Financial Frictions, Debt Tax Shields, and the Macroeconomy

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Abstract

This paper evaluates the aggregate effects of financial frictions. Previous studies indicate that removing financial frictions will stimulus investment and reduce misallocation. However, with the tax shield of debt financing, the macroeconomic implications can be different. I build a quantitative general equilibrium dynamic investment model of heterogeneous firms. Firms face two types of financial frictions: (a) borrowing constraints and (b) costly equity issuance. I propose to target the slope of investment with respect to the debt-to-EBITDA ratio to identify the borrowing constraint parameter. My quantification results show that by removing financial frictions, aggregate capital stocks grow up to about 10%, output gains at most 3%, and aggregate productivity increases about 0.2%. My counterfactual experiments demonstrate that financial frictions have a non-monotonic effect on firm-level cross-sectional moments and aggregate productivity. When firms become less financially constrained, the debt tax shield leads to excessive borrowing and exacerbate the misallocation problem.

1 Introduction

The paper revisits a classical question in macroeconomics: what are the aggregate consequences of financial frictions? It is a commonly held view that financial frictions can reduce aggregate output and total factor productivity (TFP) [1]. However, literature in macroeconomics often ignores an important distortion of the tax shield of debt financing when modeling firm investment dynamics, which is a standard presumption in the corporate finance literature. Financial frictions can have different macroeconomic implications when interacting with debt tax shields.

[1] For example, see Khan and Thomas (2013), Midrigan and Xu (2014), Moll (2014)

The tax system in the United States and around the world have been generally favored debt over equity because interest on debt is deductible against corporate tax while returns to equity (in the form of dividends or share appreciation) are not. As a result, debt financing has a tax shield or tax benefit [2]. The bias of the tax code toward debt stimulates borrowing and contributes to higher leverage. In the absence of financial frictions, debt bias creates a deadweight loss. However, with financial frictions, the impact of debt bias is ambiguous. On one hand, tax shields may improve efficiency and mitigate frictions or distortions by raising investment when firms are credit constrained. On the other hand, this may lead to excessive debt financing and exacerbate capital misallocation.

In this paper, I build a general equilibrium dynamic model in which heterogeneous firms finance investment with either equity or debt. Firms have two potential financing instruments. Firstly, they can issue one-period debt securities. Secondly, they can raise funds directly from shareholders in the event of a cash flow shortfall. The firm's financing decision is distorted by two types of financial frictions. The first is a borrowing constraint such that firms can only borrow up to the minimum possible cash flow and a fraction of their tangible capital. The second is the costly equity issuance. The tax shield of debt in my model is a nonpecuniary wedge between the discount rate and the rate on debt. The debt tax shield makes firms behave impatiently and incentivizes firms to use debt to balance the tax advantage of borrowing with current and future expected financing costs. Finally, firms' investment is subject to capital adjustment costs.

I structurally estimate the model parameters in a simulated method of moments (SMM) procedure using firm-level balance sheet data from COMPUSTAT between 1980 and 2018. Following Hennessy and Whited (2007) and Catherine et al. (2021) I use the net equity issuance rate to pin down external equity financing costs. To identify the borrowing constraints, I carefully choose the slope of investment with respect to the ratio of debt to EBITDA as my target moment. In the data, this sensitivity is -1 which means a one-unit increase in debt-to-EBITDA is on average associated with a decline in investment of 1 percentage point.

Then I conduct three counterfactual experiments. In the first experiment, I analyze the effects of financial frictions on firm characteristics. I show that there is a non-monotonic relationship between borrowing constraints and most cross-sectional moments such as leverage ratio, investment rate, and equity issuance rate. In contrast, the slope of investment to leverage is monotonically increasing with borrowing constraints. This experiment points out that using cross-sectional moments, such as the mean leverage, to pin down structural parameters may have an identification problem.

[2] On this topic "tax shield", "tax benefit", "tax advantage", and "debt bias" are used almost interchangeably in the literature.

In the second experiment, I quantify the importance of financial frictions on the aggregate economy. I find that by removing financial frictions in debt and/or equity markets, aggregate capital stocks grow about 10%, output gains 3%, and aggregate productivity increases about 0.2%. Consistent with the literature (e.g., Catherine et al. (2021)), the aggregate costs of financial frictions mainly come from an insufficient supply of capital input. Moreover, I show that similar findings of non-monotonicity hold for aggregate TFP. As financial constraints become slack, aggregate productivity increases first and then decreases while aggregate capital and output still grow.

To investigate further why the effect of financial friction is non-monotonic, in the last counterfactual analysis, I compare the macroeconomic outcomes with and without debt bias. My results uncover that in general, the tax benefit leads to over-borrowing and over-investment. Tax benefits can be regarded as a double-bladed edge. When firms are highly constrained, by reducing the cost of capital tax shield of debt expands debt capacity and boosts investment and firm growth. However, when firms are close to unconstrained, the tax shield lowers aggregate productivity and makes the misallocation problem worse.

The contribution of this paper is twofold. Firstly, to identify frictions in the debt market I propose to use the slope of investment with respect to leverage as a target moment. I show that using cross-sectional moments, such as the mean leverage ratio, will underestimate the extent of financial friction in the economy. Second, the paper demonstrates that in the presence of a tax shield of debt, the effects of financial frictions are non-monotonic at both the firm level and the aggregate level. This highlights the interaction between financial frictions and tax benefits and implies that removing financial frictions does not necessarily alleviate the resource misallocation problem.

This paper is organized as follows. Section 2 reviews related literature. Section 3 illustrates the background of tax shield and debt bias in the current tax system. Section 4 formulates a dynamic investment model with financial frictions and other distortions. Section 5 structurally estimates the model. In Section 6 I analyze the effect of financial frictions on firm characteristics and in Section 7 I examine the aggregate implications of financial frictions. Section 8 studies the model mechanism through the lens of the tax shield. And finally, Section 9 concludes.

2 Literature

My paper builds on several strands of literature. First, this paper contributes to the broad quantitative literature on the effects of financial frictions (e.g., Hennessy and Whited (2007), Buera et al. (2011), Khan and Thomas (2013), Midrigan and Xu (2014), Moll (2014), Jo and Senga (2019), and Ottonello and Winberry (2020)). The most closely related paper is

Catherine et al. (2021) which quantitatively examines the impact of collateral constraints. My model shares common features with Catherine et al. (2021): borrowing constraints, costly external equity issuance, and a tax benefit. However, there are substantive differences. First, they focus on one source of financing friction: collateral constraints, while I also analyze the effects of other distortions (e.g., costly equity issuance and taxes) and the interactions between financial frictions and taxes. Second, in model specifications, the main differences are (a) my model has decreasing returns to scale in production; (b) goods are homogeneous and the good market is perfectly competitive, allowing for easier aggregation; (3) there is no real estate in my model. Third, in estimation, Catherine et al. (2021) exploit variations in real estate prices and use a reduced-form coefficient, the sensitivity of firm-level investment to collateral values, to identify the scope of financial frictions. They find that collateral constraint induces output losses of 7.1%, and TFP (misallocation) losses of 1.4%. Instead, my estimation method uses the slope of investment with respect to the debt-to-EBITDA ratio where the computation is considerably simpler.

Second, My work relates to the extensive theoretical and empirical corporate finance literature that studies how preexisting debt affects firms' decisions to undertake new investments (e.g, Kalemli-Özcan et al. (2020), Crouzet and Tourre (2020), Barbiero et al. (2020), Jordà et al. (2020), Albuquerque (2021), and Perla et al. (2020)). The long-standing question goes back to the seminal work of Myers (1977). He hypothesizes that outstanding debt may distort investment downwards as profits primarily benefit existing debt holders but not potential new investors in the presence of default risk. The paper refers to it as a "debt overhang problem". Relative to the literature (e.g., Diamond and He (2014)), I make two simplifications in modeling. First, my model only features one type of debt instrument, the short-term debt; second, firms exit from the markets only because of an exogenous death shock. However, with costly external equity financing, the simplicity can still capture the nexus between the firm's capital structure and its investment efficiency in the absence of endogenous and/or strategic default.

I follow Crouzet and Tourre (2020) and Blickle et al. (2022) and compute the slope of investment with respect to leverage. The slope is negative on account of debt overhang effects. Here leverage is defined as the ratio of debt and EBITDA. This leverage measure relates borrowing to a proxy for cash flow which can remove some of the endogeneity associated with the firm's financing decisions. While Crouzet and Tourre (2020) uses the moment to identify the adjustment cost parameter, I use it to primarily pin down parameters governing financial frictions.

Third, my findings echo on the literature that studies misallocation of capital and aggregate productivity (e.g., Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Gilchrist et al. (2013), and Karabarbounis and Macnamara (2021)). These studies support that financing frictions lead to input misallocation. A major difference with the aforementioned works is that my paper allows for tax deductibility of interest. I contribute to the literature that financing frictions may mitigate the extent of misallocation. In the presence of other distortions such as the tax benefit of debt financing, the relationship between financial frictions and aggregate productivity is non-monotonic.

Last but not the least, this paper speaks to the literature on taxation, firm capital structure, and dynamic trade-off theory in corporate finance. The standard theory from Modigliani and Miller (1958) states that a firm shall be indifferent between various sources of financing for its projects. However, With tax deductibility of interest, firms raise debt to balance the value of interest tax shields against costs associated with financial distress and bankruptcy, which determines an optimal amount of debt in a firm's capital structure (e.g., Modigliani and Miller (1963), Miller (1977), Myers (1984), Hennessy and Whited (2005), and Li et al. (2016)). I show that the tax shield is a key assumption driving the non-monotonic results and it plays an important role to understand the real effects of financial frictions.

3 Tax Shield

In this section, I provide a brief overview of corporate debt and tax shields of debt.

The tax system in the United States and around the world has a long history that generally favors debt over equity because interest expenses on debt are tax-deductible, while a similar deduction for the cost of equity (in the form of dividends or share appreciation) is rarely ever granted (e.g., Bank (2014)). As a result, returns to equity-financed investment are taxed at both the corporate level and the shareholder level, while debt-financed investment faces only shareholder-level tax. The differential treatment creates a tax bias in favor of debt financing. Some empirical works (e.g., Feld et al. (2013), Heider and Ljungqvist (2015)) document that taxes and the debt tax shield are important drivers of firms' capital structure.

The economic rationale for tax bias for debt is related to market failures (i.e., adverse selection, agency problem, signaling) that discourage the use of external finance and lead to underleverage, suggesting a role for tax policy that favors debt (e.g., De Mooij (2012), Pozen and Goodman (2012)). However, there is a general consensus that these justifications are not convincing. In contrast, many studies argue that the tax bias for debt can distort firms' decisions on financing and investment. In particular, tax advantages for debt finance can disproportionately hurt young and innovative firms that invest heavily in R&D expenditures for lack of assets that can be easily used as collateral. Moreover, the tax shield can generate negative externalities as excess debt increases systemic risk and macroeconomic instability (e.g, Schularick and Taylor (2012), Jordà et al. (2013)).

Given the above concerns, the debt tax shield has been the subject of analysis and discussion among lawmakers. It gains renewed interest in light of the 2008 financial and economic crisis. Many governments and international organizations have started to make tax reforms and introduce various measures (e.g., Allowance for Corporate Equity (ACE) and Comprehensive Business Income Tax (CBIT)) to reduce or eliminate the tax benefit of debt. Belgium was among the very few countries in the world that neutralized the debt bias. Since 2006, Belgium allows for a notional interest deduction on equity capital. In the United States, Congress passed the Tax Cuts and Jobs Act (TCJA) in 2017. In addition to a reduction in the corporate tax rate, the Act limits interest deductibility permanently to 30% of earnings. Previously, interest expenses are generally fully deductible. At the supranational level, European Commission proposed a debt-equity bias reduction allowance to help businesses access the financing they need and to become more resilient in May 2022. The allowance on equity is deductible for 10 consecutive tax years. The proposal also introduces a reduction of debt interest deductibility by 15%.

4 The Model

In this section, I present a dynamic general equilibrium model of firm investment under financial frictions on both debt and equity financing: (1) borrowing constraints, and (2) costly external equity. The economy consists of a continuum of a unit mass of firms. Heterogeneous firms produce a homogeneous good consumed by a representative consumer. Time is discrete on an infinite horizon. The model builds on Strebulaev and Whited (2012) and it is closely related to Hennessy and Whited (2007), Katagiri (2014) and Catherine et al. (2021).

