1. Quantum Lambda Calculus (QLC - Selinger & Valiron)

Terminología: Control clásico, datos cuánticos (Heap). **Caso:** $b_1 = 0, b_2 = 1$. Estado inicial heap: $(x, y) \mapsto |\beta_{00}\rangle$.

$$
\begin{aligned}
& \texttt{SDC}\ 0\ 1\ (x, y) \\
\xrightarrow{\beta}\ & \textbf{let}\ x' = (\textbf{if}\ 1\ \textbf{then}\ X(x)\ \textbf{else}\ x)\ \textbf{in} \ldots \\
\xrightarrow{\text{if}}\ & \textbf{let}\ x' = X(x)\ \textbf{in} \ldots \\
& \text{Heap: } (x, y) \mapsto \tfrac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \text{ [cite: 630, 658]} \\
\xrightarrow{\beta}\ & \textbf{let}\ x'' = (\textbf{if}\ 0\ \textbf{then}\ Z(x')\ \textbf{else}\ x')\ \textbf{in} \ldots \\
\xrightarrow{\text{if}}\ & \textbf{let}\ x'' = x'\ \textbf{in} \ldots \\
\xrightarrow{\text{CNOT}}\ & \textbf{let}\ (x''', y') = \text{CNOT}(x'', y) \\
& \text{Heap: } (x, y) \mapsto \tfrac{1}{\sqrt{2}}(|11\rangle + |01\rangle) \text{ [cite: 652]} \\
\xrightarrow{H}\ & \textbf{let}\ x'''' = H(x''') \\
& \text{Heap: } (x, y) \mapsto |0\rangle \otimes |1\rangle \text{ [cite: 633]} \\
\xrightarrow{\text{meas}}\ & (\text{meas}(x''''), \text{meas}(y')) \Longrightarrow (0, 1) \text{ [cite: 647]}
\end{aligned}
$$

2. Lambda-S (Díaz-Caro & Malherbe)

Terminología: Tipos $\mathbb{B}$ y $S(A)$, condicional $?_{r \cdot s}$[cite: 78, 95]. **Caso:** $b_1 = |0\rangle, b_2 = |1\rangle$. Estado inicial: $\psi = \beta_{00} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$[cite: 39, 97].

$$
\begin{aligned}
& \texttt{SDC}\ |0\rangle\ |1\rangle\ \psi \\
\xrightarrow{\beta_b}\ & (H \otimes I)\ \text{CNOT}\ (Z^{|0\rangle}(X^{|1\rangle}(\tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle)))) \text{ [cite: 135]} \\
\xrightarrow{lin}\ & \ldots \tfrac{1}{\sqrt{2}}((X\,|0\rangle) \times |0\rangle + (X\,|1\rangle) \times |1\rangle) \text{ [cite: 137, 139]} \\
\xrightarrow{\beta_b}\ & \tfrac{1}{\sqrt{2}}((|0\rangle ?_{|0\rangle \cdot |1\rangle}) \times |0\rangle + (|1\rangle ?_{|0\rangle \cdot |1\rangle}) \times |1\rangle) \text{ [cite: 150, 151]} \\
\xrightarrow{\text{if}}\ & \tfrac{1}{\sqrt{2}}(|1\rangle \times |0\rangle + |0\rangle \times |1\rangle) = \tfrac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \\
\xrightarrow{Z}\ & \ldots \text{ (Identidad por } b_1 = |0\rangle) \\
\xrightarrow{\text{CNOT}}\ & \tfrac{1}{\sqrt{2}}(\text{CNOT}\,|10\rangle + \text{CNOT}\,|01\rangle) \text{ [cite: 230]} \\
\xrightarrow{rew}\ & \tfrac{1}{\sqrt{2}}(|11\rangle + |01\rangle) \\
\xrightarrow{fact}\ & (\tfrac{1}{\sqrt{2}}(|1\rangle + |0\rangle)) \times |1\rangle = |+\rangle \times |1\rangle \text{ [cite: 170, 179]} \\
\xrightarrow{H \otimes I}\ & |0\rangle \times |1\rangle = |01\rangle \text{ [cite: 276]}
\end{aligned}
$$

3. $L^{\mathbb{C}}$ (Vectorial / Lineal)

Terminología: Linealidad algebraica, Application Group[cite: 671, 678]. **Caso:** $b_1 = |0\rangle\,, b_2 = |1\rangle$. Estado: $\psi = \frac{1}{\sqrt{2}}.\,|00\rangle + \frac{1}{\sqrt{2}}.\,|11\rangle$[cite: 666, 670].

$$
\begin{aligned}
& \texttt{SDC}\ |0\rangle\ |1\rangle\ \left(\tfrac{1}{\sqrt{2}}.\,|00\rangle + \tfrac{1}{\sqrt{2}}.\,|11\rangle\right) \\
\xrightarrow{App}\ & X^{|1\rangle}\left(\tfrac{1}{\sqrt{2}}.\,|00\rangle\right) + X^{|1\rangle}\left(\tfrac{1}{\sqrt{2}}.\,|11\rangle\right) \text{ [cite: 679]} \\
\xrightarrow{Scalar}\ & \tfrac{1}{\sqrt{2}}.(X^{|1\rangle}\,|00\rangle) + \tfrac{1}{\sqrt{2}}.(X^{|1\rangle}\,|11\rangle) \text{ [cite: 684]} \\
\xrightarrow{\beta}\ & \tfrac{1}{\sqrt{2}}.\,|10\rangle + \tfrac{1}{\sqrt{2}}.\,|01\rangle \\
\xrightarrow{\text{CNOT}}\ & \tfrac{1}{\sqrt{2}}.\text{CNOT}\,|10\rangle + \tfrac{1}{\sqrt{2}}.\text{CNOT}\,|01\rangle\ \text{ [cite: 679]} \\
=\ & \tfrac{1}{\sqrt{2}}.\,|11\rangle + \tfrac{1}{\sqrt{2}}.\,|01\rangle \\
\xrightarrow{Fact}\ & \tfrac{1}{\sqrt{2}}.(|11\rangle + |01\rangle) \text{ [cite: 677, 681]} \\
\xrightarrow{H}\ & \ldots|0\rangle \otimes |1\rangle = |01\rangle\ \text{ [cite: 693]}
\end{aligned}
$$