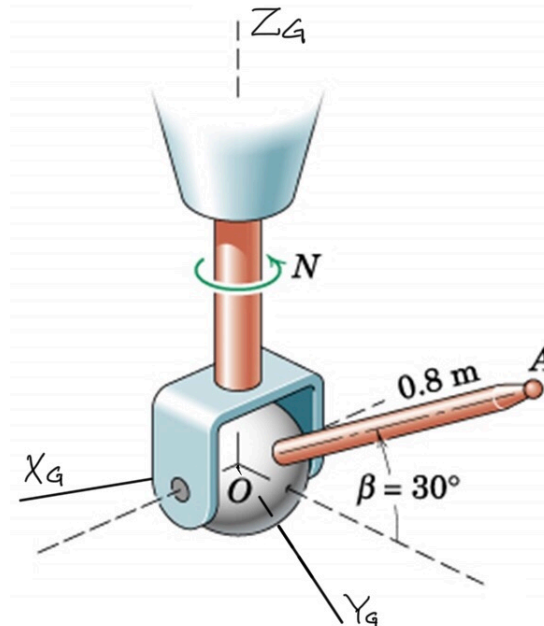
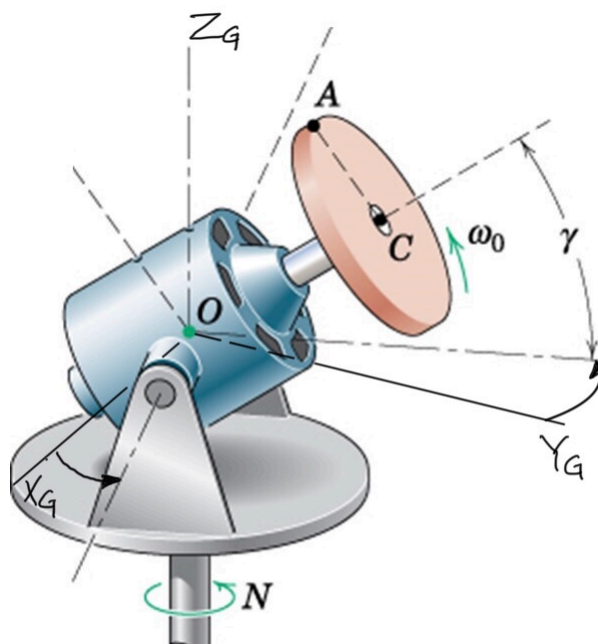


ESO209A Dynamics: **Tutorial 5**
(Week: 25 - 31 Aug. Based on L8 and L9)

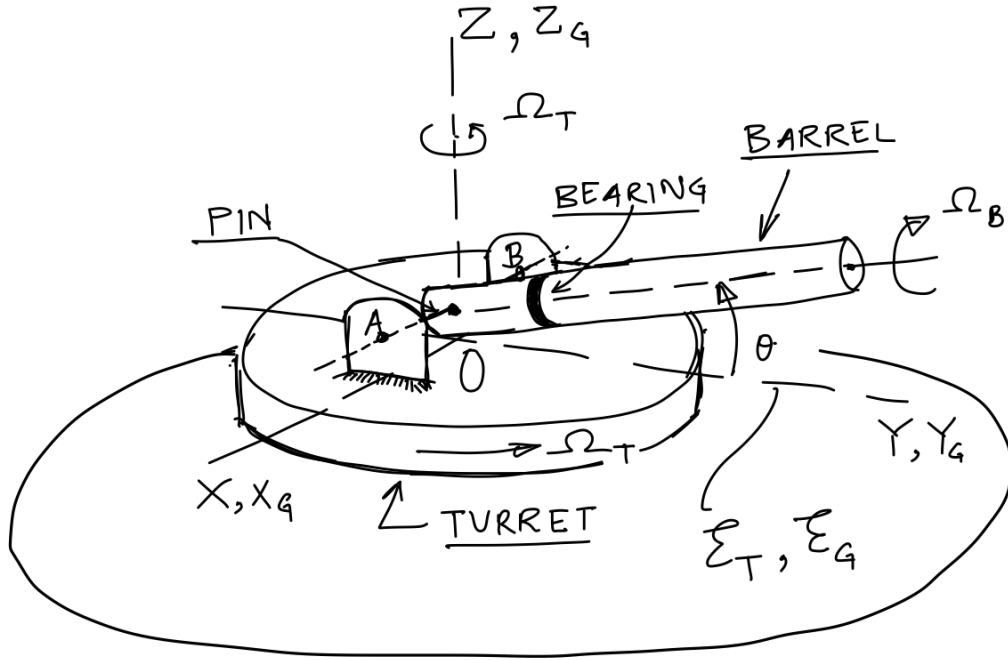
1. The arm OA is fitted in a bracket which is rotating about ground Z -axis at $N = 60$ rpm. The arm is also turning about one of its own axis at $\dot{\beta} = 2\pi$ rad/sec in the direction as shown in the figure. Find the angular velocity of the arm with respect to the ground frame when $\beta = 30^\circ$ and the $BFCS$ attached to the collar is aligned with the ground frame. Clearly identify all the rigid bodies in the system and define all the CS that you employ.



