ECO101A: Introduction to Economics

Tutorial 1 Solution

1. **Much of the demand for U.S. agricultural output has come from other countries. In 1998, the total demand for wheat was Q = 3244 - 283P. Of this, domestic demand was Q_D = 1700 - 107P. Domestic supply was Q_S = 1944 + 207P. Suppose the export demand for wheat falls by 40 percent.**
   U.S. farmers are concerned about this drop in export demand. What happens to the free market price of wheat in the United States? Do the farmers have much reason to worry?

Solution:

Given total demand, $Q = 3244 - 283P$, and domestic demand, $Q_d = 1700 - 107P$, we may subtract and determine export demand, $Q_e = 1544 - 176P$.

The initial market equilibrium price is found by setting total demand equal to supply:

$$3244 - 283P = 1944 + 207P, \text{ or}$$

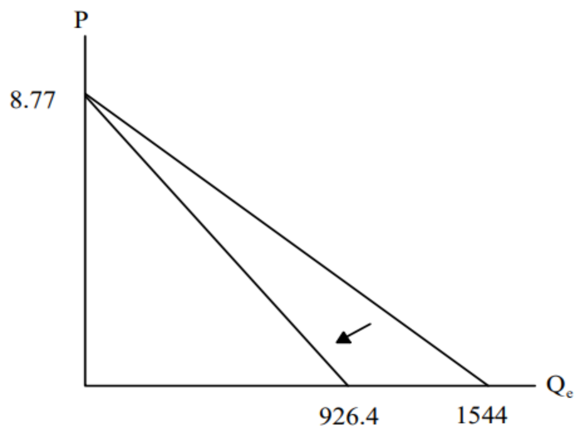
$$P = \$2.65.$$

The best way to handle the 40 percent drop in export demand is to assume that the export demand curve pivots down and to the left around the vertical intercept so that at all prices demand decreases by 40 percent, and the reservation price (the maximum price that the foreign country is willing to pay) does not change. If you instead shifted the demand curve down to the left in a parallel fashion the effect on price and quantity will be qualitatively the same, but will differ quantitatively.

The new export demand is $0.6Q_e=0.6(1544-176P)=926.4-105.6P$. Graphically, export demand has pivoted inwards as illustrated in figure 2.5a below.

Total demand becomes

$$Q_D = Q_d + 0.6Q_e = 1700 - 107P + 926.4\text{-}105.6P = 2626.4 - 212.6P.$$



Equating total supply and total demand,

$$1944 + 207P = 2626.4 - 212.6P, \text{ or } P = \$1.63,$$

which is a significant drop from the market-clearing price of $2.65 per bushel. At this price, the market-clearing quantity is 2280.65 million bushels. Total revenue has decreased from $6614.6 million to $3709.0 million. Most farmers would worry.

2. **The following table shows the average retail price of butter and the Consumer Price Index from 1980 to 2001.**

	1980	1985	1990	1995	2000	2001
CPI	**100**	**130.58**	**158.62**	**184.95**	**208.98**	**214.93**
Retail Price of butter (salted, grade AA, per lb.)	**$1.88**	**$2.12**	**$1.99**	**$1.61**	**$2.52**	**$3.30**

a. **Calculate the real price of butter in 1980 dollars. Has the real price increased/decreased/stayed the same since 1980?**

Sol:

$$\text{Real price of butter in year X} = \frac{CPI_{1980}}{CPI_{year\,X}} * nominal\ price\ in\ year\ X.$$

1980	1985	1990	1995	2000	2001
$1.88	$1.62	$1.25	$0.87	$1.21	$1.54

Since 1980 the real price of butter has decreased.

b. **What is the percentage change in the real price (1980 dollars) from 1980 to 2001?**

Sol: Percentage change in real price from 1980 to 2001 = (1.54 –1.88) / 1.88 = –0.18 = –18%.

c. **Convert the CPI into 1990 = 100 and determine the real price of butter in 1990 dollars.**

Sol:

To convert the CPI into 1990=100, divide the CPI for each year by the CPI for 1990. Use the formula from part (a) and the new CPI numbers below to find the real price of milk.

New CPI			Real price of milk		
	1980	63.1		1980	\$2.98
	1985	82.3		1985	\$2.58
	1990	100		1990	\$1.99
	1995	116.6		1995	\$1.38
	2000	131.8		2000	\$1.91
	2001	135.6		2001	\$2.43

d. What is the percentage change in the real price (1990 dollars) from 1980 to 2001? Compare this with your answer in (b). What do you notice? Explain.

Percentage change in real price from 1980 to 2001 = $\frac{2.43-2.98}{2.98} = -0.18 = -18\%$. This answer is almost identical (except for rounding error) to the answer received for part b. It does not matter which year is chosen as the base year.

3.

Solution: $|\varepsilon_P| = 3$

Price elasticity of demand is given by $\varepsilon_P = \frac{dQ}{dP} \times \frac{P}{Q}$.

Demand function is: **Q = 200 – 10P**.

