第二讲 向量的乘法

内　积

1.内积的概念与性质

2.内积的坐标形式

外　积

1.外积的概念与性质

2.外积的坐标形式

混合积

1.混合积的概念与性质

2.混合积的几何意义

内容小结

复习:

1.混合积的概念

设 $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k},\quad \vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k},$

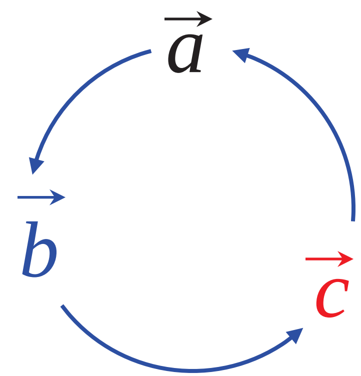
$\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k},$

$$[\vec{a}\,\vec{b}\,\vec{c}]=(\vec{a}\times\vec{b})\cdot\vec{c}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2. 混合积的性质

(1) 轮换对称性：

$$[\vec{a}\ \vec{b}\ \vec{c}]=[\vec{b}\ \vec{c}\ \vec{a}]=[\vec{c}\ \vec{a}\ \vec{b}]$$



即 $(\vec{a}\times\vec{b})\cdot\vec{c}=(\vec{b}\times\vec{c})\cdot\vec{a}=(\vec{c}\times\vec{a})\cdot\vec{b}.$

(2) 线性性 $(\vec{a}\times\vec{b})\cdot(\lambda\vec{c}+\mu\vec{d})=\lambda(\vec{a}\times\vec{b})\cdot\vec{c}+\mu(\vec{a}\times\vec{b})\cdot\vec{d}$

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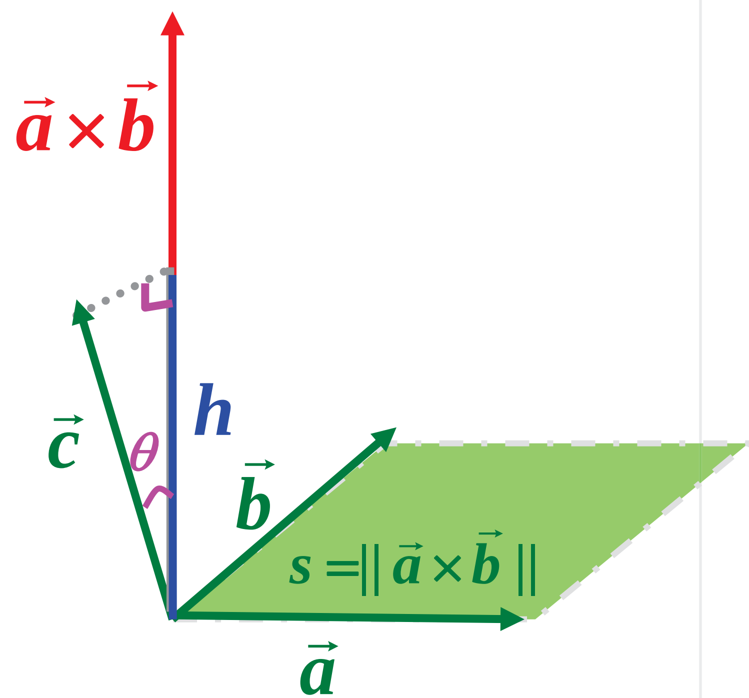
3. 混合积的几何意义

$$|[\vec{a}\ \vec{b}\ \vec{c}]| = |(\vec{a}\times\vec{b})\cdot\vec{c}|$$

$$= \|\vec{a}\times\vec{b}\| \cdot |\mathrm{Prj}_{\vec{a}\times\vec{b}}\,\vec{c}|$$