4.1 Firm

Production Technology

Firms are risk-neutral. Each firm owns predetermined capital stock k and hires labor n. It produces a homogeneous good with decreasing-returns-to-scale production technology:

$$y = zk^{\alpha}n^{\nu}, 0 < \alpha + \nu < 1 \tag{1}$$

z is a firm's idiosyncratic total factor productivity and follows a Markov chain. There is no aggregate uncertainty in the model. Labor is flexible and is hired in a competitive labor market at a wage of W. The capital accumulation of each firm is standard, $i = k' - (1-\delta)k$. The investment decision takes place before the realization of the next period's productivity z'. Therefore my model is different from Midrigan and Xu (2014) and Moll (2014) where firms rent capital in a competitive capital rental market. The capital stock depreciates at the rate δ each period. Investment is reversible and subject to a standard quadratic adjustment cost $\psi(k,i)$:

$$\psi(k,i) = \psi_1 + \frac{1}{2}\psi_0 \left(\frac{i}{k}\right)^2 k \mathbb{1}_{i \neq 0} \tag{2}$$

Firms face corporate taxation. Firms' profits net of interest payments and capital depreciation are taxed at the rate τ. Given the optimal choice of labor, the firm's earnings before an interest payment, a tax payment, and depreciation (EBITDA) is defined as follows:

$$\pi(k,z) = zk^{\alpha}n^{\nu} - Wn = (1-\nu)y \tag{3}$$

Debt Market Frictions

Each firm faces a borrowing limit on a one-period discount debt. Following Strebulaev and Whited (2012), this borrowing constraint restricts the amount of new debt level, b', by firms' value of their capital and after-tax future earnings in the worst state of the world. The borrowing constraint can thus be expressed as:

$$b' \leq (1-\tau)(\underline{z}k'^{\alpha}\underline{n}'^{\nu} - W\underline{n}') + s(1-\delta)k' \tag{4}$$

in which $\underline{z}$ is the lowest possible value that the shock z can attain.k' is the firm's future period capital. $\underline{n}$ is the corresponding optimal labor input. I assume that only a fraction of the capital stock s can be liquidated. In this specification of borrowing constraint, debt is risk-free and firms never default. Firms can always repay the debt. The state variable b takes both positive and negative values. A negative value of b denotes cash.

Consistent with corporate finance literature, I assume that debt financing has a tax shield, which creates an incentive for firms to increase their leverage. Thus, the present value of debt issued in the next period is $\frac{b'}{1+r(1-\tau)}$. r is the risk-free interest rate. The discount rate of $1+r(1-\tau)$ implies the interest deductions on the debt.

Equity Market Frictions

The costly external equity injections carry a fixed and proportional cost and are thus a more expensive source of funds than internally generated cash flows. Suppose $e_1(k,k',b,b',z)$ is the net cash flow. The cost of external cash flow is thus given by:

$$\eta(x) = (-\eta_0 + \eta_1 e_1)\mathbb{1}_{e_1<0} \tag{5}$$

where $\eta_0 > 0$ and $\eta_1 > 0$ are the fixed and linear components of the equity cost function.

For a given desired next-period capital stock k' and debt b', $e_1(k, k', b, b', z)$ is defined to be:

$$e_1(k, k', b, b', z) = (1-\tau)(y - Wn) + (1-\delta)k - b - k' - \psi(k, i) + \frac{b'}{1 + r(1-\tau)}$$

where the firm's internal fund:

$$e_{IN}(k, b, z) = (1-\tau)(y - Wn) + (1-\delta)k - b$$

The external funding requirement is equal to the desired capital stock less the internal fund:

$$\begin{aligned} e_{EX}(k, k', b, b', z) &= k' + \psi(k, i) - e_{IN}(k, b, z) - \frac{b'}{1 + r(1-\tau)} \\ &= -e_1(k, k', b, b', z) \end{aligned}$$

Assume that firms cannot retain earnings. If cash inflows exceed optimally chosen cash outflows, $e_1(k, k', b, b', z) > 0$, then the firm must pay the entire fund out to shareholders. On the other hand, if the distribution to shareholders is negative, $e_1(k, k', b, b', z) < 0$, then shareholders need to fill the gap e_{EX}. Together, debt derives value from the costly external equity and the tax benefit.

Entry and Exit

In each period, incumbent firms may exit the economy. The death shock $\pi_d \in (0, 1)$ is common across firms. Since the debt is risk-free, therefore there is no endogenous exit in the model. If the firm will continue to operate for the next period, then it may invest and/or borrow. If the firm will exit, then it keeps the residual of current production after wage bill and debt repayment. Then the cash flow $e(k, k', b, b', z)$ is defined as:

$$\begin{aligned} &e(k, k', b, b', z) \\ =& \begin{cases} e_0 = (1-\tau)(y - Wn) + (1-\delta)k - b & \text{if the firm will exit} \\ e_1 = (1-\tau)(y - Wn) + (1-\delta)k - b - k' - \psi(k, i) + \dfrac{b'}{1 + r(1-\tau)} & \text{if the firm will survive} \end{cases} \end{aligned}$$

Existing firms are replaced by an equal mass of entrants so that the total mass of production firms is fixed in each period. Entering firms are fully equity-financed with initial capital stock k_0. The initial productivity of an entrant, z_0, is randomly drawn from the ergodic distribution of z. They then proceed as incumbent firms.

Timing

At the beginning of each period, an incumbent firm is identified with a state vector (k, b, z): the predetermined capital stock k, the amount of debt carried from the previous period b, and the current period idiosyncratic productivity z. The firm makes the optimal labor choice and learns its exogenous exit status. Labor choices are static. Therefore firms with the same (k, z) will make the same labor choices, regardless of their exit shock realizations.

If the firm is assigned to exit, it simply chooses labor n to maximize its current dividend payment to shareholders. The dividends e_0 are output, less wage payment, and debt repayment, alongside the returns from capital liquidation. If the firm is continuing beyond the period, then additionally, it makes intertemporal decisions on future capital k' and borrowing b'. The current dividend payment is e_1.

For the next period, the initial state of a continuing incumbent is (k', b', z'). It starts operating, along with entering firms with initial state $(k_0, 0, z_0)$.

Firm Distribution

The distribution of firms over (k, b, z) is denoted by a probability measure μ, defined on the Borel algebra $\mathcal{S}$ by the open subsets of the product space, $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$. The evolution of the firm distribution Γ is determined in part by the actions of continuing firms and in part by entry and exit:

$$\begin{aligned} \mu' &= \Gamma(\mu) \\ \mu'(z_j) &= (1 - \pi_d) \int_{\{(k,b,z_i)|(k',b')\in A\}} \pi_{ij} d\mu(k, b, z_i) + \pi_d \chi(k_0) H(z_j), \quad \forall (A, z_j) \in \mathcal{S} \end{aligned} \tag{6}$$

where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$.

Firm Problem

Let $V(k, b, z)$ be the expected discounted value of a firm that enters with (k, b) and idiosyncratic productivity z at the beginning of the current period. Then the Bellman equation for an incumbent firm is:

$$V(k,b,z) = \max_{k',b'} \left\{ \pi_d e_0 + (1-\pi_d) \left[e_1 + \eta(e_1) + \frac{1}{1+r} \mathbb{E}\left(V(k',b',z')|z\right) \right] \right\} \tag{7}$$

subject to

$$b' \leq (1-\tau)(\underline{z} k'^{\alpha} \underline{n}'^{\nu} - W \underline{n}') + s(1-\delta)k'$$

where

$$\begin{aligned}
e_0 &= (1-\tau)(y - Wn) + (1-\delta)k - b && \text{if the firm will exit} \\
e_1 &= (1-\tau)(y - Wn) + (1-\delta)k - k' - \psi(k,i) + \frac{b'}{1+r(1-\tau)} - b && \text{if the firm will survive} \\
\eta(e_1) &= (-\eta_0 + \eta_1 e_1)\mathbb{1}_{e_1<0} \\
\psi(k,i) &= \frac{1}{2}\psi_0 \left(\frac{i}{k}\right)^2 k \mathbb{1}_{i\neq 0}
\end{aligned}$$

4.2 Household Problem

An infinitely-lived representative household holds one-period noncontingent bonds B^H and owns firms. Given the real wage W and the risk-free rate r, the household determines its current consumption C^H, hours worked N^H and new bond holdings $B^{H'}$, to maximize its lifetime expected utility:

$$V^H(B^H) = \max_{C^H, N^H, B^{H'}} \left\{ \log C^H - \varphi N^H + \beta^H V^H(B^{H'}) \right\} \tag{8}$$

subject to

$$C^H + \frac{B^{H'}}{1+r} = W N^H + B^H + T^H + \Pi^H$$

where

$$\Pi^H = \int \left(\underbrace{(1-\pi_d)\left[e_1 + \eta(e_1)\right]}_{\text{Continuing}} + \underbrace{\pi_d e_0}_{\text{Exit}} - \underbrace{\pi_d \left[k_0 - \frac{b'}{1+r(1-\tau)}\right]}_{\text{Entrant}} \right) d\mu(k,b,z)$$

Π^T is the dividend payment of the firm. T^H is the lump-sum transfer rebated to the household. β^H is the discount factor for future utility.

4.3 Equilibrium definition

Consider a stationary industrial equilibrium of the model. The equilibrium is defined by a set of value functions $\{V, V^H\}$, decision rules $\{k', b', n, B^{H'}, N^H\}$, prices $\{W, r\}$, and a measure of firms μ such that:

1. All firms optimize: V solves (7) with associated policy rules $\{k', b', n\}$.

2. The household optimizes: V^H solves (8) with associated policy rules $\{C^H, B^{H'}, N^H\}$.

3. The bond market clears:

$$B^H = \int b d\mu = \mathbf{B}$$

4. Government budget is balanced:

$$T^H = \int \left(\underbrace{\tau(y - Wn)}_{\text{Corporate income tax}} - \underbrace{\frac{\tau r b'}{(1+r)[1+r(1-\tau)]}}_{\text{Tax benefit of debt}} \right) d\mu$$

5. The good market clears:

$$\begin{aligned} \mathbf{C} &= \int y d\mu - \underbrace{\pi_d k_0}_{\text{Entrant}} + \underbrace{\pi_d \int (1-\delta) k d\mu}_{\text{Exit}} - \underbrace{(1-\pi_d) \int \Big((i+\psi) - \eta\Big) d\mu}_{\substack{\text{Continuing: investment, capital AC,} \\ \text{and equity financing cost}}} \\ &= \mathbf{Y} + \pi_d(1-\delta-\kappa_0)\mathbf{K} - (1-\pi_d)(\mathbf{I} + \mathbf{\Psi} - \mathbf{H}) \end{aligned}$$

   where aggregate output $\mathbf{Y}$, capital stock $\mathbf{K}$, investment $\mathbf{I}$, adjustment costs $\mathbf{\Psi}$, equity issuance cost $\mathbf{H}$, consumption $\mathbf{C}$. κ is the fraction of the steady-state aggregate capital stock held by each entrant and $k_0 = \kappa_0 \mathbf{K}$

6. The labor market clears:

$$N^H = \int n(k, b, z) d\mu = \mathbf{N}$$

5 Estimation

Because the model has no closed-form solution, I estimate key parameters using a Simulated Method of Moments (SMM) in this section. I estimate the model in two steps. First, I exogenously fix a subset of parameters. Second, I estimate the remaining parameters to match moments in the data.

5.1 Parameterization

I assume that firm productivity is a log-normal AR(1) process:

$$\ln(z') = \rho \ln(z) + \varepsilon'$$

where $\varepsilon' \sim \mathcal{N}(0, \sigma^2)$. The parameters (ρ, σ) of the driving process are unknowns that must be estimated. I use the procedure of Tauchen (1986) to discretize the stochastic shock into a 5-state Markov chain.

