Lecture 17

Rigid body in space: Stability; Poinsot construction; Effect of energy dissipation.

---

6 - 19 October, 2021

Stability

Stability of rigid body ($I_1 \neq I_2 \neq I_3$) in space in *pure* spin about a principal axis of inertia.

Definition: System is ***stable*** if *small perturbations* of the motion remain *small* for <u>all</u> time.

I. *BFCS* is principal CS $\{\mathscr{E}, G, \hat{\mathbf{e}}_i\}$ of $\mathbf{I}^G$.

II. *Base state*: Body spins about $\hat{\mathbf{e}}_3$ at Ω_0.

III. *Small perturbations at* $t = 0$:

$$\Omega_0\hat{\mathbf{e}}_3 \xrightarrow{pert.} \omega_1^0\hat{\mathbf{e}}_1 + \omega_2^0\hat{\mathbf{e}}_2 + (\Omega_0 + \omega_3^0)\hat{\mathbf{e}}_3, \; |\omega_i^0| \ll |\Omega_0|$$

IV. *Linearized* $\mathrm{AMB}_{/G}$: $\begin{cases} 0 = I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\Omega_0 \\ 0 = I_2\dot{\omega}_2 - (I_3 - I_1)\omega_1\Omega_0 \\ 0 = I_3\dot{\omega}_3 \end{cases}$

V. Spin about <u>major</u> / <u>minor</u> axis is *stable*:

$$\omega_1 = \omega_{01}\cos(\nu t + \phi), \; \omega_2 = \omega_{02}\sin(\nu t + \phi)$$

VI. Spin about <u>intermediate</u> axis is *unstable*:

$$\omega_{1,2} = \omega^+_{01,02}\exp(\nu t) + \omega^-_{01,02}\exp(-\nu t)$$

Application

I. Perturbation to pure spin about axes of *maximum* and *minimum* inertia remain small because these axes are stable axes of rotation.

II. Pure spin about the *intermediate* axis is unstable, hence small initial perturbations magnify with time leading to tumbling motion.

Poinsot construction

I. Rigid body in space with $I_1 > I_2 > I_3$:

1. $\mathbf{h}^G = H\hat{\mathbf{E}}_3 = \text{const.}; E_k = T = \text{const.}$
2. *BFCS* $\{\mathscr{E}, G, \hat{\mathbf{e}}_i\}$ is principal CS of $\mathbf{I}^G$

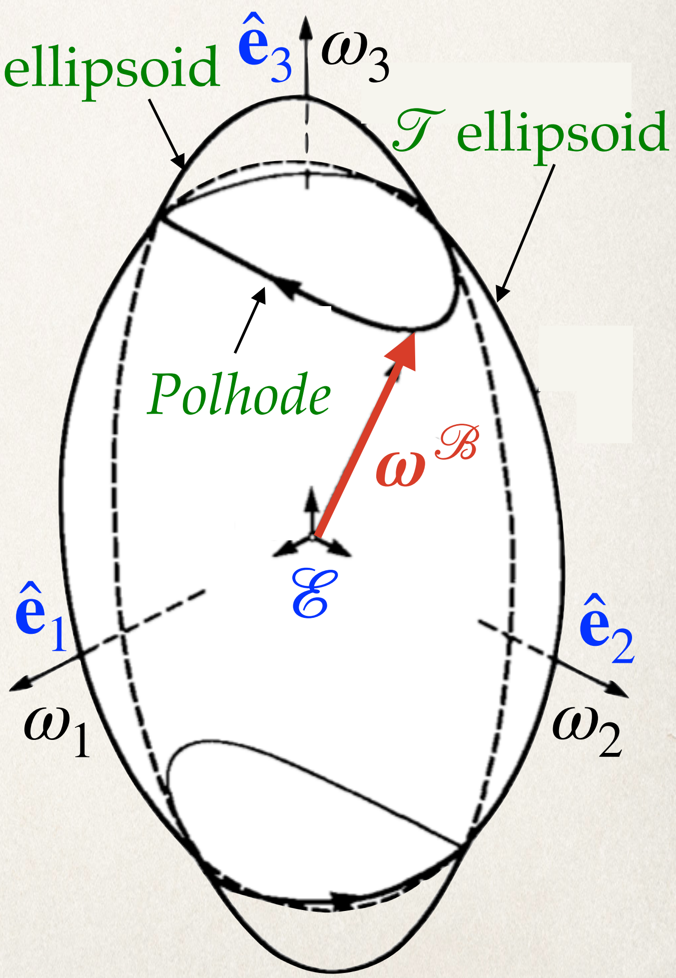
II. $\mathscr{H}$ *ellipsoid*: $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2 = H^2$.

