Lecture 15

Rigid body kinetics: Examples. Balance laws: power

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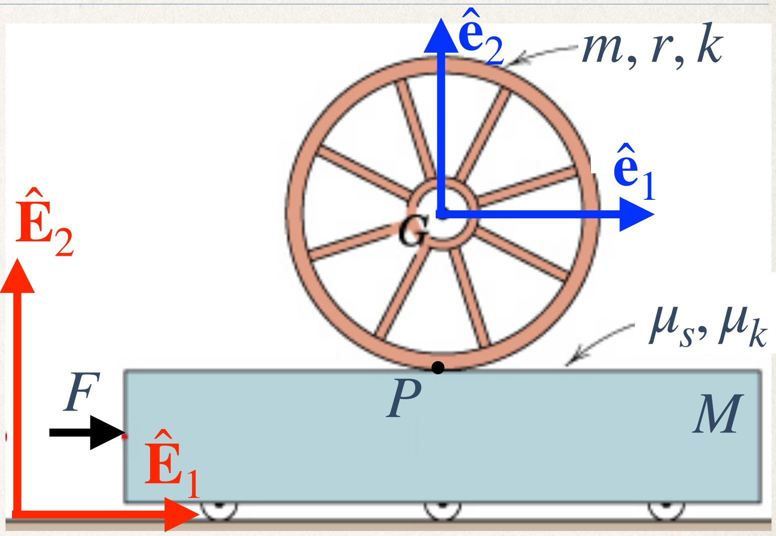
29 September - 5 October, 2021

Example 1

Find max. F which may be applied and still ensure that the wheel will not slip when it rolls.



I. FBD. Two bodies, two FBDs.

1. Assume roll *without* slip $\implies f \leqslant \mu_s N$.

II. **Kinetic analysis**.

1. LMB of wheel and carriage.
2. AMB about P_w for wheel.

III. **Kinematic analysis**.

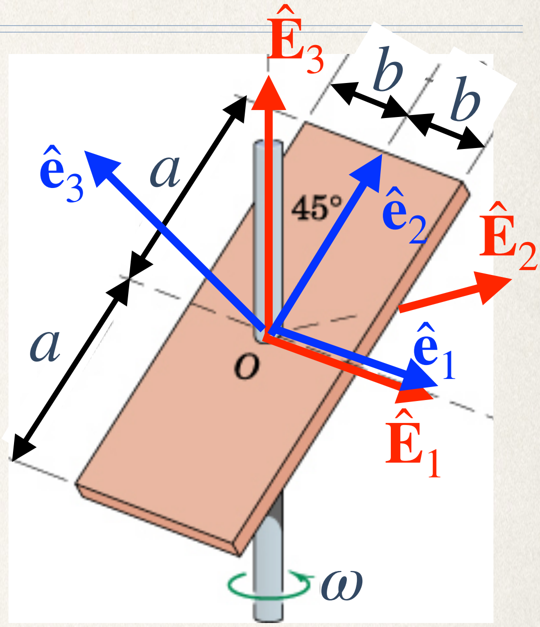
1. P_w and G on wheel.
2. *Rolling condition*: $\mathbf{a}^{P_w} \cdot \hat{\mathbf{t}} = \mathbf{a}^{P_c} \cdot \hat{\mathbf{t}}$

IV. **Assume**: No *vertical* motion of wheel.

V. *Pure rolling* if $F \leqslant \mu_s g \{m + M(1 + r^2/k^2)\}$.

Example 2

Find the forces and moments exerted by the vertical shaft on the plate. The rate $\omega = const.$



I. **FBD** of plate. *Three* forces and *three* moments act at O.

II. **Kinetic analysis**.

1. LMB: $\sum \mathbf{F} = m\mathbf{a}^O = \mathbf{0} \implies$ No force at O.
2. AMB about O: $\mathbf{M}^O = \boldsymbol{\omega}^p \times \left(\mathbf{I}^O \cdot \boldsymbol{\omega}^p\right) + \mathbf{I}^O \cdot \boldsymbol{\alpha}^p$