5.2 Predefined parameters

The model comprises 16 parameters. I externally calibrate 11 of them. I set the capital share $\alpha = 0.25$ and labor share $\nu = 0.6$, implying a decreasing return to scale of 0.85. These values are close to the values commonly used in the investment literature (e.g., Khan and Thomas (2013), Bloom et al. (2018), Jeenas (2019), and Ottonello and Winberry (2020)). The depreciation rate δ is fixed at 0.1, in line with Bloom et al. (2018) and Karabarbounis and Macnamara (2021). I use a tax rate τ of 0.2, consistent with Gomes and Schmid (2010). The value is almost the same as the current federal statutory corporate tax rate of 0.21. As employed by Khan and Thomas (2013), I set the exogenous exit rate π_d to 0.1. I use the relative initial capital stock of potential entrants to the average incumbent firm κ_0 as 0.2, which is based on estimation in Jeenas (2019). I set both the fixed investment adjustment cost ψ_1 and the fixed equity issuance cost η_0 to 0, following Catherine et al. (2021). I set the risk-free interest rate $r = 0.04$, as in Jo and Senga (2019). The value is standard in the real business cycle literature. The subjective discount factor β^H implies the long-run real interest rate. So the value is $\frac{1}{1+r} = 0.96$. Finally, I follow Bloom et al. (2018) and set the labor disutility φ at 2. Table 1 summarizes these externally calibrated parameters.

5.3 Data and Target Moments

I structurally estimate the remaining 5 parameters: the productivity persistence ρ, the standard deviation of innovation to productivity σ, the convex capital adjustment cost ψ_0, the linear equity issuance cost η_1, and the borrowing constraint s.

To calculate data moments, I employ the COMPUSTAT industrial files. I use the fundamental annual sample of nonfinancial, unregulated publicly listed US firms from 1980 to 2018. Details on the data and sample selection are provided in Appendix A.2. I choose moments that are informative about parameters. The SMM estimates parameters by minimizing the distance between model-implied moments and their empirical counterparts:

$$\hat{\theta} = \arg\min_{\theta} \tag{9}$$

I describe my model solution algorithm and structural estimation method in detail in Appendix B.

In total, I use 6 moments. Given the model is overidentified, the identification is not a one-to-one mapping between data moments and structural parameters. All of the model parameters jointly affect all of these moments in some way. Nonetheless, some moments have a greater influence on certain parameters.

Idiosyncratic productivity process (ρ, σ)

I primarily use three moments to pin down parameters governing the productivity process (ρ, σ). Following Midrigan and Xu (2014) and Catherine et al. (2021), I use 1-year and 5-year standard deviation of sales growth rate ($\sigma(\Delta y_{-1})$, $\sigma(\Delta y_{-5})$) to simultaneously estimate these two parameters. The volatility of the short-run and long-run empirical growth rates is 0.35 and 0.8 respectively in data. I also use the volatility of the debt-to-assets ratio, $\sigma(b/k)$. In data the ratio is 0.32.

Capital adjustment cost ψ_0

I choose the dispersion of the investment rate, $\sigma(i/k)$, to infer the capital adjustment cost parameter ψ_0, as in DeAngelo et al. (2011) and Eisfeldt and Muir (2016). Large adjustment costs lead the firm to a smooth investment. Adjustment cost should also have a sizeable effect on the volatility of short-term output $\sigma(\Delta y_{-1})$. Therefore, larger adjustment costs can be identified by smaller investments and short-term output volatility.

Equity market frictions: linear external equity issuance cost η_1

In the spirit of Hennessy and Whited (2007), the cost of external equity issuance parameter η_1, heavily depends on the average ratio of net positive equity issuance scaled by assets, $\mu(e/k)$, because a higher cost of external equity financing implies lower equity issuance. The value is 0.1 in the sample.

Debt market frictions: borrowing constraint s

As mentioned above, I use the slope of the investment rate with respect to the ratio of debt and EBITDA, β.

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Given that the leverage ratio has been widely used in the literature, I estimate another version of the model (Model 2) using the average leverage, $\mu(b/k)$. The leverage used here is defined as the ratio of debt and assets. Catherine et al. (2021) provides a detailed survey of the use of the moment.

5.4 Results

Table 2 reports parameter estimates and model fits for both targeted moments and non-targeted moments.

Model 1 Parameter Estimates

The estimated productivity process is persistent with $\rho = 0.872$. The estimated standard deviation of the innovation to productivity σ is 0.109. The estimated value for convex capital adjustment cost ψ_0 is 0.056. I estimate that the linear equity issuance cost $\eta_1 = 0.036$ and the borrowing constraint $s = 0.147$.

Model 1 parameter estimates are broadly comparable to existing estimates in the literature. The productivity process parameters (ρ, σ) are close to estimates (0.909 and 0.118) in Khan and Thomas (2013). The estimated adjustment cost ψ_0 is greater than 0.004 in Catherine et al. (2021) and less than 0.1519 in DeAngelo et al. (2011). The estimate of the linear cost of equity issuance η_1 is similar to 0.059 in Hennessy and Whited (2005) and somewhat smaller than 0.091 in Hennessy and Whited (2007) and Catherine et al. (2021). The borrowing constraint s in Model 1 is smaller than some other estimates in the literature (e.g., 0.25 in Catherine et al. (2021)). A possible reason for a tight s is a different specification for credit constraints. In my model, borrowing constraint is not only based on assets but also on the expected minimum earnings.

Model 2 Parameter Estimates

Compared to Model 1, the idiosyncratic TFP shocks are less persistent and less volatile ($\rho = 0.835, \sigma = 0.076$). The frictions are also smaller in Model 2. Specifically, I estimate a smaller capital adjustment cost $\psi_0 = 0.008$, a less expensive equity issuance cost $\eta_1 = 0.008$, and a more relaxed borrowing constraint $s = 0.349$.

I analyze why targeting the leverage in Model 2 leads to a less severity of financial frictions in Section 6.

Model Fit for Targeted Moments

The table shows that both models match the targeted moments reasonably well. Both models roughly match the dispersion of leverage ratio and investment rate.

Model 1 somewhat overpredicts the sales growth rate volatility. But it matches perfectly the average net equity issuance rate, and importantly, the slope of investment with respect to the debt-EBITDA ratio β.

Model 2 slightly underpredicts the average net equity issuance rate. But it matches perfectly the key moment of the average leverage ratio $\mu(b/k)$.

Model Fit for Non-targeted Moments

The two models perform differently in the fit with respect to non-targeted moments. Model 1 under-matches the leverage ratio $\mu(b/k)$. The model implies -0.051, as opposed to 0.1 from the data. Model 2 does a much worse job of matching β. The model-implied slope is -9.5 in Model 2, significantly smaller than -1 in the data.

Furthermore, by mapping a within-firm moment of slope, Model 1 leads to a successfully fit for investment autocorrelation. By contrast, Model 2 delivers a smaller value substantially. Finally, both models are able to reproduce average investment rate and equity issuance volatility.

The failure of Model 2 to match the key moment, the slope of investment to the debt-EBITDA ratio β, indicates that it is problematic to only target cross-sectional moments (i.e., average leverage). In Section 6, I discuss parameter identification and why Model 1 is better.

6 Effects of Financial Frictions on Firm Characteristics

In this section, I examine various firm characteristics in response to the change in debt and equity market frictions respectively. Specifically, I compare and contrast the effects on cross-sectional moments (e.g., the mean leverage) and the slope of investment with respect to the debt-to-EBITDA ratio β.

In Figure 1, I show that as borrowing constraints s increase, most firm-level cross-sectional moments are non-monotonic. But the slope of investment to the debt-to-EBITDA ratio is increasing with respect to debt market frictions, illustrated in Figure 3. In contrast, Figure 2 and 4 shows that the effect of equity issuance cost η_1 is mostly monotonic on firm-level characteristics.

I argue that Model 1 by targeting the slope of investment to the debt-to-EBITDA ratio can return unbiased parameter estimates. Model 2 targeting the leverage underestimates the severity of financial frictions in the economy.

6.1 The non-monotonic response of cross-sectional moments to borrowing constraints

Firm Investment

Figure 1 shows that, for both Model 1 and Model 2, the average investment rate $\mu(i/k)$ and the volatility of investment rate $\sigma(i/k)$ increase first and then decreases as the borrowing constraint parameter s increases.

The non-monotonic relationship between borrowing constraints and average investment rate suggests that when firms are financially-constrained, relaxing the borrowing constraints increases firms' access to credit, expands their debt capacity, and boosts investment. Hence investment rate rises at the beginning. However, as firms become less financially constrained at a higher value of s, the speed of investment and capital accumulation slows down. Therefore, investment rates decrease when s is large. This non-monotonicity suggests that an increase in s does not necessarily accelerate investment.

The investment rate volatility ...

Firm external equity issuance

The relationship between the average net equity issuance rate and the borrowing constraint parameter is a U-shaped curve.

The economic intuition is similar to that of the investment rate. When s increases, debt market frictions decline. Firms increasingly rely on debt financing and therefore the external equity issuance decreases. As s continues to increase and capital accumulation decelerates, the net equity issuance rate spikes up,

Firm leverage

For Model 1, the mean leverage ratio decreases first and then increases as the borrowing constraint parameter s decreases.

The economic intuition is similar to that of investment. When the borrowing constraint is binding, the capital stock grows large at a faster rate than the debt does and therefore the leverage ratio drops at the beginning. After a certain level, the speed of investment and capital accumulation slows down. But due to the tax advantage of debt, firms still have an incentive to borrow more. As a result, leverage increases afterward.

However, the bottom panel of Figure 1 shows that leverage is monotonic in the borrowing constraint s in Model 2.

6.2 The response of slope β

Figure 3 shows that the slope β is increasing monotonically with s and becomes less negative. Intuitively, as the borrowing constraint is relaxed, firms have increased access to credit and are able to borrow to invest. Investment is less depressed. So the slope is increasing and approaching zero.

6.3 Identification of borrowing constraint parameter s

The non-monotonicity relationship between cross-sectional moments (e.g., leverage) and debt market financial friction parameter s may give rise to an identification problem. s can be well identified only if leverage is large enough. The top panel of Figure 1 shows that when the moment of leverage is relatively large (e.g., 0.1), then it is well above the U shape and falls in the region where the value is monotonically increasing with s. Therefore, a larger targeted moment can avoid the identification problem and help pin down the parameter s.

On the contrary, there is no identification concern using the slope of investment in Model 1 since the response is unambiguously monotonic.

6.4 Implications on the magnitude of frictions and distortions in the economy

While parameters in both models are well identified, the implications of the magnitude of financial frictions are different.

In Model 2, the borrowing constraint parameter has to be larger in order to match a leverage ratio of 0.1 in the data. Firms face a less stringent borrowing constraint, thereby leading to a larger debt capacity. With similar model investment rates across two models, the larger model moment of leverage explains why the response of investment to preexisting debt β is more negative in Model 2.

Leverage also affects other parameters (see Appendix). Matching leverage implies that overall the magnitude of financial frictions and distortions is smaller. As a result, Model 2 generates estimates of smaller equity and debt market frictions (η_1, s) and smaller capital adjustment costs ψ_0.

6.5 Key takeaway: model choice

My analysis of the effects of financial frictions at the firm level points out that Model 2 fails badly in matching a key moment of the slope of investment to the debt-to-EBITDA ratio β. While it is able to perfectly target the empirical leverage ratio, the failure in targeting β uncovers that it underestimates financial frictions and distortions in the economy. These results justify using β as a target moment to identify parameters. Therefore, Model 1 is better.

7 The Aggregate Effects of Financial Frictions

In this section, I use the structural model to evaluate the aggregate effect of financial frictions. First, I analyze how macroeconomic variables such as capital stock and output respond without financial constraints. In this counterfactual experiment, I consider three different frictionless benchmarks and contrast the aggregate outcomes of my baseline model with Model 2 targeting the average leverage ratio. Table 3 reports percentage changes of macroeconomic variables when financial frictions are removed. I show that the impact of financial frictions at the aggregate level is also non-monotonic. Second, I analyze how financial frictions affect resource misallocation. 3 reports TFP loss.

Table 3 Column (1) shows the aggregate effects when the equity is free, $\eta_1 = 0$. I examine the effect of eliminating equity issuance costs. But firms can still benefit from the tax shield of debt financing. This is the unconstrained benchmark used in Catherine et al. (2021). Column (2) sets $s = 1$, which means that firms can pledge all value of the capital stock as collateral. Column (3) lifts frictions in both equity and debt markets by fixing $\eta_1 = 0, s = 1$. I note that this is not the first-best allocation since the economy still has distortions from taxes and capital adjustment costs.

7.1 Aggregate Gains from removing financial frictions

Model 1: baseline model

Financial frictions greatly affect output and capital. Removing equity financing costs increase aggregate capital by 1.54% and aggregate output by 0.84%. The effects are more significant when the pledgeability is equal to one. This increases 9.4% for capital stock and 2.68% for output. Without both frictions, gains are the largest, 9.88% and 2.97% respectively. Removing financial frictions also improves the labor market. Both aggregate employment and wage increase.

