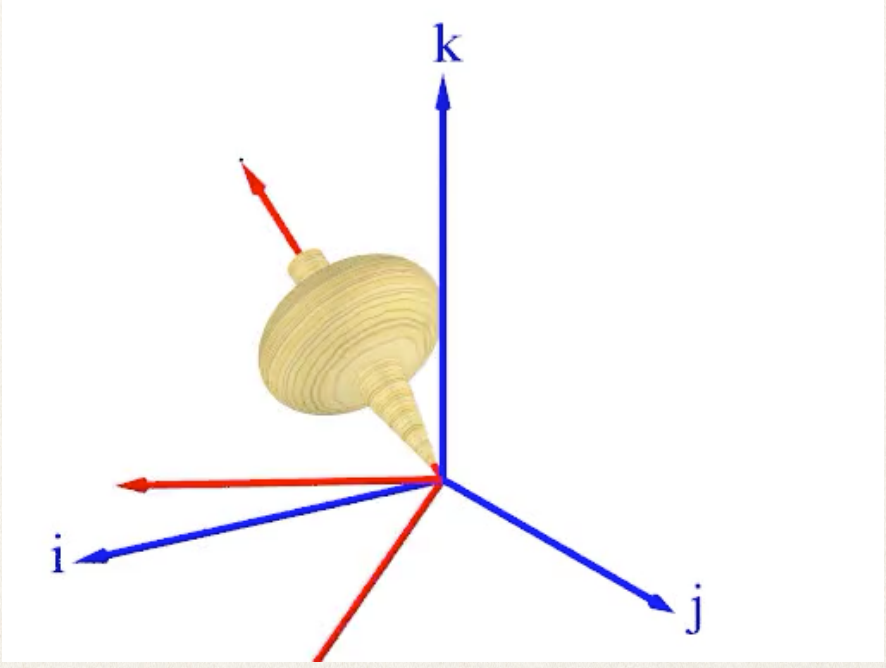
Lecture 19

More on Tops

20 - 26 October, 2021

Nutation of a top



Nutation of a top

I. **Governing equations** for $I_1 = I_2 \neq I_3$.

$$\left.\begin{aligned} \dot{\varphi}\cos\theta + \dot{\psi} &= \omega_3^0 \\ I_1(\ddot{\varphi}\sin\theta + 2\dot{\varphi}\dot{\theta}\cos\theta) - I_3\omega_3^0\dot{\theta} &= 0 \\ I_1(\ddot{\theta} - \dot{\varphi}^2\sin\theta\cos\theta) + I_3\omega_3^0\dot{\varphi}\sin\theta &= mgL\sin\theta \end{aligned}\right\} \quad (1)$$

II. *Integrate* (1b) using (1a):

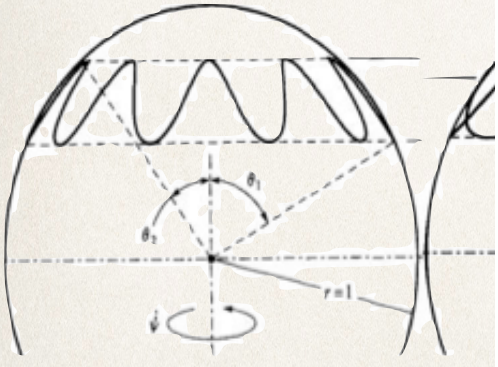
$$I_3\omega_3^0\cos\theta + I_1\dot{\varphi}\sin^2\theta = H_3 =: \mathbf{H}^O \cdot \hat{\mathbf{E}}_3 \quad (2)$$

