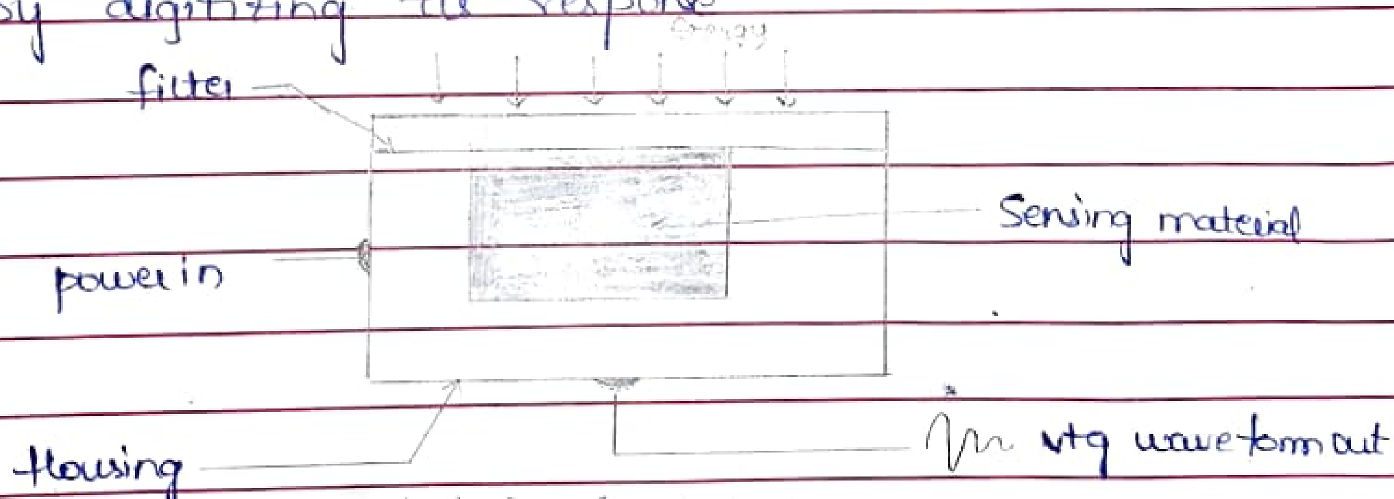
Assignment - I

1(a) Discuss the three principal sensor arrangements used to transform illumination energy into digital image.

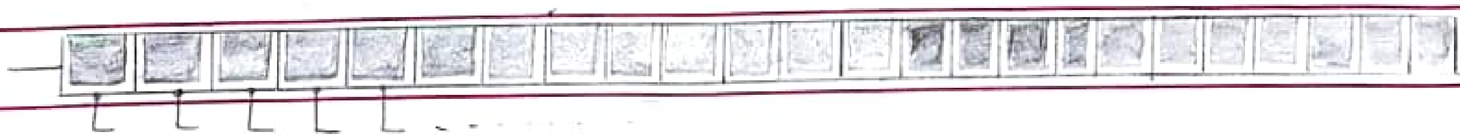
As The types of images in which we are interested are generated by the combination of an illumination source and the reflection or absorption of energy from that source by the elements of the scene being imaged. We enclose illumination and scene in quotes to emphasize the fact that they are considerably more general than the familiar situation in which a visible light source illumination may originate from a source of electromagnetic energy such as a radar, infrared, or X-ray energy. But, as noted earlier, it could originate from less traditional sources, such as a ultrasound or even a computer-generated illumination pattern.

Similarly, the scene elements could be familiar objects, but they can just as easily be molecules, buried rock formations, or a human brain. We could even image a source, such as acquiring images of the sun. Depending on the nature of the source, illumination energy is reflected from a planar surface. An example in the second category is when X-rays pass through a patient's body for the purpose of generating a diagonistic X-ray film. In some applications, the reflected or transmitted energy is focused onto a photo convertor (eg., a phosphor screen) of which converts the energy into visible light. Electron microscopy and some applications of gamma imaging use this approach.

The above fig shows the three principle sensor arrangements used to transform illumination energy into digital images. The idea is simple: Incoming energy is transformed into a vtg by the combination of input electrical power and sensor material that is responsive to the particular type of energy being detected. The o/p vtg waveform is the response of the sensor and a digital quantity is obtained from each sensor by digitizing its response.



(a) Single imaging sensor



(b) Line sensor

(c) Array sensor

1b) Explain about the simple image formation model.

A) Image Formation model:-

The above fig (c) shows individual sensors arranged in the form of 2D-array. Numerous electromagnetic and some ultrasonic sensing devices frequently are arranged in an array format. this is also the predominant arrangement found in digital cameras. A typical sensor for these cameras in a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of 4000 * 4000 elements or more. CCD Sensor are used widely in digital cameras and other light sensing instruments. the response of each sensor is proportional to the integral of light energy projected onto the Surface of the sensor, a property that is used in astronomical and other applications requiring low noise images. If the illumination is light, the front end of the imaging system is a lens, which projects the viewed scene onto the lens focal plane. as fig(b) shows the sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light recieved at each sensor. Digital and analog circuitry sweep these outputs and convert them to a video signal.

