ECO101A: Introduction to economics
Tutorial 3: Solution

1. **The production function for a product is given by q = 100KL. If the price of capital is \$120 per day and the price of labor \$30 per day, what is the minimum cost of producing 1000 units of output?**

The cost-minimizing combination of capital and labor is the one where

$$\frac{MP_L}{MP_K} = MRTS = \frac{w}{r}$$

The marginal product of labor is $\frac{\partial q}{\partial L} = \ 100K$. The marginal product of capital is $\frac{\partial q}{\partial L} = \ 100L$. Therefore, the marginal rate of technical substitution is $\frac{100K}{100L} = \frac{K}{L}$. To determine the optimal capital-labor ratio set the marginal rate of technical substitution equal to the ratio of the wage rate to the rental rate of capital:

$$\frac{K}{L} = \frac{30}{120}, \text{ or } L = 4K.$$

Substitute for L in the production function and solve where K yields an output of 1,000 units:

1,000 = (100)(K)(4K), or K = 1.58.

Because L equals 4K this means L equals 6.32.
With these levels of the two inputs, total cost is:

TC = wL + rK, or

TC = (30)(6.32) + (120)(1.58) = \$379.20.

To see if K = 1.58 and L = 6.32 are the cost minimizing levels of inputs, consider small changes in K and L. around 1.58 and 6.32. At K = 1.6 and L = 6.32, total cost is \$381.60, and at K = 1.58 and L = 6.4, total cost is \$381.6, both greater than \$379.20. We have found the cost-minimizing levels of K and L.

2. **Suppose a production function is given by *F(K, L) = KL²*, the price of capital is \$10, and the price of labor \$15. What combination of labor and capital minimizes the cost of producing any given output?**

The cost-minimizing combination of capital and labor is the one where

$$\frac{MP_L}{MP_K} = MRTS = \frac{w}{r}$$

The marginal product of labor is $\frac{\partial q}{\partial L} = \ 2KL$. The marginal product of capital is $\frac{\partial q}{\partial L} = \ L^2$.
Set the marginal rate of technical substitution equal to the input price ratio to determine the optimal capital-labor ratio:

$$\frac{2KL}{L^2} = \frac{15}{10}\ , or\ K \ = \ 0.75L$$

Therefore, the capital-labor ratio should be 0.75 to minimize the cost of producing any given output.

3. **Suppose the process of producing light-weight parkas by Polly's Parkas is described by the function:**

$$\boldsymbol{q = 10K^{0.8}(L-40)^{0.2}}$$

where q is the number of parkas produced, K the number of computerized stitching machine hours, and L the number of person-hours of labor. In addition

to capital and labor, \$10 worth of raw materials are used in the production of each parka.

We are given the production function:

$$q = F(K,L) = 10K^{0.8}(L-40)^{0.2}$$

We also know that the cost of production, in addition to the cost of capital and labor, includes \$10 of raw material per unit of output. This yields the following total cost function:

$$TC(q) = wL + rK + 10q$$

a. By minimizing cost subject to the production function, derive the cost-minimizing demands for *K* and *L* as a function of output (*q*), wage rates (*w*), and rental rates on machines (*r*). Use these results to derive the total cost function, that is costs as a function of *q*, *r*, *w*, and the constant \$10 per unit materials cost.

We need to find the combinations of K and L that will minimize this cost function for any given level of output q and factor prices r and w. To do this, we set up the Lagrangian:

$$\Phi = wL + rK + 10q - \lambda[10K^{.8}(L-40)^{.2} - q]$$

Differentiating with respect to K, L, and λ, and setting the derivatives equal to zero:

$$(1) \qquad \frac{\partial \Phi}{\partial K} = r - 10\lambda(.8)K^{-.2}(L-40)^{.2} = 0$$

$$(2) \qquad \frac{\partial \Phi}{\partial L} = w - 10\lambda K^{.8}(.2)(L-40)^{-.8} = 0$$

$$(3) \qquad \frac{\partial \Phi}{\partial \lambda} = 10K^{.8}(L-40)^{.2} - q = 0.$$

