Provable and Efficient Nonconvex Procedures for Multi-Channel Sparse Blind Deconvolution

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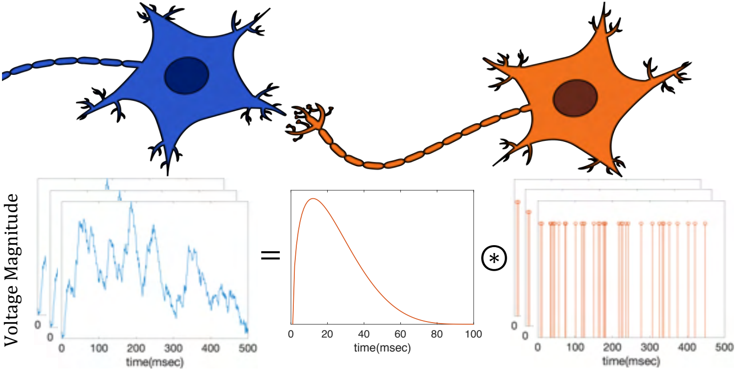
Motivation

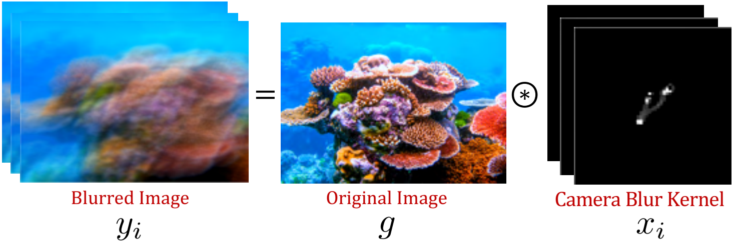
Understanding neural recordings



Neural Electric Signal y_i | Firing Pattern g | Temporal Sparse Train x_i

How to recover these temporal sparse/spike trains which indicate when the neuron is activated?

Image superresolution/deblurring



How to find the high-resolution original image and the blurring kernels simultaneously?

Formulation

Multi-channel sparse blind deconvolution (MSBD)

Problem Formulation: the i-th observed signal $\boldsymbol{y}_i \in \mathbb{R}^n$ can be expressed as:

$$\boldsymbol{y}_i = \boldsymbol{g} \circledast \boldsymbol{x}_i = \mathcal{C}(\boldsymbol{g})\boldsymbol{x}_i, \quad i = 1, \ldots, p,$$

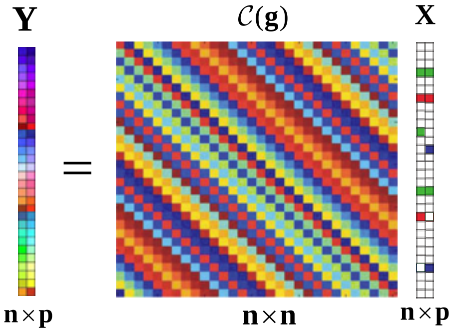
- $\boldsymbol{g}$ is a filter, and $\boldsymbol{x}_i \in \mathbb{R}^n$ is a sparse input signal.
- p is the total number of observations, and $\circledast$ denote the circulant convolution.
- $\boldsymbol{g} = [g_1, g_2, \cdots, g_n]^\top$ and circulant matrix $\mathcal{C}(\boldsymbol{g}) \in \mathbb{R}^{n \times n}$:

$$\mathcal{C}(\boldsymbol{g}) = \begin{bmatrix} g_1 & g_n & \cdots & g_2 \\ g_2 & g_1 & \cdots & g_3 \\ \vdots & \vdots & \ddots & \vdots \\ g_n & g_{n-1} & \cdots & g_1 \end{bmatrix}.$$

Multi-channel sparse blind deconvolution (MSBD)

