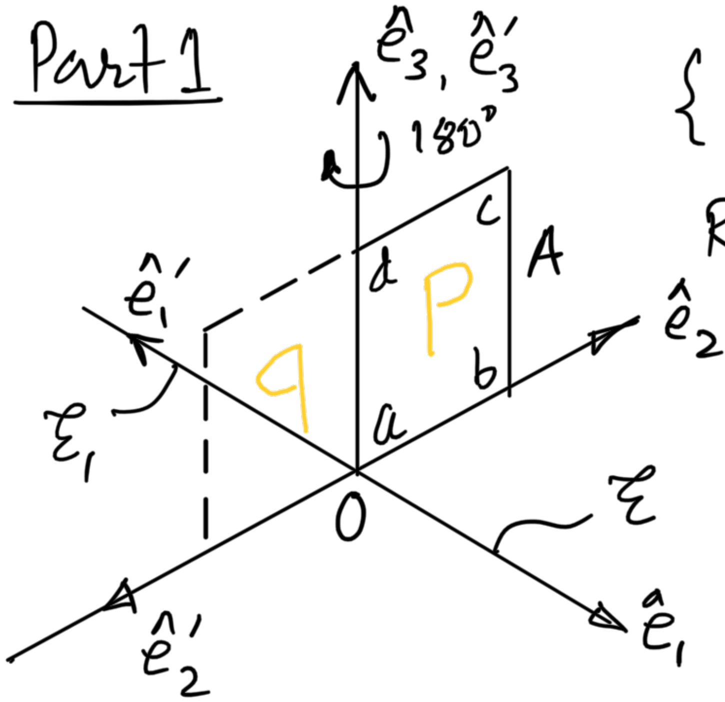
Part 1



$$\{\mathcal{E}, O, \hat{e}_i\} \xrightarrow{\underline{\underline{R}}_1} \{\mathcal{E}_1, O, \hat{e}_i'\}$$

Recall $R_{ij} = \hat{e}_j' \cdot \hat{e}_i$ —— (1)