P = 15, Putting the value of P in demand function we get Q = 50. Differentiating Q wrt. P we get $\frac{dQ}{dP} = -10$.

Substituting all these values in elasticity equation we get $|\varepsilon_P| = 3$

4.

Solution: $P^* = 7$ and $Q^* = 30$

At equilibrium $Q_d = Q_s$.

Which gives: 51 – 3P = 6P – 12 , Solving it to get P^*. Put the value of P^* in either of demand or supply function and solve for Q^*.

5.

Solution: Excess supply

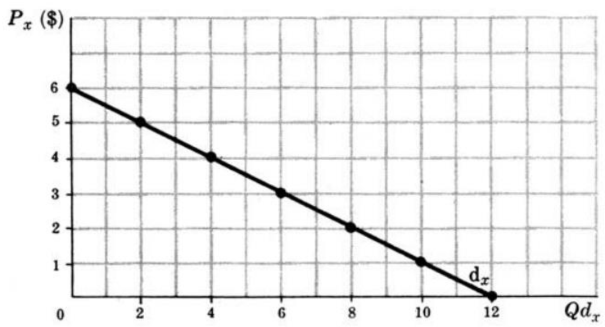
At equilibrium the supply and demand are same $Q_d = Q_s = Q^*$ (intersection of demand and supply curves) and price **P***. If price is set above **P*** then there will be supply more than **Q*** and demand will be less than **Q*** leading to excess supply.

6.

a.

P_x (\$)	6	5	4	3	2	1	0
Qd_x	0	2	4	6	8	10	12

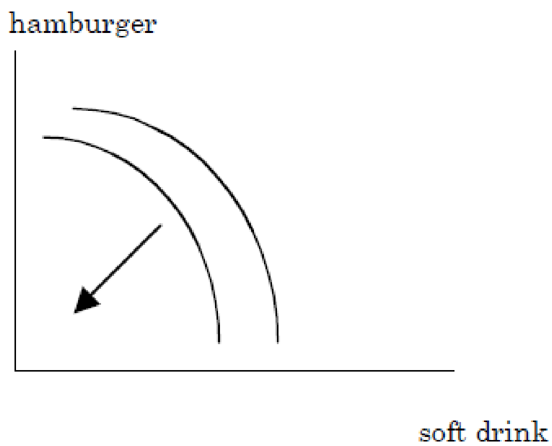
b.



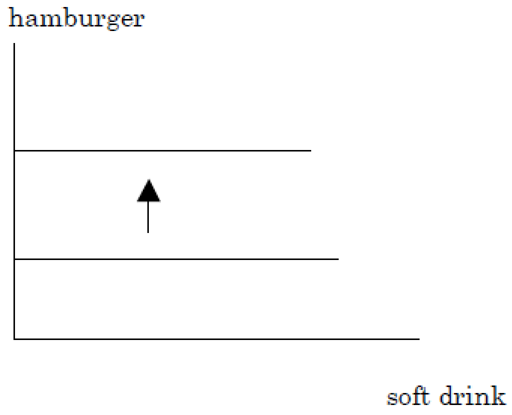
c. The maximum quantity of this commodity that the individual will ever demand per unit of time is 12 units. This occurs at a zero price. Additional units of X result in a storage and disposal problem for the individual. Thus the "relevant" points on a demand curve are all in the first quadrant.

7.

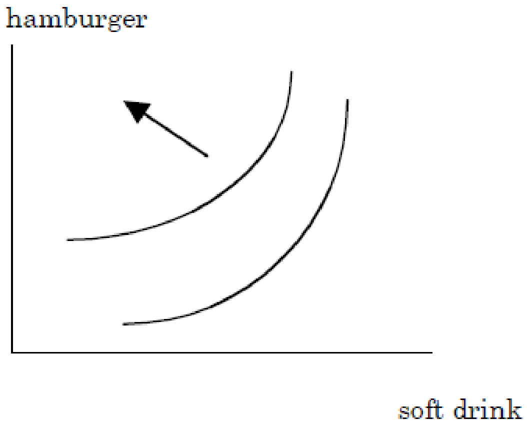
a. Since Joe dislikes both goods, his set of indifference curves will be bowed inwards towards the origin instead of outwards, as in the normal case where more is preferred to less. Given he dislikes both goods, his satisfaction is increasing in the direction of the origin. Convexity of preferences implies his indifference curves will have the normal shape in that they are bowed towards the direction of increasing satisfaction. Convexity also implies that given any two bundles between which the consumer is indifferent, the "average" of the two bundles will be in the preferred set or will leave him at least as well off.



