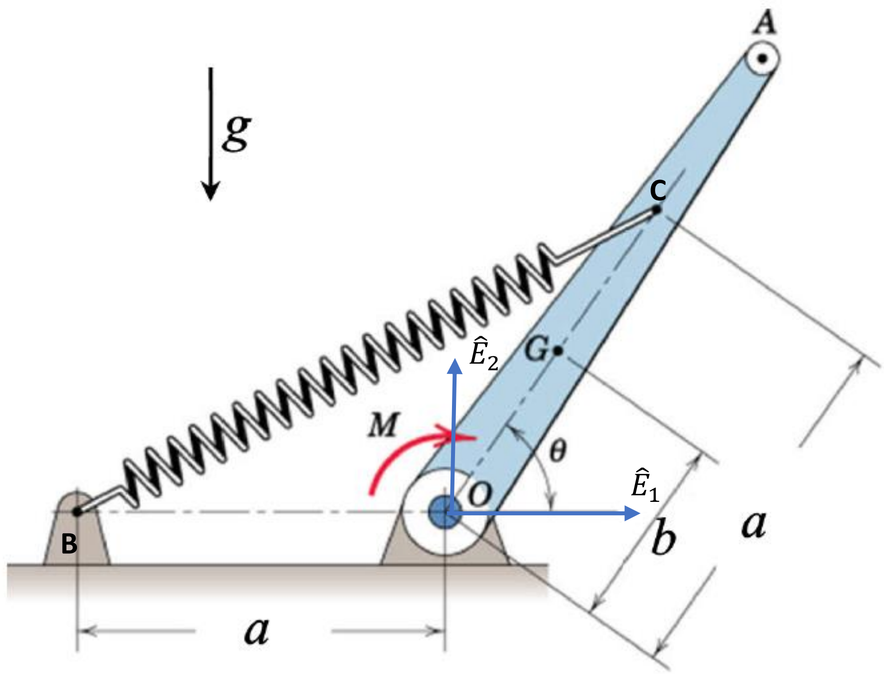
Solution:

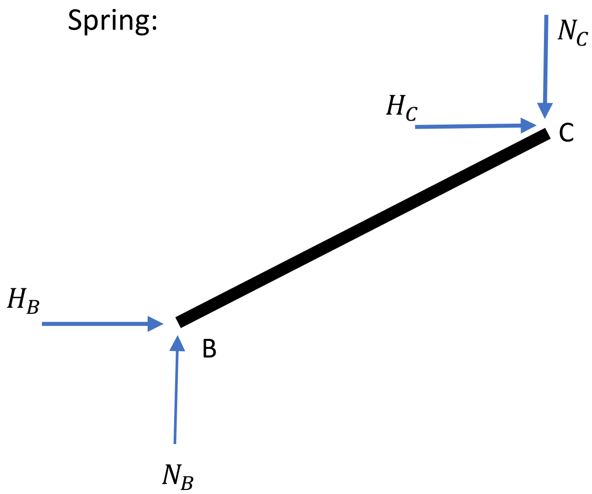
Data given: $m = 10$ kg, $\omega = 1$ rad/s when $\theta = 0^0$, $k = 700$ N/m, $b = 450$ mm, $a = 800$ mm, and radius of gyration, $r_k = 500$ mm.



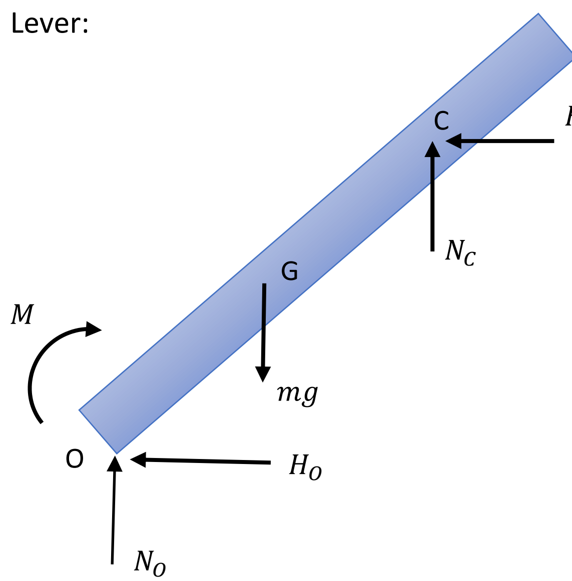
We have to find M when $\theta = 0^0$.

