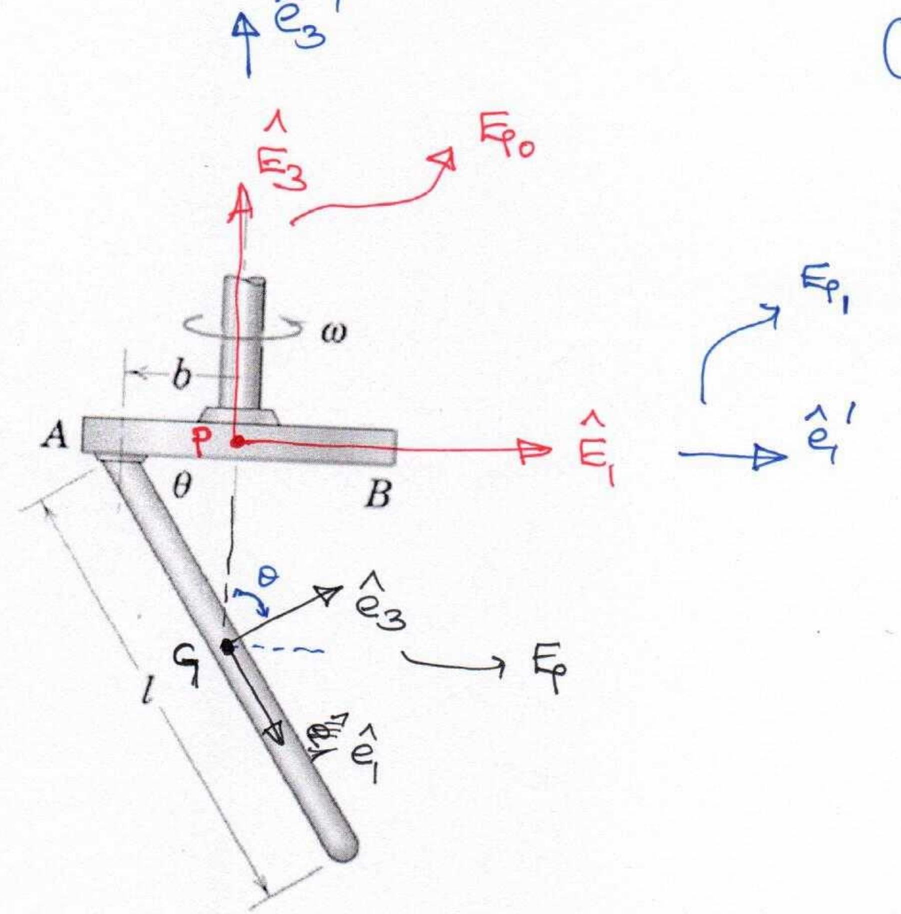
ESO 209.
Tutorial 9.

Problem 3



Given $\theta = 60^\circ$, $b = l/4$.

Determine ω such that the welded joint at A experiences zero moment.

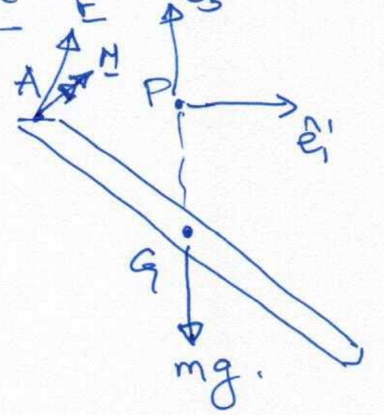
Soln: Defining Observer CS $\{E_{\varphi_0}, P, \hat{E}_i\}$ } aligned at instant of interest.

BFCS attached to AB $\{E_{\varphi_1}, P, \hat{e}_i'\}$

BFCS attached to rod $\{E_\varphi, G, \hat{e}_i\}$ $\hat{e}_2$ aligned to $\hat{E}_2$ at instant of interest

↳ this will be used only for $\underline{\underline{I}}_G$ calculation.

FBD of rod.



Reaction force at A

$\underline{F} = F_1\hat{e}_1' + F_2\hat{e}_2' + F_3\hat{e}_3'$

Reaction Moment at A

$\underline{M} = M_1\hat{e}_1' + M_2\hat{e}_2' + M_3\hat{e}_3'$

Using LMB first,

$$\sum \underline{F}_i = m\,\underline{a}_G$$

$$\underline{a}_G = \underline{\omega}_{E_{\phi 1}/E_{\phi 0}} \times (\underline{\omega}_{E_{\phi 1}/E_{\phi 0}} \times \underline{r}^{G/P}) + \cancelto{0}{\underline{\alpha}_{E_{\phi 1}/E_{\phi 0}}} \times \underline{r}^{G/P} + \cancelto{0}{\underline{a}^P}$$

$\because$ both magnitude (ω) and direction ($\hat{E}_3$) of $\underline{\omega}_{E_{\phi 1}/E_{\phi 0}}$ is fixed

$$\underline{\omega}_{E_{\phi 1}/E_{\phi 0}} = \omega \hat{E}_3$$

$$\underline{r}^{G/P} = -\frac{l}{2}\sin\theta\,\hat{E}_3$$

$$\therefore\ \underline{a}_G = 0$$

$$\therefore\ \sum \underline{F}_i = 0$$

$$\Rightarrow F_1 \hat{e}_1' + F_2 \hat{e}_2' + F_3 \hat{e}_3' - mg\,\hat{e}_3' = 0$$

$$\Rightarrow F_1 = 0 \quad F_2 = 0 \quad F_3 = mg. \Rightarrow \underline{F} = mg\,\hat{e}_3' = mg\,\hat{E}_3$$

