# ECO101A: Introduction to Economics

## Tutorial 7 Solution

1. **You are a duopolist producer of a homogeneous good. Both you and your competitor have zero marginal costs. The market demand curve is**
   ***P = 30 – Q***
   **where *Q = $Q_1$ + $Q_2$*. $Q_1$ is your output and $Q_2$ your competitor's output. Your competitor has also read this book.**
   a. **Suppose you will play this game only once. If you and your competitor must announce your output at the same time, how much will you choose to produce? What do you expect your profit to be? Explain.**

For Firm 1, total revenue will be

$$TR_1 = (30 - (Q_1 + Q_2))Q_1, \text{ or } TR_1 = 30Q_1 - Q_1^2 - Q_1Q_2.$$

Marginal revenue for Firm 1 will be the derivative of total revenue with respect to $Q_1$,

$$\frac{\partial TR}{\partial Q_1} = 30 - 2Q_1 - Q_2.$$

Because the firms share identical demand curves, the solution for Firm 2 will be symmetric to that of Firm 1:

$$\frac{\partial TR}{\partial Q_2} = 30 - 2Q_2 - Q_1.$$

To find the profit-maximizing level of output for both firms, set marginal revenue equal to marginal cost, which is zero:

$$Q_1 = 15 - \frac{Q_2}{2} \quad \text{and}$$
$$Q_2 = 15 - \frac{Q_1}{2}.$$

With two equations and two unknowns, we may solve for $Q_1$ and $Q_2$:

$$Q_1 = 15 - (0.5)\left(15 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 10.$$

By symmetry, $Q_2 = 10$.

Substitute $Q_1$ and $Q_2$ into the demand equation to determine price:

$$P = 30 - (10 + 10), \text{ or } P = \$10.$$

Since no costs are given, profits for each firm will be equal to total revenue:

$$\pi_1 = TR_1 = (10)(10) = \$100 \quad \text{and}$$
$$\pi_2 = TR_2 = (10)(10) = \$100.$$

Thus, the equilibrium occurs when both firms produce 10 units of output and both firms earn \$100. Looking back at the payoff matrix, note that the outcome (100, 100) is indeed a Nash equilibrium: neither firm will have an incentive to deviate, given the other firm's choice.

**b. Suppose you are told that you must announce your output before your competitor does. How much will you produce in this case, and how much do you think your competitor will produce? What do you expect your profit to be? Is announcing first an advantage or disadvantage? Explain briefly. How much would you pay to be given the option of announcing either first or second?**

If you must announce first, you would announce an output of 15, knowing that your competitor would announce an output of 7.5. (Note: This is the Stackelberg equilibrium.)

$$TR_1 = (30 - (Q_1 + Q_2))Q_1 = 30Q_1 - Q_1^2 - Q_1\left(15 - \frac{Q_1}{2}\right) = 15Q_1 - \frac{Q_1^2}{2}.$$

Therefore, setting $MR = MC = 0$ implies:

$$15 - Q_1 = 0, \text{ or } Q_1 = 15 \text{ and}$$
$$Q_2 = 7.5.$$

At that output, your competitor is maximizing profits, given that you are producing 15. At these outputs, price is equal to

$$30 - 15 - 7.5 = \$7.5.$$

Your profit would be

$$(15)(7.5) = \$112.5.$$

Your competitor's profit would be

$$(7.5)(7.5) = \$56.25.$$

Announcing first is an advantage in this game. The difference in profits between announcing first and announcing second is \$56.25. You would be willing to pay up to this difference for the option of announcing first.

**c. Suppose instead that you are to play the first round of a series of 10 rounds (with the same competitor). In each round, you and your competitor announce your outputs at the same time. You want to maximize the sum of your profits over the 10 rounds. How much will you produce in the first round? How much do you expect to produce in the tenth round? In the ninth round? Explain briefly.**

Given that your competitor has also read this book, you can assume that he or she will be acting rationally. You should begin with the Cournot output and continue with the Cournot output in each round, including the ninth and tenth rounds. Any deviation from this output will reduce the sum of your profits over the ten rounds.

