Q1).

A disc D of radius $R = 0.3m$ rolls on a horizontal plane without slipping such that $v_0 = -3\hat{\mathbf{E}}_1\ m/s$ and $a_0 = -6\hat{\mathbf{E}}_1 m/s^2$. Point P moves in a circular slot of radius $r = 0.2m$. At the instant shown in the figure, an observer sitting on the disc and rotating with it records the velocity and acceleration of point P to be $2\hat{\mathbf{E}}_1 m/s$ and $-10\hat{\mathbf{E}}_1 m/s2$, respectively. Determine the absolute velocity and acceleration of P.

Hint: It does not matter where the observer sits, so choose a convenient point.

Sol)

Coordinate systems-

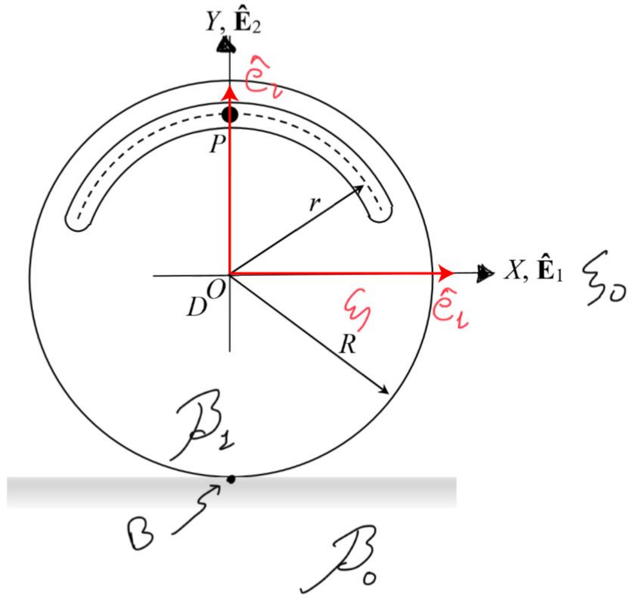

Figure 1: Schemating showing the coordinates systems

Ground frame- { $\mathscr{E}_0$, O, $\hat{\mathbf{E}}_i$ }

BFCS of disk - { $\mathscr{E}$, O, $\hat{\mathbf{e}}_i$ }

Velocity analysis

$$\mathbf{v}^P_{\mathcal{E}_0} = \mathbf{v}^P_{rel} + \boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{P/0} + \mathbf{v}^0_{\mathcal{E}_0} \tag{1}$$

where, $\mathbf{v}^P_{rel} = 2\hat{\mathbf{E}}_1$ m/s, $\mathbf{v}^0_{\mathcal{E}_0} = -3\hat{\mathbf{E}}_1$ m/s, and $\mathbf{r}^{P/0} = r\hat{\mathbf{E}}_2$.

No slip condition:-

Let point B_1 is attached to disc (body $\mathscr{B}_1$) and coincident with point of contact (B) and point B_0 is attached to ground (body $\mathscr{B}_0$).

$$\mathbf{v}^{B_1}_{\mathcal{E}_0} = \mathbf{v}^{B_0}_{\mathcal{E}_0}$$

Since, $\mathbf{v}^{B_0}_{\mathcal{E}_0} = 0$, hence, $\mathbf{v}^{B_1}_{\mathcal{E}_0} = 0$

Velocity of B_1 through point O is

$$\mathbf{v}^{B_1} = \boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{B_1/0} + \mathbf{v}^0 \tag{2}$$

Let $\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} = \omega\hat{\mathbf{E}}_3$

here, $\mathbf{r}^{B_1/0} = R(-\hat{\mathbf{E}}_2)$

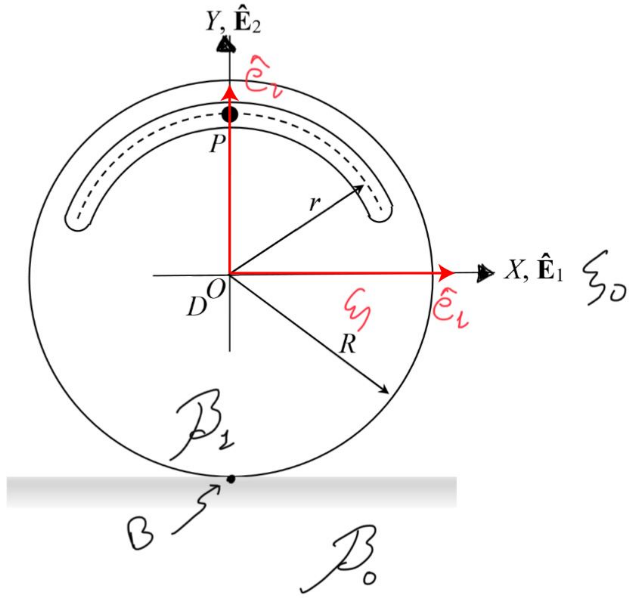

Figure 2: Schemating showing the coordinates systems

From Eqn (2)

