Lecture 11

Rigid body kinematics: Velocity analysis examples; Five-term acceleration formula; Applications

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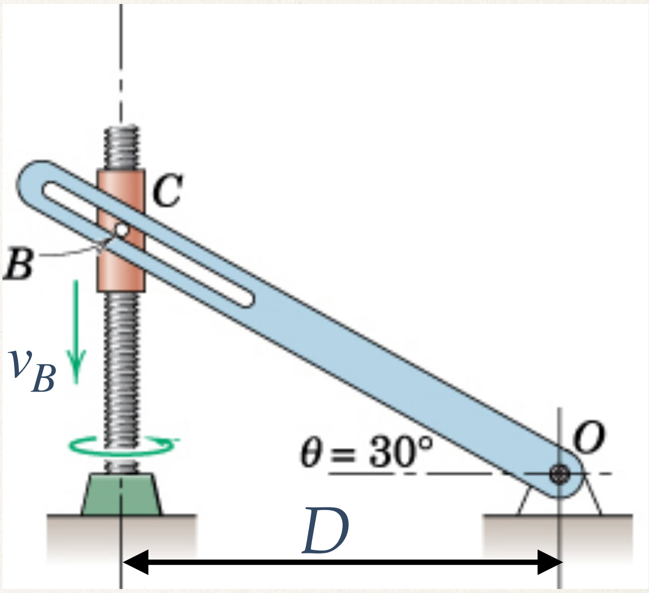
1-7 September, 2021

Example 1

Find the angular velocity of the slotted arm

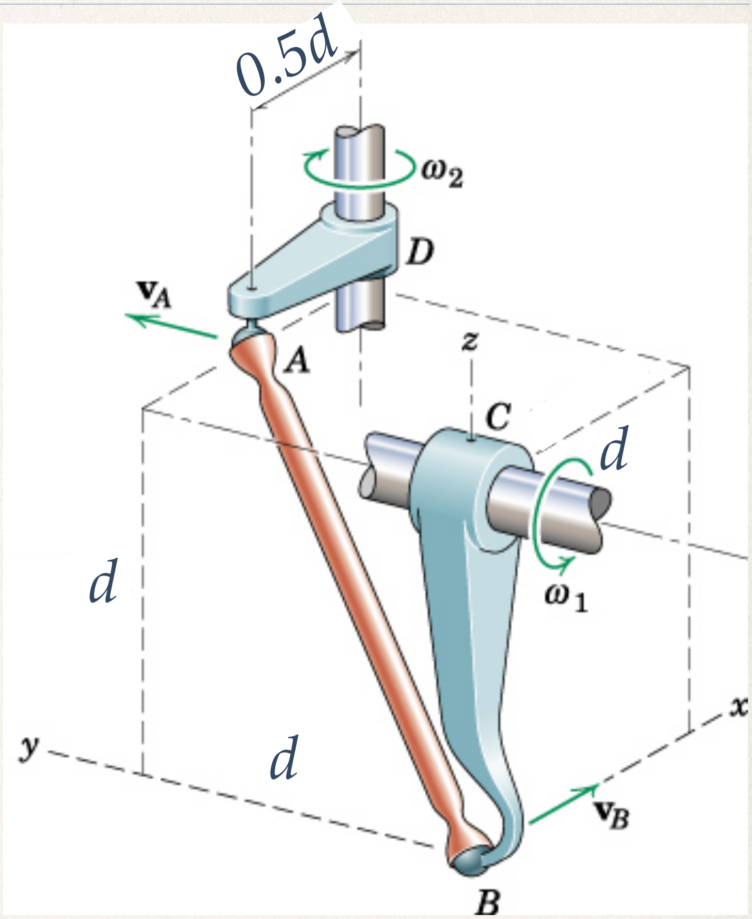


General strategy for systems with connected rigid bodies:

1. Connections impose <u>*kinematic constraints*</u>.
2. Usually partial kinematic information is given for each rigid body. *For example,* slotted arm rotates in plane; point *O* on arm is fixed; collar C moves vertically.
3. Find a convenient point whose motion is found in <u>two</u> ways, say *w.r.t.* two rigid bodies, and equate. *For example,* point *B* is on collar *C* and velocity $\mathbf{v}_B$ is known. But $\mathbf{v}_B$ can also be found by sitting in the slotted arm's *BFCS*. Comparing the two expressions of $\mathbf{v}_B$ solves the problem.