III. $\mathscr{T}$ *ellipsoid*: $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2 = 2T$.

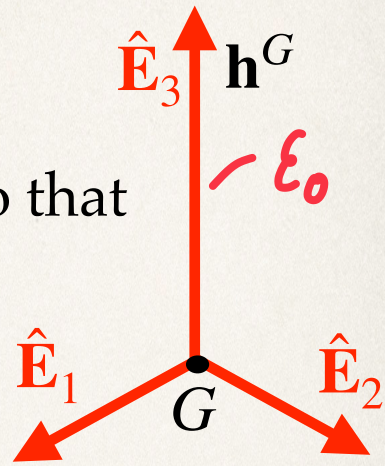
IV. Construct $\mathscr{H}$ and $\mathscr{T}$ ellipsoids in $\mathscr{E}$.

V. $\mathscr{H}$ and $\mathscr{T}$ ellipsoids will intersect along curves called *polhodes*.

1. $\boldsymbol{\omega}^{\mathscr{B}}$ vector's tip follows a *polhode*
2. $\boldsymbol{\omega}^{\mathscr{B}}$ changes in general $\Longrightarrow \boldsymbol{\alpha}^{\mathscr{B}}$ $\Longrightarrow$ *Tumbling*!



Poinsot construction



I. Given rigid body $(I_1 > I_2 > I_3)$ in space with $\mathbf{h}^G = H\hat{\mathbf{E}}_3$.

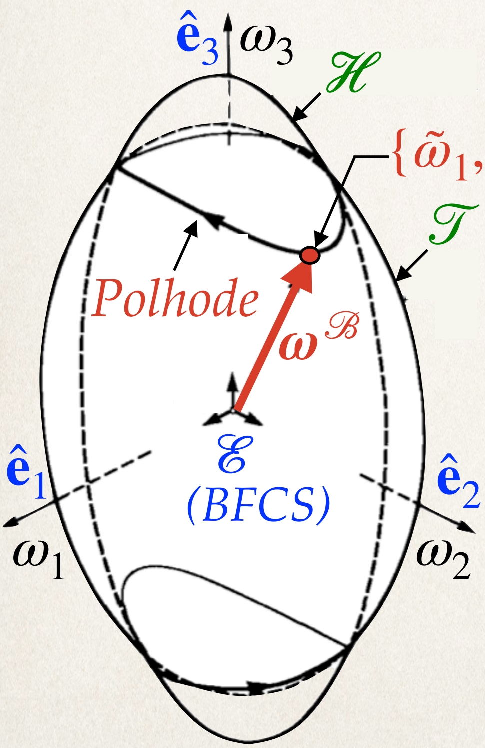
II. How to orient *BFCS w.r.t* $\mathscr{E}_0$ so that

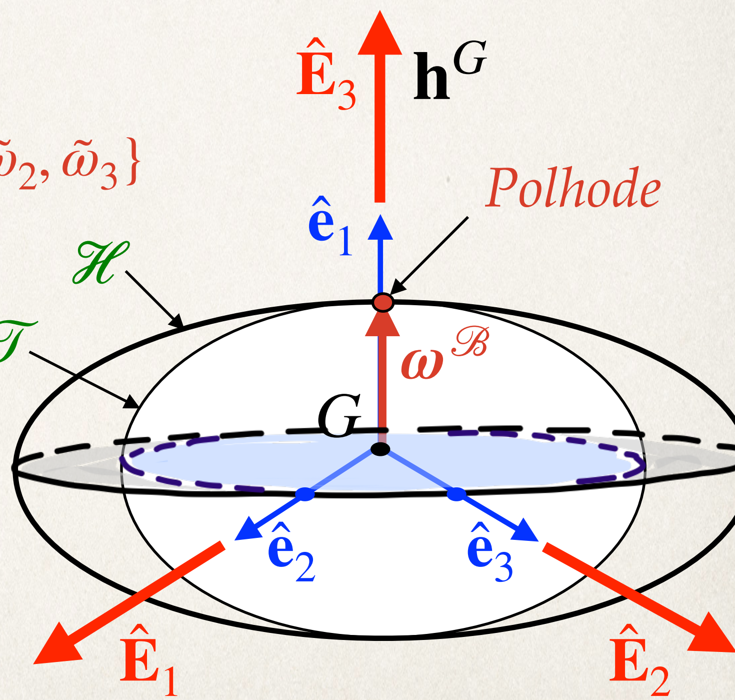
$$\mathbf{h}^G = H\hat{\mathbf{E}}_3 = \sum I_i \tilde{\omega}_i \hat{\mathbf{e}}_i \,?$$

III. Find Euler angles to relate *BFCS* and $\mathscr{E}_0$.

IV. As $\boldsymbol{\omega}^{\mathscr{B}}$ moves on *polhode, BFCS* changes orientation *w.r.t.* $\mathscr{E}_0 \Longrightarrow$ rigid body *tumbles*!

