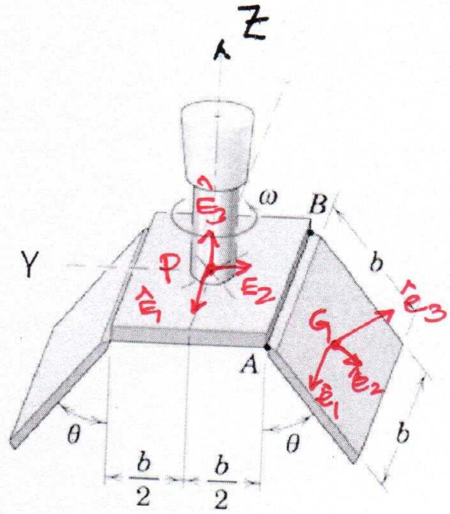
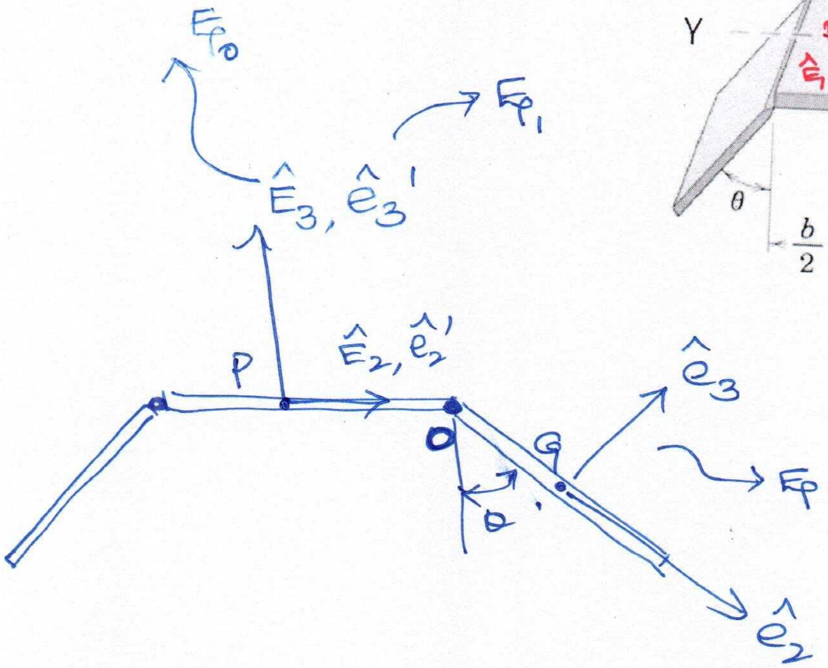
ESO 209.

Tutorial 8.

Problem 6





Three coordinate system.

1) $\{E_{\phi_0}, P, \hat{E}_i\}$ observer CS

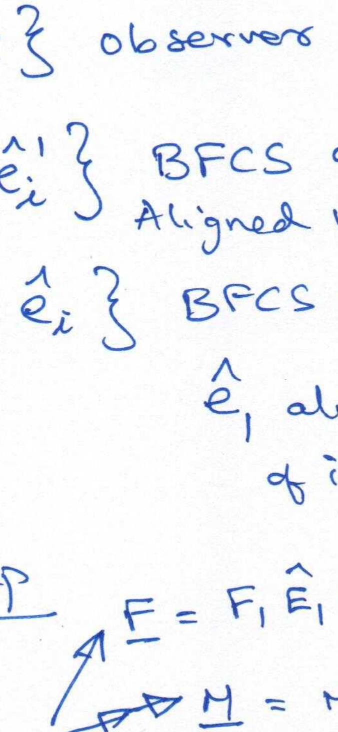
2) $\{E_{\phi_1}, P, \hat{e}_i{}'\}$ BFCS of Horizontal panel. Aligned with E_ϕ at instant of interest.