- $\boldsymbol{Y} = [\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p] \in \mathbb{R}^{n \times p}$, $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_p] \in \mathbb{R}^{n \times p}$:

$$\boldsymbol{Y} = \mathcal{C}(\boldsymbol{g})\boldsymbol{X}.$$



- **Goal:** recover both the unknown signals $\{\boldsymbol{x}_i\}_{i=1}^p$ and the kernel $\boldsymbol{g}$ from multiple observations $\{\boldsymbol{y}_i\}_{i=1}^p$

Ambiguities

- The bilinear form of the observations:

$$\boldsymbol{y}_i = (\beta \cdot \mathcal{S}_j(\boldsymbol{g})) \circledast \frac{\mathcal{S}_{-j}(\boldsymbol{x}_i)}{\beta},$$

  where $\mathcal{S}_j(\boldsymbol{z})$ is the j-th circulant shift of the vector $\boldsymbol{z}$, $\beta \neq 0$ is an arbitrary scalar.
- **Challenge:** Scaling and shift ambiguities $\rightarrow$ $\boldsymbol{g}$ and $\{\boldsymbol{x}_i\}_{i=1}^p$ are not uniquely identifiable.
- **Goal:** recover filter $\boldsymbol{g}$ and sparse inputs $\{\boldsymbol{x}_i\}_{i=1}^p$, up to scaling and shift ambiguity.

Bilinear to linear

- $\mathcal{C}(\boldsymbol{g})$ is invertible $\rightarrow$ a unique inverse filter $\boldsymbol{g}_{\text{inv}}$:

$$\mathcal{C}(\boldsymbol{g}_{\text{inv}})\mathcal{C}(\boldsymbol{g}) = \mathcal{C}(\boldsymbol{g})\mathcal{C}(\boldsymbol{g}_{\text{inv}}) = \boldsymbol{I}.$$

- **Bilinear to linear**: multiply $\mathcal{C}(\boldsymbol{g}_{\text{inv}})$ on both side,

$$\begin{aligned} \boldsymbol{y}_i &= \mathcal{C}(\boldsymbol{g})\boldsymbol{x}_i \rightarrow \\ \mathcal{C}(\boldsymbol{g}_{\text{inv}})\boldsymbol{y}_i &= \mathcal{C}(\boldsymbol{g}_{\text{inv}})\mathcal{C}(\boldsymbol{g})\boldsymbol{x}_i = \underbrace{\boldsymbol{x}_i}_{\text{sparse}} \quad i = 1, \ldots, p. \end{aligned}$$

A natural formulation

- **Exploiting the sparsity of $\{x_i\}_{i=1}^p$**: seek $\boldsymbol{h}$ that minimize the cardinality of $\mathcal{C}(\boldsymbol{h})\boldsymbol{y}_i = \mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h}$:

$$\min_{\boldsymbol{h}\in\mathbb{R}^n} \ \frac{1}{p}\sum_{i=1}^{p} \|\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h}\|_0.$$

  - $\|\cdot\|_0$ is the pseudo-ℓ_0 norm: counts the cardinality of the nonzero entries of the input vector.

- Problematic for two reasons:
  1. has a trivial solution $\boldsymbol{h} = \mathbf{0}$.
  2. the cardinality minimization is computationally intractable.

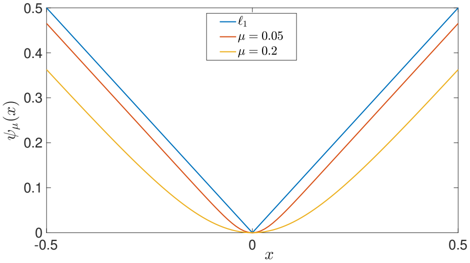
How to recover $\boldsymbol{g}_{\mathrm{inv}}$ provably and efficiently ?

