0.1086 (variance of leverage is 0.0118). [Karabarbounis and Macnamara \(2021\)](#) report a standard deviation of the gross leverage ratio of 0.42.

The investment rate volatility is 0.53, a little higher than existing estimates. Using PPEGT, the gross value property, plant, and equipment, as capital stock, [DeAngelo et al. \(2011\)](#) report that the standard deviation of investment rate is 0.1962 (variance of investment rate is 0.0385). [Karabarbounis and Macnamara \(2021\)](#) also use PPEGT and report that the standard deviation of investment rate is 0.22 and [Ottonello and Winberry \(2020\)](#) report a value of 0.33. Weighting and sample selection might explain why my data moment is more elevated.

My data moment of short-run sales growth rate volatility is 0.35 and the long-run volatility is 0.8. The estimates are similar to [Catherine et al. \(2021\)](#), which document that the volatility of 1-year and 5-year sales growth rates is 0.327 and 0.912 respectively.

My sample's average net equity issuance rate is 0.1, which is broadly consistent with the literature. [Hennessy and Whited \(2005\)](#) and [Hennessy and Whited \(2007\)](#) report a gross equity issuance rate of 4.2% and 8.9%. [Belo et al. \(2019\)](#) report that the value is 0.04. [Catherine et al. \(2021\)](#) normalize the net equity issuance by value-added and the net equity issuance rate is 0.026.

The average leverage ratio, defined as net debt to lagged assets, in my sample is 0.1. My value is similar to 0.098 in [Catherine et al. \(2021\)](#). [Crouzet and Tourre \(2020\)](#) document the ratio is 25.58%. In [Hennessy and Whited \(2007\)](#), the leverage ratio is 12.04% for their baseline estimation and 14.52% in their restricted large firm sample. [Karabarbounis and Macnamara \(2021\)](#) report the mean leverage ratio of 0.29. [Belo et al. \(2019\)](#) report a value of 0.25.

The slope of investment with respect to the ratio of debt to EBITDA in my sample is -1. My data estimate is close to -1.04 in [Crouzet and Tourre \(2020\)](#).

The non-targeted moment of the average investment rate is 0.4 in my sample. Using PPENT, my estimate is somewhat larger than previous research. For example, the average investment is 0.1868 in [DeAngelo et al. \(2011\)](#), 0.16 in [Karabarbounis and Macnamara \(2021\)](#), 0.11 in [Crouzet and Tourre \(2020\)](#), and 0.07 in [Eisfeldt and Muir \(2016\)](#). [Hennessy and Whited \(2005\)](#) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample.

In my sample, the autocorrelation of investment rate is 0.34. The value is in the range of 0.165 in [Catherine et al. \(2021\)](#), 0.17 in [Karabarbounis and Macnamara \(2021\)](#), 0.40 in [Eisfeldt and Muir \(2016\)](#), and 0.41 in [Belo et al. \(2019\)](#).

Finally, my estimate of the standard deviation of the net equity issuance rate is 0.35. My estimate is close to the moment in [Hennessy and Whited \(2007\)](#), 0.3018.

B Model

In this section, I describe the structural estimation procedure. First, for a given set of parameters, I solve the model numerically by iterating on the firm's Bellman equation, which produces the value function $V(k, b, z)$ and the policy function (k', b') . Then, I simulate the economy and search for parameters that model-generated moments could match data moments.

B.1 Numerical Solution Method

I use policy iteration to solve for the firm's problem by iterating on the Bellman equation defined in Eq. (13) until convergence.

Grid definition

I transform (32) into a discrete-state Markov chain using the method in [Tauchen \(1986\)](#). I let productivity z (in logs) have 5 points of support on the interval $[\log(\underline{z}), \log(\bar{z})] = [-3\sigma, 3\sigma]$. I let capital stock k have 100 equally-spaced (in logs) on the interval $[\log(\underline{k}), \log(\bar{k})] = [0.001, 100]$.

