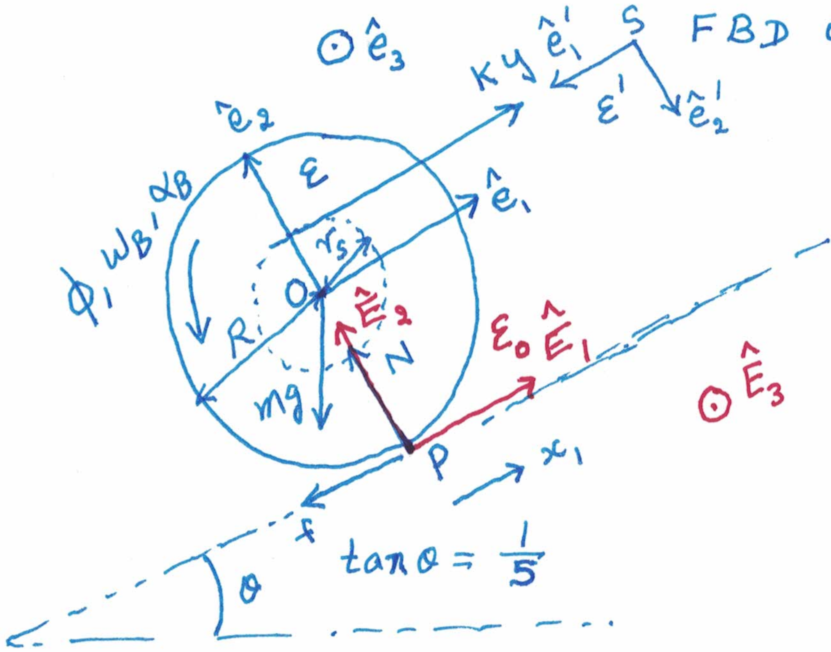
TUTORIAL-9, PROBLEM-5



FBD of double wheel

$\{\mathcal{E}_0, P, \hat{E}_i\}$ Observer frame (Inertial)

$\{\mathcal{E}, O, \hat{e}_i\}$ body-fixed frame

Define $\underline{\omega}^B = \omega_B \hat{E}_3$, $\underline{\alpha}^B = \alpha_B \hat{E}_3$

This is a 2-D planar problem. The instant shown in the figure corresponds to maximum stretch position $\left(y_0 = \frac{9R}{8}\right)$. From this point, the wheel is released. x_1 is the distance travelled by the center O along the incline in the $\mathcal{E}_0$ frame, starting from this time instant.

Velocity analysis, $\underline{V}^O = v\hat{E}_1 = \cancel{\underline{V}^{P_W}} + \underline{\omega}^B \times \underline{r}^{O/P}$

$$= -\omega_B R \hat{E}_1$$

Thus, $v = -\omega_B R$

2D planar body, using perpendicular axes theorem,

$$\left[\underline{\underline{I}}^{O}\right]_{\mathcal{E}} = m r_g^2 \begin{bmatrix} 1/2 & & 0 \\ & 1/2 & \\ 0 & & 1 \end{bmatrix}$$

From rolling condition,

$$\underline{v}^{P_W} = \underline{v}^{P_i} = 0$$

Frictional force does ~~no work~~ not do any work.

Also, no moments acting on the rigid body,

$$\underline{M}^{j} = 0$$

$$\underline{F}^{1} = -ky \, \hat{e}_1' = -\nabla\underbrace{\left(+\tfrac{1}{2}ky^2\right)}_{U_1}$$

(energy stored in the spring)

$$\underline{F}^{2} = -mg\sin\theta \, \hat{E}_1 \qquad (\underbrace{mg\cos\theta \perp \underline{v}^{O}}_{\text{no work}})$$

$$= -\nabla\underbrace{\left(mg x_1 \sin\theta\right)}_{U_2}$$

Considering energy balance,

$$E_K + \sum U_i = \text{constant.}$$

Kinetic energy,

$$E_K = \frac{1}{2} m \underline{v}^o \cdot \underline{v}^o + \frac{1}{2} \underline{\omega}^B \cdot \underline{\underline{I}}^o \cdot \underline{\omega}^B$$

$$= \frac{1}{2} m \omega_B^2 R^2 + \frac{1}{2} m r_g^2 \omega_B^2$$

Thus,

$$\frac{1}{2} m v^2 + \frac{1}{2} m \frac{r_g^2}{R^2} v^2 + m g x_1 \sin\theta + \frac{1}{2} k y^2 = \text{const.}$$

When center 'o' moves by a distance x_1, the wheel would have rotated through $\frac{x_1}{R}$ and the cord would have unwinded by $\frac{r_s x_1}{R}$. Thus,

$$r_s = \frac{3R}{8}$$

$$y = y_0 - \left(x_1 + \frac{r_s x_1}{R}\right)$$

$$y = y_0 - \frac{11 x_1}{8}$$

Initial energy is $+\frac{1}{2} k y_0^2$ and energy is conserved.

$$\frac{1}{2} m v^2 + \frac{1}{2} \frac{m r_g^2}{R^2} v^2 + m g x_1 \sin\theta + \frac{1}{2} k y^2 = +\frac{1}{2} k y_0^2$$

$\Rightarrow$

$$v^2 \frac{m(R^2+r_g^2)}{2R^2} + mgx_1\sin\theta + \cancel{\frac{1}{2}ky_0^2} - ky_0 \cdot \frac{11x_1}{8} + \frac{1}{2}k\cdot\left(\frac{11x_1}{8}\right)^2 = +\cancel{\frac{1}{2}ky_0^2}$$

$$\Rightarrow \frac{m(R^2+r_g^2)}{2R^2} v^2 = -\frac{121\,x_1^2\,k}{128} + ky_0 \cdot \frac{11x_1}{8} - mgx_1\sin\theta \quad \text{---- eq 1}$$

For maximum v, $\frac{d(v^2)}{dx_1} = 0$

$$\frac{121k}{64}x_1 - \frac{99R}{64}k + mg\sin\theta = 0 \quad \text{---- eq 2}$$

$$x_1 = \frac{99R}{121} - \frac{64\,mg\sin\theta}{121\,K}$$

$$x_1 = \frac{1}{121}\left(99R - \frac{64}{K}mg\sin\theta\right)$$

Rewriting eq. 1,
~~Substituting for x_1 in eq. 1~~

$$\frac{m(R^2+r_g^2)}{2R^2} v^2 = x_1 \overbrace{\left(\frac{99RK}{64} - \frac{121K}{64}x_1 - mg\sin\theta\right)}^{0} + \frac{121Kx_1^2}{128}$$

(before substitution making use of eq. 2 here)

$$\frac{m(R^2+r_g^2)}{2R^2} v^2 = \frac{121Kx_1^2}{128} \quad \text{---- eq 3}$$

Now, substituting for x_1 in eq. 3,

$$v^2 = \frac{\cancel{2}R^2}{m(R^2+r_g^2)} \cdot \frac{\cancel{121K}}{\underset{64}{\cancel{128}}} \frac{(99RK - 64mg\sin\theta)^2}{(121K)^{\cancel{2}}}$$

$$v_{max} = \left(\frac{99RK - 64mg\sin\theta}{88}\right)\frac{R}{\sqrt{mK(R^2+r_g^2)}}$$

Answer.