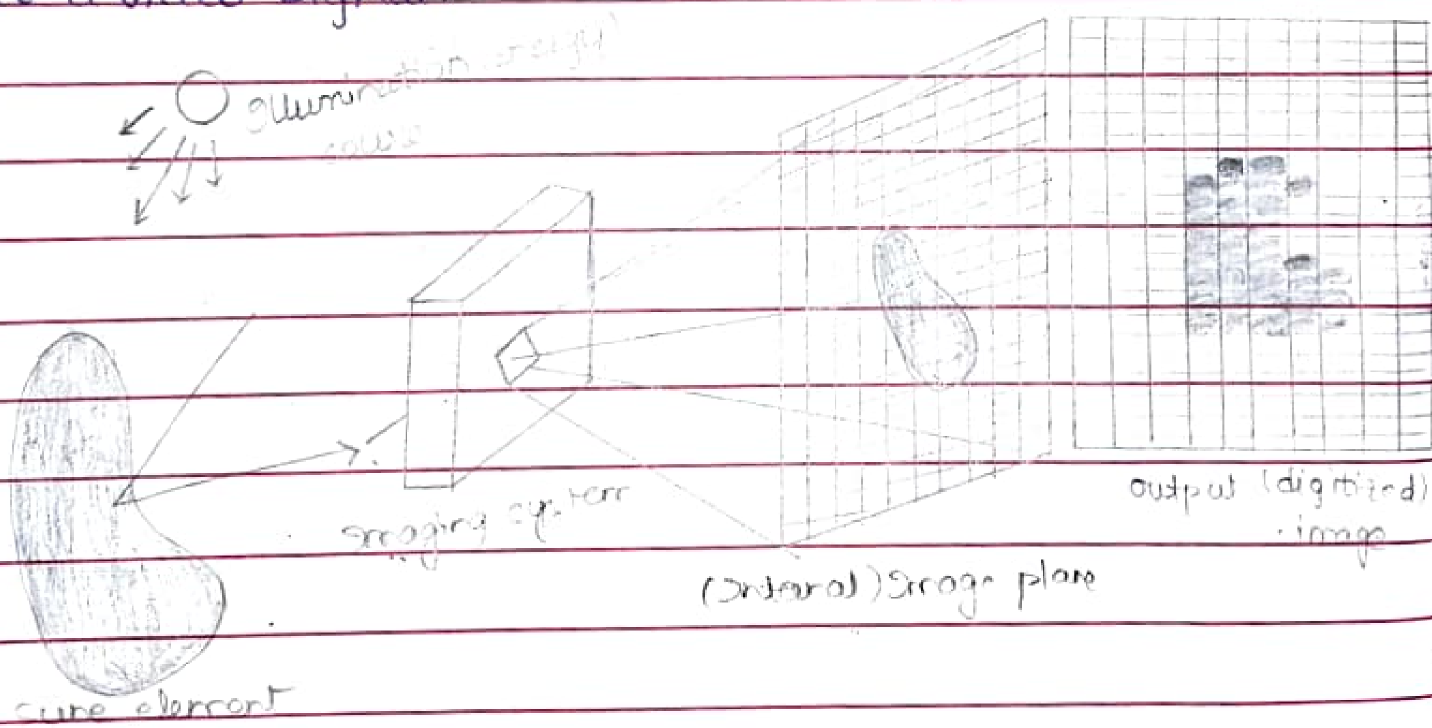


fig:- An example of the digital image formation process (a) Energy ("illumination") source (b) an element of a scene (c) Imaging system (d) Protection of the scene onto the image plane (e) Digitized image

2a) Discuss about the distance measure of a pixel with a suitable example.

As **Distance measures:-**

For pixels p, q and z with coordinates (x, y), (s, t) and (v, w) respectively, D is a distance function or metric if

(a) $D(p, q) \geq 0$ ($D(p, q) = 0$ iff $p = q$)
(b) $D(p, q) = D(q, p)$ and
(c) $D(p, z) \leq D(p, q) + D(q, z)$

The Euclidean distance b/w p and q is defined as
$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

For the distance measure, the pixels having a distance less than or equal to some value r from (x, y) are the points contained in a disk of radius r centered at (x, y)

The D_4 distance (also called city-block distance) b/w p and q is defined as

$$D_4(p,q) = |x-s| + |y-t|$$

In this case, the pixels having a D_4 distance from (x,y) less than or equal to some value r from a diamond centered at (x,y). For example, the pixels with D_4 distance ≤ 2 from (x,y) (the center point) from the following contours of constant distance

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

The pixels with $D_4 = 1$ are the 4-neighbours of (x,y)

The D_8 distance (also called chessboard distance) b/w p and q is defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

In this case, the pixels with D_8 distance from (x,y) less than or equal to some value of r form a square centered at (x,y). for example, the pixels with D_8 distance ≤ 2 from (x,y) (the center point) form the following contours of constant distance

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with $D_8 = 1$ are the 8-neighbours of (x,y). Note that the D_4 and D_8 distances b/w p and q are independent of any paths that mights exist b/w the points because these distances involves only the coordinates of the points. If we elect to consider m-adjacency, however, the D_m distance b/w two points is defined as the shortest m-path b/w the points. for instance consider the following arrangement of pixels and assume that P_3 P_2 and P_4 have value 1 and that P_1 and P_3 can have a value of 0 or 1

p_3 p_4

p_1 p_2

p

Suppose that we consider adjacency of pixels valued 1 (i.e., = {1, 4}) If p1 and p3 are 0, the length of the shortest m-path (see the defination of m-adjacency) b/w p and p4 is 2. Similar comments apply if p3 is 1 (and p1 is 0); In this case, the length of the shortest m-path also is 3. finally, if both p1 and p3 are 1 the length of the shortest m-path b/w p and p4 is 4. In this case, the path goes through the sequence of points p p1 p2 p3 p4

2b) Explain the following mathematical operations on the digital images

(i) Array versus matrix operation
(ii) Linear versus non-linear operations.

As Array v/s matrix operation:

An element wise operation involving one or more images is carried out on a pixel-by-pixel basis.

We mentioned earlier in this chapter that images can be viewed equivalently as matrices. In fact, as you will see later in this section, there are many situation in which operation b/w images are carried out using matrix theory. It is for this reason that a clear distinction must be made b/w element wise and matrix operations. For example, consider the following 2x2 images (matrices):

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

The elementwise product (often denoted using the symbol $\odot$ or $\otimes$) of these two image is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

That is, the element wise product is obtained by multiplying pairs of corresponding pixels. On the other hand, the matrix product of the images is formed using the rules of matrix multipication.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

(b) Linear Versus Non-linear Operation:-

One of the most important classification of an image processing method is whether it is linear or non linear. Consider a general operator, that produces an o/p image $g(x,y)$, from a given input image, $f(x,y)$

$$\mathcal{H}[f(x,y)] = g(x,y) \longrightarrow (1)$$

Given two arbitary constants, a and b, and two arbitary images $f_1(x,y)$ and $f_2(x,y)$. $\mathcal{H}$ is said to be a linear operator if

$$\mathcal{H}[a f_1(x,y) + b f_2(x,y)] = a\mathcal{H}[f_1(x,y)] + b\mathcal{H}[f_2(x,y)]$$
$$= a g_1(x,y) + b g_2(x,y) \longrightarrow (2)$$

This eq^n indicates that the o/p of a linear operation applied to the sum of two inputs is the same as performing the operation individually on the inputs and then summing the results. In addition, the o/p of a linear operation on a constant multiplied by an i/p is the same as the o/p of the operation due to the original input multiplied by that constant. The first property is called the property of additivity, and

The second is called the property of homogeneity. By defination, an operator that fails to satisfy eq (2) is said to be non-linear.