Since debt b is bounded above by the borrowing constraints, I set the maximal value of b with the maximal value of k , \bar{k} . Therefore $\bar{b} = (1 - \tau)(\underline{z}\bar{k}^{\alpha}\underline{n}'^{\nu} - W\underline{n}') + s(1 - \delta)\bar{k}'$. The minimum of b is chosen so that the optimal choice of debt never hits the lower endpoint. I verify ex-post and set $\underline{b} = -0.01 \times \bar{b}$. I let debt b have 40 geometrically-spaced points in the interval $[\underline{b}, \bar{b}]$.

The state space for the firm's problem is $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$.

Policy function iteration

I compute the return matrix $R(k', b', k, b, z)$ for all possible values of (k, b, z) :

$$R(k', b', k, b, z) = \pi_d e_0(k, b, z) + (1 - \pi_d) [e_1(k', b', k, b, z) + \eta(e_1(k', b', k, b, z))]$$

I set $R(\cdot)$ to “missing” when (k, b, z) are such that the borrowing constraint is violated. Given a value function $V(k, b, z)$, the policy function $(k', b') = P(k, b, z)$ solves:

$$P^*(k, b, z) = \arg \max_P \left\{ R(P(k, b, z), k, b, z) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [V(P(k, b, z), z') | z] \right\}, \forall (k, b, z)$$

I assume that firms can only choose values of (k', b') on a discrete grid, where $k' \in \mathcal{K} = \{k_1, \dots, k_{N_k}\}$ and $b' \in \mathcal{B} = \{b_1, \dots, b_{N_b}\}$. (N_k, N_b) are the number of grid points for capital and debt, respectively, and N_z is the number of grid points for productivity. Therefore the number

of states is $N_k \times N_b \times N_z = 100 \times 40 \times 5 = 20,000$. The number of choices is $N_k \times N_b = 100 \times 40 = 4,000$.

I initiate with the process with a guess V_0, P_0 and specify a solution tolerance $\varepsilon_{\text{tolerance}} > 0$. To speed up the computation, I apply the Howard improvement algorithm and iterate the policy function instead of the value function iteration.

To solve for the steady state equilibrium given a set of parameters, the algorithm proceeds as follows:

1. **Outer Loop:** Suppose the real wage is in a range of $[W_a, W_c]$. Guess the value of the real wage $W_b = (W_a + W_c)/2$ using the bisection algorithm
2. Solve the firm's problem $V(k, b, z)$ and compute the stationary distribution $\Gamma(k, b, z)$ with firm policies $(k', b') = P(k, b, z)$, for $n = 1, 2, \dots$, given real wage W_b

- (a) **Inner Loop:** Starting from the policy function $(k'_{n-1}, b'_{n-1}) = P_{n-1}(k, b, z)$ and value function $V_{n-1}(k, b, z)$ from the previous round, solve for the optimal policy P_n :

$$P_n(k, b, z) = \arg \max_p \left\{ R(P(k, b), k, b, z) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [V_{n-1}(P(k, b), z') | z] \right\}$$

- (b) Set $\tilde{V}_{n-1}^1 = V_{n-1}$. For each Howard improvement step $h = 1, \dots, H - 1$, iterate the Bellman equation without optimization:

$$\tilde{V}_{n-1}^{(h+1)}(k, b, z) = \left\{ R(P_n(k, b, z), k, b) + \frac{1 - \pi_d}{1 + r} \mathbb{E} [\tilde{V}_{n-1}^{(h)}(P_n(k, b, z), z') | z] \right\}$$

- (c) Set $V_n = \tilde{V}_{n-1}^{(H)}$

- (d) Compute the error $\|P_n - P_{n-1}\| = \max_{k, b, z} |P_n(k, b, z) - P_{n-1}(k, b, z)|$.