III. **Kinematic analysis**: $\boldsymbol{\omega}^p = \omega\hat{\mathbf{E}}_3, \boldsymbol{\alpha}^p = \mathbf{0}$.

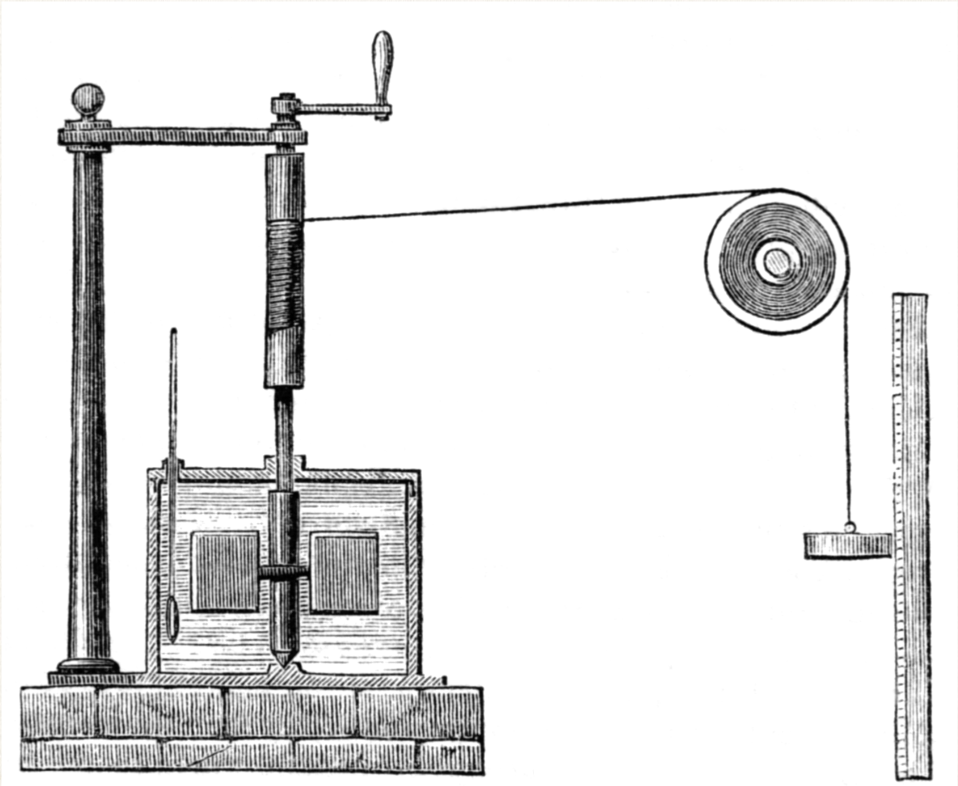
IV. Compute $\mathbf{I}^O$ in plate's BFCS $\{\mathscr{E}, O, \hat{\mathbf{e}}_i\}$.

1. Compute $\mathbf{I}^O \cdot \boldsymbol{\omega}^p$ by finding $[\boldsymbol{\omega}^p]_{\mathscr{E}}$.

V. **Answer**: $\mathbf{M}^O = m\omega^2 a^2\hat{\mathbf{E}}_1/6$.

VI. *Kinematics* is 2D, but *kinetics* is 3D !

Whatever happened to energy?



Joule's apparatus for measuring the mechanical equivalent of heat in which the "work" of the falling weight is converted into the "heat" of agitation in the water.

Power balance

I. Power input to a single rigid body:

$$\mathscr{P} = \sum \mathbf{v}^i \cdot \mathbf{F}^i + \sum \boldsymbol{\omega}^{\mathscr{B}} \cdot \mathbf{M}^j = \mathbf{v}^G \cdot \sum \mathbf{F}^i + \boldsymbol{\omega}^{\mathscr{B}} \cdot \mathbf{M}^G$$

II. *Kinetic energy* of a rigid body:

$$E_k = \frac{1}{2} m \mathbf{v}^G \cdot \mathbf{v}^G + \frac{1}{2} \boldsymbol{\omega}^{\mathscr{B}} \cdot \mathbf{I}^G \cdot \boldsymbol{\omega}^{\mathscr{B}}.$$

III. *Power balance* for a single rigid body:

$$\boxed{\mathscr{P} = \frac{\mathrm{d}E_k}{\mathrm{d}t}.}$$

IV. Special cases: *Conservative* loading.

1. In *3D*: $\mathbf{M}^j = \mathbf{0}$, $\mathbf{F}^i = -\nabla U_i \implies$

$$\boxed{E_K + \sum U_i = \text{const}.}$$

2. In *2D*: Let $\hat{\mathbf{E}}_3$ be normal to the plane.

$$\mathbf{M}^j = M_j(\theta)\hat{\mathbf{E}}_3 = -\frac{\mathrm{d}V_j}{\mathrm{d}\theta}\hat{\mathbf{E}}_3, \; \mathbf{F}^i = -\nabla U_i \implies$$

$$\boxed{E_K + \sum U_i + \sum V_j = \text{const}.}$$

Power balance

I. N rigid bodies with *frictionless* joints:

$$\mathscr{P}^{ext} = \sum_{n=1}^{N} \left(\mathbf{v}^{G_n} \cdot \sum \mathbf{F}^{i,n}_{ext} + \boldsymbol{\omega}^{\mathscr{B}_n} \cdot \mathbf{M}^{G_n}_{ext} \right) = \frac{\mathrm{d}E_k}{\mathrm{d}t}$$

$$E_k = \sum_{n=1}^{N} \left(\frac{1}{2} m_n \mathbf{v}^{G_n} \cdot \mathbf{v}^{G_n} + \frac{1}{2} \boldsymbol{\omega}^{\mathscr{B}_n} \cdot \mathbf{I}^{G_n} \cdot \boldsymbol{\omega}^{\mathscr{B}_n} \right)$$

1. *Internal* force contributions cancel out.

