# Multi-Hop Question Answering

CSE 587 Spring 2025 Final Project by Lakshay Piplani (PSU ID: ljp5718) and Durva Dev (PSU ID: dbd5616 )

Github Repository Link

#### Introduction

For our final project, we are tackling the problem of multi-hop reasoning.

#### Multi-Hop Reasoning

Multi-hop reasoning is a systematic approach to answering questions that are complex, that is, implicitly contain (and can be broken down into) sub-questions. Due to this composite nature of multi-hop questions, answering them requires the agent to execute multiple steps of inference, and at each step utilize information from different sources. Therefore, salient features of multi-hop reasoning are:

- Complex Question: A complex question that cannot be answered using a single piece of fact
- Intermediate Inferences: Multiple inferences are needed to be chained together
- Multiple Facts: Each step requires some new pieces of information

Formally, we can represent multi-hop reasoning as:

```
Answer = f_n(\cdots f_2(f_1(Query, Set of Facts_1), Set of Facts_2), \ldots, Set of Facts_n)
```

where  $f_i$  represents reasoning operations at hop i.

#### Examples

#### Example 1: Simple Two-Hop Question

**Question**: "Where was the inventor of the telephone born?"

- 1. Hop 1: Identify the inventor of the telephone Inventor = Alexander Graham Bell (from Fact 1)
- 2. Hop 2: Find birth place of Alexander Graham Bell BirthPlace = Edinburgh, Scotland (from Fact 2)

#### **Example 2: Temporal Reasoning**

Question: "Was the CEO of Company X born before the company was founded?"

- 1. Hop 1: Find founding year of Company X (1995)
- 2. Hop 2: Find birth year of current CEO (1980)
- 3. Hop 3: Compare years  $(1980 < 1995 \Rightarrow Yes)$

## Machine Learning for Multi-Hop Reasoning

Multi-hop reasoning serves as an integral benchmark for testing high-order natural language reasoning abilities of machine learning models.

#### Classical Deep Learning Approaches

Below are some example from the literature of architectures that have been tested for multi-hop reasoning:

- Non transformer-based: Schlichtkrull et al. (2018) posited an architecture that uses Gated Recurrent Units that are entity-aware to perform iterative reasoning over text passages. Sabour, Frosst, and Hinton (2017) capture relationships between entities across documents using convolution layers. They also use a routing-by-agreement mechanism which they have shown to enable multi-hop inference
- Transformer-based: Fang et al. (2020) propose Multi-Hop Graph Relation Network that uses Graph Convolutional Network with edge-type-specific message passing.
- Hybrid architecture: Qiu et al. (2019) have shown that we can use Recurrent Neural Networks to capture the context and Graph Convolutional Network to capture relation between entities.

#### Retrieval-Augmented Methods

Retrieval Augmented Generation (RAG) uses two components of multi-hop reasoning:

- Iterative Retriever: At each hop, the retriever uses the question and the accumulated relevant context to fetch further context
- Language Model: The question and the final retrieved context are used by the LM to generate the answer and optionally the reasoning chain-of-thought. Alternatively, the retriever and LM can perform alternating turns of retrieve-then-reason-then-retrieve. This method has shown to be superior to the regular retrieve once-reason once architecture.

### Language Model Fine-Tuning Methods

Latest approaches fine-tune an LM to answer complex questions requiring multi-hop reasoning. Key steps include:

- Prompt Engineering: Use an instruction template (e.g., Llama 3's chat format) to explicitly prompt the question, answer, supporting context, along with correct step-by-step reasoning, and divergent reasoning negative examples to improve robustness.
- Supervised Fine-Tuning (SFT): Train the LM via causal language modeling, where the input is the prompt and labels are the gold reasoning chains and answers.
- Evaluation: of final answer and gold chain using Exact Match, F1 scores or human review.

Common LLM training approaches include:

- **Supervised**: We give the model a prompt including instructions, the question, the facts, and the label as the final answer and the supporting chain of thought.
- Weakly Supervised: It is similar to supervised, with the exception that we give only the final answer as the label. Therefore, the "golden chain" of thought is implicit and the model should learn to create such a chain of thought implicitly.
- Reinforcement Learning: We provide positive rewards to the language model when it generates correct reasoning paths.

