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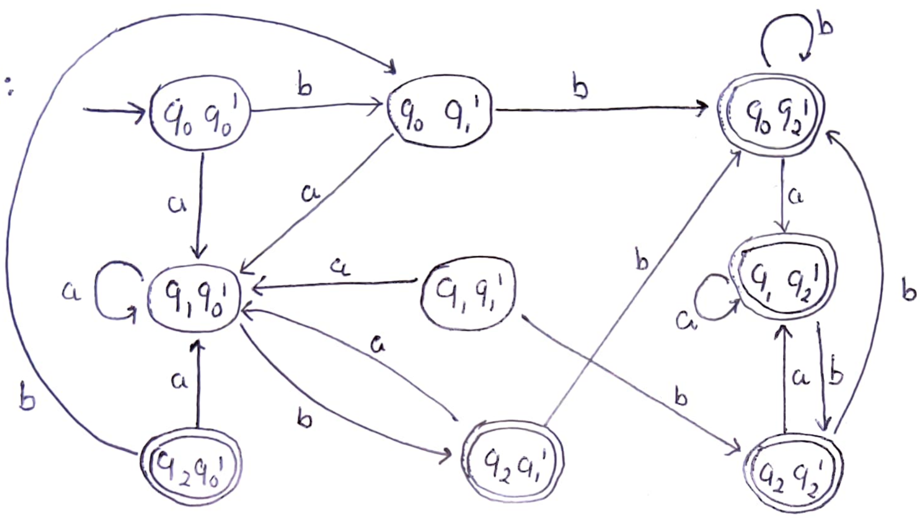
Formal Languages & Automata Theory
CS4402
End Semester Examination

CSE-1
04/05/2021

Solution 1 (i) $L(M_1) \cup L(M_2)$

δ:



$M = \{Q, \Sigma, \delta, q_0, F\}$

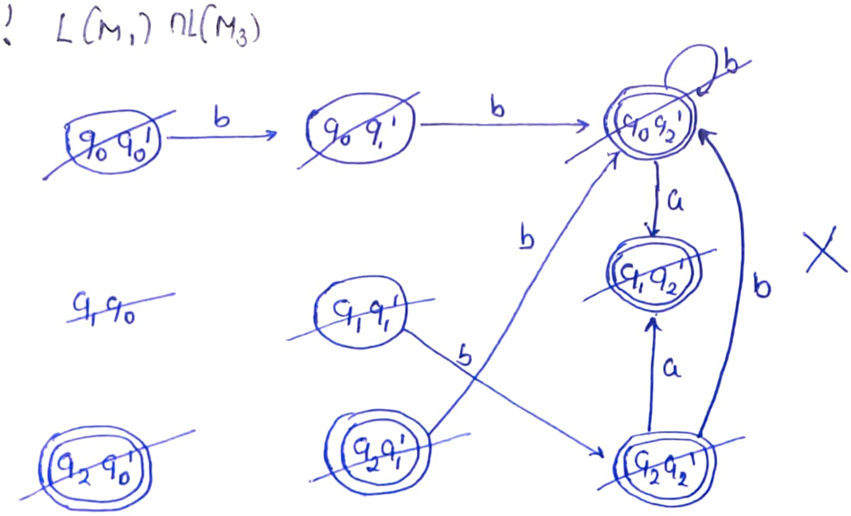
$Q = \{q_0q_0', q_0q_1', q_0q_2', q_1q_0', q_1q_1', q_1q_2', q_2q_0', q_2q_1', q_2q_2'\}$

$\Sigma = \{a, b\}$

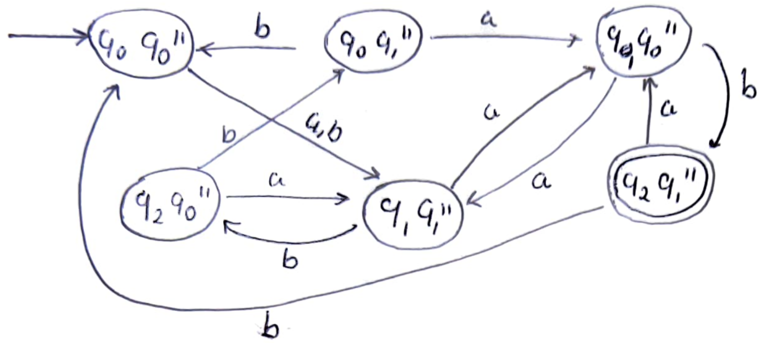
$q_0 = (q_0 q_0')$

$F = \{q_0q_0', q_2q_1', q_0q_2', q_1q_2', q_2q_2'\}$

Solution (ii) $L(M_1) \cap L(M_3)$



Solution 1) (a) ii)



