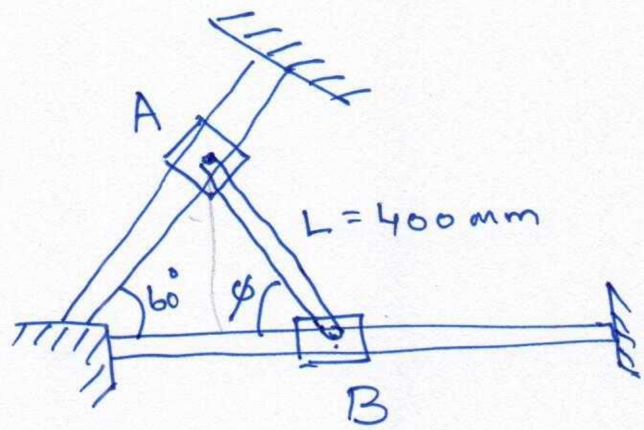
ESO209 - Tutorial 13

Problem 2



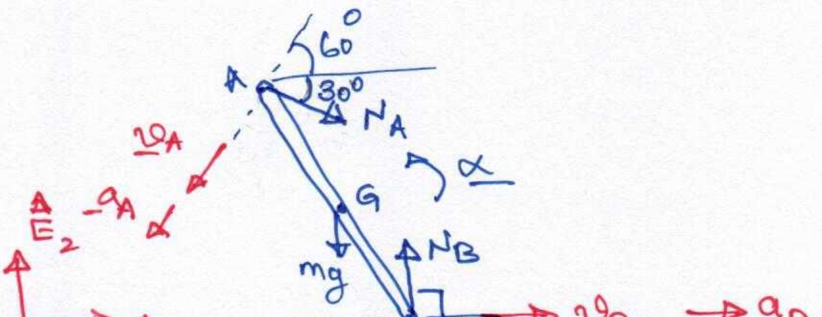
Given $\phi = 70^\circ$, friction between collars & guides is negligible

mass of collars negligible.

Say, mass of bar $= m$

To determine the angular acceleration of the bar at the instant of release.

FBD of the bar.



By virtue of the guides we can write

$$\underline{a}_A = a_A(-\cos 60^\circ \hat{E}_1 - \sin 60 \hat{E}_2) = -\frac{a_A}{2}\hat{E}_1 - \frac{\sqrt{3}a_A}{2}\hat{E}_2$$

$$\underline{a}_B = a_B \hat{E}_1$$

$\underline{\omega} = \underline{0}$ at instant of release

$\underline{\alpha} = \alpha \hat{E}_3$ (planar problem)

Thus the unknowns of the problem are

a_A , a_B , a_G , α , N_A , N_B

1 1 2 1 1 1 = 7 unknowns.

The equations we have are

1. Kinematics connecting A & B — 2
2. Kinematics connecting G & B — 2
3. LMB — 2
4. AMB — 1 (since planar problem)

7 equations.

Examining the kinematics first:

Using acceleration analysis eqns

$$\underline{a}_A = \underline{a}_B + \underline{\alpha}_B \times \underline{r}^{A/B} \qquad (\because \ \underline{\omega}_B = \underline{0} \text{ at this instant})$$

$$-\frac{a_A}{2}\hat{E}_1 - \frac{\sqrt{3}}{2}a_A\hat{E}_2 = a_B\hat{E}_1 + \alpha\hat{E}_3 \times (-L\cos\phi\,\hat{E}_1 + L\sin\phi\,\hat{E}_2)$$

$$= a_B\hat{E}_1 + (-\alpha L\cos\phi)\hat{E}_2 + (-\alpha L\sin\phi)\hat{E}_1$$

which gives the eqns

$$-\frac{a_A}{2} = a_B - \alpha L\sin\phi \quad \Rightarrow \quad -a_A = 2a_B - 2\alpha L\sin\phi$$

&

$$-\frac{\sqrt{3}}{2}a_A = -\alpha L\cos\phi \qquad -\sqrt{3}a_A = -2\alpha L\cos\phi$$

$$\Rightarrow \quad \boxed{a_A = \frac{2}{\sqrt{3}}\alpha L\cos\phi} \quad \text{— } 1$$

and

$$\boxed{a_B = \frac{1}{\sqrt{3}}\alpha L\cos\phi + \alpha L\sin\phi} \quad \text{— } 2$$

III^{ly}

$$\underline{a}_G = \underline{a}_B + \underline{\alpha}_B \times \underline{r}^{G/B}$$

$$= \left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \alpha L\sin\phi\right)\hat{E}_1$$

$$+ \alpha\hat{E}_3 \times \left(-\frac{L}{2}\cos\phi\,\hat{E}_1 + \frac{L}{2}\sin\phi\,\hat{E}_2\right)$$

$$= \left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \alpha L\sin\phi\right)\hat{E}_1 - \frac{1}{2}\alpha L\cos\phi\,\hat{E}_2 - \frac{\alpha L}{2}\sin\phi\,\hat{E}_1$$

$$\boxed{\underline{a}_G = \left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \frac{\alpha L}{2}\sin\phi\right)\hat{E}_1 - \frac{1}{2}\alpha L\cos\phi\,\hat{E}_2} \quad -(3)$$

Examining the kinetics.