#### Challenges

Key challenges in ML approaches:

- Explainability: Deep Learning architectures are a black-box, that is, it is not easy to explain why a particular output was generated by the network.
- Compositionality: Generalizing to unseen data points can be a challenge.
- Efficiency: Computational complexity of training or fine-tuning language models is very high and infeasible to perform on commodity compute resources.

### Sate-of-the-Art on Multi-Hop Question Answering

Table 1: Top 3 Models on HotpotQA Leaderboard (Full-Wiki Setting)

Rank	$\operatorname{Model}$	${ m EM/F1}$
1	Zhang et al. (2024)	72.69 / 85.04
2	PipNet (TenCent)	72.26 / 84.86
3	Yin et al. (2023)	72.07 / 84.34

#### **Dataset**

We use the HotpotQA dataset Yang et al. (2018) for our use-case of multi-hop question answering (QA), which in our opinion is well-suited for evaluating models that need to reason using multiple threads of evidence. Please note that we have used the version of dataset that contains the supporting facts and context paragraphs.

The HotpotQA dataset features natural, multi-hop questions, with strong supervision for supporting facts to enable more explainable QA systems. It is collected by a team of NLP researchers at Carnegie Mellon University, Stanford University, & Université de Montréal.

Unlike other benchmarks, the dataset was crowdsourced using Wikipedia pages as the "context" and crowd workers were shown multiple context documents and asked explicitly to come up with questions requiring reasoning about all of the documents, ensuring that the dataset covers multi-hop questions that are more natural, and are not designed with any pre-existing knowledge base schema in mind.

HotpotQA is divided into two main settings: distractor and full wiki. In this work, we focus on the distractor setting, which is easier to fit with our objective of enhancing multi-hop QA using knowledge graphs.

Since we had resource constraints that limited the scope of our experiments, we only used the train and dev splits out of all the splits (shown in Table 1) used in the actual model. We sampled and mixed both the datasets (train, dev) to get a total of 22000 examples.

Name	Desc.	Usage	# Examples
train-easy train-medium train-hard dev test-distractor test-fullwiki Total	single-hop multi-hop hard multi-hop hard multi-hop hard multi-hop hard multi-hop	training training training dev test test	18,089 56,814 15,661 7,405 7,405 7,405 112,779

Table 1: Data split. The splits *train-easy*, *train-medium*, and *train-hard* are combined for training. The distractor and full wiki settings use different test sets so that the gold paragraphs in the full wiki test set remain unknown to any models.

