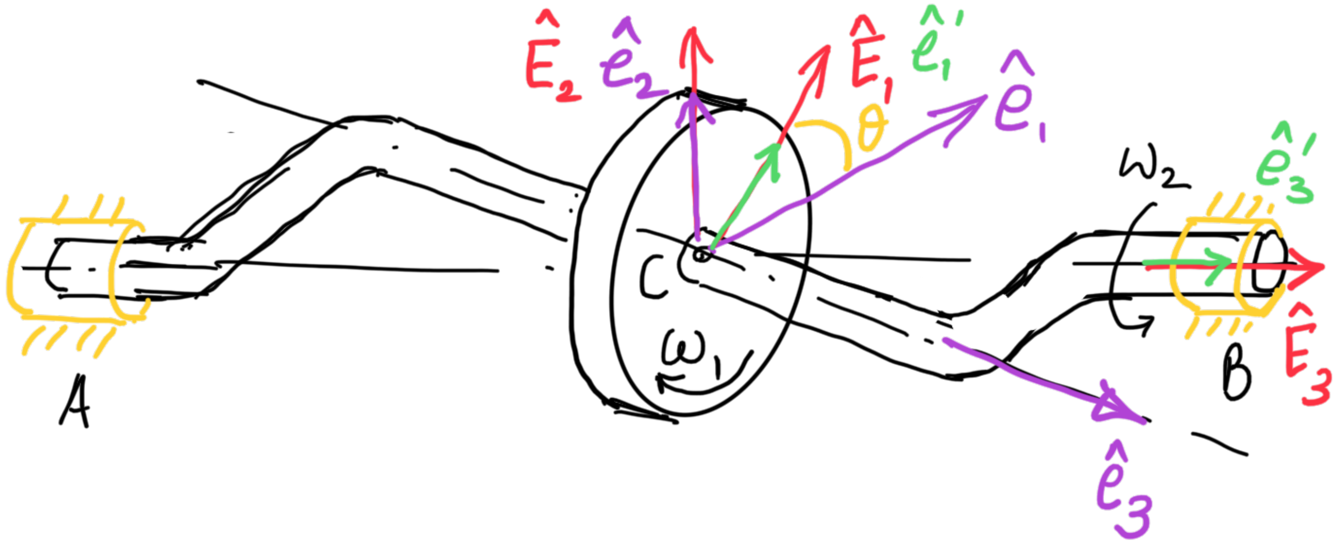
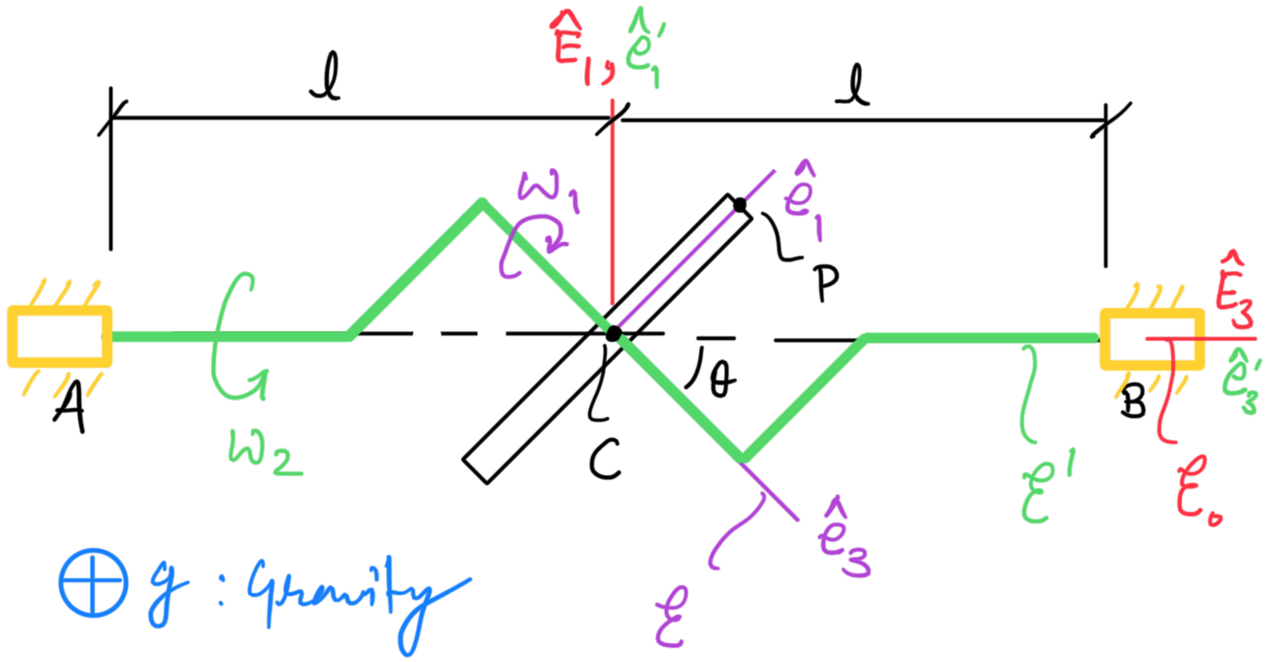