3a) Prove that seperable property of 2D-Discrete fourier Transform.

As The seperable property allows a 2D transform to be computed in two steps by successive 1D operations on rows and columns of an image.

$$F(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, e^{-j\frac{2\pi mk}{N}}\, e^{-j\frac{2\pi nl}{N}}$$

$$F(k,l) = \sum_{m=0}^{M-1} \left[\sum_{n=0}^{N-1} f(m,n)\, e^{-j\frac{2\pi nl}{N}} \right] e^{-j\frac{2\pi mk}{N}}$$

$$F(k,l) = \sum_{m=0}^{N-1} f(m,l)\, e^{-j\frac{2\pi mk}{N}}$$

$$= F(k,l)$$

Thus, peforming a 2D fourier transform is equivalent to peforming two 1D transforms as

a) performing a 1D transform on each row of image $f(m,n)$ to get $F(m,l)$

b) performing a 1D transform on each column of $F(m,l)$ to get $F(k,l)$

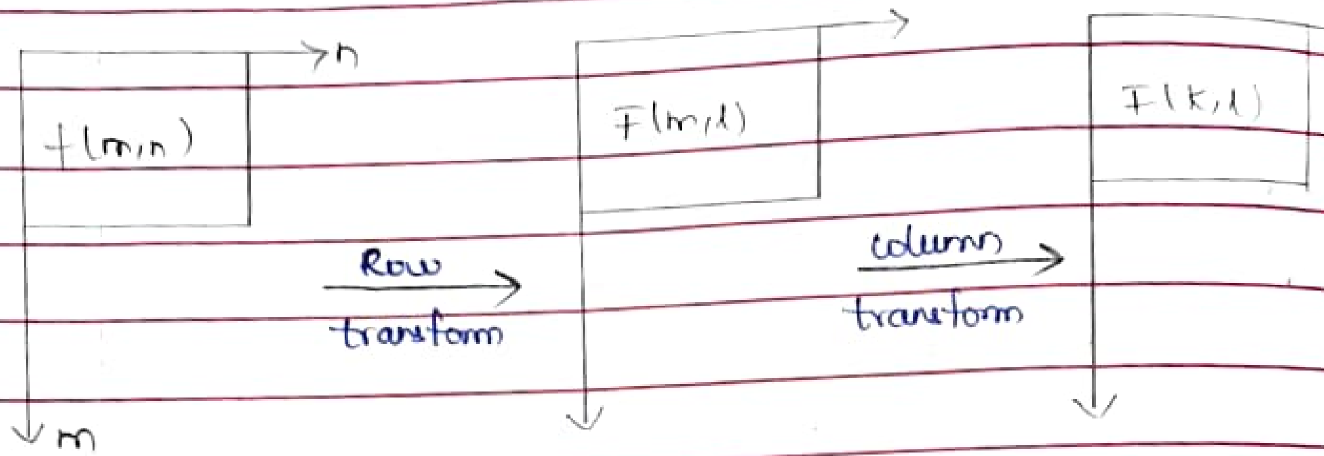


fig: computation of 2D-DFT using seperable property

3b) Prove the periodicity peropeety of 2D - Discrete fourier transform

As Periodicity Property:

The 2D DFT of a function $f(m,n)$ is said to be periodic with a period N if

$$F(k,l) \longrightarrow F(k+pN, l+qN)$$

Proof:

$$F(k+pN, l+qN) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n)\, e^{-j\frac{2\pi m(k+pN)}{N}}\, e^{-j\frac{2\pi n(l+qN)}{N}} \rightarrow ①$$

Splitting the terms $e^{-j\frac{2\pi m(k+pN)}{N}}$ and $e^{-j\frac{2\pi n(l+qN)}{N}}$ in ①

$$F(k+pN, l+qN) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n)\, e^{-j\frac{2\pi mk}{N}}\, e^{-j\frac{2\pi mpN}{N}}\, e^{-j\frac{2\pi nl}{N}}\, e^{-j\frac{2\pi nqN}{N}} \rightarrow ②$$

By taking $e^{-j2\pi mp}$ and $e^{-j2\pi nq}$ out of summation ②, we get

$$F(k+pN, l+qN) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \left[f(m,n)\, e^{-j\frac{2\pi mk}{N}}\, e^{-j\frac{2\pi nl}{N}}\right] e^{-j2\pi mp}\, e^{-j2\pi nq} \rightarrow ③$$

By taking $e^{-j2\pi mp}$ and $e^{-j2\pi nq}$ out of summation ②, we get

$$F(k+pN, l+qN) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \left[f(m,n)\, e^{-j\frac{2\pi mk}{N}}\, e^{-j\frac{2\pi nl}{N}}\right] e^{-j2\pi mp}\, e^{-j2\pi nq} \rightarrow ④$$

By substituting eq ④, we get

$$F(k+pN, l+qN) = F(k,l)\, e^{-j2\pi mp}\, e^{-j2\pi nq} \rightarrow ⑤$$

The $e^{-j2\pi mp}$ and $e^{-j2\pi nq}$ values are always 1 for any integer values of n, q, p and m.

So the product of these two exponential terms $e^{-j2\pi mp}$, $e^{-j2\pi nq}$ is also equal to '1'. By replacing $e^{-j2\pi mp}$, $e^{-j2\pi nq}$ with 1 in (5), we get

$$F(k+pN,\ l+qN) = F(k,l)\cdot 1$$

$$F(k+pN,\ l+qN) = F(k,l)$$

Step 1:- Determination of the matrix A.

Finding 4-point DFT (where $N=4$)

The formula to compute a DFT matrix of order 4 is given below:

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j2\pi kn}{4}} \quad \text{where } k = 0, 1, \ldots, 3$$

1. Finding $X(0)$

$$X(0) = \sum_{n=0}^{3} x(n) = x(0) + x(1) + x(2) + x(3)$$

2. Finding $X(1)$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$$

$$= x(0) + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + x(3)e^{-j3\pi/2}$$

$$X(4) = x(0) - jx(1) - x(2) + jx(3)$$

3. Finding $X(2)$

$$X(2) = \sum_{n=0}^{3} x(n)e^{-j\pi n}$$

$$= x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$X(2) = x(0) - x(1) + x(2) - x(3)$$

4. Finding $X(3)$

$$X(3) = \sum_{n=0}^{3} x(n)e^{-j3\pi n/2}$$

$$= x(0) + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi} + x(3)e^{-j9\pi/2}$$

$$X(3) = x(0) + jx(1) - x(2) - jx(3)$$

Collecting the coefficient of $X(0)$, $X(1)$, $X(2)$ and $X(3)$, we get

$$X[k] = A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

4a) Define KL transform and give its applications.

