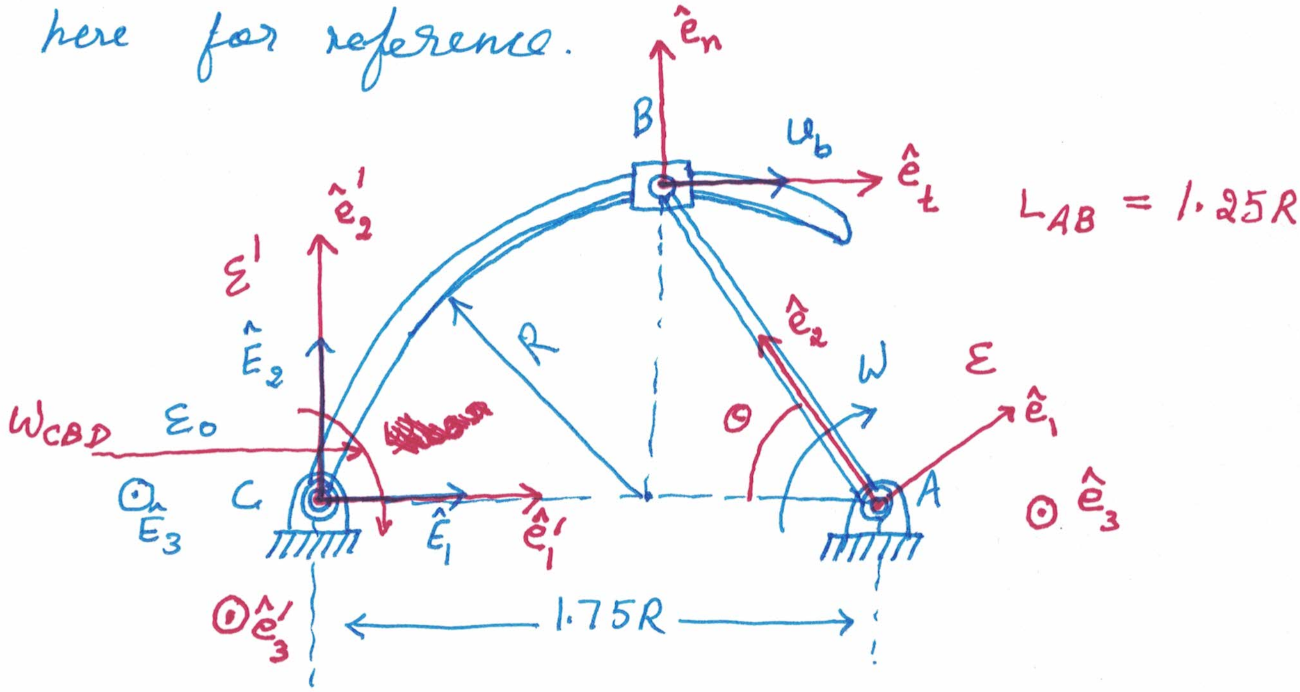
TUTORIAL-7, PROBLEM-4

This is a continuation of the problem 4 in the previous tutorial.

The particular figure along with the definition of different frames is reproduced here for reference.



Three co-ordinate systems,

$\{\mathcal{E}_0, C, \hat{E}_i\}$ — observer or ground frame

$\{\mathcal{E}, A, \hat{e}_i\}$ — frame attached to bar AB (BFCS)

$\{\mathcal{E}', C, \hat{e}_i'\}$ — frame attached to bar CBD $\rightarrow \hat{e}_2'$ tangent to CBD at C. (BFCS)

Defining unit vectors attached to B,

$\hat{e}_t$ tangent to CBD at B

$\hat{e}_n$ is perpendicular and upwards.

This is a 2-D problem.

