向量组 $\alpha_1=(1,0,0,k_1)^T,\alpha_2=(1,2,0,k_2)^T,\alpha_3=(1,2,3,k_3)^T,\alpha_4=(1,1,1,k_4)^T$，其中 k_1,k_2,k_3,k_4 是任意实数，则　　（　）

(A) 向量组 $\alpha_1,\alpha_2,\alpha_3$ 线性相关；　　(B) 向量组 $\alpha_1,\alpha_2,\alpha_3$ 线性无关；

(C) 向量组 $\alpha_1,\alpha_2,\alpha_4$ 线性相关；　　(D) 向量组 $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ 线性无关.

[解析]

(A), (B)：令 $\beta_1=\begin{pmatrix}1\\0\\0\end{pmatrix},\beta_2=\begin{pmatrix}1\\2\\0\end{pmatrix},\beta_3=\begin{pmatrix}1\\2\\3\end{pmatrix}$，则 $|\beta_1,\beta_2,\beta_3|=\begin{vmatrix}1&1&1\\0&2&2\\0&0&3\end{vmatrix}=6\neq 0$

$\Rightarrow\beta_1,\beta_2,\beta_3$ 线性无关.

又 $\alpha_1=\begin{pmatrix}\beta_1\\k_1\end{pmatrix},\alpha_2=\begin{pmatrix}\beta_2\\k_2\end{pmatrix},\alpha_3=\begin{pmatrix}\beta_3\\k_3\end{pmatrix}$ $\Rightarrow$ $\alpha_1,\alpha_2,\alpha_3$ 线性无关.

故(A)错误，(B)正确.

(C)：　类似可得,向量组 $\alpha_1,\alpha_2,\alpha_4$ 线性无关.　故(C)错误.

(D)：　$\alpha_1,\alpha_2,\alpha_3,\alpha_4$ 线性无关 $\Leftrightarrow R(A)=4\,(A=(\alpha_1,\alpha_2,\alpha_3,\alpha_4))$

$$A=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)=\begin{pmatrix}1&1&1&1\\0&2&2&1\\0&0&3&1\\k_1&k_2&k_3&k_4\end{pmatrix}\to\begin{pmatrix}1&0&0&1/2\\0&1&0&1/6\\0&0&1&1/3\\0&0&0&l\end{pmatrix}$$

$(\text{其中}\, l=6k_4-3k_1-k_2-2k_3)$

$$\Rightarrow\begin{cases}(1)\ \ l=0\Leftrightarrow R(A)=3\Rightarrow\alpha_1,\alpha_2,\alpha_3,\alpha_4\ \text{线性相关.}\\(2)\ \ l\neq 0\Leftrightarrow R(A)=4\Rightarrow\alpha_1,\alpha_2,\alpha_3,\alpha_4\ \text{线性无关.}\end{cases}$$

故(D)错误.