

# Lecture 19

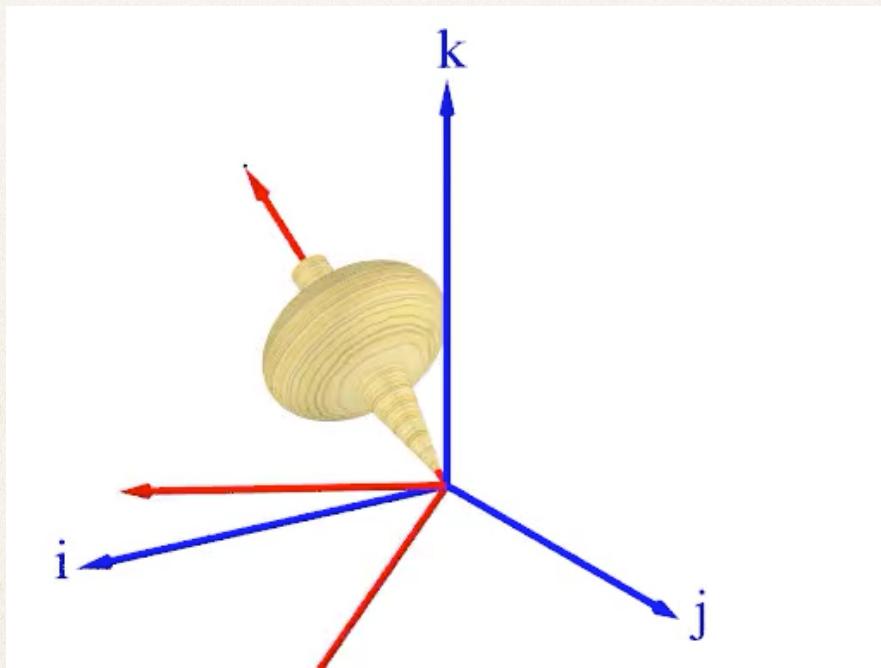
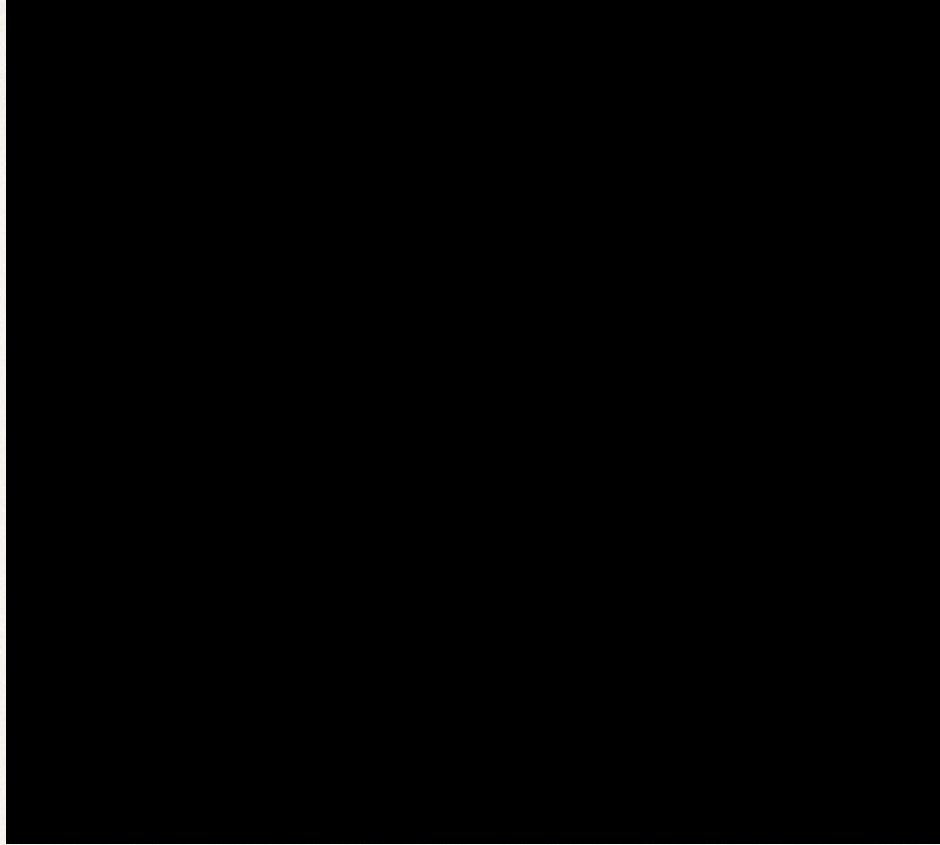
*More on Tops*

