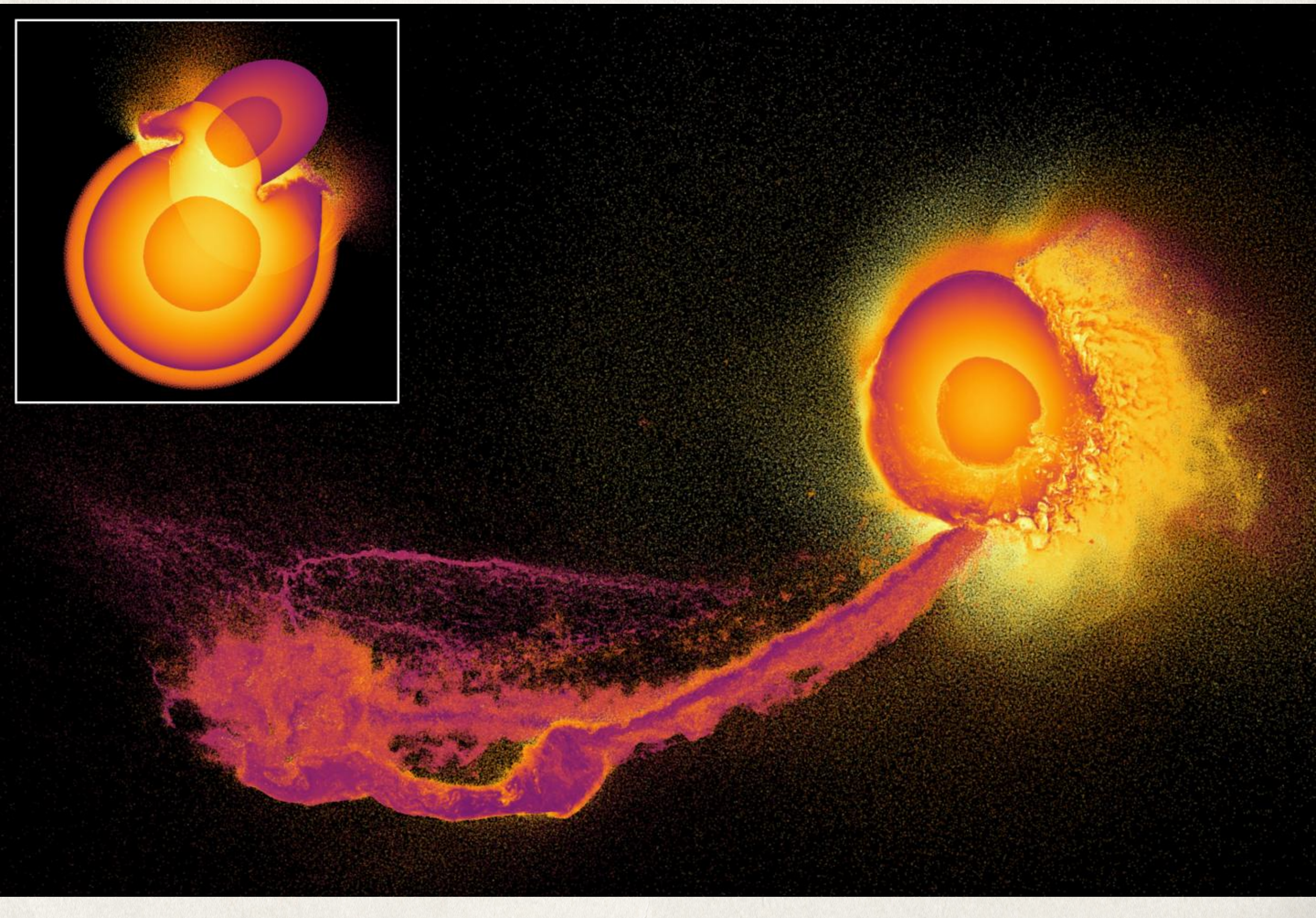
Lecture 20

Collisions: Impulse-momentum relations, elastic and inelastic collisions.

