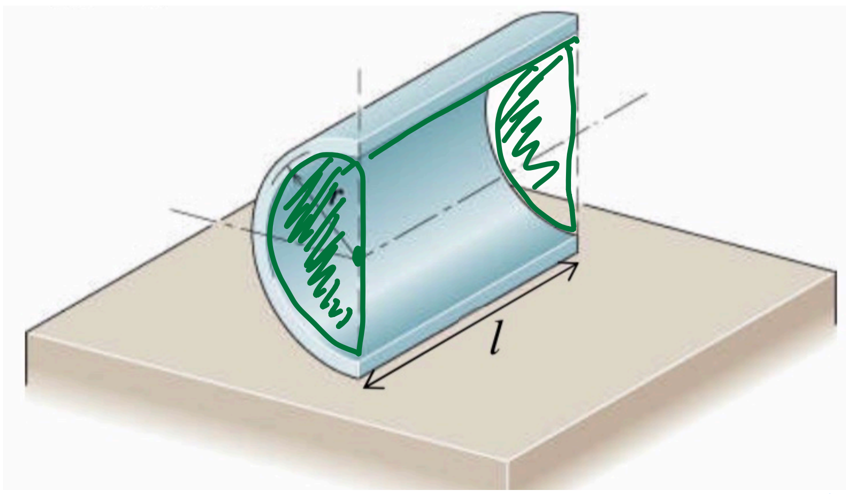
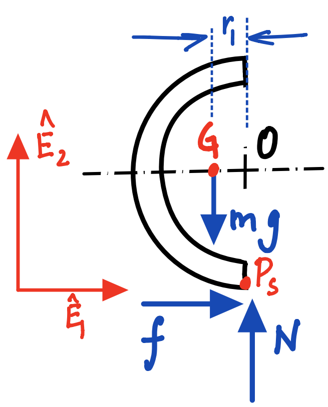
(2) A uniform thin semi-cylindrical shell of mass 10 kg, length $l = 1$ m and mean radius $r = 0.5$ m, is released from the rest position as shown. Determine the minimum coefficient of friction necessary to prevent any initial slipping of the shell.



FBD



- While the object is 3D the KINETICS is 2D (You show this)
- CM G is NOT at O. By symmetry G must be as shown (We will compute r_1 below)

LMB : $\Sigma \underline{F} = m\underline{a}^G \Rightarrow f\hat{E}_1 + (N - mg)\hat{E}_2 = m\underline{a}^G$ (1)

AMB about G: $\underline{M}^G = \underline{\omega}^B \times (\underline{\underline{I}}^G \cdot \underline{\omega}^B) + \underline{\underline{I}}^G \cdot \underline{\alpha}^B$

(2D KINETICS: $\underline{\alpha}^B = \alpha \hat{E}_3$, $\underline{\omega}^B = \omega \hat{E}_3$) $\Rightarrow (f r + N r_1)\hat{E}_3 = I_3^G \alpha \hat{E}_3$ (2)

THREE equations, FIVE (α, $\underline{a}^G$, f, N) unknowns.

The remaining TWO equations will be found by KINEMATIC ANALYSIS.

KINEMATIC ANALYSIS : Assume shell rolls without slipping

– let $\underline{a}^G = a_1\hat{E}_1 + a_2\hat{E}_2$ —— (3)

– Rolling condition #2: $\underline{a}^{P_s}\cdot\hat{E}_1 = \underline{a}^{P_G}\cdot\hat{E}_1 = 0$ (4)

(P_s is the point on shell in contact with point P_G on the ground)

$$\underline{a}^{P_s} = \underline{a}^G + \underline{\omega}^B\times\left(\underline{\omega}^B\times\underline{r}^{P_s/G}\right) + \underline{\alpha}^B\times\underline{r}^{P_s/G}$$

$$\underline{r}^{P_s/G} = r_1\hat{E}_1 - r\hat{E}_2$$

$$= a_1\hat{E}_1 + a_2\hat{E}_2 - \omega^2(r_1\hat{E}_1 - r_2\hat{E}_2) + \alpha(r_1\hat{E}_2 + r\hat{E}_1)$$

0 as given that shell is released from rest.

$$\therefore\quad \underline{a}^{P_s} = a_1\hat{E}_1 + a_2\hat{E}_2 + \alpha(r_1\hat{E}_2 + r\hat{E}_1) \text{ —— (5)}$$

Equations (4) & (5) $\Rightarrow$ $a_1 + \alpha r = 0$ —— (6)

Need one more equation.