TOP VIEW



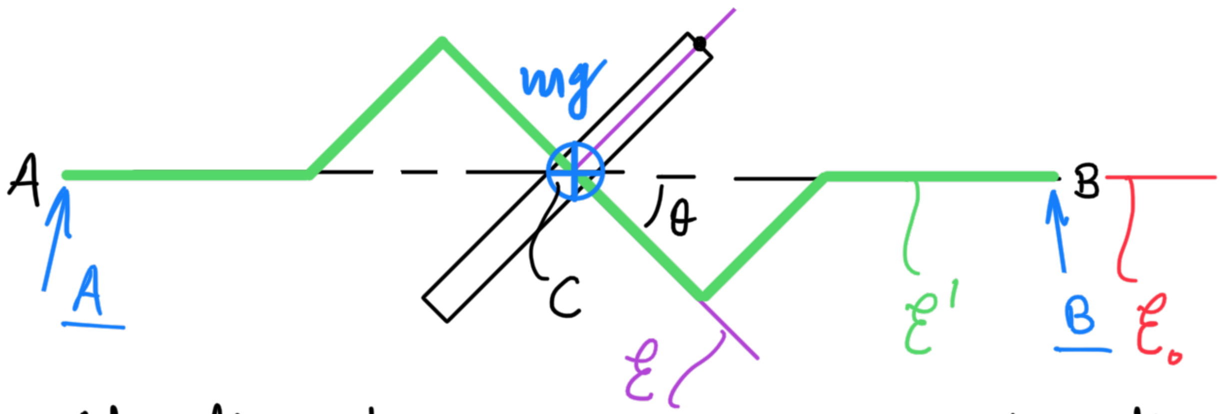
There are three frames:

(i) $(\mathcal{E}\,;\, C\,;\, \hat{e}_i)$: Fixed to the disc (P dir^{n})

(ii) $(\mathcal{E}'\,;\, C\,;\, \hat{e}'_i)$: Fixed to the bent shaft

(iii) $(\mathcal{E}_0\,;\, C\,;\, \hat{E}_i)$: Ground fixed

Step-1 Draw FBD



At this time we are assuming that the bearings at A and B are offering resistance in all three directions. Therefore, we have taken $\underline{A}$ and $\underline{B}$ in arbitrary directions.

Step-2 Applying balance laws

(i) LMB : $\sum \underline{F}_{ext} = m\,\underline{a}_c$

$$\Rightarrow \quad \underline{A} + \underline{B} - mg\,\hat{E}_2 = \underline{0} \qquad \text{— (1)}$$

Aside
C is fixed
$\therefore\ \underline{a}_c = \underline{0}.$

or $(A_1 + B_1)\hat{E}_1 + (A_2 + B_2)\hat{E}_2 + (A_3 + B_3)\hat{E}_3$
$= mg\,\hat{E}_2$

$$\Rightarrow \quad A_1 + B_1 = 0 \qquad \text{— (2a)}$$

$$A_2 + B_2 = mg \quad \text{———} \quad (2b)$$
$$A_3 + B_3 = 0 \quad \text{———} \quad (2c)$$

(ii) AMB : About pt. C. (fixed)

Note : shaft's mass is ignored.

$$\underline{M}_{/C} = \underline{\underline{I}}_{cm} \cdot \underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} + \underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \underline{\underline{I}}_{cm} \cdot \underline{\omega}_{\mathcal{E}/\mathcal{E}_0}$$
$$+ \underline{r}_{C/G} \times m \underbrace{\cancel{\underline{a}_G}}_{\underline{0}}$$

$$\therefore \underline{M}_{/C} = \underline{\underline{I}}_{cm} \cdot \underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} + \underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \underline{\underline{I}}_{cm} \cdot \underline{\omega}_{\mathcal{E}/\mathcal{E}_0}$$

$$\underline{r}_{A/C} \times \underline{A} + \underline{r}_{B/C} \times \underline{B} = \underline{\underline{I}}_{cm} \cdot \underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} +$$
$$\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \underline{\underline{I}}_{cm} \cdot \underline{\omega}_{\mathcal{E}/\mathcal{E}_0}$$
$$\text{———} \quad (3)$$

