ESO207: Data Structures and Algorithms

Programming Assignment 1

Due Date: 1st February, 2021

Total Number of Pages: 3   Total Points 150

* Submit the recurrence relations and all the codes on Moodle

Question 1. **Navigation in the country**

You have just arrived in the country of *Bahamia*, and was amazed by the jaw-dropping scenery that you saw while travelling. At that moment only, you decided to travel everyday as long as you stay in *Bahamia*!

The country of *Bahamia* can be described as follows: There are three cities in *Bahamia*, named *Alvonia*, *Borginia* and *Carpania*. Each pair of cities is connected by a road. This can be represented as shown in *Figure*1.

On *Day* 0, You are in *Alvonia*. You decided that at the start of each day, you will travel to a neighbouring city. So, if you are in *Alvonia*, you can travel to either *Borginia* or *Carpania*. Similarly, if you are in *Borginia*, you can travel to either *Alvonia* or *Carpania*.

However, at the end of Nth day, you want to be back in *Alvonia*, as you have a flight scheduled back to your home country.

Note that you start your travelling journey from *Day* 1. So at the start of *Day* 1, you will travel for the first time, and travel at the start of **each** day till the Nth day.
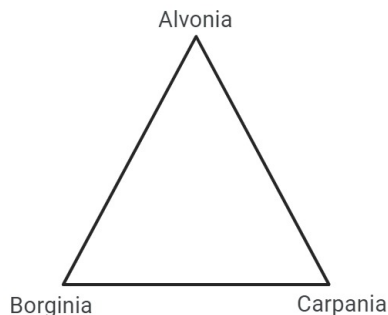

Figure 1: The country of *Bahamia*

You are now wondering, what are the total number of ways in which you can travel, such that you end up in *Alvonia* at the end of Nth day.
As the number can be quite large, you should print it modulo $1000000007(10^9 + 7)$

Example :
Let $n = 4$. Output will be 6.

Explanation :
We can represent the cities that you can travel to as strings of length N, where the ith character will represent the city that you have travelled to at the start of ith day. Also, let's use characters A, B and C to denote the cities *Alvonia*, *Borginia* and *Carpania* respectively.

Consider the sequence $BABA$. At the start of Day 1, you travelled to *Borginia*. At the start of Day 2, you travelled to city *Alvonia*. Similarly, at the start of Day 4, you travelled to city *Alvonia*, and made sure that you end up in the city A at the end of 4th day.

The following 6 strings represents the ways that you can travel:

$BABA, BACA, BCBA, CABA, CACA, CBCA$

(a) (10 points) Let $A(n)$, $B(n)$ and $C(n)$ be the total number of ways such that you end up in *Alvonia*, *Borginia* and *Carpania* at the end of nth day, respectively. Give the recurrence relations for $A(n)$, $B(n)$ and $C(n)$. Also, give the values of $A(0)$, $B(0)$ and $C(0)$.
Note that $A(N)$ is our required answer.

(b) (25 points) **Using recursive method**, write a program that output $A(N)$ for a given N.
$1 \leq N \leq 20$

(c) (35 points) **Using iterative method**, write a program that output $A(N)$ for a given N.
$1 \leq N \leq 10^5$

(d) (50 points) **Using matrix exponentiation method**, write a program that output $A(N)$ for a given N.
$1 \leq N \leq 10^{15}$

Question 2. Maximum Sum Subarray

In this problem, you are asked to implement the $\mathbf{O(N)}$ time Algorithm for the **Maximum Sum Subarray** problem as discussed in the lectures.

(a) (30 points) Let us represent the sum of the Maximum Sum Subarray by S. Write a program that output S for the given array A of length N.
Constraints:
$1 \leq N \leq 10^5$
$-10^6 \leq A_i \leq 10^6, \forall i : 1 \leq i \leq N$
Example :
Let $N = 6$ and $A = [-1, 3, -2, 4, 5, -6]$.
The output should be 10.
Explanation :
The subarray that starts at index 2 and ends at index 5, i.e., the subarray $[3, -2, 4, 5]$ is the subarray having the maximum sum. Hence, the value of S is 10.

Note :

- Parts **b, c and d** of **Question 1**, and **Question 2** have to be answered through a contest in Hackerrank. The contest has 3 challenges, each corresponding to a part. You have to submit your

code through the contest. Following is the link to the contest: `https://www.hackerrank.com/eso207a-pa1-2021-22-ii`

- For the parts **b, c and d** of **Question 1**, you will not be granted any marks if you submit your code/solution by any method other than what is expected through the question in the challenges (irrespective of the number of test cases that pass for your code). For example, if you submit matrix exponentiation method solution for the recursive code, you will get 0 marks for that part.

- Don't use memoization for the **recursive method implementation**.

- Your codes will be checked for possible plagiarism of any sorts. If we find such cases, then we will possibly award an F grade.

- Allowed Languages for challenge code submission : C, C++

- Allowed libraries : stdio.h for C and iostream for C++

- You need to submit a total of 5 files - 4 corresponding to the 4 programs (through Hackerrank), and 1 file containing solution of **part (a) and your Hackerrank username**.