Lecture 12

Rigid body kinematics: Acceleration analysis examples; Rolling motion. Example.

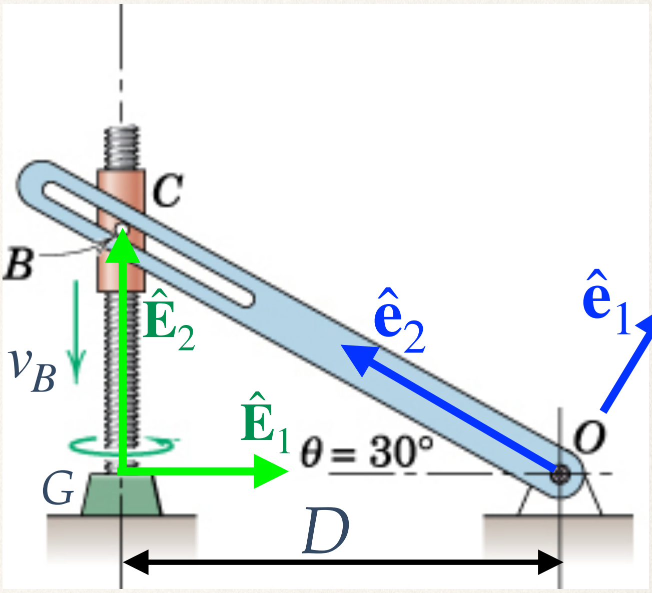
15-21 September, 2021

Example 1

Find the slotted arm's angular acceleration, given v_B = const.

Recall **general strategy** for systems with connected rigid bodies.

Acceleration analysis is done after velocity analysis.



Velocity analysis (done in Lec. 10):

$$\boldsymbol{\omega}_{arm} = -\frac{v_B \cos^2\theta}{D}\,\hat{\mathbf{E}}_3; \;\; \mathbf{v}^B_{rel} = v_B \sin\theta\,\hat{\mathbf{e}}_2 \,.$$

Acceleration analysis:

$$\boldsymbol{\alpha}_{arm} = -\frac{2v_B^2 \sin\theta \cos^3\theta}{D^2}\,\hat{\mathbf{E}}_3; \;\; \boldsymbol{a}^B_{rel} = \frac{v_B^2}{D}\cos^3\theta\,\hat{\mathbf{e}}_2 \,.$$

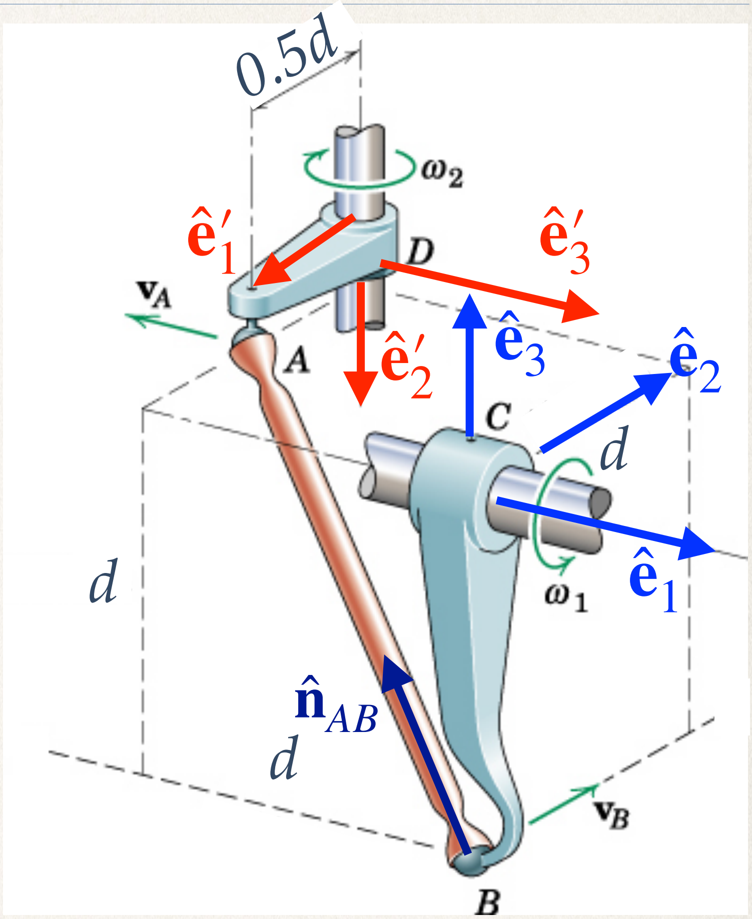
Example 2

Given $\omega_1 = const.$ find α_2 and $\boldsymbol{\alpha}^{AB}$ at this moment.

Recall: Link *AB* can rotate in an <u>arbitrary</u> manner about *AB*.