- If $\|P_n - P_{n-1}\| < \varepsilon_{\text{tolerance}}$, exit.
- If $\|P_n - P_{n-1}\| \geq \varepsilon_{\text{tolerance}}$, go back to Step 2(a) with $n = n + 1$

3. Calculate the implied value of aggregate consumption $C(W_b)$
4. If W and φC are within some tolerance of each other, $|W_b - \varphi C(W_b)| < \varepsilon_{\text{tol}}$, then I solve the model and set $W^* = W_b$. If not, then update my guess for W as follows and return to Step 1:
 - If $W_b < \varphi C(W_b)$, then wage is underestimated. I set $W_a = W_b$
 - If $W_b > \varphi C(W_b)$, then wage is overestimated. I set $W_c = W_b$

The contracting mapping theorem guarantees that there is a fixed point where the policy function converges under some regularity conditions. I set the step $H = 10$ for the Howard improvement algorithm.

With $N_m \geq N_p$, the model is over-identified. Then the test of the overidentifying restrictions of the model is:

$$J = \frac{NS}{1+S} \min_{\Theta} \hat{g}'_N \hat{W}_N \hat{g}_N \sim \chi^2(N_m - N_p)$$

Simulation and model-generated moments

Once we have solved the model for a given set of parameters, I simulate data in order to compute the simulated moments.

I simulate a balanced panel of 5,000 firms over 5,500 years, and only keep the last 50 years to ensure each firm has reached the steady-state. For each firm, I take a random draw from the distribution of productivity z and simulate a path of log

B.2 Structural Estimation Method

I use the simulated method of moments (SMM) to estimate a vector of unknown structural parameters, $\Theta^* = (\rho, \sigma, \psi_0, \eta_1, s)$. This procedure chooses parameters to minimize the distance between model-generated moments and the corresponding data moments.

SMM Estimation

Let M be the actual data moments and $m^s(\Theta)$ is a vector of moments computed from the s th simulated sample using parameters Θ , where $s = 1, \dots, S$. S is the number of simulations. N is the number of observations in actual data. The number of targeted moments $N_m = 6$. The number of parameters of interest $N_p = 5$.

The SMM estimator of Θ^* solves:

$$\begin{aligned} \hat{\Theta} &= \arg \min_{\Theta} \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S m^s(\Theta) \right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S m^s(\Theta) \right] \\ &= \arg \min_{\Theta} \hat{g}'_N \hat{W}_N \hat{g}_N \end{aligned}$$

where \hat{W}_N is an $N_m \times N_m$ arbitrary positive definite matrix that converges in probability to a deterministic positive definite matrix W .

The simulated moment estimator is asymptotically normal for fixed S . The asymptotic

distribution of Θ is given by:

$$\sqrt{N}(\hat{\Theta} - \Theta^*) \xrightarrow{d} \mathcal{N}(0, \text{avar}(\hat{\Theta}))$$

Let

$$G = \frac{\partial m(\Theta)}{\partial \Theta}, \text{ the } N_m \times N_p \text{ gradient matrix where } G_{ij} = \frac{\partial m_i(\Theta)}{\partial \Theta_j}$$

$$\Omega = \lim_{N \rightarrow \infty} \text{Var}(\sqrt{N}\hat{M}_N), \text{ the } N_m \times N_m \text{ asymptotic variance-covariance matrix of the data moments}$$

Then

$$\begin{aligned} \text{avar}(\hat{\Theta}) &= \left(1 + \frac{1}{S}\right) (G'WG)^{-1} G'W\Omega WG (G'WG)^{-1} \\ &= \left(1 + \frac{1}{S}\right) \left[\frac{\partial \hat{m}_n(\Theta)'}{\partial \Theta} W \frac{\partial \hat{m}_n(\Theta)}{\partial \Theta} \right]^{-1} \end{aligned}$$

The optimal weighting matrix is equal to the inverse of a covariance matrix that is calculated using the influence function approach of Erickson and Whited (2002): $W = \Omega^{-1}$

$$\text{avar}(\hat{\Theta}) = \left(1 + \frac{1}{S}\right) (G'WG)^{-1}$$

The weighting matrix W is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrapping with replacement on the actual data. The estimates of variance-covariance matrix is qualitatively similar to the ones computed from the Delta Method.