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*20 - 26 October, 2021*

# Nutation of a top

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# Nutation of a top

I. Governing equations for  $I_1 = I_2 \neq I_3$ .

$$\left. \begin{aligned} \dot{\phi} \cos \theta + \dot{\psi} &= \omega_3^0 \\ I_1(\ddot{\phi} \sin \theta + 2\dot{\phi}\dot{\theta} \cos \theta) - I_3\omega_3^0\dot{\theta} &= 0 \\ I_1(\ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta) + I_3\omega_3^0\dot{\phi} \sin \theta &= mgL \sin \theta \end{aligned} \right\} \quad (1)$$

II. Integrate (1b) using (1a):

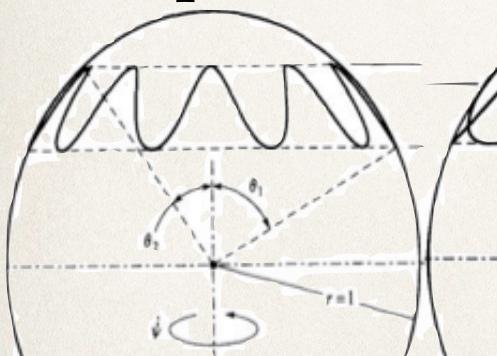
$$I_3\omega_3^0 \cos \theta + I_1\dot{\phi} \sin^2 \theta = H_3 =: \mathbf{H}^O \cdot \hat{\mathbf{E}}_3 \quad (2)$$

III. Eliminate  $\dot{\phi}$  from (2) in (1c):

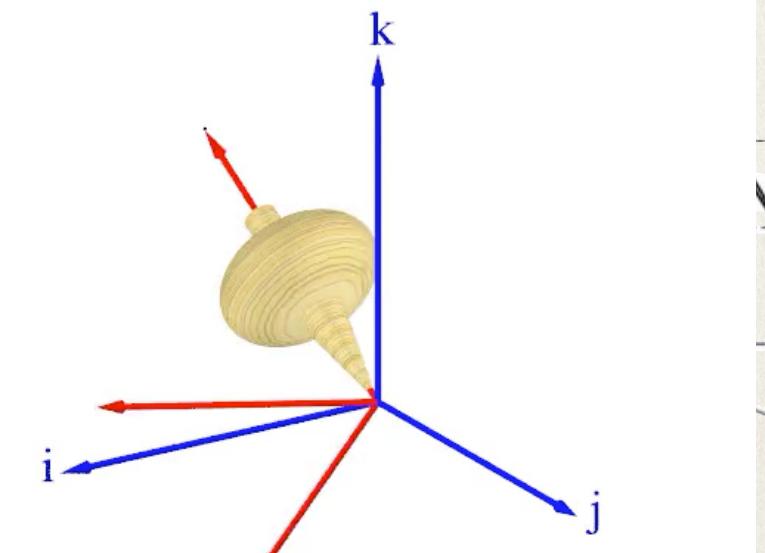
$$\ddot{\theta} + \frac{(I_3\omega_3^0 - H_3 \cos \theta)(H_3 - I_3\omega_3^0 \cos \theta)}{I_1^2 \sin^3 \theta} - \frac{mgL \sin \theta}{I_1} = 0$$

