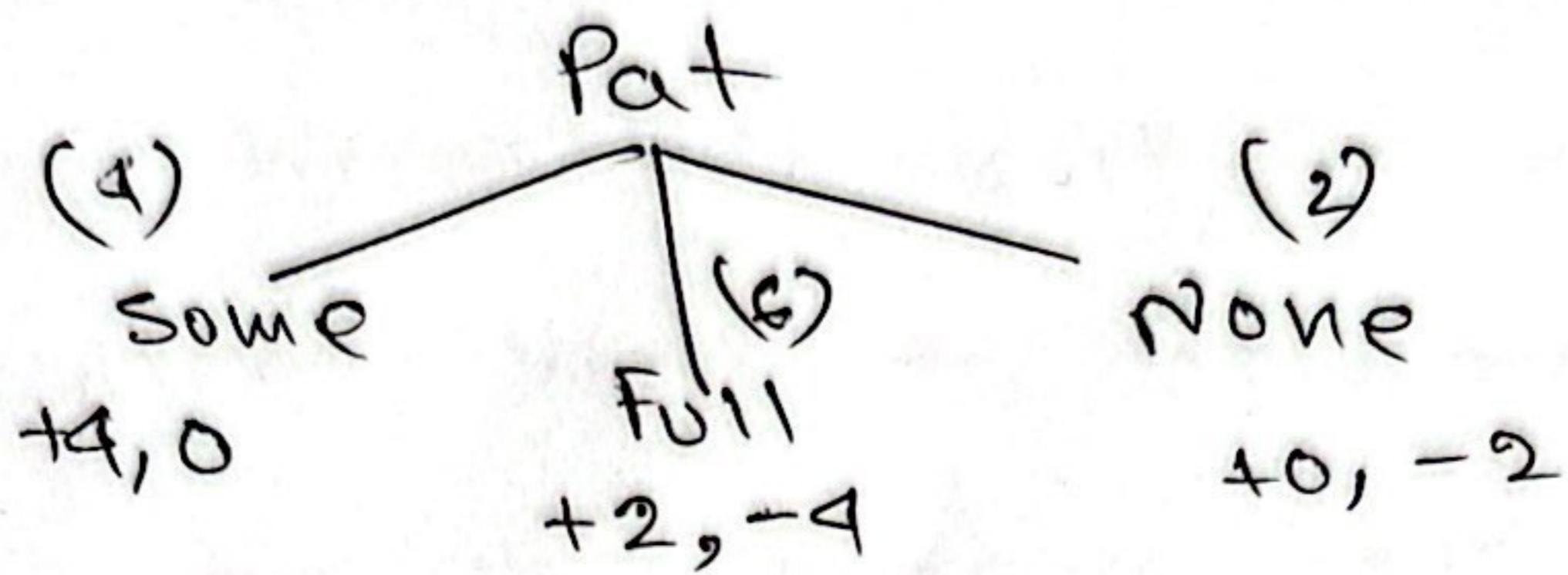


1) Information gain for Pat attribute



Before using Pat

$$+6, -6$$

$$I_1 = -\frac{6}{12} \log_2 \frac{6}{12} - \frac{6}{12} \log_2 \frac{6}{12}$$

$$= 1$$

After using Pat

$$\begin{aligned}
 I_2 &= \frac{4}{12} \times \left(-\cancel{6} - \frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} \right) \\
 &\quad + \frac{6}{12} \times \left(-\frac{2}{6} \log_2 \frac{2}{6} - \frac{4}{6} \log_2 \frac{4}{6} \right) \\
 &\quad + \frac{2}{12} \times \left(-\frac{0}{2} \log_2 \frac{0}{2} - \frac{2}{2} \log_2 \frac{2}{2} \right)
 \end{aligned}$$

$$= \frac{4}{12} \times 0 + \frac{6}{12} \times 0.92 + \frac{2}{12} \times 0$$

$$= 0.46$$

$$\therefore \text{Gain} = I_1 - I_2 = 1 - 0.46 = 0.54$$

2) Need of pruning a decision tree

- A tree might have complex and many branches but it may not generalize well to new or unseen data. So, to improve its generalization ability, we need to prune.
- To improve its model complexity we need to prune a decision tree
- Pruning can also help to reduce the computational resources
- After pruning, it may save our time to make a decision from the tree.

