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## Assignment - 2

Q. 1. Define the post correspondence problem? Mention the difference between post correspondence of modified post correspondence problem? obtain the solution for following PCP?

**Soln:** **Post Correspondence problem:-** It is a problem of deciding whether a set of domains has a match or not. The modified post correspondence (MPCP) is just like PCP that specify both the set of tiles & also a special tile.

**Difference:** The difference between PCP & MPCP is that in MPCP, a sol^n is required to start with the first string on each list.

Let's take two lists A & B of R strings, say

$A = w_1, w_2, w_3, \ldots, w_R$ & $B = x_1, x_2, x_3, \ldots x_R$

| MPCP solution | PCP solution |
|---|---|
| $w_1 w_i w_{i+1} \ldots w_j = x_1 x_i x_{i+1} \ldots x_j$ | $w_i w_{i+1} \ldots w_j = x_i x_{i+1} \ldots x_j$ |
| $\lvert w_1 w_2 \ldots w_R \rvert = \lvert x_1 x_2 x_3 \ldots x_R \rvert$ | $\lvert w_1 w_2 \ldots w_R \rvert = \lvert x_1 x_2 \ldots x_R \rvert$ |

**PCP Sol^n:** $A = \{b, babbb, ba\}$

$B = \{bbb, ba, a\}$

Assume $x_1 = b$, $x_2 = babbb$, $x_3 = ba$

$w_1 = bbb$, $w_2 = ba$, $w_3 = a$

$$|x_1 x_2 x_3| = 8$$
$$|w_1 w_2 w_3| = 6$$



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$$\therefore |x_1 x_2 x_3| \neq |w_1 w_2 w_3|$$

Solution for PCP : We see that (2,1,1,3) is a sequence of integers that solve this PCP instance, since the concatenation of babbb, b, b & ba is equal to concatenation of ba, bbb, bbb & a [i.e. $w_2 w_1 w_1 w_3$ = $x_2 x_1 x_1 x_3$ = babbb bbb ba ]

Lengths are not same

Hence it can be said that this post correspondence problem is undecidable.

Q.2. prove that the MPC problem is undecidable. with an example recursive En language grammar show how to construct set A & set B.

Solution: Example of Recursive Enumerable Language Grammar to construct set A & set B

Let $G = (\{A, B, C\}, \{a, b, c\}, S, P\}$ with productions

$S \rightarrow aABb \mid Bb \mid b.$

$Bb \rightarrow C$

$AC \rightarrow aaC$

& let $w = aaaC$

The string $w = aaaC$ is in $L(G)$ and has a derivation

$S \Rightarrow aABb \Rightarrow aAC \Rightarrow aaaC$

The sequences A & B obtained from the suggested constructions are given in fig.1 given below :-

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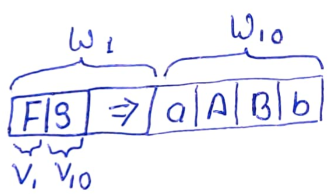
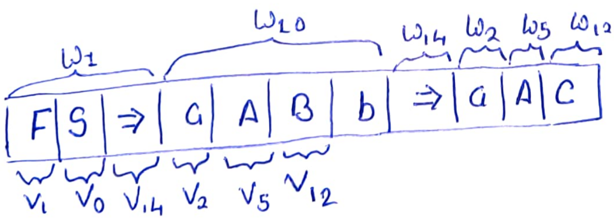
| $i$ | $w_i$ | $v_i$ |
|---|---|---|
| 1 | FS⇒ | F |
| 2 | a | a |
| 3 | b | b |
| 4 | c | c |
| 5 | A | A |
| 6 | B | B |
| 7 | C | C |
| 8 | S | S |
| 9 | E | ⇒aaaCE |
| 10 | aABb | S |
| 11 | Bbb | S |
| 12 | C | Bb |
| 13 | aaC | AC |
| 14 | ⇒ | ⇒ |

This derivation is paralled by an MPC Soln with the constructed sets can be seen in figure given below.




We want to construct an MPC Soln. So we must start with $w_1$ i.e. FS⇒. This string contains S, as to match it we have to use $v_{10}$ or $v_1$. In this instance we used $v_{10}$, this brings in $w_{10}$, leading us to second strings in the partial derivation, looking at several more steps, we see that the string $w_1 w_i w_{i+1} \dots w_j$ is always larger than the corresponding string $v_i v_i v_{i+1} \dots v_j$.

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The first is exactly one step ahead in the derivation. the Complete MPC Soln is shown in figure below:

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | S | ⇒ | a | A | B | b | ⇒ | a | A | C | ⇒ | a | a | a | C | E | |

Then the pair (A,B) has an MPC Soln if & only if $w \in L(G)$.

