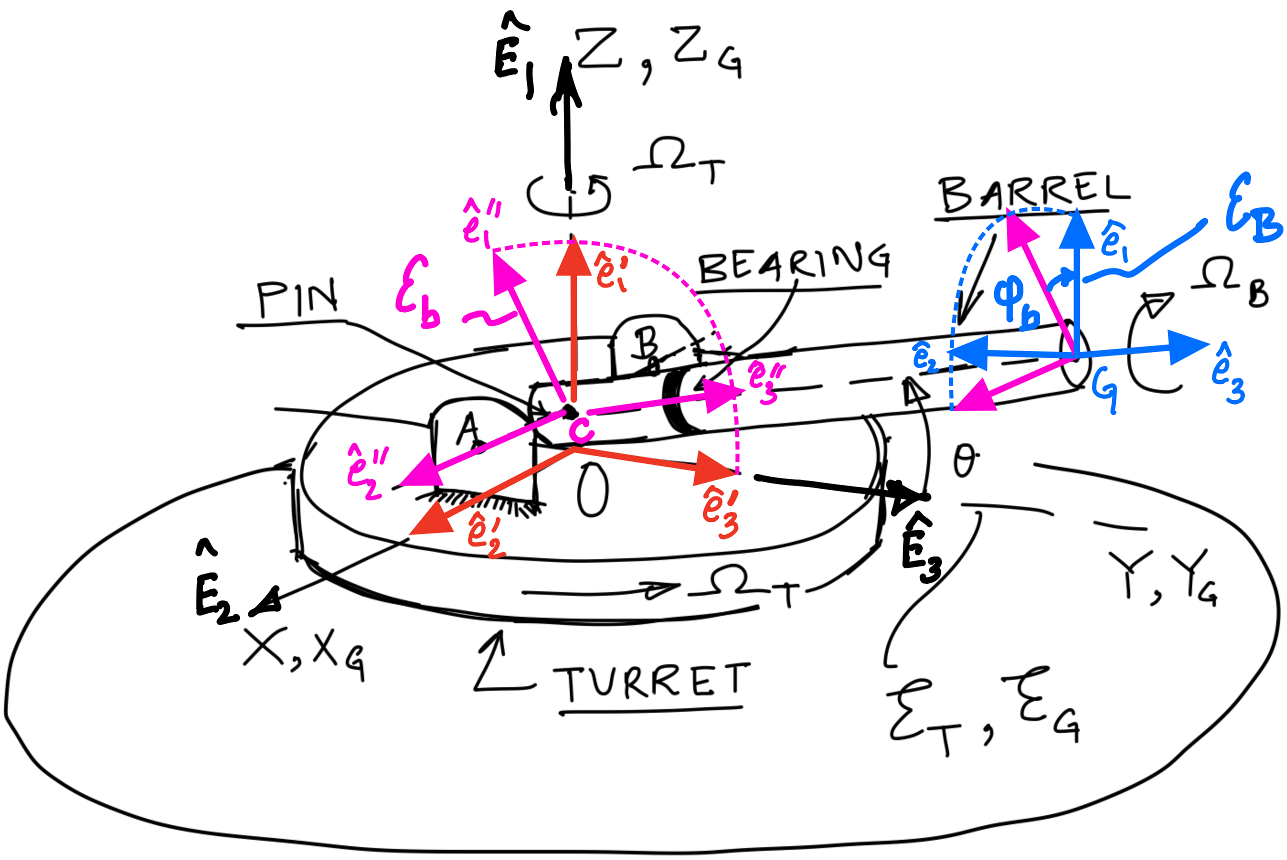
5. A barrel, spinning about its axis is mounted on a turret through trunnion pin AB. The turret itself is spinning about the ground Z_G axis, as shown in the figure. The barrel can rotate about the pin AB. For the given constant angular rates $\Omega_T = \Omega_B = 2\pi$ rad/sec and constant rate of elevation $\dot{\theta} = \dfrac{\pi}{4}$ rad/sec find the angular velocity and *angular acceleration* of the barrel as observed in the ground frame $\mathscr{E}_G$ at the instant when the turret frame $\mathscr{E}_T$ coincides with $\mathscr{E}_G$, and the barrel elevation in the YZ plane is $\theta = \pi/6$ radian.



Clearly,

$$
(*)\qquad
\begin{array}{ccc}
\{\mathcal{E}_G, O, \hat{E}_i\} & \xrightarrow{\underline{\underline{R}}(\hat{E}_1, \varphi_T)} & \{\mathcal{E}_T, C, \hat{e}_i'\} \\
 & & \Big\downarrow \underline{\underline{R}}(\hat{e}_2', \theta(t)) \\
\{\mathcal{E}_B, G, \hat{e}_i\} & \xleftarrow{\underline{\underline{R}}(\hat{e}_3'', \varphi_b)} & \{\mathcal{E}_b, C, \hat{e}_i''\}
\end{array}
$$

REMARKS

— Here the angle φ_T is not shown in the figure.

— The figure corresponds to a specific instance where we have taken $\mathcal{E}_T$ to coincide with $\mathcal{E}_G$.

— This does <u>NOT</u> make our solution any less general, because we are doing a velocity and acceleration analysis in which

(a) there is nothing special about the choice of the ground CS $\mathcal{E}_G$.