$$
\begin{aligned}
0 &= \omega\hat{\mathbf{E}}_3 \times R(-\hat{\mathbf{E}}_2) + (-3\hat{\mathbf{E}}_1) \\
&= \omega R\hat{\mathbf{E}}_1 - 3\hat{\mathbf{E}}_1 \\
\omega &= \frac{3}{R} \text{ rad/sec}
\end{aligned}
$$

$$\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} = 10\hat{\mathbf{E}}_3 \text{ rad/sec} \tag{3}$$

from eqn(1)

$$
\begin{aligned}
\mathbf{v}^P_{\mathcal{E}_0} &= 2\hat{\mathbf{E}}_1 + \frac{3}{R}\hat{\mathbf{E}}_3 \times \hat{\mathbf{E}}_2 + (-3\hat{\mathbf{E}}_1) \\
&= 2\hat{\mathbf{E}}_1 + \frac{3r}{R}(-\hat{\mathbf{E}}_1) - 3\hat{\mathbf{E}}_1 \\
\mathbf{v}^P_{\mathcal{E}_0} &= \left(-1 - \frac{3r}{R}\right)\hat{\mathbf{E}}_1
\end{aligned}
$$

$$\mathbf{v}^P_{\mathcal{E}_0} = -3\hat{\mathbf{E}}_1 m/s \tag{4}$$

Acceleration Analysis

$$\mathbf{a}^P_{\mathcal{E}_0} = \mathbf{a}^P_{rel} + \boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times (\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{P/0}) + \alpha_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{P/0} + 2\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{v}^P_{rel} + \mathbf{a}^0_{\mathcal{E}_0} \tag{5}$$

here, $\mathbf{a}^P_{rel} = -10\hat{\mathbf{E}}_1$, $\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} = \frac{3}{R}\hat{\mathbf{E}}_3$, $r^{P/0} = r\hat{\mathbf{E}}_2$, $v^P_{rel} = 2\hat{\mathbf{E}}_1$, and $a^0_{\mathcal{E}_0} = -6\hat{\mathbf{E}}_1$ m/s^2

no slip condition
Let $\hat{\mathbf{t}}$ be the tangential direction , here, $\hat{\mathbf{t}} = \hat{\mathbf{E}}_1$ then,

$$\mathbf{a}^{B_1}.\hat{\mathbf{t}} = \mathbf{a}^{B_0}.\hat{\mathbf{t}}$$
$$\text{since, } \mathbf{a}^{B_0} = 0$$
$$\text{hence, } \mathbf{a}^{B_1} \cdot \hat{\mathbf{E}}_1 = 0$$

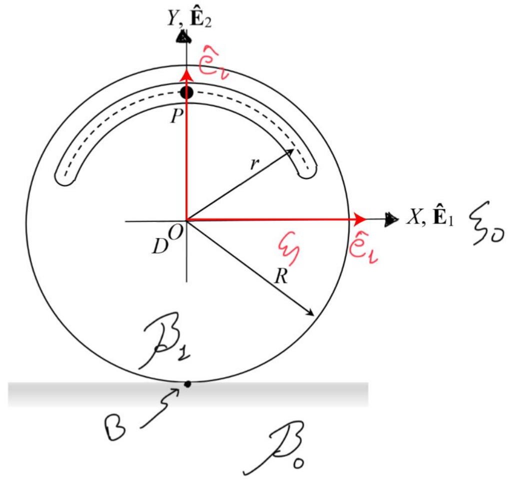

Figure 3: Schemating showing the coordinates systems

Acceleration of point B_1 through point O
relating acceleration of two point in a rigid body, B_1 and O.