III. *Eliminate* $\dot{\varphi}$ from (2) in (1c):

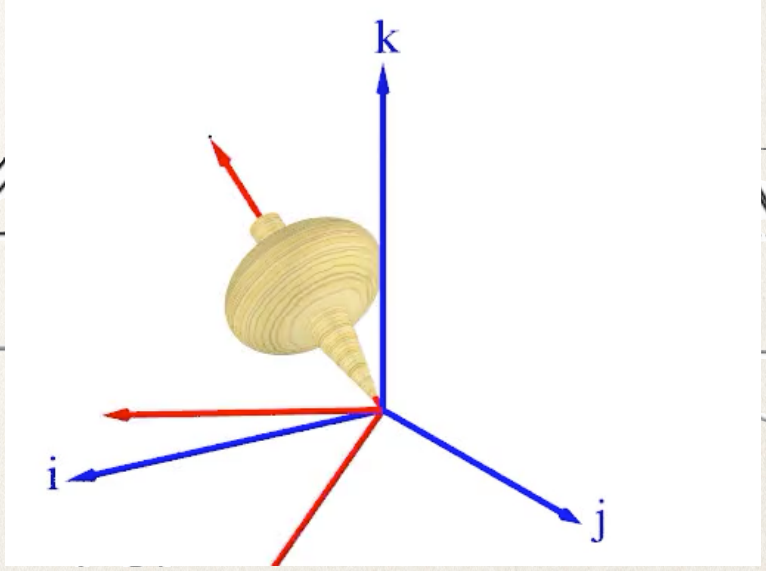
$$\ddot{\theta} + \frac{\left(I_3\omega_3^0 - H_3\cos\theta\right)\left(H_3 - I_3\omega_3^0\cos\theta\right)}{I_1^2\sin^3\theta} - \frac{mgL\sin\theta}{I_1} = 0$$

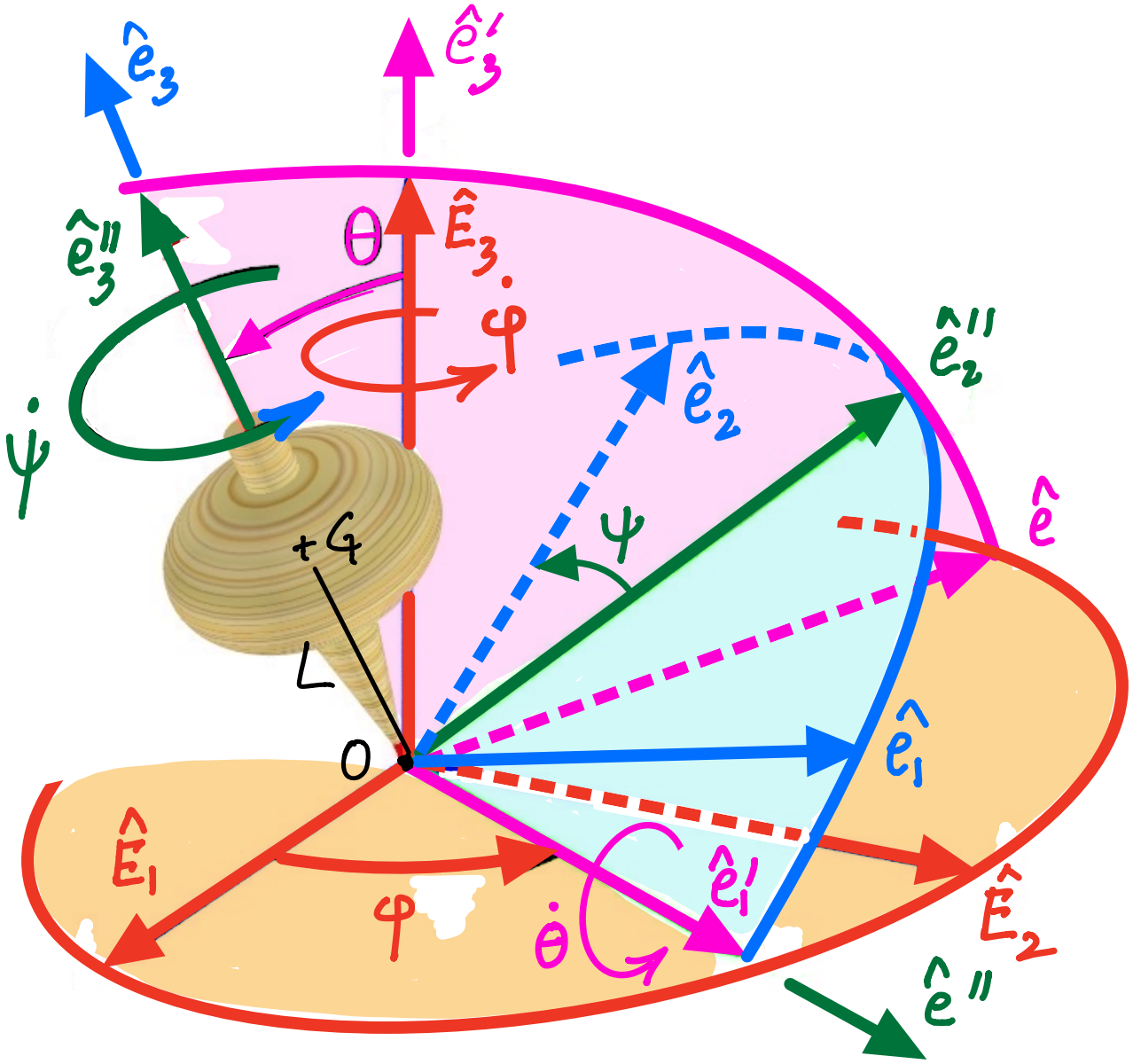
IV. Solve for $\theta(t)$ for diff. initial conditions:

Example:



unidirectional precession





$$M_3 = 0 \quad \Big| \quad \dot{\varphi} = \frac{H_3 - I_3 \omega_3^0 \cos\theta}{I_1 \sin^2\theta}$$

$\varphi(t)$ ← Need $\varphi(0)$

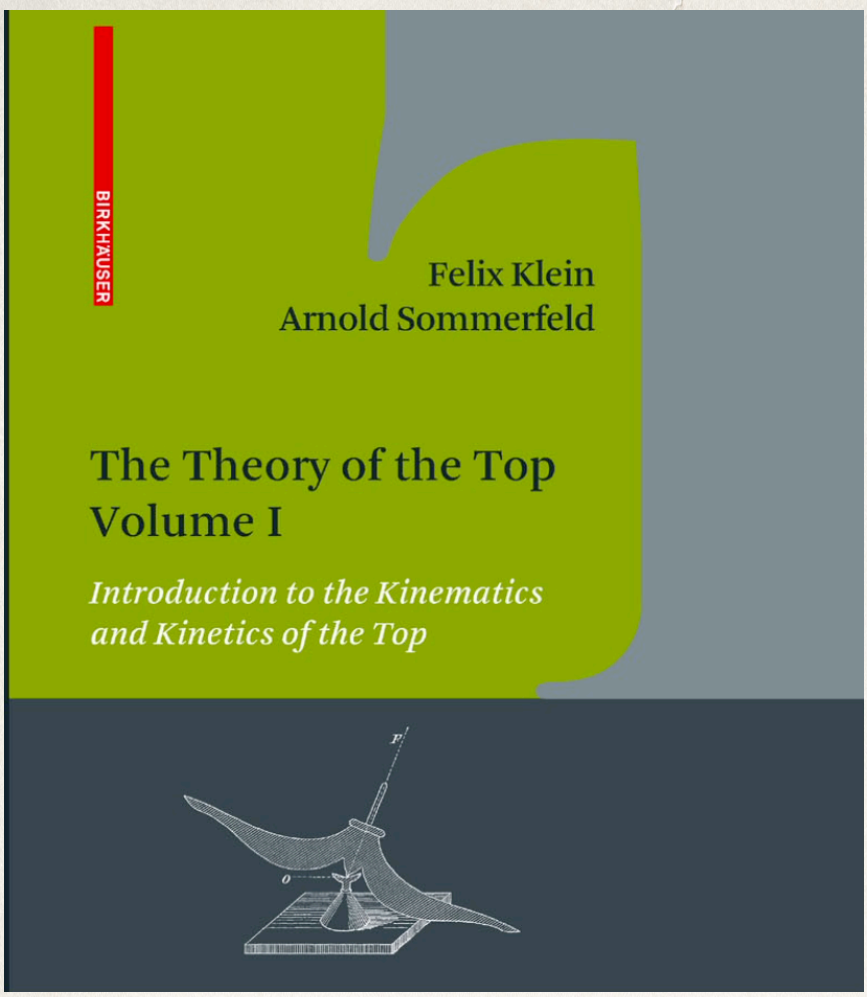
@ $t=0$ $\quad \theta(0)=\theta_0$

$\dot{\varphi}(0)=\dot{\varphi}_0 \quad \dot{\psi}(0)=\dot{\psi}_0 \Big| \theta(t)$