C Efficient Allocation and TFP Loss

Firms choose capital and labor optimally where the marginal product of capital and labor are equal to their respective costs. Since labor is static, the marginal product of labor is equal to wage W .

$$\begin{aligned} \text{MPK}_i &= \alpha \frac{y_i}{k_i} = X_i \\ \text{MPL}_i &= \nu \frac{y_i}{n_i} = W \end{aligned}$$

Then the optimal capital-labor ratio is given by

$$\frac{k_i}{n_i} = \frac{\alpha}{\nu} \frac{W}{X_i}$$

Solving for the labor input yields

$$n_i = \underbrace{z_i^{\frac{1}{1-(\alpha+\nu)}} X_i^{-\frac{\alpha}{1-(\alpha+\nu)}}}_{w_i^N} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{1-\alpha}{1-(\alpha+\nu)}} \alpha^{\frac{\alpha}{1-(\alpha+\nu)}}}_{c_N}$$

Then the optimal capital input is

$$k_i = \underbrace{z_i^{\frac{1}{1-(\alpha+\nu)}} X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}}}_{w_i^K} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{\nu}{1-(\alpha+\nu)}} \alpha^{\frac{1-\nu}{1-(\alpha+\nu)}}}_{c_K}$$

where w_i^N and w_i^K denote labor and capital wedges relative to an efficient allocation of inputs. As discussed above, MPL_i and w_i^N are the same across firms. At the efficient allocation, the marginal product of capital is also equated across firms.

Aggregate labor and capital can be expressed as

$$\begin{aligned} N &= \int n_i di = c_N \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di \\ K &= \int k_i di = c_K \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di \end{aligned}$$

The aggregate output is

$$Y = \int y_i di = (c_K^\alpha c_N^\nu) \int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di$$

Then the aggregate productivity is given by

$$\text{TFP} = \frac{Y}{K^\alpha N^\nu} = \frac{\int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di}{\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di \right)^\alpha \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di \right)^\nu} \quad (10)$$

Expressing Equation (10) in logs yields

$$\log(\text{TFP}) = \log \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} (w_i^K)^\alpha (w_i^N)^\nu di \right) - \alpha \log \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di \right) - \nu \log \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di \right) \quad (11)$$

Define $A_i = z_i^{\frac{1}{1-(\alpha+\nu)}}$. Assume that (A_i, w_i^K, w_i^N) are jointly log-normal distributed

$$\begin{bmatrix} \log(A_i) \\ \log(w_i^K) \\ \log(w_i^N) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mu_a \\ \mu_K \\ \mu_N \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{a,K} & \sigma_{a,N} \\ \sigma_{a,K} & \sigma_K^2 & \sigma_{K,N} \\ \sigma_{a,N} & \sigma_{K,N} & \sigma_N^2 \end{bmatrix} \right)$$

The second-order approximations of Equation (11) are given by

$$\begin{aligned} \log \left(\int A_i (w_i^K)^\alpha (w_i^N)^\nu di \right) &= \mu_a + (\alpha \mu_K + \nu \mu_N) + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \alpha^2 \sigma_K^2 + \frac{1}{2} \nu^2 \sigma_N^2 + \alpha \nu \sigma_{K,N} + \alpha \sigma_{a,K} + \nu \sigma_{a,N} \\ \log \left(\int A_i w_i^K di \right) &= \mu_a + \mu_K + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \sigma_K^2 + \sigma_{a,K} \\ \log \left(\int A_i w_i^N di \right) &= \mu_a + \mu_N + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \sigma_N^2 + \sigma_{a,N} \end{aligned}$$

Rearrange the above expressions. Then Equation (11) is given by

$$\log(\text{TFP}) = (1 - \alpha - \nu) \left(\mu_a + \frac{1}{2} \sigma_a^2 \right) - \frac{1}{2} \alpha (1 - \alpha) \sigma_K^2 - \frac{1}{2} \nu (1 - \nu) \sigma_N^2 + \alpha \nu \sigma_{K,N}$$

