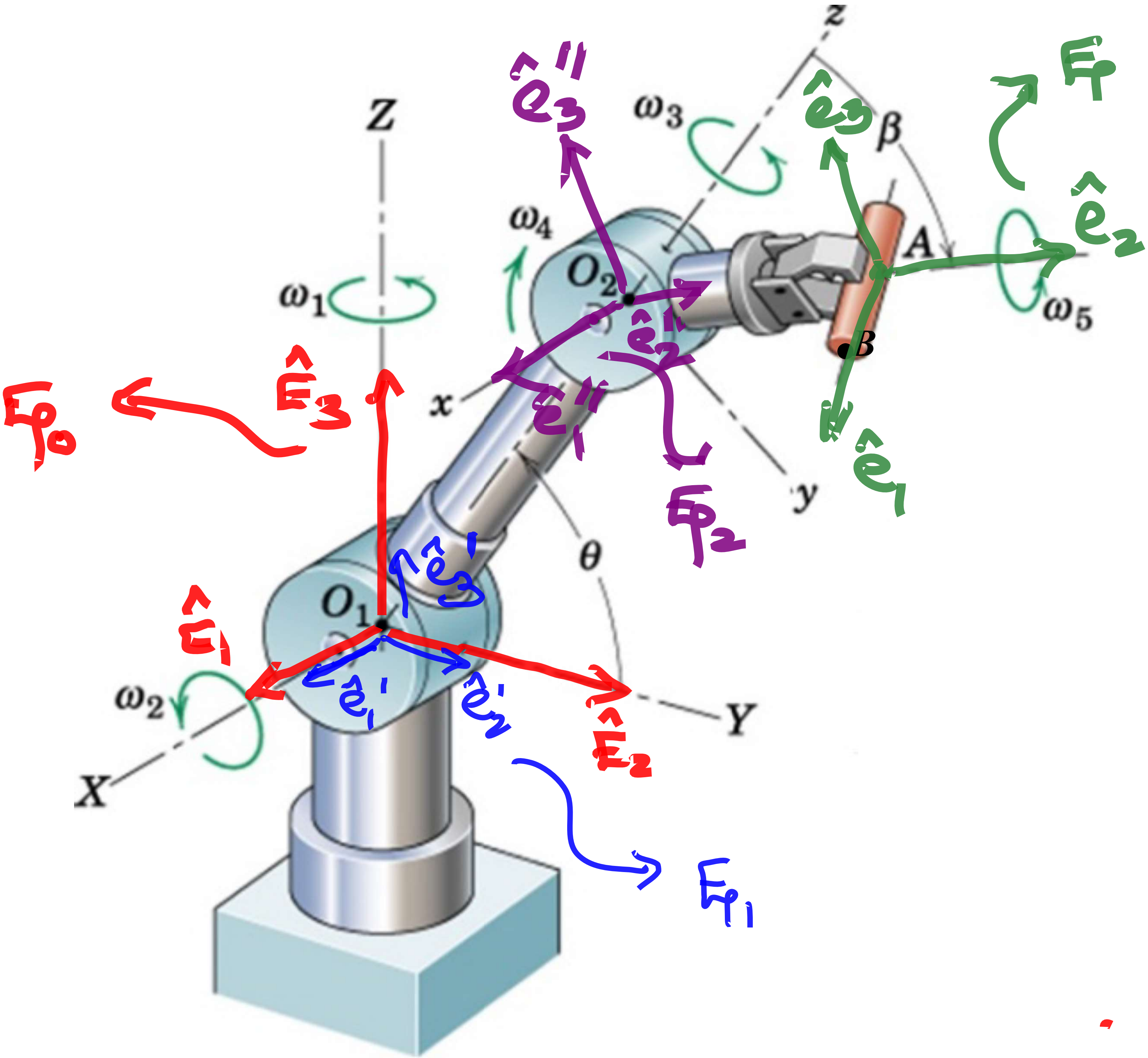


Z
ω1
ω2
ω3
ω4
ω5
O1
O2
X
Y
x
y
z
θ
β
A
B
F0
Ê3
Ê1
Ê2
ê1'
ê2'
ê3'
F1
ê1''
ê2''
ê3''
F2
ê1
ê2
ê3
F

ESO 209 - Tutorial 7

Problem 1

We define 4 coordinate systems.

