

*

Partition Equal Subset Sum (Leetcode- 416)

Given array `nums`, return true if you can partition the array into two subsets such that the sum of the elements in both subsets is equal or false otherwise.

I/P - `nums = [1, 5, 11, 5]`

O/P - true

Explanation- The array can be partitioned as $[1, 5, 5]$ and $[11]$.

Approach-

- = If sum of the elements in `nums` is odd, it means it cannot be partitioned into two subsets.
- = If sum of the elements in `nums` is even, also if there exists a subset of `nums` array whose sum is half of the sum of the elements of `nums` array, it means, it can be partitioned into two subsets.

Code using recursion

```
class Solution {
```

```
public:
```

- If sum == targetSum return 1;
- If sum < targetSum return 0;
- If index == nums.size() return 0;
- If include = solveUsingRec(nums, index+1, targetSum - nums[index]);
- If exclude = solveUsingRec(nums, index+1, targetSum);

//base cases

```
if (targetSum == 0) return 1;
```

```
if (targetSum < 0 || index >= nums.size()) {
```

- return 0;

```
bool include = solveUsingRec(nums, index+1, targetSum - nums[index]);
```

```
bool exclude = solveUsingRec(nums, index+1, targetSum);
```

```
return (include || exclude);
```

```
bool canPartition (vector<int>& nums) {
```

```
int sum = 0;
```

```
for (auto num : nums) {
```

```
sum += num;
```

```
// if odd
```

```
if (sum & 1) return 0;
```

```
// if even
```

```
return solveUsingRec(nums, 0, sum/2);
```

```
};
```

Code using memoization -

```
class Solution {
```

```
public:
```

```
bool solveUsingMemo (vector<int>& nums, int index,  
int targetSum, vector<vector<int>>& dp) {  
    //base cases  
    if (targetSum == 0) return 1;  
    if (targetSum < 0 || index >= nums.size ()) {  
        return 0;  
    }  
    if (dp[index][targetSum] != -1) {  
        return dp[index][targetSum];  
    }
```

```
    bool include = solveUsingMemo (nums, index + 1,  
                                    targetSum - nums [index]);
```

```
    bool exclude = solveUsingMemo (nums, index + 1, targetSum);
```

```
    return dp[index][targetSum] = (include || exclude);  
}
```

```
bool canPartition (vector<int>& nums) {
```

```
    int sum = 0;
```

```
    for (auto num: nums) {
```

```
        sum += num;
```

```
}
```

```
if (sum & 1) return 0;
```

```
vector<vector<int>> dp (nums.size () + 1, vector<int>(sum / 2 + 1, -1));
```

```
return solveUsingMemo (nums, 0, sum / 2, dp);
```

```
}
```

• Code using tabulation -

class Solution {

public:

```
bool solveUsingTabu(vector<int>& nums, int targetSum) {
    int n = nums.size();
    vector<vector<int>> dp(n+1, vector<int>(targetSum+1, 0));
    // if targetSum = 0 ho gaye whatever be the index, answer
    // 1 hoga -
    for (int row = 0; row <= n; row++) {
        if (dp[row][0] = 1); // base case
    }
    for (int i = n-1; i >= 0; i--) {
        for (int j = 0; j <= targetSum; j++) {
            bool include = 0;
            if (j - nums[i] >= 0) {
                include = dp[i+1][j - nums[i]];
            }
            bool exclude = dp[i+1][j];
            dp[i][j] = (include || exclude);
        }
    }
    return dp[0][targetSum];
}

bool canPartition(vector<int>& nums) {
    int sum = 0;
    for (auto num : nums) sum += num;
    if (sum & 1) return 0;
    return solveUsingTabu(nums, sum/2);
}
```

- Optimising the tabulation code space-wise \Rightarrow

$$dp[i][j] \rightarrow dp[i+1][j - \text{nums}[i]]$$

$$dp[i][j] \rightarrow dp[i+1][j]$$

By using only two rows or vectors, we can do this as the answer of current cell depends upon the answer of next row's cells only.

```
class Solution {
```

```
public:
```

```
bool solveUsingTabuSO(vector<int>& nums, int targetSum) {
    int n = nums.size();
    vector<int> next(targetSum + 1, 0);
    vector<int> curr(targetSum + 1, 0);
```

// for the targetSum = 0

curr[0] = 1; // because if targetsum gets 0

next[0] = 1; // it means subset exists

```
for (int i = n - 1; i >= 0; i--) {
```

```
    for (int j = 0; j <= targetSum; j++) {
```

bool include = 0; // not in kth position not

if (j - nums[i] >= 0) {

include = next[j - nums[i]]; // so i is included

}

bool exclude = next[j]; // not included

curr[j] = (include || exclude);

} // shifting

} // return next[targetSum];

}

```
bool canPartition (vector<int>& nums) {
```

```
    int sum = 0;
```

```
    for (auto num : nums) sum += num;
```

```
    if (sum & 1) return 0;
```

```
    return solveUsingTabuSO(nums, sum / 2);
```

} //

* Number of dice rolls with target sum -

Given n dices, each dice has k faces numbered from 1 to k . Given a target, return the number of possible ways to roll the dice, so that sum of face-up numbers equals target.