The first 2 equations imply:

$$r = 10\lambda(.8)K^{-.2}(L-40)^{.2} \quad \text{and} \quad w = 10\lambda K^{.8}(.2)(L-40)^{-.8}.$$

or

$$\frac{r}{w} = \frac{4(L-40)}{K}.$$

This further implies:

$$K = \frac{4w(L-40)}{r} \quad \text{and} \quad L - 40 = \frac{rK}{4w}.$$

Substituting the above equations for K and L-40 into equation (3) yields solutions for K and L:

$$q = 10\left(\frac{4w}{r}\right)^{.8}(L-40)^{.8}(L-40)^{.2} \quad \textit{and} \quad q = 10K^{.8}\left(\frac{rK}{4w}\right)^{.2}.$$

or

$$L = \frac{r^{.8}q}{30.3w^{.8}} + 40 \quad \textit{and} \quad K = \frac{w^{.2}q}{7.6r^{.2}}.$$

We can now obtain the total cost function in terms of only r, w, and Q by substituting these cost-minimizing values for K and L into the total cost function:

$$TC(q) = wL + rK + 10q$$

$$TC(q) = \frac{wr^{.8}q}{30.3w^{.8}} + 40w + \frac{rw^{.2}q}{7.6r^{.2}} + 10q$$

$$TC(q) = \frac{w^{.2}r^{.8}q}{30.3} + 40w + \frac{r^{.8}w^{.2}q}{7.6} + 10q.$$

b. This process requires skilled workers, who earn $32 per hour. The rental rate on the machines used in the process is $64 per hour. At these factor prices, what are total costs as a function of q? Does this technology exhibit decreasing, constant, or increasing returns to scale?

Given the values w = 32 and r = 64, the total cost function becomes:

TC(q)=19.2q+1280.

The average cost function is then given by

AC(q) = 19.2 + 1280/q.

To find returns to scale, choose an input combination and find the level of output, and then double all inputs and compare the new and old output levels.
Assume K=50 and L=60. Then $q_1= 10(50)^{0.8}(60-40)^{0.2} = 416.3$.
When K=100 and L=120, $q_2= 10(100)^{0.8}(120-40)^{0.2} = 956.4$.
Since $q_2/q_1 > 2$, the production function exhibits increasing returns to scale.

c. Polly's Parkas plans to produce 2000 parkas per week. At the factor prices given above, how many workers should the firm hire (at 40 hours per week) and how many machines should it rent (at 40 machines-hours per week)? What are the marginal and average costs at this level of production?

Given q = 2,000 per week, we can calculate the required amount of inputs K and L using the formulas derived in part a:

$$L = \frac{r^{.8}q}{30.3w^{.8}} + 40 \quad and \quad K = \frac{w^{.2}q}{7.6r^{.2}}.$$

Thus L = 154.9 worker hours and K = 229.1 machine hours. Assuming a 40 hour week, L = 154.9/40 = 3.87 workers per week, and K = 229.1/40 = 5.73 machines per week. Polly's Parkas should hire 4 workers and rent 6 machines per week.

We know that the total cost and average cost functions are given by:

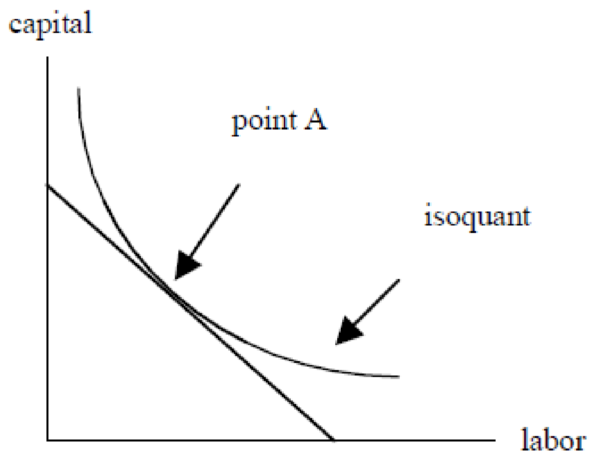
TC(q) = 19.2q + 1280
AC(q) = 19.2 + 1280/q,

