Probabilities

$0 \leq P(A) \leq 1$

$P(\emptyset) = 0$

$P(\Omega) = 1$

1) Complementary probability:

$P(\underline{A}) = 1 - P(A)$

2) Addition:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = P(A \cap B) + P(A \cap \underline{B})$

3) Conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

	B	$\underline{B}$	
A	$P(A \cap B) = P(A/B)*P(B) = P(B/A)*P(A)$	$P(A \cap \underline{B}) = P(A/\underline{B})*P(\underline{B}) = P(\underline{B}/A)*P(A)$	$P(A)$
$\underline{A}$	$P(\underline{A} \cap B) = P(\underline{A}/B)*P(B) = P(B/\underline{A})*P(\underline{A})$	$P(\underline{A} \cap \underline{B}) = P(\underline{A}/\underline{B})*P(\underline{B}) = P(\underline{B}/\underline{A})*P(\underline{A})$	$P(\underline{A})$
	$P(B)$	$P(\underline{B})$	$P(\Omega) = 1.0$

Bayes Theorem

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)}$$

Probabilities Exercises

Contingency Tables: Exercise

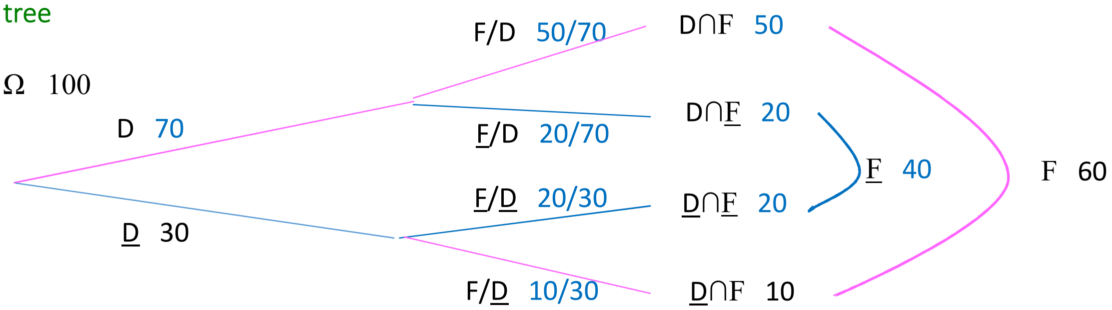
We use them when there is an idea of **simultaneity** in our data.

You are the owner of a start-up company, and you need to hire people. In order to do so, you posted an advertising on a social media and 100 candidates sent their CV: 30 of them do not have a third level degree, while the rest do, and 60 candidates speak a foreign language. Also, 10 of the candidates do not have a third level degree but they speak a foreign language. If you randomly choose a candidate:

table

	D (3rd *level)*	$\underline{D}$	
F *(foreign lang.)*	*50*	**10**	**60**
$\underline{F}$	*20*	*20*	*40*
	70	**30**	**100**

tree



$P(\underline{D}) = \frac{30}{100} = 0.3$

$P(F) = \frac{60}{100} = 0.6$

$P(\underline{D} \cap F) = \frac{10}{100} = 0.1$

a) What is the probability of choosing a person who has a third level degree?

$P(D) = 1 - P(\underline{D}) = 0.7$

b) What is the probability of choosing a person who does not speak a foreign language?

$P(\underline{F}) = 1 - P(F) = 0.4$

c) What is the probability of choosing a person who has a third level degree or speak a foreign language?

$$P(F) = P(D \cap F) + P(\underline{D} \cap F) \rightarrow P(D \cap F) = P(F) - P(\underline{D} \cap F)$$

$P(D \cup F) = P(D) + P(F) - P(D \cap F) = P(D) + P(F) - P(F) + P(\underline{D} \cap F) = P(D) + P(\underline{D} \cap F) = 0.7 + 0.1 = 0.8$

d) If we know that the person speaks a foreign language, what is the probability that also holds a third level degree?

$$P(D \cap F) = P(F) - P(\underline{D} \cap F) = 0.6 - 0.1 = 0.5$$

$$P(D/F) = \frac{P(D \cap F)}{P(F)} = \frac{0.5}{0.6} = 0.8333$$

Probabilities Exercises

Tree Diagrams: Exercise

We use them when there is an idea of **consecutiveness** in our data.