From a larger gain in capital stock than in output, the table shows that firstly, the output loss mainly comes from the loss of the input of capital stock. Removing financial frictions in either equity or debt financing market increases financially-constrained firms' access to credit. And the misallocation of input is attenuated and TFP increases.

Secondly, comparing Column (1) and Column (2), it is evident that the effect of removing equity financing costs is much smaller than the effect of removing borrowing constraints. This is because, in my baseline model, the borrowing constraint $s = 0.147$ is much tight. In contrast, the cost of equity issuance is only 0.036 per $1 of new equity issued. Therefore, the gains from removing debt market frictions are much larger quantitatively.

Thirdly, Column (3) shows that lifting financial frictions does not necessarily increase aggregate productivity. Instead, TFP gains decrease from Column (2) to Column (3) as the cost of equity issuance is removed. Figure 5 plots aggregate TFP as a function of borrowing constraints s. It can be seen that the relationship is non-monotonic. The reduced TFP gains are associated with the tax shield of debt in my model. I discuss the role of tax distortions in detail in Section 8.

Model 2

Table 3 also reports macroeconomic outcomes for Model 2 in the bottom panel. The non-monotonicity holds for more aggregate variables including capital stock, output, and employment.

The effects of relaxing borrowing constraints in Model 2 are qualitatively similar to the baseline model. Aggregate variables increase except TFP. However, when the cost of equity issuance η_1 decreases to zero, both output and input fall surprisingly and only TFP increases. It is evident that financial development may in fact reduce output and a lower cost of an equity issuance can improve resource misallocation.

7.2 Efficient allocation and TFP loss

Table 3 shows that financial frictions reduce aggregate TFP. Since there is still an effect of tax distortions and capital adjustment costs, aggregate productivity is not equal to the first-best after eliminating financial frictions. In this section, I follow procedures in Gilchrist et al. (2013) and Karabarbounis and Macnamara (2021) and compute the efficient level of aggregate TFP and the size of TFP loss.

Consider a problem faced by a social planner is to maximize aggregate output, given aggregate labor and capital:

$$Y = \max_{k_i, n_i} \int \left(z_i k_i^{\alpha} n_i^{\nu} \right) di, \; 0 < \alpha + \nu < 1$$
$$\text{subject to} \int n_i di = N \text{ and } \int k_i di = K$$

where K and N are the aggregate capital and labor stocks. The solution to this problem implies that the marginal product of labor (MPL) and the marginal product of capital (MPK) are equated across firms. Then the optimal input choices are given by:

$$n_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{N}{\Gamma} \right)$$
$$k_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \left(\frac{K}{\Gamma} \right)$$
$$\text{where } \Gamma = \int z_i^{\frac{1}{1-(\alpha+\nu)}} di$$

Under the efficient allocation, the first-best TFP is:

$$\text{TFP}^{FB} = \frac{Y}{K^{\alpha} N^{\nu}} = \Gamma^{1-(\alpha+\nu)} = \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} di \right)^{1-(\alpha+\nu)}$$

TFP loss is then defined as:

$$\text{TFP Loss} = \frac{\text{TFP}^{FB}}{\text{TFP}} - 1$$

Gilchrist et al. (2013) and Karabarbounis and Macnamara (2021) assume that (z, MPK) are jointly log-normally distributed across firms. I also make the same assumption and the relative

TFP loss due to resource misallocation is approximated by (See Appendix for details):

$$\text{Relative TFP Loss} = \log\left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}}\right)$$
$$\approx \frac{1}{2}\alpha(1-\alpha)\left(\frac{1-\nu}{1-\alpha-\nu}\right)^2 \text{Var}(\log(\text{MPK}))$$

Financial frictions will reduce TFP by increasing the dispersion in MPK across firms.

Table 3 reports TFP losses for the economy with and without financial frictions. Lifting financial frictions, TFP loss decreases from 8% to about 7.5%. Compared to the economy without both debt and equity market frictions (Column (3) $\eta_1 = 0, s = 1$), the economy with equity market frictions (Column (2) $s = 1$) has a smaller TFP loss. Similar results hold for Model 2.

8 Model Mechanism: The Impact of Tax Benefit of Debt

In Section 7, I show that eliminating financial frictions can conversely disrupt economic activity and generate misallocation. In this section, I inspect the model mechanism and investigate the role of the tax shield in firm financing decisions and the aggregate economy.

To examine the role of the tax shield, I vary the magnitude of the tax rate only on the interest deduction while fixing the tax rate on the corporate income at τ. Then the tax shield value of the interest expense is $br\tau_S$ where τ_S can be different from the baseline corporate tax rate τ. I refer τ_S as the tax shield rate. Figure 7 shows that the increase in the tax shield rate leads to a higher leverage ratio, capital stock and wage as firms have a greater incentive to borrow. The dispersion of leverage and investment also increases, which is consistent with the effect of borrowing constraints. At the aggregate level, the impact on productivity is negligible, while the aggregate output is negatively affected when τ_S increases, as shown in Figure 8.

Table 4 compares aggregate variables with and without tax shield of debt. When debt financing does not have a tax advantage, free equity implies the same capital allocations regardless of the value of borrowing constraint s (see Panel A Column (1) and Column (3)). Therefore, employment, output, and productivity are the same at the optimal level.

In the presence of the tax shield, aggregate capital and output are higher. When the economy is constrained by both debt and equity financing frictions, Column (2) shows that increased borrowing helps mitigate misallocation so that TFP is higher in Panel B. On the contrary, when firms are less financially constrained as in Column (1) and Column (3), the over-borrowing exacerbates the misallocation problem therefore TFP is lower in Panel B.

In summary, by reducing the borrowing cost of capital, tax benefit in fact relaxes credit constraints, thereby decreasing the aggregate costs of financial frictions. Therefore, if the financial friction problem is severe in the baseline economy, not accounting for the tax shield will overestimate the aggregate effects of financial frictions. On the other hand, if the financial friction problem is minor, the presence of a tax shield has an adverse impact on the macroeconomy by distorting resource allocation and reducing aggregate productivity, while it still boosts the output. As a result, not accounting for the tax shield will underestimate the aggregate effects of financial frictions.

9 Conclusion

In this paper, I estimate the aggregate effects of financial frictions. Using a dynamic general equilibrium heterogeneous firm investment model, I quantify the importance of debt and equity market frictions, in the forms of borrowing constraints and costly equity issuance. I find that in the presence of tax benefit of debt, the effects of financial frictions are non-monotonic in most firm-level cross-sectional moments. But the effect on the slope of investment with respect to leverage is increasing. When financial constraints are relaxed, the slope becomes less negative and approaches zero. Therefore, I propose to target the slope of investment to leverage, as opposed to the mean leverage ratio in the literature.

My quantification results show that aggregate variables gain from removing financial frictions: 10% for aggregate capital stock, 4% for employment, 3% for output, and 0.2% for TFP. But under the influence of tax, the effect of financial frictions on aggregate productivity is also non-monotonic. This is due to the role of the tax shield of debt in my model. My counterfactual analysis reveals that the tax-induced debt bias can reduce the negative impacts of financial frictions for credit-constrained firms by incentivizing them to borrow and invest. On the other hand, the resulting over-borrowing and over-investment may distort resource allocation and hence drag down aggregate productivity. This indicates that the effects of financial frictions depend on the interactions of various frictions and distortions.

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Table 1: Externally Calibrated Parameter Values

Parameter	Description	Value	Source
Technology			
α	Capital share	0.25	Typical in literature
ν	Labor share	0.6	Typical in literature
δ	Capital depreciation rate	0.1	Bloom et al. (2018)
τ	Corporate tax rate on profits and interest	0.2	Gomes and Schmid (2010)
π_d	Exit rate	0.1	Khan and Thomas (2013)
κ_0	Fraction of the steady-state aggregate capital stock held by each entrant	0.2	Jeenas (2019)
Financial Frictions			
ψ_1	Fixed investment adjustment cost	0	Catherine et al. (2021)
η_0	Fixed external financing/equity issuance cost	0	Catherine et al. (2021)
Preference			
φ	Labor disutility	2	Bloom et al. (2018)
β^H	Subjective discount factor	0.96	Jo and Senga (2019)
Price			
r	Risk-free interest rate	0.04	Jo and Senga (2019)

Note: The table reports the notation, description, value, and source for the set of externally calibrated parameters.

Table 2: Model Estimation Results

Panel A. Estimated Parameters

Parameter	Description	Model 1	SE	Model 2	SE
ρ	Productivity persistence	0.872	(0.0020)	0.835	(0.0011)
σ	St. Dev. of innovations to productivity	0.109	(0.0005)	0.076	(0.0008)
ψ_0	Convex investment adjustment cost	0.056	(0.0018)	0.008	(0.0046)
η_1	Linear equity issuance cost	0.036	(0.0001)	0.008	(0.0000)
s	Frac. of debt that can be collateralized	0.147	(0.0182)	0.349	(0.0031)

Panel B. Model Fit: Targeted Moments

Moment	Description	Model 1	Model 2	Data
$\sigma(b/k)$	debt rate volatility	0.365	0.300	0.32
$\sigma(i/k)$	investment rate volatility	0.478	0.556	0.53
$\sigma(\Delta y_{-1})$	1-year sales growth rate volatility	0.374	0.340	0.35
$\sigma(\Delta y_{-5})$	5-year sales growth volatility	0.938	0.806	0.8
$\mu(e/k)$	average net equity issuance rate	0.100	0.067	0.1
β	slope of i/k wrt debt/EBITDA	-0.998		-1
$\mu(b/k)$	mean leverage		0.091	0.1

Panel C. Model Fit: Non-Targeted Moments

β	slope of i/k wrt debt/EBITDA		-9.500	-1
$\mu(b/k)$	mean leverage	-0.051		0.1
$\mu(i/k)$	average investment rate	0.187	0.309	0.40
$\text{corr}(i/k, i/k_{-1})$	autocorrelation of investment rate	0.281	0.015	0.32
$\sigma(e/k)$	net equity issuance rate volatility	0.243	0.219	0.45

Notes: The table reports point estimates and standard errors (in parentheses) for each of the parameters estimated via SMM. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is the inverse of the moment covariance matrix.

Table 3: Aggregate Implications of Financial Frictions: (1) free equity (2) no collateral constraint (3) free equity and no collateral constraint

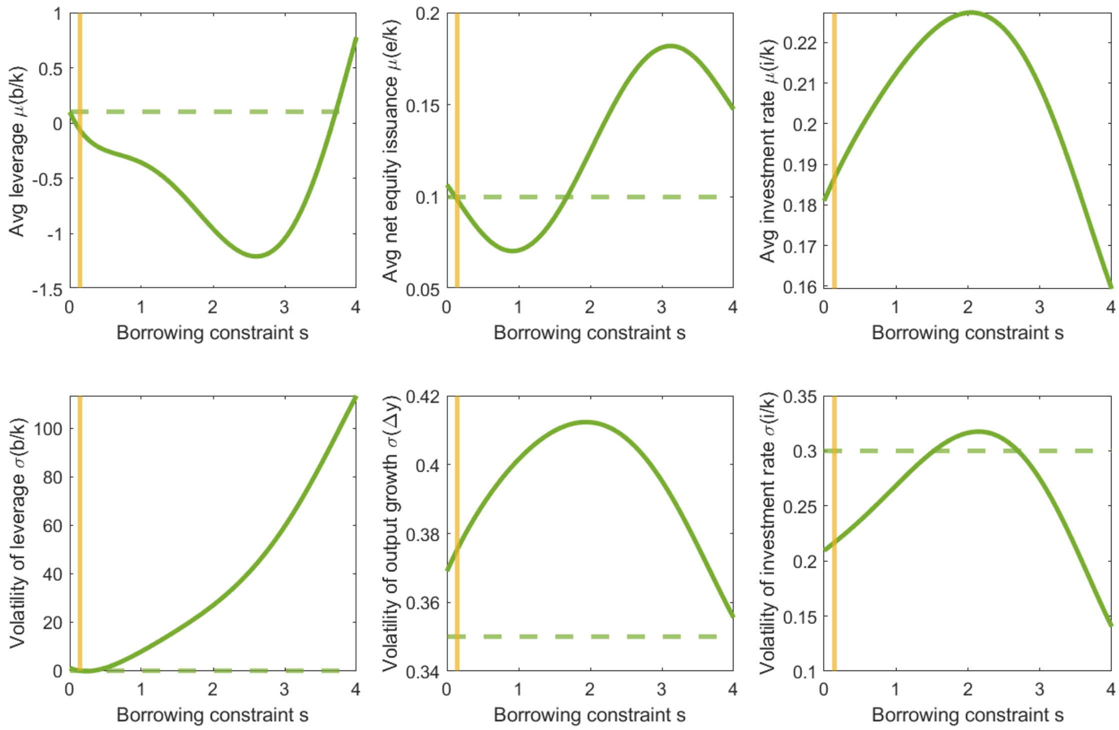
	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	Benchmark value
Model 1: targeting slope β				
$100 \times \Delta \log$ (K)	1.541	9.397	9.880	0.775
$100 \times \Delta \log$ (N)	0.402	0.212	0.498	0.310
$100 \times \Delta \log$ (Output)	0.837	2.678	2.965	0.576
$100 \times \Delta \log$(Wage)	0.435	2.466	2.466	1.114
$100 \times \Delta \log$ (TFP)	0.211	0.202	0.196	1.234
Relative TFP loss	7.659%	7.465%	7.493%	8.055%
Model 2: targeting mean leverage				
$100 \times \Delta \log$ (K)	−0.799	1.969	−0.156	0.706
$100 \times \Delta \log$ (N)	−0.874	−0.101	−0.879	0.312
$100 \times \Delta \log$ (Output)	−0.699	0.408	−0.580	0.493
$100 \times \Delta \log$(Wage)	0.175	0.509	0.299	0.949
$100 \times \Delta \log$(TFP)	0.025	−0.024	−0.014	1.082
Relative TFP loss	3.402%	3.442%	3.430%	3.431%