$M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0 q_0'', q_0 q_1'', q_1 q_0'', q_1 q_1'', q_2 q_0'', q_2 q_1''\}$

$\Sigma = \{a, b\}$

$q_0 = \{(q_0 q_1'')\}$

$F = \{q_2 q_1''\}$

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Class-Test-2

Formal Languages and Automata Theory.

Solution 1) b).

The regular expression for the set of strings having at most 1 Consecutive a's and 1 consecutive b's is given by.

Let G be the grammar and r be the regular expression of language L.

$\therefore r = (ab+ba)^* (aa+bb+aabb+bbaa)^* (ab+ba)^*$

Which will accept languages containing strings.

$L(r) = \{aa, bb, abaaaba, aabbab, \ldots\}$

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Solution 2) a)

$L = \{ (a+b)^* \mid$ the length of string is odd & the middle symbol in the string is always $a \}$

$$L = \{ (a+b)^n \, a \, (a+b)^n \mid n \geq 0 \} \quad \text{---} (1)$$

also, $G = (V, T, P, S)$

So, for language in equation(1) following CFG is possible

$$\left.\begin{aligned} S &\to XaX \\ X &\to aX \\ X &\to bX \\ X &\to \epsilon \end{aligned}\right\} P \text{ (productive Rule)}$$

$V = \{S, X\}$ ∵ Set of non-terminals

$T = \{a, b\}$ ∵ Set of terminals

$S = S \to$ Starting Symbol

$$\left[G = (\{S, X\}, \{a, b\}, \{ S \to XaX,\ X \to aX / bX,\ X \to \epsilon \}, S) \right]$$

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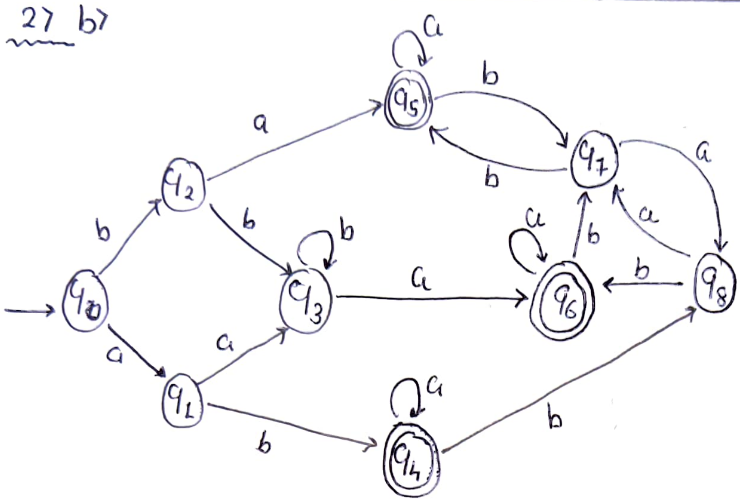
Solution 2) b)



Let G_1 & G_2 be two groups.
G_1 Containing non final states
and G_2 final states

$G_1 = \{q_0, q_1, q_2, q_3, q_7, q_8\}$

$G_2 = \{q_5, q_4, q_6\}$

G_1:

	a	b
q_0	G_1	G_2
q_1	G_1	G_2
q_2	G_2	G_1
q_3	G_2	G_1
q_7	G_1	G_2
q_8	G_1	G_2

G_2:

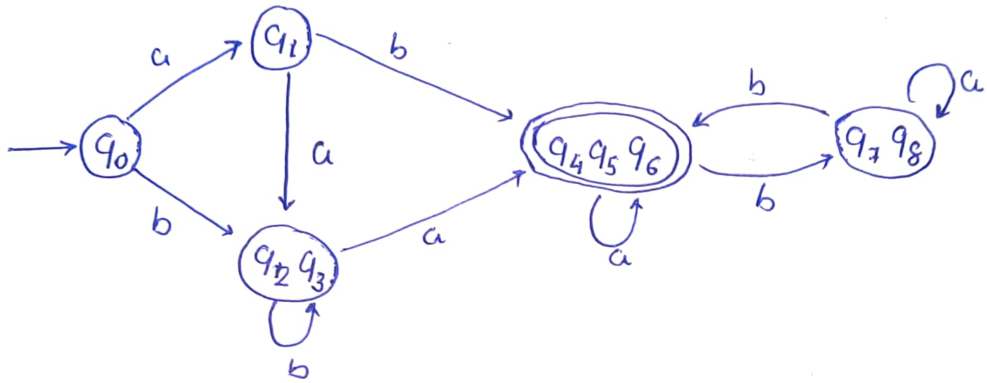
	a	b
q_4	G_2	G_1
q_5	G_2	G_1
q_6	G_2	G_1