You find two purses. In one of them there are 5 silver coins and 3 bronze coins, and in the second one there are 4 silver coins and 6 bronze coins.

a) If you picked a purse and took a coin from this one, what is the probability of picking a silver coin?

O = This is the purse number one

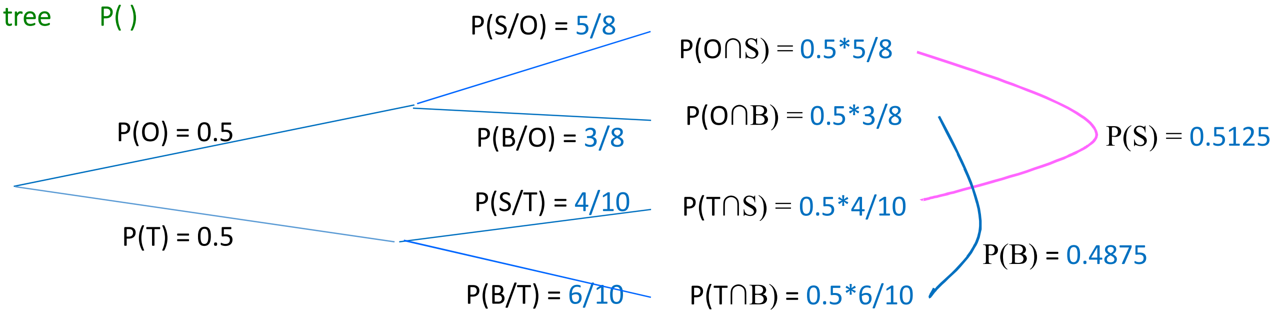
T = This is the purse number two

S = This is a silver coin

B = This is a bronze coin

Here we clearly see that there is an event that should happen before reaching to the question. First I have to pick a purse and after I have to pick the coin. This is the consecutiveness that we refer when using the trees.

tree P( )



P(S) = P(O∩S) + P(T∩S) = P(O)*P(S/O) + P(T)*P(S/T) = 0.5*5/8+0.5*4/10 = 0.5125

table

	S *(silver)*	C = $\underline{S}$ *(bronze)*	
O *(purse 1)*	5/8 **0.5= 0.3125*	3/8**0.5 = 0.1875*	**0.5**
T = $\underline{O}$ *(purse 2)*	4/10**0.5 = 0.2*	6/10**0.5 = 0.3*	**0.5**
	0.5125	*0.4875*	**1.0**

Probabilities Exercises

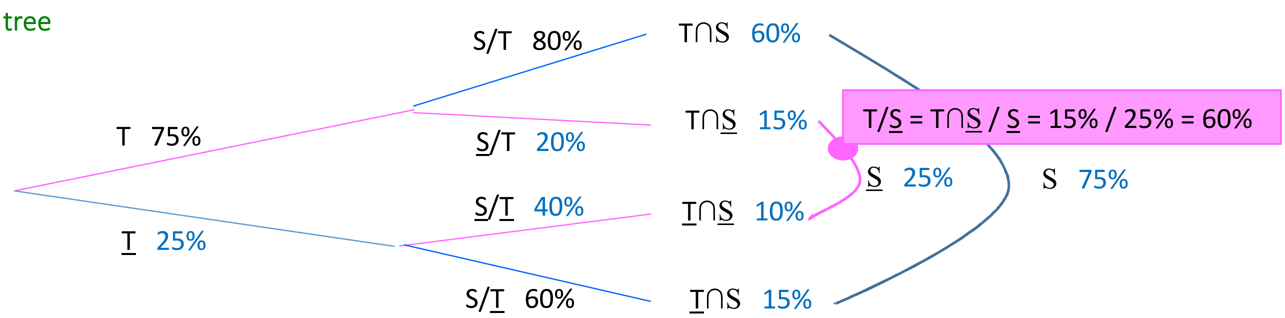
Exercise 1

A purchasing department finds that 75% of its special orders are received on time. Of those received on time, 80% fully meet the specifications. Of those who arrive late, 60% comply. A random package is chosen:

table

	S *(specification)*	$\underline{S}$ *(failed specification)*	
T *(on time)*	**0.8** x 0.75 = 0.60	*0.2 x 0.75 = 0.15*	**0.75**
$\underline{T}$ *(late)*	**0.6** x 0.25 = 0.15	*0.4 x 0.25 = 0.10*	*0.25*
	0.75	*0.25*	**1.00**

tree



$P(T) = 0.75 \qquad P(\underline{T}) = 1 - P(T) = 0.25$

$P(S/T) = 0.8 \qquad = P(S \cap T) / P(T) \qquad P(S \cap T) = P(S/T) \times P(T) = 0.8 \times 0.75 = 0.6$