31 Given, $\sigma = 10$, $\mu = 40$, $x = 45$

$$\begin{aligned} P(x) &= \frac{1}{\sqrt{2\pi} \times \sigma} \times e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\ &= \frac{1}{\sqrt{2\pi} \times 10} \times e^{-\frac{(45-40)^2}{2 \times 10^2}} \\ &= 0.0352 \quad \underline{\text{Ans}} \end{aligned}$$

MPC

$$(1) \cdot P(c|x) = \frac{P(x|c) \times P(c)}{P(x)}$$

$s = \text{sunny}, t = \text{cool}, h = \text{high}, w = \text{weak}$

$$P(\text{Yes}|s, c, h, w) = \frac{P(s, c, h, w | \text{Yes})}{P(s, c, h, w)}$$

$$P(\text{No}|s, c, h, w) = \frac{P(s, c, h, w | \text{No})}{P(s, c, h, w)}$$

For $P(\text{Yes})$ $\text{No} = 5$ total = 14
 $\text{Yes} = 9$

For Yes class

$$P(s, c, h, w | \text{Yes}) = P(s | \text{Yes}) \times P(c | \text{Yes}) \times P(h | \text{Yes}) \times P(w | \text{Yes}) \times P(\text{Yes})$$

$$= \frac{2}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{5}{9} \times \frac{9}{14}$$

$$= 0.01058$$

$$= 0.00\overline{1058}$$

$$P(S, C, H, W | N_0) = P(S|N_0) \times P(C|N_0) \times P(H|N_0) \\ \times P(W|N_0) \times P(N_0)$$

$$= \frac{3}{5} \times \frac{1}{3} \times \frac{1}{5} \times \frac{1}{7} \times \frac{5}{19} \\ = 0.0049$$

wind (No)

$$(9) \text{ Strong} = 5 = \frac{5}{5} \\ (5) \text{ Weak} = 0 = \frac{0}{5}$$

↓

$$\text{Strong} = 5 + 1 = 6 = \frac{6}{7}$$

$$\text{Weak} = 0 + 1 = 1 = \frac{1}{7}$$

$$\text{AS } 0.007055 > 0.0049$$

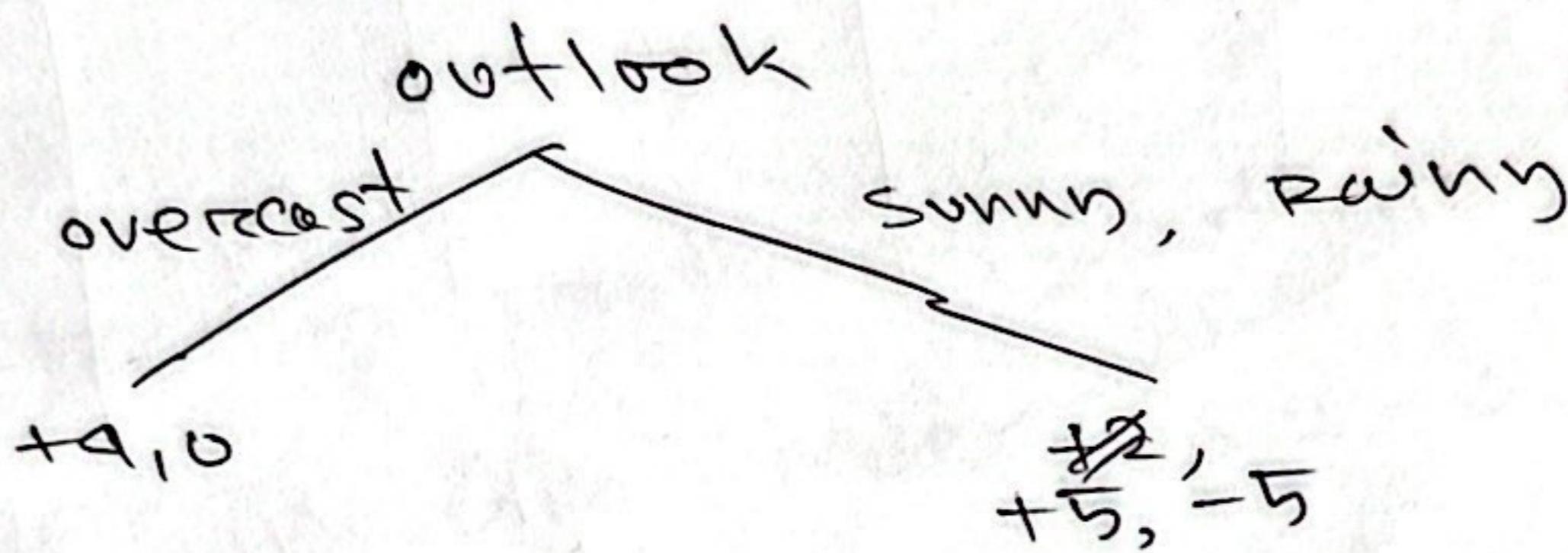
So, result is Yes.

