3. If **n** is a unit vector and **a** is some nonzero vector then show $a_i = a_k n_k n_i + \epsilon_{ijk} \epsilon_{krs} n_j a_r n_s$.

$\varepsilon$ - $\delta$ identity:

$$\varepsilon_{ijk}\,\varepsilon_{ipq} = \delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp} \quad \text{—— ①}$$

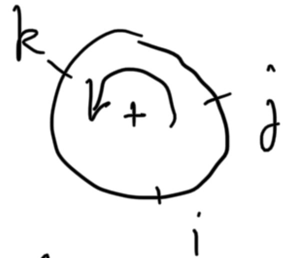
$$\varepsilon_{ijk}\,\varepsilon_{krs}\, n_j\, a_r\, n_s$$

$$= \varepsilon_{kij}\,\varepsilon_{krs}\, n_j\, a_r\, n_s$$

$$= (\delta_{ir}\delta_{js} - \delta_{is}\delta_{jr})\, n_j\, a_r\, n_s$$

$$= \delta_{ir}\delta_{js}\, n_j\, a_r\, n_s - \delta_{is}\delta_{jr} \times (n_j\, a_r\, n_s)$$

Aside




$$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$$

$$= n_s\, a_i\, n_s - n_r\, n_i\, a_r$$

$$= a_i\, n_s n_s - n_r n_i a_r$$

$$= a_i\,\underbrace{(n_1 n_1 + n_2 n_2 + n_3 n_3)}_{\shortparallel} - n_r n_i a_r$$

$\longrightarrow$ 1.0 $\Big\{$ square of norm of the unit vector

$$= a_i - n_r n_i a_r$$

$$= a_i - n_k n_i a_k$$

$$\therefore\ \varepsilon_{ijk}\,\varepsilon_{krs}\, n_j\, n_r\, a_s = a_i - n_k a_k n_i$$

Or
$$\boxed{a_i = \varepsilon_{ijk}\varepsilon_{krs} n_j n_r a_s + n_k a_k n_i}$$