3) $\{E_\phi, G, \hat{e}_i\}$ BFCS of flap (G is at CM of flap)
$\hat{e}_1$ aligned with $\hat{E}_1$ at instant of interest.

FBD of flap

$\underline{F} = F_1\hat{E}_1 + F_2\hat{E}_2 + F_3\hat{E}_3$

$\underline{M} = M_1\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3$



Linear Momentum Balance.

LMB states

$$\Sigma \underline{F}^{i} = m\, \underline{a}^G$$

$$\text{LHS} \equiv F_1 \hat{E}_1 + F_2 \hat{E}_2 + F_3 \hat{E}_3 - mg\, \hat{E}_3$$

RHS

To find $\underline{a}^G$, we use the BFCS E_{P_1} in which POG is like a rigid body (θ = const)

$$\therefore \underline{a}^G = \underline{a}^P + \underline{\omega}^B \times (\underline{\omega}^B \times \underline{r}^{G/P}) + \underline{\alpha}^B \times \underline{r}^{G/P}$$

Since P is fixed, $\underline{a}^P = 0$

$$\underline{\alpha}^B = \frac{d}{dt}(\underline{\omega}^B) = \frac{d}{dt}(\omega \hat{E}_3) = 0$$

since both ω & $\hat{E}_3$ are constant

$$\underline{r}^{G/P} = \left(\frac{b}{2} + \frac{b}{2}\sin\theta\right)\hat{E}_2 - \frac{b}{2}\cos\theta\, \hat{E}_3$$

$$\therefore \underline{a}^G = \omega \hat{E}_3 \times \left[\omega \hat{E}_3 \times \left\{\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right)\hat{E}_2 - \frac{b}{2}\cos\theta\, \hat{E}_3\right\}\right]$$

$$= \omega \hat{E}_3 \times \left[-\frac{\omega b}{2}(1+\sin\theta)\hat{E}_1\right]$$

Thus, $\underline{a}^G = -\frac{\omega^2 b}{2}(1+\sin\theta)\hat{E}_2$

Thus equating LHS & RHS we get

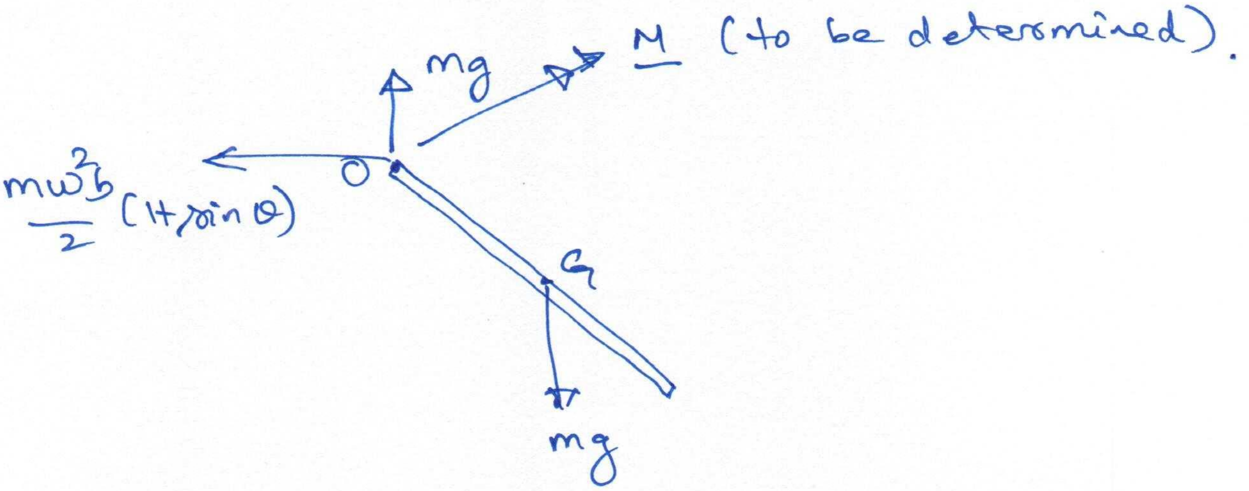
$$F_1 \hat{E}_1 + F_2 \hat{E}_2 + (F_3 - mg)\hat{E}_3 = -\frac{m\omega^2 b}{2}(1+\sin\theta)\hat{E}_2$$

Thus

$$F_1 = 0$$

$$F_2 = -\frac{m\omega^2 b}{2}(1+\sin\theta)$$

$$F_3 = mg.$$



Considering the AMB equation.