2) Gini index of outlook

Before using outlook

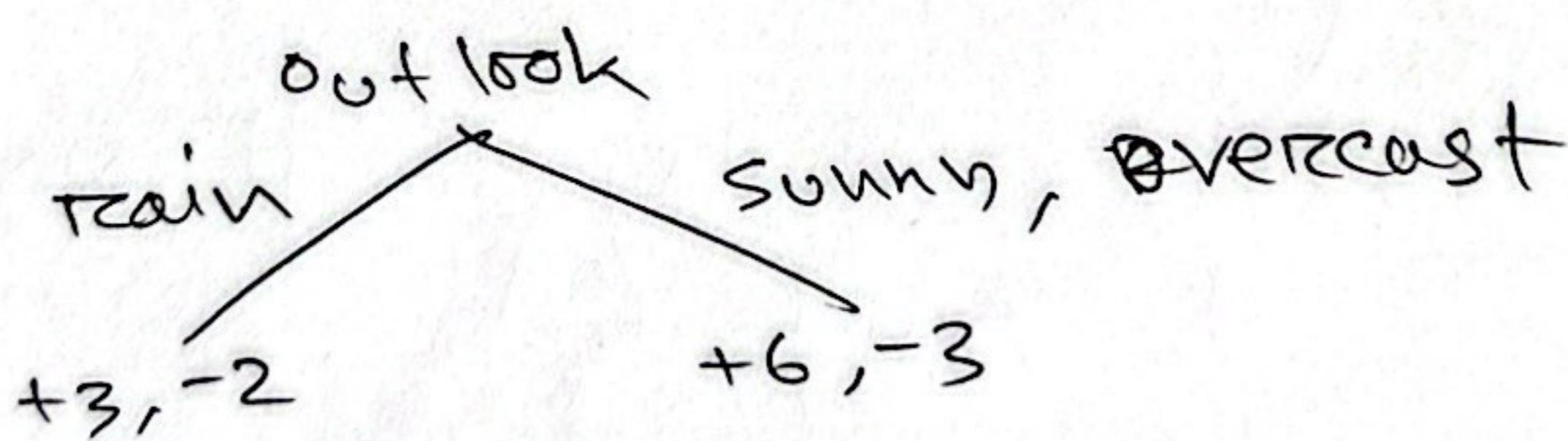
$$I_1 = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 \\ = 0.46$$