V. **Example**. *Pure* spin about $\hat{\mathbf{e}}_1$ $(\tilde{\omega}_2 = \tilde{\omega}_3 = 0)$: *Polhode* is a *point* $\Longrightarrow$ $\hat{\mathbf{e}}_1$ <u>must</u> align with $\hat{\mathbf{E}}_3$.





Poinsot construction

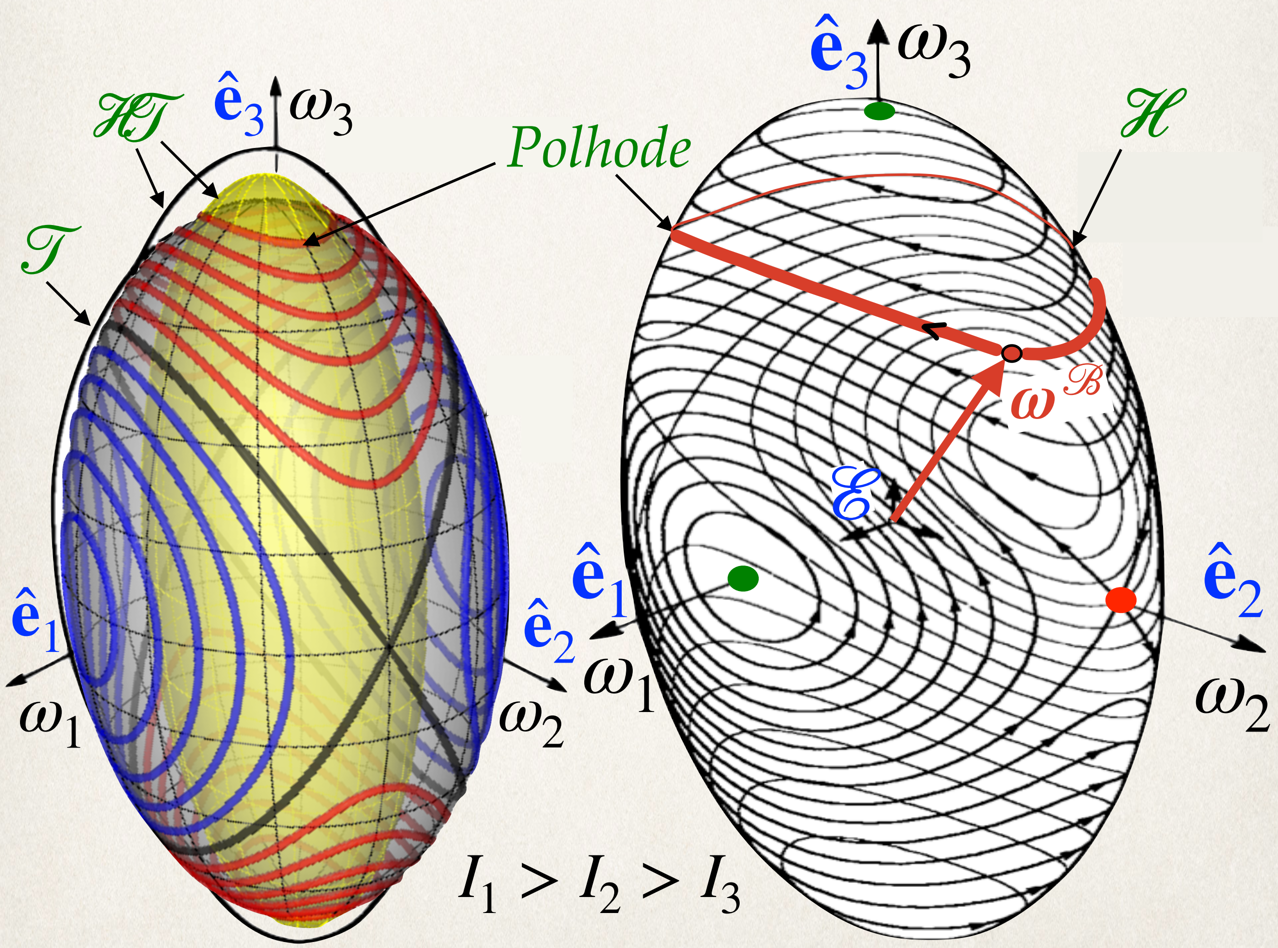
I. On $\mathscr{H}$: *Family of polhodes* generated by T.

