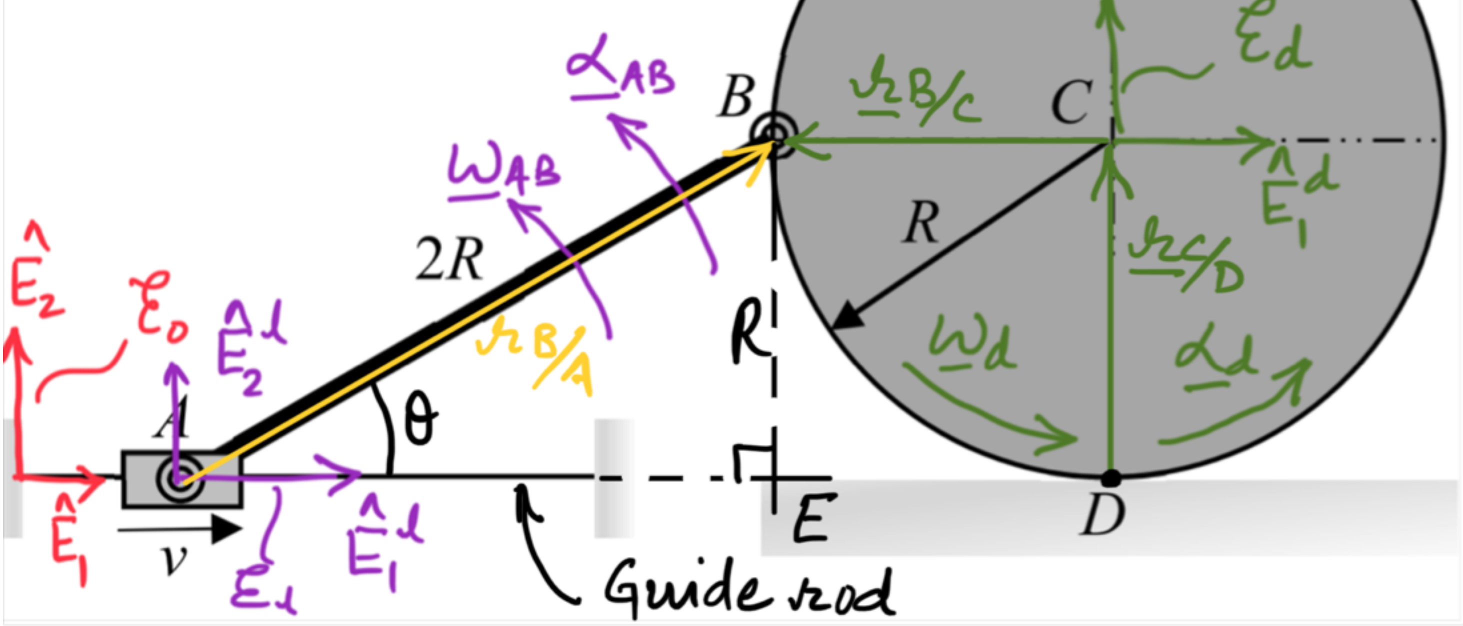
① Planar Problem

② $\alpha = \dot{\omega}$



In $\triangle$ AEB

$$\sin\theta = \frac{BE}{AB} = \frac{R}{2R} \quad \therefore \theta = 30^\circ$$

Let the angular velocity of link AB be $\underline{\omega}_{AB} = \omega_{AB}\hat{E}_3$ and that of the disc be $\underline{\omega}_d = \omega_d\hat{E}_3$. Similarly, let the angular acceleration of AB and disc be $\alpha_{AB}\hat{E}_3$ and $\alpha_d\hat{E}_3$, respectively.

Further, velocity and acceleration of point C will be in $\hat{E}_1$-direction. Let these be $v_c\hat{E}_1$ and $a_c\hat{E}_1$, respectively.

As taught to you – we will approach finding velocity and acceleration of point B first from A and then from C.