Table 4: Baseline Model: Effects of Tax Shield on Aggregate Variables

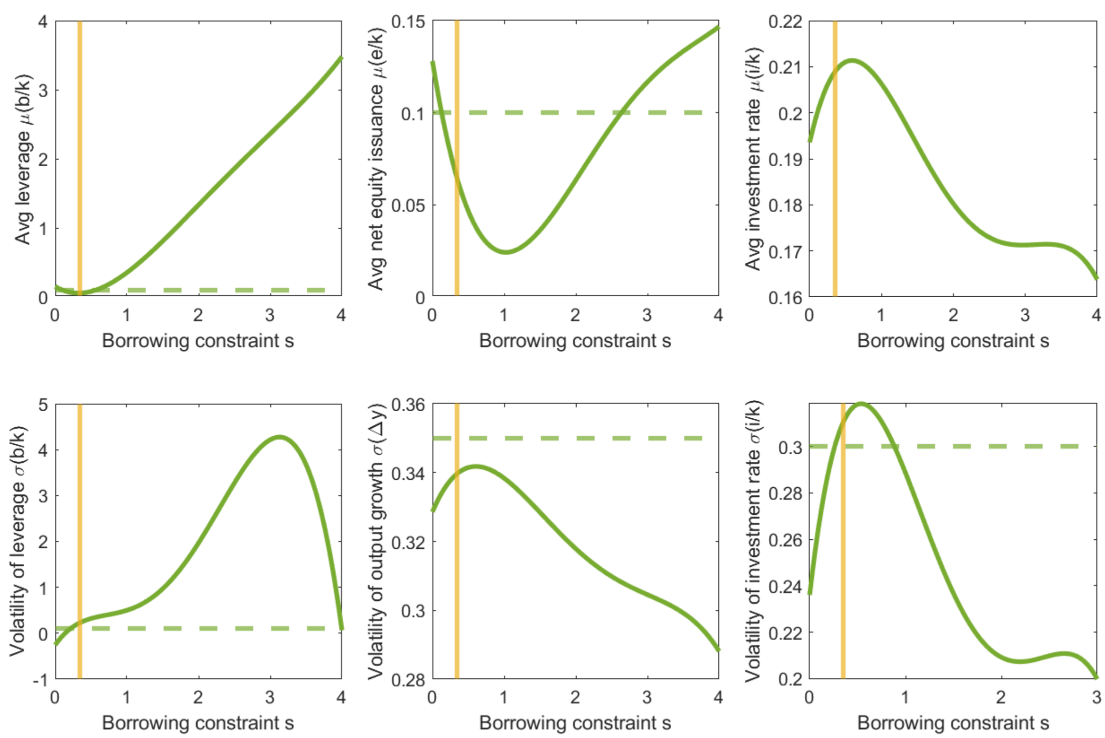
	(1) $\eta_1 = 0$	(2) $s = 1$	(3) $\eta_1 = 0, s = 1$	(4) Benchmark value
Panel A. Without Tax Shield				
K	0.788 75	0.791 56	0.788 75	0.781 21
N	0.313 23	0.313 75	0.313 23	0.314 27
Y	0.579 49	0.580 45	0.579 49	0.578 11
TFP	1.233 93	1.233 65	1.233 93	1.231 51
Panel B. With Tax Shield				
K	0.814 74	0.851 62	0.855 75	0.775 24
N	0.317 11	0.309 90	0.310 78	0.309 24
Y	0.588 50	0.586 92	0.588 60	0.571 41
TFP	1.233 87	1.233 92	1.233 84	1.231 43

Figure 1: Effect of Borrowing Constraints s on Firm Characteristics

(a) Model 1: targeting β



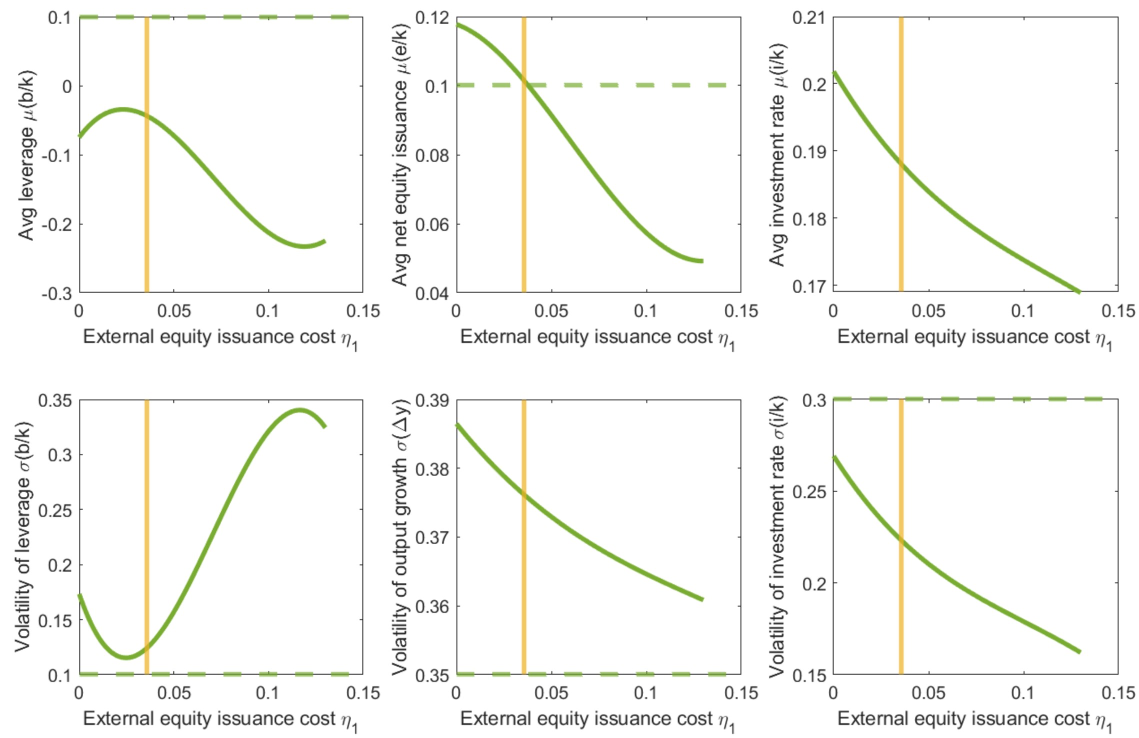
(b) Model 2: targeting $\mu(b/k)$



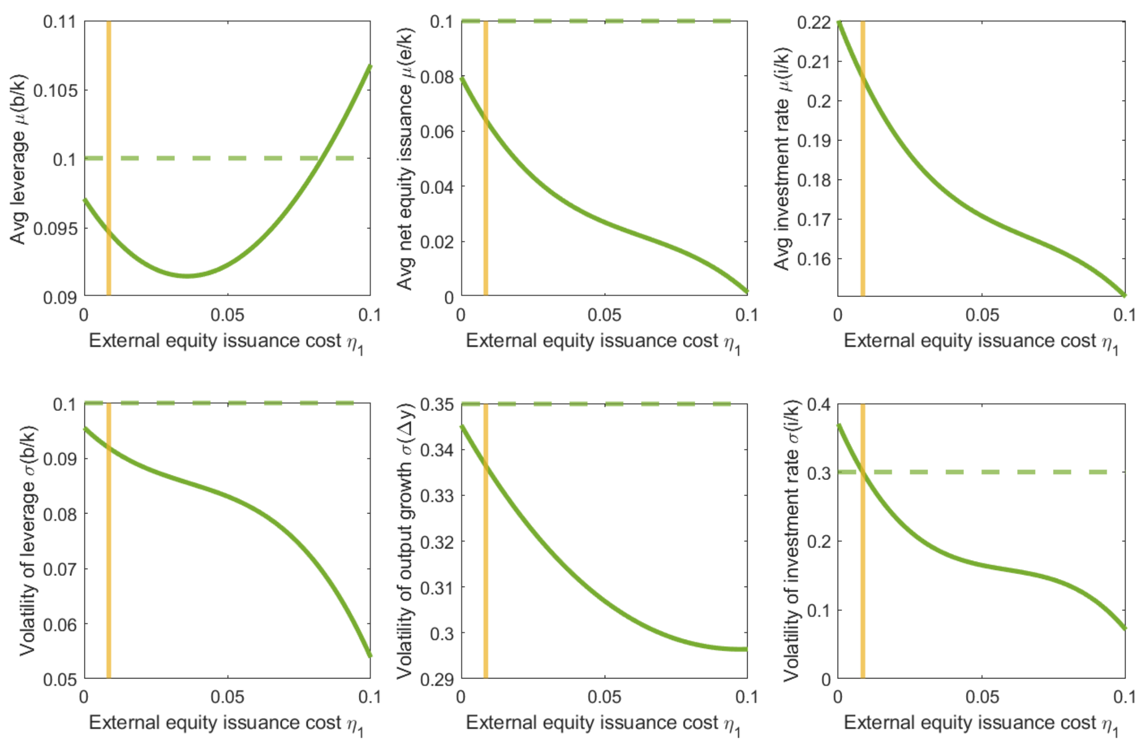
Notes: Numerical comparative statics are smoothed using a polynomial approximation.