II. *Polhode* fixed by $|\mathbf{h}^G| = H$ and $E_k = T$.

III. *Polhodes* can not intersect.

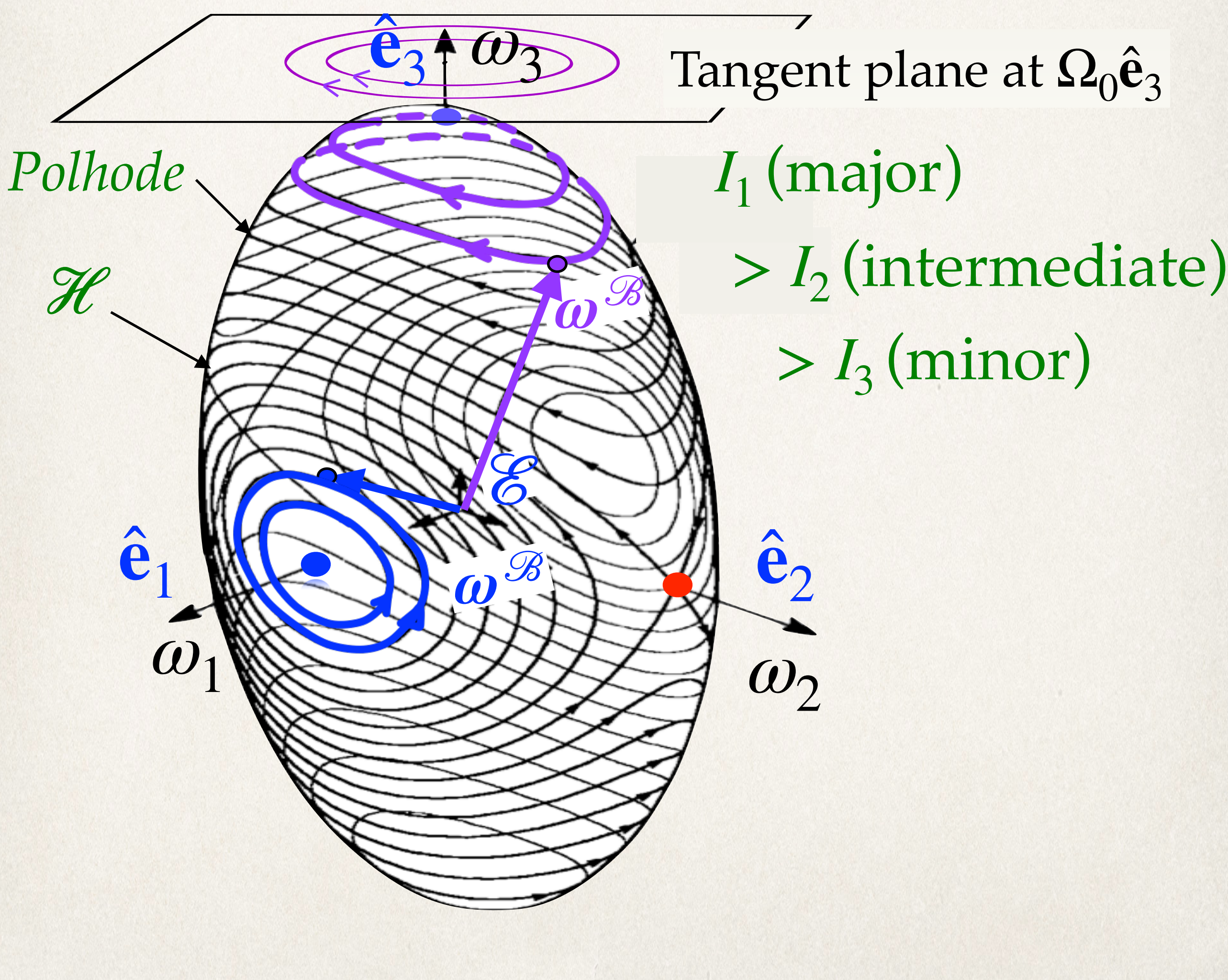
IV. For a given rigid body H and T are known $\Longrightarrow$ *Polhode* fixed

1. $\Longrightarrow \boldsymbol{\omega}^{\mathscr{B}}$ can only move on this *polhode*.



