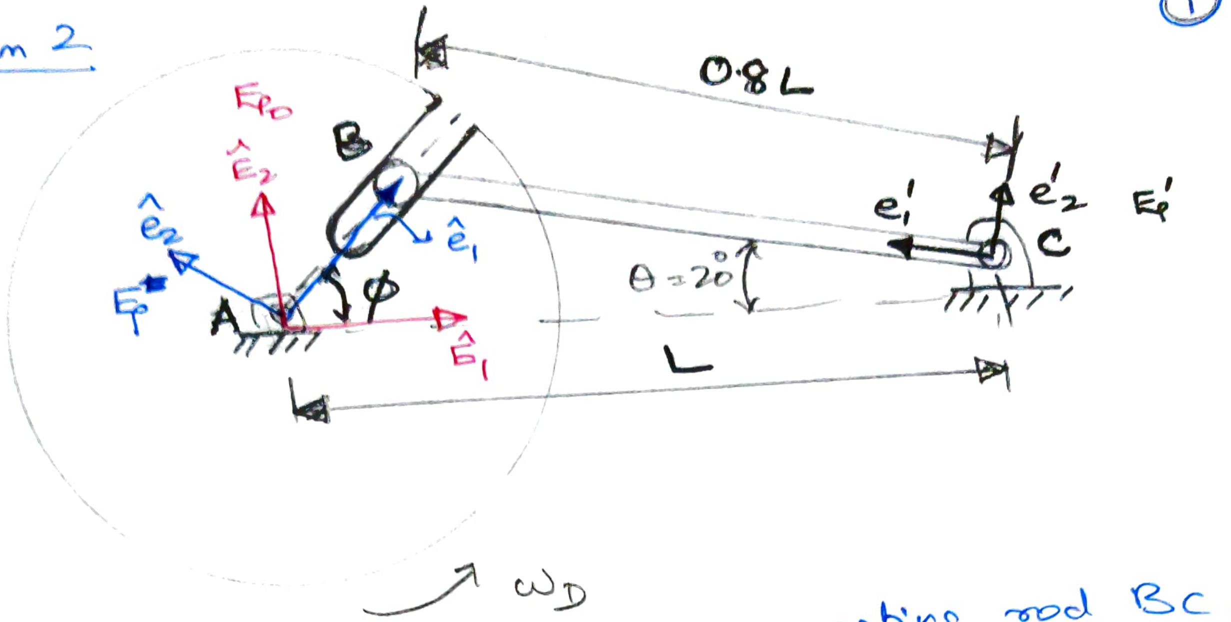
ESO209 Tutorial 6

Problem 2



To find angular velocity of connecting rod BC.
we define coordinate system

Observer CS : $\{E_{P_0}, A, \hat{E}_i\}$ in red.

Disc BFCS : $\{E_P, A, \hat{e}_i\}$ in blue.

Connecting rod (Link) BFCS : $\{E_P', C, \hat{e}_i'\}$ in black.

We choose the point B as the convenient point for analysis since it represents the joint between the disc and the link (connecting rod) BC.

We use the relation

$$\underline{v}^P = \underline{v}^P_{rel} + \underline{\omega}_{E_P/E_{P_0}} \times \underline{r}^{P/G} + v^{G/O}$$

$P \to B$, $\underline{\omega}_{E_P/E_{P_0}} = \underline{\omega}_{Disc}$,

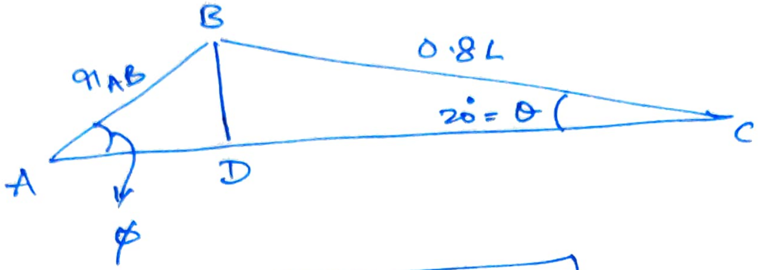
$G \to A$, $O \to A$

$$\underline{v}^B = \underline{v}^B_{rel} + \underline{\omega}_{Disc} \times \underline{r}^{B/A} + \cancel{v^{A/A}}^{\,0}$$