Given w_i^N is equalized across firms, then $\sigma_N^2 = 0$ and $\sigma_{K,N} = 0$. Therefore,

$$\log(\text{TFP}) = (1 - \alpha - \nu) \left(\mu_a + \frac{1}{2} \sigma_a^2 \right) - \frac{1}{2} \alpha (1 - \alpha) \sigma_K^2$$

The efficient allocation implies that $\sigma_K^2 = 0$. Then I can approximate the first-best TFP as

$$\log(\text{TFP}^{\text{FB}}) = (1 - \alpha - \nu) \left(\mu_a + \frac{1}{2} \sigma_a^2 \right) \quad (12)$$

The TFP loss is defined to be

$$\text{Relative TFP Loss} = \log \left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}} \right) = \frac{1}{2} \alpha (1 - \alpha) \sigma_K^2 \quad (13)$$

To solve for σ_K^2 . Recall that

$$\text{MPK}_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = w_i^K$$

Then MPK is also log-normally distributed

$$\begin{aligned} -\frac{1-\nu}{1-(\alpha+\nu)} \log(X_i) &= \log(w_i^K) \sim \mathcal{N}(\mu_K, \sigma_K^2) \\ \log(X_i) &= -\frac{1-(\alpha+\nu)}{1-\nu} \log(w_i^K) \sim \mathcal{N} \left(\mu_K \left[-\frac{1-(\alpha+\nu)}{1-\nu} \right], \sigma_K^2 \left[\frac{1-(\alpha+\nu)}{1-\nu} \right]^2 \right) \end{aligned}$$

This solves σ_K^2

$$\begin{aligned} \text{Var}(\log(\text{MPK}_i)) &= \text{Var}(\log(X_i)) = \sigma_K^2 \left[\frac{1-(\alpha+\nu)}{1-\nu} \right]^2 \\ \sigma_K^2 &= \left[\frac{1-\nu}{1-(\alpha+\nu)} \right]^2 \text{Var}(\log(\text{MPK}_i)) \end{aligned}$$

Plug σ_K^2 into Equation (13) and TFP loss is given by

$$\log \left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}} \right) = \frac{1}{2} \alpha (1 - \alpha) \left[\frac{1-\nu}{1-(\alpha+\nu)} \right]^2 \text{Var}(\log(\text{MPK}_i))$$

Table C1: Variable Definitions

Variable	Notation	Definition	Compustat data item
Capital	k	Total net value of property, plant, and equipment	PPENT
Investment	$i = k' - (1 - \delta)k$	Capital expenditures on property, plant, and equipment	CAPXV
Debt	b	Net debt computed as the sum of short-term and long-term debt minus cash	DLC+DLTT-CHE
Sale	y	Sales	SALE
EBITDA	π	Earnings before interest, taxes, depreciation and amortization	EBITDA
Net equity issuance	e	Stock sales minus cash dividends and share buybacks	SSTK - PRSTKC - DV
Investment rate	i/k		CAPXV/L.PPENT
Leverage	-	See Appendix A.1	
Net equity issuance rate	e/k	See Appendix A.1	
Sales growth rate	Δy_{-t}	See Appendix A.1	
Capital-to-sales ratio	k/y		AT/SALE

Note: The table describes the empirical counterpart of model variables. It includes the model notation, definition, and data item from Compustat for each variable.