Perturbed motion

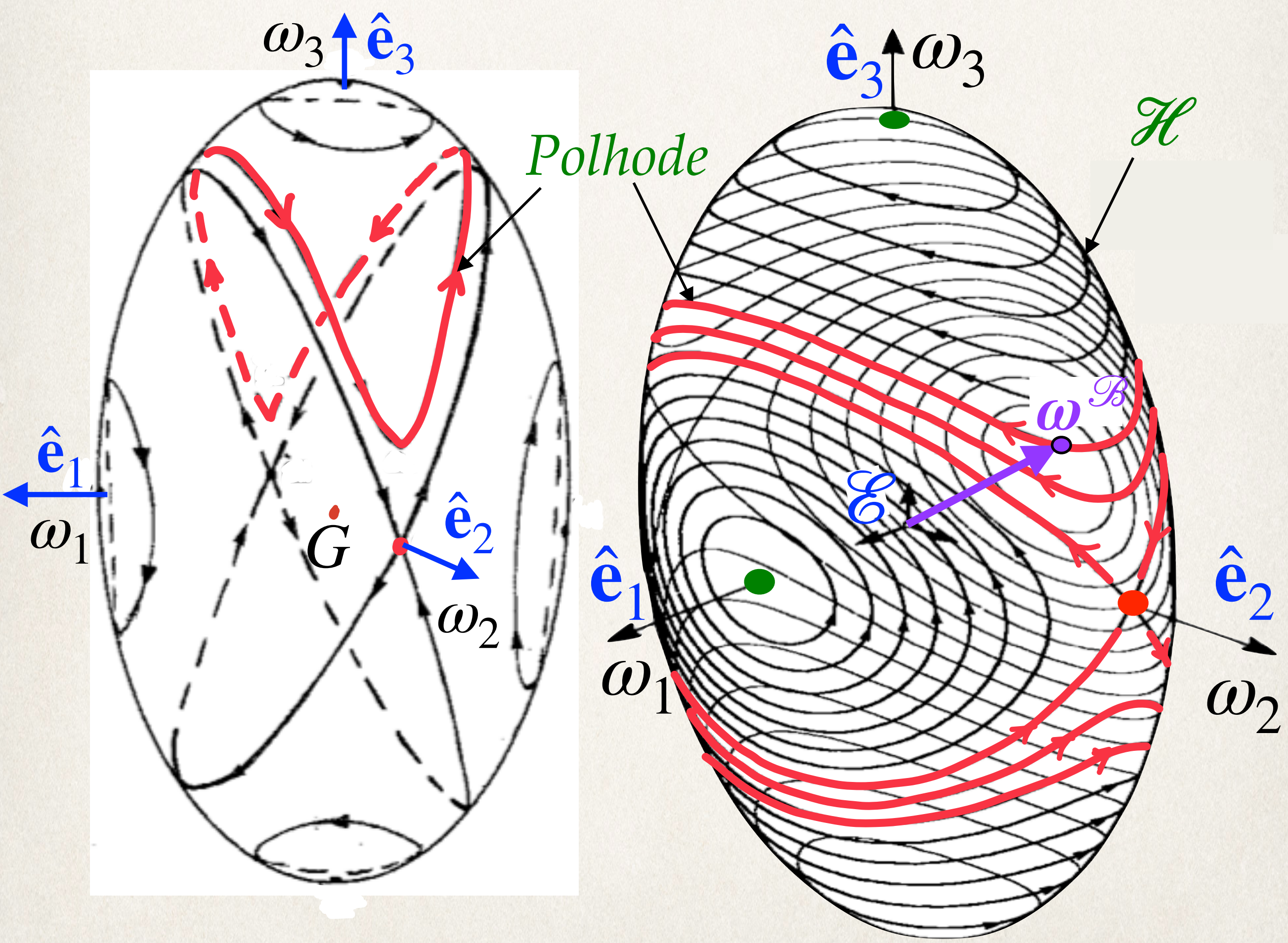
1. *Base state*: Pure spin about $\hat{\mathbf{e}}_1$ or $\hat{\mathbf{e}}_3 \Longrightarrow$ Stable.
2. **Perturbed motion**. $\boldsymbol{\omega}^{\mathscr{B}}$ moves on *polhodes* that are *centered around base state.*
3. *Small* perturbations: *Polhodes* $\approx$ *Ellipses.*
   i. **Example**. *Base state* is spin at Ω_0 about $\hat{\mathbf{e}}_3$.
      Perturbed motion: $\omega_3 = \Omega_0$,
      $\omega_1 = \omega_{01} \cos(\nu t + \phi)$, $\omega_2 = \omega_{02} \sin(\nu t + \phi)$,