$$\begin{aligned}
\mathbf{a}^{B_1}_{\mathcal{E}_0} &= \mathbf{a}^O + \boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times (\boldsymbol{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{B_1/0}) + \boldsymbol{\alpha}_{\mathcal{E}/\mathcal{E}_0} \times \mathbf{r}^{B_1/0} \\
\text{here, } \mathbf{r}^{B_1/0} &= R(-\hat{\mathbf{E}}_2) \\
\mathbf{a}^{B_1}_{\mathcal{E}_0} &= -6\hat{\mathbf{E}}_1 + \frac{3}{R}\hat{\mathbf{E}}_3 \times [\frac{3}{R}\hat{\mathbf{E}}_3 \times R(-\hat{\mathbf{E}}_2)] + \boldsymbol{\alpha}_{\mathcal{E}/\mathcal{E}_0} \times R(-\hat{\mathbf{E}}_2) \\
\text{let, } \alpha_{\mathcal{E}/\mathcal{E}_0} &= \alpha\hat{\mathbf{E}}_3 \\
\mathbf{a}^{B_1}_{\mathcal{E}_0} &= -6\hat{\mathbf{E}}_1 + \frac{3}{R}\hat{\mathbf{E}}_3 \times (3\hat{\mathbf{E}}_1) + \alpha\hat{\mathbf{E}}_3 \times R(-\hat{\mathbf{E}}_2) \\
&= \frac{9}{R}\hat{\mathbf{E}}_2 + (\alpha R - 6)\hat{\mathbf{E}}_1 \\
\mathbf{a}^{B_1}_{\mathcal{E}_0}.\hat{\mathbf{E}}_1 &= \frac{9}{R}\hat{\mathbf{E}}_2.\hat{\mathbf{E}}_1 + (\alpha R - 6)\hat{\mathbf{E}}_1.\hat{\mathbf{E}}_1
\end{aligned}$$

taking dot product with $\hat{\mathbf{E}}_1$ both sides

$$\mathbf{a}^{B_1}_{\mathcal{E}_0}.\hat{\mathbf{E}}_1 = \frac{9}{R}\hat{\mathbf{E}}_2.\hat{\mathbf{E}}_1 + (\alpha R - 6)\hat{\mathbf{E}}_1.\hat{\mathbf{E}}_1$$

since $\mathbf{a}^{B_1}_0 \cdot \hat{\mathbf{E}}_1 = 0$ and $\hat{\mathbf{E}}_2 \cdot \hat{\mathbf{E}}_1 = 0$

$$\implies \alpha = \frac{6}{R} rad/s^2 \tag{6}$$

Now, from Equation 5,

$$\begin{aligned}
\mathbf{a}^P_{\mathcal{E}_0} &= -10\hat{\mathbf{E}}_1 + \frac{3}{R}\hat{\mathbf{E}}_3 \times (\frac{3}{R}\hat{\mathbf{E}}_3 \times r\hat{\mathbf{E}}_2) + (\frac{6}{R}\hat{\mathbf{E}}_3 \times r\hat{\mathbf{E}}_2) + 2(\frac{3}{R})\hat{\mathbf{E}}_3 \times 2\hat{\mathbf{E}}_1 + (-6\hat{\mathbf{E}}_1) \\
\mathbf{a}^P_{\mathcal{E}_0} &= -10\hat{\mathbf{E}}_1 + \frac{3}{R}\hat{\mathbf{E}}_3 \times (-\frac{3r}{R}\hat{\mathbf{E}}_1) + \frac{6}{R}r(-\hat{\mathbf{E}}_1) + \frac{12}{R}\hat{\mathbf{E}}_2 - 6\hat{\mathbf{E}}_1 \\
&= -10\hat{\mathbf{E}}_1 - \frac{9r}{R^2}\hat{\mathbf{E}}_2 - \frac{6}{R}r\hat{\mathbf{E}}_1 + \frac{12}{R}\hat{\mathbf{E}}_2 - 6\hat{\mathbf{E}}_1 \\
\mathbf{a}^P_{\mathcal{E}_0} &= -20\hat{\mathbf{E}}_1 + \left(\frac{12}{R} - \frac{9r}{R^2}\right)\hat{\mathbf{E}}_2
\end{aligned}$$

$$\mathbf{a}^P_{\mathcal{E}_0} = -20\hat{\mathbf{E}}_1 + 20\hat{\mathbf{E}}_2 \quad m/s^2 \tag{7}$$