so, the marginal cost function is

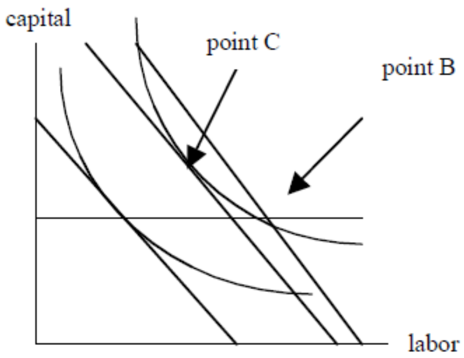
MC(q) = d TC(q) / d q = 19.2.

Marginal costs are constant at \$19.2 per parka and average costs are 19.2+1280/2000 or \$19.84 per parka.

4. **Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.**
   a. **The firm is currently producing 100 units of output, and has determined that the cost-minimizing quantities of labor and capital are 20 and 5 respectively. Graphically illustrate this situation on a graph using isoquants and isocost lines.**
   The isoquant is convex. The optimal quantities of labor and capital are given by the point where the isocost line is tangent to the isoquant. The isocost line has a slope of 1/4, given labor is on the horizontal axis. The total cost is TC=\$20*20+\$80*5=\$800, so the isocost line has the equation \$800=20L+80K. On the graph, the optimal point is point A.



   b. **The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm require? Illustrate this point on your graph and find the new cost.**
   The new level of labor is 39.2. To find this, use the production function $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$ and substitute 140 in for output and 5 in for capital. The new cost is TC=\$20*39.2+\$80*5=\$1184. The new isoquant for an output of 140 is above and to the right of the old isoquant for an output of 100. Since capital is fixed in the short run, the firm will move out horizontally to the new isoquant and new level of labor. This is point B on the graph below. This is not likely to be the cost minimizing point. Given the firm wants to produce more output, they are likely to want to hire more capital in the long run. Notice also that there are points on the new isoquant that are below the new isocost line. These points all involve hiring more capital.



c. **Graphically identify the cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.**
This is point C on the graph above. When the firm is at point B they are not minimizing cost. The firm will find it optimal to hire more capital and less labor and move to the new lower isocost line. All three isocost lines above are parallel and have the same slope.

d. **If the marginal rate of technical substitution is K / L, find the optimal level of capital and labor required to produce the 140 units of output.**
Set the marginal rate of technical substitution equal to the ratio of the input costs so that K /L = 20 / 80, or K = L / 4. Now substitute this into the production function for K, set q equal to 140, and solve for L: $140 = 10L^{\frac{1}{2}}(\frac{L}{4})^{\frac{1}{2}}$, L=28, K= 7. The new cost is TC=$20*28+$80*7 or $1120.

5. A computer company's cost function, which relates its average cost of production AC to its cumulative output in thousands of computers Q and its plant size in terms of thousands of computers produced per year q, within the production range of 10,000 to 50,000 computers is given by
AC = 10 - 0.1Q + 0.3q.

a. **Is there a learning curve effect?**

The learning curve describes the relationship between the cumulative output and the inputs required to produce a unit of output. Average cost measures the input requirements per unit of output. Learning curve effects exist if average cost falls with increases in cumulative output. Here, average cost decreases as cumulative output, Q, increases. Therefore, there are learning curve effects.

b. **Are there economies or diseconomies of scale?**

Economies of scale can be measured by calculating the cost-output elasticity, which measures the percentage change in the cost of production resulting from a one percentage increase in output. There are economies of scale if the firm can double its output for less than double the cost. There are economies of scale because the average cost of production declines as more output is produced, due to the learning effect.

c. **During its existence, the firm has produced a total of 40,000 computers and is producing 10,000 computers this year. Next year it plans to increase its production to 12,000 computers. Will its average cost of production increase or decrease? Explain.**

First, calculate average cost this year:

$AC_1 = 10 - 0.1Q + 0.3q = 10 - (0.1)(40) + (0.3)(10) = 9$.