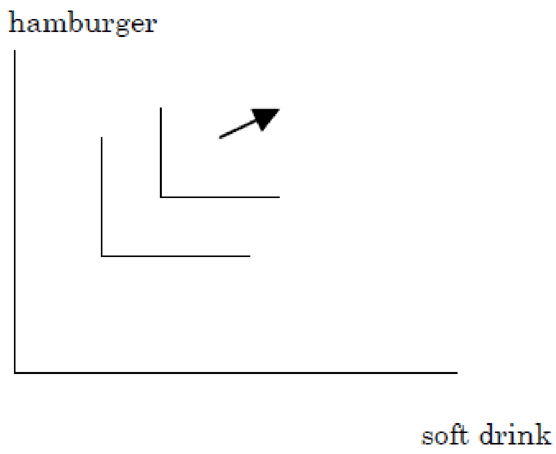
b. Since Jane can freely dispose of the soft drink if it is given to her, she considers it to be a neutral good. This means she does not care about soft drinks one way or the other. With hamburgers on the vertical axis, her indifference curves are horizontal lines. Her satisfaction increases in the upward direction.



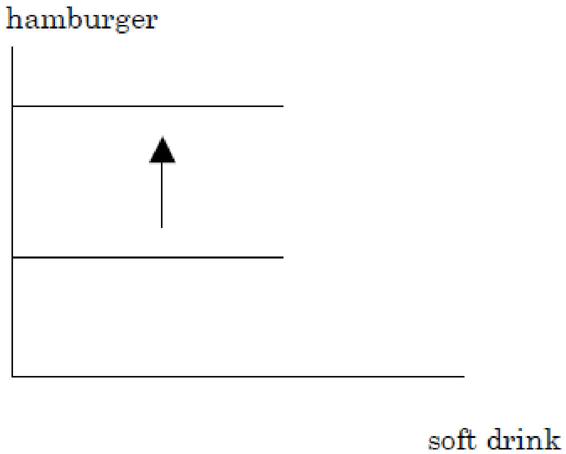
c. Since Bob will drink the soft drink in order to be polite, it can be thought of as a “bad”. When served another soft drink, he will require more hamburgers at the same time in order to keep his satisfaction constant. More soft drinks without more hamburgers will worsen his utility. More hamburgers and fewer soft drinks will increase his utility.



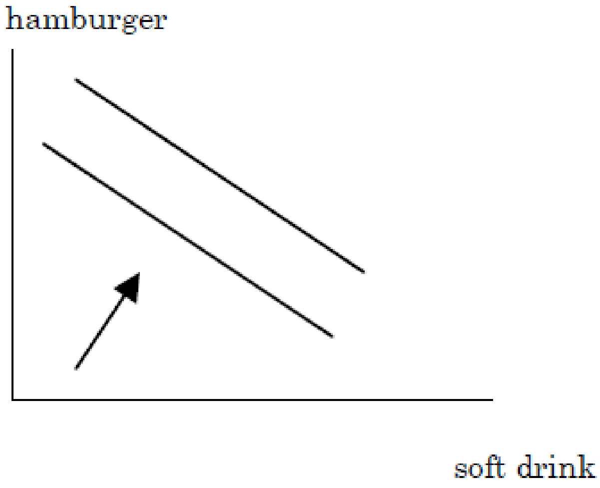
d. Molly wants to consume the two goods in a fixed proportion so her indifference curves are L-shaped. For any given amount of one good, she gets no extra satisfaction from having more of the other good. She will only increase her satisfaction if she has more of both goods.



e. Like Jane, Bill considers soft drinks to be a neutral good. Since he does not care about soft drinks one way or the other we can assume that no matter how many he has, his utility will be the same. His level of satisfaction depends entirely on how many hamburgers he has.



f. How much extra satisfaction Mary gains from an extra hamburger or soft drink tells us something about the marginal utilities of the two goods, or about her MRS. If she always receives twice the satisfaction from an extra hamburger then her marginal utility from consuming an extra hamburger is twice her marginal utility from consuming an extra soft drink. Her MRS, with hamburgers on the vertical axis, is 1/2. Her indifference curves are straight lines with a slope of 1/2.



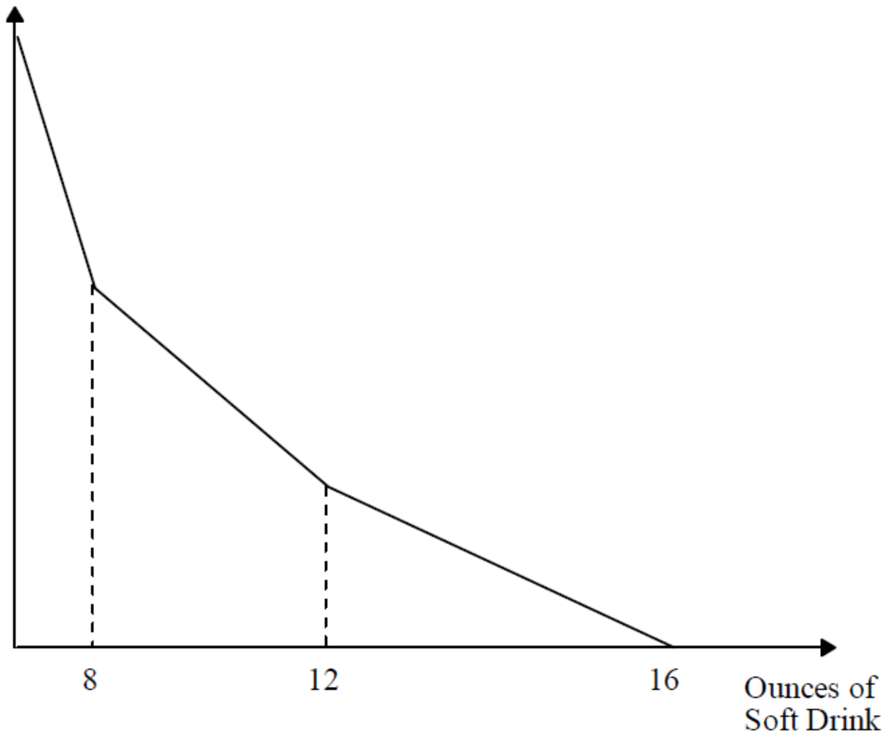
8.

