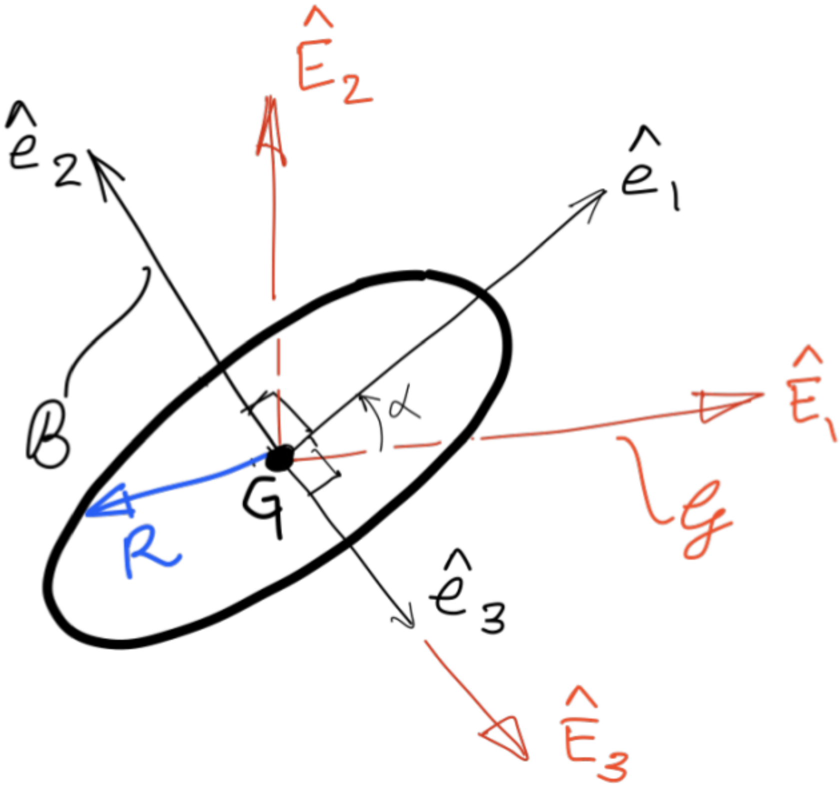


$\hat{E}_2$
$\hat{e}_2$
$\hat{e}_1$
$\mathcal{B}$
α
$\hat{E}_1$
G
R
$\mathcal{G}$
$\hat{e}_3$
$\hat{E}_3$

The mass moment of inertia tensor $\underline{\underline{I}}$ in $\{\mathcal{B}, G, \hat{e}_i\}$ about G (center of mass is known

$$[I]_{\mathcal{B}} = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{——— (1)}$$

where $k = \frac{mR^2}{4}$.

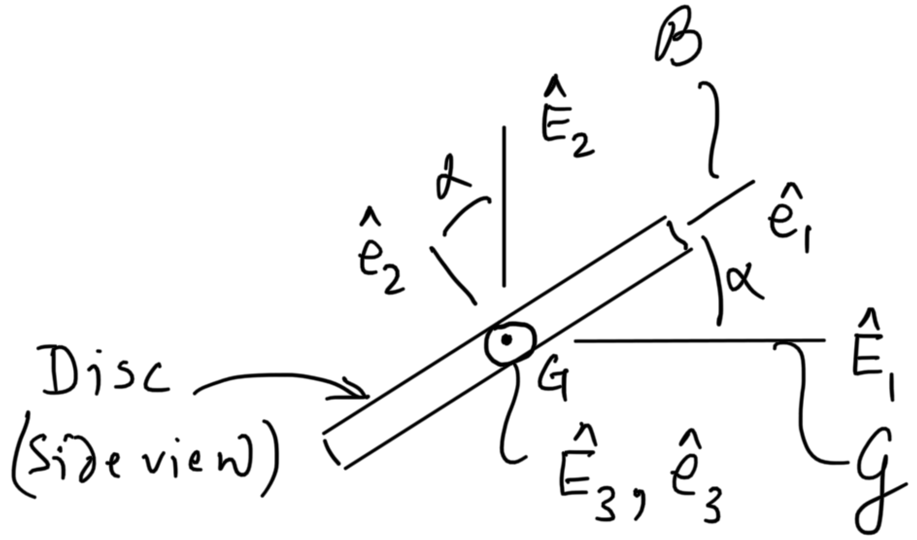
We have to compute $[I]_{\mathcal{G}}$!

Essentially we have to use the change of basis formula.

$$\{\mathcal{B}, G, \hat{e}_i\} \xrightarrow{\underline{\underline{R}}} \{\mathcal{G}, O, \hat{E}_i\}$$

Recall $R_{ij} = \hat{e}_j' \cdot \hat{e}_i$ which in this case is

$$R_{ij} = \hat{E}_j \cdot \hat{e}_i \quad \text{——— (2)}$$



$$[R]_{\mathcal{B}} = \begin{bmatrix} \hat{E}_1 \cdot \hat{e}_1 & \hat{E}_2 \cdot \hat{e}_1 & \hat{E}_3 \cdot \hat{e}_1 \\ \hat{E}_1 \cdot \hat{e}_2 & \hat{E}_2 \cdot \hat{e}_2 & \hat{E}_3 \cdot \hat{e}_2 \\ \hat{E}_1 \cdot \hat{e}_3 & \hat{E}_2 \cdot \hat{e}_3 & \hat{E}_3 \cdot \hat{e}_3 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \cos(90-\alpha) & \cos 90^\circ \\ \cos(90+\alpha) & \cos\alpha & \cos 90^\circ \\ \cos 90^\circ & \cos 90^\circ & \cos 0^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[I]_{\mathcal{G}} = [R]_{\mathcal{B}}^{T}\,[I]_{\mathcal{B}}\,[R]_{\mathcal{B}}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k & 0 & 0 \\ 0 & 2k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore [I]_{\mathcal{G}} = \begin{bmatrix} k(1+\sin\alpha) & -k\sin\alpha\cos\alpha & 0 \\ -k\sin\alpha\cos\alpha & k(1+\cos\alpha) & 0 \\ 0 & 0 & k \end{bmatrix}$$