- ***Unstable System***:
  - ***Any transient disturbance would result in a response that will persist or even blow up with time***
    - ***Eventually gets limited by the nonlinearities of the system***
  - ***Positive feedback systems are inherently unstable***
    - ***They are designed as such***, e.g., ***oscillators***
  - ***Negative feedback systems are inherently stable***

- ***However, there may be situations when they may become unstable and break out into spontaneous oscillations***
- ***Potentially dangerous situation***, and the ***system should be protected against it***

- ***How does a negative feedback system become unstable***?
  - Write the ***loop gain*** expression in ***polar form***:

    $$L(j\omega) = f(j\omega)A(j\omega) = |f(j\omega)A(j\omega)|\exp[j\phi(\omega)]$$

    $\phi(\omega)$: ***Frequency dependent phase of the system***

- Consider a ***particular frequency*** ω_x, at which $\phi(\omega_x) = 180°$
- At ω_x, L would be a ***real number*** with ***negative sign***

  $\Rightarrow$ ***The feedback turns positive at this frequency***
- ***3 conditions may arise at ω_x***:
  - $|L| < 1$:
    - $A_f(j\omega_x) > A(j\omega_x)$, but the ***system will be stable***
  - $|L| = 1$:
    - $A_f(j\omega_x) \rightarrow \infty$, and ***output will appear without any input*** $\Rightarrow$ ***Oscillator***

- $|L| > 1$:
    - $A_f(j\omega_x) < A(j\omega_x)$, but the ***output will oscillate with gradually increasing amplitude***, and will ***eventually get limited by the nonlinearities present in the system***

- Thus, for a ***negative feedback system*** to turn into a ***positive feedback one***, the ***loop gain*** (L = fA) being ***equal to or less than –1*** is a ***sufficient and necessary condition***
- ***For this to happen***, the ***magnitude of the loop gain*** (L) ***should be equal to or greater than unity***, and the ***total phase around the loop should be 180°***

The Complex Frequency & The s-Plane

- Needed to understand the concept of ***stability*** of a system
- ***s-Plane***: ***Complex Frequency Plane***
- Consider a ***sinusoidal signal*** with an ***exponential envelope***:

$$v(t) = V_M[\cos(\omega t + \phi)]\exp(\sigma t)$$

V_M: ***Amplitude***

ϕ: ***Phase***

σ: ***Coefficient*** of the ***exponential enevelope***, having ***unit*** of ***time inverse*** (similar to ***frequency***)

- For ***positive*** σ, the signal will ***keep on growing with time***
- For ***negative*** σ, the signal would ***decay exponentially all the way to zero***

- ***3 interesting scenarios***:

1. $\sigma = \omega = 0$:

   $\Rightarrow v(t) = V_M \cos\phi$

   $\Rightarrow$ ***DC signal*** (***constant***)

2. $\sigma = 0$:

   $v(t) = V_M[\cos(\omega t + \phi)]$

   $\Rightarrow$ ***Normal ac signal***

3. $\omega = 0$:

   $v(t) = [V_M \cos\phi]\exp(\sigma t)$

   $\Rightarrow$ ***Exponential signal*** (***increasing/decreasing with time*** for ***positive/negative*** $\boldsymbol{\sigma}$)

- A normal ***sinusoidal voltage*** v(t), having ***angular frequency*** ω and ***phase*** ϕ, can be represented in ***polar form*** as:

$$v(t) = V_M \exp[j(\omega t + \phi)] \qquad (1)$$

- Comparing Eq.(1) with ***scenario 3***, we note that their ***functional forms*** are the ***same***

$\Rightarrow$ σ can be thought of as a ***frequency***, and is referred to as the ***neper frequency***, with unit of ***nepers/sec***

➢ However, this definition is ***not much used***