ECO101A: Introduction to Economics

Tutorial 5 Solution

1. A firm has two factories for which costs are given by:

Factory #1: $C_1(Q_1) = 10\ Q_1^2$

Factory # 2: $C_2(Q_2) = 20\ Q_2^2$

The firm faces the following demand curve: *P = 700 - 5Q*

where Q is total output, i.e. $Q = Q_1 + Q_2$.

a. On a diagram, draw the marginal cost curves for the two factories, the average and marginal revenue curves, and the total marginal cost curve (i.e., the marginal cost of producing $Q = Q_1 + Q_2$). Indicate the profit-maximizing output for each factory, total output, and price.

The average revenue curve is the demand curve,

$$P = 700 - 5Q.$$

For a linear demand curve, the marginal revenue curve has the same intercept as the demand curve and a slope that is twice as steep:

$$MR = 700 - 10Q.$$

Next, determine the marginal cost of producing Q. To find the marginal cost of production in Factory 1, take the first derivative of the cost function with respect to Q:

$$\frac{dC_1(Q_1)}{dQ} = 20Q_1.$$

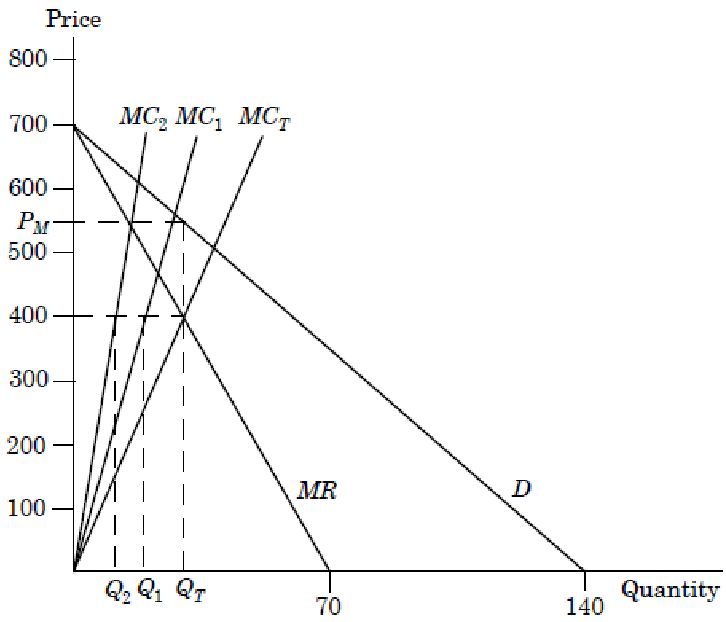
Similarly, the marginal cost in Factory 2 is

$$\frac{dC_2(Q_2)}{dQ} = 40Q_2.$$

Rearranging the marginal cost equations in inverse form and horizontally summing them, we obtain total marginal cost, MC_T:

$$Q = Q_1 + Q_2 = \frac{MC_1}{20} + \frac{MC_2}{40} = \frac{3MC_T}{40}, \text{ or}$$
$$MC_T = \frac{40Q}{3}.$$

Profit maximization occurs where $MC_T = MR$. See Figure 10.8.a for the profit-maximizing output for each factory, total output, and price.



b. Calculate the values of Q_1, Q_2, Q, and P that maximize profit.

Calculate the total output that maximizes profit, i.e., Q such that $MC_T = MR$:

$$\frac{40Q}{3} = 700 - 10Q, \text{ or } Q = 30.$$

Next, observe the relationship between MC and MR for multiplant monopolies:

$$MR = MC_T = MC_1 = MC_2.$$

We know that at $Q = 30$, $MR = 700 - (10)(30) = 400$.

Therefore,

$$MC_1 = 400 = 20Q_1, \text{ or } Q_1 = 20 \text{ and}$$

$$MC_2 = 400 = 40Q_2, \text{ or } Q_2 = 10.$$

To find the monopoly price, P_M, substitute for Q in the demand equation:

$$P_M = 700 - (5)(30), \text{ or}$$

$$P_M = 550.$$

c. Suppose labor costs increase in Factory 1 but not in Factory 2. How should the firm adjust the following (i.e., raise, lower, or leave unchanged): Output in Factory 1? Output in Factory 2? Total output? Price?

An increase in labor costs will lead to a horizontal shift to the left in MC_1, causing MC_T to shift to the left as well (since it is the horizontal sum of MC_1 and MC_2). The new MC_T curve intersects the MR curve at a lower quantity and higher marginal revenue. At a higher level of marginal revenue, Q_2 is greater than at the original level for MR. Since Q_T falls and Q_2 rises, Q_1 must fall. Since Q_T falls, price must rise.

