UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UCSC

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

First Year Examination – Semester II – 2017

SCS 1111 – Mathematical Methods II

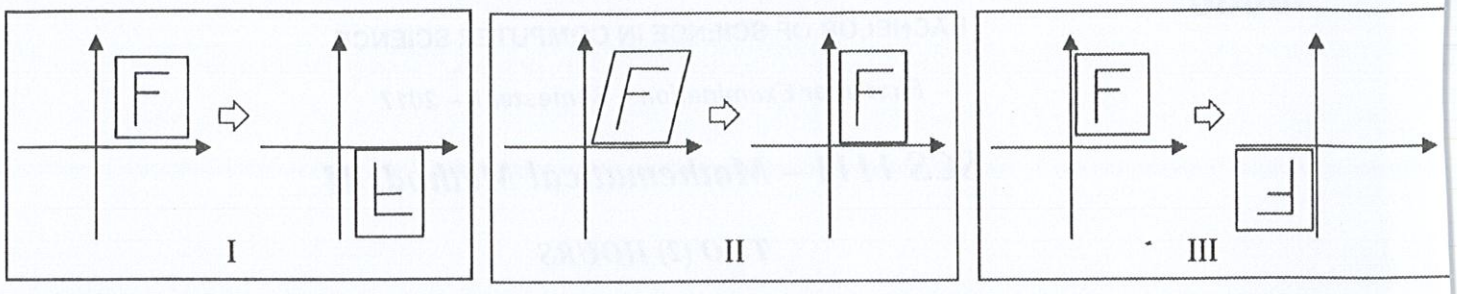
TWO (2) HOURS

Important Instructions to candidates:

1. The medium of instruction and questions is **English**.
2. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
3. Note that questions appear on both sides of the paper. If a page is not printed, please inform the supervisor immediately.
4. Write your index number on each and every page of the answer paper.
5. This paper has **4** questions and **05** pages.
6. Answer **ALL** questions. All questions carry equal marks (**25** marks).
7. Any electronic device capable of storing and retrieving text including electronic dictionaries and mobile phones are **not allowed**.
8. **Non-Programmable** calculators are **allowed**.

1. Mark the correct response with a pen on the answer sheet.

For problems (1) to (3), refer the following geometric transformations given as I, II and III, which represent the matrices A_1, A_2 and A_3 respectively.



(1) Which of the following could be A_1?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ (e) None of the above

(2) Which of the following could be A_2?

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ (e) None of the above

(3) Which of the following could be A_1A_3?

(a) $\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (e) None of the above

(4) A and B are two symmetric matrices of same order. Which of the following is necessarily symmetric?

I. AB III. ABA III. A^2BA^2

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II and III

(5) Which of the following determinant is non-zero, if $a \in \mathbb{R}$?

(a) $\begin{vmatrix} a & a^2 \\ 1-a & a-a^2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 1+a \\ 1-a & 1-a^2 \end{vmatrix}$ (c) $\begin{vmatrix} 0 & a \\ 0 & 1+a \end{vmatrix}$ (d) $\begin{vmatrix} a & -1 \\ 1 & a \end{vmatrix}$ (e) None of the above

(6) If $A = \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$, then what is the value of $|(A-2I)(A-3I)|$?

(a) 0 (b) 1 (c) -1 (d) 14 (e) None of the above

(7) Let λ and μ be the eigenvalues of two $n \times n$ matrices A and B respectively. Which of the following is always true?

I. $\lambda + \mu$ is an eigenvalue of $A + B$. II. $\lambda\mu$ is an eigenvalue of AB. III. $\lambda^2\mu$ is an eigenvalue of A^2B.

(a) I only (b) II only (c) II and III only (d) I, II and III only (e) None of I, II and III

(8) The characteristic equations of A and B (of same order) are given by $\lambda^2 - 2\lambda - 4 = 0$ and $\mu^2 - 3 = 0$ respectively. Which of the following expression is necessarily equal to the identity matrix?

(a) $A^2 + B^2 + 2A$ (b) $A^2 - B^2 - 2A$ (c) $A^2 - B^2 + 2A$ (d) $-A^2 + B^2 + 2A$

(e) None of the above

(9) Which of the following is true?