Below is an example from the dataset.

```
Г
       "Kind Hearts and Coronets",
   1
],
"context": [
   "Skindles",
           "Skindles was a hotel in Maidenhead, England, on the Buckinghamshire bank of the
               → River Thames by Maidenhead Bridge.",
           " Formerly the Orkney Arms, built in 1743, it was turned from a coaching inn into
               \hookrightarrow a fashionable hotel by William Skindle in 1833.",
           " In the 20th century, it became notorious as a place for adulterous assignations
               \hookrightarrow .",
           " Its guests included Winston Churchill and Princess Margaret, and musicians who
               → performed there included The Rolling Stones and The Strawbs.",
           " The hotel appears in the film Kind Hearts and Coronets.",
           " Skindles is mentioned in the play Journey's End by R. C. Sherriff: 'We danced a

→ bit at Skindles, and drank a lot of port and muck'."

       ]
   ],
       "Dead Clever",
       Γ
           "Dead Clever is a British black comedy film, first screened on ITV on New Year's
               \hookrightarrow Day, 2007.",
           " Written by Sally Wainwright, it stars Suranne Jones, Helen Baxendale and Dean

    → Lennox Kelly.",

           " Although officially titled \"Dead Clever\" it was subtitled \"The Life and
               \hookrightarrow Crimes of Julie Bottomley\".",
           " The music was written by BAFTA nominated TV & film music composer Sheridan
               → Tongue."
       ]
   ],
   Γ
       "Burn Burn Burn",
       "Burn Burn Burn is a 2015 British black comedy film, the directorial debut of
               "The film is a coming-of-age tale, inspired by the Jack Kerouac novel \"On the
               → Road\" published in 1957.",
           " The fictional plot follows the story of two girls, Seph (Laura Carmichael) and
               \hookrightarrow Alex (Chloe Pirrie), taking a road trip to follow the instructions of their
               \hookrightarrow close friend Dan, who has died and given them instructions where to
               → scatter his ashes.",
           " The ashes (stored in tupperware in the glove compartment) keep diminishing in

→ quantity as the trip progresses.",
           " The film had its World premiere at the BFI London Film Festival 2015."
       ]
   ],
   "Bigga than Ben",
       "Bigga than Ben is a 2008 British black comedy film written and directed by Suzie
               → Halewood.",
           " The film is based on the 1999 Russian novel of the same name."
```

```
]
],
    "Burke & Hare (2010 film)",
       "Burke & Hare is a 2010 British black comedy film, loosely based on the Burke and
           → Hare murders.",
       " Directed by John Landis, the film stars Simon Pegg and Andy Serkis as William
           → Burke and William Hare respectively.",
       " It was Landis's first feature film release in 12 years, the last being 1998's
           → \"Susan's Plan\".",
       " The film was released in the United Kingdom on 29 October 2010."
   1
],
    "A Long Way Down (film)",
    "A Long Way Down is a 2014 British black comedy film directed by Pascal Chaumeil,
           → loosely based on author Nick Hornby's 2005 novel, \"A Long Way Down\".",
       " It stars Pierce Brosnan, Toni Collette, Imogen Poots, and Aaron Paul as four
           \hookrightarrow strangers who happen to meet on the roof of a London building on New Year's
           \hookrightarrow Eve, each with the intent of committing suicide.",
       " Their plans for death in solitude are ruined when they meet as they decide to
           → come down from the roof alive \u2014 however temporary that may be."
   ]
],
    "Kind Hearts and Coronets",
    Γ
       "Kind Hearts and Coronets is a 1949 British black comedy film.",
       " It features Dennis Price, Joan Greenwood, Valerie Hobson and Alec Guinness;

→ Guinness plays nine characters.",
       " The plot is loosely based on the novel \"Israel Rank: The Autobiography of a
           " It concerns Louis D'Ascoyne Mazzini, the son of a woman disowned by her

→ aristocratic family for marrying out of her social class.",

       " After her death Louis decides to take revenge on the family, and to take the
           \hookrightarrow dukedom, by murdering the eight people ahead of him in succession to the
           → title."
   ]
],
    "Entertaining Mr Sloane (film)",
    "Entertaining Mr Sloane is a 1970 British black comedy film directed by Douglas
           → Hickox.",
       " The screenplay by Clive Exton is based on the 1964 play of the same title by
           → Joe Orton.",
       " This was the second adaptation of the play, the first having been developed for
           → British television and telecast by ITV on 15 July 1968."
   ]
],
    "The Ruling Class (film)",
       "The Ruling Class is a 1972 British black comedy film.",
       " It is an adaptation of Peter Barnes' satirical stage play \"The Ruling Class\"
```

```
\hookrightarrow which tells the story of a paranoid schizophrenic British nobleman (played
                 → by Peter O'Toole) who inherits a peerage.",
              " The film co-stars Alastair Sim, William Mervyn, Coral Browne, Harry Andrews,
                 " It was produced by Jules Buck and directed by Peter Medak."
          ]
      ],
          "Just Jim (2015 film)",
              "Just Jim is a 2015 British black comedy film written and directed by Craig
                 → Roberts in his directorial debut.",
              " The film stars Roberts as a lonely Welsh teenager who is given the chance to
                 \hookrightarrow increase his popularity when a cool American (Emile Hirsch) moves in next
                 → door."
          ]
      ]
   ],
   "type": "bridge",
   "level": "hard"
}
```

Each example includes a question along with,

- 1. Two Gold (supporting paragraphs) these are the paragraphs and the line number that contain the context required to answer the question i.e., the truth
- 2. Eight **distractor** paragraphs these are paragraphs selected to be similar, but irrelevant to answering the question, increasing the difficulty.