As Hoteling (or Karhunen-Loeve [KL] Transform):-

* The karhunen-Loeve transform or KLT was originally introduced as a series expansion for continious random process by karhunen and Loeve

* For discrete signals hotelling first studied what was called a method of principal components, which is discrete equivalent of the KL series expansion

* Consequentely, the KL transform is also called the Hotelling transform or the method of principal components.

* Let e_i and λ_i, $i = 1, 2, 3 \cdots n$, be this set of eigen vectors and corresponding eigen values of C_x, arranged in.

b) Apply the KL transform for the following image.

$$f(m,n) = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

A: Given $f(m,n) = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$$X = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 4 \\ -1 \end{bmatrix} ; \quad X_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

then $M = 2$.

$$\bar{x} = \frac{1}{M} \sum_{k=0}^{M-1} X_k = \frac{1}{2} \sum_{k=0}^{1} X_k = 1/2 \, (X_0 + X_1)$$

$$\bar{x} = \frac{1}{2} \left(\begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right) = 1/2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\bar{x}^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\bar{x}\bar{x}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$E(XX^T) = \frac{1}{M}\sum_{k=0}^{M-1} X_k X_k^T$$

$$= \frac{1}{2}\left[X_0 X_0^T + X_1 X_1^T\right]$$

$$= \frac{1}{2}\left[\begin{bmatrix}4\\-1\end{bmatrix}\begin{bmatrix}4 & -1\end{bmatrix} + \begin{bmatrix}-2\\3\end{bmatrix}\begin{bmatrix}-2 & 3\end{bmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{bmatrix}16 & -4\\-4 & 1\end{bmatrix} + \begin{bmatrix}4 & -6\\-6 & 9\end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix}20 & -10\\-10 & 10\end{bmatrix} = \begin{bmatrix}10 & -5\\-5 & 5\end{bmatrix}$$

Step2:

$$\text{conv}(x) = E(XX^T) - \bar{X}\bar{X}^T$$

$$= \begin{bmatrix}10 & -5\\-5 & 5\end{bmatrix} - \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}9 & -6\\-6 & 4\end{bmatrix}$$

Step3:

Eigen values

$$|\text{conv}(x) - \lambda I| = 0$$

$$\left|\begin{bmatrix}9 & -6\\-6 & 4\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix}9-\lambda & -6\\-6 & 4-\lambda\end{bmatrix}\right| = 0$$

$$(9-\lambda)(4-\lambda)-(-6)(-6)=0$$
$$36-9\lambda-4\lambda+\lambda^2-36=0$$
$$\lambda^2-13\lambda=0$$
$$\lambda(\lambda-13)=0$$
$$\lambda=0,13$$
$$\lambda_0=0\ ;\ \lambda_1=13$$

Step 4:-

Eigen vector

$$[\mathrm{conv}(x)-\lambda_0 I]\phi_0=0$$

$$\left[\begin{bmatrix}9 & -6\\ -6 & 4\end{bmatrix}-0\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right]\begin{bmatrix}\phi_{00}\\ \phi_{01}\end{bmatrix}=11$$

$$\begin{bmatrix}9 & -6\\ -6 & 4\end{bmatrix}\begin{bmatrix}\phi_{00}\\ \phi_{01}\end{bmatrix}=0$$

$$9\phi_{00}-6\phi_{01}=0$$
$$-6\phi_{00}+4\phi_{01}=0$$

Let $\phi_{01}=1$ and $\phi_{11}=1$

$$-6\phi_{00}+4(1)=0$$
$$\phi_{00}=\cancel{-}4/\cancel{-}6=2/3=0.66$$

$$\phi_0=\begin{bmatrix}\phi_{00}\\ \phi_{01}\end{bmatrix}=\begin{bmatrix}0.66\\ 1\end{bmatrix}$$

$$(cov(x) - \lambda_1 I)\phi_1 = 0$$

$$\left[\begin{bmatrix} 9 & -6 \\ -6 & 4 \end{bmatrix} - 13\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]\begin{bmatrix} \phi_{10} \\ \phi_{11} \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & -6 \\ -6 & -9 \end{bmatrix}\begin{bmatrix} \phi_{10} \\ \phi_{11} \end{bmatrix} = 0$$

$$-4\phi_{10} - 6\phi_{11} = 0$$

$$-6\phi_{10} - 9\phi_{11} = 0$$

$$-6\phi_{10} - 9(1) = 0$$

$$\phi_{10} = 9/-6 = -3/2 = -1.5$$

$$\phi_1 = \begin{bmatrix} \phi_{10} \\ \phi_{11} \end{bmatrix} = \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}$$

Step 5: Normalize eigen vector

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{\phi_{00}^2 + \phi_{01}^2}}\begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix}$$

$$= \frac{1}{\sqrt{0.66^2 + 1^2}}\begin{bmatrix} 0.66 \\ 1 \end{bmatrix} = 0.8346\begin{bmatrix} 0.66 \\ 1 \end{bmatrix}$$

$$T_0 = \begin{bmatrix} 0.5508 \\ 0.8346 \end{bmatrix}$$

$$\frac{\phi_1}{\|\phi_1\|} = \frac{1}{\sqrt{\phi_{10}^2+\phi_{11}^2}} \begin{bmatrix} \phi_{10} \\ \phi_{11} \end{bmatrix}$$

$$= \frac{1}{\sqrt{-1.5^2+1^2}} \begin{bmatrix} -1.5 \\ 1 \end{bmatrix} = 0.5547 \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} -0.8320 \\ 0.5547 \end{bmatrix}$$

