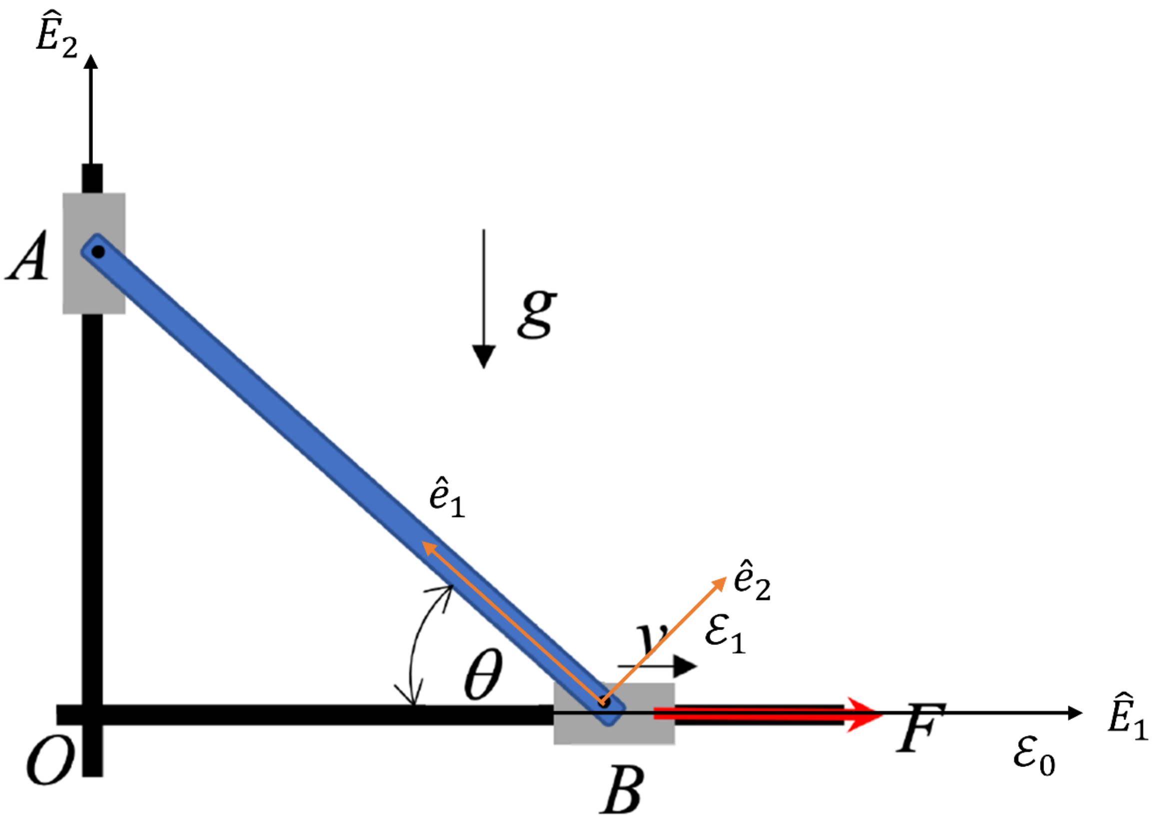
Problem 1:

Identifying all rigid bodies, they are link AB, collars and guide rods OA and OB. Now attaching BFCS



CS: $\{\mathcal{E}_0, O, \hat{E}_i\}$ as ground fixed coordinate system

CS: $\{\mathcal{E}_1, B, \hat{e}_i\}$ as BFCS of rod.

Above problem is a planar problem and collar can only move in $\hat{E}_1$ and $\hat{E}_2$ directions.

Velocity analysis:

Known

$$\underline{v}_B = v\hat{E}_1 \dots\dots\dots (1)$$

$$\underline{a}_B = 0 \dots\dots\dots (2)$$

$\underline{a}_B = 0$, because $\underline{v}_B$ is constant both in magnitude and direction.