$P(S/\underline{T}) = 0.6 \qquad = P(S \cap \underline{T}) / P(\underline{T}) \qquad P(S \cap \underline{T}) = P(S/\underline{T}) \times P(\underline{T}) = 0.6 \times 0.25 = 0.15$

a) What is the probability that it meets the specifications?

a) $P(S) = P(S \cap T) + P(S \cap \underline{T}) = 0.75 \times 0.8 + 0.25 \times 0.6 = 0.75$

b) If an order was chosen that does not meet specifications, what is the probability that it was received on time?

b) $P(T/\underline{S}) = P(T \cap \underline{S}) / P(\underline{S}) = P(T \cap \underline{S}) / (1 - P(\underline{S})) = \frac{0.75 \times 0.2}{0.25} = 0.6$

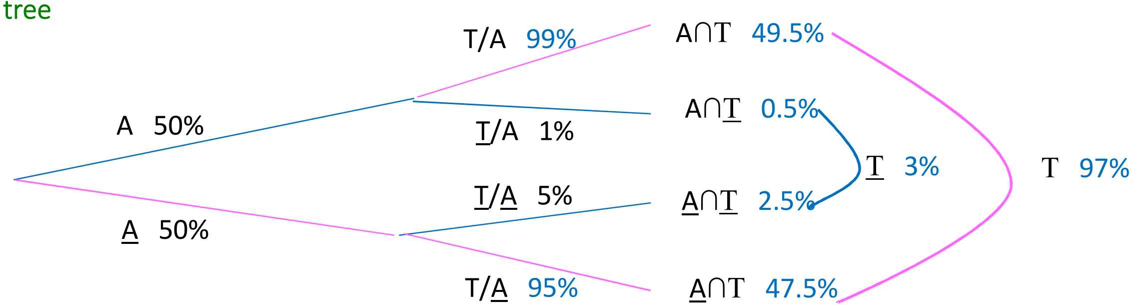
Exercise 2.

Every night James rolls a dice to decide how he is going to wake up the next day to go to college. If the number turns out to be even, he will use the alarm of his cell phone and, if not, he will use the alarm clock. 5% of the times that his cell phone alarm sounds, he falls asleep, while only 1% of the times that he uses the alarm clock, he falls asleep and then fails to get to school on time.

table

	A (alarm clock)	$\underline{A}$ (phone/even)	
T (on time)	0.5 x 0.99 = 0.495	0.5 x 0.95 = 0.475	0.97
$\underline{T}$ (falls asleep)	**0.5 x 0.01 = 0.005**	**0.5 x 0.05 = 0.025**	0.03
	0.5	**0.5**	**1.0**

tree



a) What is the probability that, on any given day, James falls asleep?

a) $P(\underline{T}) = P(\underline{T}\cap A) + P(\underline{T}\cap \underline{A}) = 0.5 \times 0.01 + 0.5 \times 0.05 = 0.03$

b) If it is known that on a certain day James attended college on time, what is the probability that he chose to wake up with the alarm on his cell phone?

b) $P(\underline{A}/T) = P(\underline{A}\cap T) / P(T) = P(\underline{A}\cap T) / (1 - P(\underline{T})) = \frac{0.5 \times 0.95}{0.97} = 0.4897$

+) If it is known that on a certain day James falls asleep, what is the probability that he chose to wake up with the alarm on his cell phone?

+) $P(\underline{A}/\underline{T}) = P(\underline{A}\cap\underline{T}) / P(\underline{T}) = \frac{0.5 \times 0.05}{0.03} = 0.8333$

Exercise 3.