### LLM Selection and Training

#### Base Language Model

We have fine-tuned the open-weights Llama-3.2-3B, a 3-billion-parameter decoder-only transformer distilled from the official Llama-3 family proposed by Grattafiori et al. (2024)

Compared to larger Llama-3 models, this 3B version can fit into low-grade GPUs, making training and inference more accessible for us.

#### LoRA Adapters

We have used rank-r = 8 LoRA adapters (lora\_alpha=16, dropout 0.05) to every q\_proj and v\_proj in the attention blocks. Furthermore, the base weights are quantized to 4-bit NF4 via bitsandbytes; only  $\approx 0.3$  M trainable parameters are updated.

#### Supervised Fine-Tuning Pipeline

Figure 1 illustrates the complete data flow. Steps are described below.

- 1. **Train-Eval-Test split**: After we retrieve the raw JSON HotPotQA data, we take 70% of the records for training and reserve 30% for final evaluation. We serialize the raw JSON data into Huggingace's Arrow Dataset for the interim. The training data is further broken into actual training and evaluation data, which is 20% of the training set and is used during training evaluation.
- 2. **Per-data point graphization**: Each data point is converted into a graph G = (V, E) where nodes  $v_i$  represent context paragraphs and edges  $e_i$  encode inter-paragraph links (golden chain edges, and common tokens edges). Therefore, there is a graph for each data point, and we use it to generate golden and divergent examples while creating the prompt.
- 3. **Prompt construction**: For every example, we generate *system*, *user* and *assistant* blocks using the official Llama-3 chat template:

<br/>
<br/>
degin\_of\_text>

<start\_header\_id>system<end\_header\_id>

You are a careful multi-hop reasoner. Think step-by-step. Your task is to:

- 1. First identify the most relevant paragraph(s) to the question
- 2. Extract key information from those paragraphs
- 3. Combine the information to form a final answer
- 4. Present your reasoning clearly with [STEP X] tags before each step

EXAMPLE QUESTION: Which company is older, EOG Resources or General Mills? PARAGRAPHS:

- 0: EOG Resources —— Founded in 1999...
- 1: General Mills —— Founded in 1866...
- [STEP 1] Check founding dates in paragraph 0 and 1
- [STEP 2] EOG founded in 1999 (paragraph 0)
- [STEP 3] General Mills founded in 1866 (paragraph 1)

FINAL: General Mills is older <eot\_id>

<start\_header\_id>user<end\_header\_id>

```
Here is the QUESTION you are supposed to answer: QUESTION: ...

PARAGRAPHS:

0: Title<sub>0</sub> || Sentences<sub>0</sub>
1: Title<sub>1</sub> || Sentences<sub>1</sub>
...

These are the relevant paragraph numbers:

[STEP 1] p<sub>0</sub>
...

FINAL: answer

The other candidate(s) are not relevant to the question. These are wrong paths.

CANDIDATE 2: (divergent)
...

CANDIDATE 3: (divergent)
...

<eot_id>
<start_header_id>assistant<end_header_id>

[STEP 1] p<sub>0</sub>
...

FINAL: answer

assistant<end_header_id>
```

We use each data-point's graph to generate three candidate chains: Candidate 1 is the *golden* reasoning chain, and two additional distractor chains (random non-gold hops) are provided as in-context negatives.

- 4. Label generation: Only the assistant section containing golden reason is used as target text. HuggingFace's tokeniser is called with text\_target set to this slice, so all preceding tokens are masked to -100 and ignored by the cross-entropy loss.
- 5. **Left padding**: We set

```
tokenizer.pad_token = tokenizer.eos_token, tokenizer.padding_side = "left".
```

The DataCollatorForSeq2Seq pads inputs and labels on the *left*, following the best practice for causal LM training.