Figure 2: Effect of Equity Issuance Cost η_1 on Firm Characteristics

(a) Model 1: targeting β



(b) Model 2: targeting $\mu(b/k)$



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

Figure 3: Effect of Borrowing Constraints s on Slope of Investment to Debt/EBITDA

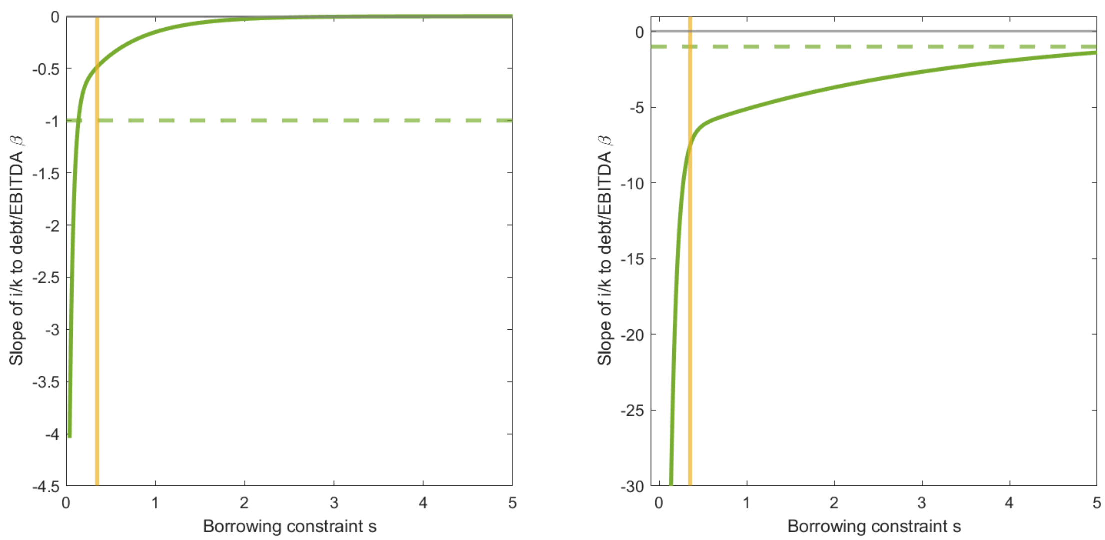


Figure 4: Effect of Equity Issuance Cost η_1 on Slope of Investment to Debt/EBITDA

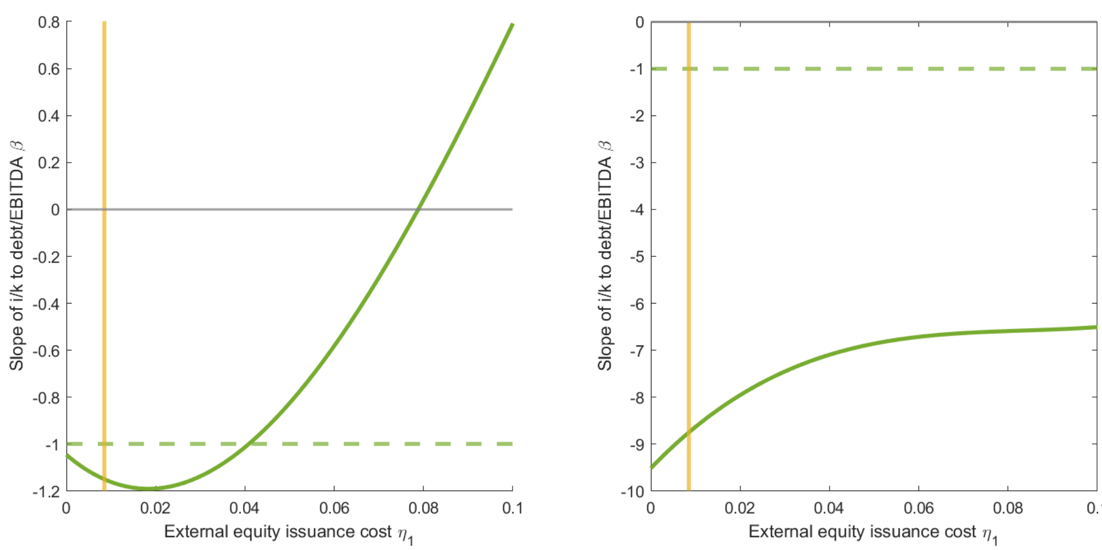
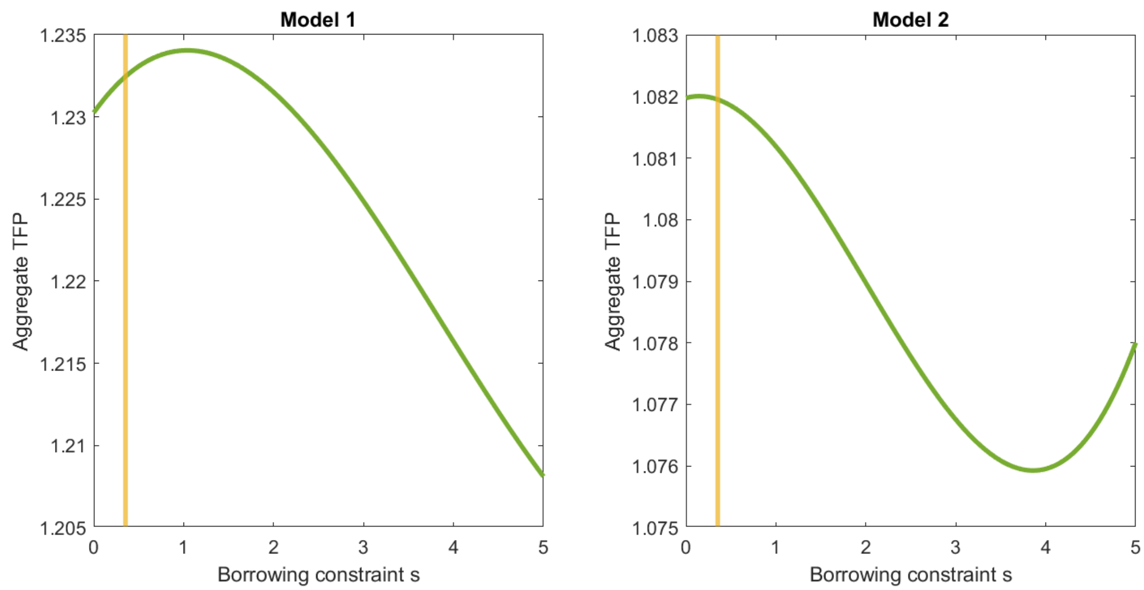
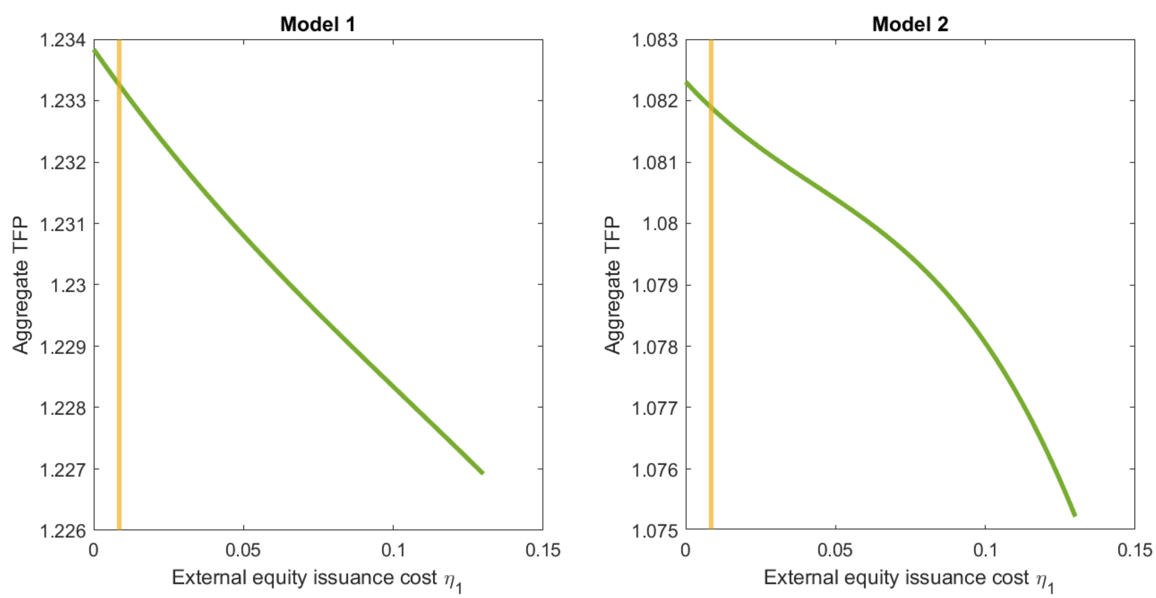


Figure 5: Effect of Borrowing Constraints s on Aggregate TF



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

Figure 6: Effect of Equity Issuance Cost η_1 on Aggregate TFP



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

Figure 7: Effect of Debt Tax Shields on Firm Characteristics

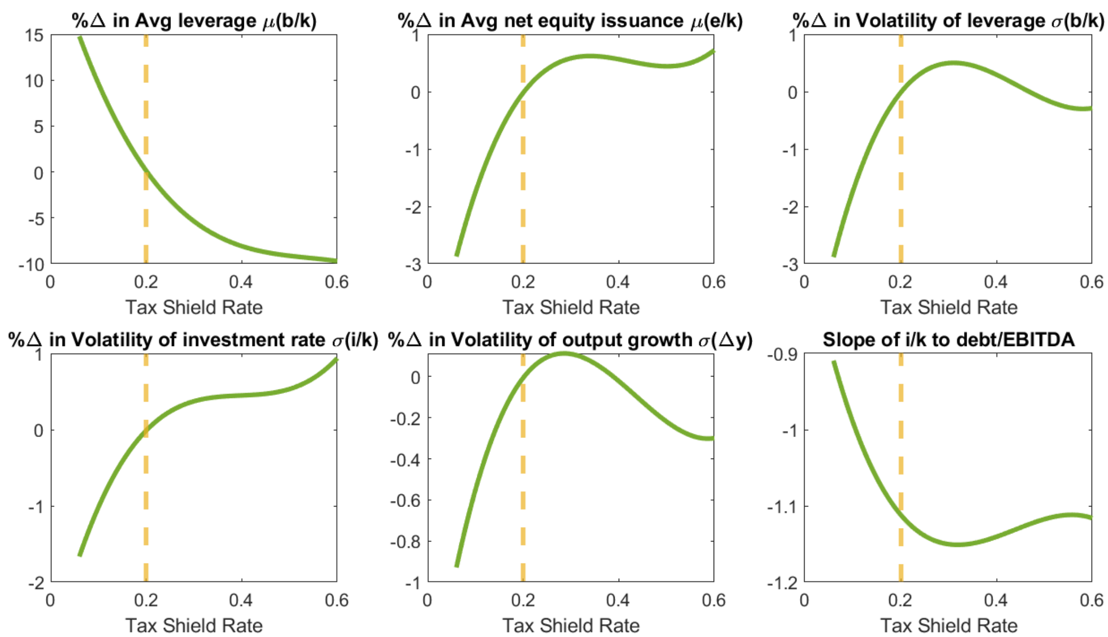
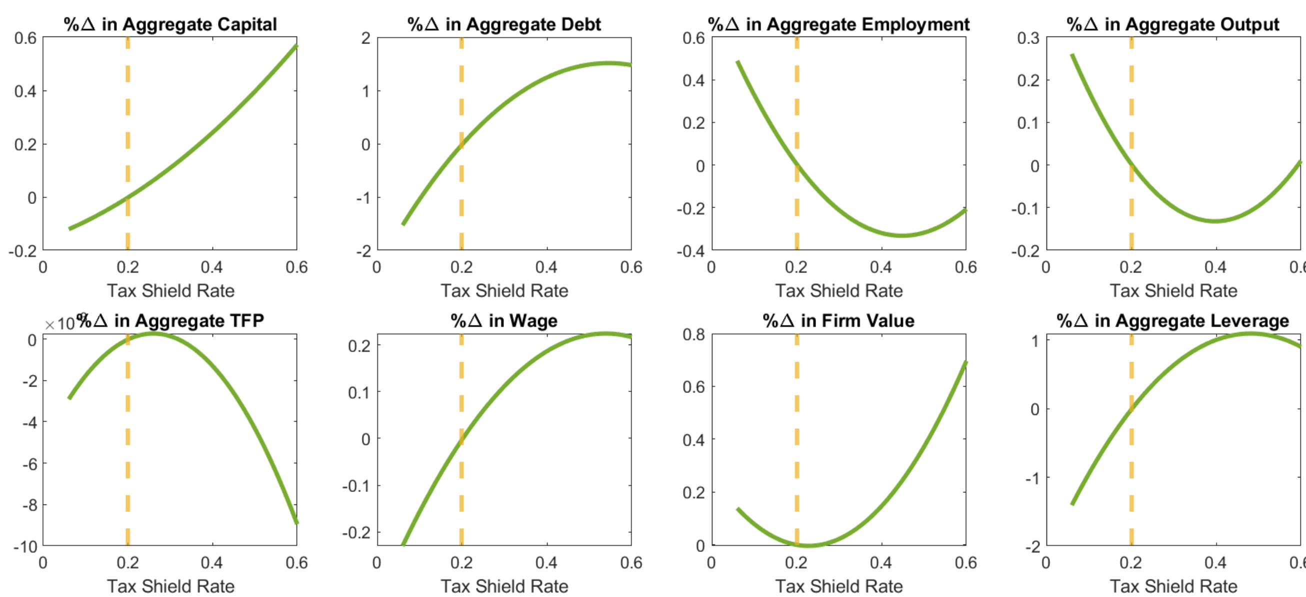


Figure 8: Effect of Debt Tax Shields on Aggregate Variables



Notes: Numerical comparative statics are smoothed using a polynomial approximation.