2. **Dayna's Doorstops, Inc. (DD), is a monopolist in the doorstop industry. Its cost is *C = 100 - 5Q + Q²*, and demand is *P = 55 - 2Q*.**

   a. **What price should DD set to maximize profit? What output does the firm produce? How much profit and consumer surplus does DD generate?**

   To maximize profits, DD should equate marginal revenue and marginal cost. Given a demand of $P = 55 - 2Q$, we know that total revenue, PQ, is $55Q - 2Q^2$. Marginal revenue is found by taking the first derivative of total revenue with respect to Q or:

$$MR = \frac{dTR}{dQ} = 55 - 4Q.$$

   Similarly, marginal cost is determined by taking the first derivative of the total cost function with respect to Q or:

$$MC = \frac{dTC}{dQ} = 2Q - 5.$$

   Equating MC and MR to determine the profit-maximizing quantity,

$$55 - 4Q = 2Q - 5, \text{ or}$$

$$Q = 10.$$

   Substituting $Q = 10$ into the demand equation to determine the profit-maximizing price:

$$P = 55 - (2)(10) = \$35.$$

   Profits are equal to total revenue minus total cost:

$$\pi = (35)(10) - (100 - (5)(10) + 10^2) = \$200.$$

   Consumer surplus is equal to one-half times the profit-maximizing quantity, 10, times the difference between the demand intercept (the maximum price anyone is willing to pay) and the monopoly price:

$$CS = (0.5)(10)(55 - 35) = \$100.$$

   b. **What would output be if DD acted like a perfect competitor and set MC = P? What profit and consumer surplus would then be generated?**

   In competition, profits are maximized at the point where price equals marginal cost, where price is given by the demand curve:

$$55 - 2Q = -5 + 2Q, \text{ or}$$

$$Q = 15.$$

   Substituting $Q = 15$ into the demand equation to determine the price:

$$P = 55 - (2)(15) = \$25.$$

   Profits are total revenue minus total cost or:

$$\pi = (25)(15) - (100 - (5)(15) + 15^2) = \$125.$$

   Consumer surplus is

$$CS = (0.5)(55 - 25)(15) = \$225.$$

   c. **What is the deadweight loss from monopoly power in part (a)?**

   The deadweight loss is equal to the area below the demand curve, above the marginal cost curve, and between the quantities of 10 and 15, or numerically

$$DWL = (0.5)(35 - 15)(15 - 10) = \$50.$$

d. Suppose the government, concerned about the high price of doorstops, sets a maximum price at $27. How does this affect price, quantity, consumer surplus, and DD's profit? What is the resulting deadweight loss?

With the imposition of a price ceiling, the maximum price that DD may charge is $27.00. Note that when a ceiling price is set above the competitive price the ceiling price is equal to marginal revenue for all levels of output sold up to the competitive level of output. Substitute the ceiling price of $27.00 into the demand equation to determine the effect on the equilibrium quantity sold:

$$27 = 55 - 2Q, \text{ or } Q = 14.$$

Consumer surplus is

$$CS = (0.5)(55 - 27)(14) = \$196.$$

$$\text{Profits} = (27)(14) - (100 - (5)(14) + 14^2) = \$152.$$

The deadweight loss is $2.00 This is equivalent to a triangle of

$$(0.5)(15 - 14)(27 - 23) = \$2$$

e. Now suppose the government sets the maximum price at $23. How does this affect price, quantity, consumer surplus, DD's profit, and deadweight loss?

With a ceiling price set below the competitive price, DD will decrease its output. Equate marginal revenue and marginal cost to determine the profit-maximizing level of output:

$$23 = -5 + 2Q, \text{ or } Q = 14.$$

With the government-imposed maximum price of $23,

$$\text{profits} = (23)(14) - (100 - (5)(14) + 14^2) = \$96.$$

Consumer surplus is realized on only 14 doorsteps. Therefore, it is equal to the consumer surplus in part d., i.e. $196, plus the savings on each doorstep, i.e.,

$$CS = (27 - 23)(14) = \$56.$$

Therefore, consumer surplus is $252. Deadweight loss is the same as before, $2.00.

f. Finally, consider a maximum price of $12. What will this do to quantity, consumer surplus, profit, and deadweight loss?

With a maximum price of only $12, output decreases even further:

$$12 = -5 + 2Q, \text{ or } Q = 8.5.$$

$$\text{Profits} = (12)(8.5) - (100 - (5)(8.5) + 8.5^2) = -\$27.75.$$

Consumer surplus is realized on only 8.5 units, which is equivalent to the consumer surplus associated with a price of $38 (38 = 55 - 2(8.5)), i.e.,

$$(0.5)(55 - 38)(8.5) = \$72.25$$

plus the savings on each doorstep, i.e.,

$$(38 - 12)(8.5) = \$221.$$

Therefore, consumer surplus is $293.25. Total surplus is $265.50, and deadweight loss is $84.50.