Our nonconvex formulation

- We propose a nonconvex optimization formulation (following [Sun, et al, 2017][1] ,[Li and Bresler, 2019][2]) :

$$\min_{\boldsymbol{h}\in\mathbb{R}^n} \; f_o(\boldsymbol{h}) = \frac{1}{p}\sum_{i=1}^{p} \underbrace{\psi_\mu(\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h})}_{\text{convex surrogate}} \quad \text{s.t} \quad \underbrace{\|\boldsymbol{h}\|_2 = 1}_{\text{nonconvex}}$$

  - Add a spherical constraint.
  - Relax to a convex smooth surrogate: $\psi_\mu(z) = \mu \log\cosh(z/\mu)$, where μ controls the smoothness of the surrogate.

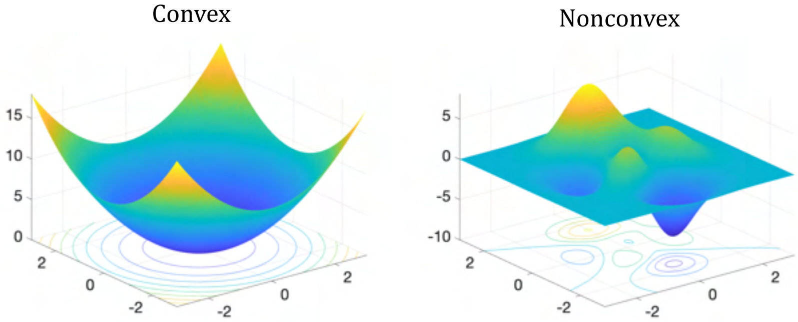


[1] Ju Sun, Qing Qu, and John Wright. "Complete Dictionary Recovery Over the Sphere I: Overview and the Geometric Picture". In: *IEEE Transactions on Information Theory* 63.2 (2017), pp. 853–884.

[2] Yanjun Li and Yoram Bresler. "Multichannel sparse blind deconvolution on the sphere". In: *IEEE Transactions on Information Theory* 65.11 (2019), pp. 7415–7436.

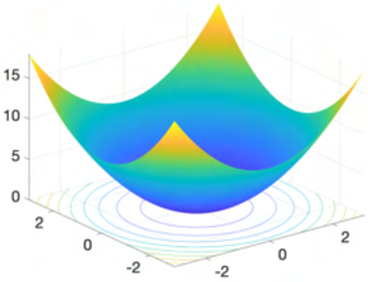
Optimization Geometry

Convex vs nonconvex: optimization geometry



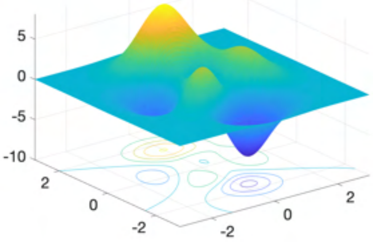
Convex vs nonconvex: optimization geometry

Convex



Unique global minimizer

Nonconvex



Convex vs nonconvex: optimization geometry

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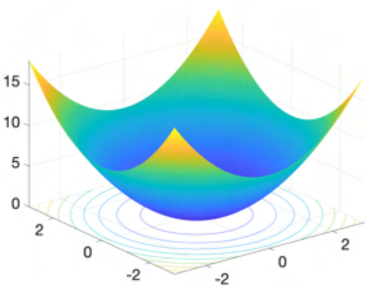

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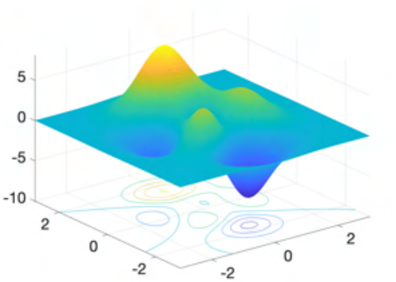


saddle points and spurious local minimizers

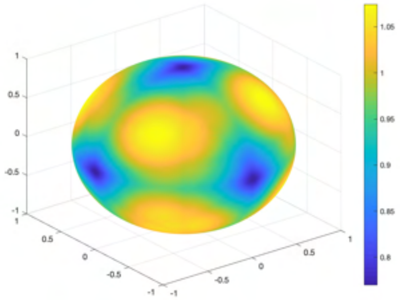
Is our objective landscape geometry of MSBD bad ?