1. Observer CS : $\{E_{p0}, O_1, \hat{E}_i\}$
2. Base joint BFCS : $\{E_{p1}, O_1, \hat{e}_i'\}$
3. Upper joint BFCS : $\{E_{p2}, O_2, \hat{e}_i''\}$
4. Arm BFCS : $\{E_p, A, \hat{e}_i\}$

Given $\omega_1 = \omega_4 = \omega_5 = 1$ rad/sec

$\omega_2 = \omega_3 = 0$ rad/sec.

$O_1O_2 \equiv L_1 = 1.2\,m$

$O_2A \equiv L_2 = 0.6\,m$

$AB \equiv L_3 = 0.2\,m$

$\theta = 60^\circ$

$\beta = 90^\circ$

We assume that AB is such that AB $\perp$ plane of page, ie $\hat{e}_1$ is aligned with $\hat{E}_1$ (and $\hat{e}_1'$ and $\hat{e}_1''$)

Velocity Analysis

We use the general eqn $\underline{v}^P = \underline{v}^P_{rel} + \underline{\omega}_B \times \underline{r}^{P/G} + \underline{v}^G$

To find the velocity of point B, we can write

$$\underline{v}^B = \cancel{\underline{v}^B_{rel}}^{\,\to 0} + \underline{\omega}_{E_P/E_{P0}} \times \underline{r}^{B/A} + \underline{v}^A$$

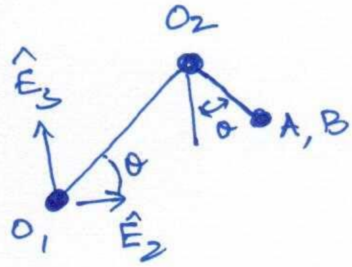
Determining the terms in the above equation

$\boxed{\underline{\omega}_{E_P/E_{P0}}}$

$$\underline{\omega}_{E_P/E_{P0}} = \underline{\omega}_{E_P/E_{P2}} + \underline{\omega}_{E_{P2}/E_{P1}} + \underline{\omega}_{E_{P1}/E_{P0}}$$

$$= \omega_5 \hat{e}_2'' - \omega_4 \hat{e}_1' + \omega_1 \hat{E}_3$$

At the instant of interest



$$\hat{e}_2'' = -\cos\theta\, \hat{E}_3 + \sin\theta\, \hat{E}_2$$

(using $\beta = 90^\circ$)

$$\hat{e}_1' = \hat{E}_1$$

$$\therefore \underline{\omega}_{E_P/E_{P0}} = \omega_5(-\cos\theta\, \hat{E}_3 + \sin\theta\, \hat{E}_2) - \omega_4 \hat{E}_1 + \omega_1 \hat{E}_3$$

$$\Rightarrow \underline{\omega}_{E_P/E_{P0}} = -\omega_4 \hat{E}_1 + \omega_5 \sin\theta\, \hat{E}_2 + (\omega_1 - \omega_5 \cos\theta)\hat{E}_3$$

$\boxed{\underline{r}^{B/A}}$

$$\underline{r}^{B/A} = L_3 \hat{e}_1 = L_3 \hat{E}_1 \quad \text{(at time instant of interest)}$$

$\boxed{\underline{v}^A}$

$$\underline{v}^A = \underline{\omega}_{E_{P2}/E_{P0}} \times \underline{r}^{A/O_2} + \underline{v}^{O_2}$$

Again, determining the individual terms,

$\boxed{\underline{\omega}^{E_{F2}/E_{F0}}}$

$$\underline{\omega}^{E_{F2}/E_{F0}} = \underline{\omega}^{E_{F2}/E_{F1}} + \underline{\omega}^{E_{F1}/E_{F0}}$$

$$= -\omega_4 \hat{e}_1' + \omega_1 \hat{E}_3$$

$$\therefore \underline{\omega}^{E_{F2}/E_{F0}} = -\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3 \quad \text{(at instant of interest)}$$

