第二章 行列式

§2.1 n阶行列式的定义

一. 一阶、二阶和三阶行列式

二. n阶行列式的定义

三. 定义计算简单的行列式

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一. 一阶、二阶和三阶行列式

$$A_{n\times n}X = b$$

$n=1$ $\quad a_{11}x = b_1 (a_{11} \neq 0)$ $\qquad x = \dfrac{b_1}{a_{11}}$

$n=2$ $\quad \begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$ $\quad (a_{11}a_{22} - a_{12}a_{21} \neq 0)$ $\quad \begin{cases} x_1 = \dfrac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} \\ x_2 = \dfrac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \end{cases}$

$n=3$ $\quad \begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$

$$x_1 = \frac{b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} - b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3}{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

$$A_{n\times n}X = b$$

$n=1$ $\quad x = \dfrac{b_1}{a_{11}}$

一阶行列式 $\quad |A| = |a_{11}| = a_{11}$

如:行列式 $|-5| = -5$, $|3| = 3$

$n=2$ $\quad \begin{cases} x_1 = \dfrac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} \\ x_2 = \dfrac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \end{cases}$

二阶行列式 $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

如: $\begin{vmatrix} 2 & -1 \\ 3 & -3 \end{vmatrix} = -6 + 3 = -3$

$n=3$ $\quad x_1 = \dfrac{b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} - b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3}{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$

……

三阶行列式 $\quad |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$

三阶行列式的记忆法

(对角线法)

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

例1. 计算三阶行列式 $D = \begin{vmatrix} 1 & 2 & -4 \\ -2 & 2 & 1 \\ -3 & 4 & -2 \end{vmatrix}$

解 $D = 1\times 2\times(-2) \quad + 2\times 1\times(-3) \quad + (-4)\times(-2)\times 4$

$-(-4)\times 2\times(-3) \;\; -2\times(-2)\times(-2) \;\; -1\times 1\times 4 \; = -14$

例2. 求解方程 $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & x \\ 4 & 9 & x^2 \end{vmatrix} = 0.$

解 方程左端

$$D = 3x^2 + 4x + 18 - 9x - 2x^2 - 12 = x^2 - 5x + 6,$$

由 $x^2 - 5x + 6 = 0$ 解得 $x = 2$ 或 $x = 3.$

[结束]