

6) OR function

linear equation:  $0.5x_1 + 0.5x_2 - 1.25 = 0$

Learning rate,  $\alpha = 0.01$

hard threshold function

$$\text{new weight} = \text{old weight} + \alpha (\text{Target} - \underbrace{\text{prediction}}_{h(n)} \times \text{input})$$

As hard threshold

$$\therefore 0.5x_1 + 0.5x_2 - 1.25 \geq 0 \quad \text{then output} = 1$$

$$0.5x_1 + 0.5x_2 - 1.25 < 0 \quad \text{then output} = 0$$

OR

	$x_1$	$x_2$	$Y$
1)	0	0	0
2)	0	1	1
3)	1	0	1
4)	1	1	1

for 1st input

$$\text{input} = 0, 0$$

$$\text{output} = 0 \quad [\text{target}]$$

$$\text{prediction} = h(n) = 0$$

$$0.5 \times 0 + 0.5 \times 0 - 1.25$$

$$= -1.25 < 0$$

$$\text{So, } h(n) = 0$$

No update



for 2nd input

$$\text{input} = 0, 1$$

$$\text{output} = 1 \quad [\text{target}]$$

$$u(x) = 0$$

$$\left| \begin{array}{l} 0 \times 0.5 + 1 \times 0.5 - 1.25 \\ = -0.75 < 0 \end{array} \right.$$

$$u(x) = 0$$

$$\therefore \text{new weight}_1 = 0.5 + 0.01(1-0) \times 0 \\ = 0.5$$

$$\text{new weight}_2 = 0.5 + 0.01(1-0) \times 1 \\ = 0.51$$

$$\underline{\text{eq}}: 0.5x_1 + 0.51x_2 - 1.25 = 0$$

for third input

$$\text{input} = 1, 0$$

$$\text{output} = 1$$

$$u(x) = 0$$

$$\left| \begin{array}{l} 0.5 \times 1 + 0.51 \times 0 - 1.25 \\ = -0.75 < 0 \\ u(x) = 0 \end{array} \right.$$

$$\text{new weight}_1 = 0.5 + 0.01(1-0) \times 1 \\ = 0.51$$

$$\text{new weight}_2 = 0.51 + 0.01(1-0) \times 0 \\ = 0.51$$

$$\underline{\text{eq}}: 0.51x_1 + 0.51x_2 - 1.25 = 0$$



For 4th input

$$\text{input} = 1, 1$$

$$\text{output} = 1$$

$$h(n) = 0$$

$$\begin{array}{l} \cancel{0.51 \times 0.5} \\ 0.51 \times 1 + 0.51 \times 1 - 1.25 = \end{array}$$

$$-0.23 < 0$$

$$h(n) = 0$$

$$\text{new weight}_1 = 0.51 + 0.01(1-0) \times 1$$

$$= 0.52$$

$$\text{new weight}_2 = 0.51 + 0.01(1-0) \times 1$$

$$= 0.52$$

equation:  $\boxed{0.52x_1 + 0.52x_2 - 1.25 = 0}$



# Sigmoid Function

$$h_w(x) = \frac{1}{1 + e^{-w \cdot x}}$$

Let  $0.5 \geq \text{class} = 1$   
 $0.5 < \text{class} = 0$

new weight

$$w_i \leftarrow w_i + \alpha (y - h_w(x)) \times h_w(x) \times (1 - h_w(x)) \times \text{input}$$

$$0.2x_1 + 0.5x_2 - 0.6 = 0$$

target = 0  
input = 0, 0

$$w_i = 0.2 \times 0 + 0.5 \times 0 - 0.6 = -0.6$$

$$h_w(x) = \frac{1}{1 + e^{-0.6}} = 0.3 < 0.5$$

$$\therefore \text{class} = 0$$

$$\text{target} = \text{class}$$

no change



derivation of sigmoid function

$$\Delta w_i \propto \frac{\delta \text{Loss}}{\delta w_i} = \frac{\delta}{\delta w_i} \underbrace{\left( y - \sum w_i x_i \right)^2}_{\text{Loss}}$$

$$= \frac{\delta}{\delta w_i} \left( y - \sum w_i x_i \right)^2$$

$$= 2 \left( y - \sum w_i x_i \right) \cdot \frac{\delta}{\delta w_i} \left( y - \sum w_i x_i \right)$$

$$= 2 \left( y - \sum w_i x_i \right) \left[ \frac{\delta}{\delta w_i} (y) - \frac{\delta}{\delta w_i} \left( \sum w_i x_i \right) \right]$$

$$= 2 \left( y - \sum w_i x_i \right) \left[ 0 - \frac{\delta}{\delta w_i} \frac{1}{1 + e^{-w \cdot x}} \right]$$

$$= -2 \left( y - \sum w_i x_i \right) \times \frac{\delta}{\delta w_i} \left( \frac{1}{1 + e^{-w \cdot x}} \right)$$

$$= -2 \left( y - \sum w_i x_i \right) \times \frac{1}{1 + e^{-w \cdot x}} \times \left( 1 - \frac{1}{1 + e^{-w \cdot x}} \right) \times \frac{\delta}{\delta w_i} (w \cdot x)$$

$$= -2 \left( y - \sum w_i x_i \right) \times \frac{1}{1 + e^{-w \cdot x}} \times \left( 1 - \frac{1}{1 + e^{-w \cdot x}} \right) \times x_i$$



$$= 2 \left( y - h \omega(n) \right) \times \frac{1}{1 + e^x} \times \left( 1 - h \omega(n) \right) \times x_i$$

input