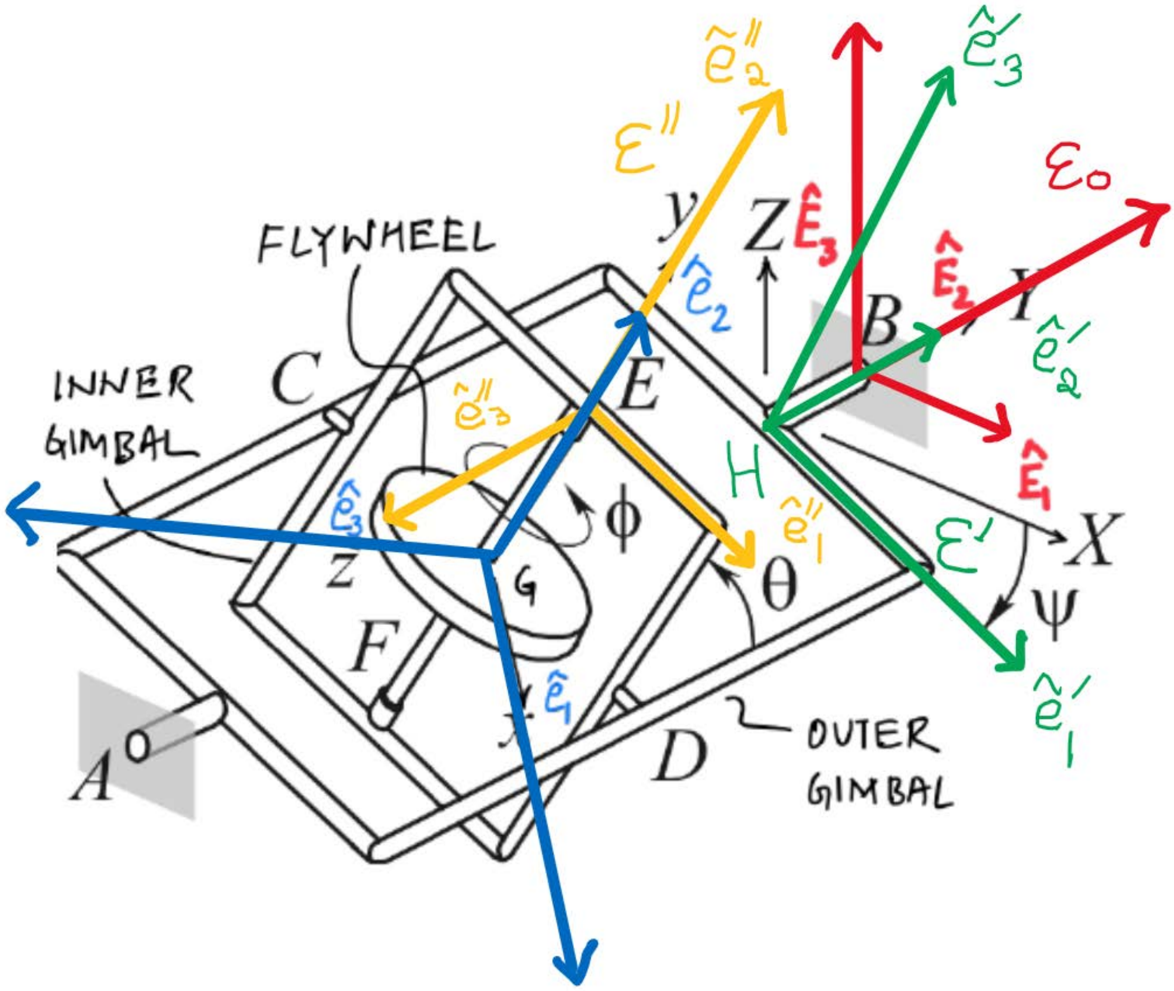

FIGURE P4

TUTORIAL-5, PROBLEM-4

Refer to figure P4 in page 4.

