

# Tutorial - 4

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AS and DS  
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$$1) T(n) = 3T(n/2) + n^2$$

$$T(n) = aT(n/b) + f(n^2)$$

$a \geq 1, b > 1$  on comparing

$$a = 3, b = 2, f(n^2)$$

now

$$C = \log_b a = \log_2 3 = 1.584$$

$$n^C = n^{1.584} < n^2$$

$$f(n) \geq n^2$$

$$\boxed{T(n) = \Theta(n^2)}$$

$$2) T(n) = 4T(n/2) + n^2$$

$$a \geq 1, b > 1$$

$$a = 4, b = 2, f(n) = n^2$$

$$C = \log_2 4 = 2$$

$$n^C = n^2 = f(n) = n^2$$

$$\boxed{T(n) = \Theta(n^2 \log_2 n)}$$

$$3) T(n) = T(n/2) + 2^n$$

$$a = 1, b = 2, f(n) = 2^n$$

$$C = \log_2 1 = 0$$

$$n^C = 1, f(n) > n$$

$$\boxed{T(n) = \Theta(2^n)}$$

$$4) T(n) = 2^n T(n/2) + n^n$$

$$a = 2^n, b = 2, f(n) = n^n$$

$$C = \log_2 2^n = n$$

$$f(n) = n^C$$

$$\boxed{T(n) = \Theta(n^n \log_2 n)}$$

$$5) T(n) = 16T(n/4) + n$$

$$a = 16, b = 4, f(n) = n$$

$$C = \log_4 16 = 2$$

$$f(n) < n^C$$

$$\boxed{T(n) = \Theta(n^2)}$$

$$6) T(n) = 2T(n/2) + n \log n$$

$$a = 2, b = 2, f(n) = n \log n$$

$$C = \log_2 2 = 1, n^C = n$$

$$n \log n > n$$

$$\boxed{T(n) = \Theta(n \log n)}$$

$$7) T(n) = 2T(n/2) + n/\log n$$

$$a = 2, b = 2, f(n) = n/\log n$$

$$C = \log_2 2 = 1$$

$$n^C = n$$

$$\frac{n}{\log n} < n$$

$$\boxed{T(n) = \Theta(n)}$$

$$8) T(n) = 2T(n/4) + n^{0.5}$$

$$a = 2, b = 4, f(n) = n^{0.5}$$

$$C = \log_4 2 = 0.5$$

$$n^C = n^{0.5}$$

$$n^{0.5} < n^{0.5}$$

$$f(n) > n^C$$

$$\boxed{T(n) = \Theta(n^{0.5})}$$

$$9) T(n) = 0.5T(n/2) + 1/n$$

$$a = 0.5, b = 2$$

$a \geq 1$ , but here  $a < 0.5$

so we can't apply

master's method.



$$10) T(n) = 16T(n/4) + n$$

$$a=16, b=4, f(n)=n$$

$$c = \log_4 16 = 2 \quad n^c = n^2$$

$$n! > n^2$$

$$T(n) = \Theta(n!)$$

$$11) 4T(n/2) + \log n$$

$$a=4, b=2, f(n) = \log n$$

$$c = \log_2 4 = 2$$

$$n^2 = n^2$$

$$T(n) = \Theta(n^2)$$

$$12) T(n) = \sqrt{n} T(n/2) + \log n$$

$$a = \sqrt{n}, b=2, f(n) = \log n$$

$$c = \log_2 \sqrt{n} = \frac{1}{2} \log n$$

$$\therefore f(n) > n^c$$

$$T(n) \neq \Theta(\log(n))$$

$$13) T(n) = 3T(n/2) + n$$

$$a=3, b=2, f(n)=n$$

$$c = \log_2 3 = 1.584$$

$$n < n^{1.58}$$

$$T(n) = \Theta(n^{1.58})$$

$$Q14) T(n) = 3T(n/3) + \sqrt{n}$$

$$a=3, b=3, f(n) = \sqrt{n}$$

$$c = \log_3 3 = 1$$

$$n^c = f(n)$$

$$T(n) = \Theta(n)$$

$$Q15) T(n) = 4T(n/2) + n$$

$$a=4, b=2, f(n)=n$$

$$c = \log_2 4 = 2$$

$$n^c > f(n)$$

$$T(n) = \Theta(n^2)$$

$$16) T(n) = 3T(n/4) + n \log n$$

$$a=3, b=4, f(n) = n \log n$$

$$c = \log_4 3 = 0.792$$

$$n^c = n^{0.792} \quad n^c < f(n)$$

$$T(n) = \Theta(n \log n)$$

$$17) T(n) = 3T(n/3) + n/2$$

$$a=3, b=3, f(n) = n/2$$

$$c=1 \quad n^c > f(n)$$

$$T(n) = \Theta(n)$$

$$18) T(n) = 6T(n/3) + n^2 \log n$$

$$a=6, b=3, f(n) = n^2 \log n$$

$$c = \log_3 6 = 1.6309$$

$$n^c < n^2 \log n$$

$$T(n) = \Theta(n^2 \log n)$$

$$19) T(n) = 4T(n/2) + n / \log n$$

$$a=4, b=2, f(n) = \frac{n}{\log n}$$

$$c = \log_2 4 = 2$$

$$\frac{n}{\log n} < n^2$$

$$T(n) = \Theta(n^2)$$

$$Q20) T(n) = 64T(n/8) - n^2 / \log n$$

$$a=64, b=8$$

$$c = \log_8 64 = 2$$

$$n^c = n^2 \quad n^2 / \log n > n^2$$

$$T(n) = \Theta(n^2 \log n)$$

$$Q21) T(n) = 7T(n/3) + n^2$$

$$a=7, b=3, c = \log_3 7 = 1.7712$$

$$n^c < f(n)$$

$$T(n) = \Theta(n^2)$$

$$Q22) T(n) = T(n/2) + n/2 - \log n$$

$$a=1, b=2, c = \log_2 1 = 0$$

$$n(2 - \log n) > n^c$$

$$T(n) = \Theta(n(2 - \log n))$$