ECO101A: Introduction to Economics
Tutorial 2: Solution

1. **A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.**

Solution: The marginal rate of technical substitution, MRTS, is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the MRTS, we need to know the rate at which one input may be substituted for the other. In this case, we do not know whether the MRTS is high or low. All we know is that it is a constant number. We need to know the marginal product of each input to determine the MRTS.

2. **The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of technical substitution of hours of labor for hours of machine capital is 1/4. What is the marginal product of capital?**

The marginal rate of technical substitution is defined at the ratio of the two marginal products. Here, we are given the marginal product of labor and the marginal rate of technical substitution. To determine the marginal product of capital, substitute the given values for the marginal product of labor and the marginal rate of technical substitution into the following formula:

$$\frac{MP_L}{MP_K} = MRTS, \quad or \quad \frac{50}{MP_K} = \frac{1}{4} \;\; or,$$

MP_K = 200 computer chips per hour.

3. **Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased, and the other factor is held constant at some level?**

a. q = 3L + 2K

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 10. If L is 4 and K is 4 then q is 20. When the inputs are doubled, output will double. Each marginal product is constant for this production function. When L increases by 1 q will increase by 3. When K increases by 1 q will increase by 2.

b. $q = (2L + 2K)^{1/2}$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 2.8. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will less than double. The marginal product of each input is decreasing. This can be determined using calculus by differentiating the production function with respect to either input, while holding the other input constant. For example, the marginal product of labor is

$$\frac{\partial q}{\partial l} = \frac{2}{2(2L + 2K)^{1/2}}$$

Since L is in the denominator, as L gets bigger, the marginal product gets smaller. If you do not know calculus, then you can choose several values for L, find q (for some fixed value of

K), and then find the marginal product. For example, if L=4 and K=4 then q=4. If L=5 and K=4 then q=4.24. If L=6 and K=4 then q= 4.47. Marginal product of labor falls from 0.24 to 0.23.

c. $q = 3LK^2$

This function exhibits increasing returns to scale. For example, if L is 2 and K is 2 then q is 24. If L is 4 and K is 4 then q is 192. When the inputs are doubled, output will more than double. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = 3(\lambda L)(\lambda K)^2 = \lambda^3 3LK^2 = \lambda^3 q.$$

Since λ is raised to a power greater than 1, we have increasing returns to scale. The marginal product of labor is constant and the marginal product of capital is increasing. For any given value of K, when L is increased by 1 unit, q will go up by $3K^2$ units, which is a constant number. Using calculus, the marginal product of capital is MPK=2*3*L*K. As K increases, MPK will increase. If you do not know calculus then you can fix the value of L, choose a starting value for K, and find q. Now increase K by 1 unit and find the new q. Do this a few more times and you can calculate marginal product. This was done in part b above, and is done in part d below.

d. $q = L^{\frac{1}{2}}K^{\frac{1}{2}}$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 2. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will exactly double. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = (\lambda L)^{\frac{1}{2}} (\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}}K^{\frac{1}{2}} = \lambda q.$$

Since λ is raised to the power 1, we have constant returns to scale. The marginal product of labor is decreasing and the marginal product of capital is decreasing. Using calculus, the marginal product of capital is

$$q = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}}$$

For any given value of L, as K increases, MPK will increase. If you do not know calculus then you can fix the value of L, choose a starting value for K, and find q. Let L=4 for example. If K is 4 then q is 4, if K is 5 then q is 4.47, and if K is 6 then q is 4.89. The marginal product of the 5th unit of K is 4.47-4=0.47, and the marginal product of the 6th unit of K is 4.89-4.47=0.42. Hence, we have diminishing marginal product of capital. You can do the same thing for the marginal product of labor.

e. $q = 4L^{\frac{1}{2}} + 4K$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 13.66. If L is 4 and K is 4 then q is 24. When the inputs are doubled, output will less than double.
The marginal product of labor is decreasing, and the marginal product of capital is constant. For any given value of L, when K is increased by 1 unit, q will go up by 4 units, which is a constant number. To see that the marginal product of labor is decreasing, fix K=1 and choose

values for L. If L=1 then q=8, if L=2 then q=9.65, and if L=3 then q=10.93. The marginal product of the second unit of labor is 9.65-8=1.65 and the marginal product of the third unit of labor is 10.93-9.65=1.28. Marginal product of labor is diminishing.