Table C2: Aggregate Effects of Financial Frictions

	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	(4) Benchmark value
Panel A. Without Tax Shield				
K	0.7888	0.7916	0.7888	0.7812
B	-0.1067	0.2664	-0.6320	-0.0572
N	0.3132	0.3137	0.3132	0.3143
Y	0.5795	0.5804	0.5795	0.5781
TFP	1.2339	1.2337	1.2339	1.2315
$\Delta\%$ (K)	0.9605	1.3162	0.9605	
$\Delta\%$ (N)	-0.3315	-0.1656	-0.3315	
$\Delta\%$ (Y)	0.2384	0.4040	0.2384	
$\Delta\%$ (TFP)	0.1963	0.1736	0.1963	
Panel B. With Tax Shield				
K	0.8147	0.8516	0.8557	0.7752
B	0.1009	0.6929	0.6943	0.0939
N	0.3171	0.3099	0.3108	0.3092
Y	0.5885	0.5869	0.5886	0.5714
TFP	1.2339	1.2339	1.2338	1.2314
$\Delta\%$ (K)	1.5409	9.3966	9.8804	
$\Delta\%$ (N)	0.4018	0.2118	0.4982	
$\Delta\%$ (Y)	0.8369	2.6782	2.9646	
$\Delta\%$ (TFP)	0.2105	0.2020	0.1956	

Table C3: Model Estimation Results

Panel A. Estimated Parameters

Parameter	Description	Model 1	SE	Model 2	SE
ρ	Productivity persistence	0.872	(0.0020)	0.835	(0.0011)
σ	St. Dev. of innovations to productivity	0.109	(0.0005)	0.076	(0.0008)
ψ_0	Convex investment adjustment cost	0.056	(0.0018)	0.008	(0.0046)
η_1	Linear equity issuance cost	0.036	(0.0001)	0.008	(0.0000)
s	Frac. of debt that can be collateralized	0.147	(0.0182)	0.349	(0.0031)

Panel B. Model Fit: Targeted Moments

Moment	Description	Model 1	Model 2	Data
$\sigma(b/k)$	debt rate volatility	0.365	0.300	0.32
$\sigma(i/k)$	investment rate volatility	0.478	0.556	0.53
$\sigma(\Delta y_{-1})$	1-year sales growth rate volatility	0.374	0.340	0.35
$\sigma(\Delta y_{-5})$	5-year sales growth volatility	0.938	0.806	0.8
$\mu(e/k)$	average net equity issuance rate	0.100	0.067	0.1
β	slope of i/k wrt debt/EBITDA	-0.998		-1
$\mu(b/k)$	mean leverage		0.091	0.1

Panel C. Model Fit: Non-Targeted Moments

β	slope of i/k wrt debt/EBITDA		-9.500	-1
$\mu(b/k)$	mean leverage	-0.051		0.1
$\mu(i/k)$	average investment rate	0.187	0.309	0.40
$\text{corr}(i/k, i/k_{-1})$	autocorrelation of investment rate	0.281	0.015	0.32
$\sigma(e/k)$	net equity issuance rate volatility	0.243	0.219	0.45

Notes: The table reports point estimates and standard errors (in parentheses) for each of the parameters estimated via SMM. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is the inverse of the moment covariance matrix. Model 1 is the Baseline model. Model 2 is the model targeting the mean leverage ratio instead of the slope of investment to leverage.

Table C4: Aggregate Implications of Financial Frictions: (1) free equity (2) no collateral constraint (3) free equity and no collateral constraint

	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	Benchmark value
Model 1: targeting slope β				
$100 \times \Delta \log (K)$	1.541	9.397	9.880	0.775
$100 \times \Delta \log (N)$	0.402	0.212	0.498	0.310
$100 \times \Delta \log (\text{Output})$	0.837	2.678	2.965	0.576
$100 \times \Delta \log (\text{Wage})$	0.435	2.466	2.466	1.114
$100 \times \Delta \log (\text{TFP})$	0.211	0.202	0.196	1.234
Relative TFP loss	7.659%	7.465%	7.493%	8.055%
Model 2: targeting mean leverage				
$100 \times \Delta \log (K)$	-0.799	1.969	-0.156	0.706
$100 \times \Delta \log (N)$	-0.874	-0.101	-0.879	0.312
$100 \times \Delta \log (\text{Output})$	-0.699	0.408	-0.580	0.493
$100 \times \Delta \log (\text{Wage})$	0.175	0.509	0.299	0.949
$100 \times \Delta \log (\text{TFP})$	0.025	-0.024	-0.014	1.082
Relative TFP loss	3.402%	3.442%	3.430%	3.431%