Second, calculate the average cost next year:

$AC_2 = 10 - (0.1)(50) + (0.3)(12) = 8.6$.

(Note: Cumulative output has increased from 40,000 to 50,000.) The average cost will decrease because of the learning effect.

6. **Suppose the long-run total cost function for an industry is given by the cubic equation $TC = a + bQ + cQ^2 + dQ^3$. Show (using calculus) that this total cost function is consistent with a *U*-shaped average cost curve for at least some values of *a, b, c, d*.**

To show that the cubic cost equation implies a *U*-shaped average cost curve, we use algebra, calculus, and economic reasoning to place sign restrictions on the parameters of the equation. These techniques are illustrated by the example below.

First, if output is equal to zero, then $TC = a$, where a represents fixed costs. In the short run, fixed costs are positive, $a > 0$, but in the long run, where all inputs are variable $a = 0$. Therefore, we restrict a to be zero.

Next, we know that average cost must be positive. Dividing *TC* by *Q*:

$$AC = b + cQ + dQ^2.$$

This equation is simply a quadratic function. When graphed, it has two basic shapes: a U shape and a hill shape. We want the *U* shape, i.e., a curve with a minimum (minimum average cost), rather than a hill shape with a maximum.

At the minimum, the slope should be zero, thus the first derivative of the average cost curve with respect to *Q* must be equal to zero. For a *U*-shaped AC curve, the second derivative of the average cost curve must be positive.

The first derivative is $c + 2dQ$; the second derivative is $2d$. If the second derivative is to be positive, then $d > 0$. If the first derivative is equal to zero, then solving for *c* as a function of *Q* and *d* yields: $c = -2dQ$. If *d* and *Q* are both positive, then *c* must be negative: $c < 0$.

To restrict *b*, we know that at its minimum, average cost must be positive. The minimum occurs when $c + 2dQ = 0$. We solve for *Q* as a function of *c* and *d*:

$Q = -\frac{c}{2d} > 0$. Next, substituting this value for *Q* into our expression for average cost, and simplifying the equation:

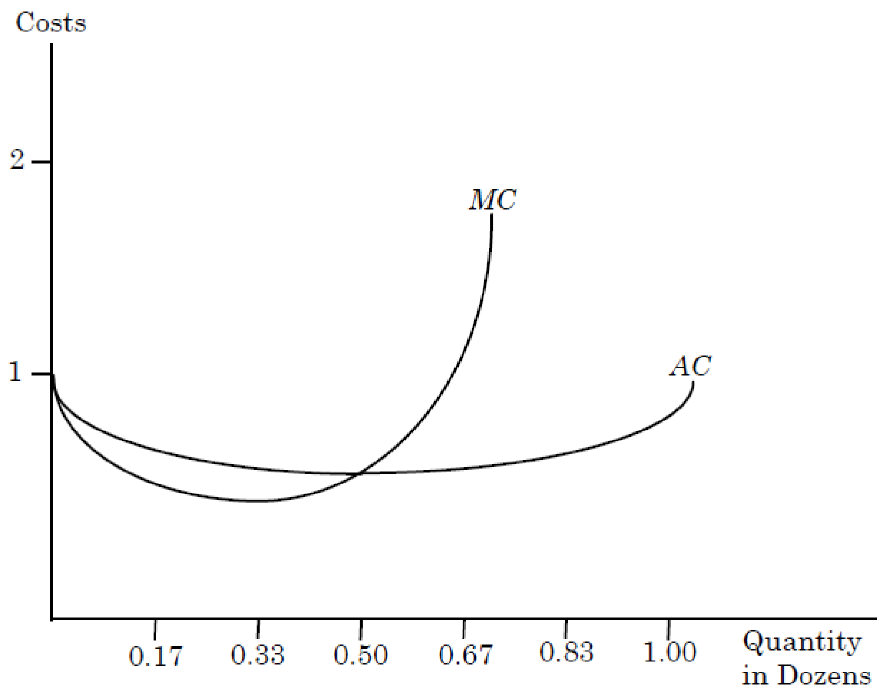
$$AC = b + cQ + dQ^2 = b + c\left(-\frac{c}{2d}\right) + dQ\left(-\frac{c}{2d}\right)^2, \text{ or}$$