Could only find $\boldsymbol{\omega}_n^{AB} = \hat{\mathbf{n}}_{AB} \times \left(\boldsymbol{\omega}^{AB} \times \hat{\mathbf{n}}_{AB}\right)$

Thus, will find $\boldsymbol{\alpha}_n^{AB} = \hat{\mathbf{n}}_{AB} \times \left(\boldsymbol{\alpha}^{AB} \times \hat{\mathbf{n}}_{AB}\right)$



Velocity analysis (Lec. 10): $\hat{\mathbf{n}}_{AB} = \dfrac{-2\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2 + 2\hat{\mathbf{e}}_3}{3}$

$$\omega_2 = \omega_1; \;\; \boldsymbol{\omega}_n^{AB} = -\frac{\omega_1}{2}\hat{\mathbf{n}}_{AB} \times \left(\hat{\mathbf{e}}_1 + 2\hat{\mathbf{e}}_2\right).$$

Acceleration analysis: $\boldsymbol{\alpha}^{CD} = \alpha_2 \hat{\mathbf{e}}_2' = -\dfrac{33\omega_1^2}{8}\hat{\mathbf{e}}_2'$

$$\boldsymbol{\alpha}_n^{AB} = -\frac{\omega_1^2}{12}\hat{\mathbf{n}}_{AB} \times \left(-33\mathbf{e}_1 + 8\hat{\mathbf{e}}_2 + 16\hat{\mathbf{e}}_3\right).$$

Rolling motion

I. **Idea**. Let a rigid body $\mathscr{B}_1$ touch another rigid body $\mathscr{B}_0$ at <u>*contact point*</u> P which traces a <u>*contact path*</u>. Call P_0 and P_1 as points on the rigid bodies $\mathscr{B}_0$ and $\mathscr{B}_1$, respectively, which are <u>coincident</u> with P *at this moment*.

II. **Definition**. A rigid body $\mathscr{B}_1$ is said to <u>***roll without slipping***</u> on a rigid body $\mathscr{B}_0$ when

$$\mathbf{v}^{P_1} = \mathbf{v}^{P_0} \qquad \textit{(Rolling condition \#1)}$$

1. *Contact point* P is <u>not</u> a material point, and $\mathbf{v}^{P_i} \neq \mathbf{v}^{P}$ in general.
2. If body $\mathscr{B}_0$ is stationary: $\mathbf{v}^{P_1} = \mathbf{v}^{P_0} = \mathbf{0}$. But the contact point's $\mathbf{v}^{P} \neq \mathbf{0}$.

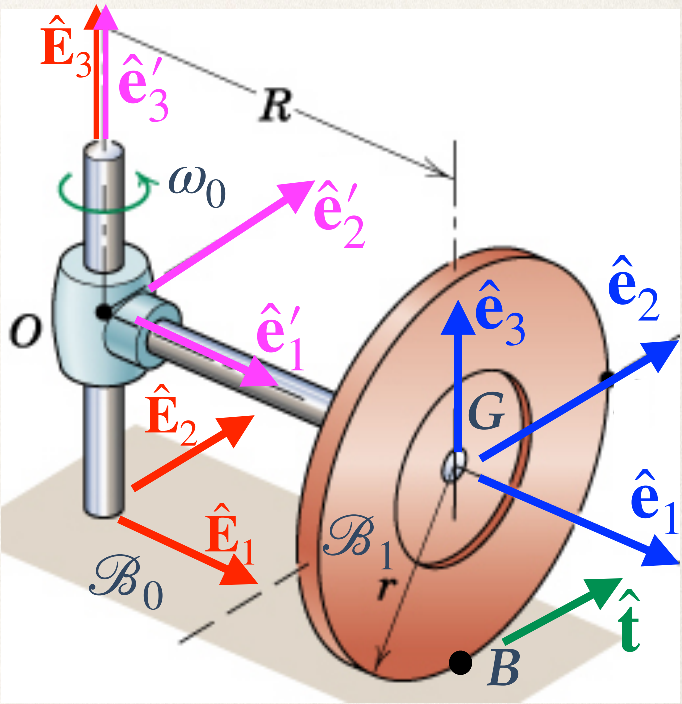
III. **Acceleration**. $\hat{\mathbf{t}}$ is tangent to *contact path* at P:

$$\mathbf{a}^{P_1} \cdot \hat{\mathbf{t}} = \mathbf{a}^{P_0} \cdot \hat{\mathbf{t}} \qquad \textit{(Rolling condition \#2)}$$

1. Note that $\mathbf{a}^{P_1} \neq \mathbf{a}^{P_0} \neq \mathbf{a}^{P}$.
2. If $\mathscr{B}_0$ is fixed $\mathbf{a}^{P_1} \cdot \hat{\mathbf{t}} = \mathbf{0}$, but $\mathbf{a}^{P_1} \neq \mathbf{0}$.

Example 3

Disc rolls without slipping. Find <u>acceleration</u> of point on the disc in contact with the ground if $\omega_0 = const.$



Call disc body $\mathscr{B}_1$ and the ground $\mathscr{B}_0$.

Then, point B_0 lies on $\mathscr{B}_0$ and B_1 is on $\mathscr{B}_1$.

$\{\mathscr{E}_0, C, \hat{\mathbf{E}}_i\}$ is ground-fixed CS; $\{\mathscr{E}', O, \hat{\mathbf{e}}_i'\}$ is BFCS of arm; $\{\mathscr{E}, G, \hat{\mathbf{e}}_i\}$ is BFCS of disc.

