UNIVERSITY OF COLOMBO, SRI LANKA

UCSC

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

BACHELOR OF COMPUTER SCIENCE
Academic Year 2006/2007 - First Year Examination – Semester 2

SCS1001 - Mathematics for Computing I

(TWO HOURS)

Answer four questions only

No of Pages = 5 | No of Questions = 5

Notations:

Z – set of integers | **N – set of positive integers**
R – set of real numbers | **∅ - (null) empty set**
S – Universal set | **R^+- set of non-negative real numbers**

1.

(a). Using truth tables, show that

(i). $p \Rightarrow q \equiv \sim q \Rightarrow \sim p$

(ii). $(\sim p \Rightarrow \sim q) \wedge (\sim q \Rightarrow \sim p) \equiv (p \vee \sim q) \wedge (q \vee \sim p)$

(iii). $\sim p \wedge (q \vee r) \equiv (\sim p \wedge q) \vee (\sim p \wedge r)$

(6 marks)

(b). Let R be the set of real numbers and, p(x) and q(x) are two predicates of variable x defined by $x < 2$ and $x \geq 2$ respectively where $x \in R$. Check the truth values of the following prepositions. Justify your answers.

(i). $\exists x\, (p(x) \vee q(x))$
(ii). $(\exists x\, p(x)) \vee (\exists x\, q(x))$
(iii). $\exists x\, (p(x) \wedge q(x))$
(iv). $(\exists x\, p(x)) \wedge (\exists x\, q(x))$
(v). $\forall x\, (p(x) \vee q(x))$
(vi). $(\forall x\, p(x)) \vee (\forall x\, q(x))$
(vii). $\forall x\, (p(x) \wedge q(x))$
(viii). $(\forall x\, p(x)) \wedge (\forall x\, q(x))$

(8 marks)

(c). Check the validity of the following argument? Justify your answer.

$$p,\ \sim(p \wedge q),\ r \Rightarrow p \vdash \sim r$$

(6 marks)

(d). Prove that the following set of statements is inconsistent.

$$p,\ \sim(p \wedge q),\ r \Rightarrow p,\ r$$

(5 marks)

2.

(a). Let A and B be any two sets. Define the following set operations using set notation (NOT Venn diagrams).

(i). $A \cap B$
(ii). $A \cup B$
(iii). $A \subseteq B$
(iv). A^c

(4 marks)

(b). Suppose A, B and C are three non-empty sets. By using algebraic method, prove that

(i). $A \cap B \subseteq A \cup B$

(ii). $(A \subseteq B \wedge B \subseteq C) \Rightarrow C^c \subseteq A^c$

(iii). $(A \subset B \wedge B \subset C) \Rightarrow C^c \subset A^c$

(21 marks)

3.

(a). When do you say that the relation ρ is an equivalence relation?

(3 marks)

(b). Let ρ be an equivalence relation and $x \in D(\rho)$. Define the equivalence class of x .

(2 marks)

(c). If ρ is an equivalence relation, show that

(i). $D(\rho) = R(\rho)$
(ii). $[x]_\rho = [y]_\rho \Rightarrow (x,y) \in \rho$
(iii). $(x,y) \in \rho \Rightarrow [x]_\rho = [y]_\rho$
(iv). $(x,y) \notin \rho \Rightarrow [x]_\rho \cap [y]_\rho = \Phi$

(20 marks)

4.

(a). Suppose f is a function and $D(f)$ is it's domain. Define the range of f.

(3 marks)

(b). When the function f is said to be one to one.

(5 marks)

(c). Let f be a function defined on $A=\{1,2,3\}$. Write down all possible functions f such that

(i). f is A into A
(ii). f is A onto A

(7 marks)

(d). Let A, B and C be three non-empty sets and, f and g be two functions. Suppose f maps A into B and g maps B into C. Show that the function, $g \circ f$ maps A into C.

(10 marks)

5.

(a). Let $A = \frac{1}{9}\begin{pmatrix} 2 & -2 & 1 \\ -1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{pmatrix}$

(i). Compute $[A(B+C)]^T$

(ii). Compute $[(B+C)A]^T$

(iii). Find $(B+C)^{-1}$

(10 marks)

(b). Let $A = \begin{pmatrix} 1 & -4 & 2 & -2 \\ 4 & 7 & -3 & 5 \\ 3 & 0 & 8 & 0 \\ -5 & -1 & 6 & 9 \end{pmatrix}$. If $|A|=2042$, find the determinant of the following matrices.

(i). $\begin{pmatrix} \frac{1}{2} & -2 & 1 & -1 \\ 2 & \frac{7}{2} & \frac{-3}{2} & \frac{5}{2} \\ 3 & 0 & 8 & 0 \\ -5 & -1 & 6 & 9 \end{pmatrix}$ (iii). $\begin{pmatrix} 1 & -4 & 2 & -2 \\ -5 & -1 & 6 & 9 \\ 4 & 7 & -3 & 5 \\ 3 & 0 & 8 & 0 \end{pmatrix}$

(ii). $\begin{pmatrix} 1 & 2 & 3 & 2 \\ 4 & 5 & 0 & 5 \\ 3 & 0 & 8 & 0 \\ -5 & 9 & 0 & 9 \end{pmatrix}$ (iv). $\begin{pmatrix} 5 & 3 & -1 & 3 \\ 4 & 7 & -3 & 5 \\ 3 & 0 & 8 & 0 \\ -5 & -1 & 6 & 9 \end{pmatrix}$

(5 marks)

(c). Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

(i). Show that A is non-singular

(ii). Find adjoint of A

(iii). Hence, solve the following set of linear equations using matrix algebra.

$$\begin{aligned} x - z &= 3 \\ y + z &= -1 \\ x + 2z &= 2 \end{aligned}$$

(10 marks)

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