$\boxed{\underline{r}^{A/O_2}}$

$$\underline{r}^{A/O_2} = L_2 \hat{e}_2''$$

$$= L_2(-\cos\theta \hat{E}_3 + \sin\theta \hat{E}_2)$$

$$\therefore \underline{r}^{A/O_2} = L_2 \sin\theta \hat{E}_2 - L_2 \cos\theta \hat{E}_3$$

$\boxed{\underline{v}^{O_2}}$

$$\underline{v}^{O_2} = \underline{\omega}^{E_{F1}/E_{F0}} \times \underline{r}^{O_2/O_1} + \cancel{\underline{v}^{O_1}}^{\,0}$$

Determining the terms,

$\boxed{\underline{\omega}^{E_{F1}/E_{F0}}}$

$$\underline{\omega}^{E_{F1}/E_{F0}} = \omega_1 \hat{E}_3$$

$\boxed{\underline{r}^{O_2/O_1}}$

$$\underline{r}^{O_2/O_1} = L_1 \cos\theta \hat{E}_2 + L_1 \sin\theta \hat{E}_3$$

Now we start substituting the terms.

$$\underline{V}^{O_2} = (\omega_1 \hat{E}_3) \times (L_1 \cos\theta \hat{E}_2 + L_1 \sin\theta \hat{E}_3)$$

$$\Rightarrow \boxed{\underline{V}^{O_2} = -\omega_1 L_1 \cos\theta \hat{E}_1}$$

$$\underline{\omega}_{E_{e2}/E_{e0}} \times \underline{r}^{A/O_2}$$

$$= (-\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3) \times (L_2 \sin\theta \hat{E}_2 - L_2 \cos\theta \hat{E}_3)$$

$$= -\omega_1 L_2 \sin\theta \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3$$

$$\begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ -\omega_4 & 0 & \omega_1 \\ 0 & L_2\sin\theta & -L_2\cos\theta \end{vmatrix}$$

Substituting in the $\underline{V}^A$ expression

$$\therefore \underline{V}^A = -\omega_1 L_2 \sin\theta \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3 - \omega_1 L_1 \cos\theta \hat{E}_1$$

$$\boxed{\underline{V}^A = -\omega_1 (L_2 \sin\theta + L_1 \cos\theta) \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3}$$

$$\underline{\omega}_{E_e/E_{e0}} \times \underline{r}^{B/A}$$

$$= \left[-\omega_4 \hat{E}_1 + \omega_5 \sin\theta \hat{E}_2 + (\omega_1 - \omega_5 \cos\theta) \hat{E}_3\right] \times L_3 \hat{E}_1$$

$$= -\omega_5 L_3 \sin\theta \hat{E}_3 + (\omega_1 - \omega_5 \cos\theta) L_3 \hat{E}_2$$

$$\therefore \underline{V}^B = -\omega_5 L_3 \sin\theta \hat{E}_3 + (\omega_1 - \omega_5 \cos\theta) L_3 \hat{E}_2 - \omega_1 (L_2 \sin\theta + L_1 \cos\theta) \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3$$

$$\underline{V}^B = -\omega_1 (L_2 \sin\theta + L_1 \cos\theta)\hat{E}_1 + \left[(\omega_1 - \omega_5 \cos\theta)L_3 - \omega_4 L_2 \cos\theta\right]\hat{E}_2 - \left[\omega_5 L_3 \sin\theta + \omega_4 L_2 \sin\theta\right]\hat{E}_3$$

Now substituting for the parameters

$\omega_1 = \omega_4 = \omega_5 = 1 \quad L_1 = 1.2 \quad L_2 = 0.6 \quad L_3 = 0.2$

$\theta = 60^\circ$

$$\underline{V}^B = -\left(0.6 \cdot \frac{\sqrt{3}}{2} + 1.2 \cdot \frac{1}{2}\right)\hat{E}_1 + \left[\left(1 - \frac{1}{2}\right)0.2 - 0.6 \cdot \frac{1}{2}\right]\hat{E}_2 - \left[0.2 \cdot \frac{\sqrt{3}}{2} + 0.6 \cdot \frac{\sqrt{3}}{2}\right]\hat{E}_3$$

$$\boxed{\underline{V}^B = -1.12\,\hat{E}_1 - 0.2\,\hat{E}_2 - 0.69\,\hat{E}_3} \quad m/s$$

To find acceleration of point B.