A Data

I obtain data on U.S. nonfinancial firms from the 2021 Standard and Poor's Compustat industrial annual fundamental files. The data is an unbalanced panel that covers 1981 to 2018.

A.1 Variable Definition

I use the beginning-of-the-period capital `PPENT` as the firms' capital stocks.

To compute the net equity issuance rate, I take the maximum of e in Table 5 and zero and normalize it by total assets (`AT`). The construction is similar to Catherine et al. (2021), which normalizes equity issuance by value-added. They approximate value added by 60% of total sales, assuming a 40% gross margin ratio.

I consider two definitions of the leverage ratio. The first is defined as the ratio of net debt to lagged assets, `(DLC+DLTT-CHE)/L.AT`. This definition is used for targeted moments: debt rate volatility $\sigma(b/k)$ and mean leverage ratio $\mu(b/k)$. The second is the debt-to-EBITDA ratio, `(DLC+DLTT-CHE)/EBITDA`, which is used to compute the slope of investment with respect to preexisting debt.

I follow Crouzet and Tourre (2020) to compute the slope of investment with respect to the debt-to-EBITDA ratio as follows:

$$\beta = \frac{\text{Cov}(i/k,\, b/\text{EBITDA})}{\text{Var}(b/\text{EBITDA})} \times 100$$

where b/EBITDA is the beginning-of-the-period value and i/k is the current-period value. The slope is computed with non-negative `EBITDA`.

I use the BEA nonresidential fixed investment implicit price deflator (from FRED) to deflate capital stock and investment. I use the gross GDP deflator to deflate other variables.

A.2 Sample Selection

We apply the following sample selection criteria:

1. Drop firm-year observation that is incorporated in the United States (`FIC` = "USA")

2. Drop firm-year observations if two-digit SIC code (`SIC`) is in the financial industry (SIC code between 6000 and 6999), utilities (SIC code between 4900 and 4999), or public administration (SIC code between 9000 and 9999)

3. Keep observations for fiscal year (`FYEAR`) between 1980 and 2018

4. Drop firm-year observations with missing assets (AT), sales (SALE), cash holdings (CHE), long-term debt (DLTT), short-term debt (DLC), capital expenditure (CAPXV), earnings before interest (EBITDA), and capital stock (PPENT)
5. Drop firm-year observations with non-positive SALE or AT
6. Drop firms that are in the data for smaller than 5 years
7. Keep observations with non-missing
8. Trim variables of interest at the top 99% and bottom 1%

Comparison of key moments to literature

The volatility of the leverage ratio (i.e., net debt-to-lagged asset) is 0.32. This is in the range of the moments in the literature. DeAngelo et al. (2011) uses gross debt, who report that the standard deviation of gross leverage is 0.1086 (variance of leverage is 0.0118). Karabarbounis and Macnamara (2021) report a standard deviation of the gross leverage ratio of 0.42.

The investment rate volatility is 0.53, a little higher than existing estimates. Using PPEGT, the gross value property, plant, and equipment, as capital stock, DeAngelo et al. (2011) report that the standard deviation of investment rate is 0.1962 (variance of investment rate is 0.0385). Karabarbounis and Macnamara (2021) also use PPEGT and report that the standard deviation of investment rate is 0.22 and Ottonello and Winberry (2020) report a value of 0.33. Weighting and sample selection might explain why my data moment is more elevated.

My data moment of short-run sales growth rate volatility is 0.35 and the long-run volatility is 0.8. The estimates are similar to Catherine et al. (2021), which document that the volatility of 1-year and 5-year sales growth rates is 0.327 and 0.912 respectively.

My sample's average net equity issuance rate is 0.1, which is broadly consistent with the literature. Hennessy and Whited (2005) and Hennessy and Whited (2007) report a gross equity issuance rate of 4.2% and 8.9%. Belo et al. (2019) report that the value is 0.04. Catherine et al. (2021) normalize the net equity issuance by value-added and the net equity issuance rate is 0.026.

The average leverage ratio, defined as net debt to lagged assets, in my sample is 0.1. My value is similar to 0.098 in Catherine et al. (2021). Crouzet and Tourre (2020) document the ratio is 25.58%. In Hennessy and Whited (2007), the leverage ratio is 12.04% for their baseline estimation and 14.52% in their restricted large firm sample. Karabarbounis and Macnamara (2021) report the mean leverage ratio of 0.29. Belo et al. (2019) report a value of 0.25.

The slope of investment with respect to the ratio of debt to EBITDA in my sample is -1. My data estimate is close to -1.04 in Crouzet and Tourre (2020).

The non-targeted moment of the average investment rate is 0.4 in my sample. Using PPENT, my estimate is somewhat larger than previous research. For example, the average investment is 0.1868 in DeAngelo et al. (2011), 0.16 in Karabarbounis and Macnamara (2021), 0.11 in Crouzet and Tourre (2020), and 0.07 in Eisfeldt and Muir (2016). Hennessy and Whited (2005) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample.

In my sample, the autocorrelation of investment rate is 0.34. The value is in the range of 0.165 in Catherine et al. (2021), 0.17 in Karabarbounis and Macnamara (2021), 0.40 in Eisfeldt and Muir (2016), and 0.41 in Belo et al. (2019).

Finally, my estimate of the standard deviation of the net equity issuance rate is 0.35. My estimate is close to the moment in Hennessy and Whited (2007) Variance of Equity Issuance/Asset, 0.3018.

B Model

In this section, I describe the structural estimation procedure. First, for a given set of parameters, I solve the model numerically by iterating on the firm's Bellman equation, which produces the value function $V(k,b,z)$ and the policy function (k',b'). Then, I simulate the economy and search for parameters that model-generated moments could match data moments.

B.1 Numerical Solution Method

I use policy iteration to solve for the firm's problem by iterating on the Bellman equation defined in Eq. (13) until convergence.

Grid definition

I transform (32) into a discrete-state Markov chain using the method in Tauchen (1986). I let productivity z (in logs) have 5 points of support on the interval $[\log(\underline{z}), \log(\overline{z})] = [-3\sigma, 3\sigma]$. I let capital stock k have 100 equally-spaced (in logs) on the interval $[\log(\underline{k}), \log(\overline{k})] = [0.001, 100]$.

Since debt b is bounded above by the borrowing constraints, I set the maximal value of b with the maximal value of k, $\overline{k}$. Therefore $\bar{b} = (1-\tau)(\underline{z}\overline{k}'^{\alpha}\underline{n}'^{\nu} - W\underline{n}') + s(1-\delta)\overline{k}'$. The minimum of b is chosen so that the optimal choice of debt never hits the lower endpoint. I verify ex-post and set $\underline{b} = -0.01 \times \bar{b}$. I let debt b have 40 geometrically-spaced points in the interval $[\underline{b}, \bar{b}]$

The state space for the firm's problem is $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$.

Policy function iteration

I compute the return matrix $R(k',b',k,b,z)$ for all possible values of (k,b,z):

$$R(k',b',k,b,z) = \pi_d e_0(k,b,z) + (1-\pi_d)\left[e_1(k',b',k,b,z) + \eta(e_1(k',b',k,b,z))\right]$$

I set $R(.)$ to "missing" when (k,b,z) are such that the borrowing constraint is violated. Given a value function $V(k,b,z)$, the policy function $(k',b') = P(k,b,z)$ solves:

$$P^*(k,b,z) = \arg\max_P \left\{R(P(k,b,z),k,b,z) + \frac{1-\pi_d}{1+r}\mathbb{E}\left[V(P(k,b,z),z')|z\right]\right\}, \forall(k,b,z)$$

I assume that firms can only choose values of (k',b') on a discrete grid, where $k' \in \mathcal{K} = \{k_1, ..., k_{N_k}\}$ and $b' \in \mathcal{B} = \{b_1, ..., b_{N_b}\}$. (N_k, N_b) are the number of grid points for capital and debt, respectively, and N_z is the number of grid points for productivity. Therefore the number of states is $N_k \times N_b \times N_z = 100 \times 40 \times 5 = 20,000$. The number of choices is $N_k \times N_b = 100 \times 40 = 4,000$.

I initiate with the process with a guess V_0, P_0 and specify a solution tolerance $\varepsilon_{\text{tolerance}} > 0$. To speed up the computation, I apply the Howard improvement algorithm and iterate the policy function instead of the value function iteration.

To solve for the steady state equilibrium given a set of parameters, the algorithm proceeds as follows:

1. **Outer Loop**: Suppose the real wage is in a range of $[W_a, W_c]$. Guess the value of the real wage $W_b = (W_a + W_c)/2$ using the bisection algorithm

2. Solve the firm's problem $V(k, b, z)$ and compute the stationary distribution $\Gamma(k, b, z)$ with firm policies $(k', b') = P(k, b, z)$, for $n = 1, 2, \dots$, given real wage W_b

   (a) **Inner Loop**: Starting from the policy function $(k'_{n-1}, b'_{n-1}) = P_{n-1}(k, b, z)$ and value function $V_{n-1}(k, b, z)$ from the previous round, solve for the optimal policy P_n:

   $$P_n(k, b, z) = \arg\max_P \left\{ R(P(k, b), k, b, z) + \frac{1 - \pi_d}{1 + r} \mathbb{E}\left[V_{n-1}(P(k, b), z') | z \right] \right\}$$

   (b) Set $\tilde{V}^1_{n-1} = V_{n-1}$. For each Howard improvement step $h = 1, \dots H - 1$, iterate the Bellman equation without optimization:

   $$\tilde{V}^{(h+1)}_{n-1}(k, b, z) = \left\{ R(P_n(k, b, z), k, b) + \frac{1 - \pi_d}{1 + r} \mathbb{E}\left[\tilde{V}^{(h)}_{n-1}(P_n(k, b, z), z') | z \right] \right\}$$

   (c) Set $V_n = \tilde{V}^{(H)}_{n-1}$

   (d) Compute the error $\|P_n - P_{n-1}\| = \max_{k,b,z} |P_n(k, b, z) - P_{n-1}(k, b, z)|$.

   - If $\|P_n - P_{n-1}\| < \varepsilon_{\text{tolerance}}$, exit.
   - If $\|P_n - P_{n-1}\| \geq \varepsilon_{\text{tolerance}}$, go back to Step 2(a) with $n = n + 1$

3. Calculate the implied value of aggregate consumption $C(W_b)$

4. If W and φC are within some tolerance of each other, $|W_b - \varphi C(W_b) < \varepsilon_{tol}|$, then I solve the model and set $W^* = W_b$. If not, then update my guess for W as follows and return to Step 1:

   - If $W_b < \varphi C(W_b)$, then wage is underestimated. I set $W_a = W_b$
   - If $W_b > \varphi C(W_b)$, then wage is overestimated. I set $W_c = W_b$

The contracting mapping theorem guarantees that there is a fixed point where the policy function converges under some regularity conditions. I set the step $H = 10$ for the Howard improvement algorithm.

With $N_m \geq N_p$, the model is over-identified. Then the test of the overidentifying restrictions of the model is:

$$J = \frac{NS}{1+S} \min_{\Theta} \hat{g}_N' \hat{W}_N \hat{g}_N \sim \chi^2(N_m - N_p)$$

Weighting matrix

Simulation and model-generated moments

Once we have solved the model for a given set of parameters, I simulate data in order to compute the simulated moments.

I simulate a balanced panel of 5,000 firms over 5,500 years, and only keep the last 50 years to ensure each firm has reached the steady-state. For each firm, I take a random draw from the distribution of productivity z and simulate a path of log

Optimization algorithm

Standard errors

B.2 Structural Estimation Method

I use the simulated method of moments (SMM) to estimate a vector of unknown structural parameters, $\Theta^* = (\rho, \sigma, \psi_0, \eta_1, s)$. This procedure chooses parameters to minimize the distance between model-generated moments and the corresponding data moments.

SMM Estimation

Let M be the actual data moments and $m^s(\Theta)$ is a vector of moments computed from the sth simulated sample using parameters Θ, where $s = 1, ..., S$. S is the number of simulations. N is the number of observations in actual data. The number of targeted moments $N_m = 6$. The number of parameters of interest $N_p = 5$.

