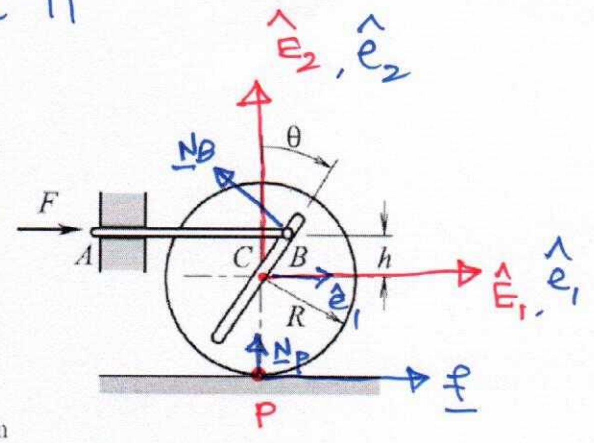
ESO 209 - Tutorial 11

(2) Horizontal force F causes the actuating rod to move to the left at the constant speed v. This rod is connected to the wheel by pin B, which may slide F through the groove. The mass of the wheel is m, the radius of gyration is κ, and μ_s and μ_k are the coefficients of static and kinetic friction, respectively, between the wheel and the ground. Friction between the pin and the groove is negligible, as is the mass of the rod. Consider the instant when $\theta = 30°$. Derive expressions for the acceleration of center C, the angular acceleration of the gear, and the force F under the assumption that (a) the wheel rolls without slipping, (b) there is slippage between the wheel and the ground.

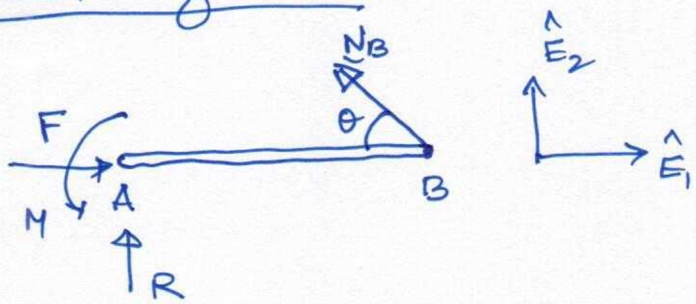


Choosing Observer CS $\{E_{P0}, C, \hat{E}_i\}$

wheel BFCS $\{F, C, \hat{e}_i\}$

Assuming reaction $\underline{N}_B$ at pin B, friction force $\underline{f}$ and reaction at point P (contact between wheel & ground) as $\underline{N}_P$.

FBD of actuating rod:



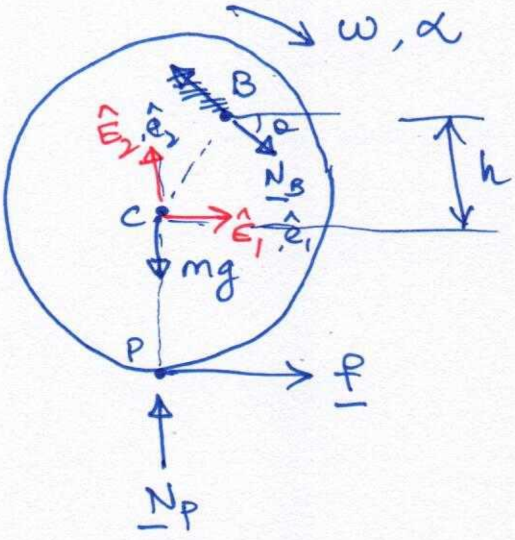
Considering only the $\hat{E}_1$ direction components,

$$\Sigma F = \overset{0}{\cancel{m}} a^G \quad \text{(Since mass is negligible)}$$

$$F - N_B \cos\theta = 0$$

$$\Rightarrow \boxed{N_B = F/\cos\theta} \quad \text{— } 1$$

FBD of wheel



$\underline{v} = v\hat{E}_1$

$\underline{a} = a\hat{E}_1$

LMB of wheel

Note: $\hat{e}_1$ & $\hat{E}_1$ and $\hat{e}_2$ & $\hat{E}_2$ are aligned at instant of interest.

$$\Sigma \underline{F} = m\underline{a}^G$$

$$LHS = \Sigma \underline{F} \equiv N_B \cos\theta\, \hat{E}_1 - N_B \sin\theta\, \hat{E}_2 - mg\,\hat{E}_2 + N_p \hat{E}_2 + f\hat{E}_1$$

$$RHS = m\underline{a}^G \equiv m\dot{v}\,\hat{E}_1$$

Equating the components of $\hat{E}_1$ & $\hat{E}_2$

$\hat{E}_1$: $\quad N_B \cos\theta + f = m\dot{v}$

using Eqn (1), we get

$$\boxed{F + f = m\dot{v}} \quad \text{— (2)}$$

$\hat{E}_2$: $\quad -N_B \sin\theta - mg + N_p = 0$

again substituting for N_B from eqn (1),

$$-F\tan\theta - mg + N_p = 0$$

$$\Rightarrow \boxed{N_p = F\tan\theta + mg} \quad \text{— (3)}$$

AMB of wheel.