$$AC = b - \frac{c^2}{4d} > 0.$$

implying $b > \frac{c^2}{4d}$. Because $c^2 > 0$ and $d > 0$, b must be positive.

In summary, for U-shaped long-run average cost curves, a must be zero, b and d must be positive, c must be negative, and $4db > c^2$. However, the conditions do not insure that marginal cost is positive. To insure that marginal cost has a U shape and that its minimum is positive, using the same procedure, i.e., solving for Q at minimum marginal cost $- c / 3d$, and substituting into the expression for marginal cost $b + 2cQ + 3dQ^2$, we find that c^2 must be less than $3bd$. Notice that parameter values that satisfy this condition also satisfy $4db > c^2$, but not the reverse.

For example, let $a = 0, b = 1, c = -1, d = 1$. Total cost is $Q - Q^2 + Q^3$; average cost is $1 - Q + Q^2$; and marginal cost is $1 - 2Q + 3Q^2$. Minimum average cost is $Q = 1/2$ and minimum marginal cost is $1/3$ (think of Q as dozens of units, so no fractional units are produced).



7. Why would a firm that incurs losses choose to produce rather than shut down?

Losses occur when revenues do not cover total costs. Revenues could be greater than variable costs, but not total costs, in which case the firm is better off producing in the short run rather than shutting down, even though they are incurring a loss. The firm should compare the level of loss with no production to the level of loss with positive production, and pick the option that results in the smallest loss. In the short run, losses will be minimized as long as the firm covers its variable costs. In the long run, all costs are variable, and thus, all costs must be covered if the firm is to remain in business.

8. Explain why the industry supply curve is not the long-run industry marginal cost curve.

In the short run, a change in the market price induces the profit-maximizing firm to change its optimal level of output. This optimal output occurs when price is equal to marginal cost, as long as marginal cost exceeds average variable cost. Therefore, the supply curve of the firm is its marginal cost curve, above average variable cost. (When the price falls below average variable cost, the firm will shut down.) In the long run, the firm adjusts its inputs so that its long-run marginal cost is equal to the market price. At this level of output, it is operating on a short-run marginal cost curve where short-run marginal cost is equal to price. As the long-run price changes, the firm gradually changes its mix of inputs to minimize cost. Thus, the long-run supply response is this adjustment from one set of short-run marginal cost curves to another.

Also note that in the long run there will be entry and the firm will earn zero profit, so that any level of output where MC>AC is not possible.

9. **In long-run equilibrium, all firms in the industry earn zero economic profit. Why is this true?**

The theory of perfect competition explicitly assumes that there are no entry or exit barriers to new participants in an industry. With free entry, positive economic profits induce new entrants. As these firms enter, the supply curve shifts to the right, causing a fall in the equilibrium price of the product. Entry will stop, and equilibrium will be achieved, when economic profits have fallen to zero.