After using outlook



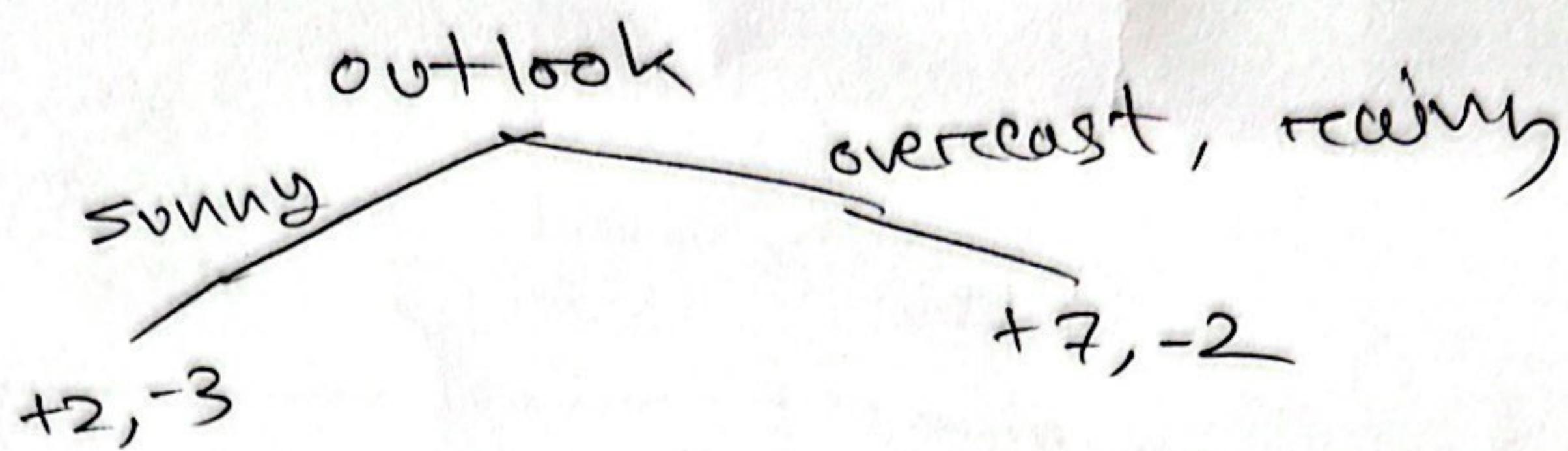
$$I_2 = \frac{9}{14} \left\{ 1 - \left(\frac{4}{9}\right)^2 - \left(\frac{5}{9}\right)^2 \right\} + \frac{10}{14} \left\{ 1 - \left(\frac{5}{10}\right)^2 - \left(\frac{5}{10}\right)^2 \right\} \\ = 0.36$$

$$\therefore \text{change in Gini index} = 0.46 - 0.36 \\ = 0.10$$



$$I_2 = \frac{9}{14} \left\{ 1 - \left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2 \right\} + \frac{9}{14} \left\{ 1 - \left(\frac{6}{9}\right)^2 - \left(\frac{3}{9}\right)^2 \right\} \\ = 0.46$$