(b) the spin rates Ω_T, Ω_B & $\dot{\theta}$ do NOT depend upon the orientations φ_T.

From the flowchart (*) and the figure in which all CS are shown, we can write down the angular velocity of the barrel:

$$\underline{\omega}_B = \underline{\omega}_{\mathcal{E}_B/\mathcal{E}_G} = \underline{\omega}_{\mathcal{E}_B/\mathcal{E}_b} + \underline{\omega}_{\mathcal{E}_b/\mathcal{E}_T} + \underline{\omega}_{\mathcal{E}_T/\mathcal{E}_G}.$$

$$= -\Omega_B \hat{e}_3'' + \dot{\theta}\,\hat{e}_2' + \Omega_T \hat{E}_1 \qquad \text{——— (1)}$$

This is a MIXED vector and we will unmix it later to get the final answer in $\mathcal{E}_G$.

Meanwhile, the angular acceleration of the barrel:

$$\underline{\alpha}_B = \frac{d\underline{\omega}_B}{dt} = \frac{d}{dt}\left(-\Omega_B \hat{e}_3'' + \dot{\theta}\hat{e}_2' + \Omega_T \hat{E}_1\right)$$

(2) $$= -\Omega_B \dot{\hat{e}}_3'' + \dot{\theta}\dot{\hat{e}}_2'$$

- $\Omega_B, \dot{\theta}, \Omega_T$ are given to be constant
- $\hat{E}_i$ are fixed

How to compute $\dot{\hat{e}}_3''$ & $\dot{\hat{e}}_2'$?

We have shown in an earlier lecture that, for any unit vector $\hat{f}$ of a ROTATING CS $\mathcal{F}$,

$$\dot{\hat{f}} = \underline{\omega}_{\mathcal{F}/\mathcal{E}_G} \times \hat{f}$$

where $\underline{\omega}_{\mathcal{F}/\mathcal{E}_G}$ is the angular velocity of the CS $\mathcal{F}$ w.r.t. $\mathcal{E}_G$

Therefore, $$\left.\begin{aligned} \dot{\hat{e}}_2' &= \underline{\omega}_{\mathcal{E}_T/\mathcal{E}_G} \times \hat{e}_2' \\ \text{and} \quad \dot{\hat{e}}_3'' &= \underline{\omega}_{\mathcal{E}_b/\mathcal{E}_G} \times \hat{e}_3'' \end{aligned}\right\} \text{——— (3)}$$

From the flowchart, figure and inculating (1) we have

$$\left.\begin{aligned} \underline{\omega}_{\mathcal{E}_T/\mathcal{E}_G} &= \Omega_T \hat{E}_1 \\ \text{and} \quad \underline{\omega}_{\mathcal{E}_b/\mathcal{E}_G} &= \underline{\omega}_{\mathcal{E}_b/\mathcal{E}_T} + \underline{\omega}_{\mathcal{E}_T/\mathcal{E}_G} = \dot{\theta}\hat{e}_2' + \Omega_T \hat{E}_1 \end{aligned}\right\} (4)$$

Combining (3) & (4) and substituting in (2):

$$\underline{\alpha}_B = -\Omega_B\left(\dot\theta\,\hat{e}_2' + \Omega_T\hat{E}_1\right)\times\hat{e}_3'' + \dot\theta\,\Omega_T\hat{E}_1\times\hat{e}_2'$$

Finally, to compute the various cross-products we will use the CURRENT configuration shown:

$$\hat{e}_2'\times\hat{e}_3'' = \hat{e}_1'' \;;\; \hat{E}_1\times\hat{e}_3'' = -\cos\theta\,\hat{E}_2 \;;\; \hat{E}_1\times\hat{e}_2' = \hat{E}_3$$

Then, $\underline{\alpha}_B = -\Omega_B\dot\theta\,\hat{e}_1'' + \Omega_B\Omega_T\cos\theta\,\hat{E}_2 + \dot\theta\,\Omega_T\hat{E}_3$.

Now, again from the figure $\hat{e}_1'' = \cos\theta\,\hat{E}_1 - \sin\theta\,\hat{E}_3$.

$$\therefore \boxed{\begin{aligned}\underline{\alpha}_B &= -\Omega_B\dot\theta\cos\theta\,\hat{E}_1 + \Omega_B\Omega_T\cos\theta\,\hat{E}_2\\ &\quad + \dot\theta\left(\Omega_B\sin\theta + \Omega_T\right)\hat{E}_3\end{aligned}} \qquad (5)$$

Similarly, we can express the barrel's angular velocity in the CURRENT configuration:

$$\boxed{\begin{aligned}\underline{\omega}_B &= -\Omega_B\underbrace{\left(\cos\theta\,\hat{E}_3 + \sin\theta\,\hat{E}_1\right)}_{\hat{e}_3''} + \dot\theta\,\hat{E}_2 + \Omega_T\hat{E}_1\\ &= \left(-\Omega_B\sin\theta + \Omega_T\right)\hat{E}_1 + \dot\theta\,\hat{E}_2 - \Omega_B\cos\theta\,\hat{E}_3\end{aligned}} \qquad (6)$$

QUESTION: Can you get $\underline{\alpha}_B$ in (5) by differentiating $\underline{\omega}_B$ in (6)? If not, then why not?