1. Using index notation write the expression for $\cos\theta$ and $\sin\theta$, where θ is the angle between vectors **a** and **b**.

Solution:

Part 1(a)

$$\mathbf{a} = a_i\hat{e}_i \text{ and } \mathbf{b} = b_j\hat{e}_j$$

Dot product of **a** and **b**

$$\mathbf{a}.\mathbf{b} = a_i\hat{e}_i \,.\, b_j\hat{e}_j$$

$$|\mathbf{a}||\mathbf{b}|\cos\theta = a_i b_j \hat{e}_i.\hat{e}_j = a_i b_j \delta_{ij}$$

using contraction

$$|\mathbf{a}||\mathbf{b}|\cos\theta = a_i b_i$$

$$\cos\theta = \frac{a_i b_i}{|\mathbf{a}||\mathbf{b}|}$$

Since $|\mathbf{a}| = \sqrt{a_m a_m}$ and $|\mathbf{b}| = \sqrt{b_n b_n}$, therefore

$$\cos\theta = \frac{a_i b_i}{\left(\sqrt{a_m a_m}\right)\left(\sqrt{b_n b_n}\right)}$$

1. Using index notation write the expression for $\cos\theta$ and $\sin\theta$, where θ is the angle between vectors **a** and **b**.

Solution:

Part 1(b)

$$\mathbf{a} = a_i\hat{e}_i \text{ and } \mathbf{b} = b_j\hat{e}_j$$

vector product of **a** and **b**

$$\mathbf{a}\times\mathbf{b} = a_i\hat{e}_i \times b_j\hat{e}_j$$

$$|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{e}_k = a_ib_j\hat{e}_i \times \hat{e}_j = a_ib_j\epsilon_{ijk}\hat{e}_k$$

using contraction

$$|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{e}_k = a_ib_j\epsilon_{ijk}\hat{e}_k$$

($\hat{e}_k$ is unit vector perpendicular to the plane of **a** and **b**)

$$\sin\theta = \frac{a_ib_j\epsilon_{ijk}}{|\mathbf{a}||\mathbf{b}|}$$

Since $|\mathbf{a}| = \sqrt{a_ma_m}$ and $|\mathbf{b}| = \sqrt{b_nb_n}$, therefore

$$\sin\theta = \frac{a_ib_j\epsilon_{ijk}}{\left(\sqrt{a_ma_m}\right)\left(\sqrt{b_nb_n}\right)}$$