Recognizing that this is a planar 2-D problem, we can write the $\hat{E}_3$ components of AMB eqn as.

$$\Sigma \underline{M}_G = \underline{\underline{I}}_G \cdot \underline{\alpha}^{E_\phi / E_0}$$

$$\Sigma M_G \text{ in } \hat{E}_3 = fR - N_B \cdot \underbrace{\frac{h}{\cos\theta}}_{\text{distance } BC}$$

$$\left[\underline{\underline{I}}_G\right]_{E_\phi} = \begin{bmatrix} \frac{mk^2}{2} & 0 & 0 \\ 0 & \frac{mk^2}{2} & 0 \\ 0 & 0 & mk^2 \end{bmatrix}$$

$$\left[\underline{\alpha}^{E_\phi / E_0}\right]_{E_\phi} = \begin{bmatrix} 0 \\ 0 \\ -\alpha \end{bmatrix}$$

substituting $N_B = F/\cos\theta$ from 1

$$\therefore \boxed{fR - \frac{Fh}{\cos^2\theta} = -mk^2\alpha} \quad \text{— } 4$$

The unknowns are f, $\dot{v}$, α and we have equations 2 & 4.

The missing equation is that of kinematics ~~that~~ of the wheel rolling without slipping, which by now can be easily written as

$$v = R\omega \Rightarrow \boxed{\dot{v} = R\dot{\omega} = R\alpha} \quad \text{— } 5$$

(2D-planar problem : ~~all~~ $\underline{\omega}$ & $\underline{\alpha}$ vectors are in $\hat{E}_3$ direction)

Note: Eqn 3 is not very helpful here. However, Eqn. 3 helps in determining the value of f at which the wheel begins to slip.

Eliminating f from Eqns (2) & (4), we get

$$\underbrace{(m\ddot{x} - F)}_{f} R - \frac{Fh}{\cos^2\theta} = -mk^2\alpha$$

Using equation (5) for $\ddot{x}$

$$(mR\alpha - F)R - \frac{Fh}{\cos^2\theta} = -mk^2\alpha$$

$$\Rightarrow \quad mR^2\alpha - FR - \frac{Fh}{\cos^2\theta} = -mk^2\alpha$$

$$\Rightarrow \quad m(R^2+k^2)\alpha = F\left(R + \frac{h}{\cos^2\theta}\right)$$

$$\Rightarrow \quad \boxed{\alpha = \frac{F}{m}\frac{(R + h/\cos^2\theta)}{(R^2+k^2)}} \quad \text{--- (6)}$$

From (5) & (6), we get

$$\boxed{a^G = \ddot{x} = \frac{FR\,(R + h/\cos^2\theta)}{(R^2+k^2)}} \quad \text{--- (7)}$$

Case 2 when the wheel starts slipping, the kinematic relation is no longer applicable.

and $\quad f = -\mu_K N_p$

Using Equation (3)

$$f = -\mu_K(F\tan\theta + mg)$$

Using the LMB eqn 2, we get

$$m\ddot{x} = F - \mu_K(F\tan\theta + mg)$$

$$\therefore \boxed{\ddot{x} = \frac{1}{m}\left[F(1-\mu_K\tan\theta) - \mu_K mg\right]} \quad -(8)$$

Using the AMB eqn (4), we get

$$fR - \frac{Fh}{\cos^2\theta} = -mk^2\alpha$$

$$\left[-\mu_K(F\tan\theta + mg)\right]R - \frac{Fh}{\cos^2\theta} = -mk^2\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{1}{mk^2}\left[\mu_K FR\tan\theta + \mu_K mgR + \frac{Fh}{\cos^2\theta}\right]} \quad -(9)$$

8 & 9 are the expressions for acceleration of centre C & angular acceleration α (in the negative $\hat{E}_3$ direction) for the wheel slipping condition.