a. Bridget receives a utility of 10*10*5=500 from this bundle. The indifference curve is represented by the equation 10FC=500 or FC=50. Some bundles on this indifference curve are (5,10), (10,5), (25,2), and (2,25). Erin receives a utility of .2*10*10*5*5=500 from the bundle (10,5). Her indifference curve is represented by the equation $500 = 0.20\ F^2C^2$, or 50=FC. This is the same indifference curve as Bridget. Both indifference curves have the normal, convex shape.

b. For each person, plug in F=15 and C=8 into their respective utility functions. For Bridget, this gives her a utility of 1200, so her indifference curve is given by the equation 10FC=1200, or FC=120. Some bundles on this indifference curve are (12,10), (10,12), (3,40), and (40,3). For Erin, this bundle gives her a utility of 2880, so her indifference curve is given by the equation $2880 = 0.20\ F^2C^2$, or FC=120. This is the same indifference curve as Bridget.

c. They have the same preferences because for any given bundle they have the same level of utility. This means that they will rank all bundles in the same order. Note however, that it is not necessary that they receive the same level of utility to have the same set of preferences. All that is necessary is that they rank the bundles in the same order.

9. First notice that as the size of the drink increases, the price per ounce decreases. When she buys the 8-ounce soft drink she pays $1.50 / 8 oz = $ 0.19 per oz. When she buys the 12-ounce size she pays $0.17 per ounce, and when she buys the 16-ounce size, she pays $0.14 per ounce. Given that there are three different prices per ounce of soft drink, the budget line will have two kinks in it, as illustrated below. Notice that at each kink, the slope of the budget line gets flatter (due to the decreasing cost per ounce relative to the "other good" on the vertical axis).



10. First, we need to calculate F and C, which make up the bundle of food and clothing which maximizes Meredith's utility given 1990 prices and her income in 1990. Use the hint to simplify the problem: Since she spends equal amounts on both goods, $P_F F = P_C C$. Or, you can derive this same equation mathematically: With this utility function, $MU_C = \Delta U/\Delta C = F$, and $MU_F = \Delta U/\Delta F = C$.

    To maximize utility, Meredith chooses a consumption bundle such that $MU_F/MU_C = P_F/P_C$, which again yields $P_F F = P_C C$.

    From the budget constraint, we also know that:
    $P_F F + P_C C = Y$.

    Combining these two equations and substituting the values for the 1990 prices and income yields the system of equations:
    $C = F$ and $C + F = 1{,}200$.
    Solving these two equations, we find that:
    $C = 600$ and $F = 600$.

    Laspeyres Index

    The Laspeyres index represents how much more Meredith would have to spend in 1995 versus 1990 if she consumed the same amounts of food and clothing in 1995 as she did in 1990. That is, the Laspeyres index for 1995 (L) is given by:

    $L = 100\,(Y')/Y$

where Y’ represents the amount Meredith would spend at 1995 prices consuming the same amount of food and clothing as in 1990. In 1995, 600 clothing and 600 food would cost ($3)(600)+($2)(600)=$3000.

Therefore, the Laspeyres cost-of-living index is:
L = 100($3000/$1200) = 250.

Ideal Index

The ideal index represents how much Meredith would have to spend on food and clothing in 1995 to get the same amount of utility as she had in 1990. That is, the ideal index for 1995 (I) is given by:

$I = 100(Y'')/Y$, where $Y'' = P'_F F' + P'_C C' = 2F' + 3C'$

where F' and C' are the amount of food and clothing that give Meredith the same utility as she had in 1990. F' and C' must also be such that Meredith spends the least amount of money at 1995 prices to attain the 1990 utility level.

The bundle (F',C') will be on the same indifference curve as (F,C) so F'C'=FC=360,000 in utility. If Meredith’s income is adjusted in 1995 so that the bundle (F',C') is maximizing her utility given her income, then the indifference curve at this point will be tangent to the budget line with slope $-(P'_F/P'_C)$, where P'_F and P'_C are the prices of food and clothing in 1995.