27 October - 2 November, 2021

Collisions



Impulse-momentum

Collision at t_0: Find kinematics *just after* the collision, given kinematics *just before*.

1. *LMB*: $\sum \mathbf{F} = m\dot{\mathbf{v}}^G$; $AMB_{/O}$: $\mathbf{M}^O = \dot{\mathbf{h}}^O$
2. Rigid body collisions are *instantaneous*.
3. *Integrate* these eqns. from $t_0 - \delta t$ to $t_0 + \delta t$.
4. *Assume* system configuration remains <u>unchanged</u> during integration.
5. *Force impulse* at time t_0: $\mathbf{I}_f = \int_{t_0-\delta t}^{t_0+\delta t} \mathbf{F} \mathrm{d}t$
6. *Moment impulse* at t_0 about O:

$$\mathbf{I}_m^O = \int_{t_0-\delta t}^{t_0+\delta t} \mathbf{M}^O \mathrm{d}t = \int_{t_0-\delta t}^{t_0+\delta t} \left(\sum \mathbf{M}^j + \mathbf{r}^{i/O} \times \mathbf{F}^i \right) \mathrm{d}t$$

Impulse-momentum relations

LMB: $\sum \mathbf{I}_f = m\mathbf{v}^G(t + \delta t) - m\mathbf{v}^G(t - \delta t)$

$AMB_{/O}$: $\mathbf{I}_m^O = \mathbf{h}^O(t + \delta t) - \mathbf{h}^O(t - \delta t)$

Problem formulation

I. **FBD** during collision with *only* impulses.

   1. *Instantaneous* collisions, so δt is small.
   2. *Bounded* forces impart *no* impulse.

II. Relate <u>post</u>-collision and <u>pre</u>-collision kinematics by utilizing *impulse-momentum relations*

   1. in *directions* where force and moment impulses are known or are zero;
   2. about *points* where moment impulses are known or are zero.

III. In *systems* with <u>many</u> bodies, internal impulses cancel out. Therefore,

Impulse-momentum relations for system

LMB: $$\sum \mathbf{I}_f^{ext} = \mathbf{p}_{sys}(t + \delta t) - \mathbf{p}_{sys}(t - \delta t)$$

AMB$_{/O}$: $$\mathbf{I}_m^{O,ext} = \mathbf{h}_{sys}^{O}(t + \delta t) - \mathbf{h}_{sys}^{O}(t - \delta t)$$

Impact models

Impulse-momentum relations may ***not*** give enough equations. Need *impact models.*

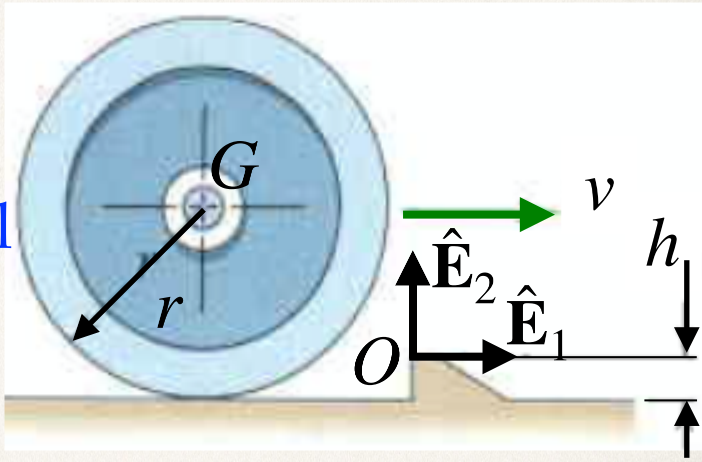
1. *Newton's restitution model*. At each point of contact in a collision: $v_f = -ev_i$
   i. $0 \leqslant e \leqslant 1$: restitution coefficient
   ii. v_i : normal velocity of *approach.*
   iii. v_f : normal velocity of *separation.*
   iv. $\Leftrightarrow$ *balance of energy* in *single*-point collisions: $e = 1 \implies$ collision conserves energy, else not.
   v. e depends on impact velocity (*CV Raman*) and system geometry.
   vi. Used mainly for <u>normal</u> impact.
   vii. Can lead to **un**physical results.

