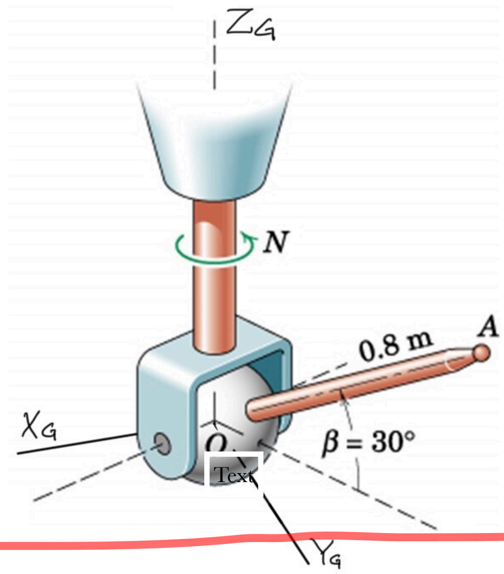
ESO209A: Dynamics
Tutorial - 05

1. The arm OA is fitted in a bracket which is rotating about ground Z-axis at $N = 60$ rpm. The arm is also turning about one of its own axis at $\dot{\beta} = 2\pi$ rad/sec in the direction as shown in the figure. Find the angular velocity of the arm with respect to the ground frame. Clearly identify all the rigid bodies in the system and define all the CS that you employ.

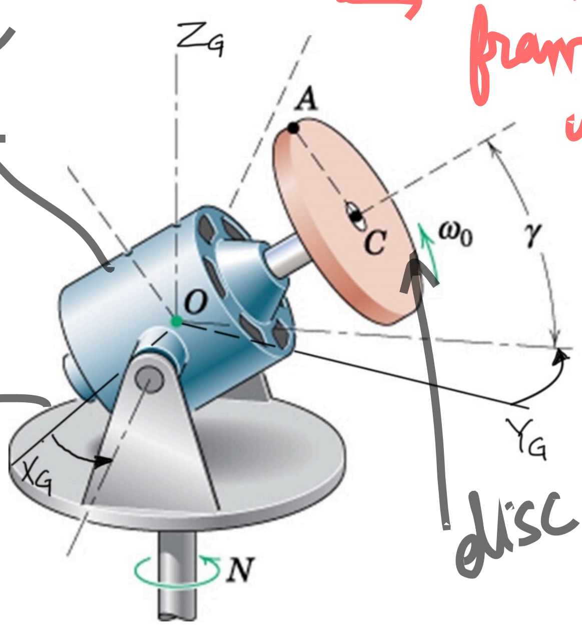


2. A disc is mounted on the shaft of a motor which in turn is mounted on a turn table. The turn table rotates about the ground fixed Z-axis at $N = 60$ rpm and the motor spins about its axis at 180 rpm. For $\gamma = 30°$ determine the angular velocity of the disc as measured in the ground frame. Clearly identify all the rigid bodies in the system and define all the CS that you employ.