II. *Conservative* system

1. In *3D*: $\mathbf{M}^{j,n}_{ext} = \mathbf{0}$, $\mathbf{F}^{i,n}_{ext} = -\nabla U_{in} \implies$

$$E_K + \sum_{n=1}^{N} \sum_{i} U_{ik} = \text{const}\,.$$

2. In *2D*: Let $\hat{\mathbf{E}}_3$ be normal to the plane.

$$\mathbf{M}^{j,n} = M_{jn}(\theta)\hat{\mathbf{E}}_3 = -\frac{\mathrm{d}V_{jn}}{\mathrm{d}\theta}\hat{\mathbf{E}}_3,\ \mathbf{F}^{i,n} = -\nabla U_{in} \implies$$

$$E_K + \sum_{n=1}^{N} \sum_{i} U_{in} + \sum_{n=1}^{N} \sum_{j} V_{jn} = \text{const}\,.$$

Power balance

I. Power balance is *not* an independent balance in mechanics.

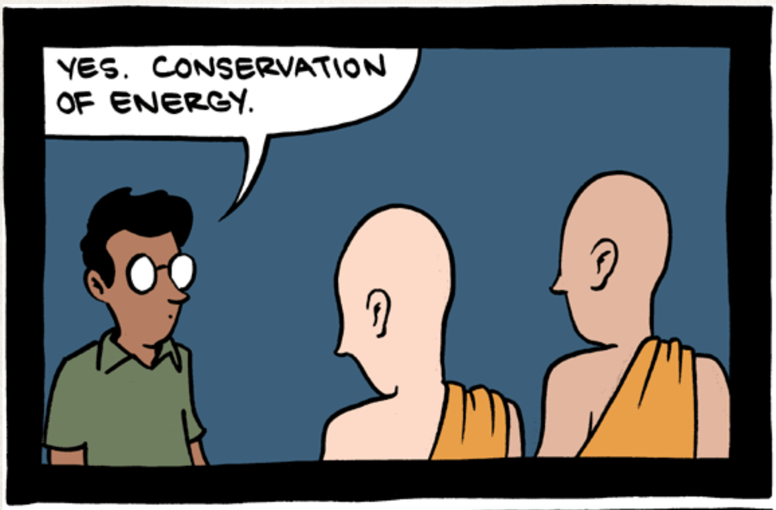
II. It is derived from *LMB* and *AMB*.

III. This is because in our study so far energy in only mechanical in nature.

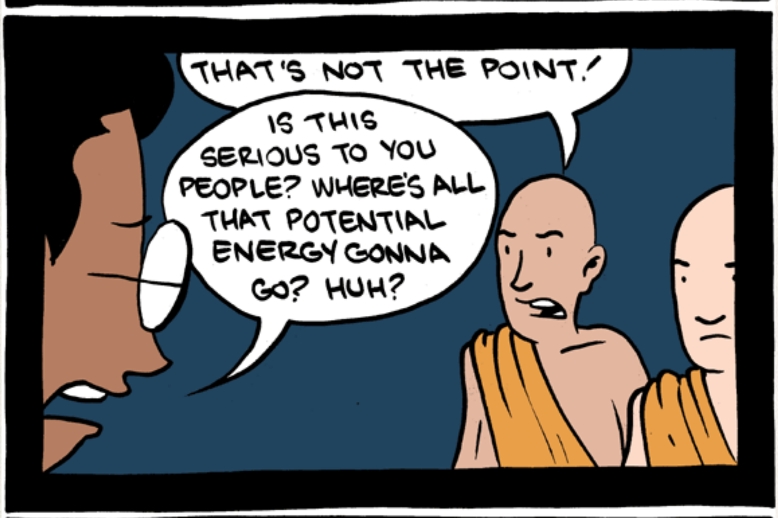
IV. We ignore thermodynamics and its connection with mechanics

V. In *thermo*-mechanics this is *not* the case, and power balance is an independent postulate.

Philosophy…



…and thermo-mechanics…



… may *not* mix well. Caution advised.