4. **The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $q = 10K^{0.6}L^{0.4}$.**

a. If both companies use the same amounts of capital and labor, which will generate more output?

Let Q be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then, according to their production functions,

$$q = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

and

$$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

Because $q = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same level of output. In fact if K>L then q_2>q.

b. Assume that capital is limited to 9 machine hours but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

With capital limited to 9 machine units, the production functions become $q = 30L^{0.5}$ and $q_2 = 37.372L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	q Firm 1	MP_L Firm 1	q Firm 2	MP_L Firm 2
0	0.0	___	0.00	___
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	58.00	8.69
4	60.00	8.04	65.07	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.

5. **A firm pays its accountant an annual retainer of $10,000. Is this an economic cost?**

Explicit costs are actual outlays. They include all costs that involve a monetary transaction. An implicit cost is an economic cost that does not necessarily involve a monetary transaction, but still involves the use of resources. When a firm pays an annual retainer of $10,000, there is a monetary transaction. The accountant trades his or her time in return for money. Therefore, an annual retainer is an explicit cost.

6. **The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?**

Opportunity costs are measured by comparing the use of a resource with its alternative uses. The opportunity cost of doing accounting work is the time not spent in other ways, i.e., time such as running a small business or participating in leisure activity. The economic, or opportunity, cost of doing accounting work is measured by computing the monetary amount that the owner's time would be worth in its next best use.

7. **Please explain whether the following statements are true or false.**

a. **If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.**

This is True. Since there is no monetary transaction, there is no accounting, or explicit, cost. However, since the owner of the business could be employed elsewhere, there is an economic cost. The economic cost is positive and reflecting the opportunity cost of the owner's time. The economic cost is the value of the next best alternative, or the amount that the owner would earn if he took the next best job.

b. **A firm that has positive accounting profit does not necessarily have positive economic profit.**

True. Accounting profit considers only the explicit, monetary costs. Since there may be some opportunity costs that were not fully realized as explicit monetary costs, it is possible that when the opportunity costs are added in, economic profit will become negative. This indicates that the firm's resources are not being put to their best use.

c. **If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.**

False. The opportunity cost measures the value of the worker's time, which is unlikely to be zero. Though the worker was temporarily unemployed, the worker still possesses skills, which have a value and make the opportunity cost of hiring the worker greater than zero. In addition, since opportunity cost is the equivalent of the worker's next best option, it is possible that the worker might have been able to get a better job that utilizes his skills more efficiently. Alternatively, the worker could have been doing unpaid work, such as care of a child or elderly person at home, which would have had a value to those receiving the service.

8. **Suppose a chair manufacturer finds that the marginal rate of technical substitution of capital for labor in his production process is substantially greater than the ratio of the rental rate on machinery to the wage rate for assembly-line labor. How should he alter his use of capital and labor to minimize the cost of production?**

To minimize cost, the manufacturer should use a combination of capital and labor so the rate at which he can trade capital for labor in his production process is the same as the rate at which he can trade capital for labor in external markets. The manufacturer would be better off if he increased his use of capital and decreased his use of labor, decreasing the marginal rate of technical substitution, MRTS. He should continue this substitution until his MRTS equals the ratio of the rental rate to the wage rate. The MRTS in this case is equal to MPK/MPL. As the manufacturer uses more K and less L, the MPK will diminish and the MPL will increase, both of which will decrease the MRTS until it is equal to the ratio of the input prices (rental rate on capital divided by wage rate).

9. **Assume the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.**

Marginal cost can be increasing while average variable cost is either increasing or decreasing. If marginal cost is less (greater) than average variable cost, then each additional unit is adding less (more) to total cost than previous units added to the total cost, which implies that the AVC declines (increases). Therefore, we need to know whether marginal cost is greater than average variable cost to determine whether the AVC is increasing or decreasing.