2. Many other models. *Open question* still.

<u>Best model so far</u>: *Rakshit & Chatterjee* (2015)

Example 1

Find velocity of wheel's CM *just* after it hits the step. Wheel has mass m and radius of gyration k.



1. *Post-collision*: Wheel's CM velocity is $\mathbf{v}^G$ and angular velocity $\boldsymbol{\omega}^f = \omega_f \hat{\mathbf{E}}_3$.
2. **FBD** $\Longrightarrow$ unknown force impulses in both horizontal and vertical directions at O.
3. So, write *Impulse-momentum* relation corresponding to $AMB_{/O}$:

$$\underbrace{m\mathbf{r}^{G/O} \times v\hat{\mathbf{E}}_1 + \mathbf{I}^G \cdot \boldsymbol{\omega}^i}_{\mathbf{h}^O(t-\delta t)} = \underbrace{m\mathbf{r}^{G/O} \times \mathbf{v}^G + \mathbf{I}^G \cdot \boldsymbol{\omega}^f}_{\mathbf{h}^O(t+\delta t)}$$

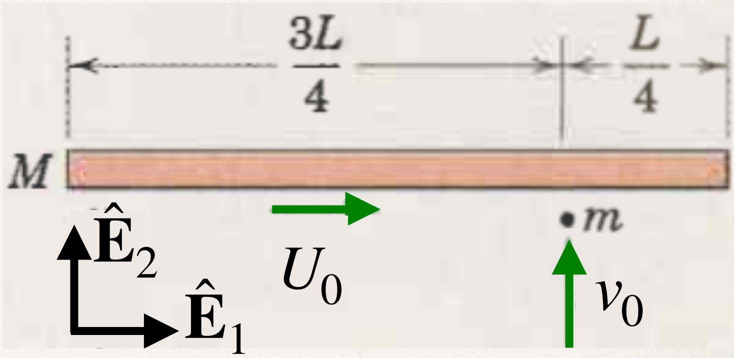
4. 3 unknowns, 1 eqn. Need *impact model*:
   - $e = 0 \implies$ *Wheel* **sticks** to step $\Longrightarrow$

$$\mathbf{v}^G = \boldsymbol{\omega}^f \times \mathbf{r}^{G/O}$$

5. Solve: $\omega = -(k^2 + r^2 - rh)v/(k^2 + r^2)r$

Example 2

Particle P (mass m) hits the rod. Find the post-collision rotation rate of rod.