$\{q_0\}$ $\quad \underbrace{\{q_1, q_7, q_8\}}_{G_{12}}$ $\quad \underbrace{\{q_2\ q_3\}}_{G_{13}}$ $\quad \underbrace{\{q_4, q_5, q_6\}}_{G_2}$

	a	b
q_1	G_{13}	G_2
q_1	G_{12}	G_2
q_8	G_{12}	G_2

Statès :-

$\{q_0\}, \{q_1\}, \{q_7, q_8\}, \{q_2, q_3\}, \underbrace{\{q_4, q_5, q_6\}}_{\text{final states...}}$



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Solution: 2) c)

$L = \{xy \mid x, y \in (a+b)^*$ & y is either x or $x^R\}$

$L = \{abab, abba, \epsilon, abaaba, \ldots\}$

Let String $S = abab$, $n = 2$

$\{|w| \geq 2\}$

$\underbrace{a}_{x}\underbrace{b}_{y}\underbrace{ab}_{z}$

$\{|xy| \leq 2\}$

No consider $a(b)^i ab$

for $i = 0$

String $= aab$

$aab \notin L$

or

for $i = 2$

String $= abbab$

also abbab is not of the form xx or xx^R

Hence $abbab \notin L$

Therefore, the given language is not regular.

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Solution 3>
a>

Language $L = \{a^m b^{(m+n)} c^n \mid m,n \geq 1\}$

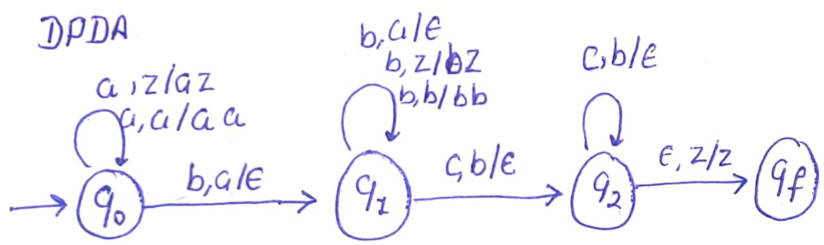
$L = \{abbc, abbbcc, aabbbbcc, \ldots\}$

Here we need to maintain the order of a's, b's and c's. Thus we need a Stack along with state diagram. The count of a's, b's and c's is maintained by Stack.

We will take 3 stack alphabets $\Gamma = \{a, b, z\}$

z = Starting Symbol.

Required DPDA



Tuples.

$F = \{q_f\}$

$Q = \{q_0, q_1, q_2, q_f\}$

$Q_0 = q_0$

$\Gamma = \{a, b, z\}$

$\Sigma = \{a, b, c\}$

$M = (\{q_0, q_1, q_2, q_f\}, \{a, b, c\}, \{a, b, z\}, \delta, q_0, z, \{q_f\})$

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Solution 3>
b> To prove: The intersection of two Context free languages is not a context free language

Proof: Let L_1 & L_2 are two context free languages

Example: say $L_1 = \{a^n b^n c^m \mid n \geq 0 \text{ and } m \geq 0\}$
say $L_2 = \{a^m b^n c^n \mid n \geq 0 \text{ and } m \geq 0\}$

also $L_3 = L_1 \cap L_2$
which is $L_3 = \{a^n b^n c^n \mid n \geq 0\}$
need not to be context free.

Explanation: $L1$ says number of a's should be equal to number of b's and L_2 says number of b's should be equal to number of c's. Their intersection says both conditions need to be true, but push down automata can compare only two as we know. So it cannot be accepted by push down automata, hence not context free.

So CFL are not closed under intersection.

