Multicomponent Oil Preheating System

A project based on heat transfer

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Contributions

This project was made successful due to the efforts of (in no specific order):

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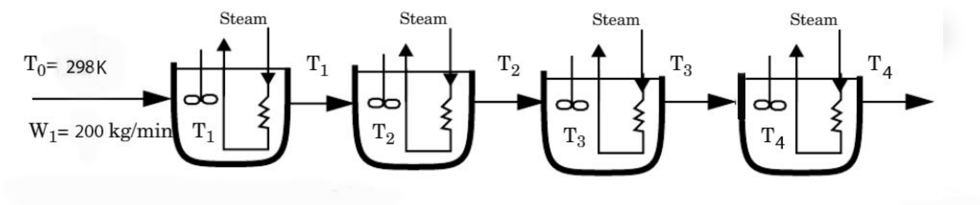
Introduction to Problem Statement

Some changes were made in the original problem statement as suggested by the tutor.



A system contains 4 tanks arranged in series, where each tank contains 2500 kilograms of multicomponent oil solution at a temperature of 25°C. We are preheating the oil solution before it is sent to a distillation column for separation. The heat capacity of the oil is 2.1 kJ/kg. In each tank, saturated steam at a temperature of 250°C condenses within the coils immersed in the oil. The heat from the steam is consumed to heat up the oil. The oil is fed into the first tank at a rate of 200 kg/min and then overflows into the 2^{nd}, 3^{rd} & 4^{th} tanks at the same flow rate. We know the rate at which heat is transferred to the oil from the steam $[Q = UA(T_{\text{steam}} - T)]$ where UA is overall heat coefficient = 18.69 units. The temperature inside the tanks is well mixed, meaning the temperature is uniform throughout each tank. The outlet stream temperature from each tank is equal to the temperature within that particular tank.

The steam increases the temperatures of the tanks and after some time, the system system will reach steady state. **We are interested in calculating the steady state temperatures of the tanks as well as solving for T vs t using numerical method(s) for solving ODEs.**

Solution & Results

We can obtain the differential equations by using the energy balance for the tanks.

For Tank 1, the input is the energy coming with oil with flow rate, W_1 and temperature T_0 & the energy supplied by the steam. Output is the energy taken with oil leaving the tank with flow rate, W_1 and temperature T_1. There is no heat generation or consumption in this system. So applying energy balance for the tank, we get the equation

$\frac{d}{dt}(mC_pT_1) = W_1C_pT_0 + Q - W_1C_pT_1 = UA(T_{\text{steam}} - T_1) - W_1C_p(T_0 - T_1)$

Similarly, for the other 3 tanks we get the equations:

$\frac{d}{dt}(mC_pT_2) = UA(T_{\text{steam}} - T_2) - W_1C_p(T_1 - T_2)$

$\frac{d}{dt}(mC_pT_3) = UA(T_{\text{steam}} - T_3) - W_1C_p(T_2 - T_3)$

$\frac{d}{dt}(mC_pT_4) = UA(T_{\text{steam}} - T_4) - W_1C_p(T_3 - T_4)$

Now, at the steady state, the derivative $\frac{dT_i}{dt}$ will be 0, $\forall\ i$. So we end up getting the following system of linear equations (converted into matrix form)

$$\begin{bmatrix} UA + W_1C_p & 0 & 0 & 0 \\ W_1C_p & -(UA + W_1C_p) & 0 & 0 \\ 0 & W_1C_p & -(UA + W_1C_p) & 0 \\ 0 & 0 & W_1C_p & -(UA + W_1C_p) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} W_1C_pT_o + UAT_{\text{steam}} \\ -UAT_{\text{steam}} \\ -UAT_{\text{steam}} \\ -UAT_{\text{steam}} \end{bmatrix}$$

To solve this system of linear equations we have applied the **Gauss elimination** algorithm as well as **Jacobi's Method** algorithm because these methods are **highly efficient, numerically stable and robust** compared to Cramer's Rule. In the code, we wrote these algorithms as functions which takes the A & B matrix as inputs (for AX = B form) and gave our above matrices as the inputs.

The solutions of this system of linear equations gave us T_1, T_2, T_3 & T_4 which are the steady state temperatures of the tanks. Our first task is accomplished till now.