Step 6:- KL Transformation of normalized eigen vector

$$T = (T_0 \quad T_1)$$

$$T = \begin{bmatrix} 0.5508 & -0.8320 \\ 0.8346 & 0.5547 \end{bmatrix}$$

step 7: Kl transformation of input matrix

$$y = T(x)$$

$$y_0 = T(X_0)$$

$$y_0 = \begin{bmatrix} 0.5508 & -0.8320 \\ 0.8346 & 0.5547 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.0352 \\ 2.7837 \end{bmatrix}$$

$$y_1 = T(X_1)$$

$$y_1 = \begin{bmatrix} 0.5508 & -0.8320 \\ 0.8346 & 0.5547 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$y_1 = \begin{bmatrix} -3.5976 \\ -0.0051 \end{bmatrix}$$

Step 8: Reconstruct the i/p values

$$x = y \cdot T^T$$

$$T^T = \begin{bmatrix} 0.5508 & 0.8346 \\ -0.8320 & 0.5547 \end{bmatrix}$$

$$x_0 = T^T y_0$$

$$= \begin{bmatrix} 0.5508 & 0.8346 \\ -0.8320 & 0.5547 \end{bmatrix} \begin{bmatrix} 3.0352 \\ 2.7837 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 3.9950 \\ -0.9811 \end{bmatrix}$$

$$x_1 = T^T y_1$$

$$= \begin{bmatrix} 0.5508 & 0.8346 \\ -0.8320 & 0.5547 \end{bmatrix} \begin{bmatrix} -3.5976 \\ -0.0051 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} -1.9858 \\ 2.9903 \end{bmatrix}$$

5a) Define histogram and draw the histogram four basic image types.

As **Histogram:-**

* Histogram manipulation basically modifies the histogram of an input image so as to improve the visual quality of the image. In order to understand histogram manipulation, it is necessary that one should have some basic knowledge about the histogram of the image.

* The histogram of an image is a plot of the no. of occurences of gray levels in the image against the gray-level values. the histogram provides a convinent summary of the intensities in an image, but it is unable to convey any information regarding spatial relationships b/w pixels. the histogram provides more insight about image contrast and brightness.

Let r_k, for $k = 0, 1, 2 \ldots L-1$, denote the intensities of an L-level digital image, $f(x,y)$. The unnormalized histogram of f is defined as

$h(r_k) = n_k$ for $k = 0, 1, 2 \ldots L-1$

where n_k is the no. of pixels in f with intensity r_k, and the subdivisions of the intensity scale are called histogram bins. Similarly, the normalized histogram of f is defined as

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

where, as usual M and N are the number of image rows and columns, respectively

* Histogram shape is related to image appearance. For example, fig 1. shows images with four basic intensity characteristics: dark, light, low contrast and high contrast; the image histograms are also shown.

* we note in the dark image that most populated histogram bins are concentrated on the lower (dark) end of the intensity scale. Similarly, the most populated bins of the image histograms are also shown.

* we note in the dark image that the most populated histogram bins are connected on the lower (dark) end of the intensity scale.

* An image with low contrast has a narrow histogram located typically towards the middle of the intensity scale. Finally, we see that the components of the histogram of the high-contrast image cover a wide range of the intensity scale. and the distribution of pixels is not too far from uniform, with few bins being much higher than the other.

* Intuitively, it is resonable to conclude that an image whose pixels tends to occupy the entire range of possible intensity levels and, tend to be distributed uniformly, will have an appearance of high contrast and will exhibit a large variety of gray tones

* The net effect will be an image that shows a great deal of gray level detail and has a high dynamic range

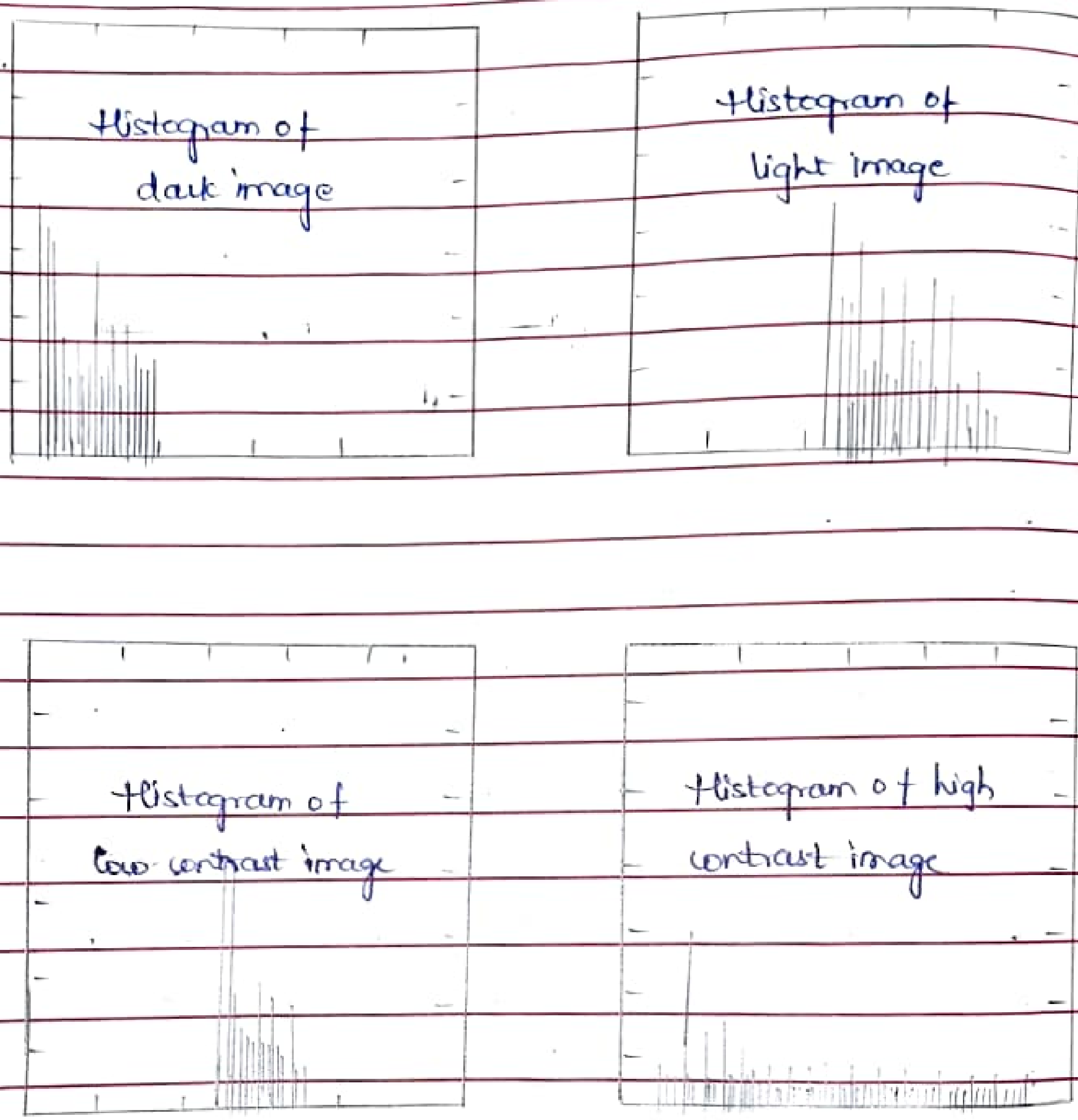


fig 1:- four image types and their corresponding histograms (a) dark ; (b) light ; (c) low contrast ; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