Our Optimization Geometry

Benign geometry in the orthogonal case

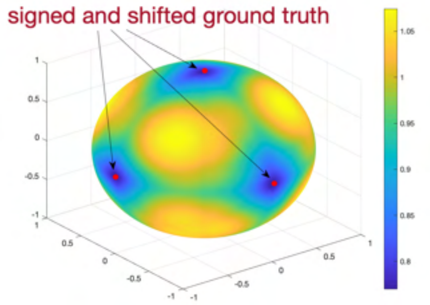
$$\min_{\boldsymbol{h}\in\mathbb{R}^n} \; f_o(\boldsymbol{h}) = \frac{1}{p}\sum_{i=1}^{p} \psi_\mu(\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h}) \quad \text{s.t} \quad \|\boldsymbol{h}\|_2 = 1$$

- The landscape of the loss value $f_o(\boldsymbol{h})$ with respect to $\boldsymbol{h}$:
  - $\mathcal{C}(\boldsymbol{g}) = \boldsymbol{I}$.
  - $n = 3, p = 30$.



Benign geometry in the orthogonal case

- The landscape of the loss value $f_o(\boldsymbol{h})$ with respect to $\boldsymbol{h}$:
  - $\mathcal{C}(\boldsymbol{g}) = \boldsymbol{I}$.
  - $2n = 6$ ground truth $\{\pm \boldsymbol{e}_i\}_{i=1}^{3}$



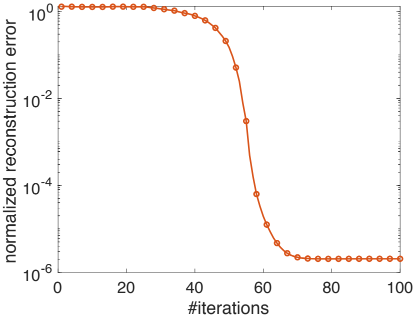
- **Benign geometry**: $2n$ local minimizers are approximately all shift and signed variants of the ground truth ($\{\pm \boldsymbol{e}_i\}_{i=1}^{3}$), and symmetrically distributed over the sphere.

Manifold gradient descent (MGD)

- Manifold gradient descent:

$$\boldsymbol{h}_{t+1} := \frac{\boldsymbol{h}_t - \eta_t \partial f_o(\boldsymbol{h}_t)}{\|\boldsymbol{h}_t - \eta_t \partial f_o(\boldsymbol{h}_t)\|_2},$$

  where η_t is the stepsize, $\partial f_o(\boldsymbol{h}) = (\boldsymbol{I} - \boldsymbol{h}\boldsymbol{h}^\top)\nabla f_o(\boldsymbol{h})$, and $\nabla f_o(\boldsymbol{h})$ is the Euclidean gradient of $f_o(\boldsymbol{h})$.
- With random initialization, $n = 128, p = 16$.



Surprising success of nonconvex optimization

Theoretical guarantee

Can we establish theoretical guarantee for the simple and efficient MGD based on nonconvex optimization formulation?

Yes. The statistical model will help !

Main Theoretical Results

Assumptions

- **Inputs are sparse**: the inputs $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_p]$ is under Bernoulli-Gaussian[3] model $\mathrm{BG}(\theta)$.
  - Each entry x in $\boldsymbol{X}$ is an i.i.d variable satisfing $x = \Omega \cdot z$, where Ω is a Bernoulli variable with parameter θ and $z \sim \mathcal{N}(0, 1)$.
- $\mathcal{C}(\boldsymbol{g})$ **is invertible**[4]: ensure the identifiability of the filter $\boldsymbol{g}$.
  - The condition number of $\mathcal{C}(\boldsymbol{g})$ is κ, i.e.

$$\kappa = \sigma_1(\mathcal{C}(\boldsymbol{g}))/\sigma_n(\mathcal{C}(\boldsymbol{g}))$$

.