10. **Why do firms enter an industry when they know that in the long run economic profit will be zero?**

Firms enter an industry when they expect to earn economic profit. These short-run profits are enough to encourage entry. Zero economic profits in the long run imply normal returns to the factors of production, including the labor and capital of the owners of firms. For example, the owner of a small business might experience positive accounting profits before the foregone wages from running the business are subtracted from these profits. If the revenue minus other costs is just equal to what could be earned elsewhere, then the owner is indifferent to staying in business or exiting.

11. **Suppose a competitive industry faces an increase in demand (i.e., the demand curve shifts upward). What are the steps by which a competitive market insures increased output? Will your answer change if the government imposes a price ceiling?**

If demand increases with fixed supply, price and profits increase. The price increase induces the firms in the industry to increase output. Also, with positive profit, firms enter the industry, shifting the supply curve to the right. This results in a new equilibrium with a higher quantity produced and a price that earns all firms zero economic profit. With an effective price ceiling, profit will be lower than without the ceiling, reducing the incentive for firms to enter the industry. With zero economic profit, no firms enter and there is no shift in the supply curve.

12. **Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by $C = 200 + 2q^2$, where q is the level of**

output and C is total cost. (The marginal cost of production is 4q. The fixed cost of production is $200.)

a. **If the price of watches is $100, how many watches should you produce to maximize profit?**

Profits are maximized where marginal cost is equal to marginal revenue. Here, marginal revenue is equal to $100; recall that price equals marginal revenue in a competitive market: 100 = 4q, or q = 25.

b. **What will the profit level be?**

Profit is equal to total revenue minus total cost:

$\pi = (100)(25) - (200 + 2*25^2) = \$1050$.

c. **At what minimum price will the firm produce a positive output?**

A firm will produce in the short run if the revenues it receives are greater than its variable costs. Remember that the firm’s short-run supply curve is its marginal cost curve above the minimum of average variable cost. Here, average variable cost is

$VC/q = 2q^2/q = 2q$. Also, MC is equal to 4q. So, MC is greater than AVC for any quantity greater than 0. This means that the firm produces in the short run as long as price is positive.

13. **A firm produces a product in a competitive industry and has a total cost function $TC = 50 + 4q + 2q^2$ and a marginal cost function $MC = 4 + 4q$. At the given market price of *$20,* the firm is producing 5 units of output. Is the firm maximizing profit? What quantity of output should the firm produce in the long run?**

If the firm is maximizing profit, then price will be equal to marginal cost. P=MC results in P=20=4+4q=MC, or q=4. The firm is not maximizing profit, since it is producing too much output. The current level of profit is

profit = 20*5-(50+4*5+2*5*5) = –20,

and the profit maximizing level is

profit = 20*4-(50+4*4+2*4*4) = –18.

Given no change in the price of the product or the cost structure of the firm, the firm should produce q=0 units of output in the long run since at the quantity where price is equal to marginal cost, economic profit is negative. The firm should exit the industry.

14. **Suppose the cost function is $C(q)=4q^2+16$.**

a. **Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost.**

Variable cost is that part of total cost that depends on q ($4q^2$) and fixed cost is that part of total cost that does not depend on q (16).

$$VC = 4q^2$$

$$FC = 16$$

$$AC = C(q)/q = 4q + 16/q$$

$$AVC = VC/q = 4q$$

$$AFC = FC/q = 16/q$$

b. Show the average cost, marginal cost, and average variable cost curves on a graph.

Average cost is u-shaped. Average cost is relatively large at first because the firm is not able to spread the fixed cost over very many units of output. As output increases, average fixed costs will fall relatively rapidly. Average cost will increase at some point because the average fixed cost will become very small and average variable cost is increasing as q increases. Average variable cost will increase because of diminishing returns to the variable factor labor. MC and AVC are linear, and both pass through the origin. Average variable cost is everywhere below average cost. Marginal cost is everywhere above average variable cost. If the average is rising, then the marginal must be above the average. Marginal cost will hit average cost at its minimum point.

c. Find the output that minimizes average cost.