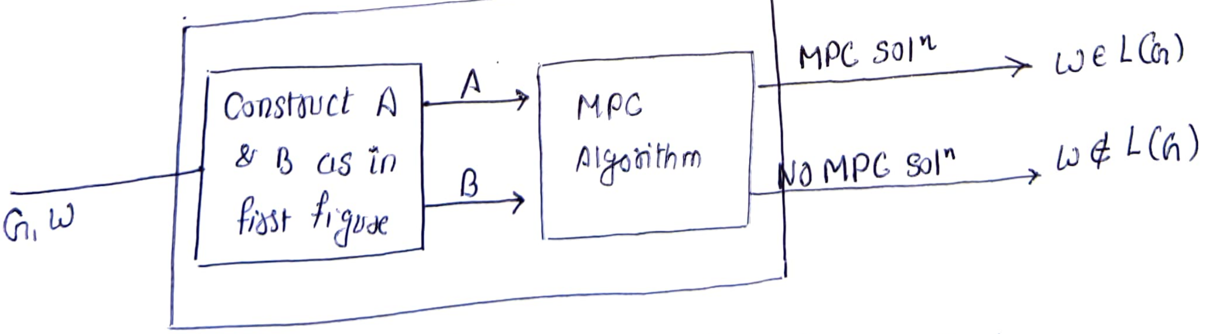
→ The MPC problem is undecidable.

proof: Given any unrestricted grammar $G = (V, T, S, P)$ & $w \in T^+$ we Construct the set A & set B as suggested above. The pair (A,B) has an MPC Soln if and only if $w \in L(G)$

Suppose now we assume that the modified post Correspond ence problem is decidable. We Can construct then an algorithm for the membership problem of G as shown in fig. 4. An algorithm for constructing A from B from G & w.

Clearly exists, but a membership algorithm for in & w does not exist

Membership Algorithm




we must therefore conclude that there can't be any algorithm for deciding the modified post Correspond ence problem. Hence, MPC problem is Undecidable.

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Q.3> Consider the following grammar for a context free language L:

$G = (\{S, A, B\}, \{0,1\}, \{S \to A1B, A \to 0A, A \to \epsilon, B \to 1B, B \to 0B, B \to \epsilon\}, S)$

Check the string w = 011 0110 is a member of the L(G) or not using the CYK algorithm.

Soln:

P: $S \to A1B$
$A \to 0A / \epsilon$
$B \to 1B / 0B / \epsilon$

CYK algorithm works on CNF and the given grammar is not in CNF, so conversion required.

Elimination of $\epsilon$-production, then unit-production the useless production.

$S \to A1B / 1B / A1 / 1$
$A \to 0A / 0$
$B \to 1B / 0B / 0 / 1$

Create a new production as

$X_1 \to 1,\ X_2 \to 0$

Now, $S \to AX_1B / X_1B / AX_1 / 1$

$A \to X_2A / 0$

$B \to X_1B / X_2B / 0 / 1$

for $S \to AX_1B \Rightarrow S \to X_3B$
$X_3 \to AX_1$

Now, given grammar is in Chomsky normal Form.

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$S \to X_3B \mid X_1B \mid AX_1 \mid 1$

$X_3 \to AX_1$

$A \to X_2A \mid 0$

$B \to X_1B \mid X_2B \mid 0 \mid 1$

Applying CYK Algorithm,

| | | | | | | |
|---|---|---|---|---|---|---|
| $S, B$ | | | | | | |
| $S, B$ | $S, \cancel{X_3} B$ | | | | | |
| $S, B$ | $S, \cancel{X_3}, B$ | $S, \cancel{X_3}, B$ | | | | |
| $S, B$ | $S, \cancel{X_3}, B$ | $S, \cancel{X_3} B$ | $S, B$ | | | |
| $S, B$ | $S, \cancel{X_3}, B$ | $S, \cancel{X_3}, B$ | $S, B$ | $S, \cancel{X_3}, B$ | | |
| $S, B$ | $S, \cancel{X_3}, B$ | $S, \cancel{X_3} B$ | $S, B$ | $S, \cancel{X_3}, B$ | $S, \cancel{X_3}, B$ | |
| $A, B, X_2$ | $S, B, X_1$ | $S, B, X_1$ | $A, B, X_2$ | $S, B, X_1$ | $S, B, X_1$ | $A, B, X_2$ |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |

Step: 1

| | |
|---|---|
| $A \to 0$ | $S \to 1$ |
| $B \to 0$ | $B \to 1$ |
| $X_2 \to 0$ | $X_1 \to 1$ |

Step: 2 String of length: 2

0 1

$(A, B, X_2)\ (S, B, X_1)$

$\underbrace{AX_1}_{S},\ \underbrace{X_2B}_{B}$

1 1

$(S, B, X_1),\ (X_1, B, S)$

$\underbrace{X_1B}$

$S, \cancel{X_3}, B$

0 1 0

$(S, B, X_1)\ (X_2, B, A)$

$(X, B), (X, A)$

$S, \cancel{X_3}, B$

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Step 3: String of length 3.