[3] Qing Qu et al. "Analysis of the Optimization Landscapes for Overcomplete Representation Learning". In: *arXiv preprint arXiv:1912.02427* (2019).

[4] Yanjun Li, Kiryung Lee, and Yoram Bresler. "A unified framework for identifiability analysis in bilinear inverse problems with applications to subspace and sparsity models". In: *arXiv preprint arXiv:1501.06120* (2015).

Main results

- Distance metric to measure the success recovery:

$$\mathrm{dist}(\boldsymbol{h}, \boldsymbol{g}_{\mathrm{inv}}) = \min_{j \in [n]} \|\boldsymbol{g}_{\mathrm{inv}} \pm \mathcal{S}_j(\boldsymbol{h})\|_2.$$

Theorem (Shi and Chi, 2019)

Instate the assumptions above, for $\theta \in (0, \frac{1}{3})$*, when* μ *is small enough, with* $O(\log n)$ *random initializations, the output* $\hat{\boldsymbol{h}}$ *of MGD with a proper step size will satisfy:*

$$\mathrm{dist}(\hat{\boldsymbol{h}}, \boldsymbol{g}_{\mathrm{inv}}) \lesssim \frac{\kappa^4}{\theta^2}\sqrt{\frac{n}{p}}$$

in polynomial iterations, provided $p \gtrsim \frac{\kappa^8 n^{4.5} \log^4 p \log^2 n}{\theta^4}$

Prior work

Table: Comparison with existing methods for solving MSBD

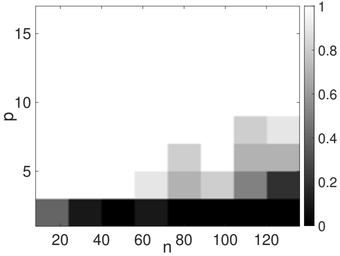
Methods	[Wang and Chi, 2016]	[Li and Bresler, 2019]	**Ours**
Assumptions	**filter $\boldsymbol{g}$ spiky** $\&$ **$\mathcal{C}(\boldsymbol{g})$ invertible**, $\boldsymbol{X} \sim \mathrm{BG}(\theta)$	$\mathcal{C}(\boldsymbol{g})$ invertible, $\boldsymbol{X} \sim \mathrm{BR}(\theta)$	**$\mathcal{C}(\boldsymbol{g})$ invertible**, $\boldsymbol{X} \sim \mathrm{BG}(\theta)$
Formulation	Convex $\min_{\boldsymbol{e}_1^\top \boldsymbol{h}=1} \|\mathcal{C}(\boldsymbol{h})\boldsymbol{Y}\|_1$	Nonconvex $\max_{\|\boldsymbol{h}\|_2=1} \|\mathcal{C}(\boldsymbol{h})\boldsymbol{R}\boldsymbol{Y}\|_4^4$	Nonconvex $\min_{\|\boldsymbol{h}\|_2=1} \psi_\mu(\mathcal{C}(\boldsymbol{h})\boldsymbol{R}\boldsymbol{Y})$
Algorithm	linear programming	*noisy* MGD	*vanilla* MGD
Recovery Condition	$\boldsymbol{\theta \in O(1/\sqrt{n})}$, $p \geq O(n)$	$\theta \in O(1)$, $\boldsymbol{p \geq O(n^9)}$	$\boldsymbol{\theta \in O(1)}$, $\boldsymbol{p \geq O(n^{4.5})}$

- For order of p, assuming θ, κ are constants, the order of sample complexity p is shown up to logarithmic factors.

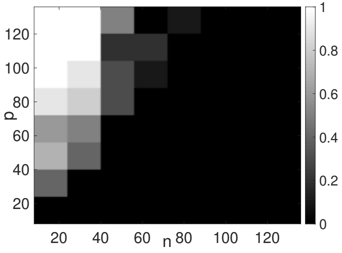
Practical Experiment Results

Numerical experiments: synthetic data

- Success rate of recovering the filter $\boldsymbol{g}$:
  - 10 Monte Carlo for success rate $\in [0, 1]$.
  - Fix sparsity $\theta = 0.3$.