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Solution 3) c)

P: $S \to SS$
$S \to aSb$
$S \to ab$

Step 1: Convert CFG to CNF

$S \to SS$
$X \to aS$
$X_1 \to a$
$X_2 \to b$
$S \to ab$
$Y \to aab$
$S \to YX_2$

Step 2: Renaming

$S \to A_1$
$Y \to A_2$
$X_2 \to A_3$
$X_1 \to A_4$

Step 3: $\begin{bmatrix} A_1 \to A_1A_1 \\ A_1 \to A_2A_3 \end{bmatrix}$

$A_2 \to aA_1$
$A_3 \to b$
$A_4 \to a$
$A_1 \to ab$
$A_2 \to aab$

$\because \begin{bmatrix} A \to A\alpha / \beta \\ A \to \beta A' / \beta \\ A' \to \alpha A' / \alpha \end{bmatrix}$

Step 4: $A_1 \to \overset{A}{A_1}\overset{\alpha}{A_1} / \overset{\beta_1}{ab}$

applying lemma 2

$A \to A\alpha / \beta$
$A \to \beta A' / \beta$

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$A' \rightarrow \alpha A' / \alpha$

$A_1 \rightarrow abZ / ab$

$Z \rightarrow A_1 Z / A_1$

Step 5: $Z \rightarrow abZZ \ / abZ \ / ab$

A_2 is already in GNF then

Substituting A_2 : $A_1 \rightarrow aabA_3 / aA_1A_3 /$

So all productions in GNF

$$
\left\{
\begin{array}{l}
A_1 \rightarrow abZ / ab \\
Z \rightarrow abZZ / abZ / ab \\
A_1 \rightarrow aabA_3 / aA_1A_3 \\
A_2 \rightarrow aA_1 \\
A_3 \rightarrow b \\
A_4 \rightarrow a \\
A_2 \rightarrow aab \\
A_1 \rightarrow ab
\end{array}
\right.
$$

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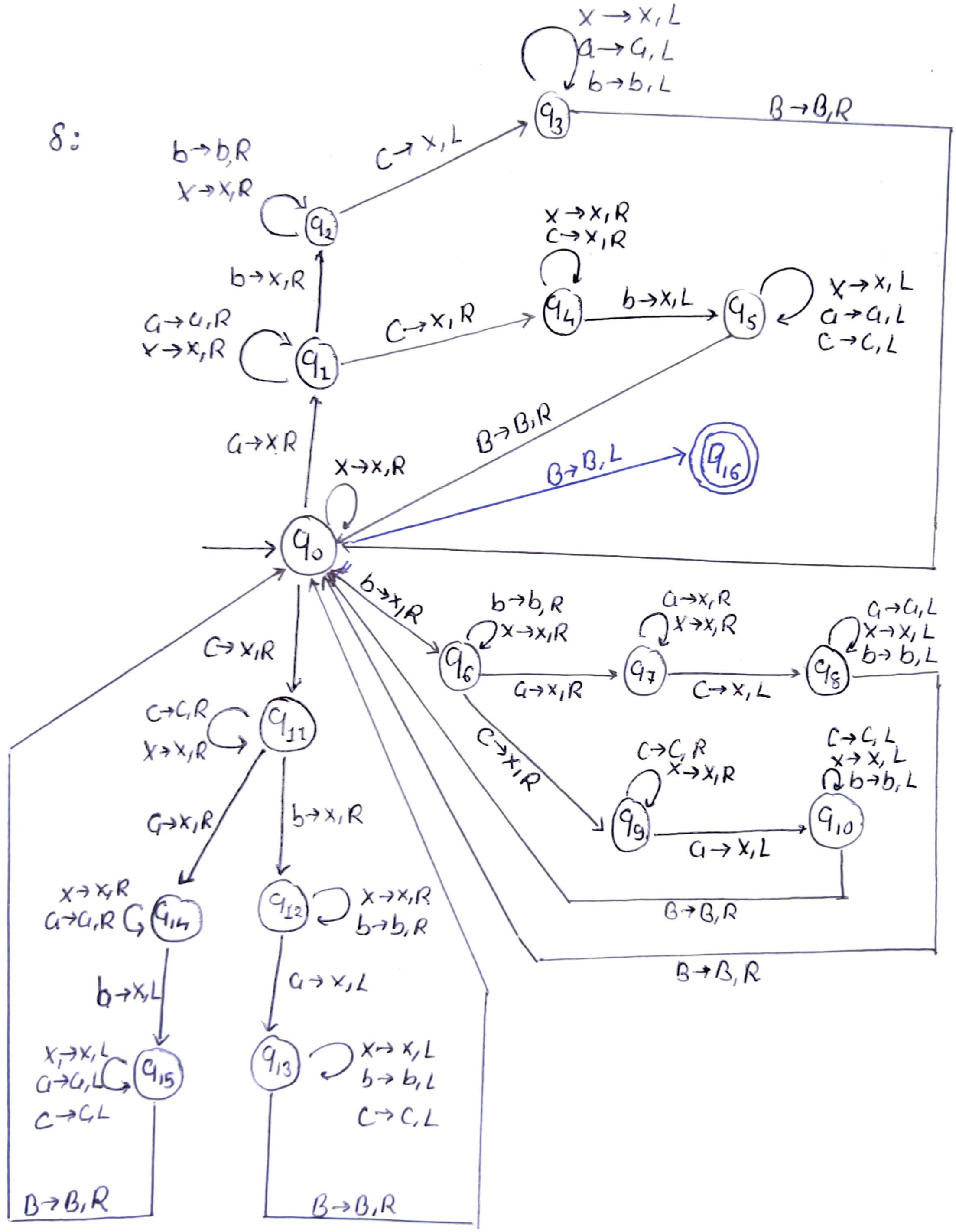
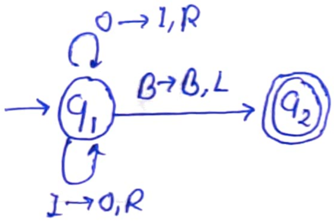
Solution: 4) a) $L = \{ w \in (a+b+c)^* \mid n_a(w) = n_b(w) = n_c(w) \}$