Perturbed motion

1. *Base state*: Pure spin about $\hat{\mathbf{e}}_2 \Longrightarrow$ Unstable.
2. *Perturbed motion* <u>deviates</u> from pure spin.
3. *Polhodes* near $\hat{\mathbf{e}}_2$ go *far* away from $\hat{\mathbf{e}}_2$. Thus, $\boldsymbol{\omega}^{\mathscr{B}}$ different from pure spin $\Rightarrow$ ***Tumbling***!
4. *Base state* is a **Saddle Point**. *Perturbed motion*: $\omega_{1,3} = \omega^{+}_{01,03}\exp(\nu t) + \omega^{-}_{01,03}\exp(-\nu t)$

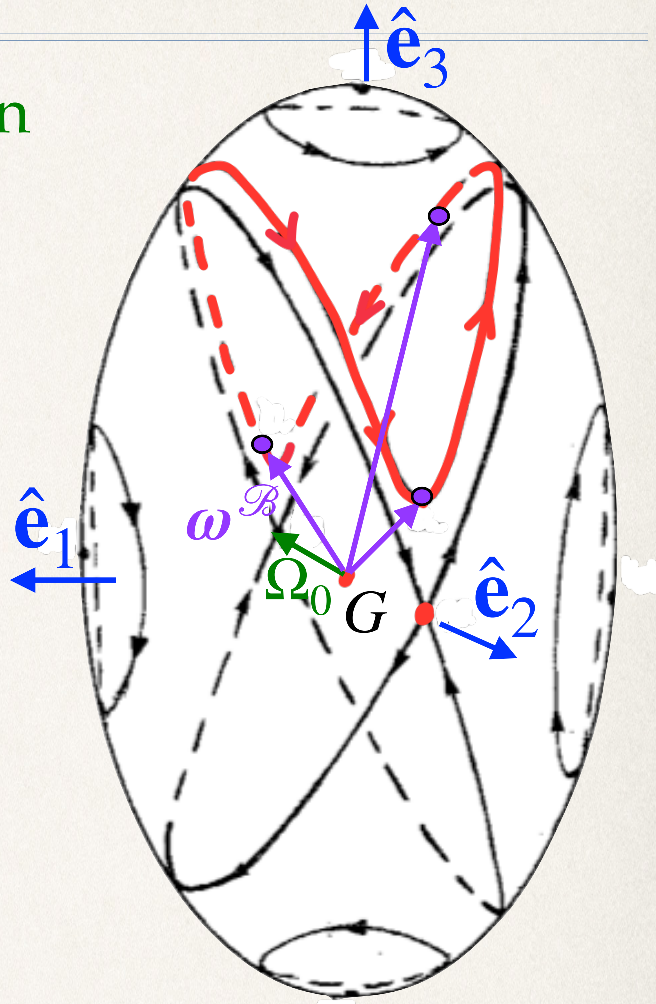
I_1 (major) > I_2 (intermediate) > I_3 (minor)