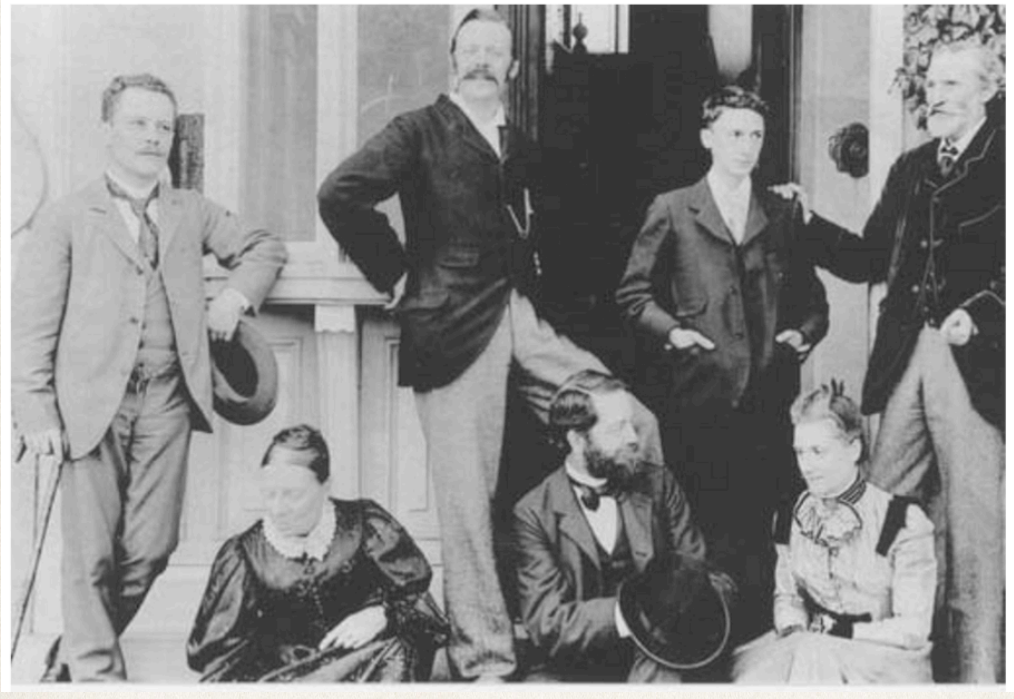
$\dot{\theta}(0)=\dot{\theta}_0$

$\dot{\psi} = \omega_3^0 - \dot{\varphi}\cos\theta \Big| \Rightarrow \psi(t)$

$\psi(0)=\psi_0$



FOUR more volumes



F Klein: At par with Hilbert and Poincare.

A Sommerfeld: 84 Nobel prize nominations!

Meanwhile tops come in square as well — look it up!