$$= S\,h = V$$



3. 混合积的几何意义

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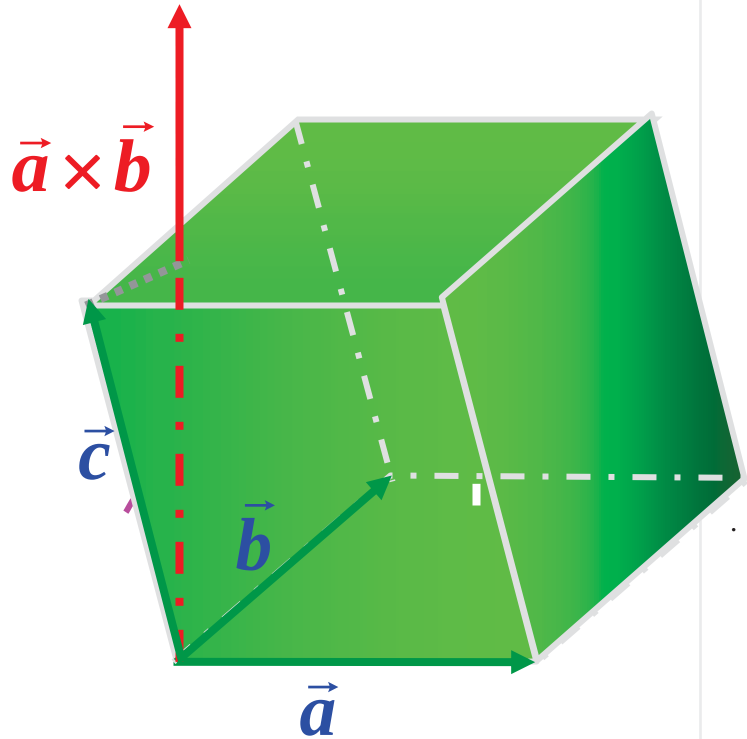
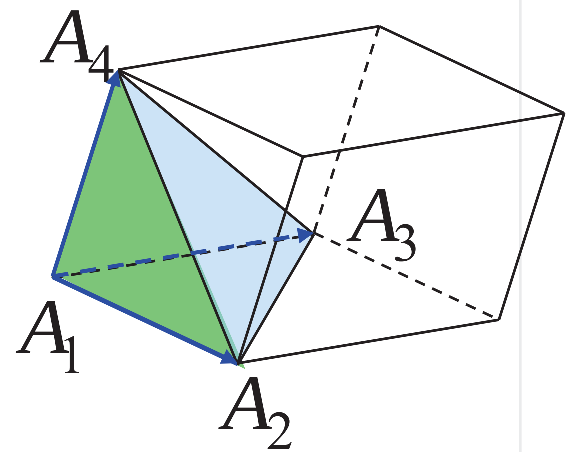


三向量 $\vec{a}$、$\vec{b}$、$\vec{c}$ 共面

$$\Leftrightarrow\ [\vec{a}\,\vec{b}\,\vec{c}] = 0.$$

例 已知一四面体的顶点 $A_k(x_k, y_k, z_k)$ $(k = 1,2,3,4)$ 求该四面体体积 .

解 已知四面体的体积等于以向量 $\overrightarrow{A_1A_2}, \overrightarrow{A_1A_3}, \overrightarrow{A_1A_4}$ 为棱的平行六面体体积的 $\frac{1}{6}$, 故



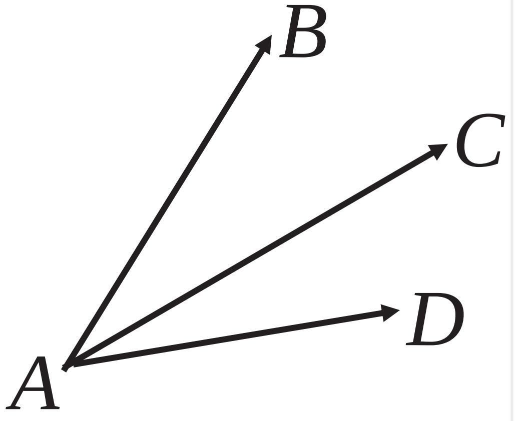
$$V = \frac{1}{6}\left| [\overrightarrow{A_1A_2}\ \overrightarrow{A_1A_3}\ \overrightarrow{A_1A_4}] \right|$$

$$= \frac{1}{6}\left| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} \right|$$

主要内容

混合积的几何意义

练习. 证明四点 $A(1,1,1)$, $B(4,5,6)$, $C(2,3,3)$, $D(10,15,17)$共面.



解 因

$$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 2 \\ 9 & 14 & 16 \end{vmatrix} = 0$$

故 $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ 四点共面.

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