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Assignment-1

* To solve the Recurrence Relation using substitution method :-

$$T(n) = \begin{cases} 3T\left(\frac{n}{2}\right) + n & n>1 \\ 1 & n=1 \end{cases}$$

$$T(n) = 3T\left(\frac{n}{2}\right)+2 \quad \text{--(i)}$$

Substituting value of $T\left(\frac{n}{2}\right)$ in Eq^n (i) we get

$$T(n) = 3\left[3T\left(\frac{n}{2^2}\right)+\left(\frac{n}{2}\right)\right]+n$$

$$= 9\left[T\left(\frac{n}{4}\right)\right] + 3\left(\frac{n}{2}\right)+n$$

$$\vdots$$

Similarly doing substitution till we get $T(1)=1$

also $T(n) = 3^k\, T\left(\frac{n}{2^k}\right) + 3^{k-1}\left(\frac{n}{2^{k-1}}\right) + 3^{k-2}\left(\frac{n}{2^{k-2}}\right) + \cdots\cdots + \frac{3n}{2} + n$

Here $\frac{n}{2^k} = 1$

$[\log n = k]$

So, Substituting

$$T(n) = 3^{\log n}\, T(1) + 3^{\log n-1}\left(\frac{n}{2^{\log n-1}}\right) + \cdots\cdots + \frac{3n}{2}+n$$

$$= 3^{\log n} + n\left[\left(\frac{3}{2}\right)^{\log n-1} + \left(\frac{3}{2}\right)^{\log n-2} + \cdots\cdots + \left(\frac{3}{2}\right)^{0}\right]$$

$$= \left[\frac{\left(\frac{3}{2}\right)^{\log n} - 1}{\frac{3}{2}-1}\right] \times n + 3^{\log_2 n}$$

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$$= 2n.\left(\left(\frac{3}{2}\right)^{\log_2 n} - 1\right) + n^{\log_2 3}$$

$$T(n) = 2n\cdot n^{\log_2 3/2} - 2n + n^{\log_2 3}$$

$$= 2n\left(n^{\log 3} - \log_2 2\right) - 2n + n^{\log_2 3}$$

$$= 2n.n^{\log 3 - 1} - 2n + n^{\log_2 3}$$

$$= 3n^{\log_2 3} - 2n$$

[so for big-oh value $T(n) = O(n^{\log_2 3})$]

Using Master Theorem:

$T(n) = 3T\left(\frac{n}{2}\right) + n$

Given $a=3, b=2, K=1, p=0$

Hence checking $\log_b a = \log_2 3 > 1$

Hence Case-1 Can be applied i.e. $\log_b a+\epsilon$

$\therefore T(n) = O(n^{\log_b a})$

$= O(n^{\log_2 3})$