(a) Ours

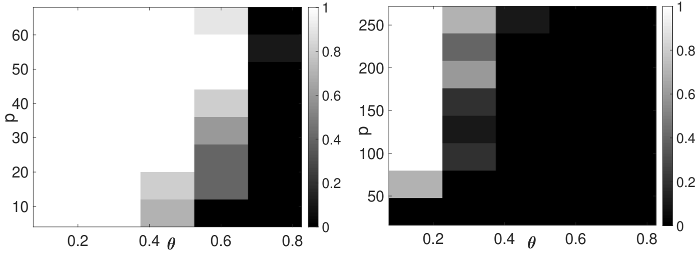


(b) [Li and Bresler, 2019]

Figure: Requirement of sample complexity p with respect to n.

Numerical experiments: synthetic data

- Success rate of recovering the filter $\boldsymbol{g}$:
  - 10 Monte Carlo for success rate $\in [0, 1]$.
  - Fix $n = 64$.

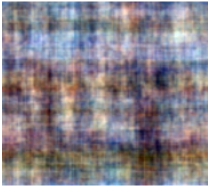


(a) Ours (b) [Li and Bresler, 2019]

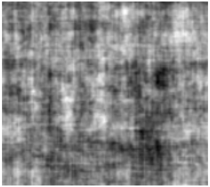
Figure: Requirement of sample complexity p with respect to θ.

Numerical experiments: blind image deconvolution

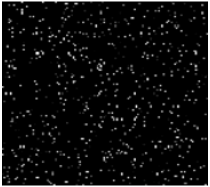
- Experimental setting:
  - The filter size is $n = 128 \times 128$.
  - The number of observations is $p = 1000$.
  - Sparsity level $\theta = 0.1$: $\boldsymbol{X} \in \mathrm{BG}(\theta)$



(a) Observation (RGB)



(b) Observation (R)



(c) Sparse input

Numerical experiments: blind image deconvolution

Comparisons of the recovered filter $\boldsymbol{g}$:

(d) True image

(e) Recovery via ours

(f) Recovery via [Li, et al., 2019]

Summary so far

- Introduction of our nonconvex approach for MSBD.
- Main results with comparisons to prior work.
  - Theoretical improvement on sample complexity p.
  - Practical much better performance.
- **Proof of our theoretical results.**

Proof Pipeline

Proof pipeline

- $\mathcal{C}(\boldsymbol{g})$ **is orthogonal**:

  1. one good subset of interest: benign geometry in the subset around one signed and shifted ground truth.
  2. $2n$ good subsets: Symmetry $\rightarrow$ benign geometry in $2n$ subsets of interest.
  3. Success recovery guarantee: convergence guarantee of MGD to the ground truth when initialized in these subsets.
  4. Random initialization: Subsets of interest are large enough.

- **Extend to** $\mathcal{C}(\boldsymbol{g})$ **is invertible**: by pre-conditioning $\boldsymbol{R}$.

Proof pipeline

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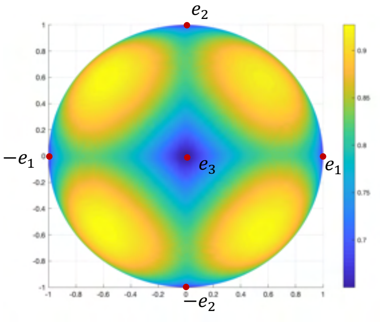
Subsets of Interest

Subsets of interest

$\mathcal{C}(\boldsymbol{g}) = \boldsymbol{I} \rightarrow$ shifted and sign-permuted copies of the ground truth $\{\pm \boldsymbol{e}_i\}_{i=1}^n$.

