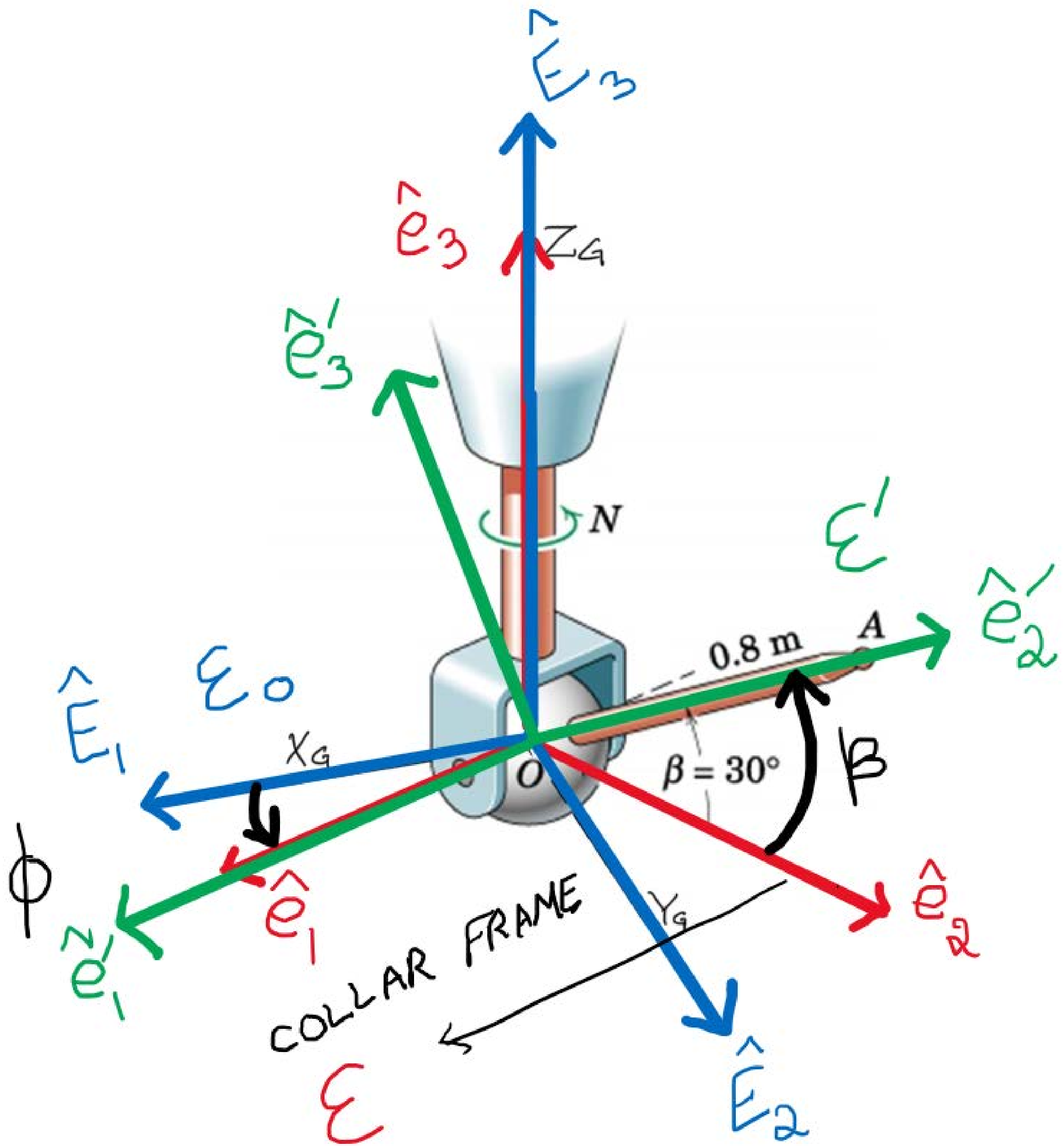
$\hat{E}_3$
$\hat{e}_3$
Z_G
$\hat{e}_3'$
N
$\mathcal{E}'$
$\hat{e}_2'$
0.8 m
A
$\mathcal{E}_0$
$\hat{E}_1$
X_G
O
$\beta = 30°$
β
ϕ
$\hat{e}_1$
$\hat{e}_2$
$\hat{e}_1'$
COLLAR FRAME
Y_G
$\mathcal{E}$
$\hat{E}_2$

FIGURE P1

TUTORIAL - 5, PROBLEM - 1

Refer to figure P1 in page ①.

ϕ is defined such that $\dot{\phi} = N$

$$N = 60 \text{ rpm} = \frac{60 * 2\pi}{60} = \underline{\underline{2\pi \text{ rad/s}}}$$

$$\{\mathcal{E}_0, O, \hat{E}_i\} \xrightarrow{\underline{\underline{R}}_0(\hat{E}_3, \phi(t))} \{\mathcal{E}, O, \hat{e}_i\} \xrightarrow{\underline{\underline{R}}_1(\hat{e}_1, \beta(t))} \{\mathcal{E}', O, \hat{e}'_i\}$$

ground frame — collar frame — arm frame

$$\hat{e}_i = \underline{\underline{R}}_0 \cdot \hat{E}_i \quad \text{------------} \quad ②$$

$$e'_i = \underline{\underline{R}}_1 \cdot \hat{e}_i$$

We know that,

$$[\underline{\underline{R}}_0]_{\mathcal{E}_0} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{-------} \quad ①$$

and,

$$[\underline{\underline{R}}_1]_{\mathcal{E}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix}$$

Angular velocity vector, (by definition)

$$\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = \underline{\omega}_{\mathcal{E}'/\mathcal{E}} + \underline{\omega}_{\mathcal{E}/\mathcal{E}_0}$$

$$= \dot{\beta}\,\hat{e}_1 + \dot{\phi}\,\hat{E}_3$$

given $\beta = 30°$, $\phi = 0$, $\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0}$ is required, expressed in the ground frame ($\mathcal{E}_0$).

From equation (2),

$$\hat{e}_1 = \underline{\underline{R}}_0 \cdot \hat{E}_1 = \left[\underline{\underline{R}}_0\right]_{\mathcal{E}_0} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathcal{E}_0}$$

Note that $\hat{E}_1$ in the ground frame is equivalent to the column vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$$= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi \\ \sin\phi \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathcal{E}_0} \quad (\text{for } \phi = 0)$$

$$= \hat{E}_1$$

Thus, $$\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = \dot{\phi}\,\hat{E}_3 + \dot{\beta}\,\hat{E}_1$$

given $\dot{\beta} = 2\pi$ rad/s and we ~~know~~ have found that $\dot{\phi} = 2\pi$ rad/s.

Hence, $$\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = 2\pi\,\hat{E}_3 + 2\pi\,\hat{E}_1$$