- 6. **Parameter-Efficient Fine-Tuning**: As discussed earlier, we apply LoRA adapters to the layers of the base model, which itself remains unchanged. The LoRA weights themselves are 4-bit quantized to accelerate training at the cost of decreased performance.
- 7. **Optimization**. Training parameters are:
  - Epochs = 2
  - batch = 1, gradient accumulation = 12. We keep batch size as 1 because larger batch sizes were not fitting onto our hardware
  - AdamW optimizer
  - Mixed-precision FP16
  - The evaluation split (20%) monitors perplexity at each epoch

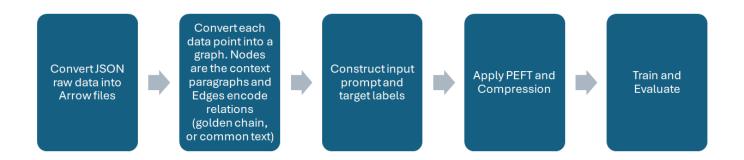


Figure 1: Training Pipeline

### **Experiments and Evaluation**

#### Experimental Setup

**Dataset splits**: As discussed earlier, HotpotQA JSON data is randomly partitioned 70:30 into **train** and **test** folds, preserving the label distribution. During fine-tuning we further carve the training slice into 80:20 train / eval using Dataset.train\_test\_split (seed = 42). The final outcomes of the experiments below are reported on the held-out test split.

**Inference controller**: We use the below inference methods:

- 1. **Greedy (1-step).** The model receives the prompt (question + context paragraphs) and generates a single FINAL: answer without explicit reasoning steps.
- 2. Adaptive Graph-of-Thought (AGoT). We also use AGoT inference strategy, as proposed by Pandey et al. (2025) Our controller expands a reasoning DAG, branching only when the Shannon entropy of the next-hop logits exceeds  $\tau = 1.3$  bits. We cap the search at four hops to prevent loops.

All inference runs are performed with the 4-bit base weights and rank-8 LoRA adapters; decoding uses do\_sample=False.

### Inference Prompt

We gave the following prompt to the model for inference.

```
"<|begin_of_text|>",
"<|start_header_id|>system<|end_header_id|>",
"You are a careful multi-hop reasoner. Think step-by-step.",
"Your task is to:"
"1. First identify the most relevant paragraph(s) to the question",
"2. Extract key information from those paragraphs",
"3. Combine the information to form a final answer",
"4. Present your reasoning clearly with [STEP X] tags before each step",
"5. End with FINAL: ",
"""
```

```
EXAMPLE QUESTION: Which company is older, EOG Resources or General Mills?
PARAGRAPHS:
0: EOG Resources || Founded in 1999...
1: General Mills || Founded in 1866...

[STEP 1] Check founding dates in paragraph 0 and 1
[STEP 2] EOG founded in 1999 (paragraph 0)
[STEP 3] General Mills founded in 1866 (paragraph 1)
FINAL: General Mills is older
"""

"<|eot_id|>"
prompt.extend([
"<|start_header_id|>user<|end_header_id|>",
f"Here is the QUESTION you are supposed to answer: {question}",
"PARAGRAPHS:"
<|eot_id|>"<|start_header_id|>assistant<|end_header_id|>\n")
"FINAL"
```

#### **Experiments**

We ran multiple experiments to study the performance of the HotpotQA dataset on multiple models. Figures show the loss, gradient norm and learning rate.

Training Metrics Analysis

- 1. Llama-3.2-3B quantized (4-bit) finetuned vs pre-trained
- 2. Llama-3.2-1B quantized (4-bit) finetuned vs pre-trained

#### Loss Curve **Gradient Norm** Learning Rate Schedule Training Loss Evaluation Loss Stability Threshold 14 0.7 0.6 0.5 0.4 0.3 10 Loss Gradient 0.50 0.25 0.50 0.75 1.00 1.25 1.50 1.75 0.25 0.75 1.00 1.25 1.50 1.75 2.00 0.25 1.00 1.25 1.50 1.75 Epoch Epoch Epoch

Figure 2: Plots for LLama-3.2-3B 4-bit quantized

We tried to fine-tune the model with the 8-bit quantized option but faced significant difficulties in training due to memory constraints.

Overall, the training for both models seems to be stable

- (a) Loss decreases steadily over two epochs, although we suspect that we did not reach full convergence due to early stopping
- (b) Gradient norm flattens after inital updates, suggesting stable learning
- (c) Learning rate (log) is generally stable, supporting smooth optimization.

