Properties of DTFT :—

① Central Ordinate property:-

$x[n] = x[e^{jw}]$

then,

$$\sum_{n=-\alpha}^{\alpha} x[n] = x[e^{j0}]$$

Proof:-

$$x[e^{jw}] = \sum_{n=-\alpha}^{\alpha} x[n]e^{-jwn}.$$

$$x[e^{j0}] = \sum_{n=-\alpha}^{\alpha} x[n]e^{-j(0)n}.$$

$$= \sum_{n=-\alpha}^{\alpha} x[n]$$

Ilys,

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} x[e^{jw}]dw = x[0]$$

$$\Rightarrow \int_{-\pi}^{\pi} x(e^{jw})dw = 2\pi x(0)$$

② Linear Property :-

$x_1[n] \longrightarrow x_1[e^{jw}]$

$x_2[n] \longrightarrow x_2[e^{jw}]$

Then,

$$ax_1[n] + bx_2[n] \longrightarrow ax_1[e^{jw}] + bx_2[e^{jw}]$$

$$= y[n] \qquad\qquad = Y[e^{jw}]$$

$$\rightarrow Y[e^{jw}] = \sum_{n=-\alpha}^{\alpha} y[n]e^{-jwn}.$$

$$= \sum_{n=-\alpha}^{\alpha} [ax_1(n] + bx_2[n]\} e^{-jwn}$$

$$= a\sum_{n=-\alpha}^{\alpha} x_1[n]e^{-jwn} + b\sum_{n=-\alpha}^{\alpha} x_2[n]e^{-jwn}$$

3. Inverse Property:-

$$x[n] \longrightarrow X[e^{j\omega}]$$

$$y(n) = x[-n] \longrightarrow X(e^{-j\omega}] = Y[e^{j\omega}]$$

Proof:

$$Y[e^{j\omega}] = \sum_{n=-\infty}^{\infty} x[-n]\, e^{-j\omega n}$$

$-n = p$:

$$= \sum_{p=-\infty}^{\infty} x(p]\, e^{-j\omega(-p]}$$

$$= \sum_{p=-\infty}^{\infty} x(p]\, e^{-jp[-\omega]}$$

4. Time delay Property:-

$$x[n] \longleftrightarrow X[e^{j\omega}]$$

then

$$x[n-n_0] \rightarrow e^{-j\omega n_0} X[e^{j\omega}]$$

$$\downarrow \qquad\qquad \downarrow$$

$$y(n) \qquad\qquad Y(e^{j\omega})$$

Proof:

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n]\, e^{-j\omega n}$$

$n - n_0 = p$

$$= \sum_{n=-\infty}^{\infty} x(n-n_0]\, e^{-j\omega(n)}$$

$$= \sum_{p=-\infty}^{\infty} x(p)\, e^{-j\omega(p+n_0)}$$

$$= \sum_{p=-\infty}^{\infty} x[p]\, e^{-j\omega p} \cdot e^{-j\omega n_0}$$

$$= e^{-j\omega n_0} \sum_{p=-\infty}^{\infty} x(p)\, e^{-j\omega p}$$

$$= X^{-j\omega n_0} X(e^{j\omega})$$

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Similarly,

$$x(n+n_0) \longrightarrow e^{j\omega n_0} X(e^{j\omega})$$

(5) frequency delay:-

$$x[n] \longrightarrow X[e^{j\omega}]$$

$$e^{j\omega_0 n} x(n) \Rightarrow X(e^{j(\omega-\omega_0)})$$

Proof:- $$Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} e^{j\omega_0 n} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{\alpha} x(n) e^{-j(\omega-\omega_0)n}$$

Similarly,

$$e^{-j\omega_0 n} x(n) \longrightarrow X(e^{j(\omega+\omega_0)})$$

(6) multiplication property:-

$$x_1[n] \longrightarrow X_1[e^{j\omega}]$$

$$x_2[n] \longrightarrow X_2[e^{j\omega}]$$

then,

$$y(n) = x_1(n) . x_2(n) \longrightarrow \frac{1}{2\pi} \underbrace{\int_{-\pi}^{\pi} X_1(e^{j\lambda}) X_2(e^{j(n-\lambda)}) d\lambda}_{Y(e^{j\omega})}$$