Using $MU_F'/MU_C' = P_F'/P_C'$ we know that 2F' = 3C'.
We now have two equations: F'C'=360,000 and 2F' = 3C'.

Solving for F' yields:
$F'[(2/3)F'] = 360{,}000$ or $F' = \sqrt{[(3/2)360{,}000)]} = 734.8$.

From this, we obtain C':
C' = (2/3)F' = (2/3)734.8 = 489.9.

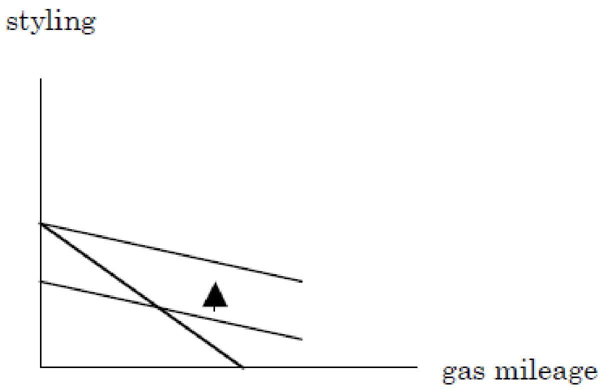
In 1995, the bundle of 734.8 food and 489.9 clothing would cost $2939.60 and Meredith would still get 360,000 in utility.

We can now calculate the ideal index:
I = 100($2939.60/$1200) = 244.9.

11.

a. For every $5,000 she spends on style the index rises by one so the most she can achieve is a car with a style index of 5. For every $2,500 she spends on gas mileage, the index rises by one so the most she can achieve is a car with a gas mileage index of 10. The slope of her “budget line” is -1/2.

b. If Brenda always receives three times as much satisfaction from an extra unit of styling as she does from an extra unit of gas mileage then she is willing to trade on unit of styling for three units of gas mileage, and still maintain the same level of satisfaction. This is her MRS or the slope of her indifference curves, which is constant. Since the MRS is 1/3 and the slope of her budget line is -1/2, Brenda will choose all styling. You can also compute the marginal utility per dollar for styling and gas mileage and note that styling will be higher. In the graph below, she will move up to the highest possible indifference curve where she chooses all styling and no gas mileage.



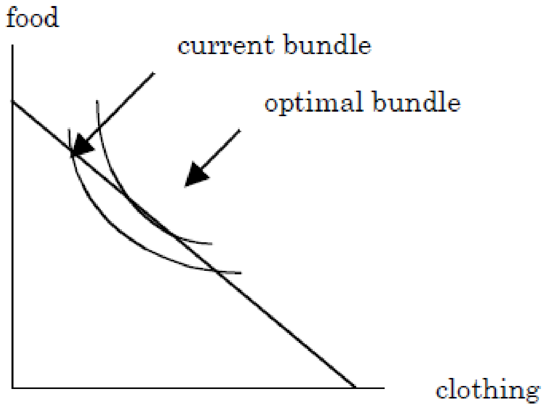
c. To find the optimal value of each index, set MRS equal to the price ratio of 1/2 and cross multiply to get S=2G.
Now substitute into the budget 5000S+2500G=25000 to get G=2 and S=4.

d. To find the optimal value of each index set MRS equal to the price ratio of 1/2 and cross multiply to get G=6S. Now substitute into the budget 5000S+2500G=25000 to get G=7.5 and S=1.25.

12.

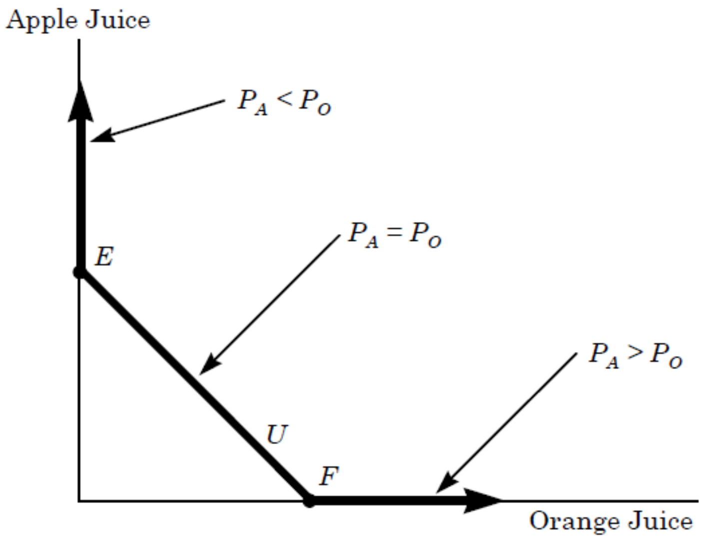
a. Utility is maximized when MRS (food for clothing) equals P_C/P_F, the price ratio. Given that clothing is on the horizontal axis and food is on the vertical axis, then the price ratio is the slope of the budget line, which is price of clothing divided by the price of food or -5.

b. In absolute value terms, the slope of his indifference curve at this non-optimal bundle is greater than the slope of his budget line. He is willing to give up more food than he has to at market prices to obtain one more unit of clothing. He will therefore find it optimal to give up some food in exchange for clothing.