— Consider the motion of O. Think of O as a part of the cylindrical shell. This is because O remains at the <u>same</u> distance from all points on the shell

Clearly $\quad \underline{a}^O \cdot \hat{E}_2 = 0$ ———— (7)

as O remains at a distance r from the ground.

As O & G are on the rigid shell :

$$\underline{a}^O = \underline{a}^G + \cancel{\underline{\omega}^B \times (\underline{\omega}^B \times \underline{r}^{O/G})} + \underline{\alpha}^B \times \underline{r}^{O/G}$$

from (3) $\quad$ 0

$$\Rightarrow \underline{a}^O = a_1 \hat{E}_1 + a_2 \hat{E}_2 + \alpha r_1 \hat{E}_2$$

Using (7): $\quad a_2 + \alpha r_1 = 0$ ———— (8)

Equations (6) & (8) are the required TWO more equations.

Collecting (1), (2), (6) & (8)

$$\underbrace{f = m a_1 \, ; \quad N - mg = m a_2 \, ;}_{\text{LMB}} \qquad \overbrace{f r + N r_1 = I_3^G \alpha}^{\text{AMB/G}}$$

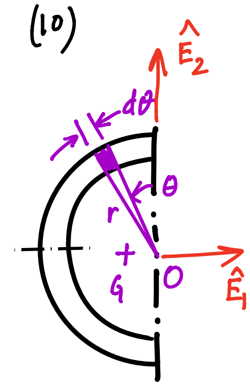
KINEMATIC ANALYSIS : $\quad a_1 + \alpha r = 0 \, ; \quad a_2 + \alpha r_1 = 0$

Solving for f & N :

(9) $$f = \frac{mg(m r r_1)}{I_3^G + m(r^2 + r_1^2)} \qquad N = mg\left(1 - \frac{m r_1^2}{I_3^G + m(r^2 + r_1^2)}\right)$$

Now we must have $f \leq \mu N$

$$\Rightarrow \boxed{\mu_{min} = f/N = \frac{m r r_1}{I_3^G + m r^2}} \qquad (10)$$



All we need now is r_1 & I_3^G.

- From $\underline{r}^{G/O} = \frac{\int \rho \underline{r}\, dV}{m}$ we obtain

$$- r_1 \hat{E}_1 = - \frac{\int_0^\pi r \sin\theta\, \rho r\, d\theta\, \hat{E}_1 L}{m} = - \frac{\rho r^2 (-\cos\theta)\Big|_0^\pi}{m} \hat{E}_1 L$$

$$\Rightarrow r_1 = \frac{2\rho r^2 L}{m} = \frac{2\rho r^2 L}{\pi r \rho L} = 2r/\pi \qquad (11)$$

- $$\underline{\underline{I}}^O = \underline{\underline{I}}^G + m\left(|\underline{r}^{G/O}|^2 \underline{\underline{1}} - \underline{r}^{G/O} \otimes \underline{r}^{G/O}\right) \qquad \text{(// axis theorem)}$$

It is easy to guess/compute that $\hat{e}_3 = \hat{E}_3$ is the principal axis of $\underline{\underline{I}}^O$ & $\underline{\underline{I}}^G$

$$\Rightarrow I_3^O = I_3^G + m r_1^2$$

(Easy to see that $I_3^O = m r^2$ as we have mass m at a distance r from O)

$$\Rightarrow m r^2 = I_3^G + m r_1^2$$

$$\Rightarrow I_3^G = m(r^2 - r_1^2) = m r^2 (1 - 4/\pi^2) \qquad (12)$$

Combine (10) – (12) to get $\boxed{\mu_{min} = \frac{\pi}{\pi^2 - 2}}$ ∎