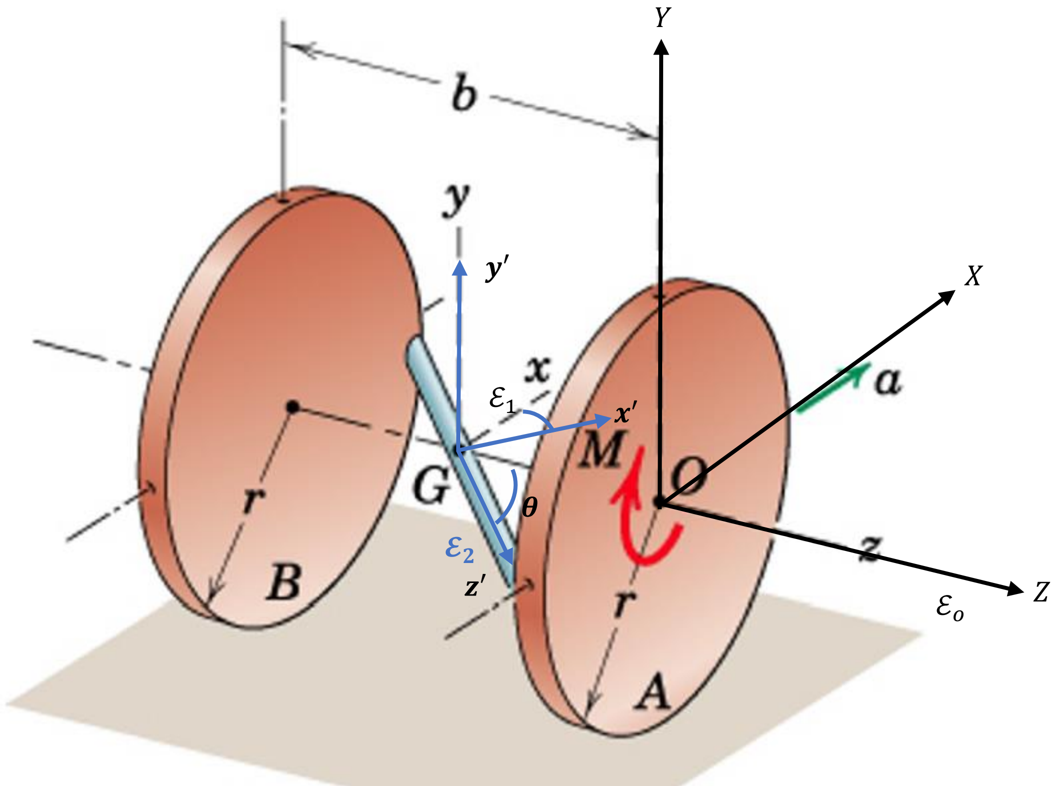
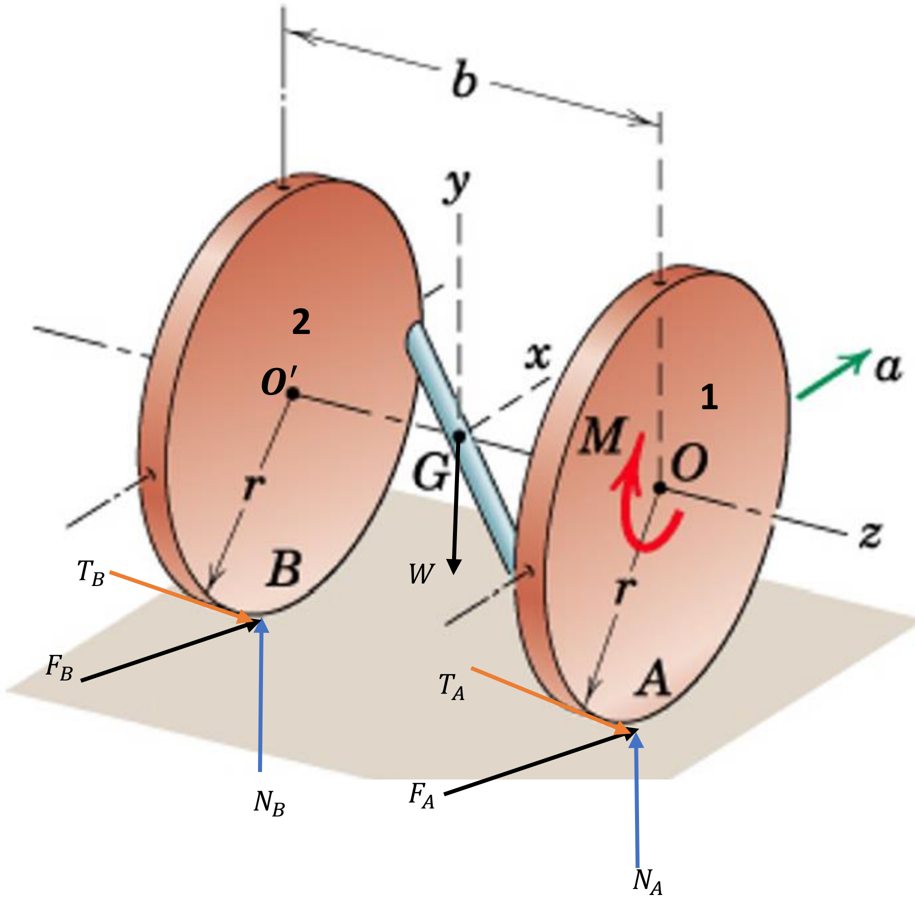
Problem 3:



At the instant shown, ground fixed BFCS $\{\mathcal{E}_o, O, \hat{E}_i\}$, BFCS $\{\mathcal{E}_1, G, \hat{e}_i\}$ is attached at point G. At the instant shown we assumed that this $\mathcal{E}_1$ and $\mathcal{E}_o$ are parallel. One more BFCS for inclined rod $\{\mathcal{E}_2, G, \hat{e}_i'\}$.

Information given in the problem statement, Two circular discs, each of mass m_1. Mass of rod is m_2. The assembly is at rest when a Torque M is applied to a disc. Free body diagram of assembly is drawn.



Given $\underline{a}_G = a\hat{e}_1$

At point A and B, where discs are in contact with ground, we assume that motion is pure rolling. Therefor tangential components of acceleration at A and B. since, assembly starts from rest therefore $\underline{\omega}_{system} = 0$ and normal components of acceleration will also be zero. Therefore $\underline{a}_A = \underline{a}_B = 0$.

We know $\underline{\alpha}_{system} = \alpha\hat{e}_3$ and $\underline{a}_G = a\hat{e}_1$. Where α can be given as $\frac{a}{r}$ from condition of no slip.

To obtain the ground reactions apply LMB and AMB.

1. Apply LMB

$$\sum \underline{F}_{ext} = m_{system}\,\underline{a}_G$$

$$F_A\hat{e}_1 + N_A\hat{e}_2 + T_A\hat{e}_3 + F_B\hat{e}_1 + N_B\hat{e}_2 + T_B\hat{e}_3 - W\hat{e}_2 = (2m_1 + m_2)a\hat{e}_1$$

Where $W = (2m_1 + m_2)g$

Now comparing coeff. of $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$.

$$F_A + F_B = (2m_1 + m_2)a \ldots\ldots (1)$$

$$N_A + N_B = (2m_1 + m_2)g \ldots\ldots (2)$$

$$T_A + T_B = 0 \ldots\ldots (3)$$

2. Apply AMB

Since $\underline{\omega}_{system} = 0, \underline{a}_A = \underline{a}_B = 0$ therefore writing total moment of point A

$$\underline{M}^A = \underline{\underline{I}}^A \cdot \underline{\alpha}_{system}$$

$$-F_B b\hat{e}_2 + N_B b\hat{e}_1 - W\frac{b}{2}\hat{e}_1 - M\hat{e}_3 = \underline{\underline{I}}^A \cdot \alpha\hat{e}_3$$

Now comparing components of $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$.

$$N_B b - (2m_1 + m_2)g\frac{b}{2} = I_{A,xz}\alpha \ldots\ldots (4)$$

$$-F_B b = I_{A,yz}\alpha \ldots\ldots (5)$$

$$-M = I_{A,zz}\alpha \ldots\ldots (6)$$

To obtain ground reactions N_A, N_B, F_A and F_B, we can solve Eq. (1), (2), (4) and (5). For that we need $I_{A,xz}$ and $I_{A,yz}$.

Now calculating $I_{A,xz}$ and $I_{A,yz}$.

$$I_{A,xz} = I_{A,xz}\big|_{disc\ 1} + I_{A,xz}\big|_{disc\ 2} + I_{A,xz}\big|_{rod} \ldots\ldots (7)$$

$$I_{A,yz} = I_{A,yz}\big|_{disc\ 1} + I_{A,yz}\big|_{disc\ 2} + I_{A,yz}\big|_{rod} \ldots\ldots (8)$$

Now obtaining mass moment of inertia tensor at point A. for that we can use parallel axis theorem.

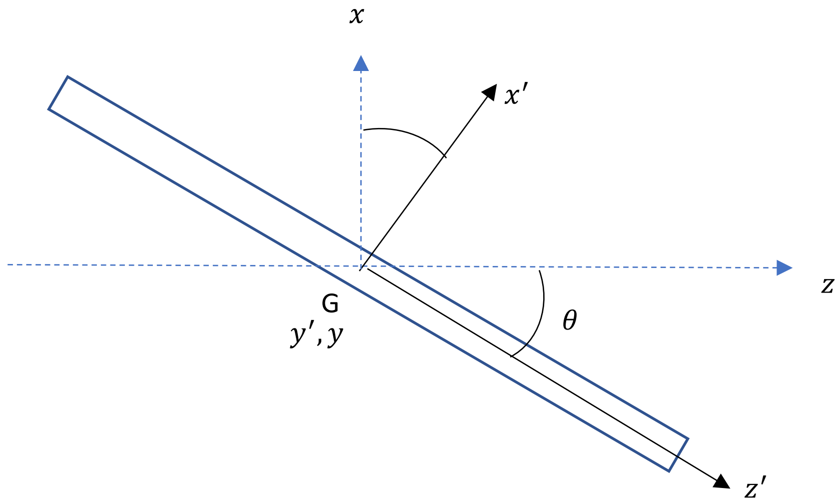
For disc 1 at point O

$$[I]_{O,disc\ 1} = \frac{m_1 r^2}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \ldots\ldots (9)$$