FBD:

Spring:



Lever:



Observations:

1. Spring is massless.
2. All joints are frictionless.
3. N_c, H_c are internal forces.
4. N_B, H_B, N_O, H_O are external forces but do no work as $\underline{v}^O = \underline{v}^B = 0$.
5. External forces/Moments are mg, M and spring forces.

For 2D conservative system

$$E_k + \sum_{n=1}^{N} \sum_{i} U_{in} + \sum_{n=1}^{N} \sum_{j} V_{jn} = \text{constant} \quad \ldots\ldots (1)$$

Clearly $N = 2$. Kinetic energy of the system is given as

$$E_k = \frac{1}{2} m \left|\underline{v}^G\right|^2 + \frac{1}{2} \underline{\omega}^B \cdot \underline{\underline{I}}^G \cdot \underline{\omega}^B$$

For 2D system,

$$E_k = \frac{1}{2} m \left|\underline{v}^G\right|^2 + \frac{1}{2} I_3^G \omega_B^2$$

where, $\underline{\omega}^B = \dot{\theta} \hat{E}_3$ at any instant of time.

For $\underline{v}^G$, performing kinematics

Consider lever OA, since both points O and G are on same body therefore

$$\underline{v}^G = \underline{v}^O + \underline{\omega}^B \times \underline{r}_{G/O}$$

and $$\underline{r}_{G/O} = b \cos\theta \hat{E}_1 + b \sin\theta \hat{E}_2$$

this gives:

$$\underline{v}^G = 0 + \dot{\theta} \hat{E}_3 \times \left(b \cos\theta \hat{E}_1 + b \sin\theta \hat{E}_2\right)$$

$$\Longrightarrow \quad \underline{v}^G = b\dot{\theta} \cos\theta\, \hat{E}_2 - b\dot{\theta} \sin\theta\, \hat{E}_1$$

$$\Longrightarrow \quad \left|\underline{v}^G\right|^2 = b^2 \dot{\theta}^2$$

Therefore,

$$E_k = \tfrac{1}{2} m b^2 \dot{\theta}^2 + \tfrac{1}{2} I_3^G \dot{\theta}^2 \quad \ldots\ldots (2)$$

where $I_3^G = m r_k^2$.

Scalar potentials due to external forces and moments,

$$\sum_{n=1}^{2}\sum_{i} U_{in} = mgb\sin\theta + \frac{1}{2}kx^2 \;\; \dots\dots(3)$$

where x is displacement of spring at any instant.

$$\sum_{n=1}^{2}\sum_{j} V_{jn} = -M\theta \;\; \dots\dots(4)$$

Now using Eqs. (2), (3) and (4) into Eq. (1), we get

$$\frac{1}{2}mb^2\dot{\theta}^2 + \frac{1}{2}I_3^G\dot{\theta}^2 + mgb\sin\theta + \frac{1}{2}kx^2 - M\theta = \text{constant} \;\; \dots\dots(5)$$

For obtaining the value of 'constant', using initial condition.

At $\theta = 90^0$, $\dot{\theta} = 0$ rad/s and $x = 0$.

From Eq. (5), we get

$$\text{constant} = mgb - \frac{M\pi}{2}$$

then Eq. (5) becomes

$$\frac{1}{2}mb^2\dot{\theta}^2 + \frac{1}{2}I_3^G\dot{\theta}^2 + mgb\sin\theta + \frac{1}{2}kx^2 - M\theta = mgb - \frac{M\pi}{2} \;\; \dots\dots(6)$$

We have to find M when $\theta = 0^0$.

At $\theta = 90^0$, spring is unstretched and its length is $\sqrt{2}$a, and when $\theta = 0^0$, spring stretched, and its length become 2a. so $x = 2a - \sqrt{2}a$.

Use Eq. (6) when $\theta = 0^0$.

$$\begin{aligned}
&\frac{1}{2}(10)(0.45)^2(1)^2 + \frac{1}{2}(10)(0.5)^2(1)^2 \\
&\qquad + (10)(9.81)(0.45)\sin 0^0 + \frac{1}{2}(700)\left(2(0.8) - \sqrt{2}(0.8)\right)^2 - M(0) \\
&\qquad = (10)(9.81)(0.45) - \frac{M\pi}{2}
\end{aligned}$$

$$M = 22.28\ \text{Nm}$$