We can use the general 5-term formula for acceleration:

$$\underline{a}^P = \underline{a}^P_{rel} + \underline{\omega}_B \times (\underline{\omega}_B \times \underline{r}^{P/G}) + \underline{\alpha}_B \times \underline{r}^{P/G}$$
$$+ 2\underline{\omega}_B \times \underline{v}^P_{rel} + \underline{a}^G$$

In this problem, the BFCS are all attached to rigid bodies and hence the $\underline{a}^P_{rel}$ & $\underline{v}^P_{rel}$ terms are equal to zero.

Thus we use,

$$\underline{a}^P = \underline{\omega}_B \times (\underline{\omega}_B \times \underline{r}^{P/G}) + \underline{\alpha}_B \times \underline{r}^{P/G} + \underline{a}^G$$

Now finding the acceleration of point B.

$$\underline{a}^B = \underline{\omega}_{E_P/E_{P0}} \times (\underline{\omega}_{E_P/E_{P0}} \times \underline{r}^{B/A}) + \underline{\alpha}_{E_P/E_{P0}} \times \underline{r}^{B/A} + \underline{a}^A$$

where,

$$\underline{\omega}_{E_P/E_{P0}} = \cancel{\omega_4}\ \omega_5 \hat{e}_2'' - \omega_4 \hat{e}_1' + \omega_1 \hat{E}_3$$

$$\underline{r}^{B/A} = L_3 \hat{e}_1$$

To find $\underline{\alpha}_{E_P/E_{P0}}$, we use,

$$\underline{\alpha}_{E_P/E_{P0}} = \frac{d}{dt}(\underline{\omega}_{E_P/E_{P0}})$$
$$= \frac{d}{dt}(\omega_5 \hat{e}_2'' - \omega_4 \hat{e}_1' + \omega_1 \hat{E}_3)$$

~~ie~~

$$\Rightarrow \underline{\alpha}_{E\phi/E\phi_0} = \cancelto{0}{\frac{d\omega_5}{dt}} \cdot \hat{e}_2'' + \omega_5 \cdot \frac{d\hat{e}_2''}{dt} - \cancelto{0}{\frac{d\omega_4}{dt}} \cdot \hat{e}_1' - \omega_4 \cdot \frac{d\hat{e}_1'}{dt} + \cancelto{0}{\frac{d\omega_1}{dt}} \hat{E}_3 + \cancelto{0}{\omega_1 \frac{d\hat{E}_3}{dt}}$$

$$= \omega_5 \cdot \frac{d\hat{e}_2''}{dt} - \omega_4 \cdot \frac{d\hat{e}_1'}{dt}$$

$$= \omega_5 \left(\underline{\omega}_{E\phi_2/E\phi_0} \times \hat{e}_2'' \right) - \omega_4 \left(\underline{\omega}_{E\phi_1/E\phi_0} \times \hat{e}_1' \right)$$

at the instant of interest.

$$= \omega_5 \left[\left(-\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3 \right) \times \left(\sin\theta \, \hat{E}_2 - \cos\theta \, \hat{E}_3 \right) \right]$$

$$- \omega_4 \left[\omega_1 \hat{E}_3 \times \hat{E}_1 \right]$$

$$\begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ -\omega_4 & 0 & \omega_1 \\ 0 & \sin\theta & -\cos\theta \end{vmatrix} = -\omega_1 \sin\theta \, \hat{E}_1 - \omega_4 \cos\theta \, \hat{E}_2 - \omega_4 \sin\theta \, \hat{E}_3$$

$$\therefore \quad \underline{\alpha}_{E\phi/E\phi_0} = \omega_5 \left(-\omega_1 \sin\theta \, \hat{E}_1 - \omega_4 \cos\theta \, \hat{E}_2 - \omega_4 \sin\theta \, \hat{E}_3 \right) - \omega_4 \left(\omega_1 \hat{E}_2 \right)$$

$$\boxed{\underline{\alpha}_{E\phi/E\phi_0} = -\omega_1 \omega_5 \sin\theta \, \hat{E}_1 - \left(\omega_4 \omega_5 \cos\theta + \omega_1 \omega_4 \right) \hat{E}_2 - \omega_4 \omega_5 \sin\theta \, \hat{E}_3}$$