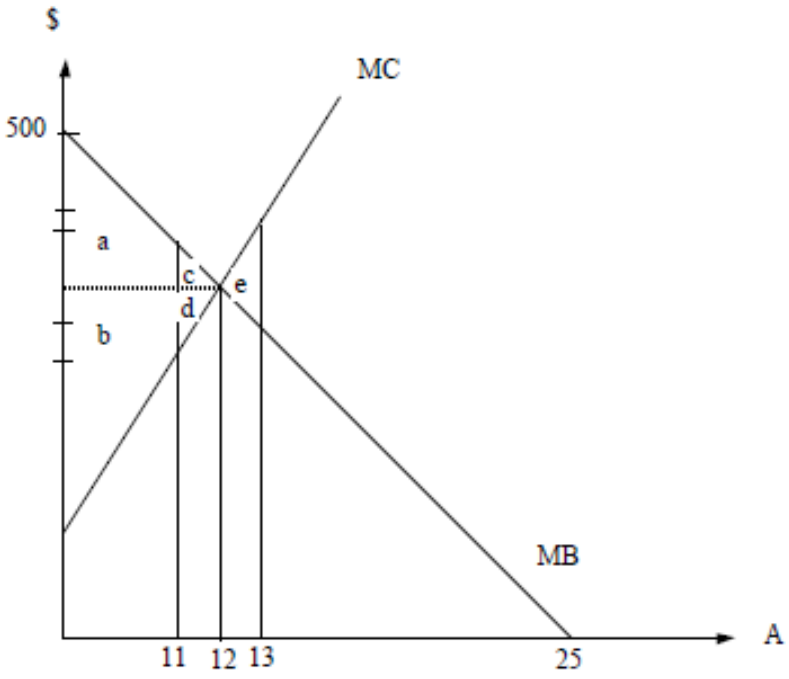
**d. Once again you will play a series of 10 rounds. This time, however, in each round your competitor will announce its output before you announce yours. How will your answers to (c) change in this case?**

If your competitor always announces first, it might be more profitable to behave by reacting "irrationally" in a single period. For example, in the first round your competitor will announce an output of 15. Rationally, you would respond with an output of 7.5. If you behave this way in every round, your total profits for all ten rounds will be \$562.50. Your competitor's profits will be \$1,125. However, if you respond with an output of 15 every time your competitor announces an output of 15, profits will be reduced to zero for both of you in that period. If your competitor fears, or learns, that you will respond in this way, he or she will be better off by choosing the Cournot output of 10, and your profits after that point will be \$75 per period. Whether this strategy is profitable depends on your opponent's

expectations about your behaviour, as well as how you value future profits relative to current profits.

(Note: A problem could develop in the last period, however, because your competitor will know that you realize that there are no more long-term gains to be had from behaving strategically. Thus, your competitor will announce an output of 15, knowing that you will respond with an output of 7.5. Furthermore, knowing that you will not respond strategically in the last period, there are also no long-term gains to be made in the ninth period from behaving strategically. Therefore, in the ninth period, your competitor will announce an output of 15, and you should respond rationally with an output of 7.5, and so on.)

2. **Assume that scientific studies provide you with the following information concerning the benefits and costs of sulphur dioxide emissions:**
   **Benefits of abating (reducing) emissions: *MB=500-20A***
   **Costs of abating emissions: *MC=200+5A***
   **where A is the quantity abated in millions of tons and the benefits and costs are given in dollars per ton.**
   a. **What is the socially efficient level of emissions abatement?**
      To find the socially efficient level of emissions abatement, set marginal benefit equal to marginal cost and solve for A:
      500-20A=200+5A
      A=12.
   b. **What are the marginal benefit and marginal cost of abatement at the socially efficient level of abatement?**
      Plug A=12 into the marginal benefit and marginal cost functions to find the benefit and cost:
      MB=500-20(12)=260
      MC=200+5(12)=260.
   c. **What happens to net social benefits (benefits minus costs) if you abate 1 million more tons than the efficient level? 1 million fewer?**
      Net social benefits are the area under the marginal benefit curve minus the area under the marginal cost curve. At the socially efficient level of abatement this is equal to area **a+b+c+d** in Figure or
      0.5(500-200)(12)=1800 million dollars.
      If you abate 1 million more tons then the net social benefit is area **a+b+c+d-e** or
      1800-0.5(265-240)(1)=1800-12.5=1787.5 million dollars.
      If you abate 1 million less tons then the net social benefit is area **a+b** or
      0.5(500-280)(11)+(280-255)(11)+0.5(255-200)(11)=1787.5 million dollars.
   d. **Why is it socially efficient to set marginal benefits equal to marginal costs rather than abating until total benefits equal total costs?**
      It is socially efficient to set marginal benefit equal to marginal cost rather than total benefit equal to total cost because we want to maximize net benefits, which are total benefit minus total cost. Maximizing total benefit minus total cost means that at the margin, the last unit abated will have an equal cost and benefit. Choosing the point where total benefit is equal to total cost will result in too much abatement and would be analogous to choosing to produce where total revenue was equal to total cost. If total revenue was always equal to total cost by choice, then there would never be any profit. In the case of abatement, the more we abate, the costlier it is. Given that funds will tend to be scarce, dollars should be allocated to abatement only so long as the benefit of the last unit of abatement is greater than or equal to the cost of the last unit of abatement.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