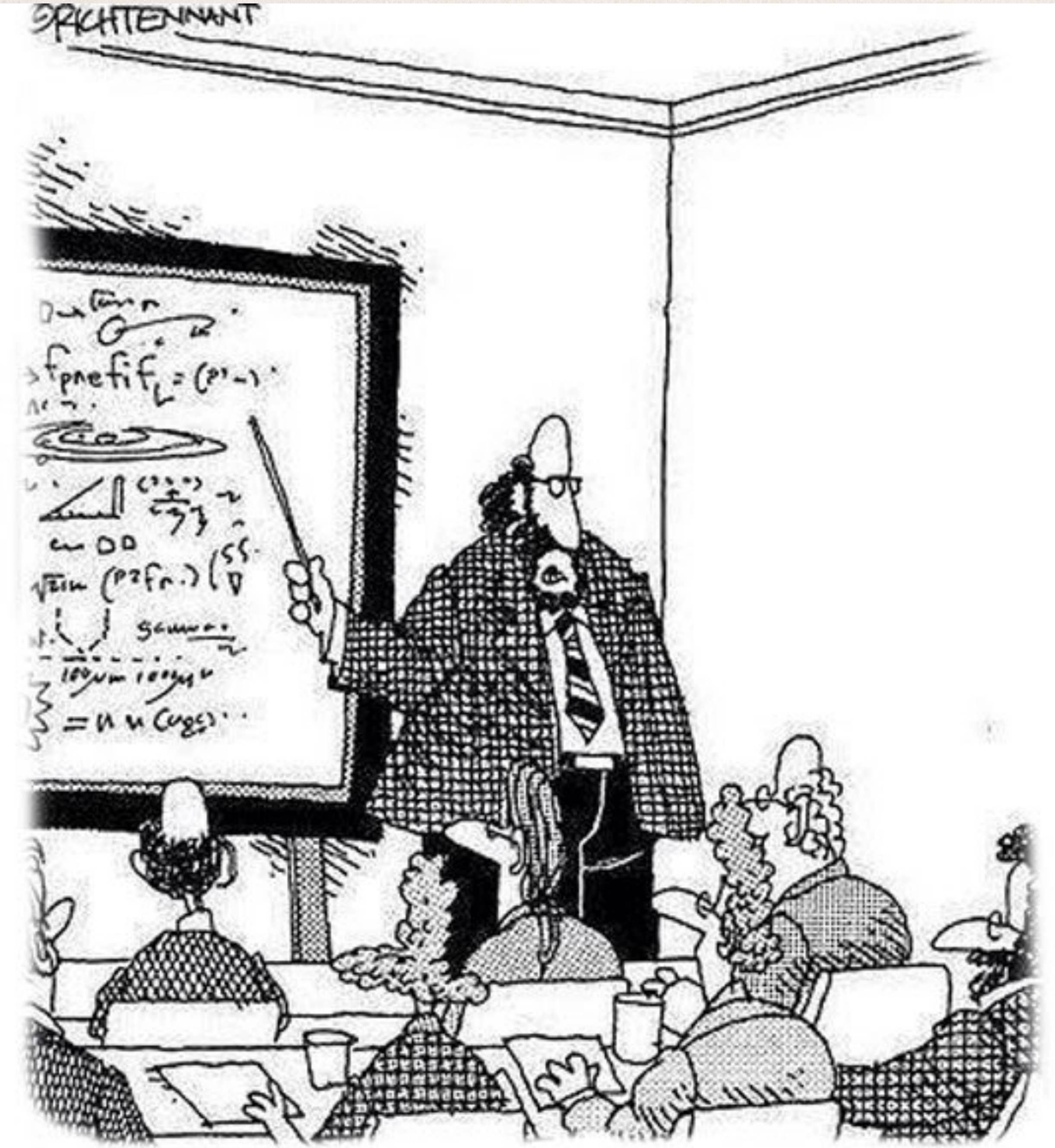


1. *Post-collision*: Rod's CM velocity is $\mathbf{v}^G$ and angular velocity $\boldsymbol{\omega} = \omega\hat{\mathbf{E}}_3$. P has vel. $\mathbf{v}^P$.
2. **FBD** $\Longrightarrow$ NO external impules. So can write all *Impulse-momentum* relations:

LMB: $$\underbrace{mv_0\hat{\mathbf{E}}_2 + MU_0\hat{\mathbf{E}}_1}_{\mathbf{p}_{sys}(t-\delta t)} = \underbrace{m\mathbf{v}^P + M\mathbf{v}^G}_{\mathbf{p}_{sys}(t+\delta t)}$$

$AMB_{/G}$: $$\underbrace{m\mathbf{r}^{O/G} \times v_0\hat{\mathbf{E}}_2}_{\mathbf{h}_{sys}(t-\delta t)} = \underbrace{m\mathbf{r}^{O/G} \times \mathbf{v}^P + \mathbf{I}^G \cdot \boldsymbol{\omega}}_{\mathbf{h}_{sys}(t+\delta t)}$$

3. 5 unknowns, 3 eqns. Need *impact model*:
   - $e = 0 \implies$ P **sticks** to the rod $\Longrightarrow$
   $$\mathbf{v}^P = \mathbf{v}^G + \boldsymbol{\omega} \times \mathbf{r}^{O/G}$$
4. Solve: $\omega = 12mv_0/(7m + 4M)L$



"Along with 'Antimatter,' and 'Dark Matter,' we've recently discovered the existence of 'Doesn't Matter,' which appears to have no effect on the universe whatsoever."

Dynamics matters!