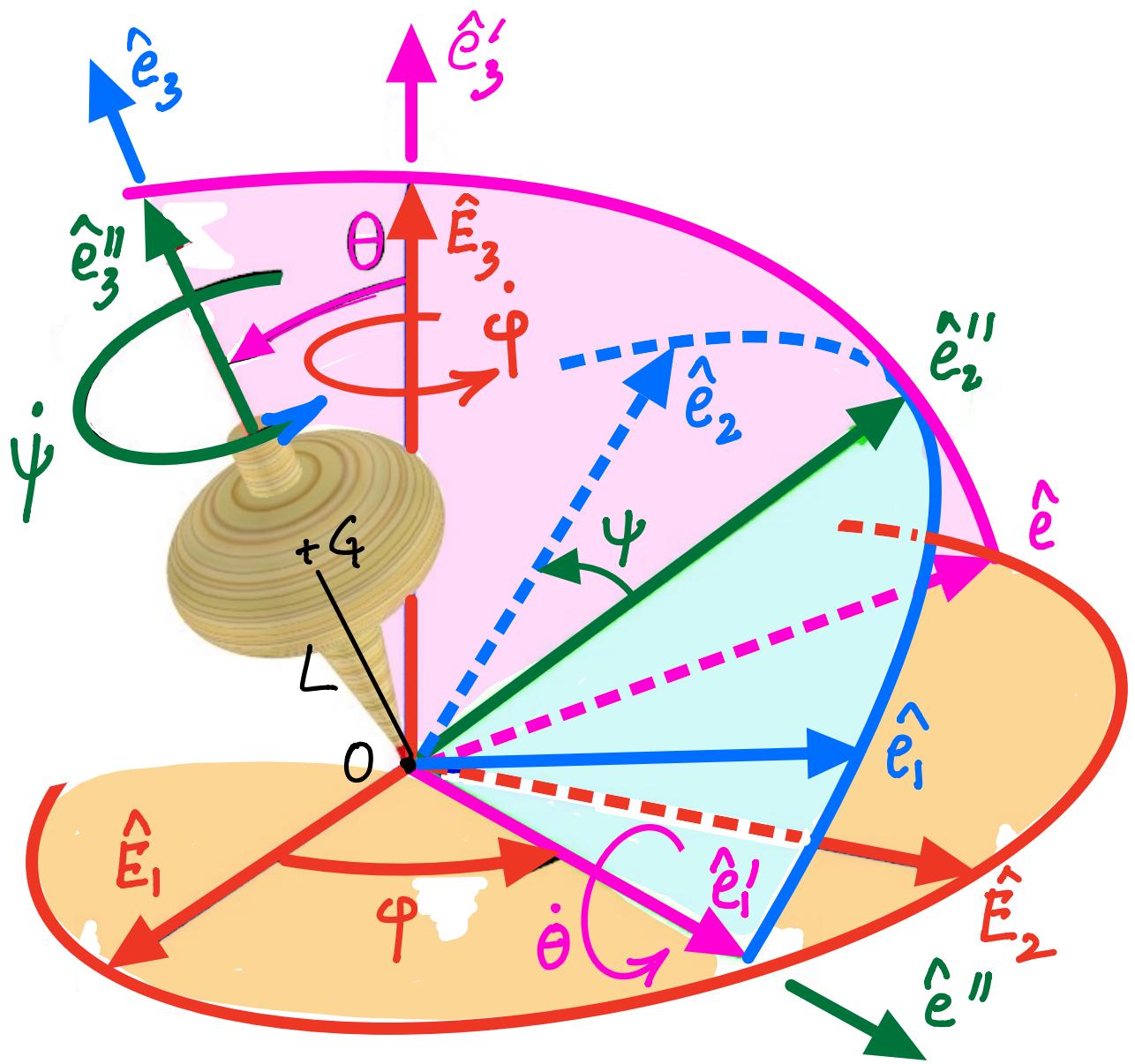
IV. Solve for  $\theta(t)$  for diff. initial conditions:

**Example:**



unidirectional precession





$$M_3 = 0 \quad | \quad \dot{\varphi} = \frac{H_3 - I_3 \omega_3^0 \cos \theta}{I_1 \sin^2 \theta}$$

