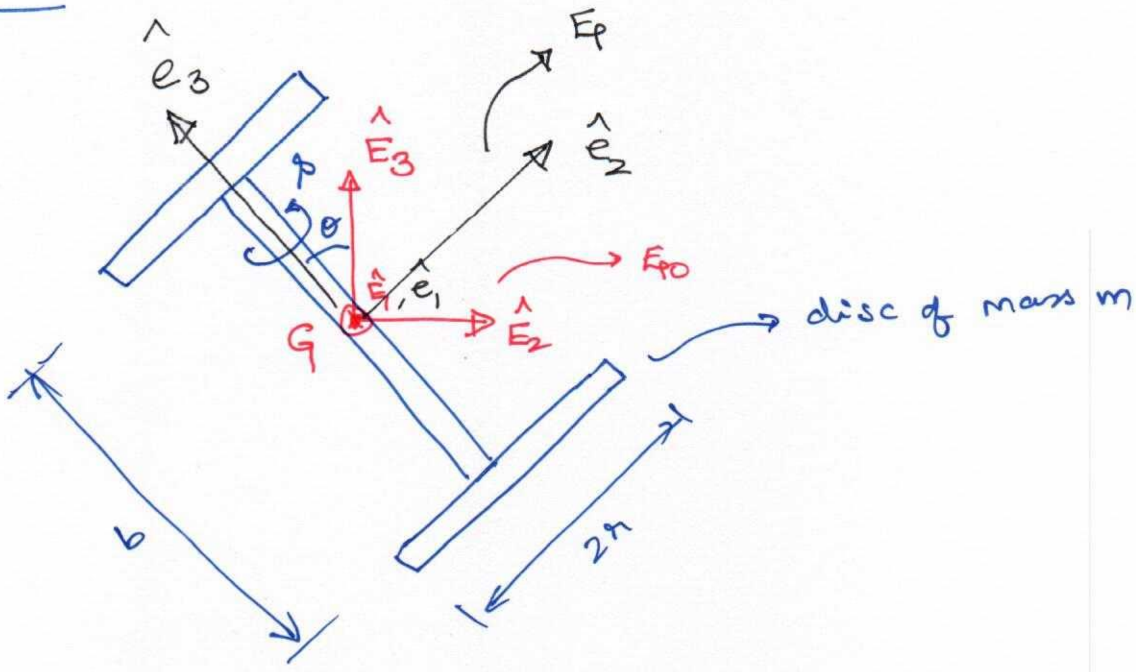
ESO209 - Tutorial 10

Problem - 5



<u>Note</u>: The spin rate in the problem is given as 'p', but we denote it as 's' here (to be consistent with lecture notes).

We choose the observer CS $\{E_{\phi 0}, G, \hat{E}_i\}$ such that G is centre of mass, $\hat{E}_3$ is along the angular momentum vector of the system.

ie
$$\underline{h}^G = H\hat{E}_3$$

We then choose the BFCS $\{E_\phi, G, \hat{e}_i\}$ such that $\hat{e}_3$ is along the axis of symmetry (one of the principal axes), $\hat{e}_1$ is aligned with $\hat{E}_1$ at instant of interest and $\hat{e}_2$ is $\perp$ to $\hat{e}_3$ & $\hat{e}_1$. (as shown in fig).

Angular momentum vector

$$[\underline{h}_G]_{E_p} = [\underline{\underline{I}}]_{E_p} \cdot [\underline{\omega}_B]_{E_p}$$

$$= \begin{bmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

where $\underline{\omega}_B = \omega_x \hat{e}_1 + \omega_y \hat{e}_2 + \omega_z \hat{e}_3$

~~form the BFCS~~

$\cancel{\omega_x = 0}$, $\cancel{\omega_y}$

$$[\underline{h}_G]_{E_p} = \begin{bmatrix} I_0 \omega_x \\ I_0 \omega_y \\ I_3 \omega_z \end{bmatrix}$$

$$\Rightarrow \quad \underline{h}_G = I_0 \omega_x \hat{e}_1 + I_0 \omega_y \hat{e}_2 + I_3 \omega_z \hat{e}_3 \quad — (1)$$

We have also defined $\hat{E}_3$ such that

$$\underline{h}_G = H \hat{E}_3$$

$$= H(\cos\theta\, \hat{e}_3 + \sin\theta\, \hat{e}_2) \quad — (2)$$

~~Which~~ Equating the components of $\hat{e}_1, \hat{e}_2, \hat{e}_3$ of (1) & (2), we get

$$\omega_x = 0$$

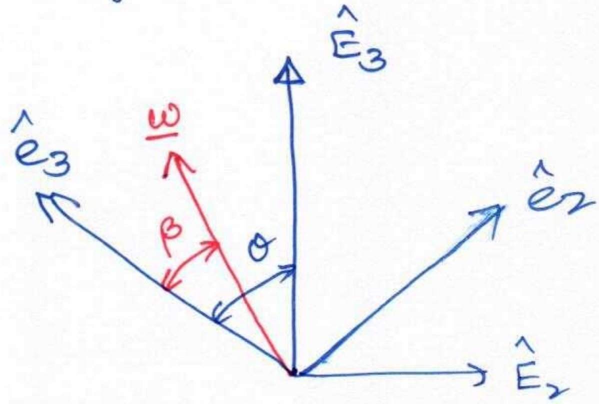
$$I_0 \omega_y = H \sin\theta \quad — (3)$$

$$I_3 \omega_z = H \cos\theta \quad — (4)$$

Dividing 3 & 4, we get

$$\tan\theta = \frac{I_0}{I_3}\,\frac{\omega_y}{\omega_z}.$$

Considering the $\underline{\omega}$ closely,



Since $\omega_x = 0$ $\underline{\omega}$ lies in the $\hat{E}_2 - \hat{E}_3$ $(\hat{e}_2 - \hat{e}_3)$ plane

$\therefore$ $\frac{\omega_y}{\omega_z} = \tan\beta$, where β is the angle $\underline{\omega}$ makes with its spin axis $\hat{e}_3$

$$\therefore \quad \tan\theta = \frac{I_0}{I_3}\tan\beta.$$

The no precession condition essentially means $\theta = \beta_{(=0)}$, which means that $I_0 = I_3$.

$$I_0 = \underbrace{2}_{\text{2 discs}} * \Big[\underbrace{\tfrac{1}{4}mr^2}_{\text{MI about its axis in the plane of disc}} + m\left(\tfrac{b}{2}\right)^2\Big] = \tfrac{1}{2}mr^2 + \tfrac{1}{2}mb^2$$

Using ∥ axis theorem and finding MI about G.

$$I_3 = 2 * \left[\tfrac{1}{2}mr^2\right] = mr^2$$

$$\Rightarrow I_0 = I_3 \Longrightarrow \tfrac{1}{2}mr^2 + \tfrac{1}{2}mb^2 = mr^2 \Rightarrow \boxed{b = r}$$