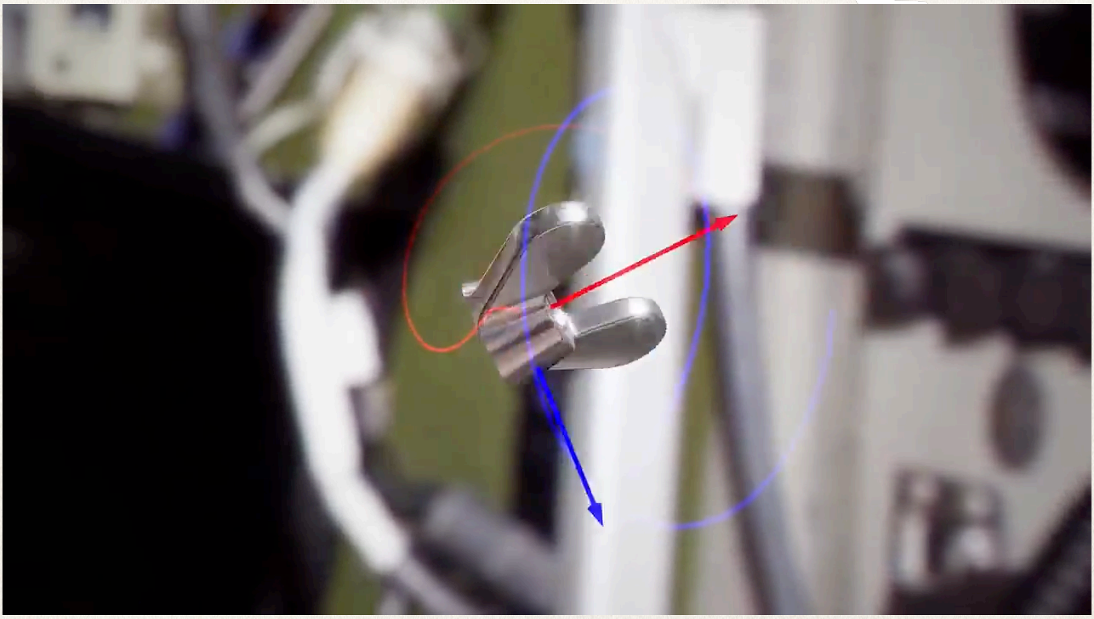


Application

I. *Base state*: Pure spin Ω_0 along intermediate axis $-\hat{\mathbf{e}}_2$.

II. When *perturbed*, $\boldsymbol{\omega}^{\mathscr{B}}$ can traverse a polhode in which $\boldsymbol{\omega}^{\mathscr{B}}$ points nearly along $-\hat{\mathbf{e}}_2$ to aligning along $\hat{\mathbf{e}}_2$!





Energy dissipation

I. Consider rigid body $(I_1 > I_2 > I_3)$.

II. For <u>fixed</u> H, *pure* rotation about

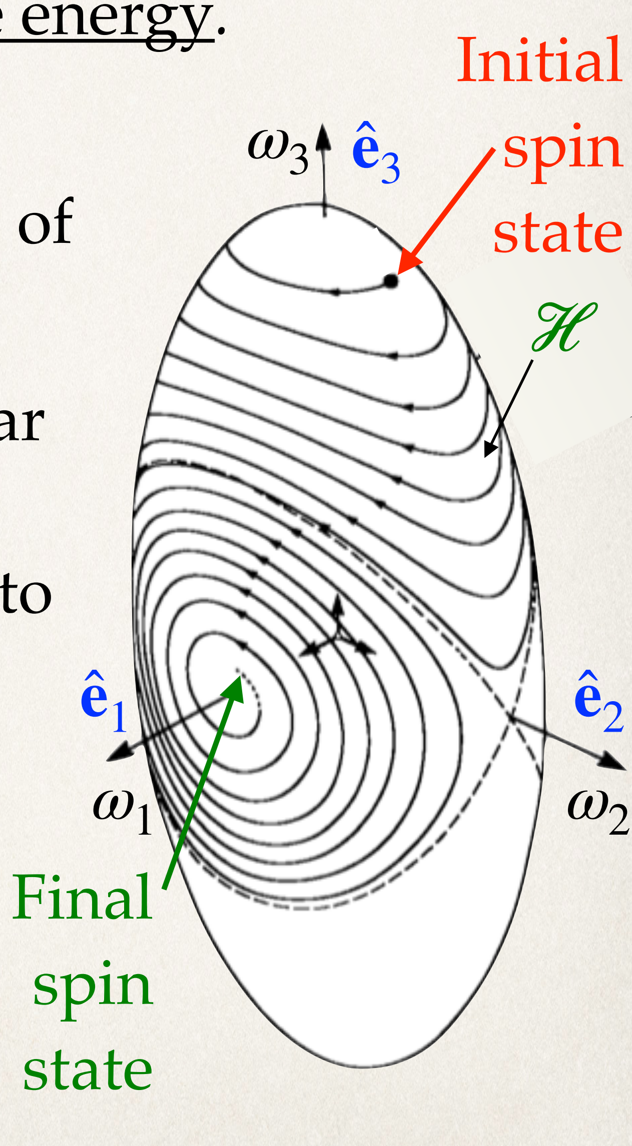
1. $\hat{\mathbf{e}}_1$ has *minimum* KE: $T_{\min} = H^2/I_1$;
2. $\hat{\mathbf{e}}_3$ has *minimum* KE: $T_{\max} = H^2/I_3$.

III. *All* bodies <u>dissipate energy</u>.

IV. **Energy dissipation** drives body to state of *minimum* energy.

V. Body rotating in near pure spin about $\hat{\mathbf{e}}_3$ (high KE) is driven to rotate near $\hat{\mathbf{e}}_1$(low KE).