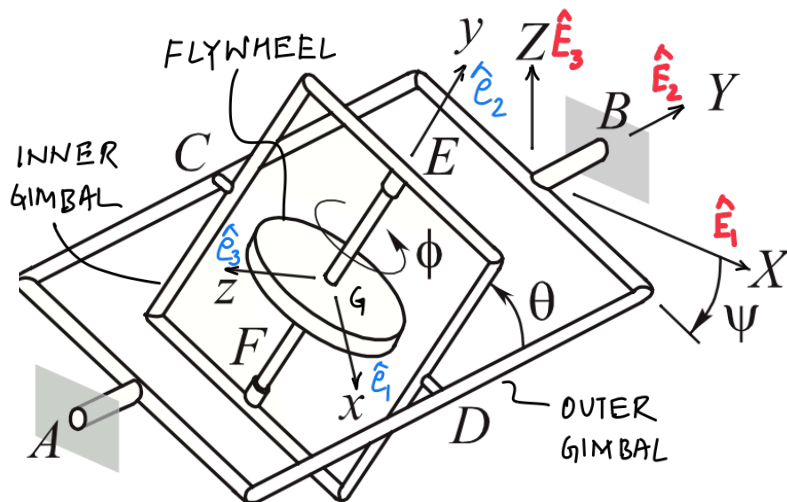
2. A disc is mounted on the shaft of a motor which in turn is mounted on a turn table. The turn table rotates about ground fixed Z -axis at $N = 60$ rpm and the motor spins about its axis at 180 rpm. For $\gamma = 30^\circ$ determine the angular velocity of the disc in ground frame when the $BFCS$ of the turntable is aligned with the ground frame. Clearly identify all the rigid bodies in the system and define all the CS that you employ.



3. A barrel, spinning about its axis is mounted on a turret through trunnion pin AB . The turret itself is spinning about the ground Z_G axis, as shown in the figure. The barrel can rotate about the pin AB . For the given constant angular rates $\Omega_T = \Omega_B = 2\pi$ rad/s and constant rate of elevation $\dot{\theta} = \pi/4$ rad/s find the angular velocity of the barrel as observed in the ground frame \mathcal{E}_G . at the instant when the turret frame \mathcal{E}_T coincides with \mathcal{E}_G , and the barrel elevation in the YZ plane is $\theta = \pi/6$ radian.



4. The three-axis gyroscope consists of an outer gimbal that may rotate by the rate $\dot{\psi}$ about the AB axis relative to a fixed frame $\{\mathcal{E}_0, B, \hat{\mathbf{E}}_i\}$ and an inner gimbal that may rotate by angle $\theta(t)$ about the CD axis relative to the outer gimbal. The flywheel rotates relative to the inner gimbal at the angle $\dot{\phi}$. The $BFCS \{\mathcal{E}, G, \hat{\mathbf{e}}_i\}$ of the flywheel coincides with the fixed CS \mathcal{E}_0 when $\phi = \theta = \psi = 0^\circ$. Find the angular velocity of the flywheel in the ground frame when $\psi = 30^\circ$ and $\theta = 90^\circ$.



5. The end of link A is welded to the yoke which is attached to the collar C . The pivoting axis of the yoke is denoted by $\hat{\mathbf{n}}$. The collar may rotate about axis $\hat{\mathbf{h}}$ of the fixed shaft. Link A and its yoke can thus rotate *only* about the $\hat{\mathbf{n}}$ and $\hat{\mathbf{h}}$ directions, but not about any direction which is not in the plane formed by $\hat{\mathbf{n}}$ and $\hat{\mathbf{h}}$. For example, the link A cannot rotate about the direction $\hat{\mathbf{e}}$ along the link. Show that, regardless of the motion of the other end (not shown) of the link A , the angular velocity $\boldsymbol{\omega}$ of the link A and its yoke must satisfy the relation

$$\boldsymbol{\omega} \cdot \hat{\mathbf{h}} \times (\hat{\mathbf{e}} \times \hat{\mathbf{h}}) = 0.$$

