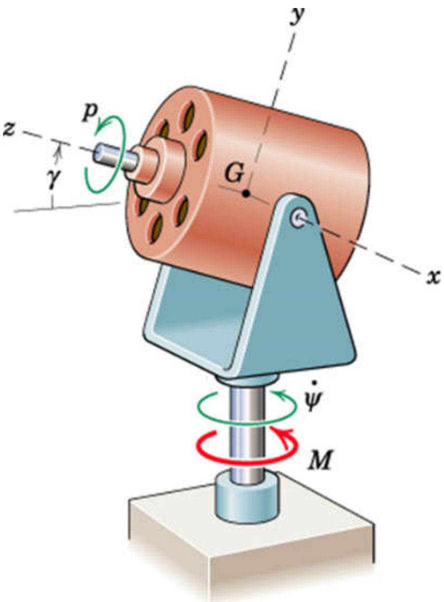
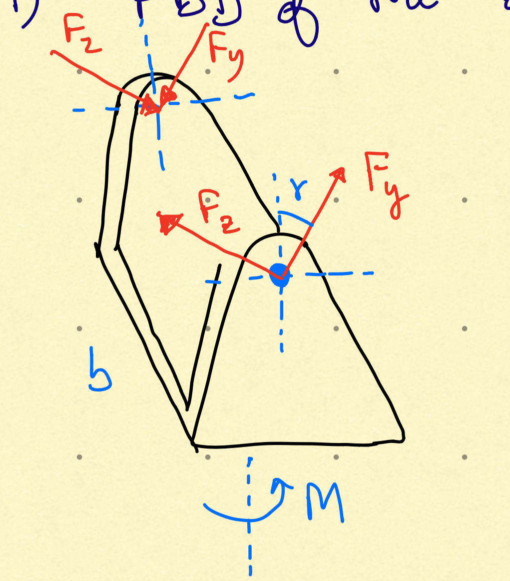
(3) The housing of an electric motor is free to rotate about -axis, which passes through the centroid of the armature, which is spinning at a constant rate , as shown. The radius of gyration of the armature about -axis is $\kappa$ and that about -axis is $\kappa$. Determine $\ddot{\psi}$ when a torque $\gamma$ is applied as shown. Assume $\dot{\gamma} = \dot{\psi} = 0$.



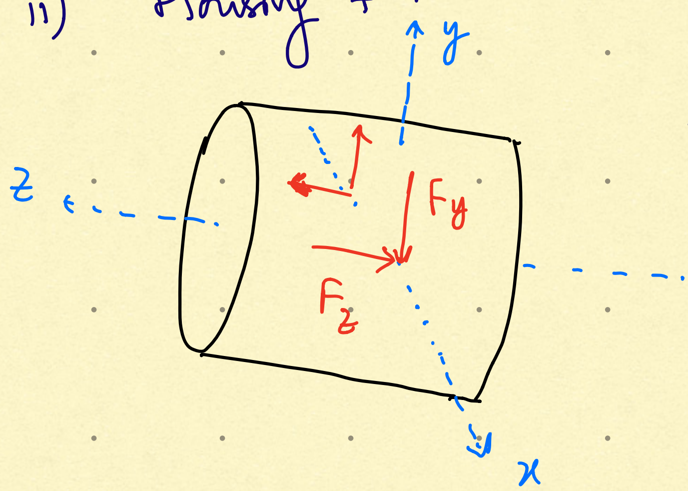
# Solution to #3, Tutorial 11

i) FBD of the bracket




$$M = F_z b \cos\gamma - F_y b \sin\gamma \qquad (1)$$

ii) Housing + Rotor




Recall the following equations from lecture 17, Part I:

$$M_1 = I_1 \dot{\omega}_1 - (I_2 - I_3)\omega_2 \omega_3$$

$$M_2 = I_2 \dot{\omega}_2 - (I_3 - I_1)\omega_1 \omega_3$$

$$M_3 = I_3 \dot{\omega}_3 - (I_1 - I_2)\omega_1 \omega_2$$

Here, $\omega_1 = \dot{\varphi} \sin\theta \sin\psi + \dot{\theta} \cos\psi$

$\omega_2 = \dot{\varphi} \sin\theta \cos\psi - \dot{\theta} \sin\psi$

$\omega_3 = \dot{\varphi} \cos\theta + \dot{\psi}$.

Various terms have meanings as discussed in the lecture.

We have a symmetric body here, such that $I_1 = I_2$.

Moreover, as discussed in the Lecture 17 (p 1), $\psi$ plays no role and is taken as $\psi = 0$ (but $\dot{\psi} \neq 0$). With this we get,

$$\omega_1 = \dot{\theta} \,, \quad \omega_2 = \dot{\varphi} \sin\theta \,, \quad \omega_3 = \dot{\varphi} \cos\theta + \dot{\psi}$$

and $$\dot{\omega}_1 = \ddot{\theta} + \dot{\varphi} \sin\theta \, \dot{\psi} \,, \quad \dot{\omega}_2 = \ddot{\varphi} \sin\theta + \dot{\varphi} \dot{\theta} \cos\theta - \dot{\theta} \dot{\psi} .$$

Substituting these back into the Euler Equations, we get:

$$M_1 = I_1 \left( \ddot{\theta} - \dot{\varphi}^2 \sin\theta \cos\theta \right) + I_3 \dot{\varphi} \sin\theta \, \omega_3 \quad (2)$$

$$M_2 = I_1 \left( \ddot{\varphi} \sin\theta + 2 \dot{\varphi} \dot{\theta} \cos\theta \right) - I_3 \dot{\theta} \, \omega_3 \quad (3)$$

$$M_3 = I_3 \dot{\omega}_3 \quad (4)$$

These are the equations that we will use now.

For the present problem, we have:

$$\theta = \pi/2 - \gamma \quad , \quad \dot{\varphi} = 0 \quad , \quad \dot{\psi} = p \ (\text{const.})$$

$$\dot{\theta} = 0, \ \ddot{\theta} = 0 \quad , \quad \ddot{\varphi} = \text{to be determined?}$$

$$I_1 = m k_x^2 \quad , \quad I_3 = m k_z^2$$

$$M_1 = 0$$

$$M_2 = F_z b$$

$$M_3 = -F_y b$$

Substituting these in ②, ③, ④ we have

② ⇒ $\quad 0 = 0 \quad$ (trivial)

③ ⇒ $\quad F_z b = m k_x^2 (\ddot{\varphi} \cos\gamma) \qquad$ ⑤

④ ⇒ $\quad -F_y b = m k_z^2 (\ddot{\varphi} \sin\gamma) \qquad$ ⑥

Recalling ①, we get,

$$\boxed{\ddot{\varphi} = \frac{M}{m(k_x^2 \cos^2\gamma + k_z^2 \sin^2\gamma)}}$$