To find $\underline{a}^A$

$$\underline{a}^A = \underline{\omega}_{E\varphi_2/E\varphi_0} \times \left(\underline{\omega}_{E\varphi_2/E\varphi_0} \times \underline{r}^{A/O_2}\right) + \underline{\alpha}_{E\varphi_2/E\varphi_0} \times \underline{r}^{A/O_2} + \underline{a}^{O_2}$$

$$\underline{\omega}_{E\varphi_2/E\varphi_0} = -\omega_4 \hat{e}_1' + \omega_1 \hat{E}_3$$

$$\underline{r}^{A/O_2} = L_2 \hat{e}_2''$$

$$\underline{\alpha}_{E\varphi_2/E\varphi_0} = \frac{d}{dt}\left(\underline{\omega}_{E\varphi_2/E\varphi_0}\right)$$

$$= \frac{d}{dt}\left(-\omega_4 \hat{e}_1' + \omega_1 \hat{E}_3\right)$$

$$= -\overset{0}{\cancel{\frac{d\omega_4}{dt}}} \cdot \hat{e}_1' + (-\omega_4)\frac{d\hat{e}_1'}{dt} + \overset{0}{\cancel{\frac{d\omega_1}{dt}}}\hat{E}_3 + \omega_1 \overset{0}{\cancel{\frac{d\hat{E}_3}{dt}}}$$

$$= -\omega_4 \frac{d\hat{e}_1'}{dt}$$

$$\Rightarrow \underline{\alpha}_{E\varphi_2/E\varphi_0} = -\omega_4\left(\underline{\omega}_{E\varphi_1/E\varphi_0} \times \hat{e}_1'\right) = -\omega_4\left(\omega_1 \hat{E}_3 \times e_1'\right) = -\omega_1\omega_4\left(\hat{E}_3 \times \hat{E}_1\right) = -\omega_1\omega_4 \hat{E}_2$$

To find $\underline{a}^{O_2}$

$$\underline{a}^{O_2} = \underline{\omega}_{E\varphi_1/E\varphi_0} \times \left(\underline{\omega}_{E\varphi_1/E\varphi_0} \times \underline{r}^{O_2/O_1}\right) + \underline{\alpha}_{E\varphi_1/E\varphi_0} \times \underline{r}^{O_2/O_1} + \underline{a}^{O_1}$$

$$\underline{\omega}_{E\varphi_1/E\varphi_0} = \omega_1 \hat{E}_3$$

$$\underline{r}^{O_2/O_1} = L_1 \cos\theta\, \hat{e}_2' + L_1 \sin\theta\, \hat{e}_3'$$

$$\underline{\alpha}_{E\varphi_1/E\varphi_0} = \frac{d}{dt}\left(\underline{\omega}_{E\varphi_1/E\varphi_0}\right)$$

$$\therefore \quad \underline{\alpha}_{E_{\phi1}/E_{\phi0}} = \frac{d}{dt}\left(\omega_1 \hat{e}_3'\right)$$

$$= \cancel{\frac{d}{dt}\omega_1}^{\,0}\, \hat{e}_3' + \omega_1 \frac{d\hat{e}_3'}{dt}$$

$$= \omega_1 \left(\underline{\omega}_{E_{\phi1}/E_{\phi0}} \times \hat{e}_3' \right)$$

$$= \omega_1 \left(\omega_1 \hat{E}_3 \times \hat{e}_3' \right)$$

$$= \omega_1 \left(\omega_1 \hat{E}_3 \times \hat{E}_3 \right)$$

$$= 0$$

$$\underline{a}^{O_1} = 0 \quad (\because O_1 \text{ is fixed})$$

$$\therefore \underline{a}^{O_2} = \omega_1 \hat{e}_3' \times \left(\omega_1 \hat{e}_3' \times \{ L_1 \cos\theta \hat{e}_2' + L_1 \sin\theta \hat{e}_3' \} \right)$$