Now writing velocity of Point A, since both Point A and B on rod therefore

$$\underline{v}_A = \underline{v}_B + \underline{\omega}_{rod} \times \underline{r}_{A/B}$$

We know $\underline{v}_A = v_A\hat{E}_2$, $\underline{\omega}_{rod} = \omega\hat{E}_3$ and $\underline{\alpha}_{rod} = \alpha\hat{E}_3$. Substituting them

$$v_A\hat{E}_2 = v\hat{E}_1 + \omega\hat{E}_3 \times \underline{r}_{A/B}$$

Where $\underline{r}_{A/B} = l\,\hat{e}_1$ and $\hat{e}_1 = -\cos\theta\,\hat{E}_1 + \sin\theta\hat{E}_2$ so $\underline{r}_{A/B} = -l\cos\theta\,\hat{E}_1 + l\sin\theta\hat{E}_2$.

$$v_A\hat{E}_2 = v\hat{E}_1 + \omega\hat{E}_3 \times \left(-l\cos\theta\,\hat{E}_1 + l\sin\theta\hat{E}_2\right)$$

$$v_A\hat{E}_2 = (v - l\omega\sin\theta)\hat{E}_1 - \omega l\cos\theta\hat{E}_2$$

Now comparing coefficients of $\hat{E}_1$ and $\hat{E}_2$

For $\hat{E}_2$

$$v - l\omega\sin\theta = 0$$

$$\omega = \frac{v}{l\sin\theta} \dots\dots..(3)$$

Acceleration analysis:

Acceleration of point A

$$\underline{a}_A = \underline{a}_B + \underline{\omega}_{rod} \times \left(\underline{\omega}_{rod} \times \underline{r}_{A/B}\right) + \underline{\alpha}_{rod} \times \underline{r}_{A/B}$$

We know, $\underline{a}_A = a_A\hat{E}_2$, then

$$a_A\hat{E}_2 = 0 + \omega\hat{E}_3 \times \left(\omega\hat{E}_3 \times \left(-l\cos\theta\,\hat{E}_1 + l\sin\theta\hat{E}_2\right)\right) + \alpha\hat{E}_3 \times \left(-l\cos\theta\,\hat{E}_1 + l\sin\theta\hat{E}_2\right)$$

$$a_A\hat{E}_2 = (\omega^2 l\cos\theta - \alpha l\sin\theta)\hat{E}_1 + (-\omega^2 l\sin\theta - \alpha l\cos\theta)\hat{E}_2$$

Now comparing coefficients of $\hat{E}_1$ and $\hat{E}_2$

For $\hat{E}_1$

$$\omega^2 l\cos\theta - \alpha l\sin\theta = 0$$

$$\alpha = \frac{\omega^2\cos\theta}{\sin\theta} = \omega^2\cot\theta$$

Substitute ω from eq. (3)

$$\alpha = \frac{v^2}{l^2\sin\theta^2}\cot\theta \dots\dots.(4)$$

Now by using ω and α, we will find $\underline{a}_G$

$$\underline{a}_G = \underline{a}_B + \underline{\omega}_{rod} \times \left(\underline{\omega}_{rod} \times \underline{r}_{G/B}\right) + \underline{\alpha}_{rod} \times \underline{r}_{G/B}$$

Where $\underline{r}_{G/B} = \frac{l}{2}\hat{e}_1$ or $\underline{r}_{G/B} = -\frac{l}{2}\cos\theta\,\hat{E}_1 + \frac{l}{2}\sin\theta\hat{E}_2$

$$\underline{\omega}_{rod} = \omega \hat{E}_3 = \frac{v}{l \sin\theta} \hat{E}_3$$