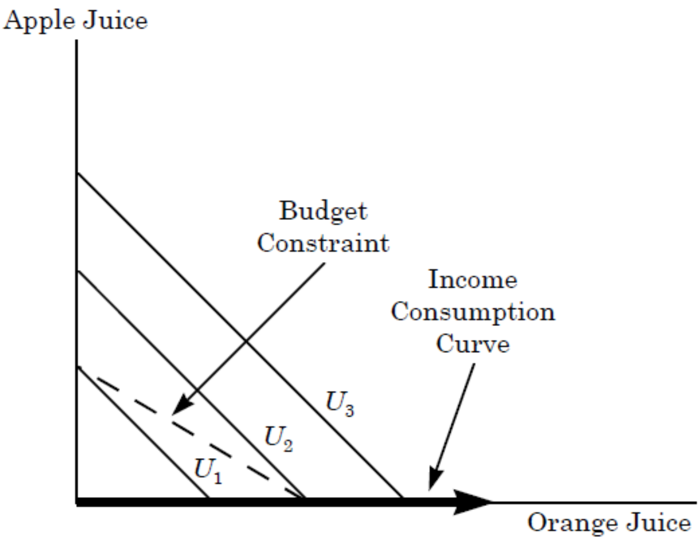


13. Orange juice and apple juice are known to be perfect substitutes. Draw the appropriate price-consumption (for a variable price of orange juice) and income consumption curves.

We know that the indifference curves for perfect substitutes will be straight lines. In this case, the consumer will always purchase the cheaper of the two goods. If the price of orange juice is less than that of apple juice, the consumer will purchase only orange juice and the price consumption curve will be on the "orange juice axis" of the graph (point F). If apple juice is cheaper, the consumer will purchase only apple juice and the price consumption curve will be on the "apple juice axis" (point E). If the two goods have the same price, the consumer will be indifferent between the two; the price consumption curve will coincide with the indifference curve (between E and F). See the figure below.

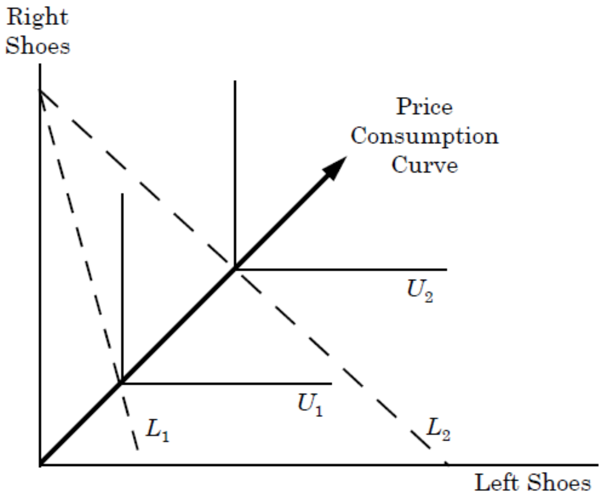


Assuming that the price of orange juice is less than the price of apple juice, the consumer will maximize her utility by consuming only orange juice. As the level of income varies, only the amount of orange juice varies. Thus, the income consumption curve will be the "orange juice axis" in the figure below.

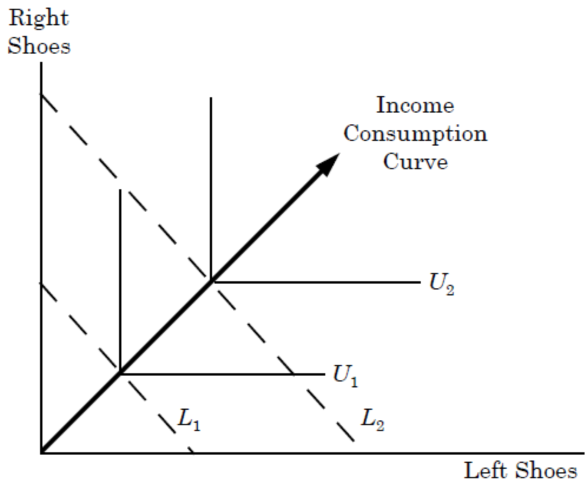


14. **Left shoes and right shoes are perfect complements. Draw the appropriate price consumption and income-consumption curves.**

For goods that are perfect complements, such as right shoes and left shoes, we know that the indifference curves are L-shaped. The point of utility maximization occurs when the budget constraints, L_1 and L_2 touch the kink of U_1 and U_2. See the following figure.



In the case of perfect complements, the income consumption curve is also a line through the corners of the L-shaped indifference curves. See the figure below.



