

设向量组 $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ 的秩为 3, 且满足 $\alpha_1 + 2\alpha_3 - 3\alpha_5 = 0, \alpha_2 = 2\alpha_4$
则该向量组的最大无关组是 ()

- (A) $\alpha_1, \alpha_3, \alpha_5$; (B) $\alpha_1, \alpha_2, \alpha_3$; (C) $\alpha_2, \alpha_4, \alpha_5$; (D) $\alpha_1, \alpha_2, \alpha_4$.

[解析]

$$R(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = 3$$

$\alpha_{i_1}, \alpha_{i_2}, \alpha_{i_3}$ 线性无关 $\Rightarrow \alpha_{i_1}, \alpha_{i_2}, \alpha_{i_3}$ 为最大无关组

(A): $\alpha_1 + 2\alpha_3 - 3\alpha_5 = 0 \Rightarrow \alpha_1, \alpha_3, \alpha_5$ 线性相关. 故(A)错误.

(C), (D): $\alpha_2 = 2\alpha_4 \Rightarrow \alpha_2, \alpha_4$ 线性相关 $\Rightarrow \begin{cases} \alpha_2, \alpha_4, \alpha_5 \text{ 线性相关} \\ \alpha_1, \alpha_2, \alpha_4 \text{ 线性相关} \end{cases}$. 故(C), (D)错误.

$$\begin{aligned} \text{(B): } \alpha_2 = 2\alpha_4 &\Rightarrow \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \text{ 与 } \alpha_1, \alpha_2, \alpha_3, \alpha_5 \text{ 等价} & R(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = 3 \\ \alpha_1 + 2\alpha_3 - 3\alpha_5 = 0 &\Rightarrow \alpha_1, \alpha_2, \alpha_3, \alpha_5 \text{ 与 } \alpha_1, \alpha_2, \alpha_3 \text{ 等价} & \Rightarrow R(\alpha_1, \alpha_2, \alpha_3) \end{aligned}$$

$\Rightarrow \alpha_1, \alpha_2, \alpha_3$ 线性无关. 故(B)正确.