$$+ 0 + 0$$

$$= \omega_1 \hat{e}_3' \times \left(- \omega_1 L_1 \cos\theta \, \hat{e}_1' \right)$$

$$\underline{a}^{O_2} = - \omega_1^2 L_1 \cos\theta \, \hat{e}_2'$$

Substituting for all the terms in $\underline{a}^A$,

$$\therefore \quad \underline{a}^A = \underline{\omega}_{E_{\phi2}/E_{\phi0}} \times \left(\underline{\omega}_{E_{\phi2}/E_{\phi0}} \times \underline{r}^{A/O_2} \right) + \underline{\alpha}_{E_{\phi2}/E_{\phi0}} \times \underline{r}^{A/O_2} + \underline{a}^{O_2}$$

$$= \left(-\omega_4 \hat{e}_1' + \omega_1 \hat{E}_3 \right) \times \left[\left(-\omega_4 \hat{e}_1' + \omega_1 \hat{E}_3 \right) \times \left(L_2 \hat{e}_2'' \right) \right]$$

$$+ \left(-\omega_1 \omega_4 \hat{E}_2 \right) \times \left(L_2 \hat{e}_2'' \right) + \left(-\omega_1^2 L_1 \cos\theta \, \hat{e}_2' \right)$$

Writing all basis vectors in terms of the observer CS at the time of interest,

$$\underline{a}^A = (-\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3) \times \left[(-\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3) \times (L_2 \sin\theta \hat{E}_2 - L_2 \cos\theta \hat{E}_3) \right.$$

$$- (\omega_1 \omega_4 \hat{E}_2) \times (L_2 \sin\theta \hat{E}_2 - L_2 \cos\theta \hat{E}_3)$$

$$- \omega_1^2 L_1 \cos\theta \hat{E}_2$$

$$= (-\omega_4 \hat{E}_1 + \omega_1 \hat{E}_3) \times (-\omega_1 L_2 \sin\theta \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3)$$

$$\begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ -\omega_4 & 0 & \omega_1 \\ 0 & L_2 \sin\theta & -L_2 \cos\theta \end{vmatrix} = -\omega_1 L_2 \sin\theta \hat{E}_1 - \omega_4 L_2 \cos\theta \hat{E}_2 - \omega_4 L_2 \sin\theta \hat{E}_3$$

$$+ \omega_1 \omega_4 L_2 \cos\theta \hat{E}_1$$

$$- \omega_1^2 L_1 \cos\theta \hat{E}_2$$

$$\begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ -\omega_4 & 0 & \omega_1 \\ -\omega_1 L_2 \sin\theta & -\omega_4 L_2 \cos\theta & -\omega_4 L_2 \sin\theta \end{vmatrix} = \omega_1 \omega_4 L_2 \cos\theta \hat{E}_1 - (\omega_4^2 L_2 \sin\theta + \omega_1^2 L_2 \sin\theta) \hat{E}_2 + \omega_4^2 L_2 \cos\theta \hat{E}_3$$

$$= 2\omega_1 \omega_4 L_2 \cos\theta \hat{E}_1$$

$$- (\omega_1^2 L_1 \cos\theta + \omega_4^2 L_2 \sin\theta + \omega_1^2 L_2 \sin\theta) \hat{E}_2$$

$$+ \omega_4^2 L_2 \cos\theta \hat{E}_3$$

Substituting above expression of $\underline{a}^A$ in the expression for $\underline{a}^B$,

$$\underline{a}^B = \underline{\omega}_{E_e/E_0} \times (\underline{\omega}_{E_e/E_0} \times \underline{r}^{B/A}) + \underline{\alpha}_{E_e/E_0} \times \underline{r}^{B/A} + \underline{a}^A$$

$$= (\omega_5 \hat{e}_2'' - \omega_4 \hat{e}_1' + \omega_1 \hat{E}_3) \times \left[(\omega_5 \hat{e}_2'' - \omega_4 \hat{e}_1' + \omega_1 \hat{E}_3) \times L_3 \hat{e}_1 \right]$$

