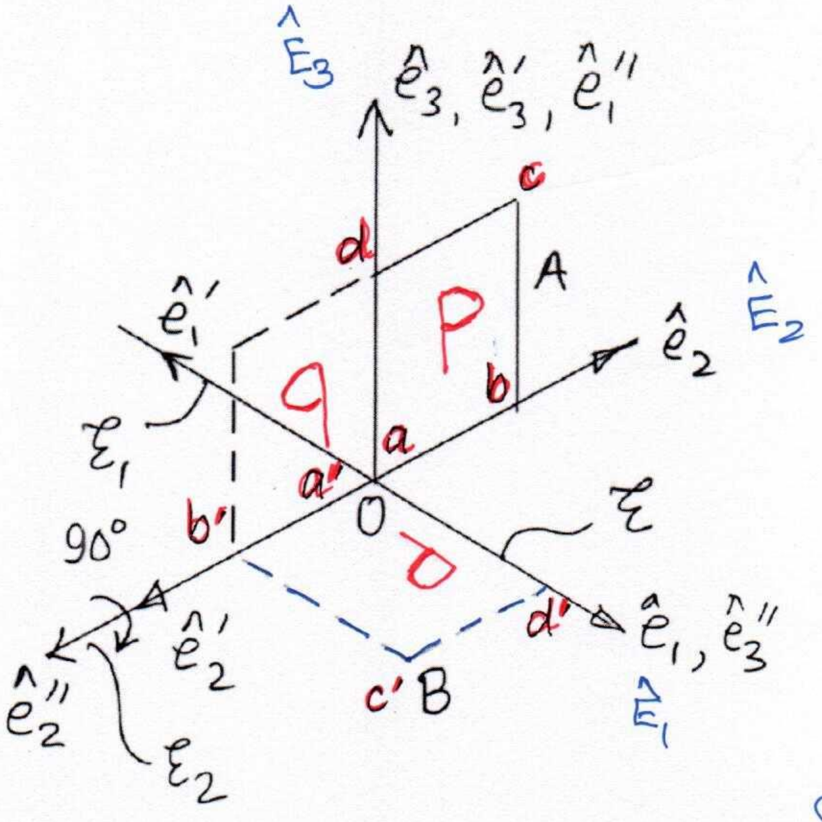
ESO 209

Tutorial 4, Problem #1

1)



The initial position of the body-fixed CS $\{\mathcal{B}, 0, \hat{e}_i\}$ is aligned to CS $\{E_{P_1}, 0, \hat{E}_i\}$. The final CS is $\{E_{P_2}, 0, \hat{e}_i''\}$. The rotation tensor for this is given by $\underline{\underline{R}}$ where the matrix of $\underline{\underline{R}}$ in E_{P_1} is given by

$$[\underline{\underline{R}}]_{E_{P_1}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The principal values of $\underline{\underline{R}}$ are

$$\lambda_1 = -1 \ , \quad \lambda_2 = -1 \ , \quad \lambda_3 = +1$$

and the corresponding principal vectors in E_{p_1} are

$$[\hat{v}_1]_{E_{p_1}} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad [\hat{v}_2]_{E_{p_1}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \qquad [\hat{v}_3]_{E_{p_1}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

We need to find the rotation axis $\hat{n}$.

From Euler's theorem,

$$\underline{\underline{R}} \cdot \hat{n} = \hat{n}$$

($\because$ the axis of rotation itself will be unaffected by the rotation tensor)

This corresponds to the principal value $\lambda_3 = 1$.

$\therefore$ the $\boxed{\hat{n} = \hat{v}_3 = \frac{1}{\sqrt{2}} \hat{e}_1 + \frac{1}{\sqrt{2}} \hat{e}_3}$

