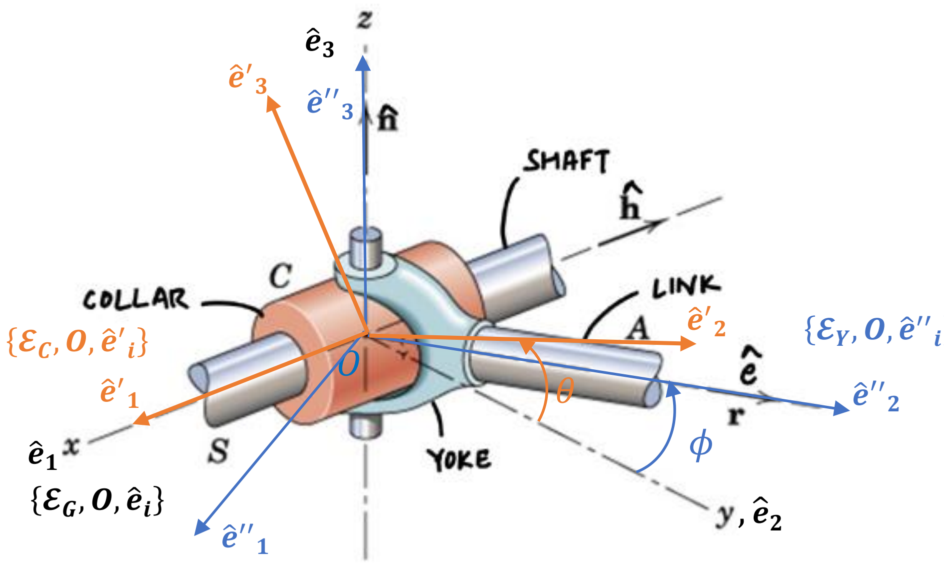
Problem 5:

5. The end of link A is welded to the yoke which is attached to the collar C. The pivoting axis of the yoke is denoted by $\hat{\mathbf{n}}$. The collar may rotate about axis $\hat{\mathbf{h}}$ of the fixed shaft. Link A and its yoke can thus rotate *only* about the $\hat{\mathbf{n}}$ and $\hat{\mathbf{h}}$ directions, but not about any direction which is not in the plane formed by $\hat{\mathbf{n}}$ and $\hat{\mathbf{h}}$. For example, the link A cannot rotate about the direction $\hat{\mathbf{e}}$ along the link. Show that, regardless of the motion of the other end (not shown) of the link A, the angular velocity $\boldsymbol{\omega}$ of the link A and its yoke must satisfy the relation

$$\boldsymbol{\omega} \cdot \hat{\mathbf{h}} \times (\hat{\mathbf{e}} \times \hat{\mathbf{h}}) = \mathbf{0}\,.$$



Solution:

By reading problem statement, we know that link A (or yoke) has no angular velocity component in the $\hat{h} \times \hat{n}$.

From figure, ground frame $\{\mathcal{E}_G, O, \hat{e}_i\}$ is attached to the shaft, BFCS $\{\mathcal{E}_C, O, \hat{e}'_i\}$ is attached to collar and BFCS $\{\mathcal{E}_Y, O, \hat{e}''_i\}$ is attached to yoke (or link).

angular velocity of yoke (or link) in $\mathcal{E}_Y$

$$\underline{\omega}_{yoke/\mathcal{E}_Y} = \Omega_y \hat{e}''_3 = \Omega_y \hat{n}, \qquad (\hat{e}''_3 = \hat{e}_3 = \hat{n})$$

Now, angular velocity of yoke (or link) in $\mathcal{E}_C$

$$\underline{\omega}_{yoke/\mathcal{E}_C} = \underline{\omega}_{yoke/\mathcal{E}_Y} + \underline{\omega}_{\mathcal{E}_Y/\mathcal{E}_c} = \Omega_y \hat{n}, \quad \underline{\omega}_{\mathcal{E}_Y/\mathcal{E}_c} = 0$$

For angular velocity of yoke (or link) in $\mathcal{E}_G$

$$\underline{\omega}_{yoke/\mathcal{E}_G} = \underline{\omega}_{yoke/\mathcal{E}_C} + \underline{\omega}_{\mathcal{E}_C/\mathcal{E}_G}$$

now, angular velocity of collar in $\mathcal{E}_G$

$$\underline{\omega}_{\mathcal{E}_C/\mathcal{E}_G} = -\Omega_c \hat{e}'_1 = \Omega_c \hat{h}, \qquad (\hat{e}'_1 = \hat{e}_1 = -\hat{h}\ )$$

For angular velocity of yoke (or link) in $\mathcal{E}_G$

$$\underline{\omega}_{yoke/\mathcal{E}_G} = \underline{\omega}_{yoke/\mathcal{E}_C} + \underline{\omega}_{\mathcal{E}_C/\mathcal{E}_G}$$

$$\underline{\omega}_{yoke/\mathcal{E}_G} = \Omega_y \hat{n} + \Omega_c \hat{h}$$

clearly no components in $\hat{h} \times \hat{n}$.

now let's prove

$$\underline{\omega}_{yoke/G}.\left(\hat{h} \times (\hat{e} \times \hat{h})\right) = 0 \ldots\ldots\ldots\ldots (1)$$

now consider

$$\hat{h} \times (\hat{e} \times \hat{h}) = (\hat{h}.\hat{h})\hat{e} - (\hat{h}.\hat{e})\hat{h}$$

since $\hat{h}.\hat{h} = 1$,

$$\hat{h} \times (\hat{e} \times \hat{h}) = \hat{e} - (\hat{h}.\hat{e})\hat{h}$$

consider eq. (1) again

$$\underline{\omega}_{yoke/G}.\left(\hat{h} \times (\hat{e} \times \hat{h})\right) = (\Omega_y \hat{n} + \Omega_c \hat{h}).(\hat{e} - (\hat{h}.\hat{e})\hat{h})$$

$$\underline{\omega}_{yoke/G}.\left(\hat{h} \times (\hat{e} \times \hat{h})\right) = (\Omega_y \hat{n} + \Omega_c \hat{h}).\hat{e} - (\Omega_y \hat{n} + \Omega_c \hat{h}).(\hat{h}.\hat{e})\hat{h}$$

since $\hat{n}.\hat{e} = 0$. (Note: $\hat{h}.\hat{e}$ is a constant)

$$\underline{\omega}_{yoke/G}.\left(\hat{h} \times (\hat{e} \times \hat{h})\right) = \Omega_c(\hat{h}.\hat{e}) - \Omega_y \hat{n}.\hat{h}(\hat{h}.\hat{e}) - \Omega_c \hat{h}.\hat{h}(\hat{h}.\hat{e})$$

since $\hat{n}.\hat{h} = 0$

$$\underline{\omega}_{yoke/G}.\left(\hat{h} \times (\hat{e} \times \hat{h})\right) = \Omega_c(\hat{h}.\hat{e}) - \Omega_c(\hat{h}.\hat{e}) = 0$$

Proved.