<u>Velocity analysis</u>:

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_{AB} \times \underline{r}_{B/A}, \quad \text{—— (1)}$$

and

$$\underline{v}_B = \underline{v}_C + \underline{\omega}_d \times \underline{r}_{B/C} \quad \text{—— (2)}$$

Substituting $\underline{r}_{B/A} = R(\sqrt{3}\,\hat{E}_1 + \hat{E}_2)$ and $\underline{r}_{B/C} = -R\hat{E}_1$ in (1) and (2), respectively, we get

$$\underline{v}_B = v_A \hat{E}_1 + \omega_{AB}\hat{E}_3 \times R(\sqrt{3}\,\hat{E}_1 + \hat{E}_2)$$

$$= v_A \hat{E}_1 + \sqrt{3}\,\omega_{AB} R\,\hat{E}_2 - \omega_{AB} R\,\hat{E}_1$$

$$\therefore \underline{v}_B = (v_A - \omega_{AB} R)\hat{E}_1 + \sqrt{3}\,\omega_{AB} R\,\hat{E}_2 \quad \text{— (3)}$$

and

$$\underline{v}_B = v_C \hat{E}_1 + \omega_d \hat{E}_3 \times (-R\hat{E}_1)$$

Or $\underline{v}_B = v_C \hat{E}_1 - \omega_d R\,\hat{E}_2$ —— (4)

Equating (3) and (4) we get

$$v_C = v_A - \omega_{AB} R \quad \text{—— (5)}$$

and $\sqrt{3}\,\omega_{AB} \cancel{R} = -\omega_d \cancel{R}$ —— (6)

Unknowns ; v_C, ω_{AB}, ω_d — 03

Equations: — 02

We need one more equation.

Let us write the velocity of point C relative to the point of contact D.

$$\underline{v}_C = \underline{v}_D + \omega_d \hat{E}_3 \times (\underline{r}_{C/D})$$

$$= \underline{0} + \omega_d \hat{E}_3 \times (R \hat{E}_2)$$

$$= -\omega_d R \hat{E}_1$$

Or $v_C = -\omega_d R$ —— (7)

Using (5), (6), and (7)

$$\boxed{\begin{aligned} \underline{\omega}_d &= \frac{-\sqrt{3}\, v}{R(1+\sqrt{3})} \hat{E}_3, \quad \underline{\omega}_{AB} = \frac{v}{R(1+\sqrt{3})} \hat{E}_3 \quad \text{and} \\ \underline{v}_c &= \frac{\sqrt{3}\, v}{(1+\sqrt{3})} \hat{E}_1 \end{aligned}}$$

Acceleration analysis:

Consider AB : Note pts. A and B are on the same rigid body $\therefore$ $\underline{v}_{rel} = \underline{0}$ and $\underline{a}_{rel} = \underline{0}$.

$$\therefore \underline{a}_B = \underline{a}_A + \underline{\alpha}_{AB} \times \underline{r}_{B/A} + \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{B/A}) \quad \text{—— (8)}$$

Since $\underline{v}_A$ is constant $\therefore \underline{a}_A = \underline{0}$

$$\therefore \underline{a}_B = \alpha_{AB} \hat{E}_3 \times (\sqrt{3} R \hat{E}_1 + R \hat{E}_2) + \omega_{AB} \hat{E}_3 \times \left(-\omega_{AB} R \hat{E}_1 + \sqrt{3}\, \omega_{AB} R \hat{E}_2\right)$$

$$= \alpha_{AB} R \left[\sqrt{3}\, \hat{E}_2 - \hat{E}_1\right] + \omega_{AB}^2 R \left[-\hat{E}_2 - \sqrt{3}\, \hat{E}_1\right]$$

$$\underline{a}_B = -\left(\alpha_{AB} R + \sqrt{3}\, \omega_{AB}^2 R\right) \hat{E}_1 + \left(\sqrt{3}\, \alpha_{AB} R - \omega_{AB}^2 R\right) \hat{E}_2 \quad \text{—— (9)}$$