The SMM estimator of Θ^* solves:

$$\begin{aligned}\hat{\Theta} &= \arg\min_{\Theta} \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^{S} m^s(\Theta) \right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^{S} m^s(\Theta) \right] \\ &= \arg\min_{\Theta} \hat{g}_N' \hat{W}_N \hat{g}_N\end{aligned}$$

where $\hat{W}_N$ is an $N_m \times N_m$ arbitrary positive definite matrix that converges in probability to a deterministic positive definite matrix W.

The simulated moment estimator is asymptotically normal for fixed S. The asymptotic distribution of Θ is given by:

$$\sqrt{N}(\hat{\Theta} - \Theta^*) \xrightarrow{\text{d}} \mathcal{N}(0, \text{avar}(\hat{\Theta}))$$

Let

$G = \dfrac{\partial m(\Theta)}{\partial \Theta}$, the $N_m \times N_p$ gradient matrix where $G_{ij} = \dfrac{\partial m_i(\Theta)}{\partial \Theta_j}$

$\Omega = \lim_{N \to \infty} \text{Var}(\sqrt{N}\hat{M}_N)$, the $N_m \times N_m$ asymptotic variance-covariance matrix of the data moments

Then

$$\begin{aligned}
\text{avar}(\hat{\Theta}) &= \left(1 + \frac{1}{S}\right)(G'WG)^{-1}G'W\Omega WG(G'WG)^{-1} \\
&= \left(1 + \frac{1}{S}\right)\left[\frac{\partial \hat{m}_n(\Theta)'}{\partial \Theta} W \frac{\partial \hat{m}_n(\Theta)}{\partial \Theta}\right]^{-1}
\end{aligned}$$

The optimal weighting matrix is equal to the inverse of a covariance matrix that is calculated using the influence function approach of Erickson and Whited (2002): $W = \Omega^{-1}$

$$\text{avar}(\hat{\Theta}) = \left(1 + \frac{1}{S}\right)(G'WG)^{-1}$$

The weighting matrix W is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrapping with replacement on the actual data. The estimates of variance-covariance matrix is qualitatively similar to the ones computed from the Delta Method.

C First-Best TFP and TFP Loss

In this section, I derive expressions for the first-best TFP and relative TFP loss.

Firms choose capital and labor optimally where the marginal product of capital and labor are equal to their respective costs. Since labor is static, the marginal product of labor is equal to wage W.

$$\mathrm{MPK}_i = \alpha \frac{y_i}{k_i} = X_i$$
$$\mathrm{MPL}_i = \nu \frac{y_i}{n_i} = W$$

Then the optimal capital-labor ratio is given by

$$\frac{k_i}{n_i} = \frac{\alpha}{\nu} \frac{W}{X_i}$$

Solving for the labor input yields

$$n_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \underbrace{X_i^{-\frac{\alpha}{1-(\alpha+\nu)}}}_{w_i^N} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{1-\alpha}{1-(\alpha+\nu)}} \alpha^{\frac{\alpha}{1-(\alpha+\nu)}}}_{c_N}$$

Then the optimal capital input is

$$k_i = z_i^{\frac{1}{1-(\alpha+\nu)}} \underbrace{X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}}}_{w_i^K} \underbrace{\left(\frac{\nu}{W}\right)^{-\frac{\nu}{1-(\alpha+\nu)}} \alpha^{\frac{1-\nu}{1-(\alpha+\nu)}}}_{c_K}$$

where w_i^N and w_i^K denote labor and capital wedges relative to an efficient allocation of inputs. As discussed above, MPL_i and w_i^N are the same across firms. At the efficient allocation, the marginal product of capital is also equated across firms.

Aggregate labor and capital can be expressed as

$$N = \int n_i di = c_N \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di$$
$$K = \int k_i di = c_K \int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di$$

The aggregate output is

$$Y = \int y_i di = (c_K^{\alpha} c_N^{\nu}) \int z_i^{\frac{1}{1-(\alpha+\nu)}} \left(w_i^K\right)^{\alpha} \left(w_i^N\right)^{\nu} di$$

Then the aggregate productivity is given by

$$\text{TFP} = \frac{Y}{K^{\alpha} N^{\nu}} = \frac{\int z_i^{\frac{1}{1-(\alpha+\nu)}} \left(w_i^K\right)^{\alpha} \left(w_i^N\right)^{\nu} di}{\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di\right)^{\alpha} \left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di\right)^{\nu}} \tag{10}$$

Expressing Equation (10) in logs yields

$$\log(\text{TFP}) = \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} \left(w_i^K\right)^{\alpha} \left(w_i^N\right)^{\nu} di\right) - \alpha \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^K di\right) - \nu \log\left(\int z_i^{\frac{1}{1-(\alpha+\nu)}} w_i^N di\right) \tag{11}$$

Define $A_i = z_i^{\frac{1}{1-(\alpha+\nu)}}$. Assume that $\left(A_i, w_i^K, w_i^N\right)$ are jointly log-normal distributed

$$\begin{bmatrix} \log(A_i) \\ \log(w_i^K) \\ \log(w_i^N) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_a \\ \mu_K \\ \mu_N \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{a,K} & \sigma_{a,N} \\ \sigma_{a,K} & \sigma_K^2 & \sigma_{K,N} \\ \sigma_{a,N} & \sigma_{K,N} & \sigma_N^2 \end{bmatrix}\right)$$

The second-order approximations of Equation (11) are given by

$$\begin{aligned} \log\left(\int A_i \left(w_i^K\right)^{\alpha} \left(w_i^N\right)^{\nu} di\right) &= \mu_a + (\alpha\mu_K + \nu\mu_N) + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha^2\sigma_K^2 + \frac{1}{2}\nu^2\sigma_N^2 + \alpha\nu\sigma_{K,N} + \alpha\sigma_{a,K} + \nu\sigma_{a,N} \\ \log\left(\int A_i w_i^K di\right) &= \mu_a + \mu_k + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\sigma_k^2 + \sigma_{a,K} \\ \log\left(\int A_i w_i^N di\right) &= \mu_a + \mu_N + \frac{1}{2}\sigma_a^2 + \frac{1}{2}\sigma_N^2 + \sigma_{a,N} \end{aligned}$$

Rearrange the above expressions. Then Equation (11) is given by

$$\log(\text{TFP}) = (1-\alpha-\nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) - \frac{1}{2}\alpha(1-\alpha)\sigma_K^2 - \frac{1}{2}\nu(1-\nu)\sigma_N^2 + \alpha\nu\sigma_{K,N}$$

Given w_i^N is equalized across firms, then $\sigma_N^2 = 0$ and $\sigma_{K,N} = 0$. Therefore,

$$\log(\text{TFP}) = (1-\alpha-\nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) - \frac{1}{2}\alpha(1-\alpha)\sigma_K^2$$

The efficient allocation implies that $\sigma_K^2 = 0$. Then I can approximate the first-best TFP as

$$\log(\text{TFP}^{\text{FB}}) = (1-\alpha-\nu)\left(\mu_a + \frac{1}{2}\sigma_a^2\right) \tag{12}$$

The TFP loss is defined to be

$$\text{Relative TFP Loss} = \log\left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}}\right) = \frac{1}{2}\alpha(1-\alpha)\sigma_K^2 \tag{13}$$

To solve for σ_K^2. Recall that

$$\text{MPK}_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = X_i^{-\frac{1-\nu}{1-(\alpha+\nu)}} = w_i^K$$

Then MPK is also log-normally distributed

$$-\frac{1-\nu}{1-(\alpha+\nu)}\log(X_i) = \log(w_i^K) \sim \mathcal{N}(\mu_K, \sigma_K^2)$$
$$\log(X_i) = -\frac{1-(\alpha+\nu)}{1-\nu}\log(w_i^K) \sim \mathcal{N}\left(\mu_K\left[-\frac{1-(\alpha+\nu)}{1-\nu}\right], \sigma_K^2\left[\frac{1-(\alpha+\nu)}{1-\nu}\right]^2\right)$$

This solves σ_K^2

$$\text{Var}(\log(\text{MPK}_i)) = \text{Var}(\log(X_i)) = \sigma_K^2\left[\frac{1-(\alpha+\nu)}{1-\nu}\right]^2$$
$$\sigma_K^2 = \left[\frac{1-\nu}{1-(\alpha+\nu)}\right]^2 \text{Var}(\log(\text{MPK}_i))$$

Plug σ_K^2 into Equation (13) and TFP loss is given by

$$\log\left(\frac{\text{TFP}^{\text{FB}}}{\text{TFP}}\right) = \frac{1}{2}\alpha(1-\alpha)\left[\frac{1-\nu}{1-(\alpha+\nu)}\right]^2 \text{Var}(\log(\text{MPK}_i))$$

D Quantitative Robustness: Aggregate Implications with Alternative Parameter Values

Table 6 reports various parameter robustness checks. Starting at benchmark point estimates from Table 2 and externally calibrated parameter values from Table 1, I vary the magnitude of a single parameter up and down to alternative values used in literature and compare the implications for a range of macroeconomic aggregates, while keeping other parameters fixed. Each row corresponds to a different robustness check. In the last row, I modify multiple parameter values so that they are consistent with Catherine et al. (2021).

I also examine the the response of aggregate TFP to the change of borrowing constraints. Figure shows that overall the changes are qualitatively similar to the baseline results.

Table 5: Variable Definitions

Variable	Notation	Definition	Compustat data item
Capital	k	Total net value of property, plant, and equipment	`PPEGT`
Investment	i	Capital expenditures on property, plant, and equipment	`CAPX`
Debt	b	Net debt computed as gross debt minus cash	`DLC+DLTT-CHE`
Sale	y	Sales	`SALE`
EBITDA	π	Earnings before interest, taxes, depreciation and amortization	`EBITDA`
Net equity Issuance	e	Net equity issues computed as stock sales minus cash dividends and share buybacks	`SSTK - PRSTKC - DV`
Investment rate	i/k		`(DLC+DLTT-CHE)/L.PPEGT`
t-year sales growth rate	Δy_{-t}		$\log(\texttt{SALE}) - \log(\texttt{SALE}_{-t})$

Table 6: Robustness to Alternative Parameters

		Source	K	B	N	Y	TFP	U
Benchmark			0.775	0.094	0.309	0.571	1.231	−1.213
Low capital revenue elasticity	$\alpha = .21$	Ottonello and Winberry (2020)	0.602	0.071	0.308	0.528	1.189	−1.283
High capital revenue elasticity	$\alpha = .28$	Jo and Senga (2019)	0.952	0.113	0.317	0.633	1.279	−1.133
Low labor revenue elasticity	$\nu = .5$	Bloom et al. (2018)	0.754	0.105	0.258	0.541	1.143	−1.162
High labor revenue elasticity	$\nu = .64$	Ottonello and Winberry (2020)	0.814	0.081	0.337	0.617	1.304	−1.194
Low depreciation	$\delta = .06$	Midrigan and Xu (2014)	1.128	0.137	0.292	0.605	1.229	−1.058
High depreciation	$\delta = .15$	Hennessy and Whited (2007)	0.538	0.044	0.325	0.538	1.234	−1.349
Low initial capital	$\kappa_0 = .1$	Khan and Thomas (2013)	0.772	0.093	0.306	0.568	1.231	−1.200
High initial capital	$\kappa_0 = .23$		0.784	0.095	0.312	0.576	1.232	−1.215
Firm exit rate	$\pi_d = .08$		0.794	0.096	0.318	0.584	1.232	−1.231
Low risk-free interest rate	$r = .03$		0.853	0.104	0.313	0.589	1.232	−1.192
High risk-free interest rate	$r = .06$		0.657	0.076	0.310	0.550	1.233	−1.250
Low labor disutility	$\varphi = 1.28$	Katagiri (2014)	1.088	0.134	0.486	0.816	1.232	−0.860
High labor disutility	$\varphi = 2.48$	Jo and Senga (2019)	0.649	0.073	0.250	0.481	1.231	−1.387
Catherine et al. (2021)			1.108	0.203	0.301	0.638	1.279	−1.053

Note: The table reports aggregate capital, debt, employment, output, consumption, productivity, and utility. Different rows report these values for robustness checks varying the value of the indicated parameter, holding all other parameters fixed at their benchmark values from Table 1 and Table 2. The last row set parameter values consistent with Catherine et al. (2021) where $\rho = 0.851, \sigma = 0.131, \psi_0 = 0.004, \eta_1 = 0.091, s = 0.25, \delta = 0.06, r = 0.03, \pi_d = 0.08, \tau = 0.33$.