Sol:- $$Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} x_1(n) x_2(n) e^{-j\omega n}$$

$$x_1(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\omega}) e^{j\omega n} d\omega$$

$$x_1(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\lambda}) e^{j\lambda n} . d\lambda$$

$$Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\lambda}) e^{j\lambda n} d\lambda \right) x_2(n) e^{-j\omega n}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ X_1(e^{-j\lambda}) \sum_{n=-\alpha}^{\alpha} x_2(n) e^{-j(\omega-\lambda)n} \right\} d\lambda$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\lambda})\, X_2(e^{j(\omega-\lambda)})\, d\lambda$$

(7) Convolution property :-

$$x_1(n) \rightarrow X_1(e^{j\omega})$$

$$x_2(n) \rightarrow X_2(e^{j\omega})$$

then

$$y(n) = x_1(n) * x_2(n) \rightarrow X_1(e^{j\omega}) . X_2(e^{j\omega})$$

$$Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} \{ x_1(n) * x_2(n) (e^{-j\omega n}) \}$$

$$= \sum_{n=-\alpha}^{\alpha} \sum_{m=-\alpha}^{\alpha} x_1(m)\, x_2(n-m)\, e^{-j\omega m}\, e^{-j\omega n} .\, e^{j\omega m}$$

$$= \sum_{n=-\alpha}^{\alpha} x_1(m) e^{-j\omega m} \sum_{n=-\alpha}^{\alpha} x_2(n-m) e^{-j(n-m)}$$

$$\underline{n-m=p}$$

$$= X_1(e^{j\omega}) . X_2(e^{j\omega})$$

(8) Conjugate property :-

$$x(n) \longrightarrow X(e^{j\omega})$$

$$x^*(n) \longrightarrow X_1^*(e^{-j\omega})$$

Proof :-
$$Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} x^*(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{\alpha} (x(n)\, e^{j\omega n})^*$$

$$= \left[\sum_{n=-\alpha}^{\alpha} x(n) .\, e^{-j(-\omega)n} \right]^*$$

$$= X^* (e^{-j\omega})$$

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9.

$x(n) \longrightarrow X(e^{j\omega})$

$n x(n) \rightarrow j\frac{d}{d\omega}X(e^{j\omega})$

Proof: $Y(e^{j\omega}) = \sum_{n=-\alpha}^{\alpha} n \cdot x(n)e^{-j\omega n}$

$\left(\frac{d e^{-j\omega n}}{d\omega} = (-jn)e^{-j\omega n} \right)$

$= \sum_{n=-\alpha}^{\alpha} x(n)\left(\frac{j}{j}\right)e^{-j\omega n}$

$= j\sum_{x=-\alpha}^{\alpha} x(n)(-jn)e^{-j\omega n}$

$= j\sum_{n=-\alpha}^{\alpha} x(n)\frac{d}{d\omega}(e^{-j\omega n})$

(10) Difference Property:-

$\frac{d}{dt}x(t) = \frac{x(t_1)-x(t_2)}{t_1-t_2}$

$\downarrow$

$x(n)-x(n-1)$

$x(n) \longrightarrow X(e^{j\omega})$

then

$x(n)-x(n-m) \longrightarrow (1-e^{-j\omega m})X(e^{j\omega})$

(11) Accumulation Property:-

$\sum_{k=-\alpha}^{\alpha} x(k) = \frac{1}{1-e^{-j\omega n}} + \pi X(e^{j\omega})\sum_{m=-\alpha}^{\alpha}\delta(\omega-2\pi m)$

$\sum_{k=-\alpha}^{\alpha} x(k) = \frac{X(e^{j\omega})}{1-e^{j\omega n}} + \underbrace{\pi X(e^{j\omega})\sum_{m=-\alpha}^{\alpha}\delta(\omega-2\pi m)}_{\text{DC value}}$

$Y(n) = \sum_{n=0}^{0}\delta(k)$