I/P - $n=2$, $k=6$, target=7

O/P - 6

Explanation - There are 6 ways to get sum = 7 →

$$\{1, 6\}, \{2, 5\}, \{3, 4\}, \{4, 3\}, \{5, 2\}, \{6, 1\}$$

Approach -

We are using here explore all ways pattern. for 2 dices and 6 faces each, dice1 has 6 choices and for every choice of dice 1, dice 2 has 6 choices.

So there are 6×6 possible choices out of which 6 are there with the target sum.

• Code using recursion -

```
class Solution
```

```
public:
```

```
int MOD = 1000000007;
```

```
int solveUsingRec (int n, int k, int target, int diceUsed,
```

```
//base cases
```

```
if (diceUsed == n && currSum == target) return 1;
```

```
if (diceUsed != n && currSum == target ||
```

```
diceUsed == n && currSum != target) return 0;
```

```
int ans = 0;
```

```
for (int face = 1; face <= k; face++) {
```

```
int recAns = 0;
```

```
if (currSum + face <= target) {
```

```
recAns = solveUsingRec (n, k, target, diceUsed + 1,
```

```
(currSum + face) % MOD);
```

```
ans = (ans % MOD + recAns % MOD) % MOD;
```

```
}
```

```
return ans;
```

```
int numRollsToTarget (int n, int k, int target) {
```

```
int diceUsed = 0;
```

```
int currSum = 0;
```

```
return solveUsingRec (n, k, target, diceUsed, currSum);
```

```
}
```

```
};
```

Code using memoization -

```
class Solution {
public:
    int MOD = 1000000007;
    int solveUsingMemo(int n, int k, int target, int diceUsed,
                       vector<vector<int>>&dp, int currSum) {
        //base cases
        if (diceUsed == n && currSum == target) {
            return 1;
        }
        if (diceUsed == n && currSum != target || diceUsed != n &&
            currSum == target) return 0;
        if (dp[diceUsed][currSum] != -1) return dp[diceUsed][currSum];
        int ans = 0;
        for (int face = 1; face <= k; face++) {
            int recAns = 0;
            if (currSum + face <= target) {
                recAns = solveUsingMemo(n, k, target, diceUsed + 1, dp,
                                         currSum + face) % MOD;
            }
            ans = (ans % MOD + recAns % MOD) % MOD;
        }
        return dp[diceUsed][currSum] = ans;
    }

    int numRollsToTarget(int n, int k, int target) {
        int diceUsed = 0;
        int currSum = 0;
        vector<vector<int>> dp(n + 1, vector<int>(target + 1, -1));
        return solveUsingMemo(n, k, target, diceUsed, dp, currSum);
    }
};
```

• Code using tabulation -

```
class Solution {
public:
    int solveUsingTabu( int n, int k, int target) {
        vector<vector<int>> dp( n+1, vector<int>( target+1, 0));
        dp[n][target] = 1;
        for (int diceUsed=n-1; diceUsed >= 0; diceUsed--) {
            for (int currSum=target; currSum >= 0; currSum--) {
                int ans = 0;
                for (int face=1; face <= k; face++) {
                    int recAns = dp[diceUsed+1][currSum+face];
                    ans = (ans % MOD + recAns % MOD) % MOD;
                }
                dp[diceUsed][currSum] = ans;
            }
        }
        return dp[0][0];
    }

    int numRollsToTarget( int n, int k, int target) {
        int diceUsed = 0;
        int currSum = 0;
        return solveUsingTabu(n, k, target);
    }
};
```

$dp[diceUsed][currSum] \rightarrow dp[diceUsed+1][currSum+face]$

- Space optimisation in tabulation code -

class Solution {

public:

int MOD = 1000000007;

int solveUsingTabSO (int n, int k, int target) {

vector<int> next (target + 1, 0);

vector<int> curr (target + 1, 0);

for (int diceUsed = n - 1; diceUsed >= 0; diceUsed--) {

for (int currSum = target; currSum >= 0; currSum--) {

int ans = 0;

for (int face = 1; face <= k; face++) {

if (currSum + face <= target) {

ans = next[currSum + face];

ans = (ans % MOD + recAns % MOD) % MOD;

curr[currSum] = ans;

if shifting

next = curr;

} // target + face to i + 1 to target + face

return next[0];

int numRollsToTarget (int n, int k, int target) {

int diceUsed = 0;

int currSum = 0;

return solveUsingTabSO (n, k, target);

} ;