Example 2

Given ω_1, find ω_2 and $\boldsymbol{\omega}^{AB}$ in this configuration.



Note: Even when stationary, link AB can rotate in an arbitrary manner about AB, because of spherical joints at hinges A and B.

Implication: We can write

$$\boldsymbol{\omega}^{AB} = \underbrace{\left(\boldsymbol{\omega}^{AB} \cdot \hat{\mathbf{n}}_{AB}\right) \hat{\mathbf{n}}_{AB}}_{\boldsymbol{\omega}_a^{AB},\ \text{along } AB} + \underbrace{\hat{\mathbf{n}}_{AB} \times \left(\boldsymbol{\omega}^{AB} \times \hat{\mathbf{n}}_{AB}\right)}_{\boldsymbol{\omega}_n^{AB},\ \text{normal to } AB}$$

1. The **Note** above says that we can not expect to find $\boldsymbol{\omega}_a^{AB} = \left(\boldsymbol{\omega}_{AB} \cdot \hat{\mathbf{n}}_{AB}\right) \hat{\mathbf{n}}_{AB}$.
2. Will only find $\boldsymbol{\omega}_n^{AB} = \hat{\mathbf{n}}_{AB} \times \left(\boldsymbol{\omega}_{AB} \times \hat{\mathbf{n}}_{AB}\right)$.

Acceleration analysis

Relating acceleration of point *P* in two CS.

1. *P*'s acceleration is $\mathbf{a}^P_{\mathscr{E}}$ ($=:$ $\mathbf{a}^P_{rel}$) *w.r.t.* rigid body $\mathscr{B}$ with BFCS $\{\mathscr{E}(t), G, \hat{\mathbf{e}}_i(t)\}$.
2. $\mathscr{B}$ rotates at $\boldsymbol{\omega}_{\mathscr{B}} := \boldsymbol{\omega}_{\mathscr{E}/\mathscr{E}_0}$ and $\boldsymbol{\alpha}_{\mathscr{B}} = \boldsymbol{\alpha}_{\mathscr{E}/\mathscr{E}_0}$ *w.r.t.* CS $\{\mathscr{E}_0, O, \hat{\mathbf{E}}_i\}$.

Find *P*'s acceleration *w.r.t.* $\mathscr{E}_0$, *i.e.* $\mathbf{a}^P_{\mathscr{E}_0} =: \mathbf{a}^P$

In general, $\mathscr{E}_0$ and $\mathscr{E}$ have *<u>different</u>* origins:

a. $\mathscr{E}$'s origin is at $\mathbf{r}^{G/O}(t)$ *w.r.t.* *O*, and

b. $\mathscr{E}$'s origin has acceleration $\mathbf{a}^G_{\mathscr{E}_0} =: \mathbf{a}^G$:

$$\boxed{\begin{aligned}\mathbf{a}^P = \mathbf{a}^P_{rel} + \boldsymbol{\omega}_{\mathscr{B}} \times \left(\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{r}^{P/G}\right) \ \ldots \\ \ldots + \boldsymbol{\alpha}_{\mathscr{B}} \times \mathbf{r}^{P/G} + 2\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{v}^P_{rel} + \mathbf{a}^G .\end{aligned}}$$

c. $\mathbf{a}^G = 0$, if $\mathscr{E}_0$ and $\mathscr{E}$ have *<u>same</u>* origins.

"Five-term" formula

$$\mathbf{a}^P = \mathbf{a}^P_{rel} + \boldsymbol{\omega}_{\mathscr{B}} \times \left(\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{r}^{P/G}\right) + \boldsymbol{\alpha}_{\mathscr{B}} \times \mathbf{r}^{P/G} + \ldots$$
$$\ldots + 2\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{v}^P_{rel} + \mathbf{a}^G .$$

Acceleration $\mathbf{a}^P$ of P in $\{\mathscr{E}_0, O, \hat{\mathbf{E}}_i\}$ is found by measuring P's motion in $\{\mathscr{E}(t), G, \hat{\mathbf{e}}_i(t)\}$ which rotates at $\boldsymbol{\omega}_{\mathscr{B}}$ and $\boldsymbol{\alpha}_{\mathscr{B}}$ *and* translates *w.r.t.* $\mathscr{E}_0$.