Always, $\hat{E}_3 = \hat{e}_3 = \hat{e}'_3$

At the time instant shown,

$$\hat{e}'_1 = \hat{E}_1 = \hat{e}_t \qquad \text{--------- (1)}$$
$$\hat{e}'_2 = \hat{E}_2 = \hat{e}_n$$

We have the following,

$$\underline{\omega}_{AB} = -\omega \hat{E}_3 \quad ; \quad \underline{\omega}_{CBD} = -\omega_{CBD} \hat{E}_3$$

Using velocity analysis, we found that (at the time instant shown)

$$\omega_{CBD} = -0.75\,\omega,$$

$$\underline{v}^B_{rel} = 1.75\, R\omega\, \hat{e}_t \quad \text{and} \qquad \text{------- (2)}$$

$$\underline{v}^B = 0.75\, R\omega\, \hat{E}_2 + R\omega\, \hat{E}_1$$

- For acceleration analysis, again choosing B as the convenient point to determine $\underline{a}^B$ when observed from two different BFCS. Using the relation between the accelerations of two points on the same rigid body, we can write $\underline{a}^B$ in terms of the BFCS of bar AB, as follows, (Lecture 11)

Acceleration of B w.r.t. ground frame,

$$\underline{a}^B = \cancel{\underline{a}^A}_{0} + \left(\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \left(\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \underline{r}^{B/A}\right)\right) + \cancel{\underline{\alpha}_{\mathcal{E}/\mathcal{E}_0}}_{0} \times \underline{r}^{B/A}$$

A is the origin of the CS '$\mathcal{E}$'.

Note that,

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} = \underline{\omega}_{AB} = -\omega \hat{E}_3$$

$$\underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} = \frac{d}{dt}\left(\underline{\omega}_{AB}\right) = \frac{d}{dt}\left(-\omega \hat{E}_3\right) = 0$$

It is given that ω is constant, and $\hat{E}_3$ is not changing.

$$\underline{r}^{B/A} = L_{AB}\,\hat{e}_2 = 1.25R\,\hat{e}_2.$$

Thus,

$$\underline{a}^B = -\omega \hat{E}_3 \times \left(-\omega \hat{E}_3 \times 1.25R\,\hat{e}_2\right)$$

$$= \omega \hat{E}_3 \times \left(1.25R\omega\left(-\hat{e}_1\right)\right)$$

since, $\hat{E}_3 = \hat{e}_3$ always

$$\underline{a}^B = -\underline{1.25R\omega^2}\,\hat{e}_2 \quad \text{------} \quad (3)$$

Using the five-term formula, we can also write $\underline{a}^B$ in terms of the BFCS of bar CBD as follows, (Lecture 11)

$$P \rightarrow B, \quad G \rightarrow C, \quad O \rightarrow C$$

$$\underline{a}^B = \underline{a}^B_{rel} + \underline{\omega}_{CBD} \times \left(\underline{\omega}_{CBD} \times \underline{r}^{B/C}\right) + \underline{\alpha}_{\mathcal{E}'/\mathcal{E}_0} \times \underline{r}^{B/C} + 2\underline{\omega}_{CBD} \times \underline{v}^B_{rel} + \cancel{\underline{a}^{C/O}}$$

- Note that $\underline{\omega}_{CBD} = \underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = +0.75\,\omega\,\hat{E}_3$ (at the given time instant) only

- From ②, $\underline{v}^B_{rel} = 1.75\,R\,\omega\,\hat{e}_t$

- ~~Angular~~ And, angular acceleration of CBD is defined as,

$$\underline{\alpha}_{\mathcal{E}'/\mathcal{E}_0} = \frac{d}{dt}\left(\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0}\right) = \underline{\alpha}_{CBD}$$

- Also, at the given time instant the position vector $\underline{r}^{B/C} = R\hat{E}_1 + R\hat{E}_2$

Thus, for bar CBD,

$$\underline{a}^B = \underline{a}^B_{rel} + 0.75\,\omega\,\hat{E}_3 \times \left(0.75\,\omega\,\hat{E}_3 \times (R\hat{E}_1 + R\hat{E}_2)\right) + \ldots$$

$$\cdots + \underline{\alpha}_{CBD} \times \left(R\hat{E}_1 + R\hat{E}_2\right) \quad \text{------} \quad (4)$$

$$+ 2 * 0.75 w \hat{E}_3 \times \left(1.75 R w \hat{e}_t\right)$$

Now, equating (3) & (4),

$$\cancel{-1.25 R w^2 \hat{e}_2 = \underline{a}^B_{rel} + 0.75 w \hat{E}_3 \times (0.75 R w \hat{E}_2 * 0.75 Rw}$$

$$-1.25 R w^2 \hat{e}_2 = \underline{a}^B_{rel} + 0.75 w \hat{E}_3 \times \left[0.75 R w \left(\hat{E}_2 - \hat{E}_1\right)\right]$$

$$+ \underline{\alpha}_{CBD} \times \left(R\hat{E}_1 + R\hat{E}_2\right) \quad \text{......} \quad (5)$$

$$+ \frac{21}{8} R w^2 \hat{E}_2$$

(since at the given time instant, $\hat{e}_t = \hat{E}_1$)