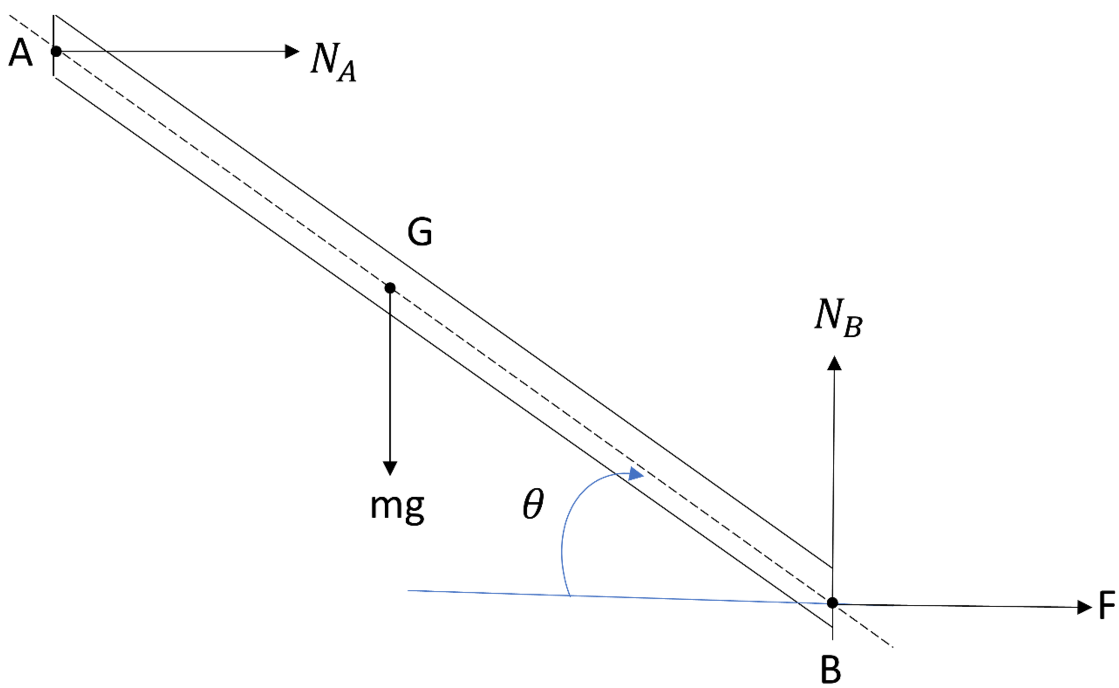
$$\underline{\alpha}_{rod} = \alpha \hat{E}_3 = \frac{v^2}{l^2 \sin\theta^2} \cot\theta \hat{E}_3$$

Now substitute in $\underline{a}_G$

$$\underline{a}_G = -\frac{\omega^2 l}{2 \sin\theta} \hat{E}_2$$

Now for finding $\underline{F}$

Drawing FBD of rod



Where $\underline{N_A}$ and $\underline{N_B}$ are force reactions at point A and Point B

Apply LMB

$$\sum \underline{F}_{ext} = m \underline{a}_G$$

$$(F + \underline{N_A}) \hat{E}_1 + (\underline{N_B} - mg) \hat{E}_2 = -\frac{m\omega^2 l}{2 \sin\theta} \hat{E}_2$$

Now comparing coeff. Of $\hat{E}_1$ and $\hat{E}_2$

$$F + \underline{N}_A = 0$$

$$\underline{N}_A = -F \dots \dots (5)$$

$$\underline{N}_B - mg = -\frac{m\omega^2 l}{2\sin\theta}$$

$$\underline{N}_B = mg - \frac{m\omega^2 l}{2\sin\theta} \qquad \dots \dots (6)$$

Apply AMB

Writing Total moment about point B

$$\underline{M}^B = \underline{r}_{G/B} \times m\underline{a}_G + \underline{\omega}_{rod} \times \left(I^G \cdot \underline{\omega}_{rod}\right) + I^G \cdot \underline{\alpha}_{rod}$$

Since it is planar problem therefore $\underline{\omega}_{rod} \times \left(I^G \cdot \underline{\omega}_{rod}\right) = 0$

$$\underline{M}^B = \underline{r}_{G/B} \times m\underline{a}_G + I^G \cdot \underline{\alpha}_{rod}$$

$$\left(-\underline{N}_A l\sin\theta + mg\frac{l}{2}\cos\theta\right)\hat{E}_3$$
$$= \left(-\frac{l}{2}\cos\theta\,\hat{E}_1 + \frac{l}{2}\sin\theta\hat{E}_2\right) \times m\left(-\frac{\omega^2 l}{2\sin\theta}\hat{E}_2\right) + \frac{ml^2}{12}\left(\frac{v^2}{l^2\sin\theta^2}\cot\theta\right)\hat{E}_3$$

From eq. (5), $\underline{N}_A = -F$

$$\left(Fl\sin\theta + mg\frac{l}{2}\cos\theta\right)\hat{E}_3 = \frac{m}{12}\left(\frac{v^2}{\sin\theta^2}\cot\theta\right)\hat{E}_3 + \left(\left(-\frac{l}{2}\cos\theta\right)\left(-\frac{ml}{2\sin\theta}\right)\left(\frac{v^2}{l^2\sin\theta^2}\right)\right)\hat{E}_3$$

$$Fl\sin\theta + mg\frac{l}{2}\cos\theta = \frac{m}{12}\frac{v^2\cot\theta}{\sin\theta^2} + \frac{mv^2\cot\theta}{4\sin\theta^2} = \frac{mv^2\cot\theta}{3\sin\theta^2}$$

$$F = \frac{mv^2\cot\theta}{3l\sin\theta^3} - \frac{mg}{2}\cot\theta$$