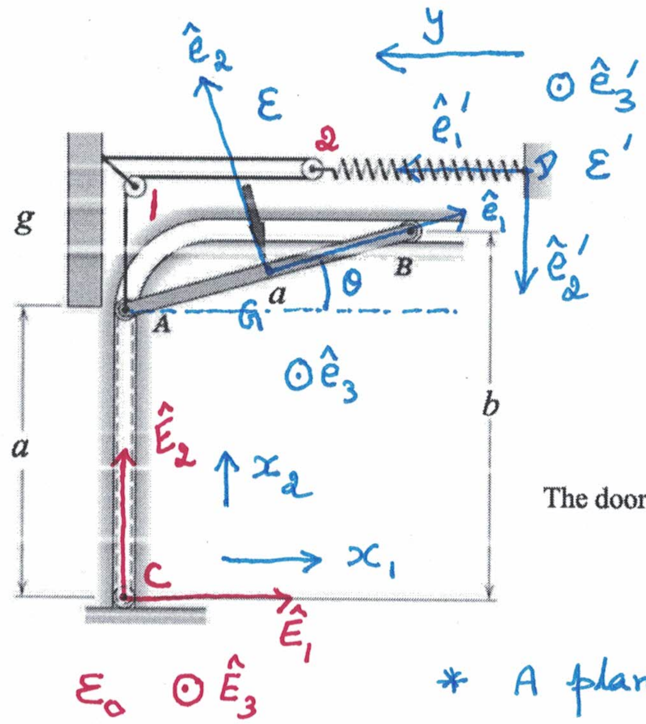
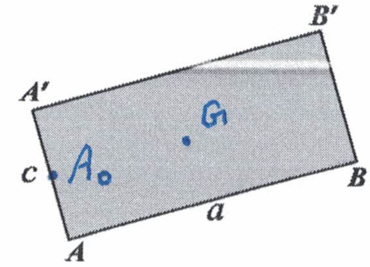
TUTORIAL-10, PROBLEM-1





The door as viewed from the direction of the blue arrow.

* A planar problem with 2D rotations

* Origin of the observer frame (C) corresponds to the point where A_0 will be, when the door hits the floor.

↓ midpoint of ~~AA~~ AA'

* Origin of the BFCS (G) corresponds to the CG of the door.

Coordinate systems,

$$\{\mathcal{E}_0, C, \hat{E}_i\} \qquad \{\mathcal{E}, G, \hat{e}_i\} \qquad \{\mathcal{E}', D, \hat{e}'_i\}$$

(observer) (BFCS) [A spring frame which is fixed to the right top wall at the point D.]

At any time instant,

$$\underline{\omega}^{AB} = \omega_{AB}\hat{E}_3$$

→ Angular velocity about CG.

$$\underline{V}^{G} = V_1\hat{E}_1 + V_2\hat{E}_2$$

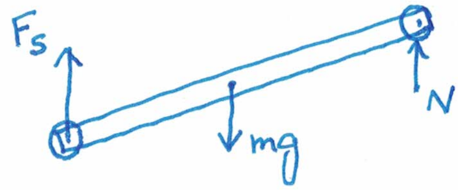
Given tiny end-rollers, mass of the end-rollers can be ignored. Also, the frictional force acting on the door at A ~~as~~ and B will be zero.

Also, neglecting friction in the two pulleys.

Hence, conservative system.

Using power balance method.

FBD of the door,



Note,

$$\sin\theta = \frac{b-a}{a} \quad \text{------ (1)}$$ (at the starting rest position)

No moments acting on the rigid body, $\underline{M}^j = 0$

Hence, no V_j.

The reaction force N will not do any work, since motion of B is always perpendicular to the force direction.

Only two forces contribute towards energy change,

$$\underline{F}^1 = -mg\,\hat{E}_2 = -\nabla\overbrace{(mg\,x_2)}^{U_1}$$

2

$$\underline{F}^2 = -F_s\,\hat{e}_1' = -ky\,\hat{e}_1' = -\nabla\underbrace{\left(\tfrac{1}{2}ky^2\right)}_{U_2}$$

$y \rightarrow$ extension of the two springs starting from the initial rest position of the door. That is, $y_0 = 0$ at the starting rest position.

Note that, $y = \dfrac{a - x_A}{2}$, where x_A is the position of A. $\left(\underline{x}_A = x_A\,\hat{E}_2\right)$

Initial, when $x_A = a$, $y_0 = 0$

Final, when $x_A = 0$, $y = \frac{a}{2}$

(just before hitting the floor)

If E_K is the kinetic energy of the door,

$$E_K + U_1 + 2U_2 = \text{constant}.$$

Initial, $E_K^i = 0$, $U_1 = mg\left(a + \frac{a}{2}\sin\theta\right)$,

$$U_2 = \tfrac{1}{2}ky_0^2 = 0$$

Final, $E_K^f = \frac{1}{2}mV_2^2$, $U_1 = mg\frac{a}{2}$,

$$U_2 = \tfrac{1}{2}k\left(\tfrac{a}{2}\right)^2$$

Note that, $\underline{W}^{f}_{AB} = 0$

and $\underline{V}^{G}$ at final state of the door

will be just $-V_2 \hat{E}_2$.

Thus, equating the total energy at initial and final states,

$$mg\left(a + \frac{a}{2}\sin\theta\right) = \frac{1}{2} m V_2^2 + mg\frac{a}{2} + \cancel{2} * \frac{1}{\cancel{2}} k \frac{a^2}{4}$$

Using equation ① and rearranging,

$$\frac{1}{2} m V_2^2 = mg\left(a + \frac{b-a}{2}\right) - mg\frac{a}{2} - \frac{k a^2}{4}$$

$$= mg\frac{b}{2} - \frac{k a^2}{4}$$

i.e. $$V_2^2 = gb - \frac{k a^2}{2m}$$

$$V_2 = \sqrt{gb - \frac{k a^2}{2m}} \quad \text{------} \quad (3)$$

given $a = 3\,m$, $b = 3.5\,m$, $k = 600\ N/m$, and $m = 150\ kg$.

Taking $g = 9.8\ m/s^2$,

$$V_2 = 4.037\ m/s$$

Since same rigid body without any angular velocity, $\underline{V}^{A} = \underline{V}^{A'} = \underline{V}^{G}$