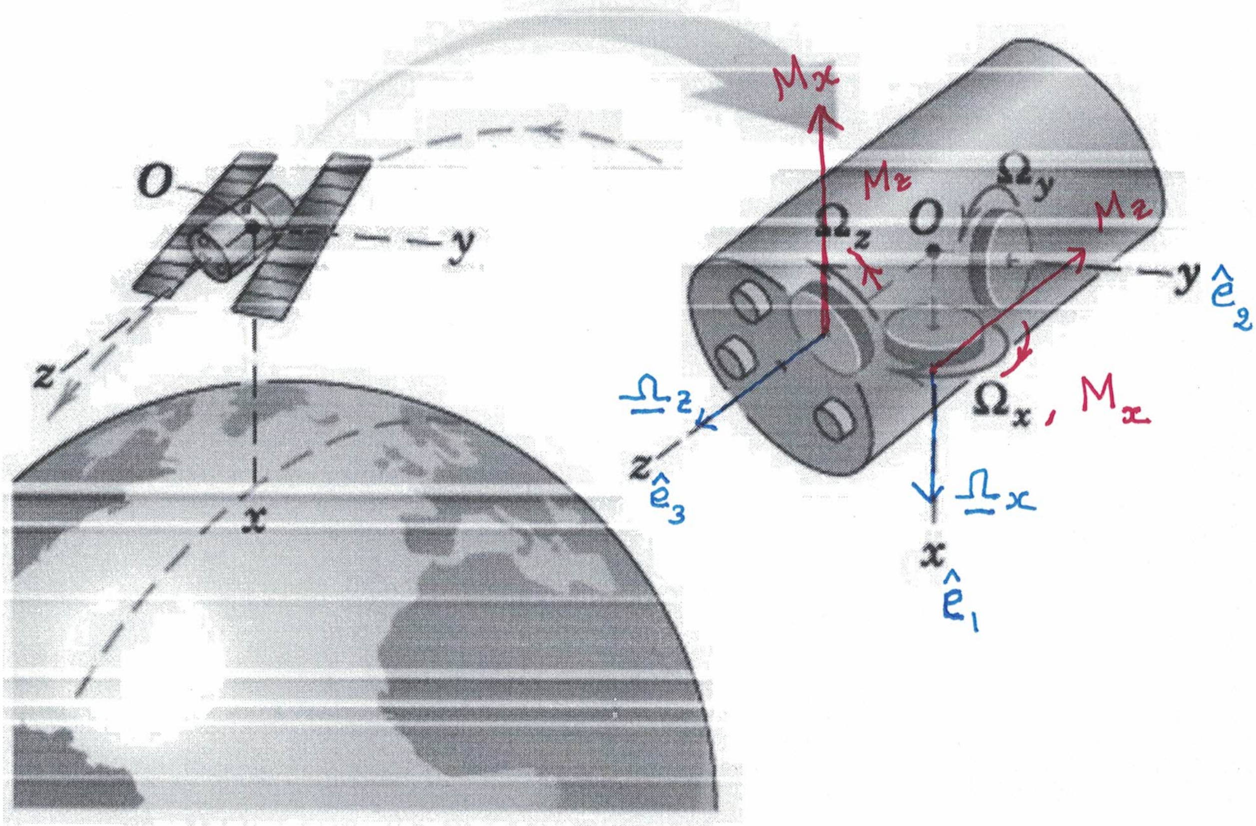

Orbital period, τ

Angular velocity, $\underline{\omega}^S = \omega_y \hat{e}_2 = \frac{2\pi}{\tau} \hat{e}_2$
$= \omega \hat{e}_2$

Points to be noted,

– The reaction-wheel attitude control system is fixed to the satellite. So, the attitude control system along with the satellite can be considered as a single system.

– Thus the individual torques M_x, M_y, M_z, that are being applied on the corresponding reaction wheels, are internal moments, as far as the satellite system is considered.

- Hence an equal and opposite moment will act on the rest of the satellite.

For example, if M_x is the shaft ~~torque~~ torque applied on the reaction-wheel 'X' to increase/decrease its angular speed, an equal and opposite moment M_x is acting on the reaction wheel 'Z' as a gyroscopic moment (refer to figure). ~~This moment will~~

The direction of this gyroscopic moment is along negative x axis.