$$\text{Change in Gini index} = 0.46 - 0.46 = 0.$$

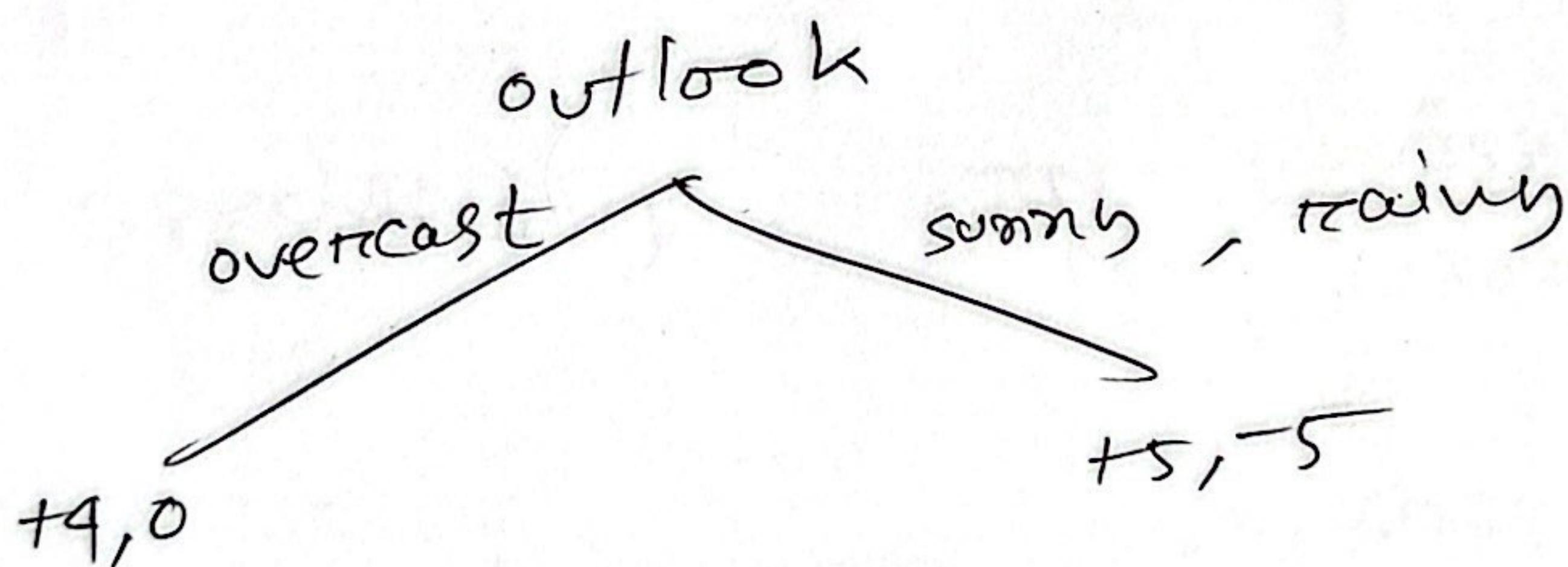


$$I_2 = \frac{5}{14} \left\{ 1 - \left(\frac{3}{5}\right)^2 - \left(\frac{2}{5}\right)^2 \right\} + \frac{9}{14} \left\{ 1 - \left(\frac{7}{9}\right)^2 - \left(\frac{2}{9}\right)^2 \right\}$$

$$= 0.39$$

$$\text{change in Gini Index} = 0.46 - 0.39 \\ = 0.07$$

so, for the root of outlook the
change in Gini index is = 0.36



		Define		
		Yes	No	
actual class	Yes	6959	96	7500
	No	412	2588	3000

FP FN

TN

$$1) \text{ accuracy} = \frac{6959 + 2588}{10000}$$

$$2) \text{ error} = 1 - \text{accuracy}$$

$$3) \text{ sensitivity} = \frac{\text{TP}}{\text{P}} = \frac{6959}{7500}$$

$$4) \text{ specificity} = \frac{\text{TN}}{\text{N}} = \frac{2588}{3000}$$

$$5) \text{ precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} = \frac{6959}{6959 + 412}$$

$$6) \text{ recall} = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{\text{TP}}{\text{P}} = \frac{6959}{7500}$$

~~$$7) F = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$~~

31 Rule

outlook = sunny

Class
NO

function

D₁⁺, D₂⁻, D₈
D₉⁺, D₁₁⁺

$$\text{accuracy} = \frac{3}{5} \times 100\% \\ = 60\%$$

NO = 3
YES = 2
Total = 5

$$\text{coverage} = \frac{5}{14} \times 100\% \\ = 35.71\%$$

[D₁⁻ D₂⁻ D₈⁻] TN
D₈⁺ D₁₁⁺] FN

$$\text{FP (False Positive)} = 0 \quad P_u \\ \text{FN (False Negative)} = 2 \quad F_u$$

Rule:

i) outlook = sunny

+2 →

Total

+ = 9
- = 5

after apply rules

(+2, -3) for Q1Q2 class = NO

so, +3, -2

$$\text{Gain} = 3 \left(\log_2 \frac{3}{3+2} - \log_2 \frac{5}{5+5} \right)$$

$$= 1.073$$

$$= 2.25$$

As negative
class
so, +5, -9

+9, -5 +5, +9

outlook = sunny \rightarrow humidity = High

class = NO

+3, -2

+3, 0

F.G₁ = -0.3

F.G₂ = ?