$$\underline{v}^B_{rel} = v_{rel} \cdot \hat{e}_1$$

$$\underline{\omega}_{Disc} = \omega_D \hat{e}_3 = \omega_D \hat{E}_3$$

$$\underline{r}^{B/A} = r_{AB} \cdot \hat{e}_1$$



$$\boxed{r_{AB}\sin\phi = 0.8L\sin\theta} \quad \text{—— (1)}$$

$$\Rightarrow \quad r_{AB} = \frac{0.8L\sin\theta}{\sin\phi}$$

∵ AC = L

Also, $\underbrace{r_{AB}\cos\phi}_{AD} + \underbrace{0.8L\cos\theta}_{DC} = \underbrace{L}_{AC}$ $\longrightarrow$ (2)

Squaring (1) & (2) and adding them

$$\therefore \quad (r_{AB}\sin\phi)^2 + (r_{AB}\cos\phi)^2 = (0.8L\sin\theta)^2 + L^2(1-0.8\cos\theta)^2$$

$$\Rightarrow \quad r_{AB}^2 = L^2(0.64\sin^2\theta + 1 + 0.64\cos^2\theta - 1.6\cos\theta)$$

$$= L^2(1.64 - 1.6\cos\theta)$$

Substituting $\theta = 20°$

$$= L^2(1.64 - 1.6 * 0.9397)$$

$$\Rightarrow \quad \boxed{r_{AB} = 0.369\,L}$$

Dividing (1) by (2)

$$\tan\phi = \frac{0.8L\sin\theta}{L(1-0.8\cos\theta)} = \frac{0.8\sin\theta}{(1-0.8\cos\theta)} = 1.102$$

$$\Rightarrow \quad \boxed{\phi = 47.8°}$$

Continuing to use the variables

$$\underline{v}^B = v_{rel}\cdot\hat{e}_1 + \omega_D \hat{e}_3 \times r_{AB}\cdot\hat{e}_1$$

$$\underline{v}^B = v_{rel}\cdot\hat{e}_1 + \omega_D\cdot r_{AB}\cdot\hat{e}_2 \quad \text{———} \quad (3)$$

Determining $\underline{v}^B$ using the BFCS, E_P'

We use the relation of two points on a rigid body

$$\underline{v}^B = \underline{v}^A + \underline{\omega}_B \times \underline{r}^{B/A}$$

in this case,

$$B \rightarrow B, \quad A \rightarrow C, \quad \underline{\omega}_B = \underline{\omega}_{link},$$

$$\Rightarrow \underline{v}^B = \cancel{\underline{v}^C} + \underline{\omega}_{link} \times \underline{r}^{B/C}$$

0

$$= \underline{\omega}_{link} \times (0.8L\,\hat{e}_1')$$

$$\underline{\omega}_{link} = \dot{\theta}\,\hat{e}_3'$$

$$\Rightarrow \underline{v}^B = \dot{\theta}\,\hat{e}_3' \times 0.8L\,\hat{e}_1'$$

$$\underline{v}^B = 0.8L\,\dot{\theta}\,\hat{e}_2' \quad \text{———} \quad (4)$$

Equating (3) & (4) gives us two scalar equations in two unknowns v_{rel} and $\dot{\theta}$.
However they are not in the same CS.
Hence they need to be brought to a common CS which in this case is the observer CS, E_{P0}

We use the coordinate transformation results presented in Lecture 5, which states:

If there is a rotation tensor $\underline{\underline{R}}$ such that

$$\{E_P, O, \hat{e}_i\} \underset{\underline{\underline{R}}^T}{\overset{\underline{\underline{R}}}{\rightleftarrows}} \{E_{P_1}, O, \hat{e}_i'\}$$

then

$$[\underline{a}]_{E_{P_1}} = [\underline{\underline{R}}]^T_{E_P} \cdot [\underline{a}]_{E_P}$$

or $$[\underline{a}]_{E_P} = [\underline{\underline{R}}]_{E_P} [\underline{a}]_{E_{P_1}}$$

