ECO101A: Introduction to Economics

Tutorial 6 solution

1. **Demand for light bulbs can be characterized by $Q = 100 - P$, where Q is in millions of lights sold, and P is the price per box. There are two producers of lights: *Everglow* and *Dimlit*. They have identical cost functions:**

$$C_i = 10Q_i + \frac{1}{2}Q_i^2, (i = E, D); Q = Q_E + Q_D$$

a. **Unable to recognize the potential for collusion, the two firms act as short-run perfect competitors. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?**

Given that the total cost function is $C_i = 10Q_i + 1/2Q_i^2$, the marginal cost curve for each firm is $MC_i = 10 + Q_i$. In the short run, perfectly competitive firms determine the optimal level of output by taking price as given and setting price equal to marginal cost. There are two ways to solve this problem. One way is to set price equal to marginal cost for each firm so that:

$$P = 100 - Q_1 - Q_2 = 10 + Q_1$$
$$P = 100 - Q_1 - Q_2 = 10 + Q_2.$$

Given we now have two equations and two unknowns, we can solve for Q_1 and Q_2. Solve the second equation for Q_2 to get

$$Q_2 = \frac{90 - Q_1}{2},$$

and substitute into the other equation to get

$$100 - Q_1 - \frac{90 - Q_1}{2} = 10 + Q_1.$$

This yields a solution where Q_1=30, Q_2=30, and P=40. You can verify that P=MC for each firm. Profit is total revenue minus total cost or

$$\Pi = 40 * 30 - (10 * 30 + 0.5 * 30 * 30) = \$450 \text{ million}.$$

The other way to solve the problem and arrive at the same solution is to find the market supply curve by summing the marginal cost curves, so that QM=2P-20 is the market supply. Setting supply equal to demand results in a quantity of 60 in the market, or 30 per firm since they are identical.

b. Top management in both firms is replaced. Each new manager independently recognizes the oligopolistic nature of the light bulb industry and plays Cournot. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

To determine the Cournot-Nash equilibrium, we first calculate the reaction function for each firm, then solve for price, quantity, and profit. Profits for Everglow are equal to TR_E - TC_E, or

$$\pi_E = (100 - Q_E - Q_D)Q_E - (10Q_E + 0.5Q_E^2) = 90Q_E - 1.5Q_E^2 - Q_E Q_D.$$

The change in profit with respect to Q_E is

$$\frac{\partial \pi_E}{\partial Q_E} = 90 - 3Q_E - Q_D.$$

To determine Everglow's reaction function, set the change in profits with respect to Q_E equal to 0 and solve for Q_E:

$$90 - 3Q_E - Q_D = 0, \text{ or}$$

$$Q_E = \frac{90 - Q_D}{3}.$$

Because Dimlit has the same cost structure, Dimlit's reaction function is

$$Q_D = \frac{90 - Q_E}{3}.$$

Substituting for Q_D in the reaction function for Everglow, and solving for Q_E:

$$Q_E = \frac{90 - \frac{90 - Q_E}{3}}{3}$$

$$3Q_E = 90 - 30 + \frac{Q_E}{3}$$

$$Q_E = 22.5.$$

By symmetry, $Q_D = 22.5$, and total industry output is 45.

Substituting industry output into the demand equation gives P:

$$45 = 100 - P, \text{ or } P = \$55.$$

Substituting total industry output and P into the profit function:

$$\Pi_i = 22.5 * 55 - (10 * 22.5 + 0.5 * 22.5 * 22.5) = \$759.375 \text{ million}.$$

c. Suppose the Everglow manager guesses correctly that Dimlit has a Cournot conjectural variation, so Everglow plays Stackelberg. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Recall Everglow's profit function:

$$\pi_E = (100 - Q_E - Q_D)\, Q_E - (10Q_E + 0.5Q_E^2)$$

If Everglow sets its quantity first, knowing Dimlit's reaction function $\left(\text{i.e., } Q_D = 30 - \frac{Q_E}{3}\right)$, we may determine Everglow's reaction function by substituting for Q_D in its profit function. We find

$$\pi_E = 60Q_E - \frac{7Q_E^2}{6}.$$

To determine the profit-maximizing quantity, differentiate profit with respect to Q_E, set the derivative to zero and solve for Q_E:

$$\frac{\partial \pi_E}{\partial Q_E} = 60 - \frac{7Q_E}{3} = 0, \text{ or } Q_E = 25.7.$$

Substituting this into Dimlit's reaction function, we find $Q_D = 30 - \frac{25.7}{3} = 21.4$. Total industry output is 47.1 and $P = \$52.90$. Profit for Everglow is \$772.29 million. Profit for Dimlit is \$689.08 million.

d. If the managers of the two companies collude, what are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

If the firms split the market equally, total cost in the industry is $10Q_T + \frac{Q_T^2}{2}$; therefore, $MC = 10 + Q_T$. Total revenue is $100Q_T - Q_T^2$; therefore, $MR = 100 - 2Q_T$. To determine the profit-maximizing quantity, set $MR = MC$ and solve for Q_T:

$$100 - 2Q_T = 10 + Q_T, \text{ or } Q_T = 30.$$

This means $Q_E = Q_D = 15$.

Substituting Q_T into the demand equation to determine price:

$$P = 100 - 30 = \$70.$$

The profit for each firm is equal to total revenue minus total cost:

$$\pi_i = (70)(15) - \left((10)(15) + \frac{15^2}{2}\right) = \$787.50 \text{ million.}$$

2. **Two competing firms are each planning to introduce a new product. Each will decide whether to produce Product A, Product B, or Product C. They will make their choices at the same time. The resulting payoffs are shown below.**

   We are given the following payoff matrix, which describes a product introduction game:

		Firm 2		
		A	*B*	*C*
	A	-10,-10	0,10	10,20
Firm 1	*B*	10,0	-20,-20	-5,15
	C	20,10	15,-5	-30,-30

   a. Are there any Nash equilibria in pure strategies? If so, what are they?

   There are two Nash equilibria in pure strategies. Each one involves one firm introducing Product A and the other firm introducing Product C. We can write these two strategy pairs as (A, C) and (C, A), where the first strategy is for player 1. The payoff for these two strategies is, respectively, (10,20) and (20,10).

   b. If both firms use maximin strategies, what outcome will result?

   Recall that maximin strategies maximize the minimum payoff for both players. For each of the players the strategy that maximizes their minimum payoff is A. Thus (A,A) will result, and payoffs will be (-10,-10). Each player is much worse off than at either of the pure strategy Nash equilibrium.

   c. If Firm 1 uses a maximin strategy and Firm 2 knows, what will Firm 2 do?

   If Firm 1 plays its maximin strategy of A, and Firm 2 knows this then Firm 2 would get the highest payoff by playing C. Notice that when Firm 1 plays conservatively, the Nash equilibrium that results gives Firm 2 the highest payoff of the two Nash equilibria.