The minimum average cost quantity is where MC is equal to AC:

$$AC = 4q + 16/q = 8q = MC$$

$$16/q = 4q$$

$$16 = 4q^2$$

$$4 = q^2$$

$$2 = q.$$

d. At what range of prices will the firm produce a positive output?

The firm will supply positive levels of output as long as $P=MC>AVC$, or as long as the firm is covering its variable costs of production. In this case, marginal cost is everywhere above average variable cost so the firm will supply positive output at any positive price.

e. At what range of prices will the firm earn a negative profit?

The firm will earn negative profit when $P=MC<AC$, or at any price below minimum average cost. In part c above we found that the minimum average cost quantity was $q=2$. Plug $q=2$ into the average cost function to find $AC=16$. The firm will therefore earn negative profit if price is below 16.

f. At what range of prices will the firm earn a positive profit?

In part e we found that the firm would earn negative profit at any price below 16. The firm therefore earns positive profit as long as price is above 16.

15. Suppose you are given the following information about a particular industry:

$Q^D = 6500 - 100P$ Market demand

$Q^S = 1200P$ Market supply

$C(q) = 722 + q^2 / 200$ Firm total cost function

Assume that all firms are identical, and that the market is characterized by pure competition.

a. Find the equilibrium price, the equilibrium quantity, the output supplied by the firm, and the profit of the firm.

Equilibrium price and quantity are found by setting market supply equal to market demand, so that $6500\text{-}100P=1200P$. Solve to find $P=5$ and substitute into either equation to find $Q=6000$. To find the output for the firm set price equal to marginal cost so that $5=2q/200$ and $q=500$. Profit of the firm is total revenue minus total cost or

$$\Pi = pq - C(q) = 5(500) - 722 - 500^2/200 = 528.$$

Notice that since the total output in the market is 6000, and the firm output is 500, there must be 6000/500=12 firms in the industry.

b. Would you expect to see entry into or exit from the industry in the long-run? Explain. What effect will entry or exit have on market equilibrium?

Entry because the firms in the industry are making positive profit. As firms enter, the supply curve for the industry will shift down and to the right and the equilibrium price will fall, all else the same. This will reduce each firm's profit down to zero until there is no incentive for further entry.

c. What is the lowest price at which each firm would sell its output in the long run? Is profit positive, negative, or zero at this price? Explain.

In the long run the firm will not sell for a price that is below minimum average cost. At any price below minimum average cost, profit is negative and the firm is better off selling its fixed resources and producing nzero output. To find the minimum average cost, set marginal cost equal to average cost and solve for q:

$$2q/200 = 722/q + q/200$$

$$q/200 = 722/q$$

$$q^2 = 722(200)$$

$$q = 380$$

$$AC\ (q = 380) = 3.8.$$

Therefore, the firm will not sell for any price less than 3.8 in the long run.

d. What is the lowest price at which each firm would sell its output in the short run? Is profit positive, negative, or zero at this price? Explain.

The firm will sell for any positive price, because at any positive price marginal cost will be above average variable cost ($AVC=q/2000$). Profit is negative as long as price is below minimum average cost, or as long as price is below 3.8.

16. Suppose that a competitive firm has a total cost function $C(q) = 450 +15q + 2q^2$ and a marginal cost function $MC(q) =15 + 4q$. If the market price is $P=\$115$ per unit, find

the level of output produced by the firm. Find the level of profit and the level of producer surplus.

The firm should produce where price is equal to marginal cost so that $P=115=15+4q=MC$ and $q=25$. Profit is $\Pi = 115(25) - 450 - 15(25) - 2(25^2) = 800$. Producer surplus is profit plus fixed cost, which is 1250. Note that producer surplus can also be found graphically by calculating the area below the price line and above the marginal cost (supply) curve, so that $PS=0.5 * (115-15) * 25=1250$.