Now using AMB about point P.

$$\sum \underline{M}_i^P = \underline{r}^{G/P} \times m\cancelto{0}{\underline{a}^G} + \underline{\omega}_{E_{\phi 1}/E_{\phi 0}} \times (\underline{\underline{I}}^G \cdot \underline{\omega}_{E_{\phi 1}/E_{\phi 0}}) + \underline{\underline{I}}^G \cdot \cancelto{0}{\underline{\alpha}_{E_{\phi 1}/E_{\phi 0}}}$$

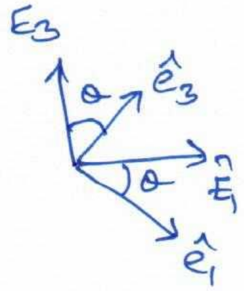
$$= \underline{\omega}_{E_{\phi 1}/E_{\phi 0}} \times (\underline{\underline{I}}^G \cdot \underline{\omega}_{E_{\phi 1}/E_{\phi 0}})$$

Since it is convenient to express $\underline{\underline{I}}^G$ in a CS which is aligned to the principal axes, we will used CS$\{E_\phi, G, \hat{e}_i'\}$.

$$[\underline{\underline{I}}^G]_{E_\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{ml^2}{12} & 0 \\ 0 & 0 & \frac{ml^2}{12} \end{bmatrix}$$

To determine

$\underline{\underline{I}}^G \cdot \underline{\omega}_{E_{F1}/E_{F0}}$, we will need to express $\underline{\omega}_{E_{F1}/E_{F0}}$ also in the CS $\{E_F, G, \hat{e}_i\}$.



$$\underline{\omega}_{E_{F1}/E_{F0}} = \omega \hat{E}_3$$

$$= \omega(-\sin\theta\,\hat{e}_1 + \cos\theta\,\hat{e}_3)$$

$$\therefore \left[\underline{\underline{I}}^G \cdot \underline{\omega}_{E_{F1}/E_{F0}}\right]_{E_F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & ml^2/12 & 0 \\ 0 & 0 & \frac{ml^2}{12} \end{bmatrix} \begin{bmatrix} -\omega\sin\theta \\ 0 \\ \omega\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \frac{ml^2}{12}\omega\cos\theta \end{bmatrix}$$

$$\therefore \underline{\omega}_{E_{F1}/E_{F0}} \times \left(\underline{\underline{I}}^G \cdot \underline{\omega}_{E_{F1}/E_{F0}}\right)$$

$$= \left(-\omega\sin\theta\,\hat{e}_1 + \omega\cos\theta\,\hat{e}_3\right) \times \left(\frac{ml^2}{12}\omega\cos\theta\,\hat{e}_3\right)$$

$$= (-\omega\sin\theta)\left(\frac{ml^2}{12}\omega\cos\theta\right)(-\hat{e}_2)$$

$$= \frac{ml^2\omega^2}{12}\sin\theta\cos\theta\,\hat{e}_2 \longrightarrow \text{RHS.}$$

Finding LHS.

$$\underline{M} = \underline{r}_A \times \underline{F} + \underline{M} + \underline{r}_G \times (-mg\hat{E}_3)$$

Recognizing that the CS E_{θ_1} and E_{θ_0} are aligned

$$\underset{\sim}{M} = (-b\hat{E}_1 \times mg\hat{E}_3) + M_1\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3 + (-\tfrac{l}{2}\sin\theta\,\hat{E}_3) \times (-mg\hat{E}_3)$$

$$= mg\,l/_4\,\hat{E}_2 + M_1\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3 + 0$$

Equating LHS & RHS, we get

$$M_1\hat{E}_1 + (M_2 + mg\,l/_4)\hat{E}_2 + M_3\hat{E}_3 = \frac{ml^2\omega^2}{12}\sin\theta\cos\theta\,\hat{e}_2$$

$$= \frac{ml^2\omega^2}{12}\sin\theta\cos\theta\,\hat{E}_2$$

$$\Rightarrow \quad M_1 = 0, \quad M_3 = 0$$

and $M_2 + mg\dfrac{l}{4} = \dfrac{ml^2\omega^2}{12}\sin\theta\cos\theta$

Now we are required to find ω such that moment is zero. M_1, M_3 are zero. M_2 also $= 0$

$$\therefore \quad mg\frac{l}{4} = \frac{ml^2\omega^2}{12}\sin\theta\cos\theta$$

Since $\theta = 60^\circ$

$$\Rightarrow \quad mg\frac{l}{4} = \frac{ml^2\omega^2}{12}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}$$

$$\Rightarrow \quad \omega^2 = \cancel{4\sqrt{3}}\ 4\sqrt{3}\,\frac{g}{l}$$

$$\Rightarrow \quad \boxed{\omega = 2\sqrt{\frac{\sqrt{3}g}{l}}}$$