二. n阶行列式的定义

1. 二、三阶行列式的规律观察

$$\begin{vmatrix} a_{11} & a_{12} \\ \boxed{a_{21}} & \boxed{a_{22}} \end{vmatrix} = a_{11}a_{22} + a_{12}(-1)a_{21} = a_{11}A_{11} + a_{12}A_{12}$$

$$A_{11} = (-1)^{1+1}|a_{22}|,\ A_{12} = (-1)^{1+2}|a_{21}|$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \underline{a_{11}a_{22}a_{33}} + \underline{a_{12}a_{23}a_{31}} + \underline{a_{13}a_{21}a_{32}}$$

$$-\underline{a_{11}a_{23}a_{32}} - \underline{a_{12}a_{21}a_{33}} - \underline{a_{13}a_{22}a_{31}}$$

$$= a_{11}(\underline{a_{22}a_{33} - a_{23}a_{32}}) - a_{12}(\underline{a_{21}a_{33} - a_{23}a_{31}}) + a_{13}(\underline{a_{21}a_{32} - a_{22}a_{31}})$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

A_{11}, A_{12}, A_{13} 分别称为 a_{11}, a_{12}, a_{13} 的代数余子式.

2. n阶行列式的定义

$$A=(a_{ij})_{n\times n} \longrightarrow \det A=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

(1) 当$n=1$时, $\det A=\det(a_{11})=a_{11}$;

(2) 当$n\geq 2$时,

$$\det A=a_{11}A_{11}+a_{12}A_{12}+\cdots+a_{1n}A_{1n},$$

其中 $A_{1j}=(-1)^{1+j}M_{1j}$, M_{1j}为划去A的第1行第j列后所得的$n-1$阶行列式,称为a_{1j}的余子式, A_{1j}称为a_{1j}的代数余子式. 记号 $\det A$, $|A|$

	行列式	矩阵
(1)	数	数表
(2)	D_n	$A_{m\times n}$
(3)	$\vert\ \ \vert$	$(\ \ ),[\ \ ]$
(4)	$\vert A\vert=\det A \leftarrow$ ------------	$A_{n\times n}$

[结束]