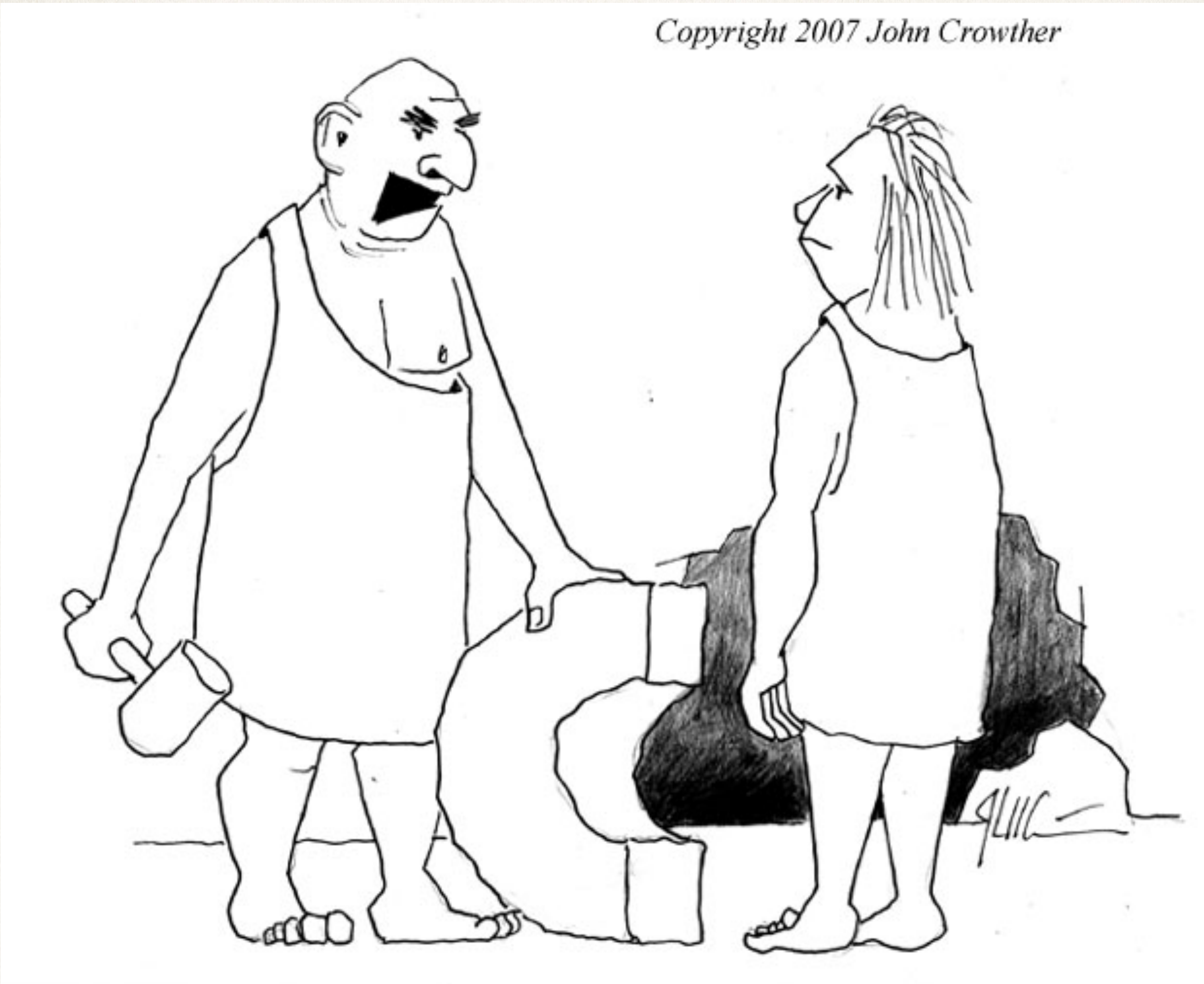
Velocity analysis: *Focus on B_1*. Rolling condition when $\mathscr{B}_0$ is stationary: $\mathbf{v}^{B_1} = \mathbf{v}^{B_0} = \mathbf{0}$

$$\Longrightarrow \ \boldsymbol{\omega}^{\mathscr{B}_1} = \omega_0\left(\hat{\mathbf{E}}_3 - R\hat{\mathbf{e}}_1'/r\right) .$$

Acceleration analysis. $\mathbf{a}^{B_1} = \omega_0^2 R\left(R\hat{\mathbf{E}}_3/r - \hat{\mathbf{E}}_1\right)$

Satisfies rolling conditions when $\mathscr{B}_0$ is stationary: $\mathbf{a}^{B_1} \cdot \hat{\mathbf{t}} = \mathbf{a}^{B_0} \cdot \hat{\mathbf{t}} = \mathbf{0}$. But $\mathbf{a}^{B_1} \neq \mathbf{0}$!

End of rigid body kinematics.



"It's not perfect I know, but I'm sure I'm on to something. All I'm asking is for you to have a little faith in me."

But, the course's best may yet come.