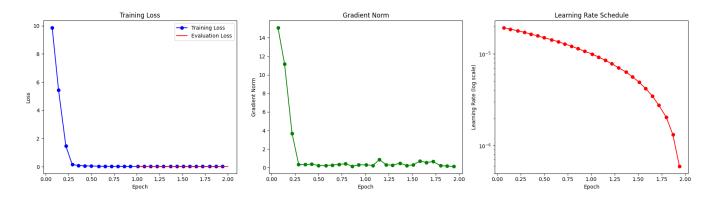


Figure 3: Plots for LLama-3.2-1B 4-bit quantized

#### Llama-3.2-3B

The figure 4 shows that fine-tuning the model on our dataset drastically increases the number of exact matches and also improves the F1 score.

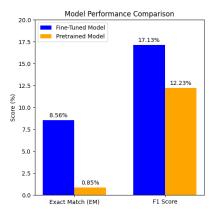


Figure 4: Fine-tuning improves Matches

The figures below show the prompts give by the fine-tuned model, the "RAW GENERATED TEXT" is the answer given by the model. For manual verification, we checked the first few prompts. For the first prompt in the figure below, the model gives the correct output "SOHA ALI KHAN".

```
Figure 1. Seeder 1. Disserting Pleader 1. July Place (1972) and the control of th
```

However, for this prompt, the model is unable to give the correct answer, the correct answer according to the dataset being 1993. Instead of giving the correct answer, the year the Dealey Plaza was given a "Historic Landmark" status, it ends up answering the year in which President John F. Kennedy's motorcade passes the plaza which is 1963.

#### LLama-3.2-1B

The 1B fine-tuned model is not able to answer most prompts correctly as shown.

```
The control of the co
```

Most examples are gibberish and end up repeating the questions.

#### **Evaluation Metrics**

- Exact Match (EM): A prediction is considered correct if its lower-cased, punctuation-stripped span matches HOTPOTQA's reference answer string.
- Token-level F1: The harmonic mean of token precision and recall, again using the official HotpotQA normaliser.

Metrics are computed with the evaluate implementation of the squad\_v2 script; the required input schema is {"id", "prediction\_text", "no\_answer\_probability"} for predictions and {"id", "answers" for references.

#### **Findings**

- 1. Bigger models handle divergence better; less prone to overfitting
- 2. More structured prompts yield clearer, more complete steps and training with positive and negative-examples leads to better model performance

### Future Directions and Learnings

We believe we could further improve the architecture through the below methods.

- Critique fine tuning: The architecture can be improved by implementing a critique fine tuning module after the supervised fine tuning. For this, we can use any LLM (the same Llama model or any other LM) to critique the output generated by our model when it is prompted to perform multi-hop reasoning given only the question and context paragraphs. Thus, the labels on which the model will be trained are the same as before, that is, gold facts and final answer, the only difference being we do not give candidates or the gold facts to the LM; instead, it learns to generate the gold chain and answer. Therefore, our current fine tuning module could teach the model to differentiate between good and bad chains, while this additional module can teach it to generate the correct chain.
- Increased Epochs: Training with multiple passes over data can help our model to better learn the underlying representation .
- Model Tuning: We relied on a conservative model in order to ensure we could train our LM under hardware constraints. However, we could try slightly heavier models like 7B variant, no LoRA, or no quantizations in order to unlock greater performance.

We also learned new techniques as part of the project

- Implementing PEFT and Compression: Having read the theory about PEFT and Compression techniques, we challenged to use these tools to reduce training time.
- **Prompting Templates**: We learned that different LLMs are trained to accept different prompt templates and tokens. So, while Chat Markup Language may suit GPT-models, Llama has its own markup language that we used to optimize generation.
- Hardware resourced needed: We realized that fine-tuning our model, even with PEFT and Compression, was not possible on commodity CPU. Therefore, we explored different options like cloud compute and on-premise GPU to run our workloads. Furthermore, we ensured that padding was done to generate token sequences that have length as multiples of 8. This helped map the data more efficiently onto GPU for matrix operations.

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