Considering AMB about P

$$\Sigma \underline{M}^P = \underline{r}^{G/P} \times m\underline{a}^G + \underline{\omega}^B \times (\underline{\underline{I}}_G \cdot \underline{\omega}^B) + \underline{\underline{I}}_G \cdot \underline{\alpha}^B$$

we have already established the following.

$$\underline{r}^{G/P} = \frac{b}{2}(1+\sin\theta)\hat{E}_2 - \frac{b}{2}\cos\theta\,\hat{E}_3$$

$$\underline{a}^G = -\frac{\omega^2 b}{2}(1+\sin\theta)\hat{E}_2$$

$$\underline{\omega}^B = \omega\hat{E}_3$$

$$\underline{\alpha}^B = 0$$

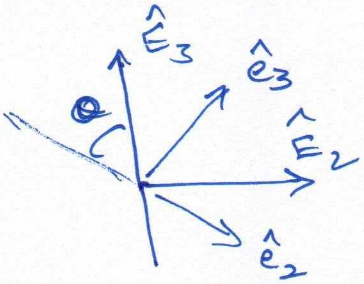
we need $\underline{\underline{I}}_G$

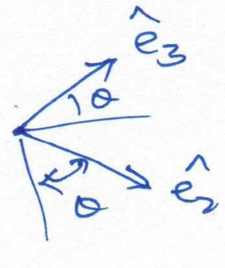
It is convenient to determine $\underline{\underline{I}}_G$ in the BFCS E_p where the axes are along the principal axes of the inertial tensor.

Using established moments of inertia of a plate about its axes of symmetry, we can say

$$[\underline{\underline{I}}_G]_{E_p} \doteq \begin{bmatrix} \frac{mb^2}{12} & 0 & 0 \\ 0 & \frac{mb^2}{12} & 0 \\ 0 & 0 & \frac{mb^2}{6} \end{bmatrix}$$

To find $\underline{\underline{I}}_G \cdot \underline{\omega}_B$, it thus helps to have $\underline{\omega}_B$ also in the BFCS E_p





$$\underline{\omega}_B = \omega \hat{E}_3$$

$$= \omega\left[-\cos\theta\, \hat{e}_2 + \sin\theta\, \hat{e}_3\right]$$

$$\therefore [\underline{\underline{I}}_G \cdot \underline{\omega}_B]_{E_p} = \begin{bmatrix} \frac{mb^2}{12} & 0 & 0 \\ 0 & \frac{mb^2}{12} & 0 \\ 0 & 0 & \frac{mb^2}{6} \end{bmatrix} \begin{bmatrix} 0 \\ -\omega\cos\theta \\ \omega\sin\theta \end{bmatrix}$$

$$\therefore \left[\underline{\underline{I}}_G \cdot \underline{\omega}_B\right]_{E_p} = \begin{bmatrix} 0 \\ -\dfrac{mb^2\omega}{12}\cos\alpha \\ \dfrac{mb^2\omega}{6}\sin\alpha \end{bmatrix}$$

ie $\underline{\underline{I}}_G \cdot \underline{\omega}_B = -\dfrac{mb^2\omega}{12}\cos\alpha\, \hat{e}_2 + \dfrac{mb^2\omega}{6}\sin\alpha\, \hat{e}_3$

Now to find $\underline{\omega}^B \times (\underline{\underline{I}}_G \cdot \underline{\omega}^B)$