For disc 2 at point O′

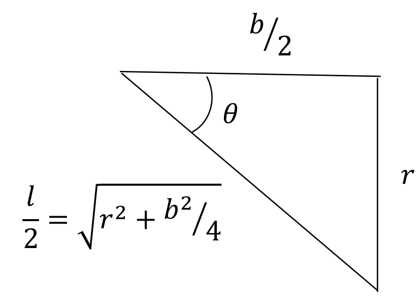
$$[I]_{O',disc\ 2} = \frac{m_1 r^2}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \ldots\ldots (10)$$

For the rod at point G



$$\theta = \tan^{-1}\left(\frac{2r}{b}\right), \sin\theta = \frac{r}{\sqrt{r^2 + {b^2}/{4}}}, \cos\theta = \frac{{b}/{2}}{\sqrt{r^2 + {b^2}/{4}}},$$

here l is assumed length of rod.



We know

$$[I]_{G,rod}\Big|_{\mathcal{E}_2} = \frac{m_2 l^2}{12}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ldots\ldots (11)$$

To get $[I]_{G,rod}\Big|_{\mathcal{E}_1}$, we will use rotation matrix $R\{\hat{e}_2, \theta\}$. where

$$[R]_{\mathcal{E}_2} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Then

$$[I]_{G,rod}\Big|_{\mathcal{E}_1} = [R]_{\mathcal{E}_2}^T [I]_{G,rod}\Big|_{\mathcal{E}_2} [R]_{\mathcal{E}_2}$$

$$[I]_{G,rod}\Big|_{\mathcal{E}_1} = \frac{m_2 l^2}{12}\begin{bmatrix} (\cos\theta)^2 & 0 & \sin\theta\cos\theta \\ 0 & 1 & 0 \\ \sin\theta\cos\theta & 0 & (\sin\theta)^2 \end{bmatrix} \ldots\ldots (12)$$

To get $\underline{\underline{I}}^A$ we will use now parallel axis theorem

For disc 1

$$\underline{\underline{I}}^{A}\Big|_{disc\ 1} = \underline{\underline{I}}^{O}\Big|_{disc\ 1} + m_1\left(\left|\underline{r}^{O/A}\right|^2 \underline{\underline{1}} - \underline{r}^{O/A} \otimes \underline{r}^{O/A}\right)$$

Where $\underline{r}^{O/A} = -r\hat{e}_2$

$$[I^A]_{disc\ 1} = \frac{m_1 r^2}{4}\begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \ldots\ldots (13)$$

For disc 2

$$\underline{\underline{I}}^{A}\Big|_{disc\ 2} = \underline{\underline{I}}^{O'}\Big|_{disc\ 2} + m_1\left(\left|\underline{r}^{O'/A}\right|^2 \underline{\underline{1}} - \underline{r}^{O'/A} \otimes \underline{r}^{O'/A}\right)$$

Where $\underline{r}^{O'/A} = b\hat{e}_3 - r\hat{e}_2$

$$[I^A]_{disc\ 2} = \frac{m_1 r^2}{4}\begin{bmatrix} 1 - \frac{4}{r^2}(b^2 + r^2) & 0 & 0 \\ 0 & 1 - \frac{4b^2}{r^2} & -\frac{4b}{r} \\ 0 & -\frac{4b}{r} & -2 \end{bmatrix} \ldots\ldots (14)$$

For rod

$$\underline{\underline{I}}^{A}\Big|_{rod} = \underline{\underline{I}}^{G}\Big|_{rod} + m_2\left(\left|\underline{r}^{G/A}\right|^2 \underline{\underline{1}} - \underline{r}^{G/A} \otimes \underline{r}^{G/A}\right)$$

Where $\underline{r}^{G/A} = \frac{b}{2}\hat{e}_3 - r\hat{e}_2$

$$[I^A]_{rod} = \frac{m_2}{6}\begin{bmatrix} -b^2 - 6r^2 & 0 & br \\ 0 & 2r^2 - b^2 & 3br \\ br & 3br & -4r^2 \end{bmatrix} \ldots\ldots (15)$$

Use Eq. (13), (14) and (15) into Eq. (7) and (8)

$$I_{A,xz} = \frac{m_2 br}{6}, I_{A,yz} = -m_1 br + \frac{m_2 br}{2}$$

Now use Eq. (1), (2), (4) and (5)

$$N_A = m_1 g + \frac{m_2}{6}(3g + a)$$

$$N_B = m_1 g + \frac{m_2}{6}(3g - a)$$

$$F_A = m_1 a + \frac{3m_2 a}{2}$$

$$F_B = m_1 a - \frac{m_2 a}{2}$$

Assume there is no resisting force in z direction then from Eq. (3), $T_A = T_B = 0$.