LMB gives

$$\Sigma F = m\underline{a}^G$$

$$\cancel{N_A(\cos\theta\,\hat{E}_1} \quad N_A(\cos 30^\circ\,\hat{E}_1 - \sin 30\,\hat{E}_2) - mg\hat{E}_2 + N_B\hat{E}_2$$

$$= m\left[\left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \frac{\alpha L}{2}\sin\phi\right)\hat{E}_1 - \frac{1}{2}\alpha L\cos\phi\,\hat{E}_2\right]$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}N_A\hat{E}_1 + \left(-\frac{N_A}{2} - mg + N_B\right)\hat{E}_2 = m\left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \frac{\alpha L}{2}\sin\phi\right)\hat{E}_1$$

$$- \frac{1}{2}m\alpha L\cos\phi\,\hat{E}_2$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}N_A = m\left(\frac{1}{\sqrt{3}}\alpha L\cos\phi + \frac{\alpha L}{2}\sin\phi\right)$$

and $\quad -\frac{N_A}{2} - mg + N_B = -\frac{1}{2}m\alpha L\cos\phi$.

ie

$$\boxed{N_A = m\left(\frac{2}{3}\alpha L\cos\phi + \frac{1}{\sqrt{3}}\alpha L\sin\phi\right)} \quad — (4)$$

and

$$N_B = \frac{N_A}{2} + mg - \frac{1}{2}m\alpha L\cos\phi$$

$$= m\left(\frac{1}{3}\alpha L\cos\phi + \frac{1}{2\sqrt{3}}\alpha L\sin\phi\right) + mg - \frac{1}{2}m\alpha L\cos\phi$$

$$\boxed{N_B = -\frac{m\alpha L\cos\phi}{6} + \frac{m\alpha L\sin\phi}{2\sqrt{3}} + mg} \quad — (5)$$

AMB about ~~CG~~ Centre of Mass, G, gives

$$\Sigma \underline{M} = I_3\alpha\hat{E}_3$$

$$LHS \Rightarrow \underline{r}^{A/G} \times \underline{N}_A + \underline{r}^{B/G} \times \underline{N}_B$$

$$\Sigma\underline{M} = \left(-\frac{L}{2}\cos\phi\hat{E}_1 + \frac{L}{2}\sin\phi\hat{E}_2\right) \times \left(N_A\cos 30^\circ\hat{E}_1 - N_A\sin 30\hat{E}_2\right)$$

$$+ \left(\frac{L}{2}\cos\phi\hat{E}_1 - \frac{L}{2}\sin\phi\hat{E}_2\right) \times \left(N_B\hat{E}_2\right)$$

$$= \left(\frac{1}{4}N_A L\cos\phi - \frac{\sqrt{3}}{4}N_A L\sin\phi + \frac{1}{2}N_B L\cos\phi\right)\hat{E}_3$$

$\therefore$ the AMB eqn gives

$$\boxed{\frac{1}{4}N_A L\cos\phi - \frac{\sqrt{3}}{4}N_A L\sin\phi + \frac{1}{2}N_B L\cos\phi = I_3\alpha} \quad — (6)$$

Substituting (4) & (5) in (6), we get

$$\frac{1}{4}\left[m\left(\frac{2}{3}\alpha L\cos\phi+\frac{1}{\sqrt{3}}\alpha L\sin\phi\right)\right]L\cos\phi$$

$$-\frac{\sqrt{3}}{4}\left[m\left(\frac{2}{3}\alpha L\cos\phi+\frac{1}{\sqrt{3}}\alpha L\sin\phi\right)\right]L\sin\phi$$

$I_3=\frac{1}{12}mL^2$

$$+\frac{1}{2}\left[-\frac{m\alpha L\cos\phi}{6}+\frac{m\alpha L\sin\phi}{2\sqrt{3}}+mg\right]L\cos\phi=\frac{mL^2}{12}\alpha$$

~~[illegible]~~

$$mL^2\alpha\left[\frac{1}{6}\cos^2\phi+\frac{1}{4\sqrt{3}}\sin\phi\cos\phi-\frac{1}{2\sqrt{3}}\sin\phi\cos\phi-\frac{1}{4}\sin^2\phi\right.$$

$$\left.-\frac{1}{12}\cos^2\phi+\frac{1}{4\sqrt{3}}\sin\phi\cos\phi\right]+\frac{1}{2}mgL\cos\phi=\frac{mL^2\alpha}{12}$$

$$\therefore\ \frac{mL^2\alpha}{12}\left[1-\left(2\cos^2\phi+\cancel{\sqrt{3}\sin\phi\cos\phi}-\cancel{2\sqrt{3}\sin\phi\cos\phi}-3\sin^2\phi\right.\right.$$

$$\left.\left.-\cos^2\phi+\cancel{\sqrt{3}\sin\phi\cos\phi}\right)\right]=\frac{1}{2}mgL\cos\phi$$

$$\Rightarrow\ \frac{mL^2\alpha}{12}\left[1-\left(\cos^2\phi-3\sin^2\phi\right)\right]=\frac{1}{2}mgL\cos\phi$$

$$\Rightarrow\ \frac{mL^2\alpha}{12}\left[4\sin^2\phi\right]=\frac{1}{2}mgL\cos\phi$$

$$\Rightarrow\ \boxed{\alpha=\frac{3}{2}\frac{g}{L}\frac{\cos\phi}{\sin^2\phi}}$$