If we throw 3 coins, find the probability of getting:

$P(A) = \frac{number\ of\ A\ events}{number\ of\ all\ events}$

a) Two heads.

$\boxed{1xHHH, 3xHHT, 3xHTT, 1xTTT \ \ \sum\ 8}$

a) $P(HHT) = \frac{3}{8} = 0.375$

b) Two tails.

b) $P(HTT) = P(HHT)$

Exercise 4.

We roll a dice, and we want to know the probability of:

$P(A) = \frac{number\ of\ A\ events}{number\ of\ all\ events}$

$\boxed{1,2,3,4,5,6 \ \sum 6}$

a) Getting an even number.

a) $P(2,4,6) = \frac{3}{6} = 0.5$

b) Getting a multiple of 6.

b) $P(6) = \frac{1}{6} = 0.1667$

c) A number greater than 4.

c) $P(56) = \frac{2}{6} = 0.3333$

Exercise 5.

An urn contains 8 red balls, 5 yellow balls and 7 green balls. If we randomly picked one, what is the probability of:

$\boxed{8 \text{ red, } 5 \text{ yellow, } 7 \text{ green} \ \sum 20}$

a) Getting a red ball?

a) $P(R) = \frac{8}{20} = 0.40$

b) Getting a green ball?

b) $P(G) = \frac{7}{20} = 0.35$

c) Getting a yellow ball?

c) $P(Y) = \frac{5}{20} = 0.25$

d) Not getting a red ball?

d) $P(\underline{R}) = 1 - P(R) = 0.60$

e) Not getting a yellow ball?

e) $P(\underline{Y}) = 1 - P(Y) = 0.75$

f) If we know that the ball is not red, what is the probability of getting a green one?

f) $P(G/\underline{R}) = P(G \cap \underline{R}) / P(\underline{R}) = P(G) / P(\underline{R}) = \frac{0.35}{0.6} = 0.5833$

Exercise 6.

Being $P(A) = 0.50$, $P(\underline{B}) = 0.30$ and $P(A \cup \underline{B}) = 0.70$, Calculate $P(A \cap B)$.

	B	$\underline{B}$	%
A	*$P(A \cap B) = 0.4$* 0.70	*0.1*	**0.50**
$\underline{A}$	*0.3*	*0.2*	*0.5*
	0.7	**0.30**	**1.00**

table

3 pink boxes = $P(A \cup \underline{B}) = P(A \cap B) + P(\underline{B}) = 0.70$

$P(A \cap B) = P(A \cup \underline{B}) - P(\underline{B})$ = {3 pink boxes} – {2 pink boxes} = $0.70 - 0.30 = 0.40$

You do not need to know $P(A)$ to solve the problem!!!

formulas

$P(A) = P(A \cap B) + P(A \cap \underline{B})$

$P(A \cup \underline{B}) = P(A) + P(\underline{B}) - P(A \cap \underline{B}) = P(A \cap B) + P(A \cap \underline{B}) + P(\underline{B}) - P(A \cap \underline{B})$

==

$P(A \cap B) = P(A \cup \underline{B}) - P(\underline{B}) = 0.70 - 0.30 = 0.40$

Exercise 7.

In a classroom of 40 students, 10 of them are boys. Between the boys, 3 can speak another language while 50% of the girls can speak a foreign language. If we randomly pick a student:

	B *(boy)*	$\underline{B}$ *(non boy, girl)*	
M *(multilingual)*	**3**	50% $\underline{B}$ = *15*	*18*
$\underline{M}$ *(non multilingual)*	*7*	50% $\underline{B}$ = *15*	*22*
	10	*30*	**40**

$\sum = 40$

$B = 10$ $\quad\quad$ $\underline{B} = \sum - B = 40 - 10 = 30$

$B \cap M = 3$

$M \cap \underline{B} = \underline{M} \cap \underline{B} = 50\% \times \underline{B} = 15$

a) What is the probability of this to be a girl?

$P(\underline{B}) = \frac{30}{40} = 0.75$

b) What is the probability of this to speak a foreign language?

$P(M) = \frac{18}{40} = 0.45$

c) If the student does not speak a foreign language, what is the probability of this person to be a boy?

$P(B/\underline{M}) = B \cap \underline{M} \,/\, \underline{M} = \frac{7}{22} = 0.3182$