$$\{\mathcal{E}_0, B, \hat{E}_i\} \xrightarrow{\underline{\underline{R}}_0(\hat{E}_2, \psi(t))} \{\mathcal{E}', H, \hat{e}_i'\} \xrightarrow{\underline{\underline{R}}_1(\hat{e}_1', \theta(t))} \{\mathcal{E}'', E, \hat{e}_i''\} \xrightarrow{\underline{\underline{R}}_2(\hat{e}_2'', \phi(t))} \{\mathcal{E}, G, \hat{e}_i\}$$

ground frame (red)

outer gimbal frame (green)

Inner gimbal frame (yellow)

Flywheel frame (blue)

We have,

$$\hat{e}_i' = \underline{\underline{R}}_0 \cdot \hat{E}_i \quad \text{------} \quad (1)$$

$$\hat{e}_i'' = \underline{\underline{R}}_1 \cdot \underline{\underline{R}}_0 \cdot \hat{E}_i \quad \text{------} \quad (2)$$

$$\hat{e}_i = \underline{\underline{R}}_2 \cdot \underline{\underline{R}}_1 \cdot \underline{\underline{R}}_0 \cdot \hat{E}_i \quad \text{------} \quad (3)$$

$$\underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = \dot{\psi}\,\hat{E}_2$$

$$\underline{\omega}_{\mathcal{E}''/\mathcal{E}'} = \dot{\theta}\,\hat{e}_1'$$

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}''} = \dot{\phi}\,\hat{e}_2''$$

Also we know,

$$[\underline{\underline{R}}_0]_{\mathcal{E}_0} = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix}$$

$$[\underline{\underline{R}}_1]_{\mathcal{E}'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

and $$\left[\underline{\underline{R_2}}\right]_{\mathcal{E}''} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \quad \text{—— (4)}$$

Using vector addition, the angular velocity vector of the flywheel w.r.t. ground frame is given by

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} = \underline{\omega}_{\mathcal{E}/\mathcal{E}''} + \underline{\omega}_{\mathcal{E}''/\mathcal{E}'} + \underline{\omega}_{\mathcal{E}'/\mathcal{E}_0}$$

$$= \dot{\phi}\,\hat{e}_2'' + \dot{\theta}\,\hat{e}_1' + \dot{\psi}\,\hat{E}_2$$

From equation (1),

$$\hat{e}_1' = \underline{\underline{R_0}} \cdot \hat{E}_1$$

$$= \left[\underline{\underline{R_0}}\right]_{\mathcal{E}_0} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathcal{E}_0}$$

$$= \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{e}_1' = \begin{bmatrix} \cos\psi \\ 0 \\ -\sin\psi \end{bmatrix} \equiv (\cos\psi)\,\hat{E}_1 - (\sin\psi)\,\hat{E}_3 \quad \text{------- (5)}$$

From equation (2),

$$\hat{e}_2'' = \underline{\underline{R}}_1 \cdot \underline{\underline{R}}_0 \cdot \hat{E}_2$$

$$= \left[\underline{\underline{R}}_1\right]_{\mathcal{E}_0} \left[\underline{\underline{R}}_0\right]_{\mathcal{E}_0} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{\mathcal{E}_0} \quad \text{(in matrix form)}$$

$$= \left[\underline{\underline{R}}_0\right]_{\mathcal{E}_0} \left[\underline{\underline{R}}_1\right]_{\mathcal{E}'} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{\mathcal{E}_0} \quad \text{(from lectures)}$$

$$= \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\hat{e}_2'' = \begin{bmatrix} \sin\psi \sin\theta \\ \cos\theta \\ \cos\psi \sin\theta \end{bmatrix}_{\mathcal{E}_0} \equiv (\sin\psi \cdot \sin\theta)\,\hat{E}_1 + (\cos\theta)\,\hat{E}_2 + (\cos\psi \sin\theta)\,\hat{E}_3 \quad \text{------ (6)}$$

For $\psi = 30^\circ$, $\theta = 90^\circ$, using equations (5) & (6), we have,

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} = \dot{\psi}\hat{E}_2 + \dot{\theta}\left(\frac{\sqrt{3}}{2}\hat{E}_1 - \frac{1}{2}\hat{E}_3\right) + \dot{\phi}\left(\frac{1}{2}\hat{E}_1 + \frac{\sqrt{3}}{2}\hat{E}_3\right)$$