Tutorial -6 ①

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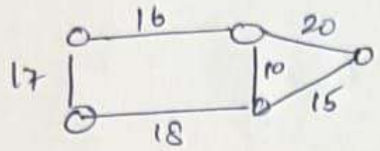
AI and DS
10

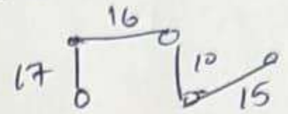
1> **Minimum Spanning Tree :-**

- A spanning Tree of an undirectional graph is a subgraph that is a tree and joined by all vertices. One of these tree which has minimum total cost would be its minimum spanning tree.

ex:-



Minimum cost spanning tree:-

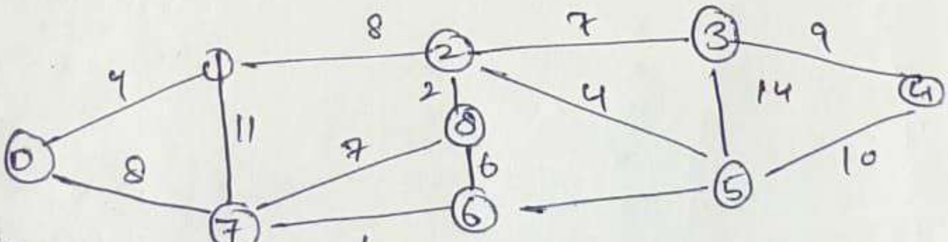


- **Application of MST:-**
  - It has direct applications in the design of networks including computer networks, telecommunication networks, transportation networks, etc.

2>

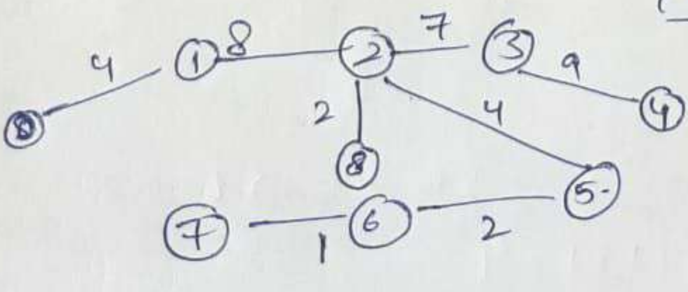
	Prism's Algo.	Kruskal's Algo.	Dijkstra's Algo.	Bellman Ford's Algo
TC	$O(V^2)$	$O(E \log V)$	$O(V+E\log V)$	$O(VE)$
SC	$O(V+E)$	$O(\|E\|+\|V\|)$	$O(V^2)$	$O(V^2)$

3>



Prism's Algo

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	∞	∞	∞	∞	∞	~~∞~~	∞
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$	~~10~~	$\boxed{4}$	~~8~~	8	$\boxed{2}$
				$\boxed{9}$		$\boxed{2}$	~~7~~	
							$\boxed{1}$	



Min weight = 37

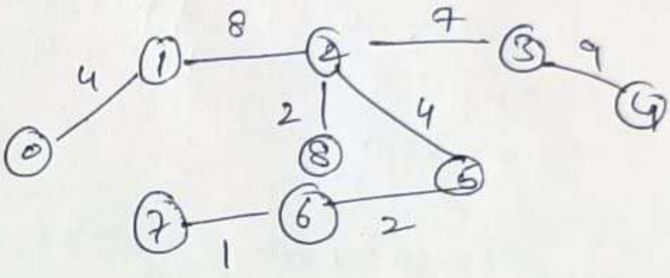
Parent :

0	1	2	3	4	5	6	7	8
-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~
	0	1	2				~~0~~	2
					2	~~8~~	~~8~~	
				~~5~~				
				3		5	6	

Parent : -1 0 1 2 3 2 5 6 2

Kruskal's Algo

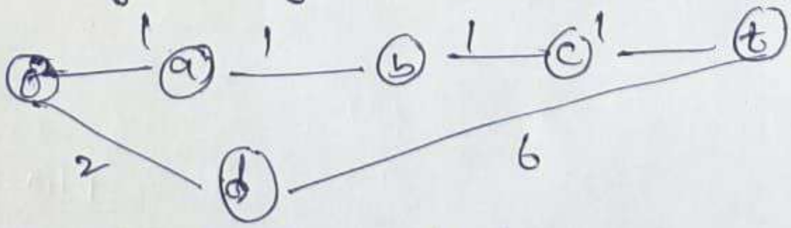
U	V	W	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	×
7	8	7	×
2	3	7	✓
1	2	8	✓
0	7	8	×
3	4	9	✓
5	4	10	×
1	7	11	×
3	5	14	×



weight = 37

4) (i) If 10 units is added to each edge, the overall weight of the path may change

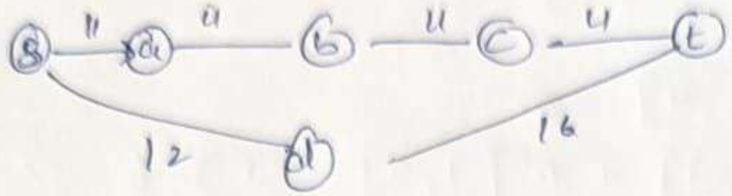
ex:



Shortest path is s → a → b → c → t

weight → 1+1+1+1 = 4

now if 10 units weight is added to each edge

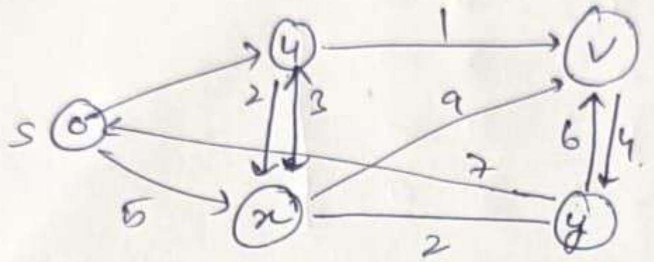


Shortest path changed to $s \rightarrow d \rightarrow t$

weight = 28

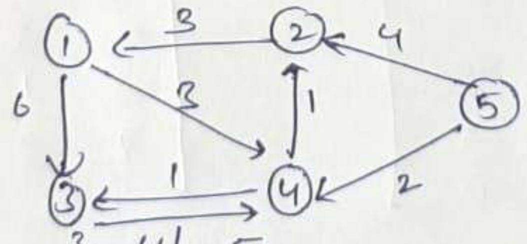
(ii) Multiplying the weight of each edge by 10 will have no change / impact on the shortest path.

5)



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	7.

6) All pair shortest path Algorithm – Floyd Warshall



$$A^0 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0 = [2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3+6 = 9$

$9 < \infty$

<u>Similarly</u>, $A^0[2,4] = \infty$

$A^0[2,1] + A^0[1,4] = 3+3 = 6 \Rightarrow 6 \not< \infty$

$A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = \cancel{3+\infty}\ 3+\infty$

$$A^2 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2+3] = \infty + 9$ $\quad 6 < \infty + 9$

$$A^3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \qquad A^5 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix}$$