δ:



Solution: 4> b>



For the above TM, any two transitions can be:

i> $\delta(q_1, 0) \rightarrow (q_1, 1, R)$

ii> $\delta(q_1, 1) \rightarrow (q_1, 0, R)$

For encoding above into binary strings.
We can use following representation

For states,

$q_1 \rightarrow 0$ & $q_2 \rightarrow 00$

For input alphabets,

$0 \rightarrow 0$ & $1 \rightarrow 00$

For direction

$L \rightarrow 0$ & $R \rightarrow 00$

So for transition i.> we have binary string as follows:

010110100100

and similarly for transition ii> 0100110101100

Two valid binary strings for above TM:

010110100100 & 0100110101100

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Solution 4> c>:

$L_1 = \{a^n b^n c^n \mid n \geq 1\}$ Clearly L_1 is recursive language.

$L_2 = \{a^p \mid p$ is a prime number$\}$ Clearly L_2 is not regular

$L_d = \{w \mid w$ is not accepted by $M_i\}$.

$\overline{L_1}, \overline{L_2}$ & $\overline{L_d}$ are Complement languages of L_1, L_2 and L_2 respectively

$L_3 = L_1 \cup L_2$

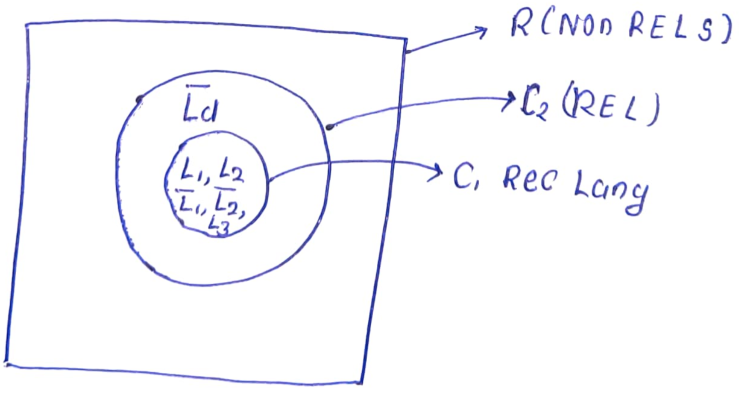
L_4 is Recursive Enumerable language & complement also REC language.

$\overline{L_1} \Rightarrow$ Rec Lang

$\overline{L_2} \Rightarrow$ Rec Lang

$L_3 = L_1 \cup L_2 =$ Rec Lang

L_4 Rec lang.



Inside $C_1 \rightarrow L_1, L_2, \overline{L_1}, \overline{L_2}, L_3, L_4$

$C_2 \rightarrow \overline{L_d}$

$R \rightarrow L_d$.