Our next task is to solve the modelled differential equation system for the tank using numerical method. It will help us seeing the trend of how temperature varies as time passes, it will also help us see if the steady state temperatures achieved here is consistent with what we got using Gauss Elimination & Jacobi's Method.

We are using **Euler's Explicit Method** to solve the system of ODEs. It is a iterative method to solve ODEs numerically.

$$y_{i+1} = y_i + hf(x_i, y_i)$$

where $f(x_i, y_i)$ is the value of $\frac{dy}{dx}$ for a given ODE, where y is the dependent variable and x is the independent variable & h is the step size of independent variable, the value by which we increase the independent variable while marching increasingly.

In our case, the dependent variable is T, it depends on t, time passed since the system is switched on. To choose an appropriate step size, we did numerical analysis of the system.

Consider the ODE for Tank-2, we can see $\frac{dT_2}{dt} = k - 0.08T_2$, giving us the choice of choosing a h which is less than a very large number. Similarly, it is applicable to other tanks as well. So, we can choose our step size as 0.1 min. We have shown the plot from t=0 minutes to t=150 minutes.

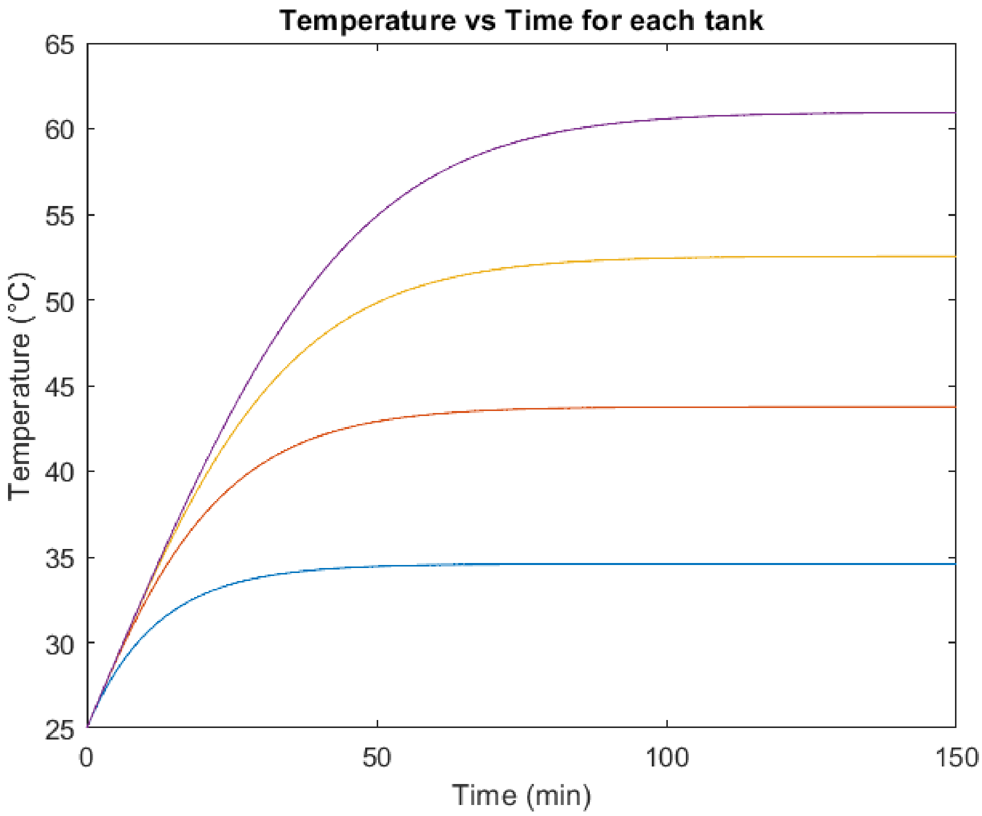


Figure 1: Plot Of T vs t for Tank-4 from top to Tank-1 at the bottom

We can see the plot is increasing and achieving a steady state as we expected earlier in the problem statement section.

Conclusion

The steady state temperatures (rounded off to two decimal places) of the tanks are

Tank-1 : 34.59°C

Tank-2 : 43.76°C

Tank-3 : 52.55°C

Tank-4 : 60.96°C

We can see the above solution is consistent with the plot of T vs t