I. $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}^{2017} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$ II. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{2017} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ III. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{2017} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(a) I only (b) II only (c) II and III only (d) I, II and III only (e) None of I, II and III

(10) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, what is e^A ?

(a) $\begin{bmatrix} 0 & e \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (e) None of the above

2. Mark the correct response with a pen on the answer sheet.

(11) What is $\vec{u} - 3\vec{v}$, if $\vec{u} = (1, -1)$ and $\vec{v} = (-1,2)$?

(a) $(4, -1)$ (b) $(-4,5)$ (c) $(4, -7)$ (d) $(2, -7)$ (e) None of the above

(12) What is the distance between the two vectors $(4, -4)$ and $(1, -8)$ in $\mathbb{R}^2$?

(a) 2 (b) 3 (c) 5 (d) 8 (e) None of the above

(13) If X is a non-empty linearly independent subset of $\mathbb{R}$, what is the number of elements in X?

(a) 0 (b) 1 (c) 2 (d) 3 (e) Data is insufficient

(14) For which of the following scalar x, the Eucliden space $\mathbb{R}^2$ is not spanned by $\{(x, 1,), (1, -2)\}$?

(a) $x = 1$ (b) $x = -1$ (c) $x = -0.5$ (d) $x = -0.25$ (e) None of the above

(15) Which of the following sets of vectors span $\mathbb{R}^3$?

I. $\{(1,1,0)(1,2,1),(2,3,1)\}$

II. $\{(1,1,0),(-1,-3,2),(0,1,-1)\}$

III. $\{(1,2,1),(2,1,1),(-2,-4,-2)\}$

(a) I only (b) I and II only (c) I and II only (d) I and III only (e) None of I, II and III

(16) What is the number of subspaces of $\mathbb{R}$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of the above

(17) If (3,4) is a vector in a non-trivial subspace U of $\mathbb{R}^2$, which of the following is another vector in U?

(a) $(1,-2)$ (b) $(2,0)$ (c) $(-6,-8)$ (d) $(-5,-9)$ (e) None of the above

(18) Which of the following defines a basis for $\mathbb{R}^3$?

I. $\{(1,0,0),(0,1,0),(0,0,1)\}$ II. $\{(1,1,-1),(0,1,-1),(0,1,1)\}$ III. $\{(1,0,1),(1,0,-1),(1,0,0)\}$

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II and III

(19) Let U_1 and U_2 be subspaces of a vector space V so that U_1 has more elements than U_2 and $U_1 \cup U_2$ is a subspace of V. Which of the following set is necessarily empty?

I. $U_1 \cap U_2$ II. $U_1 \cap {U_2}^C$ III. $U_2 \cap {U_1}^C$

(a) I only (b) II only (c) III only (d) I, II and III (e) None of I, II and III

(20) Which of the following mappings is a linear transformation from $\mathbb{R}^2$ to itself?

I. $T_1(c_1,c_2) = (2c_2,\ c_1 + c_2)$ II. $T_2(c_1,c_2) = ({c_2}^2, {c_1}^2)$ III. $T_3(c_1,c_2) = (c_1 + c_2, 0)$

(a) T_1 only (b) T_2 only (c) T_1 and T_2 only (d) T_1 and T_3 only (e) T_1, T_2 and T_3

3. (a) Justify or falsify:

If A, B and C are three arbitrary square matrices of same order satisfying, $AB = AC$, then $B = C$.

(b) Diagonalise the matrix $\begin{bmatrix} -1 & 4 \\ 0 & 1 \end{bmatrix}$.

(c) Using row/column operations, show that, $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$.

4. (a) Solve the system

$$x + y + z = 5$$
$$2x + 3y + 5z = 8$$
$$4x + 5z = 2$$

by using Gauss-Jordan elimination.

(b) Justify or falsify:

$\left\{\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} : a, b, c \in \mathbb{R} \text{ and } a + b = c\right\}$ forms a subspace of $\mathbb{R}^{2\times 2}$ (the vector space of all 2×2 real matrices).

(c) Write down the matrix representation of the linear transformation $T(x, y, z) = (x + z, y - z)$ from $\mathbb{R}^3$ *to* $\mathbb{R}^2$, with respect to the natural bases.

End of the question paper