5b) Explain the procedure for histogram process and uses of histogram.

A: Let r_k, for $k = 0, 1, 2 \ldots L-1$, denote the intensities of an L-level digital image, $f(x,y)$. The unnormalized histogram of f is defined as

$$h(r_k) = n_k \quad \text{for } k = 0, 1, 2 \ldots L-1 \longrightarrow (1)$$

where n_k is the no. of pixels in f with intensity r_k, and the subdivisions of the intensity scale are called histogram bins. Similarly, the normalized histogram of f is defined as

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN} \longrightarrow (2)$$

where, as usual, M and N are the no. of image rows and columns, respectively. Mostly, we work with normalized histograms, which we refer to simply as histograms or image histograms. The sum of $p(r_k)$ for all values of k is always 1. The corresponding components of $p(r_k)$ are estimates of the probabilities of intensity levels occuring in an image

Uses of Histogram:

- Statistics obtained directly from an image histogram can be used for image enhancement. Let r denote a discrete random variable representing intensity values in the range $[0, L-1]$, and let $p(r_i)$ denote the normalized histogram component corresponding to intensity value r_i.

- For an image with intensity levels in the range $[0, L-1]$, the nth moment of r about its mean m is defined as

$$\mu_n = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

where m is given by

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

The mean is a measure of avg intensity and the variance (or standard deviation, σ) given by

$$\sigma^2 = \mu_2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$ is a measure of image contrast

There are two uses of the mean and variance of enhancement purposes. The global mean and variance are computed over an entire image and are useful for gross adjustments in overall intensity and contrast.

→ A more powerful use of these parameters is in local enhancement, where the local mean and variance are used as the basis of making changes that depend on image characteristics in a neighbourhood about each pixel in an image

—: Assignment - II :-

1(a) Explain the method of converting colours from RGB to HSI.

Ans Given an image in RGB color format, the H component of each RGB pixel is obtained using the equation

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360-\theta & \text{if } B > G \end{cases} \longrightarrow ①$$

with

$$\theta = \cos^{-1}\left\{ \frac{\frac{1}{2}[(R-G)+(R-B)]}{[(R-G)^2+(R-B)(G-B)]^{1/2}} \right\} \longrightarrow ②$$

The saturation component is given by

$$S = 1 - \frac{3}{R+G+B}\left(\min(R,G,B)\right) \longrightarrow ③$$

Finally, the intensity component is obtained from the eq^n

$$I = \frac{1}{3}(R+G+B) \longrightarrow ④$$

1b) Illustrate the method of converting color from HSI to RGB.

A: Given values of HSI in the interval [0, 1], we now want to find the corresponding RGB values in the same range. The applicable equations depend on the values of H. There are three sectors of interest corresponding to the 120° intervals in the seperation of primaries.

We begin by multiplying H by 360°, which returns the hue to its original range of [0°, 360°)

RG sector ($0° \le H \le 120°$): when H is in the sector, the RGB components are given by the eq^n

$$B = I(1-S)$$

$$R = I\left[1 + \frac{S\cos H}{\cos(60° - H)}\right]$$

$$G = 3I - (R+B)$$

GB sector ($120° \le H \le 240°$): If the given value of H is in this sector, we first subtract 120° from it:

$$H = H - 120^\circ$$

Then, the RGB component are

$$R = I(1-S)$$

$$G = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$$

and

$$B = 3I - (R+G)$$

BR sector ($240^\circ \le H \le 360^\circ$): finally, if H is in the range, we subtract 240° from it:

$$H = H - 240^\circ$$

Then, the RGB components are

$$G = I(1-S)$$

$$B = I\left[1 + \frac{S\cos H}{\cos(60^\circ - H)}\right]$$

and

$$R = 3I - (G+B)$$

2a) Draw the degradation / restoration model in image processing and describe the each part presented on it.

As The fig shows, the degradation process is modeled as a degradation function that, together with an additive noise term, operates on an input image $f(x,y)$ to produce a degraded image $g(x,y)$

* The given $g(x,y)$, some knowledge about the degradation function H, and some knowledge about the additive noise term $\eta(x,y)$. the objective of restoration is to obtain an estimate $\hat{f}(x,y)$ of the original image.

* The estimate should be as close as possible to the original input image and, in general, the more we know about H and η, the closer $\hat{f}(x,y)$ will be to $f(x,y)$. The degraded image is given in the spatial domain by

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

where $h(x,y)$ is the spatial representation of the degradation function and, the symbol $*$ indicates convolution.

* Convolution in the spatial domain is equal to multiplication in the freq. domain, hence

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

where the terms in capital letters are the fourier transforms of the corresponding terms in above eq^n