I. FIVE terms in above formula:

1. $\mathbf{a}^P_{rel}$: P's acceleration *measured in* $\mathscr{E}$.
2. $\boldsymbol{\omega}_{\mathscr{B}} \times \left(\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{r}^{P/G}\right)$: *Centripetal* acceleration
3. $\boldsymbol{\alpha}_{\mathscr{B}} \times \mathbf{r}^{P/G}$: *Angular* acceleration
4. $2\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{v}^P_{rel}$: *Coriolis* acceleration
5. $\mathbf{a}^G$: G's acceleration *measured in* $\mathscr{E}_0$.

II. $\boldsymbol{\omega}_{\mathscr{B}} \times \left(\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{r}^{P/G}\right) + \boldsymbol{\alpha}_{\mathscr{B}} \times \mathbf{r}^{P/G}$: Measurement in rotating $\mathscr{E}$ misses obvious rotational effects

III. $2\,\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{v}^P_{rel}$: Two sources — measurement of *rate of change* of $\mathbf{v}^P_{rel}$ and $\mathbf{r}^{P/G}$ in rotating $\mathscr{E}$.

Application

I. **Example 1**. *Relating accelerations of two points on the same rigid body*. Let A and B be points on a rigid body $\mathscr{B}$, rotating at $\boldsymbol{\omega}_{\mathscr{B}}$ and $\boldsymbol{\alpha}_{\mathscr{B}}$ *w.r.t.* $\{\mathscr{E}_0, O, \hat{\mathbf{E}}_i\}$. Let $\mathbf{a}^A$ be acceleration of A in $\mathscr{E}_0$.

Find $\mathbf{a}^B$ measured *w.r.t.* $\mathscr{E}_0$.

Answer: $\mathbf{a}^B = \mathbf{a}^A + \boldsymbol{\omega}_{\mathscr{B}} \times \left(\boldsymbol{\omega}_{\mathscr{B}} \times \mathbf{r}^{B/A}\right) + \boldsymbol{\alpha}_{\mathscr{B}} \times \mathbf{r}^{B/A}$

II. **Example 2**. Find acceleration of the rod's end.

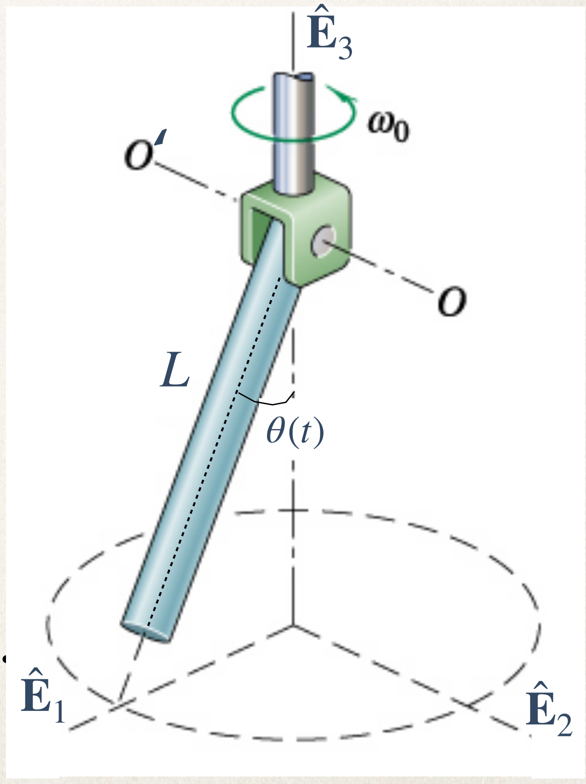
Answer:

$$\Big\{ -\left(\omega_0^2 + \dot{\theta}^2\right)\sin\theta + \ldots$$

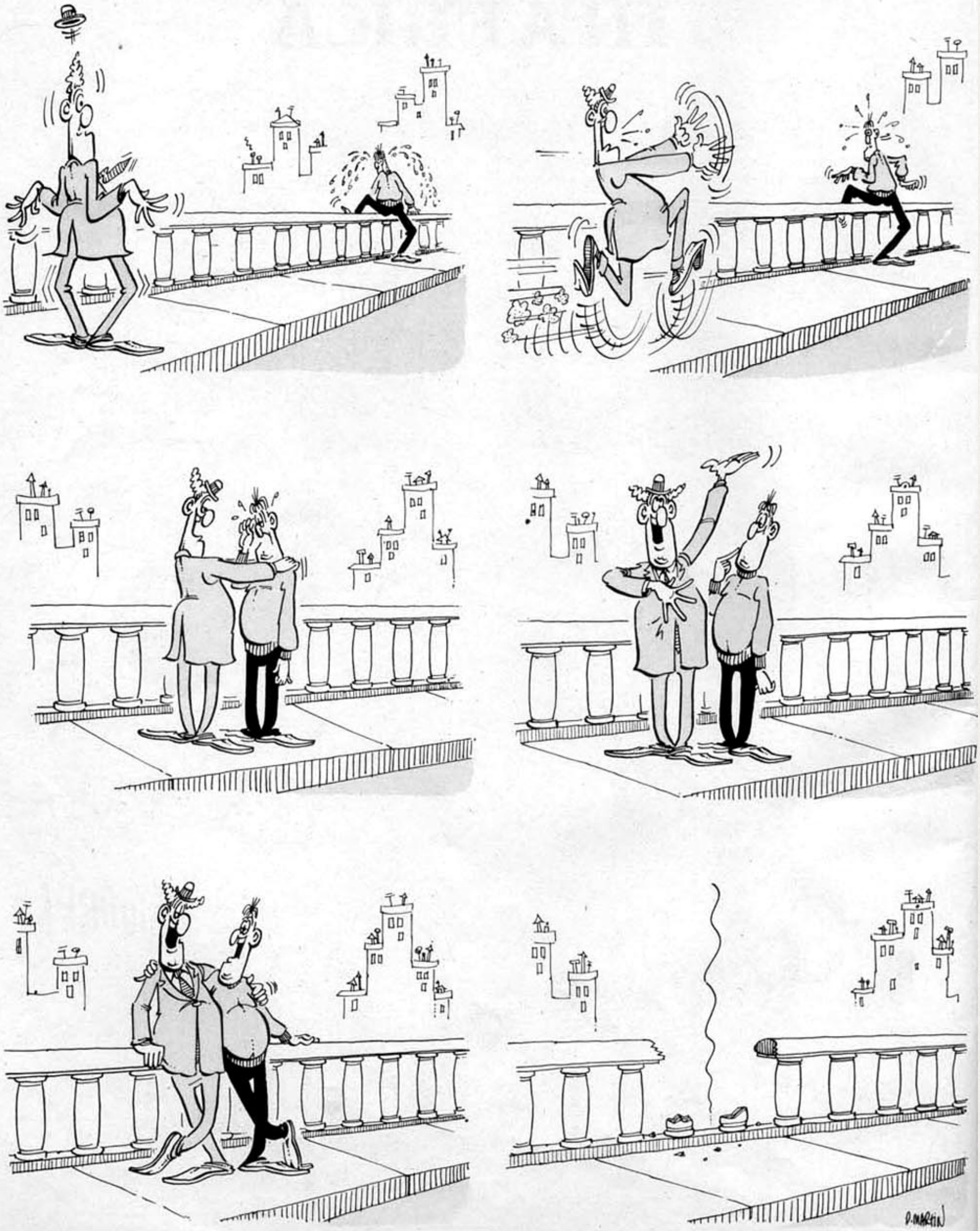
$$\ddot{\theta}\cos\theta \Big\} L\hat{\mathbf{E}}_1 + \ldots$$

$$2\omega_0\dot{\theta}L\cos\theta\hat{\mathbf{E}}_2 + \ldots$$

$$\left(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right)L\hat{\mathbf{E}}_3 .$$



ONE DAY ON THE BRIDGE



Don’t cheat, nor allow others.
Be careful who you help/know.