The above equation is a vector equation in 2D; hence leads to two scalar equations.

It appears as if we have four unknowns (components (2 each) of $\underline{a}^B_{rel}$ and $\underline{\alpha}_{CBD}$).

But note that, like we did for the velocity analysis, $\underline{a}^B_{rel}$ can have component only along $\hat{e}_t$ and $\underline{\alpha}_{CBD}$ can have component only about $\hat{E}_3$. Thus, we define

$$\underline{a}^B_{rel} = a_b \hat{e}_t \quad \& \quad \underline{\alpha}_{CBD} = -\alpha_{CBD} \hat{E}_3$$

P 6

α_{CBD} is in the same direction as ω_{CBD}, and as per the figure, I have taken clockwise to be positive.

Using this information about $\underline{a}^B_{rel}$ and $\underline{\alpha}_{CBD}$ in equation 5, we have

$$-1.25 R\omega^2 \hat{e}_2 = a_b \hat{e}_t + \frac{9}{16} R\omega^2 \left(-\hat{E}_1 - \hat{E}_2\right)$$

$$- \alpha_{CBD} \hat{E}_3 \times \left(R\hat{E}_1 + R\hat{E}_2\right)$$

$$+ \frac{21}{8} R\omega^2 \hat{E}_2 \quad \cdots\cdots 6$$

Note that at the given time instant,

$$\hat{e}_t = \hat{E}_1 \quad \text{and} \quad \hat{e}_2 = -\cos\theta\, \hat{E}_1 + \sin\theta\, \hat{E}_2$$

where $\cos\theta = \frac{0.75R}{1.25R} = \frac{3}{5}$

$$\sin\theta = \frac{R}{1.25R} = \frac{4}{5}$$

So 6 becomes,

$$-1.25 R\omega^2 \left(-\frac{3}{5}\hat{E}_1 + \frac{4}{5}\hat{E}_2\right)$$

$$= a_b \hat{E}_1 - \frac{9}{16} R\omega^2 \left(\hat{E}_1 + \hat{E}_2\right) + \frac{21}{8} R\omega^2 \hat{E}_2$$

$$- R\alpha_{CBD} \left(\hat{E}_2 - \hat{E}_1\right) \quad \cdots\cdots 7$$

$$(7)\cdot\hat{E}_1 \Rightarrow \frac{3}{4}R\omega^2 = a_b - \frac{9}{16}R\omega^2 + R\alpha_{CBD} \quad \text{------ } (8)$$

$$(7)\cdot\hat{E}_2 \Rightarrow -R\omega^2 = -\frac{9}{16}R\omega^2 + \frac{21}{8}R\omega^2 - R\alpha_{CBD} \quad \text{------ } (9)$$

From equation (9),

$$\alpha_{CBD} = \omega^2 - \frac{9}{16}\omega^2 + \frac{21}{8}\omega^2$$

$$= \frac{7+21*2}{16}\omega^2$$

$$= \frac{49}{16}\omega^2.$$

Thus, $\underline{\alpha}_{CBD} = -\frac{49}{16}\omega^2\hat{E}_3$

(angular acceleration of CBD)

Now, from equation (8),

$$a_b = \frac{3}{4}R\omega^2 + \frac{9}{16}R\omega^2 - \frac{49}{16}\omega^2 R$$

$$= \frac{3*4+9-49}{16}\omega^2 R$$

$$= -\frac{7}{4}\omega^2 R.$$

Thus, $\underline{a}^B_{rel} = -\frac{7}{4}\omega^2 R\,\hat{e}_t$

(acceleration of B relative to bar CBD)