In our case, first examining equation 3

$\{E_P, O, \hat{e}_i\}$ is $\{E_{P_0}, A, \hat{E}_i\}$

and $\{E_{P_1}, O, \hat{e}_i'\}$ is $\{E_P, A, \hat{e}_i\}$

$$[\underline{\underline{R}}_1]_{E_P} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\overset{\nearrow v^B}{\underline{a}}]_{E_{P_2}} = \begin{bmatrix} v_{rel} \\ \omega_D r_{AB} \\ 0 \end{bmatrix}$$

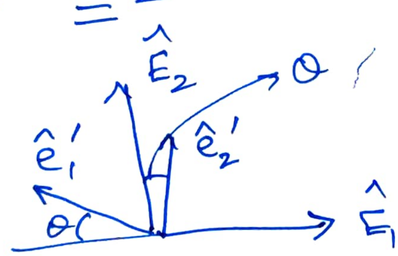
$$\therefore [\overset{\nearrow v^B}{\underline{a}}]_{E_{P_0}} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{rel} \\ \omega_D r_{AB} \\ 0 \end{bmatrix}$$

$$[\underline{v}^B]_{E_{p0}} = \begin{bmatrix} v_{rel}\cos\phi - \omega_D r_{AB}\sin\phi \\ v_{rel}\sin\phi + \omega_D r_{AB}\cos\phi \\ 0 \end{bmatrix}$$

which can be written as

$$\underline{v}^B = (v_{rel}\cos\phi - \omega_D r_{AB}\sin\phi)\hat{E}_1 + (v_{rel}\sin\phi + \omega_D r_{AB}\cos\phi)\hat{E}_2 \quad (5)$$

Now considering eqn (4), to write it using the basis vectors of CS E_{p0}, ie, $\hat{E}_i$, we use the rotation tensor from E_{p0} to E_p'. Lets call that tensor $\underline{\underline{R}}_2$



Using the relation $R_{ij} = \hat{e}_j' \cdot \hat{E}_i$, we get

$$[\underline{\underline{R}}_2]_{E_{p0}} = \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As per Equation (4)

$$[\underline{v}^B]_{E_p'} = \begin{bmatrix} 0 \\ 0.8L\dot{\theta} \\ 0 \end{bmatrix}$$

Using the coordinate transformation we write

$$[\underline{v}^B]_{E_{p_0}} = [\underline{R}]_{E_{p_0}} [\underline{v}^B]_{E_p'}$$

$$= \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8L\dot{\theta} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8L\dot{\theta}\sin\theta \\ 0.8L\dot{\theta}\cos\theta \\ 0 \end{bmatrix}$$

which can be written as

$$\underline{v}^B = 0.8L\dot{\theta}\sin\theta\,\hat{E}_1 + 0.8L\dot{\theta}\cos\theta\,\hat{E}_2 \quad \leftarrow (6)$$

Equations (5) & (6) represent the same vector $\underline{v}^B$ in the observer CS. Hence we get the following pair of equations by equating the components of $\hat{E}_1$ and $\hat{E}_2$.

$$(7) \leftarrow \quad 0.8L\dot{\theta}\sin\theta = v_{rel}\cos\phi - \omega_D r_{AB}\sin\phi$$

$$(8) \leftarrow \quad 0.8L\dot{\theta}\cos\theta = v_{rel}\sin\phi + \omega_D r_{AB}\cos\phi$$

we thus have two eqns in two unknowns, viz $\dot{\theta}$ and v_{rel}

To find $\dot{\theta}$, multiplying (7) * $\sin\phi$ and (8) * $\cos\phi$ and subtracting, we get

$$0.8L\dot{\theta}(\cos\theta\cos\phi - \sin\theta\sin\phi)$$
$$= \omega_D r_{AB}\cancel{(\cos^2\phi + \sin^2\phi)}$$

$$\Rightarrow \quad 0.8L\dot{\theta}\cos(\theta+\phi) = \omega_D r_{AB}$$

$$\Rightarrow \quad \dot{\theta} = \frac{\omega_D r_{AB}}{0.8L\cos(\theta+\phi)}$$

substituting $r_{AB} = 0.369L$ & $\theta = 20^\circ$, $\phi = 47.8^\circ$

$$\dot{\theta} = \frac{0.369\cancel{L}}{0.8\cancel{L}\cos(67.8)} \cdot \omega_D$$

$$\boxed{\dot{\theta} = 1.22\,\omega_D}$$