$\therefore R_{11} = \hat{e}_1' \cdot \hat{e}_1$

$R_{12} = \hat{e}_2' \cdot \hat{e}_1$

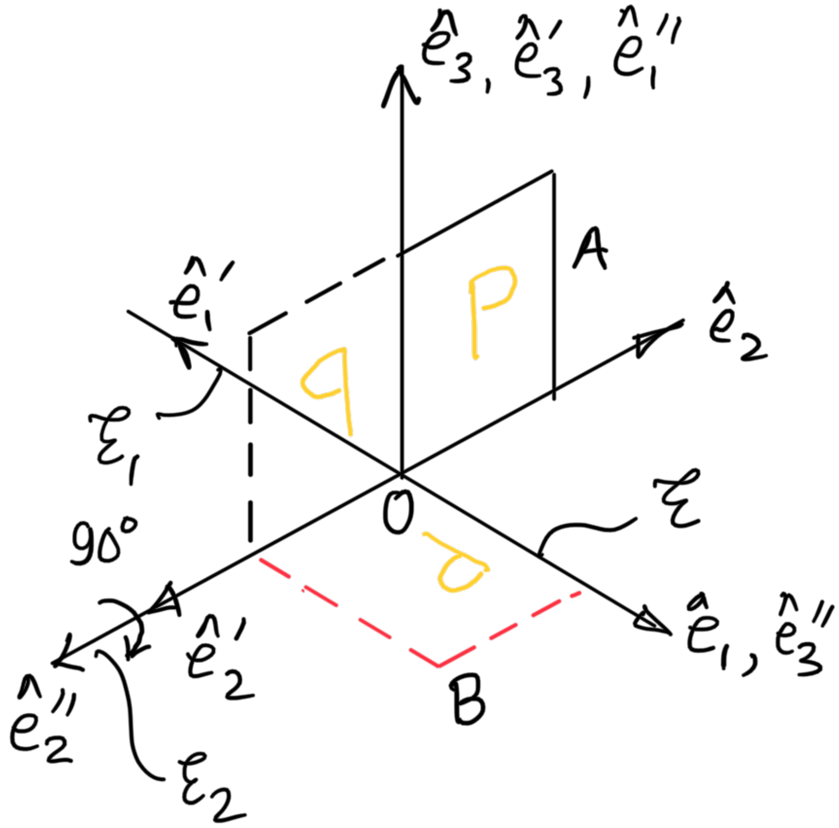
$R_{13} = \hat{e}_3' \cdot \hat{e}_1$

$\vdots \qquad \vdots$

$R_{33} = \hat{e}_3' \cdot \hat{e}_3$

Note we are computing elements of $\underline{\underline{R}}_1$ in $\mathcal{E}$ frame

$$\therefore [R_1]_{\mathcal{E}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \text{ ———— } (2)$$



Next,

$$\{\mathcal{E}_1, O, \hat{e}_i'\} \xrightarrow{\underline{\underline{R}}_2} \{\mathcal{E}_2, O, \hat{e}_i''\}.$$

Using 1 we can write components of $\underline{\underline{R}}_2$ in $\mathcal{E}_1$ frame as follows

$$[R_2]_{\mathcal{E}_1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \qquad (3)$$

Now, to compute composite rotation

$$\underline{\underline{R}} = \underline{\underline{R}}_2 \cdot \underline{\underline{R}}_1, \qquad (4)$$

we use the frame $\mathcal{E}$. Therefore Eq. 4 will be written as

$$[R]_{\mathcal{E}} = [R_2]_{\mathcal{E}} [R_1]_{\mathcal{E}}. \qquad (5)$$

We note that components of $\underline{\underline{R}}_2$ are computed in $\mathcal{E}_1$ (see Eq. 3). Therefore, we will transform $[R_2]_{\mathcal{E}_1}$ to $[R_2]_{\mathcal{E}}$ using the following rule for change of basis

$$[R_2]_{\mathcal{E}_1} = [R_1]_{\mathcal{E}}^T [R_2]_{\mathcal{E}} [R_1]_{\mathcal{E}}. \qquad (6)$$

Using orthogonality of $\underline{R}$ we re-write Eq. (6) as

$$[R_1]_{\mathcal{E}}[R_2]_{\mathcal{E}_1}[R_1]^T_{\mathcal{E}} = [R_2]_{\mathcal{E}} . \quad \text{———} \quad (7)$$

substituting $[R_2]_{\mathcal{E}}$ from Eq. (7) into Eq. (5)

$$[R]_{\mathcal{E}} = [R_1]_{\mathcal{E}}[R_2]_{\mathcal{E}_1}\underbrace{[R_1]^T_{\mathcal{E}}[R_1]_{\mathcal{E}}}_{[1]_{\mathcal{E}}} .$$

$$\therefore \; [R]_{\mathcal{E}} = [R_1]_{\mathcal{E}}[R_2]_{\mathcal{E}_1} . \quad \text{———} \quad (8)$$

Note: The order of multiplication in Eqs. (4) and (8) is opposite.

Using Eqs. (2) and (3) in Eq. (8) we get

$$[R]_{\mathcal{E}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} . \quad \text{———} \quad (9)$$

Part 2. $\{\mathcal{E}, O, \hat{e}_i\} \xrightarrow{\underline{\underline{R}}} \{\mathcal{E}_2, O, \hat{e}_i''\}$

Once again using Eq. 1 we can write

$$[R]_{\mathcal{E}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

This is the same as found in Eq. 9.

Important: In this problem we have been rotating the body-fixed CS to the plate.

$$\{\mathcal{E}, O, \hat{e}_i\} \xrightarrow{\underline{\underline{R_1}}} \{\mathcal{E}_1, O, \hat{e}_i'\} \xrightarrow{\underline{\underline{R_2}}} \{\mathcal{E}_2, O, \hat{e}_i''\}.$$

We will be making use of the concepts employed here when learning Euler's angles for rotation of a rigid body.