Step - 3   Kinematics

$$\underline{r}_{A/C} = -l\,\hat{E}_3 \; ; \; \underline{r}_{B/C} = l\,\hat{E}_3 \quad \text{———} \quad (4)$$

CP = R (radius of the disc)

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}'} = -\omega_1\,\hat{e}_3 \; ; \; \underline{\omega}_{\mathcal{E}'/\mathcal{E}_0} = \omega_2\,\hat{E}_3$$

$$\therefore \ \underline{\omega}_{\mathcal{E}/\mathcal{E}_0} = -\omega_1 \hat{e}_3 + \omega_2 \hat{E}_3 \quad \text{———} \ (5)$$

Note: ω_1 and ω_2 are constants.

<u>Computation of $\underline{\alpha}_{\mathcal{E}/\mathcal{E}_0}$:</u>

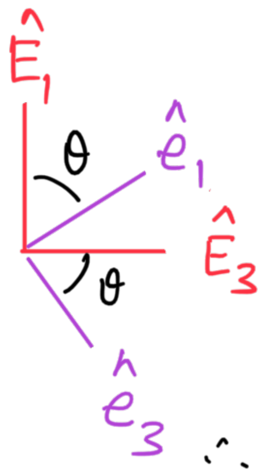
$$\begin{aligned}
\underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} &= \frac{d}{dt}\left(\underline{\omega}_{\mathcal{E}/\mathcal{E}_0}\right) \\
&= \frac{d}{dt}\left(-\omega_1 \hat{e}_3 + \omega_2 \hat{E}_3\right) \\
&= -\omega_1 \frac{d\hat{e}_3}{dt} + \underline{0} \\
&= -\omega_1 \left(\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} \times \hat{e}_3\right) \\
&= -\omega_1 \left(-\omega_1 \hat{e}_3 + \omega_2 \hat{E}_3\right) \times \hat{e}_3 \\
&= -\omega_1 \omega_2 \hat{E}_3 \times \hat{e}_3
\end{aligned}$$

<u>Aside</u>

ω_2: const.

$\hat{E}_3$: fixed

<u>Aside</u>



$$\left.\begin{aligned}
\hat{E}_3 &= \hat{e}_1 \sin\theta + \hat{e}_3 \cos\theta \\
\hat{E}_1 &= \hat{e}_1 \cos\theta - \hat{e}_3 \sin\theta
\end{aligned}\right\} - (6)$$

$$\begin{aligned}
\therefore \ \underline{\alpha}_{\mathcal{E}/\mathcal{E}_0} &= -\omega_1 \left(\omega_2 \sin\theta \ \hat{e}_1 \times \hat{e}_3\right) \\
&= \omega_1 \omega_2 \sin\theta \ \hat{e}_2 \quad \text{———} \ (7)
\end{aligned}$$

Further, using transformations ⑥

$$\underline{\omega}_{\mathcal{E}/\mathcal{E}_0} = -\omega_1 \hat{e}_3 + \omega_2 (\hat{e}_1 \sin\theta + \hat{e}_3 \cos\theta)$$
$$= \omega_2 \sin\theta\, \hat{e}_1 + (\omega_2 \cos\theta - \omega_1) \hat{e}_3 \qquad \text{⑧}$$

Now we will write ③ in $\mathcal{E}$, because $\underline{\underline{I}}_{cm}$ in $\mathcal{E}$ is diagonal and fixed. This eliminates computation of $\frac{d}{dt} \underline{\underline{I}}_{cm}$ when calculating $\dot{\underline{h}}$.