- $2n$ **subsets of interest**: around copies of the ground truth $\{\pm \boldsymbol{e}_i\}_{i=1}^n$:

$$\mathcal{S}_\xi^{(i\pm)} = \left\{ \boldsymbol{h} : h_i \gtrless 0, \frac{h_i^2}{\|\boldsymbol{h}_{\setminus\{i\}}\|_\infty^2} \geqslant 1 + \xi \right\}, \quad i \in [n], \xi > 0.$$



$n = 3$

Subsets of interest

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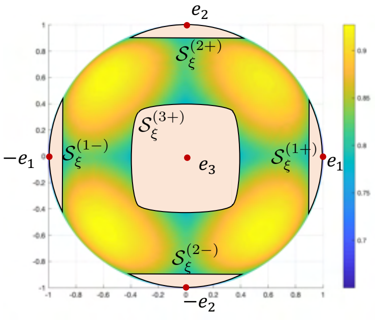

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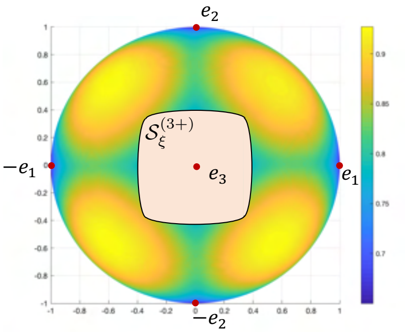
- **Focus on** $\mathcal{S}_\xi^{(n+)}$:



Geometry in $\mathcal{S}_{\xi}^{(n+)}$

Geometry of the population loss

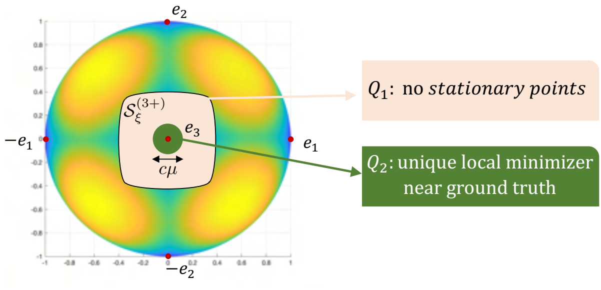
Population loss: $\mathbb{E}(f_o(\boldsymbol{h})) = \mathbb{E}\left[\frac{1}{p}\sum_{i=1}^{p}\psi_\mu(\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h})\right]$

Theorem (Shi and Chi, 2019)

WLOG, suppose $\mathcal{C}(\boldsymbol{g}) = \boldsymbol{I}$. *When* μ *is small enough, for* $\boldsymbol{h} \in \mathcal{S}_\xi^{(n+)}$, *the population loss satisfies:*

(large directional gradient) $\quad \boldsymbol{h} \in \mathcal{Q}_1$,

(strong convexity) $\quad \boldsymbol{h} \in \mathcal{Q}_2$.

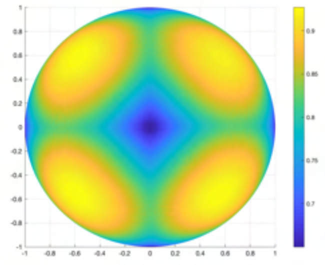


$n = 3$

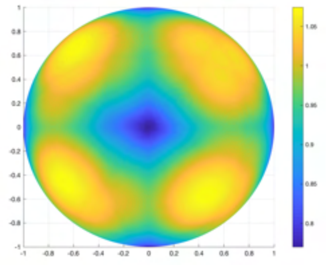
Statistical model helps: *population loss is smooth and good!*

Geometry: population loss to empirical loss

- Similar geometry of population and empirical loss:



(a) population $\mathbb{E}(f_o(\boldsymbol{h}))$
Good!



(b) empirical $f_o(\boldsymbol{h})$
?

How can we relate the properties of empirical loss to those of the population loss?

Uniform convergence of gradients and Hessians

- **Good geometry of empirical loss**:
  - Reparametrization: $\phi_o(\boldsymbol{w}) = f_o(\boldsymbol{h})$, where $\boldsymbol{w} = \boldsymbol{h}_{1:n-1}$.