$\underbrace{01}\overline{1}$

$(S, B)\ (S, B, X_1)$

$\times$

$0\ \overline{11}$

$(A, B, X_2)\ (S, \cancel{X_3}, B)$

$\underbrace{AX_3}_{S}, \underbrace{X_2B}_{B}$

$\overline{11}\ \underline{0}$

$(S, \cancel{X_3}, B)\ (A, B, X_2)$

$\underline{1}\ \overline{10}\ \times$

$(S, B, X_1)\ (S, \cancel{X_3}, B)$

$X_1B \rightarrow S, \cancel{X_3}, B$

$\overline{10}\ \underline{1}$

$(S, \cancel{X_3}, B)\ (S, B, X_1)$

$\times$

$\underline{1}\ \overline{01}$

$(S, B, X_1)\ S, B$

$X_1B \rightarrow S, \cancel{X_3}, B$

Step 4: length 4.

$\overline{011}\ \underline{0}$

$(S, B)\ (A, B, X_2)$

$\times$

$\underline{01}\ \overline{10}$

$(S, B)\ (S, \cancel{X_3}, B)$

$A\cancel{X_3}, X_2B$

$\downarrow$

$(S, B)$

$\underline{0}\ \overline{110}$

$(A, B, X_2)\ (S, \cancel{X_3}, B)$

$AX_3, X_2B$

$\downarrow\ \downarrow$

$(S, B)$

$\overline{110}\ \underline{1}$

$(S, \cancel{X_3}, B)\ (S, B, X_1)$

$\times$

$\underline{11}\ \overline{01}$

$(S, \cancel{X_3}, B)\ (S, B)$

$\times$

$\underline{1}\ \overline{101}$

$(S, B, X_1)\ (S, \cancel{X_3}, B)$

$X_1B$

$\downarrow$

$(S, \cancel{X_3}, B)$

$\overline{101}\ \underline{1}$

$(S, \cancel{X_3}, B)\ (S, B, X_1)$

$\times$

$\underline{10}\ \overline{11}$

$\underline{1}\ \overline{011}$

$(S, B, X_1)\ (S, B)$

$X_1B$

$\downarrow$

$(S, \cancel{X_3}, B)$

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Step 5: length of String 5.

0 1101

$(A, B, X_2)\ (S, \cancel{X_3}, B)$

$\downarrow$

$(S, B)$

---

01 101

$(S, B)\ (S, \cancel{X_3}, B)$

X

---

011 01

$(S, B)\ (S, B)$

X

---

0110 1

$(S, B)\ (S, B, X_1)$

X

---

1 1011

$(S, B, X_1)\ (S, \cancel{X_3}, B)$

$X_1, B$

$\downarrow$

$(S, B, \cancel{X_3})$

---

11 011

$(S, \cancel{X_3}, B) \cdot (S, B)$

X

---

110 11

$(S, \cancel{X_3}, B)\ (S, \cancel{X_3}, B)$

X

---

110 1

$(S, \cancel{X_3}, B)\ (S, B, X_1)$

X

---

1 0110

$(S, B, X_1) \cdot (S, B)$

$X_1 B \rightarrow (S, B, \cancel{X_3})$

---

10 110

$(S, \cancel{X_3}, B) \cdot (S, \cancel{X_3}, B)$

X

---

101 10

$(S, \cancel{X_3}, B)\ (S, \cancel{X_3}, B)$

X

---

1101 0

$(S, \cancel{X_3}, B)\ (A, B, X_2)$

X

Step 6: String of length 6.

0 11011

$(A, B, X_2)\ (S, \cancel{X_3}, B)$

$A\cancel{X_3},\ X_2 B$

$\downarrow$

$\boxed{(S, B)}$

---

~~0~~1 10110

$(S, B, X_1)\ (S, \cancel{X_3}, B)$

$X_1 B$

$\downarrow$

$\boxed{(S, B, \cancel{X_3})}$

---

01 1011

$(S, B)\ (S, \cancel{X_3}, B)$

X

---

11 0110

$(S, \cancel{X_3}, B)\ (S, B)$

X

---

011 011

$(S, B)\ (S, B)$

X

---

110 110

$(S, \cancel{X_3}, B)\ (S, \cancel{X_3}, B)$

X

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0110 | 11

$(S,B)$ $(S,\cancel{X_3},B)$

x

01101 | 1

$(S,B)$ $(S,B,X_1)$

x

1101 | 10

$(S,\cancel{X_3},B)$ $(S,\cancel{X_3},B)$

x

11011 | 0

$(S,\cancel{X_3},B)$ $(A,B,X_2)$

**Step 7:** String of length 7.

0 | 110110

0 | 11011 0

$(A,B,X_2)$ $(S,\cancel{X_3},B)$

$\boxed{(S,B)}$

01 | 10110

$(S,B)$ $(S,\cancel{X_3},B)$

x

011 | 0110

$(S,B)$ $(S,B)$

x

0110 | 110

$(S,B)$ $(S,\cancel{X_3},B)$

x

01101 | 10

$(S,B)$ $(S,\cancel{X_3},B)$

x

011011 | 0

$(S,B)$ $(A,B,X_2)$

Here → S in top row of the table

S is starting non-terminal

w is in Context Free Grammar 'G'.