15. **The director of a theatre company in a small college town is considering changing the way he prices tickets. He has hired an economic consulting firm to estimate the demand for tickets. The firm has classified people who go the theatre into two groups and has come up with two demand functions. The demand curves for the general public (Q_{gp}) and students (Q_s) are given below.**

$$Q_{gp} = 500 - 5P$$

$$Q_s = 200 - 4P$$

a. **Graph the two demand curves on one graph, with P on the vertical axis and Q on the horizontal axis. If the current price of tickets is $35, identify the quantity demanded by each group.**

Both demand curves are downward sloping and linear. For the general public, the vertical intercept is 100 and the horizontal intercept is 500. For the students, the vertical intercept is 50 and the horizontal intercept is 200. The general public demands $Q_{gp} = 500 - 5(35) = 325$ tickets and the students demand $Q_s = 200 - 4(35) = 60$ tickets.

b. **Find the price elasticity of demand for each group at the current price and quantity.**

The elasticity for the general public is $\varepsilon_{gp} = \frac{-5(35)}{325} = -0.54$ and the elasticity for the students is $\varepsilon_{gp} = \frac{-4(35)}{60} = -2.33$. If the price of tickets increases by one percent, then the general public will demand .54% fewer tickets and the students will demand 2.33% fewer tickets.

c. **Is the director maximizing the revenue he collects from ticket sales by charging $35 for each ticket? Explain.**

No, he is not maximizing revenue since neither one of the calculated elasticities is equal to –1. Since demand by the general public is inelastic at the current price, the director could increase the price and quantity demanded would fall by a smaller amount in percentage terms, causing revenue to increase. Since demand by the students is elastic at the current price, the director could decrease the price and quantity demanded would increase by a larger amount in percentage terms, causing revenue to increase.

d. **What price should he charge each group if he wants to maximize revenue collected from ticket sales?**

To figure this out, find the formula for elasticity, set it equal to –1, and solve for price and quantity. For the general public:

$$\varepsilon_{gp} = \frac{-5(P)}{Q} = -1$$

5P = Q = 500 - 5P

P = 50

Q = 250.

For the students:

$$\varepsilon_{s} = \frac{-4(P)}{Q} = -1$$

4P = Q = 200 - 4P

P = 25

Q = 100.

16. **Suppose the income elasticity of demand for food is 0.5, and the price elasticity of demand is –1.0. Suppose also that Felicia spends \$10,000 a year on food, the price of food is \$2, and her income is \$25,000.**
    a. **If a sales tax on food were to cause the price of food to increase to \$2.50, what would happen to her consumption of food? (Hint: Since a large price change is involved, you should assume that the price elasticity measures an arc elasticity, rather than a point elasticity.)**

The price of food increases from \$2 to \$2.50, so arc elasticity should be used:

$$E_P = \left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{\frac{P_1 + P_2}{2}}{\frac{Q_1 + Q_2}{2}}\right).$$

We know that E_P = -1, P = 2, ΔP = 0.5, and Q=5000. We also know that Q_2, the new quantity, is Q + ΔQ. Thus, if there is no change in income, we may solve for ΔQ:

$$-1 = \left(\frac{\Delta Q}{0.5}\right)\left(\frac{\frac{2 + 2.5}{2}}{\frac{5{,}000 + (5{,}000 + \Delta Q)}{2}}\right).$$

By cross-multiplying and rearranging terms, we find that ΔQ = -1,000. This means that she decreases her consumption of food from 5,000 to 4,000 units.

b. Suppose that she is given a tax rebate of $2,500 to ease the effect of the sales tax. What would her consumption of food be now?

A tax rebate of $2,500 implies an income increase of $2,500. To calculate the response of demand to the tax rebate, use the definition of the arc elasticity of income.

$$E_I = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{\frac{I_1 + I_2}{2}}{\frac{Q_1 + Q_2}{2}}\right).$$

We know that $E_I = 0.5$, I = 25,000, ΔI = 2,500, Q = 4,000 (from the answer to 4.a).

Assuming no change in price, we solve for ΔQ.

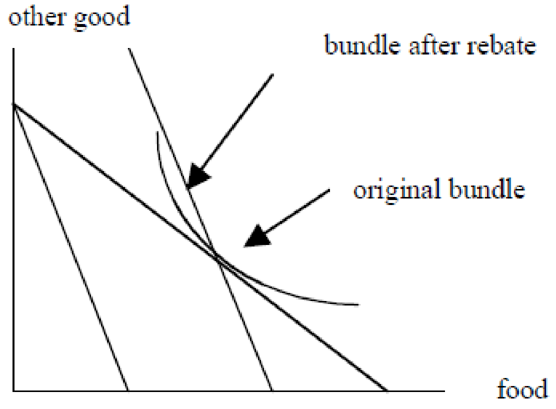
$$0.5 = \left(\frac{\Delta Q}{2{,}500}\right)\left(\frac{\frac{25{,}000 + 27{,}500}{2}}{\frac{4{,}000 + (4{,}000 + \Delta Q)}{2}}\right).$$

By cross-multiplying and rearranging terms, we find that ΔQ = 195 (approximately). This means that she increases her consumption of food from 4,000 to 4,195 units.

c. Is she better or worse off when given a rebate equal to the sales tax payments? Draw a graph and explain.

