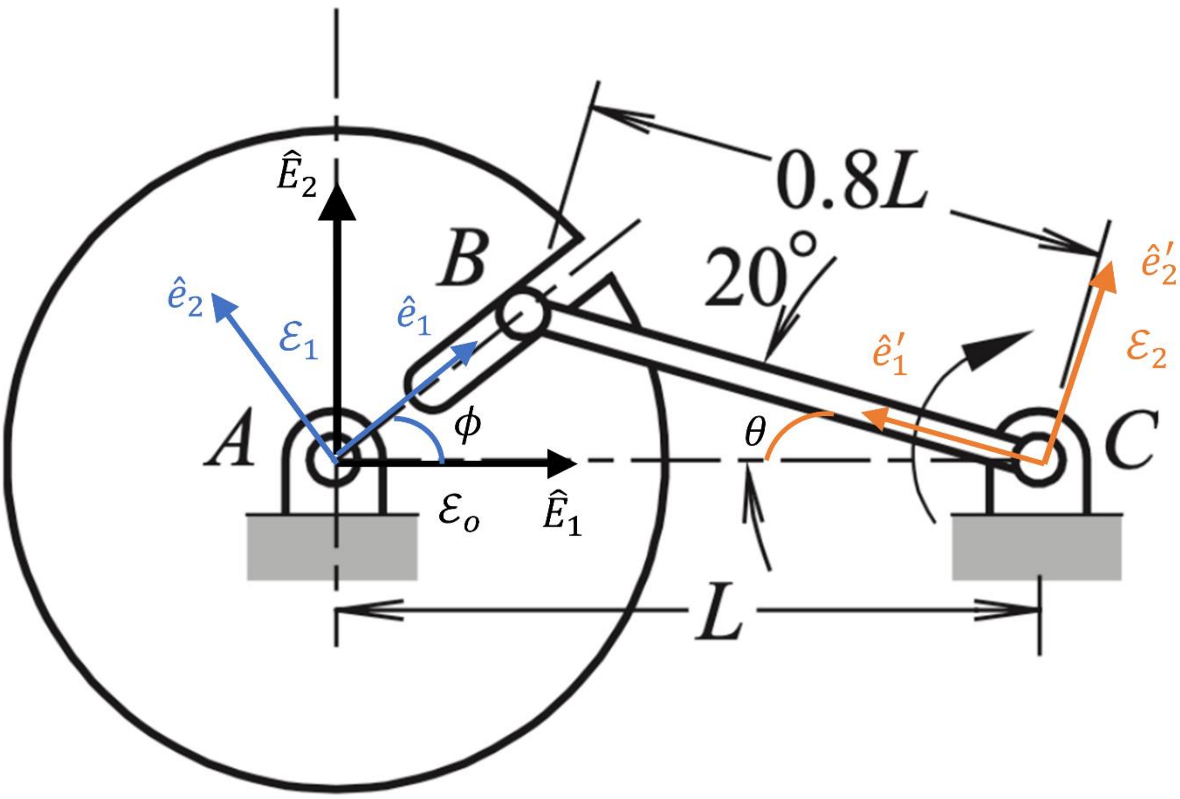
Problem 3:



First, identifying all rigid bodies and then we attached BFCS at joint A, $\{\mathcal{E}_o, A, \hat{E}_i\}$, BFCS for disc, $\{\mathcal{E}_1, A, \hat{e}_i\}$ and BFCS for connecting rod at point C as $\{\mathcal{E}_2, C, \hat{e}_i'\}$.

Velocity analysis: already solved in problem 2 of Tutorial 6.

We will choose a convenient point 'B' because it is joint between disc and connecting rod and its velocity can be obtained by from observer sitting on disc and connecting rod.

$\{\mathcal{E}_o, A, \hat{E}_i\}$: observer $\{\mathcal{E}_1, A, \hat{e}_i\}$: disc BFCS

$$\underline{v}^B = \underline{v}^B_{rel} + \underline{\omega}_{\mathcal{E}_1/\mathcal{E}_o} \times \underline{r}^{B/A} + \underline{v}^{A/A} \quad \ldots\ldots\ldots\ldots(1)$$

Clearly $\underline{v}^{A/A} = 0$; velocity of A measured in $\mathcal{E}_o$. $\underline{\omega}_{\mathcal{E}_1/\mathcal{E}_o} = \underline{\omega}_{DISC}$

$$\underline{v}^B = \underline{v}^B_{rel} + \underline{\omega}_{DISC} \times \underline{r}^{B/A} \quad \ldots\ldots\ldots(2)$$

$\underline{v}^B_{rel} = v_{rel}\hat{e}_1$; $\underline{\omega}_{DISC} = \omega_D \hat{e}_3$ and $\underline{r}^{B/A} = r_{AB}\hat{e}_1$.

