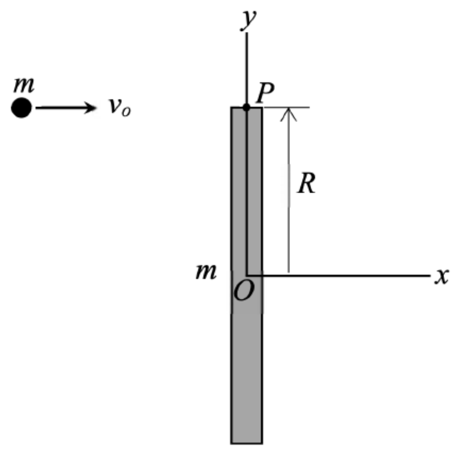
Problem:

(1) A thin uniform circular disc-type satellite of mass m and radius R is spinning at a constant angular rate of ω_0 in space about its body-fixed x-axis, as shown. A space debris of equal mass but of a point size hits the satellite normal to its plane with the velocity v_0 as shown. Subsequently, the debris gets embedded into it at point P. Find the angular velocity vector immediately after the collision. What impact model is being used here? The figure shows the side view of the satellite.



Solution: we have attached the ground fixed CS: $\{\mathcal{E}_o, O, \hat{E}_i\}$ and satellite BFCS: $\{\mathcal{E}_1, O, \hat{e}_i\}$.

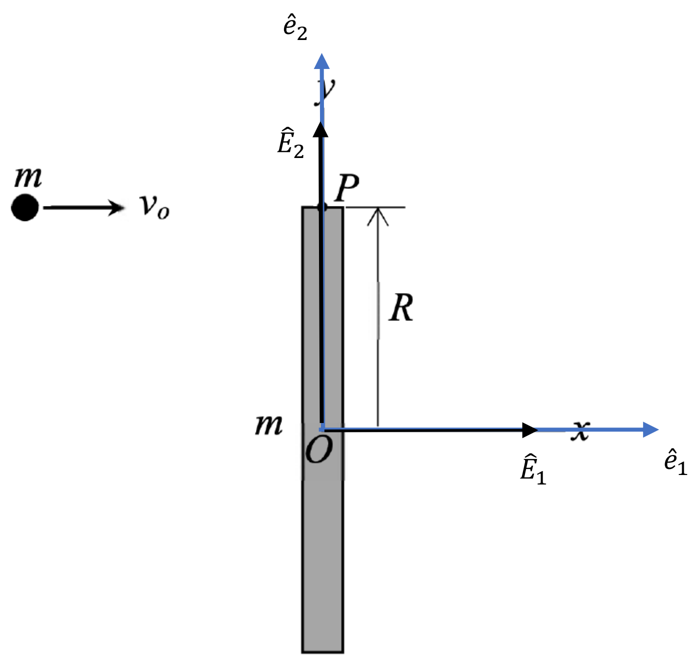


Fig. 1

FBD: as given in problem that debris gets embedded into satellite therefore, $e = 0$. Weights are not shown as they are bounded forces.

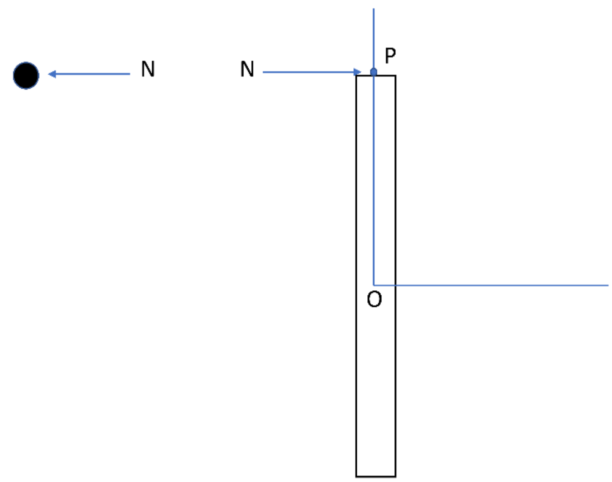


Fig.2

Let mass of debris as m_d and mass of satellite as m_s and other parameters are given as

$\underline{v}_b^s$: velocity of satellite just before impact.

