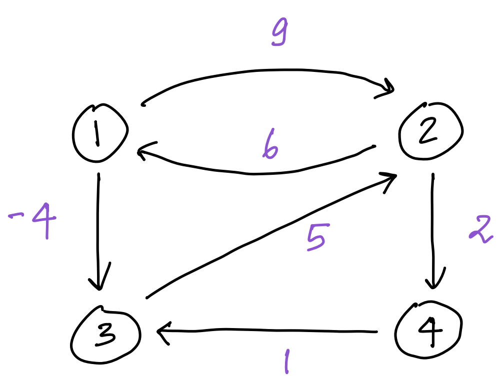
FLOYD WARSHALL ALGORITHM



all pair shortest path

single source

we'll have to apply Dijkstra algorithm 4 times here
(for 1, 2, 3 and 4)

DP approach (recursive)

Works on a weighted graph with even neg. weights

WORKING PRINCIPLE

initial distance matrix

distance from 1 to 2

as there is no direct edge from 1 to 4

$$D^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & \infty & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

distance from 2 to 1

{ all the diagonal elements will be 0 }

as distance between n_i and n_i will be 0

Now considering 1 as the middle element, construct D^1 matrix using D^0

$$D^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & \infty & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

working column & row

$$D^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & 2 & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

distance from 2 to 3 via 1

To fill D^1, consider

$D^0[2,3]$		$D^0[2,1] + D^0[1,3]$	
∞	$>$	$6 - 4 = 2$	UPDATE
$D^0[2,4]$		$D^0[2,1] + D^0[1,4]$	
2	$<$	$6 + \infty = \infty$	DON'T UPDATE

Now for D^2 (shortest paths via 2) consider D^1

$$D^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & \infty \\ 2 & 6 & 0 & 2 & 2 \\ 3 & \infty & 5 & 0 & \infty \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

$$D^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & 11 \\ 2 & 6 & 0 & 2 & 2 \\ 3 & 11 & 5 & 0 & 7 \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

Now for D^3 consider D^2

$$D^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & -4 & 11 \\ 2 & 6 & 0 & 2 & 2 \\ 3 & 11 & 5 & 0 & 7 \\ 4 & \infty & \infty & 1 & 0 \end{array}$$

$$D^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & -4 & 3 \\ 2 & 6 & 0 & 2 & 2 \\ 3 & 11 & 5 & 0 & 7 \\ 4 & 12 & 6 & 1 & 0 \end{array}$$

Now, for D^4, use D^3

$$D^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & -4 & 3 \\ 2 & 6 & 0 & 2 & 2 \\ 3 & 11 & 5 & 0 & 7 \\ 4 & 12 & 6 & 1 & 0 \end{array}$$

$$D^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & -4 & 3 \\ 2 & 6 & 0 & 2 & 2 \\ 3 & 11 & 5 & 0 & 7 \\ 4 & 12 & 6 & 1 & 0 \end{array}$$

SOLUTION

$$D^k[i,j] = \min\{D^{k-1}[i,j],\ D^{k-1}[i,k] + D^{k-1}[k,j]\}$$

Better to use this algorithm for a dense graph and consists of negative weights