Felicia is likely to be better off after the rebate. The amount of the rebate is enough to allow her to purchase her original bundle of food and other goods. Recall that originally, she consumed 5000 units of food. When the price went up by fifty cents per unit, she needed an extra 5000*$0.50=$2,500 to afford the same quantity of food without reducing the quantity of the other goods consumed. This is the exact amount of the rebate. However, she did not choose to return to her original bundle. We can therefore infer that she found a better bundle that gave her a higher level of utility.

In the graph below, when the price of food increases, the budget line will pivot inwards. When the rebate is given, this new budget line will shift outwards. The bundle after the rebate is on that part of the new budget line that was previously unaffordable, and that lies above the original indifference curve.



17. **Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings Q is given by P = 15 – 0.5 Q.**
    a. **Draw the demand curve for bridge crossings.**

The demand curve is linear and downward sloping. The vertical intercept is 15 and the horizontal intercept is 30.

    b. **How many people would cross the bridge if there were no toll?**

At a price of zero, the quantity demanded would be 30.

    c. **What is the loss of consumer surplus associated with a bridge toll of $5?**

If the toll is $5 then the quantity demanded is 20. The lost consumer surplus is the area below the price line of $5 and to the left of the demand curve. The lost consumer surplus can be calculated as (5*20) + 0.5(5*10) = $125.

18. **Show that two utility functions given below generate the identical demand functions for goods X and Y:**
a. U(X, Y) = log(X) + log(Y)
b. U(X, Y) = $(XY)^{0.5}$

If we show that the two utility functions are equivalent, then we know that the demand functions derived from them are identical. We may show their equivalence by identifying a suitable transformation from one set of numbers into another set without changing their order. Taking the logarithm of U(X, Y) = $(XY)^{0.5}$ we obtain:
Log U(X, Y) = 0.5log(X) + 0.5log(Y).
Now multiply both sides by 2:
2(log U(X, Y) = log(X) + log(Y).
Therefore, the two utility functions are equivalent and will yield identical demand functions.

However, we will solve for the demand functions in both cases to show that they are the same.
a. To find the demand functions for X and Y, corresponding to U(X, Y) = log(X) + log(Y), given the usual budget constraint, write the Lagrangian:

$$\Phi = \log(X) + \log(Y) - \lambda(P_X X + P_Y Y - I)$$

Differentiating with respect to X, Y, λ, and setting the derivatives equal to zero:

$$\frac{\partial\Phi}{\partial X} = \frac{1}{X} - \lambda P_X = 0$$

$$\frac{\partial\Phi}{\partial Y} = \frac{1}{Y} - \lambda P_Y = 0$$

$$\frac{\partial\Phi}{\partial\lambda} = P_X X + P_Y Y - I = 0.$$

The first two conditions imply that $P_X X = \frac{1}{\lambda}$ and $P_Y Y = \frac{1}{\lambda}$.

The third condition implies that $\frac{1}{\lambda} + \frac{1}{\lambda} - I = 0$, or $\lambda = \frac{2}{I}$.

Substituting this expression into $P_X X = \frac{I}{\lambda}$ and $P_Y Y = \frac{I}{\lambda}$ gives the demand functions:

$$X = \left(\frac{0.5}{P_X}\right) I \quad \text{and} \quad Y = \left(\frac{0.5}{P_Y}\right) I.$$

Notice that the demand for each good depends only on the price of that good and on income, not on the price of the other good.

b. To find the demand functions for X and Y, corresponding to $U(X,Y) = (XY)^{0.5}$ given the usual budget constraint, first write the Lagrangian:

$$\Phi = 0.5(\log X) + (1 - 0.5)\log Y - \lambda(P_X X + P_Y Y - I)$$

Differentiating with respect to X, Y, λ and setting the derivatives equal to zero:

$$\frac{\partial \Phi}{\partial X} = \frac{0.5}{X} - \lambda P_X = 0$$

$$\frac{\partial \Phi}{\partial Y} = \frac{0.5}{Y} - \lambda P_Y = 0$$

$$\frac{\partial \Phi}{\partial \lambda} = P_X X + P_Y Y - I = 0.$$

The first two conditions imply that $P_X X = \frac{0.5}{\lambda}$ and $P_Y Y = \frac{0.5}{\lambda}$.

Combining these with the budget constraint gives: $\frac{0.5}{\lambda} + \frac{0.5}{\lambda} - I = 0$ or $\lambda = \frac{1}{I}$.

Now substituting this expression into $P_X X = \frac{0.5}{\lambda}$ and $P_Y Y = \frac{0.5}{\lambda}$ gives the demand functions:

$$X = \left(\frac{0.5}{P_X}\right) I \quad \text{and} \quad Y = \left(\frac{0.5}{P_Y}\right) I.$$