二. 行列式性质4、性质5

性质4（行列式的初等变换）

(1) 将A的某一行乘以数k得到A_1，则

$$\det A_1 = k(\det A)$$

(2) 将A的某一行的$k(\neq 0)$倍加到另一行得到A_2,则

$$\det A_2 = \det A$$

(3) 交换A的两行得到A_3,则 $\det A_3 = -\det A$

证

(1) 将$\det A_1$, $\det A$分别按乘以数k的那一行展开之即得

(2)

$$\det A_2 = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{i1} & \cdots & a_{in} \\ \cdots & \cdots & \cdots \\ a_{j1}+ka_{i1} & \cdots & a_{jn}+ka_{in} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{i1} & \cdots & a_{in} \\ \cdots & \cdots & \cdots \\ a_{j1} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{i1} & \cdots & a_{in} \\ \cdots & \cdots & \cdots \\ ka_{i1} & \cdots & ka_{in} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \det A + k \cdot 0 = \det A$$

(3)

$$\det A_3=\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{j1} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots \\ a_{i1} & \cdots & a_{in} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}\begin{array}{c} \\ \\ i\text{行} \\ \\ j\text{行} \\ \\ \\ \end{array} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{j1} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots \\ a_{j1}+a_{i1} & \cdots & a_{jn}+a_{in} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

$$=\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ -a_{i1} & \cdots & -a_{in} \\ \cdots & \cdots & \cdots \\ a_{j1}+a_{i1} & \cdots & a_{jn}+a_{in} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ -a_{i1} & \cdots & -a_{in} \\ \cdots & \cdots & \cdots \\ a_{j1} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = -\det A$$

推论 $\det A$的某两行元素对应成比例 $\Rightarrow \det A = 0$

应用:

(1) A是n阶矩阵

$$\det(kA) = k^n(\det A)$$

(2) 初等矩阵的行列式:

$$\det(E_{ij}) = \det(E_{ij}I) = -\det I = -1$$

$$\det E_i(c) = c \neq 0;$$

$$\det E_{ij}(c) = 1.$$

(3) 初等矩阵与任一方阵A乘积的行列式:

$$\det(E_{ij}A) = -\det A = (\det E_{ij})(\det A),$$

$$\det(E_i(c)A) = c(\det A) = (\det E_i(c))(\det A),$$

$$\det(E_{ij}(c)A) = \det A = (\det E_{ij}(c))(\det A).$$

设E是初等矩阵,则:

$$\det(EA) = (\det E)(\det A)$$

设$E_1, E_2, \ldots, E_t$,是初等矩阵,则:

$$\det(E_1E_2\cdots E_tA) = (\det E_1)\cdots(\det E_t)(\det A)$$

例4

$$A=\begin{pmatrix}1&2&3\\2&2&3\\1&1&1\end{pmatrix}$$

$$|A|=\begin{vmatrix}1&2&3\\2&2&3\\1&1&1\end{vmatrix}=\begin{vmatrix}1&2&3\\0&-2&-3\\0&-1&-2\end{vmatrix}=\begin{vmatrix}1&2&3\\0&-2&-3\\0&0&-\frac{1}{2}\end{vmatrix}=1$$

$$|2A|=\begin{vmatrix}2&4&6\\4&4&6\\2&2&2\end{vmatrix}=2\cdot2\cdot2\begin{vmatrix}1&2&3\\2&2&3\\1&1&1\end{vmatrix}=8$$

$$2|A|\neq|2A|$$

一般,

$$|kA_{n\times n}|=k^n|A|\neq|kA|.$$