$$F.G_2 = 3 \left(\log_2 \frac{3}{3+0} - 10 \vartheta_2 \frac{3}{3+2} \right)$$

$$= \cancel{\sum}^{2.211} \underline{\text{Am}}$$

$$4) \overline{e\pi\pi}(M_1) = \frac{(30.5 + 32.2 + 20.7 + 20.6 + 31.0 + 41.0 + 27.7 \\ + 26.0 + 21.5 + 26.0)}{10}$$

$$= 27.72$$

$$\overline{e\pi\pi}(M_2) = \frac{(22.9 + 14.5 + 19.6 + 20.7 + 20.4 \\ + 22.2 + 19.9 + 16.2 + 35.0)}{10}$$

$$= \cancel{19.03} \quad 21.28$$

$$(\overline{e\pi\pi}(M_1) - \overline{e\pi\pi}(M_2)) = 27.72 - \cancel{19.03} \\ = \cancel{8.69} \\ = 6.44$$

$$e\pi\pi(M_1) - e\pi\pi(M_2) = 30.5 - 22.9 = 8.1$$

$$32.2 - 14.5 = 17.7$$

$$20.7 - 22.9 = -1.7$$

$$20.6 - 19.8 = 1$$

$$31.0 - 20.7 = 10.3$$

$$41.0 - 20.4 = 20.3$$

$$27.7 - 22.2 = 5.5$$

$$26.0 - 19.9 = 6.1$$

$$21.5 - 16.2 = 5.3$$

$$26.0 - 35.0 = -9$$

$$\text{var} \approx (M_1 - M_2)^2$$

$$= \frac{1}{K} \sum \left[(\bar{\text{err}}(M_1)_i - \bar{\text{err}}(M_2)_i) - (\bar{\bar{\text{err}}} M_1 - \bar{\bar{\text{err}}} M_2) \right]^2$$

$$= \frac{1}{K} \left[(8.1 - 6.44)^2 + (17.7 - 6.44)^2 + (-1.7 - 6.44)^2 \right. \\ \left. + (1 - 6.44)^2 + (10.3 - 6.44)^2 + (20.3 - 6.44)^2 \right. \\ \left. + (5.5 - 6.44)^2 + (6.6 - 6.44)^2 \right. \\ \left. + (5.3 - 6.44)^2 + (-9 - 6.44)^2 \right]$$

$$\approx \frac{1}{10} \times 673.31$$

$$= 67.331$$

$$t = \frac{\bar{\text{err}}(M_1) - \bar{\text{err}}(M_2)}{\sqrt{\frac{\text{var} \approx (M_1 - M_2)^2}{K}}}$$

$$\approx \frac{6.44}{\sqrt{\frac{67.331}{10}}}$$

$$\approx 2.482$$

for 9 degrees of freedom (10 round-1)
and 5% significance level,

the critical value, $z = 1.26$

$$\therefore t = 2.481 > z = 1.26$$

so, null hypothesis is rejected,
so, M_1 and M_2 has difference
and one is better.

$$\text{As } \overline{\text{err}}(M_1) > \overline{\text{err}}(M_2)$$

so M_2 is better

5)

Total example = 100

every example has equal weight =

$$\therefore w_i = \frac{1}{100} = 0.01$$

Given

accuracy = 90%.

~~∴~~ error = 10%.

\therefore error rate = 0.1

\therefore update weight for correctly

classified example is

$$\text{new } w_i = \frac{\text{error}(w_i)}{1 - \text{error}(w_i)} \times w_i$$

$$= \frac{0.1}{1 - 0.1} \times \frac{1}{100}$$

$$= 0.001$$

Normalization

(1) new weight for correctly classified

$$= 0.001 \times \frac{1}{0.001 \times 90 + 0.01 \times 10}$$

$$= 0.001 \times \frac{1}{0.19} = 0.0053$$

(2) new weight for misclassified,

$$= 0.01 \times \frac{1}{0.001 \times 90 + 0.01 \times 10}$$

$$= 0.01 \times \frac{1}{0.19} = 0.053$$

(3) weight of the classifier

$$= \log \left(\frac{1 - \text{error}(M_i)}{\text{error}(M_i)} \right)$$

$$= \log \left(\frac{1 - 0.1}{0.1} \right)$$

$$= 0.959$$

6) OR function

Linear equation: $0.5x_1 + 0.5x_2 - 1.25 = 0$

Learning rate, $\alpha = 0.01$

Hard threshold function

$$\text{new weight} = \text{old weight} + \alpha (\text{Target} - \frac{\text{prediction}}{h(n)} \times \text{input})$$