when the turn table frame is aligned with the ground frame

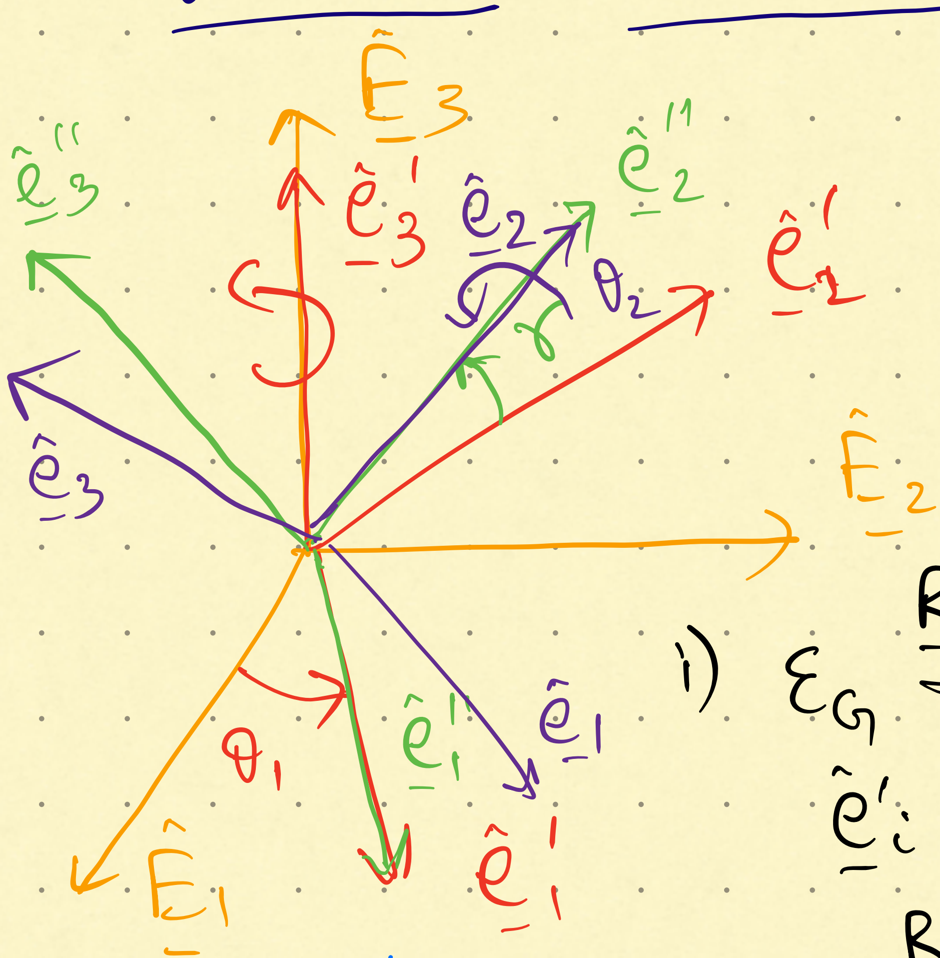
motor

turntable



disc

Tutorial 5 Problem 2



i) $\mathcal{E}_G \xrightarrow{\underline{R}_1(\hat{\underline{E}}_3, \theta_1)} \mathcal{E}_1$

$\hat{\underline{e}}_i' = \underline{R}_1 \hat{\underline{E}}_i$

ii) $\mathcal{E}_1 \xrightarrow{\underline{R}_2(\hat{\underline{e}}_1', \gamma)} \mathcal{E}_2$

$\hat{\underline{e}}_i'' = \underline{R}_2 \hat{\underline{e}}_i'$

iii) $\mathcal{E}_2 \xrightarrow{\underline{R}_3(\hat{\underline{e}}_2'', \theta_2)} \mathcal{E}_3$

$\hat{\underline{e}}_i = \underline{R}_3 \hat{\underline{e}}_i''$

Angular velocities

$\underline{\omega}_{10} = N \hat{\underline{E}}_3$

$\underline{\omega}_{21} = \underline{0}$

$\underline{\omega}_{32} = \omega_0 \hat{\underline{e}}_2''$

$\underline{\omega}_{30} = \underline{\omega}_{10} + \underline{\omega}_{21} + \underline{\omega}_{32}$

$= N \hat{\underline{E}}_3 + \omega_0 \hat{\underline{e}}_2''$

$\hat{\underline{e}}_2'' = \underline{R}_2 \hat{\underline{e}}_2' = \underline{\underline{R}_2 \underline{R}_1 \hat{\underline{E}}_2}$

$$[\hat{\underline{e}}_2''\,]_{\mathcal{E}_1} = [\underline{R}_2]_{\mathcal{E}_1} [\hat{\underline{e}}_2'\,]_{\mathcal{E}_1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \cos\gamma \\ \sin\gamma \end{bmatrix}$$

$$\hat{\underline{e}}_2'' = \cos\gamma \, \hat{\underline{e}}_2' + \sin\gamma \, \hat{\underline{e}}_3'$$

We calculate $\underline{\omega}_{30}$ when $\underline{R}_1 = \underline{1}$

$$\hat{\underline{e}}_2' = \hat{\underline{E}}_2 , \quad \hat{\underline{e}}_3' = \hat{\underline{E}}_3$$

$$\underline{\omega}_{30} = N \hat{\underline{E}}_3 + \omega_0 \hat{\underline{e}}_2''$$

$$= N \hat{\underline{E}}_3 + \omega_0 \left(\cos\gamma \, \hat{\underline{E}}_2 + \sin\gamma \, \hat{\underline{E}}_3 \right)$$