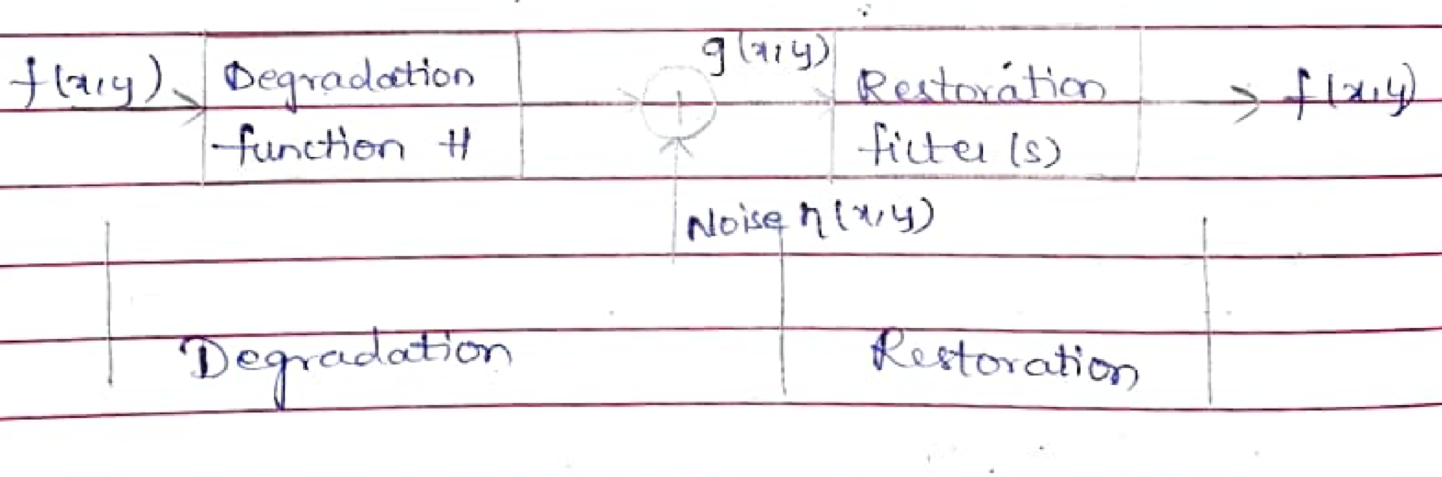


fig: model of the image degradation/restoration process

2b) Explain the rayleigh noise and gamma noise with proper PDF expression.

A: The PDF of rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

* The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b/4}$$

$$\sigma^2 = b(4-\pi)/4$$

* The PDF of erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

where the parameter are such that $a > 0$, b is a positive integer, and "!" indicates factorial. The mean and variance of this density are given by

$$\mu = b/a$$
$$\sigma^2 = b/a^2$$

3a) Explain the method of the least mean square filters for image restoration.

As The inverse filtering approach has poor performance. The wiener filtering approach uses the degradation function and statistical characteristics of noise into the restoration process.

The objective is to find an estimate of the uncorrupted image, such that the mean square error b/w them is minimized

* The error measure is given by

$$e^2 = E\{[f(x) - \hat{f}(x)]^2\}$$

where $E\{\cdot\}$ is the expected value of the argument

* we assume that the noise and the image are uncorrelated one or the order other has zero mean. The gray levels in the estimate are a linear functions

of the levels in the degraded image.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)}\right] G(u,v)$$

where $H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)\, H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2$ = power spectrum of noise

$S_f(u,v) = |F(u,v)|^2$ = power spectrum of the underated image

→ The power spectrum of the undegraded image is rarely known. An approach used frequently when these quantities are not known or cannot be estimated then the expression used is

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + k} \right] G(u,v)$$

where k is a specified constant

3b) Discuss the method of constrained least square restoration for image Restoration.

A) The wiener filter has a disadvantage that we need to know the power spectra of the undegraded image and noise. the constrained least square filtering requires only the

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

knowledge of only the mean and variance of the noise. these parameters usually can be calculated from a given degraded image this is the advantage with this method. this method produces a optimal result. this method requires the optimal criteria which is important we express the

$$g = Hf + \eta$$

* The optimally criteria for restoration is based on the measure of smoothness, such as the second derivative of an image (Laplacian). The minimum of a criterion function C defined as

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} | \nabla^2 f(x,y) |^2$$

* Subject to the constraint

$$\| g - H\hat{f} \|^2 = \| \eta \|^2$$

* The frequency domain solution to this optimization problem is given by

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

* where γ is a parameter that must be adjusted so that the constraint is satisfied.

* $P(u,v)$ is the fourier transform of Laplacian operator

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

4a) Explain the spatial and temporary redundancy with suitable example.

A) Spatial Redundancy:

* Spatial Redundancy represents the statistical correlation b/w neighbouring pixels in an image.

* Spatial redundancy implies that there is a relationship b/w neighbouring pixels in an image.

* It is not necessary to represent each pixel in an image independently.

* Instead a pixel can be predicted from its neighbouring pixels.

* Removing spatial redundancy through prediction is the basic principles of different coding which is widely employed in image & video compression.

Temporal Redundancy:-

* Temporal redundancy is the statistical correlation b/w pixels from successive frames in a video sequence. The temporal redundancy is also called interframe redundancy

* Motion compensated predictive coding is employed to reduce temporal redundancy. Removing a large amount of temporal redundancy leads to efficient video compression

* Image contain information which are ignored by the human visual system.

* Most 2-D intensity arrays contain information that is ignored by human visual system and/or extraneous to the intended use of the image

* It is redundant in the sense that is not used.

4b) Evaluate average length, compression and coding redundancy if the computer generated image has the intensity distribution shown in table. If a natural 8-bit code is used to represent its 4 possible intensities

Intensities (r_k)	$r_{87} = 87$	$r_{128} = 128$	$r_{186} = 186$	$r_{255} = 255$	r_k for $k \neq 87, 128, 186, 255$
probabilities (P_k)	0·25	0·47	0·25	0·03	0

As Entropy is defined as

$$\tilde{H} = -\sum_{k=0}^{L-1} P_k \log_2 P_k$$

$$= -\left[0.25 \log_2 0.25 + 0.47 \log_2 0.47 + 0.25 \log_2 0.25 + 0.03 \log_2 0.03\right]$$

$$= -\left[0.25(-2) + 0.47(-1.09) + 0.25(-2) + 0.03(-5.06)\right]$$

$= 1.6614$ bits / pixels

Note:-

$$\log_2 0.25 = \frac{\log_{10} 0.25}{\log_{10} 2} = \frac{-0.6020}{0.3010} = -2$$

$$\log_2 0.47 = \frac{\log_{10} 0.47}{\log_{10} 2} = \frac{-0.327}{0.3010} = -1.09$$

5a) Explain the procedure for arithmetic coding with suitable example.