VI. This is called *nutational damping*.

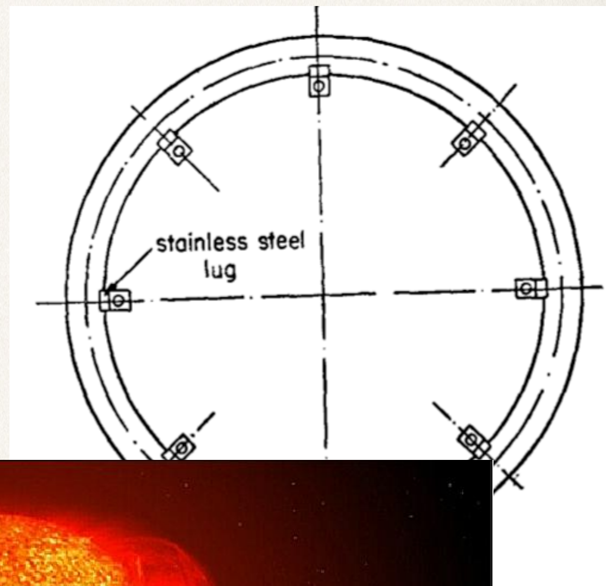


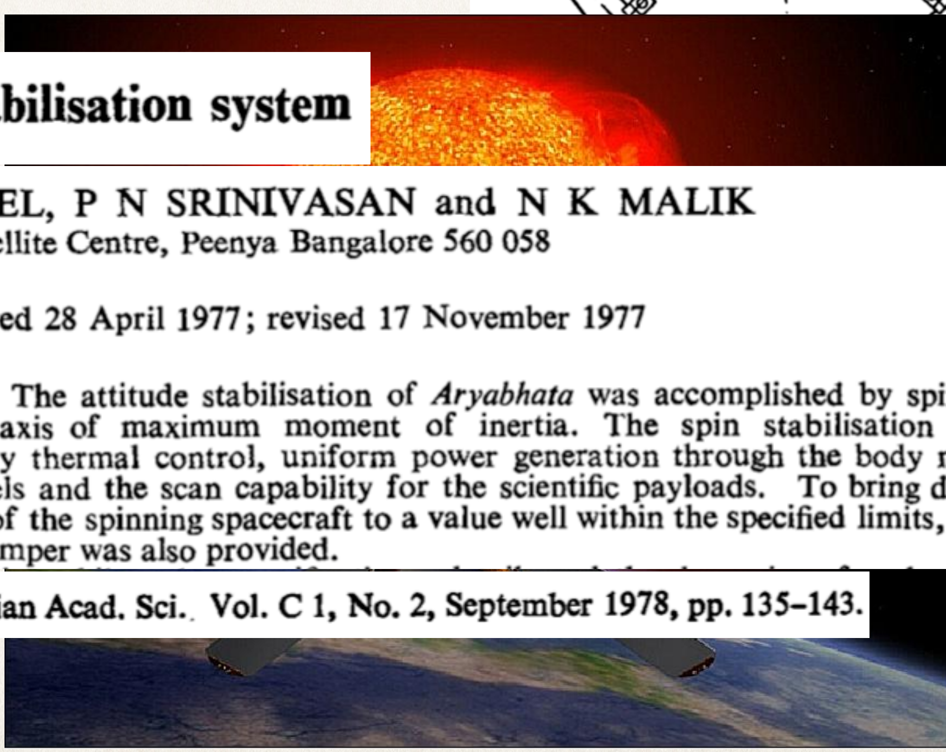
Application

I. **Asteroids**. Explains why many are in pure spin about axis of maximum inertia.

II. **Satellite nutation dampers**. To keep orientation of a satellite fixed.

1. *Challenging engineering*: Want maximum damping with space and weight constraints.





The stabilisation system

P S GOEL, P N SRINIVASAN and N K MALIK
ISRO Satellite Centre, Peenya Bangalore 560 058

MS received 28 April 1977; revised 17 November 1977

Abstract. The attitude stabilisation of *Aryabhata* was accomplished by spinning it about its axis of maximum moment of inertia. The spin stabilisation ensures satisfactory thermal control, uniform power generation through the body mounted solar panels and the scan capability for the scientific payloads. To bring down the nutation of the spinning spacecraft to a value well within the specified limits, a fluid-in-tube damper was also provided.

Proc. Indian Acad. Sci. Vol. C 1, No. 2, September 1978, pp. 135–143.

It's getting harder and harder to diagnose paranoia.