NATIONAL INSTITUTE OF TECHNOLOGY, PATNA

END-SEMESTER EXAMINATION, MAY 2021

Program: B. Tech CS **DEPT: CSE**

Code: CS4403 **Semester: 4th** **Subject: Design and Analysis of Algorithms**

Time: 2 Hours **Marks: 40**

Instruction: Marks will be deducted for unnecessary writing. Answer all the Questions with brief and precise.

Q1. Let A be an n×n two-dimensional array with all distinct elements, in which all rows and all columns are sorted in ascending order from smaller to larger indices. Given a key x, your task is to find out whether x is present in A.

(i) Can you design a O(n)-time algorithm for the problem. If yes then state the algorithm and analyze the time complexity. If no then justify. **[Marks:2 + 7 = 9]**

Q2. Consider a complete undirected graph with vertex set {1, 2, 3, 4,5}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}. What is the minimum possible weight of a spanning tree T in this graph such that vertex 1 is a leaf node in the tree T? **[Marks:9]**

0	1	8	1	4
1	0	12	4	9
8	12	0	7	3
1	4	7	0	2
4	9	3	2	0

Q3. What is the time complexity of Dijkstra's single source shortest-path algorithm? Suppose we run Dijkstra's single source shortest-path algorithm on the adjacent matrix of Q1 with vertex 1 as the source node. In what order do the nodes get included into the set of vertices for which the shortest distances are finalized? **[Marks: 2+7=9]**

Q4. An inversion of an array A[1..n] of n distinct integer elements is a pair (i, j) such that $i < j$ and $A[i] > A[j]$. Your task is to determine the number of inversions present in an array. For example, the array A[1..8] = {4,8,9,3,7,6,2,5} has a total of 18 inversions. In particular, the element-pair (1,4) (since A[1] = 4 and A[4] = 3, so $1 < 4$ but $A[1] = 4 > 3 = A[4]$) presents one such inversion in A[1..8]. Answer the following.

(i) Design an algorithm to solve the problem which runs in $\Theta(n^2)$ time. Also, deduce the given time-complexity from your algorithm. **[Marks: 7+2=9]**

Q5.
(i) The problem of determining whether there exists a cycle in an undirected graph is in P.
(ii) The problem of determining whether there exists a cycle in an undirected graph is in NP.
(iii) If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A.

Which of the above statements are TRUE? Justify your answer. **[Marks: (2+2)]**

Options: (A) (i), (ii) and (iii); (B) (i) and (ii) only; (C) (ii) and (iii) only; (D) (i) and (iii) only

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