Again using $\underline{\omega}^B = -\omega\cos\alpha\, \hat{e}_2 + \omega\sin\alpha\, \hat{e}_3$

$$\underline{\omega}^B \times (\underline{\underline{I}}_G \cdot \underline{\omega}^B) = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ 0 & -\omega\cos\alpha & \omega\sin\alpha \\ 0 & -\dfrac{mb^2\omega}{12}\cos\alpha & \dfrac{mb^2\omega}{6}\sin\alpha \end{vmatrix}$$

$$= \left[-\frac{mb^2\omega^2}{6}\sin\alpha\cos\alpha + \frac{mb^2\omega^2}{12}\sin\alpha\cos\alpha\right]\hat{e}_1$$

$$= -\frac{mb^2\omega^2}{12}\sin\alpha\cos\alpha\, \hat{e}_1$$

At the instant of interest $\hat{e}_1$ is aligned to $\hat{E}_1$

$$\therefore \underline{\omega}^B \times (\underline{\underline{I}}_G \cdot \underline{\omega}^B) = -\frac{mb^2\omega^2}{12}\sin\alpha\cos\alpha\, \hat{E}_1$$

2^{nd} term of RHS.

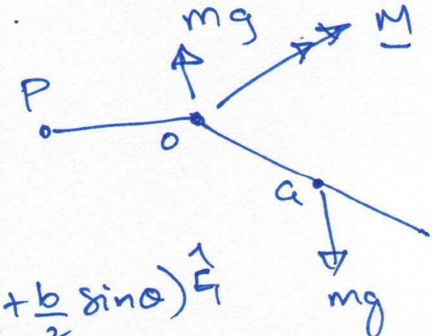
To find the 1st term of RHS

$$\underline{r}^{G/P} \times m\,\underline{a}^{G} = \begin{vmatrix} \hat{E}_1 & \hat{E}_2 & \hat{E}_3 \\ 0 & \frac{b}{2}(1+\sin\alpha) & -\frac{b}{2}\cos\alpha \\ 0 & -\frac{m\omega^2 b}{2}(1+\sin\alpha) & 0 \end{vmatrix}$$

$$= -\frac{m\omega^2 b^2}{4}(1+\sin\alpha)\cos\alpha\,\hat{E}_1$$

$$\therefore \text{RHS} \equiv -\frac{m\omega^2 b^2}{4}(1+\sin\alpha)\cos\alpha\,\hat{E}_1 - \frac{m b^2\omega^2}{12}\sin\alpha\cos\alpha\,\hat{E}_1$$

$$= \left[-\frac{m\omega^2 b^2}{4}\cos\alpha - \frac{m\omega^2 b^2}{3}\sin\alpha\cos\alpha\right]\hat{E}_1$$



$\underline{\underline{\text{LHS}}} \equiv \Sigma \underline{M}^{i}$ about P

$$= mg\cdot\frac{b}{2}\cdot\hat{E}_1 - mg\left(\frac{b}{2}+\frac{b}{2}\sin\alpha\right)\hat{E}_1 + \cancel{M_1}^{\,0}\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3$$

(because hinge)

$$= -\frac{mgb}{2}\sin\alpha\,\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3$$

Equating LHS & RHS.

$$-\frac{mgb}{2}\sin\theta\,\hat{E}_1 + M_2\hat{E}_2 + M_3\hat{E}_3 = \left[-\frac{m\omega^2 b^2}{4}\cos\theta - \frac{m\omega^2 b^2}{3}\sin\theta\cos\theta\right]\hat{E}_1$$

$$\Rightarrow \quad M_2 = 0 \;; \quad M_3 = 0$$

and
$$-\frac{\cancel{m}gb}{2}\sin\theta = -\cancel{m}\omega^2 b^2\cos\theta\left(\frac{1}{4} + \frac{\sin\theta}{3}\right)$$

$$\Rightarrow \quad \frac{6g\tan\theta}{b(3+4\sin\theta)} = \omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{6g\tan\theta}{b(3+4\sin\theta)}}$$