Computational Methods: Lab 1 - Wave equation

Irene Moulitsas, Cranfield University

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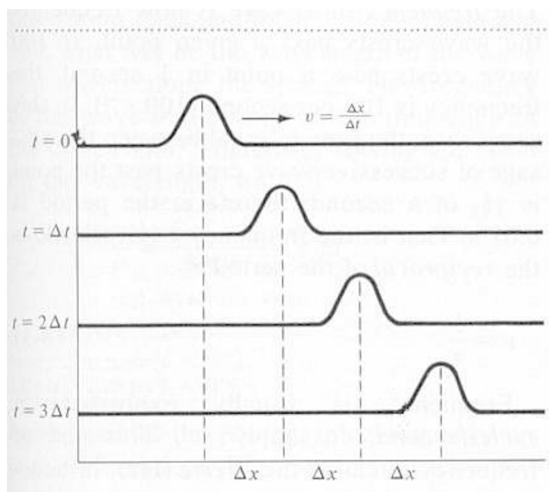

Introduction

In this lab we will examine the application of numerical schemes we have discussed in the lectures to a linear advection equation. In order to do this, we will consider the following problem:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = 0$$

$$x \in [-40, 40]$$

$$u = 1$$

With two initial/boundary condition sets:

$$f_0(x,0) = \frac{1}{2}(sign(x) + 1.)$$

$$f(-40,t) = 0$$

$$f(40,t) = 1$$

and

$$f_1(x,0) = \frac{1}{2}exp\left(-x^2\right)$$

$$f(-40,t) = 0$$

$$f(40,t) = 0$$

The analytical solutions for these initial conditions are given by:

$$f_0(x,t) = \frac{1}{2}(sign(x-t) + 1.)$$

and

$$f_1(x,t) = \frac{1}{2}exp\left(-(x-t)^2\right)$$

respectively.

Task

- Write a C/C++/FORTRAN program which solves the above problem with the initial conditions f_0 and f_1 on a uniform grid containing 100 points in x , using:
  - Upwind scheme
  - Central difference scheme
  - Lax scheme
  - Leapfrog scheme
- Solve the problem and output the numerical and exact solutions at time:
  - $t = 5$

- $t = 10$
- $t = 20$

- The solution should use the *the optimal* value of Courant number.
- Explain the results based on the truncation error of the methods used.
- Increase the grid size to 200, and then 400 points in x. How is the accuracy of your solution affected?