- Similarly, the shaft torque M_z acting on the reaction-wheel 'Z' (along positive z direction) leads to a gyroscopic moment M_z on the reaction-wheel 'X' along the negative z axis.

- Given that at $t=0$, $\Omega_z = \Omega_0$ and $\Omega_x = \Omega_y = 0$

- The angular velocity of the satellite about its y axis is a result of the precession caused by the two gyroscopic moments.

- Moment of inertia of each reaction wheel is I.

Considering actuation of reaction wheel 'X',

$$M_x = I \dot{\Omega}_x \quad \text{------} \quad (1)$$

At the same time, $-M_x$ acting on the reaction wheel 'Z' is supposed to produce a constant precession rate of $\omega_y = \omega = \frac{2\pi}{\tau}$.

This moment is acting about the point O. ~~Thus~~

Thus, using the gyroscope equation for symmetric rotors with high spin going through steady precession,

$$\underline{M}^O = I_3 \, \underline{\dot{\phi}} \times \underline{\dot{\psi}}$$

(~~from~~ for reaction wheel Z)

Here $\underline{M}^O = -M_x \hat{e}_1$

$$I_3 = I$$

$$\underline{\dot{\phi}} = \omega \hat{e}_2$$

$$\underline{\dot{\psi}} = \Omega_z \hat{e}_3.$$

Thus, $-M_x \hat{e}_1 = I\left(\omega \hat{e}_2 \times \Omega_z \hat{e}_3 \right)$

i.e., $$M_x = -I\,\Omega_z\,\omega \quad \text{--------} \quad (2)$$

Similarly for reaction-wheel 'Z',

$$M_z = I\,\dot{\Omega}_z \quad \text{-------} \quad (3)$$

and

$$-M_z\,\hat{e}_3 = I\left(\omega\,\hat{e}_2 \times \Omega_x\,\hat{e}_1\right)$$

i.e. $$M_z = I\,\Omega_x\,\omega \quad \text{--------} \quad (4)$$

From (1) & (2), $$\cancel{I}\,\dot{\Omega}_x = -\cancel{I}\,\Omega_z\,\omega \quad \text{-------} \quad (5)$$

From (3) & (4) $$\cancel{I}\,\dot{\Omega}_z = +\cancel{I}\,\Omega_x\,\omega \quad \text{-------} \quad (6)$$

substituting for Ω_x in (5) from (6),

$$\frac{\ddot{\Omega}_z}{\omega} = -\Omega_z\,\omega$$

i.e., $$\ddot{\Omega}_z + \Omega_z\,\omega^2 = 0 \quad \text{------} \quad (7)$$

substituting for Ω_z in (6) from (5),

$$-\frac{\ddot{\Omega}_x}{\omega} = \Omega_x\,\omega$$

i.e., $$\ddot{\Omega}_x + \Omega_x\,\omega^2 = 0 \quad \text{-------} \quad (8)$$

Solution to 7 & 8 will be of this form,

$$\Omega_z = A_z \cos\omega t + B_z \sin\omega t$$

$$\Omega_x = A_x \cos\omega t + B_x \sin\omega t$$

given, $\Omega_z = \Omega_0$ at $t=0$ $\Rightarrow$ $A_z = \Omega_0$

$\Omega_x = 0$ at $t=0$ $\Rightarrow$ $A_x = 0$

Substituting $\Omega_x = B_x \sin\omega t$ in 5,

$$B_x \omega \cancel{\sin}\overset{\cos}{}\,\omega t + \Omega_z \omega = 0$$

at $t=0$, $\quad B_x = -\Omega_0.$

$\Omega_z = \Omega_0$

Substituting $\Omega_z = \Omega_0 \cos\omega t + B_z \sin\omega t$

in 6,

$$-\omega\Omega_0 \sin\omega t + B_z \omega \cos\omega t - \Omega_x \omega = 0$$

at $t=0$, $\quad B_z = 0$

$\Omega_x = 0$

Thus, $\Omega_x(t) = -\Omega_0 \sin\omega t$

$\Omega_z(t) = \Omega_0 \cos\omega t$

~~Moto~~ Motor torques on the shaft,

$$M_x = -I\Omega_0 w \cos wt$$

$$M_z = -I\Omega_0 w \sin wt$$

Reaction-wheel 'y' has no rotation,

$$\Omega_y = 0 \quad \& \quad M_y = 0$$

Graphs,