Now consider disc:

Pts. B and C are on the same rigid body.

$$\underline{a}_B = \underline{a}_C + \underline{\alpha}_d \times \underline{r}_{B/C} + \underline{\omega}_d \times (\underline{\omega}_d \times \underline{r}_{B/C}) \quad \text{—— (10)}$$

$$= a_C \hat{E}_1 + \alpha_d \hat{E}_3 \times (-R \hat{E}_1) + \omega_d \hat{E}_3 \times (\omega_d \hat{E}_3 \times (-R \hat{E}_1))$$

$$= a_C \hat{E}_1 - \alpha_d R \hat{E}_2 + \omega_d \hat{E}_3 \times (-\omega_d R \hat{E}_2)$$

$$= a_c\hat{E}_1 - \alpha_d R\hat{E}_2 + \omega_d^2 R\hat{E}_1$$

$$= (a_c + \omega_d^2 R)\hat{E}_1 - \alpha_d R\hat{E}_2 \quad \text{——— (11)}$$

Now equating (9) and (11) we obtain

$$a_c + \omega_d^2 R = -(\alpha_{AB}R + \sqrt{3}\,\omega_{AB}^2 R) \quad \text{— (12)}$$

and

$$\sqrt{3}\,\alpha_{AB}\cancel{R} - \omega_{AB}^2\cancel{R} = -\alpha_d\cancel{R} \quad \text{——— (13)}$$

In (12) and (13) there are three unknowns: a_c, α_{AB} and α_d.

Eq. (7) which is true for any time can be used to find

$$\underline{a}_c = \dot{\underline{v}}_c = -R\dot{\omega}_d\hat{E}_1 = -R\alpha_d\hat{E}_1$$

$\therefore$ Eq. (12) can be written as

$$-\cancel{R}\alpha_d + \omega_d^2\cancel{R} = -\alpha_{AB}\cancel{R} - \sqrt{3}\,\omega_{AB}^2\cancel{R}$$

or $$-\alpha_d + \omega_d^2 = -\alpha_{AB} - \sqrt{3}\,\omega_{AB}^2 \quad \text{— (14)}$$

substituting α_{AB} in (14) from (13)

$$-\alpha_d + \omega_d^2 = -\left[\frac{-\alpha_d + \omega_{AB}^2}{\sqrt{3}}\right] - \sqrt{3}\,\omega_{AB}^2$$

$$= \frac{\alpha_d}{\sqrt{3}} - \frac{\omega_{AB}^2}{\sqrt{3}} - \sqrt{3}\,\omega_{AB}^2$$

or $\omega_d^2 + \omega_{AB}^2\left(\frac{4}{\sqrt{3}}\right) = \alpha_d\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$

From (6) $\omega_{AB} = -\omega_d/\sqrt{3}$

$$\therefore\ \omega_d^2 + \frac{\omega_d^2}{3} \times \frac{4}{\sqrt{3}} = \alpha_d\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$$

$$\omega_d^2\left(\frac{3\sqrt{3}+4}{3\cancel{\sqrt{3}}}\right) = \alpha_d\left(\frac{1+\sqrt{3}}{\cancel{\sqrt{3}}}\right)$$

$$\therefore\ \alpha_d = \left(\frac{3\sqrt{3}+4}{3(1+\sqrt{3})}\right)\omega_d^2$$

$$\therefore\ \underline{a_c} = -\left[\frac{3\sqrt{3}+4}{3(1+\sqrt{3})}\right]\omega_d^2 R\,\hat{E}_1$$

$$= -\left[\frac{3\sqrt{3}+4}{\cancel{3}(1+\sqrt{3})}\right]\frac{\cancel{3}v^2 \cancel{R}}{R^{\cancel{2}}(1+\sqrt{3})^2}\hat{E}_1$$

$$\boxed{\underline{a_c} = -\frac{(4+3\sqrt{3})}{(1+\sqrt{3})^3}\frac{v^2}{R}\hat{E}_1}$$