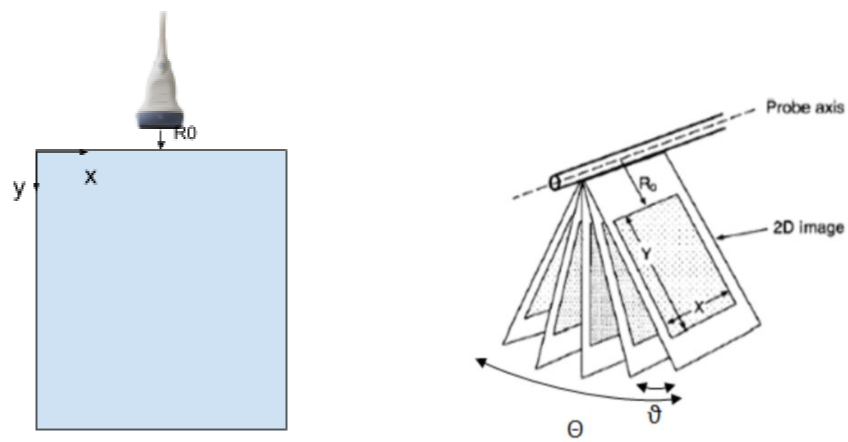
Imagine you are supposed to build a virtual 3d ultrasound machine. The ultrasound machine has a 2d probe which can be rotated iso centrically around probe center to generate fan based 3d ultrasound images. Take a look at fig. 1. to understand the single ultrasound frame geometry, x, y are horizontal and vertical image directions while R0 is the distance from the isocenter to the image frame. The probe of your virtual machine generates images of physical size Xus, Yus while the physical spacing of the images generated are sx, sy in same units. R0 is in same units. **For simplicity assume R0 = 0 for your calculations.**

Now, rotating the probe around iso center around horizontal image (x) direction generates a 2d image frames which generates fan based image data set, i.e. 2d frames acquired in 3d space.



R_0	Distance from probe axis to the image plane
X	Width of the image frame
Y	Height of the image frame
Θ	FOV
ϑ	Angle per Frame

Fig 1

Imagine, the probe is located at (0,0,0), i.e. origin of the usual 3d coordinate system, (x,y,z) and your probe is able to generate images as depicted in the fig. 2 where Xus, Yus is the ultrasound local coordinate system. **R0 = 0 in the fan based geometry below.**

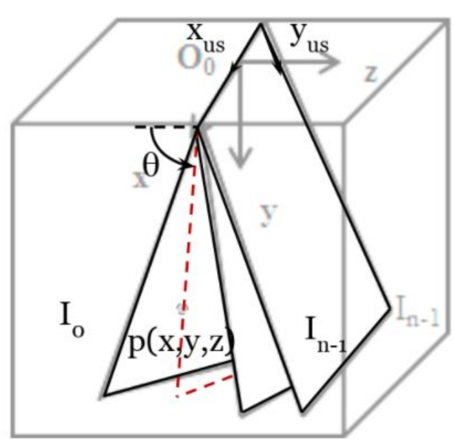


Fig. 2

1. Imagine you have a sphere of located at (0, Yp, 0) in the 3d space with a radius of R, find the condition for R to ensure you can fully image the sphere using your fan based ultrasound system, i.e. this depends on Xus, Yus, Yp, FOV (theta).
2. Now we are going to generate images using our virtual us system. Imagine your system is capable of generating images with physical size Xus, Yus = 10cm and spacing of sx, sy = 0.1cm. Imagine FOV = 120 degrees and the angle per frame = 2 degrees. The sphere is 2 cm in radius and located at (0, 6, 0) cm. Write a program to generate images using python/matlab for one rotation in FOV which is you will generate 60 images, and make the pixel values 255 in the regions where sphere is visible and 0 otherwise.

   Now let's do something interesting!

3. Imagine you are given the image that set you just generated with their corresponding angulation in the FOV which ofc you know from the way you generated images from 0-120 degrees with 2 degree steps. Let's generate the 3d ultrasound volume which is imaged by those frames, i.e. generating the virtual sphere in 3d. Use the following method to do so.
   a. First create a 3d matrix/array to contain the image frames in 3d, use the spacing of (0.1,0.1,0.1) in x, y, z. Now you have a 3d image.
   b. For each voxel (i,j,k) find the corresponding image frame, frames (if doesn't directly map to a frame).
   c. Do a bilinear distance weighted interpolation to find the voxel value of the voxel you are evaluating. Repeating this for all voxels you might be able to get the approximate sphere in 3d.