2. Simplify the following:
   (a) $\delta_{ij}(a_{ij} - a_{ji})$,
   (b) $\delta_{ip}\delta_{jq}a_p b_j c_q$, and
   (c) $(\delta_{ij} + a_{ij})(\delta_{ij} - a_{ij})$.

Solution:

(a) Simplify $\delta_{ij}(a_{ij} - a_{ji})$

use contraction

i and *j* are dummy indices

$$\delta_{ij}(a_{ij} - a_{ji}) = \delta_{ij}\,a_{ij} - \delta_{ij}\,a_{ji}$$

$$= a_{ii} - a_{jj}$$

$$= a_{ii} - a_{ii}$$

$$= 0$$

(b) Simplify $\delta_{ip}\delta_{jq}a_p b_j c_q$

Using contraction

$$\delta_{ip}\delta_{jq}a_p b_j c_q = \delta_{ip}a_p\delta_{jq}b_j c_q$$

$$= a_i b_q c_q$$

$$= a_i(b_1 c_1 + b_2 c_2 + b_3 c_3)$$

(c) Simplify $(\delta_{ij} + a_{ij})(\delta_{ij} - a_{ij})$

$$(\delta_{ij} + a_{ij})(\delta_{ij} - a_{ij}) = \delta_{ij}\delta_{ij} - \delta_{ij}a_{ij} + a_{ij}\delta_{ij} - a_{ij}a_{ij}$$

Using contraction

$$= \delta_{ii} - a_{ii} + a_{ii} - a_{ij}a_{ij}$$

$$= 3 - a_{ij}a_{ij}$$