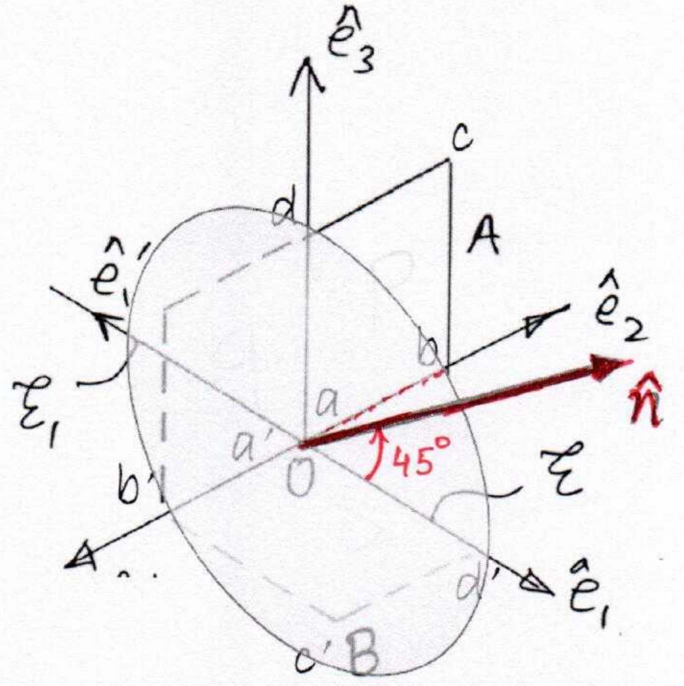
the angle θ of rotation is obtained as.

$$\theta = \cos^{-1}\left[\left\{ tr(\underline{\underline{R}}) - 1 \right\} / 2 \right]$$

$$= \cos^{-1}\left[\left\{ -1 - 1 \right\} / 2 \right] = \cos^{-1}(-1) = 180^\circ$$

$\therefore$ $\boxed{\theta = 180^\circ}$

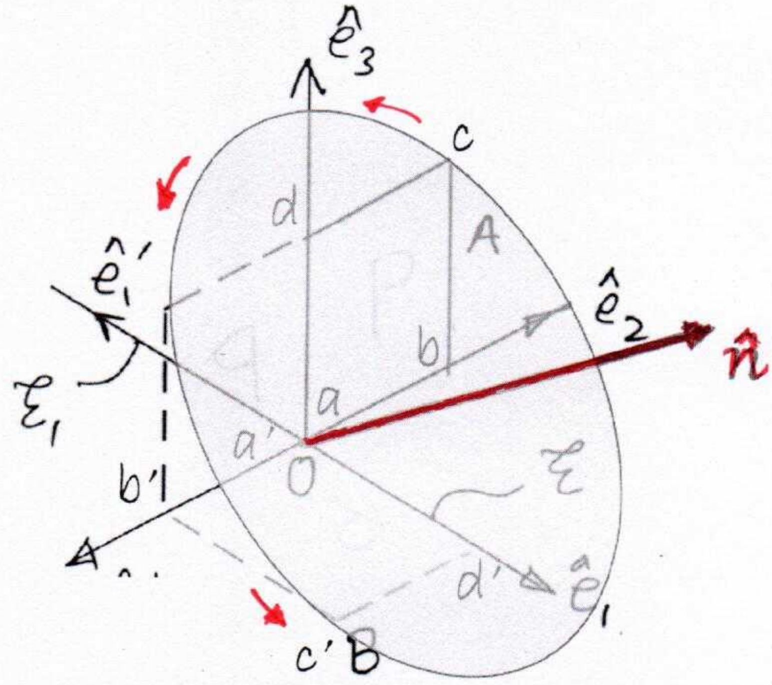
Checking to see if $\hat{n}$ & θ can be physically visualized.



we can try to imagine the rotation of the rectangle abcd to the rectangle a'b'c'd' through the rotations of individual points a', b', c', d'.

Imagine a circular disc of radius ab rotated by 45° about ab ie the axis $\hat{e}_2$. Now one can rotate ab along the plane of the disc by 180° and it will reach a'b'.

|||^{ly} for point c.



Imagine a circular disc parallel to the disc previously shown whose normal is $\hat{n}$. The disc contains the point c at the edge and the point c' is nothing but the other end of the diameter cc'. So c has rotated by ~~and~~ an angle $\theta = 180^\circ$.

And |||^{ly} one can imagine d and d' too.

Thus when the rectangle abcd is rotated by an angle $\theta = 180^\circ$ with the rotation axis $\hat{n}$, we get the rectangle a'b'c'd'