Theorem (Shi and Chi, 2019)

Under the setting, for $\boldsymbol{h}(\boldsymbol{w}) \in \mathcal{S}_\xi^{(n+)}$ for some small $t_1, t_2 > 0$:

$$\mathbb{P}\left[\sup_{\boldsymbol{h}(\boldsymbol{w})\in\mathcal{Q}_1} \left| \underbrace{\frac{\boldsymbol{w}^\top \nabla\phi_o(\boldsymbol{w})}{\|\boldsymbol{w}\|_2}}_{\text{empirical}} - \underbrace{\frac{\boldsymbol{w}^\top \nabla\mathbb{E}\phi_o(\boldsymbol{w})}{\|\boldsymbol{w}\|_2}}_{\text{population}} \right| \geq t_1\right] \leq 2\exp(-Cn),$$

$$\mathbb{P}\left[\sup_{\boldsymbol{h}(\boldsymbol{w})\in\mathcal{Q}_2} \| \underbrace{\nabla^2\phi_o(\boldsymbol{w})}_{\text{empirical}} - \underbrace{\nabla^2\mathbb{E}\phi_o(\boldsymbol{w})}_{\text{population}} \| \geq t_2\right] \leq \exp(-Cn),$$

provided $p \gtrsim O(n^{4.5})$.

- Proof is based on concentration inequalities and covering numbers.

Orthogonal case to general case

- $\mathcal{C}(\boldsymbol{g})$ is orthogonal
- **Extend to $\mathcal{C}(\boldsymbol{g})$ that is invertible**: by pre-conditioning $\boldsymbol{R}$.

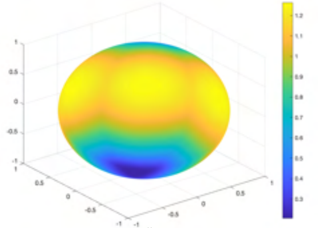
Benign geometry in general case

- The pre-conditioned problem:

$$\min_{\boldsymbol{h}\in\mathbb{R}^n} \; f(\boldsymbol{h}) = \frac{1}{p}\sum_{i=1}^{p} \psi_\mu(\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{R}\boldsymbol{h}) \quad \text{s.t} \quad \|\boldsymbol{h}\|_2 = 1$$

  - The pre-conditioning matrix is given as:

$$\boldsymbol{R} = \left[\frac{1}{\theta n p}\sum_{i=1}^{p} \mathcal{C}(\boldsymbol{y}_i)^\top \mathcal{C}(\boldsymbol{y}_i)\right]^{-1/2}.$$



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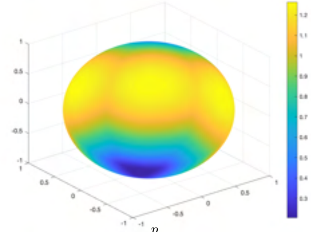
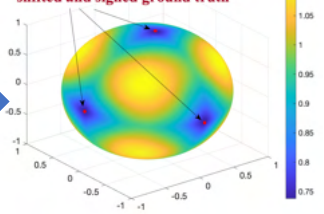


$f_o(\boldsymbol{h}) = \frac{1}{p}\sum_{i=1}^{p} \psi_\mu(\mathcal{C}(\boldsymbol{y}_i)\boldsymbol{h})$

Add $\boldsymbol{R}$



pre-conditioned $f(\boldsymbol{h})$

Conclusion

- We propose a novel nonconvex approach for MSBD problem based on MGD with random initializations.
- Under mild statistical model for sparse inputs, we provide theoretical characterizations for benign geometric landscape of the loss function $\rightarrow$ ensures the global convergence of MGD.
- Comparisons with prior work:
  1. significant improvement of sample complexity p: from $p \gtrsim O(n^9) \rightarrow p \gtrsim O(n^{4.5})$.
  2. better practical performance in a much larger range of the sparsity level.
- Future work: design a provable nonconvex procedure for self-calibrated compressive sensing.

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Thank you!

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