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UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UCSC

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

First Year Examination – Semester II – 2018

SCS 1110 – Discrete Mathematics (R1)

TWO (2) HOURS

Important Instructions to candidates:

1. The medium of instruction and questions is **English**.
2. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
3. Note that questions appear on both sides of the paper. If a page is not printed, please inform the supervisor immediately.
4. Write your index number on each and every page of the Answer paper.
5. This paper has **04** questions and **04** pages.
6. Answer **ALL** questions. All questions carry equal marks (25 marks).
7. Any electronic device capable of storing and retrieving text including electronic dictionaries and mobile phones are **not allowed**.
8. **Non-Programmable or Programmable** calculators are **not allowed**.

Question 1

(a). Construct a truth table for the compound proposition $(p \land q) \rightarrow (p \rightarrow q)$.

[4 Marks]

(b). State the converse, inverse, and contrapositive, of the conditional statement "If it is a sunny summer day, then I will go to the beach".

[3 Marks]

(c). Let p be the proposition "You drive over 70 km per hour" and q be the proposition "You get a speeding ticket". Write the following propositions using p, q and logical connectives:

i. You will get a speeding ticket if you drive over 70 km per hour.
ii. You drive over 70 km per hour, but you do not get a speeding ticket.
iii. You do not drive over 70 km per hour.
iv. If you do not drive over 70 km per hour, then you will not get a speeding ticket.
v. Whenever you get a speeding ticket, you are driving over 70 km per hour.

[5 Marks]

(d). Let L(x, y) be the predicate "x loves y," where the domain for both x and y consists of all students at the UCSC. Express each of these statements in terms of L(x, y), quantifiers, and logical connectives:

i. Everybody loves Nimal.
ii. Nobody loves Nimal.
iii. Nobody loves everybody.
iv. Everybody loves somebody.
v. There is somebody whom no one loves.

[5 Marks]

(e). Let $C(x)$ and $F(x,y)$ be predicates " x has a computer " and " x and y are friends" respectively, where domain for both x and y consists of all students at the UCSC. Translate the proposition $\forall x(C(x) \lor \exists y(C(y) \land F(x, y)))$ into English.

[4 Marks]

(f). Find the disjunctive normal form of the formula $(\neg p \lor \neg q) \rightarrow (p \leftrightarrow \neg q)$.

[4 Marks]

Question 2

(a). Test the validity of the following argument:

Anyone performs well is either intelligent or a good actor.
If someone is intelligent, then he/she can count from 1 to 10.
Nimal performs well.
Nimal cannot count from 1 to 10.
Therefore, not everyone is both intelligent and a good actor.

[10 Marks]

(b). Prove or disprove the following statements:

i. If n is an integer and n^2 is odd, then n is odd.

ii. If x is a non-zero real number, then $x^2 + \frac{1}{x^2} \geq 2$.

iii. If x is a real number, then x^2 is a positive real number.

[3x5= 15 Marks]

Question 3

(a). Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5\}$, and $C = \{0, 3, 6, 9\}$ be three sets. What are

i. $A \cup B \cup C$;

ii. $A \cap B \cap C$;

iii. $A \cap (B \cup C)$;

iv. $A - B$; and

v. $A \times C$.

[10 Marks]

(b). Let A, B, and C be three sets. Show that

i. $(\overline{A \cap B}) = \bar{A} \cup \bar{B}$; and

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

[9 Marks]

(c). Let f *and* g be two functions from the set of real numbers (($\mathbb{R}$) to itself defined respectively by $f(x) = x^2 + 1$ and $g(x) = x + 2$ for all $x \in \mathbb{R}$. Find the composition of functions $f \circ g$, and $g \circ f$.

[6 Marks]

Question 4

(a). Let $A = \{1, 2, 3, 12, 15\}$, and R be a binary relation defined on A by
$R = \{(a, b) \mid a \text{ divides } b\}$.

i. Show that R is a partial order relation. That is; show that R is reflexive, anti-symmetric, and transitive. **[6 Marks]**

ii. Draw the corresponding Hasse diagram for R. **[4 Marks]**

iii. Find the least element and greatest element if they exist. **[2 Marks]**

(b). What is meant by a relation defined on a set A is an equivalence relation? **[3 Marks]**

i. Let R be a relation defined on the set of integers ($\mathbb{Z}$) by
$x\, R\, y$ if and only if $5 \mid (x - y)$.
Show that R is an equivalence relation. **[6 Marks]**

ii. Find the equivalence classes of 2 and 13 with respect to the relation R. **[4 Marks]**