四. 矩阵乘法的运算规律

- $(AB)C = A(BC)$
- $k(AB) = (kA)B = A(kB)$
- $A(B+C) = AB + AC$
- $(B+C)A = BA + CA$

证明: $(AB)C = A(BC)$

证: 设$A_{m\times n}, B_{n\times p}, C_{p\times s}$.

$$((AB)C)_{ij} = \sum_{k=1}^{p}\left(\sum_{l=1}^{n} a_{il}b_{lk}\right)c_{kj} = \sum_{k=1}^{p}\sum_{l=1}^{n} a_{il}b_{lk}c_{kj}$$

$$\left(A\left(BC\right)\right)_{ij} = \sum_{l=1}^{n} a_{il}\left(\sum_{k=1}^{p} b_{lk}c_{kj}\right)$$

$$= \sum_{l=1}^{n}\sum_{k=1}^{p} a_{il}b_{lk}c_{kj} = \sum_{k=1}^{p}\sum_{l=1}^{n} a_{il}b_{lk}c_{kj}$$

所以，$(AB)C = A(BC)$

[结束]