$\Downarrow$

$\varphi(t)$       Need  $\varphi(0)$

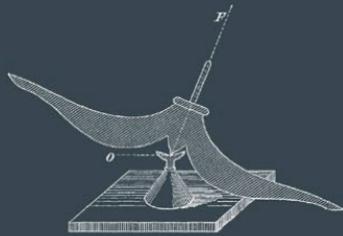
$$\begin{array}{ll} @ t=0 \quad \theta(0) = \theta_0 \\ \dot{\varphi}(0) = \dot{\varphi}_0 \quad \dot{\psi}(0) = \dot{\psi}_0 \\ \dot{\theta}(0) = \dot{\theta}_0 \end{array} \quad \left. \begin{array}{l} \theta(t) \\ \psi(t) \end{array} \right|$$

$$\begin{array}{l} \dot{\psi} = \omega_3^0 - \dot{\varphi} \sin \theta \\ \psi(0) = \psi_0 \end{array} \quad \left. \begin{array}{l} \theta(t) \\ \psi(t) \end{array} \right|$$

Felix Klein  
Arnold Sommerfeld

## The Theory of the Top Volume I

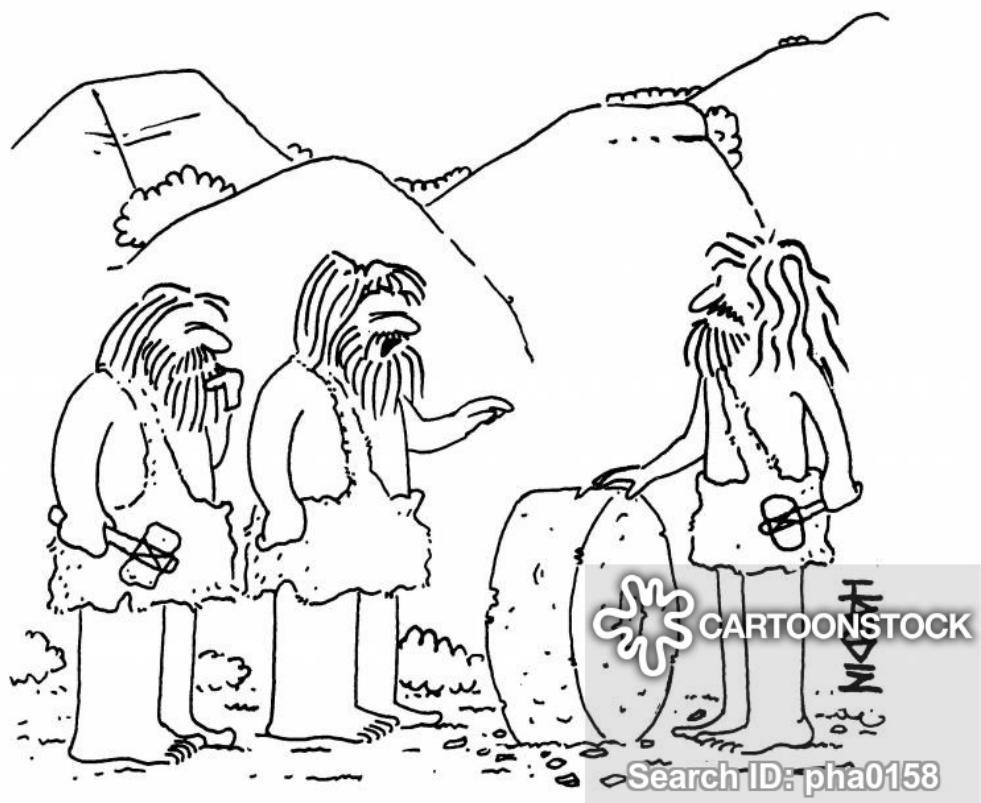
*Introduction to the Kinematics  
and Kinetics of the Top*



*FOUR more  
volumes*



*F Klein: At par with Hilbert and Poincaré.  
A Sommerfeld: 84 Nobel prize nominations!*



"This 'wheel' thing of yours—Does it have to be round or will any shape do?"

Meanwhile tops come in square as well — look it up!