Writing directly from solution of problem 2 of Tutorial 6

$r_{AB} = 0.369L$ and $\phi = 47.8^0$

$$\underline{v}^B = v_{rel}\hat{e}_1 + \omega_D \hat{e}_3 \times r_{AB}\hat{e}_1$$

$$\underline{v}^B = v_{rel}\hat{e}_1 + \omega_D r_{AB}\hat{e}_2 \ldots\ldots(3)$$

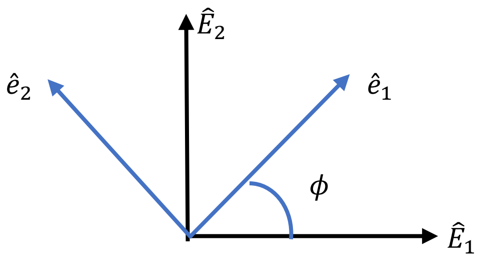
Now writing velocity of point B using $\{\mathcal{E}_2, C, \hat{e}_i'\}$. Since both B and C are on the same body

$$\underline{v}^B = \underline{v}^C + \underline{\omega}_{\mathcal{E}_2/\mathcal{E}_o} \times \underline{r}^{B/C}$$

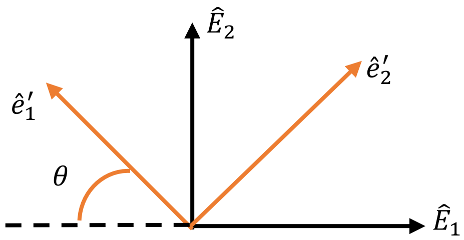
Clearly $\underline{v}^C = 0$; velocity of point C observed in $\mathcal{E}_o$. $\underline{\omega}_{\mathcal{E}_2/\mathcal{E}_o} = \underline{\omega}_{link} = \omega_{link}\hat{e}_3'$ and $\underline{r}^{B/C} = r_{BC}\hat{e}_1'$. Given $r_{BC} = 0.8L$

$$\underline{v}^B = \omega_{link}\hat{e}_3' \times r_{BC}\hat{e}_1' = \omega_{link}r_{BC}\hat{e}_2' \dots\dots\dots (4)$$

From figure



$$\hat{e}_1 = \hat{E}_1\cos\phi + \hat{E}_2\sin\phi$$
$$\hat{e}_2 = \hat{E}_2\cos\phi - \hat{E}_1\sin\phi$$



$$\hat{e}_1' = \hat{E}_2\sin\theta - \hat{E}_1\cos\theta$$
$$\hat{e}_2' = \hat{E}_1\sin\theta + \hat{E}_2\cos\theta$$

Use eq. (3) and eq. (4) for v_{rel} and ω_{link} (see solution of problem 2 of Tutorial 6), we get

$$\omega_{link} = 1.22\ \omega_D$$
$$v_{rel} = 0.9L\ \omega_D$$

acceleration analysis:

$\{\mathcal{E}_o, A, \hat{E}_i\}$: observer $\{\mathcal{E}_1, A, \hat{e}_i\}$: disc BFCS

$$\underline{a}^B = \underline{a}^B_{rel} + \underline{\omega}_{\mathcal{E}_1/\mathcal{E}_o} \times \left(\underline{\omega}_{\mathcal{E}_1/\mathcal{E}_o} \times \underline{r}^{B/A}\right) + \underline{\alpha}_{\mathcal{E}_1/\mathcal{E}_o} \times \underline{r}^{B/A} + 2\underline{\omega}_{\mathcal{E}_1/\mathcal{E}_o} \times \underline{v}^B_{rel} + \underline{a}^A$$