$$+ \left[(-\omega_1 \omega_5 \sin\theta \hat{E}_1 - (\omega_4 \omega_5 \cos\theta + \omega_1 \omega_4) \hat{E}_2 - \omega_4 \omega_5 \sin\theta \hat{E}_3) \times L_3 \hat{e}_1 \right]$$

$$+ \underline{a}^A$$

writing the BFCS basis vectors in terms of observer basis vectors at the instant of interest

$$\underline{a}^B = \left[\omega_5(\sin\theta\,\hat{E}_2 - \cos\theta\,\hat{E}_3) - \omega_4\hat{E}_1 + \omega_1\hat{E}_3\right] \times$$

$$\left[\left\{\omega_5(\sin\theta\,\hat{E}_2 - \cos\theta\,\hat{E}_3) - \omega_4\hat{E}_1 + \omega_1\hat{E}_3\right\} \times L_3\hat{E}_1\right]$$

$$+ \left[\left(-\omega_1\omega_5\sin\theta\,\hat{E}_1 - \{\omega_4\omega_5\cos\theta + \omega_1\omega_4\}\hat{E}_2 - \omega_4\omega_5\sin\theta\,\hat{E}_3\right] \times L_3\hat{E}_1$$

$$+ 2\omega_1\omega_4 L_2\cos\theta\,\hat{E}_1 - (\omega_1^2 L_1\cos\theta + \omega_4^2 L_2\sin\theta + \omega_1^2 L_2\sin\theta)\hat{E}_2 + \omega_4^2 L_2\cos\theta\,\hat{E}_3$$

$$= \left[\omega_5(\sin\theta\,\hat{E}_2 - \cos\theta\,\hat{E}_3) - \omega_4\hat{E}_1 + \omega_1\hat{E}_3\right] \times$$

$$\left[-\omega_5 L_3\sin\theta\,\hat{E}_3 - \omega_5 L_3\cos\theta\,\hat{E}_2 + \omega_1 L_3\hat{E}_2\right]$$

$$+ \left[+(\omega_4\omega_5\cos\theta + \omega_1\omega_4)L_3\hat{E}_3 - \omega_4\omega_5 L_3\sin\theta\,\hat{E}_2\right]$$

$$+ 2\omega_1\omega_4 L_2\cos\theta\,\hat{E}_1 - (\omega_1^2 L_1\cos\theta + \omega_4^2 L_2\sin\theta + \omega_1^2 L_2\sin\theta)\hat{E}_2 + \omega_4^2 L_2\cos\theta\,\hat{E}_3$$

$$\begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ -\omega_4 & \omega_5\sin\theta & \omega_1 - \omega_5\cos\theta \\ 0 & \omega_1 L_3 - \omega_5 L_3\cos\theta & -\omega_5 L_3\sin\theta \end{vmatrix}$$

$$= \left[-\omega_5^2 L_3 \sin^2\theta - (\omega_1 L_3 - \omega_5 L_3 \cos\theta)(\omega_1 - \omega_5 \cos\theta)\right]\hat{E}_1$$

Denoting it as vector $\underline{D}$

$$- \left[\omega_4 \omega_5 L_3 \sin\theta\right]\hat{E}_2 + \left[-\omega_4(\omega_1 L_3 - \omega_5 L_3 \cos\theta)\right]\hat{E}_3$$

$$\underline{a}^B = \underline{D} + 2\omega_1\omega_4 L_2 \cos\theta\, \hat{E}_1$$

$$-(\omega_4\omega_5 L_3 \sin\theta + \omega_1^2 L_1 \cos\theta + \omega_4^2 L_2 \sin\theta + \omega_1^2 L_2 \sin\theta)\hat{E}_2$$

$$+(\omega_4\omega_5 L_3 \cos\theta + \omega_1\omega_4 L_3 + \omega_4^2 L_2 \cos\theta)\hat{E}_3$$

The final expression can be obtained by substituting the numerical values for ~~ω_1, ω~~ all the parameters.