<u>RHS of ③</u> <u>in $\mathcal{E}$</u>

$$= \left(\frac{mR^2}{4} \hat{e}_1 \otimes \hat{e}_1 + \frac{mR^2}{4} \hat{e}_2 \otimes \hat{e}_2 + \frac{mR^2}{2} \hat{e}_3 \otimes \hat{e}_3\right) \cdot (\omega_1 \omega_2 \sin\theta\, \hat{e}_2)$$
$$+ (\omega_2 \sin\theta\, \hat{e}_1 + (\omega_2 \cos\theta - \omega_1) \hat{e}_3) \times$$
$$\left(\frac{mR^2}{4} \hat{e}_1 \otimes \hat{e}_1 + \frac{mR^2}{4} \hat{e}_2 \otimes \hat{e}_2 + \frac{mR^2}{2} \hat{e}_3 \otimes \hat{e}_3\right) \cdot$$
$$(\omega_2 \sin\theta\, \hat{e}_1 + (\omega_2 \cos\theta - \omega_1) \hat{e}_3)$$

$$= \frac{mR^2}{4}\omega_1\omega_2\sin\theta\,\hat{e}_2 + \Big(\omega_2\sin\theta\,\hat{e}_1 +$$

$$(\omega_2\cos\theta - \omega_1)\,\hat{e}_3\Big) \times \Big(\frac{mR^2}{4}\omega_2\sin\theta\,\hat{e}_1$$

$$+ \frac{mR^2}{2}(\omega_2\cos\theta - \omega_1)\,\hat{e}_3\Big)$$

$$= \frac{mR^2}{4}\omega_1\omega_2\sin\theta\,\hat{e}_2 - \omega_2\sin\theta\,\frac{mR^2}{2}\cdot$$

$$(\omega_2\cos\theta - \omega_1)\,\hat{e}_2 + \omega_2\sin\theta\,\frac{mR^2}{4}\cdot$$

$$(\omega_2\cos\theta - \omega_1)\,\hat{e}_2$$

$$= \Big(\cancel{\frac{mR^2}{4}\omega_1\omega_2\sin\theta} - \frac{mR^2}{2}\omega_2^2\sin\theta\cos\theta$$

$$+ \frac{mR^2}{2}\omega_1\omega_2\sin\theta + \frac{mR^2}{4}\omega_2^2\sin\theta\cos\theta$$

$$- \cancel{\frac{mR^2}{4}\omega_1\omega_2\sin\theta}\Big)\hat{e}_2$$

$$= \frac{mR^2}{2}\omega_2\sin\theta\left(\omega_1 - \frac{1}{2}\omega_2\cos\theta\right)\hat{e}_2$$

We recall that $\hat{e}_2 = \hat{E}_2$

$\therefore$ RHS of ③ in $\mathcal{E}_0$ is simply

$$= \frac{mR^2}{2}\omega_2\sin\theta\left(\omega_1 - \frac{1}{2}\omega_2\cos\theta\right)\hat{E}_2 \qquad ⑨$$

Now let us write LHS of 3 in $\mathcal{E}_0$

$$-l\hat{E}_3 \times (A_1\hat{E}_1 + A_2\hat{E}_2 + A_3\hat{E}_3) + l\hat{E}_3 \times (B_1\hat{E}_1 + B_2\hat{E}_2 + B_3\hat{E}_3)$$

$$= l(A_2 - B_2)\hat{E}_1 + l(B_1 - A_1)\hat{E}_2 \quad \text{——— (10)}$$

Equating 9 and 10 yields

$$(A_2 - B_2)l = 0$$

or $\quad A_2 - B_2 = 0$

or $\quad A_2 = B_2 \quad \text{——— (11a)}$

and

$$l(B_1 - A_1) = \frac{mR^2}{2}\omega_2 \sin\theta\left(\omega_1 - \frac{\omega_2}{2}\cos\theta\right)$$

$$\therefore B_1 - A_1 = \frac{mR^2}{2l}\omega_2 \sin\theta\left(\omega_1 - \frac{\omega_2}{2}\cos\theta\right) \quad \text{——— (11b)}$$

Using $A_1 = -B_1$ from 2a in 11b we get

$$\boxed{B_1 = \frac{mR^2}{4l}\omega_2 \sin\theta\left(\omega_1 - \frac{\omega_2}{2}\cos\theta\right)}$$

$$\boxed{A_1 = \frac{-mR^2}{4l} \omega_2 \sin\theta \left(\omega_1 - \frac{\omega_2}{2}\cos\theta\right)}$$

Using (11a) in (2b) results in

$$\boxed{A_2 = B_2 = \frac{mg}{2}}$$

From (2c) we have $A_3 = -B_3$. We need another equation to determine A_3 and B_3. In the absence of any other information e.g. details of the bearing-shaft interaction etc. these two components of $\underline{A}$ and $\underline{B}$ will remain undetermined in "rigid body" world.