Clearly $\underline{a}^A = 0$; acceleration of A measured in $\mathcal{E}_o$. $\underline{a}^B_{rel} = a_{rel}\hat{e}_1$, $\underline{\alpha}_{\mathcal{E}_1/\mathcal{E}_o} = \underline{\alpha}_{DISC} = \alpha_D\hat{e}_3$.

$$\underline{a}^B = a_{rel}\hat{e}_1 + \omega_D\hat{e}_3 \times (\omega_D\hat{e}_3 \times r_{AB}\hat{e}_1) + \alpha_D\hat{e}_3 \times r_{AB}\hat{e}_1 + 2\omega_D\hat{e}_3 \times v_{rel}\hat{e}_1$$

$$\underline{a}^B = a_{rel}\hat{e}_1 - \omega_D^2 r_{AB}\hat{e}_1 + \alpha_D r_{AB}\hat{e}_2 + 2\omega_D v_{rel}\hat{e}_2 \dots\dots..(5)$$

Now writing acceleration of point B using $\{\mathcal{E}_2, C, \hat{e}_i'\}$. Since both B and C are on the same body

$$\underline{a}^B = \underline{a}^C + \underline{\omega}_{\mathcal{E}_2/\mathcal{E}_o} \times \left(\underline{\omega}_{\mathcal{E}_2/\mathcal{E}_o} \times \underline{r}^{B/C}\right) + \underline{\alpha}_{\mathcal{E}_2/\mathcal{E}_o} \times \underline{r}^{B/C}$$

Clearly $\underline{a}^C = 0$; acceleration of C measured in $\mathcal{E}_o$. $\underline{\alpha}_{\mathcal{E}_2/\mathcal{E}_o} = \underline{\alpha}_{link} = \alpha_{link}\hat{e}_3'$

$$\underline{a}^B = \omega_{link}\hat{e}_3' \times (\omega_{link}\hat{e}_3' \times r_{BC}\hat{e}_1') + \alpha_{link}\hat{e}_3' \times r_{BC}\hat{e}_1'$$

$$\underline{a}^B = -\omega_{link}^2 r_{BC}\hat{e}_1' + \alpha_{link}r_{BC}\hat{e}_2' \dots\dots(6)$$

Use eq. (5) and (6), two unknowns, a_{rel} and α_{link}.

Substitute $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_1'$ and $\hat{e}_2'$ in eq. (5) and (6) and then solving for a_{rel} and α_{link} by comparing components of $\hat{E}_1$ and $\hat{E}_2$.

Values known

$$\omega_{link} = 1.22\ \omega_D$$

$$v_{rel} = 0.9L\ \omega_D$$

$$\phi = 47.8^0$$

$$\theta = 20^0$$

$$r_{AB} = 0.369L$$

$$r_{BC} = 0.8L$$

$$\underline{a}^B = (0.8L\omega_{link}^2 \cos\theta + 0.8L\alpha_{link} \sin\theta)\hat{E}_1 + (-0.8L\omega_{link}^2 \sin\theta + 0.8L\ \alpha_{link} \cos\theta)\hat{E}_2$$

$$\underline{a}^B = (a_{rel} \cos\phi - 0.369L\ \omega_D^2 \cos\phi - 0.369L\alpha_D \sin\phi - 2\omega_D v_{rel} \sin\phi)\hat{E}_1 + (a_{rel} \sin\phi - 0.369L\omega_D^2 \sin\phi + 0.369L\alpha_D \cos\phi + 2\omega_D v_{rel} \cos\phi)\hat{E}_2$$

comparing components of $\hat{E}_1$ and $\hat{E}_2$.

$$0.8L\omega_{link}^2 \cos\theta + 0.8L\alpha_{link} \sin\theta = a_{rel} \cos\phi - 0.369L\ \omega_D^2 \cos\phi - 0.369L\alpha_D \sin\phi - 2\omega_D v_{rel} \sin\phi \ \ldots\ldots (7)$$

$$-0.8L\omega_{link}^2 \sin\theta + 0.8L\ \alpha_{link} \cos\theta = a_{rel} \sin\phi - 0.369L\omega_D^2 \sin\phi + 0.369L\alpha_D \cos\phi + 2\omega_D v_{rel} \cos\phi \ \ldots\ldots . (8)$$

Answer:

$$\alpha_{link} = 9.536\omega_D^2 - 0.2847\alpha_D$$

$$a_{rel} = 7.89L\omega_D^2 - 0.3L\alpha_D$$