As

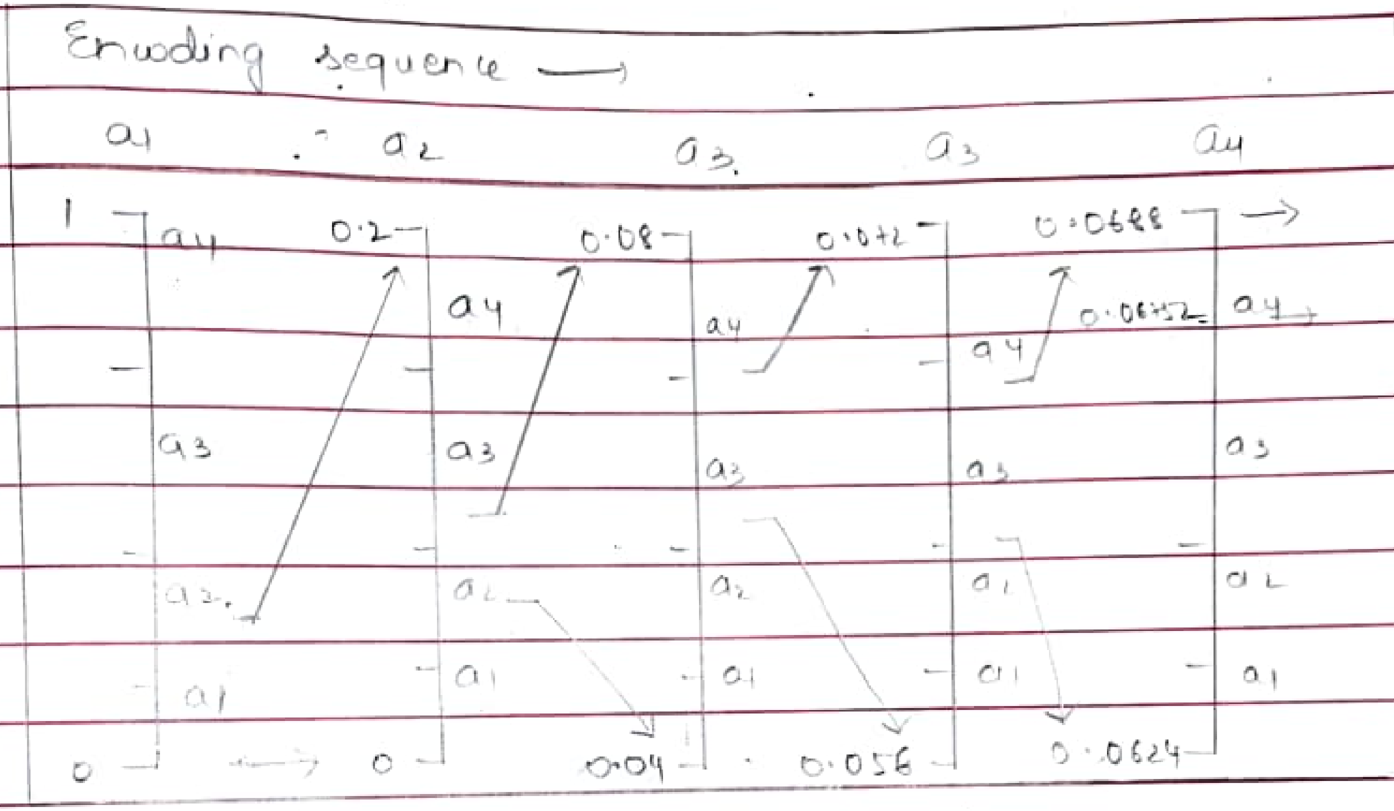


fig: Arithmetic coding procedure

* The above fig illustrate the basic arithmatic coding process. Here, a five-symbol sequence or message, $a_1 a_2 a_3 a_4$, from a four-symbol source is coded. At the start of the coding process, the message is assumed to occupy the entire half-open interval [0, 1). As table, shows, this interval is intially subdivided into four regions based on the probabilities of each source symbol. Symbol a_1, for example, is associated with subinterval [0, 0.2). Because it is the

first symbol of the message being coded, the message interval is initially narrowed to [0, 0.2). Thus in fig [0,0.2) is expanded to the full height of the figure and its end points labeled by the values of the narrowed range. The narrowed range is then subdivided in accordance with the original source symbol probabilities and the process continues with the next message symbol.

Source Symbol	Probability	Initial Subinterval
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)

Table: Arithmetic coding example

In this manner, symbol a_2 narrows the subinterval to [0.04, 0.08), a_3 further narrows it to [0.056, 0.072) and so on. The final message symbol, which must be reserved as a special end-of message indicator, narrows the range to [0.06752, 0.0688). Of course, any number within this subinterval - for example, 0.068 - can be used to represent the message.

In this arithmetically coded message of fig, three decimal digits per source symbol and compares favourably with the entropy of the source, which is 0.58 decimal digits or 10-ary units/symbol. As the length of the sequence being coded increases, the resulting arithmetic code approaches the bound established by the noiseless coding theorem

5b) Summarize the procedure of bit plane coding with the suitable example.

A: An effective technique for reducing an image's inter pixel redundancies is to process the image's bit planes individually. The technique, called bit-plane coding, is based on the concept of decomposing a multilevel (monochrome or color) image into a series of binary images and compressing each binary image via one of several well-known binary compression method.

Bit-plane decomposition:-

The gray levels of an m-bit gray-scale image can be represented in the form of the base 2 polynomial

$$a_{m-1}2^{m-1} + a_{m-2}2^{m-2} + \dots + a_1 2^1 + a_0 2^0$$

Based on this property, a simple method of decomposing the image into a collection of binary images is to seperate the m coefficient of the polynomial into m-1 bit planes. The zeroth-order bit plane is generated by collecting the a_0 bits of each pixels, while (m-1)st order bit plane contains the a_{m-1}, bits or coefficients.

An alternative decomposition approach (which reduces the effect of small gray-level variations) is to first represent the image by an m-bit gray code. The m-bit gray code $g_{m-1} \dots g_2 g_1 g_0$ that corresponds to the polynomial in eqn ⑨, above can be computed from

$$g_i = a_i \oplus a_{i+1} \qquad 0 \leq i \leq m-2$$

$$g_{m-1} = a_{m-1}$$

The accomplished by placing the lossy encoder's predictor within a feedback loop, where its input, denoted f'_n, is generated as a function of past predictions and the corresponding quantized errors. that is

$$\dot{f}_n = \dot{e}_n + \hat{f}_n$$

This closed loop configuation prevents error buildup at the decoder's output. that the o/p of the decoder also is given by the above eqn.