As hard threshold

$$\therefore 0.5x_1 + 0.5x_2 - 1.25 \geq 0 \text{ then output} = 1$$

$$\therefore 0.5x_1 + 0.5x_2 - 1.25 < 0 \text{ then output} = 0$$

OR

	x_1	x_2	y
1)	0	0	0
2)	0	1	1
3)	1	0	1
4)	1	1	1

For 1st input

$$\text{input} = 0, 0$$

$$\text{output} = 0 \text{ [target]}$$

$$\text{prediction} =$$

$$h(n) = 0$$

$$0.5 \times 0 + 0.5 \times 0 - 1.25$$

$$= -1.25 < 0$$

$$\text{so, } h(n) = 0$$

NO update

for 2nd input

$$\text{input} = 0, 1$$

$$\text{output} = 1 \text{ (target)}$$

$$6 \times 0.5 \times 0.5 - 0.25 = -0.75 < 0$$

$\therefore h(x) = 0$ [don't match with target]

so update needed

$$w_1 = 0.5 + 0.01(1-0) \times 0 = 0.5$$

$$w_2 = 0.5 + 0.01(1-0) \times 1 = 0.51$$

$$w_0 = -1.25 + 0.01(1-0) \times 1 = -1.24$$

equation: $0.5x_1 + 0.51x_2 - 1.24 = 0$

for 3rd input

$$\text{input} = 1, 0$$

$$\text{output} = 1 \text{ (target)}$$

$$0.5 \times 1 + 0.51 \times 0 - 1.24 = -0.74 < 0$$

$\therefore h(x) = 0$ [don't match with target]

update needed

$$w_1 = \cancel{0.5 \times 1 + 0.5} + 0.5 + 0.01(1-0) \times 1 = 0.51$$

$$w_2 = 0.51 + 0.01(1-0) \times 0 = 0.51$$

$$w_0 = -1.24 + 0.01(1-0) \times 1 = -1.23$$

so, equation: $0.51x_1 + 0.51x_2 - 1.23 = 0$

4+n input

input = 0, 1
output = 1 [target]

$$0.51x_1 + 0.51x_1 - 1.23 = -0.21 < 0$$

$u(n) = 0$ [don't match with target]

update needed

$$w_1 = 0.51 + 0.01(1-0)x_1 = 0.52$$

$$w_2 = 0.51 + 0.01(1-0)x_1 = 0.52$$

$$w_0 = -1.23 + 0.01(1-0)x_1 = -1.22$$

equation: $0.52x_1 + 0.52x_2 - 1.22 = 0$

31 derivation of sigmoid function

$$\Delta w_i \propto \frac{\delta \text{loss}}{\delta w_i}$$

$$\text{Loss} = (y - h\omega(x))^2$$

$$= \frac{\partial}{\partial w_i} (y - h\omega(x))^2$$

$$h\omega(x) = \frac{1}{1 + e^{-w \cdot x}}$$

$$= 2(y - h\omega(x)) \left[\frac{\partial}{\partial w_i} (y - h\omega(x)) \right]$$

$$= 2(y - h\omega(x)) \left[0 - \frac{\delta}{\delta w_i} h\omega(x) \right]$$

$$= -2(y - h\omega(x)) \times \frac{\delta}{\delta w_i} \left(\frac{1}{1 + e^{-w \cdot x}} \right)$$

$$= -2(y - h\omega(x)) \times \frac{1}{1 + e^{-w \cdot x}} \times \left(1 - \frac{1}{1 + e^{-w \cdot x}} \right) \times \frac{\delta w \cdot x}{\delta w_i}$$

$$= -2(y - h\omega(x)) \times h\omega(x) \times (1 - h\omega(x)) \times x_i$$

$$= \alpha (y - h\omega(x)) \times h\omega(x) \times (1 - h\omega(x)) \times x_i$$