$\underline{v}_b^d$: velocity of debris just before impact.

$\underline{v}_a^s$: velocity of satellite just after impact.

$\underline{v}_a^d$: velocity of debris just before impact.

$\underline{\omega}_b^s$: angular velocity of satellite just before impact.

$\underline{\omega}_a^s$: angular velocity of satellite just after impact.

Now writing all *impulse-momentum relations,*

First writing conservation of Linear momentum

$$p_{sys}(t-\delta t) = p_{sys}(t+\delta t) \dots\dots (1)$$

Using Eq. (1)

$$m_s\underline{v}_b^s + m_d\underline{v}_b^d = m_s\underline{v}_a^s + m_d\,\underline{v}_a^d,$$

Given: $\underline{v}_b^d = v_o\hat{e}_1, \underline{v}_b^s = \underline{0}$ and $m_s = m_d = m$. Then

$$\underline{v}_a^s + \underline{v}_a^d = v_o\hat{e}_1 \dots\dots (2)$$

Using impact model, $e = 0$, as after impact debris get embedded into satellite at point P, therefore

$$\underline{v}_a^d = \underline{v}_a^s + \underline{\omega}_a^s \times \underline{r}_{P/O} \dots\dots (3)$$

where, $\underline{r}_{P/O} = R\hat{e}_2$ and let, $\underline{\omega}_a^s = \omega_1\hat{e}_1 + \omega_2\hat{e}_2 + \omega_3\hat{e}_{3.}$ Then from Eq. (3) and (2)

$$\underline{v}_a^s = \left(\frac{v_o + \omega_3 R}{2}\right)\hat{e}_1 - \frac{\omega_1 R}{2}\hat{e}_3 \ldots\ldots(4)$$

From Eq. (4) and (2)

$$\underline{v}_a^d = \left(\frac{v_o - \omega_3 R}{2}\right)\hat{e}_1 + \frac{\omega_1 R}{2}\hat{e}_3 \ldots\ldots(4)$$

Now writing conservation of angular momentum about point O

$$\underline{h}_O^{sys}(t-\delta t) = \underline{h}_O^{sys}(t+\delta t) \ldots\ldots(5)$$

Using Eq. (5)

$$\underline{h}_O^{sys}(t-\delta t) = \underline{h}_O^s(t-\delta t) + \underline{h}_O^d(t-\delta t),$$

$$\underline{h}_O^{sys}(t-\delta t) = \underline{\underline{I}}_O \cdot \underline{\omega}_b^s + \underline{r}_{O/O} \times m_s \underline{v}_b^s + \underline{r}_{P/O} \times m_d \underline{v}_b^d\ ,$$

$$\underline{h}_O^{sys}(t+\delta t) = \underline{h}_O^s(t+\delta t) + \underline{h}_O^d(t+\delta t),$$

$$\underline{h}_O^{sys}(t+\delta t) = \underline{\underline{I}}_O \cdot \underline{\omega}_a^s + \underline{r}_{P/O} \times m_d \underline{v}_a^d.$$

This gives:

$$\underline{\underline{I}}_O \cdot \underline{\omega}_b^s + +\underline{r}_{P/O} \times m_d \underline{v}_b^d = \underline{\underline{I}}_O \cdot \underline{\omega}_a^s + \underline{r}_{P/O} \times m_d \underline{v}_a^d \ldots(6)$$

where

$$\underline{\underline{I}}_O = \frac{mr^2}{4}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\underline{\omega}_b^s = \omega_o\hat{e}_1,$$

From Eq. (6) and (4), and comparing components of $\hat{e}_1, \hat{e}_2$ and $\hat{e}_3$, we get

$$\omega_1 = \frac{\omega_o}{2}, \omega_2 = 0, \text{and } \omega_3 = -\frac{2v_o}{3R}$$

Then

$$\underline{\omega}_a^s = \frac{\omega_o}{2}\hat{e}_1 - \frac{2v_o}{3R}\hat{e}_{3.}$$