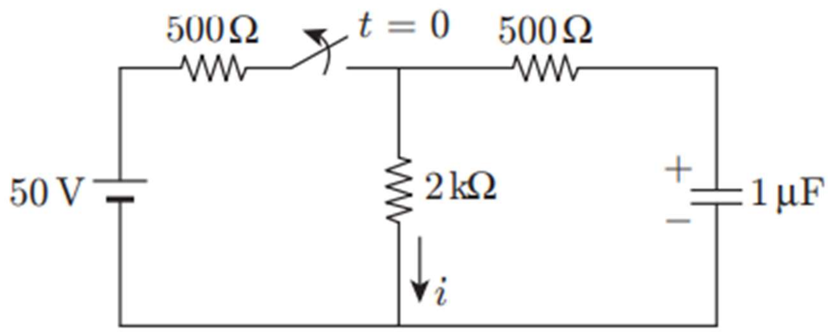
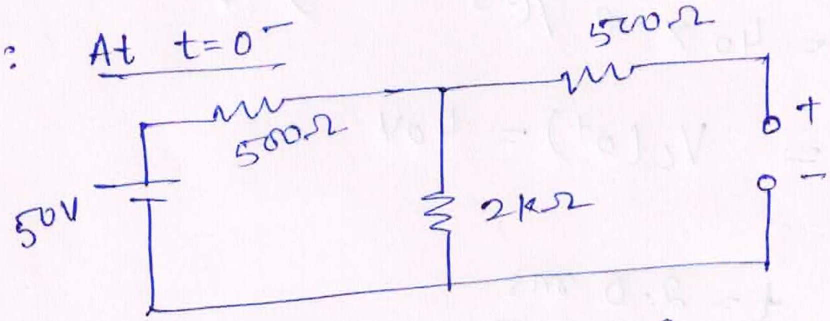
1. **For the circuit, shown in the figure below, the switch is closed for a long time and it is opened at t = 0. Determine $V_C(0^+)$, $i(0^+)$ and V_C (2 ms).**



Solution -

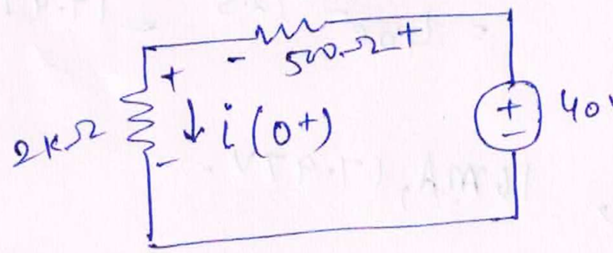
Soln:

At $t=0^-$



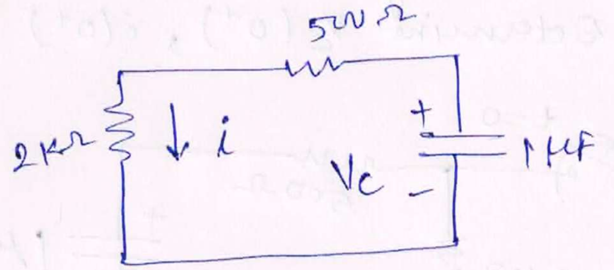
$$V_c(0^-) = \frac{50}{2.5\times10^3}\times 2\times10^3 = 40V = V_c(0^+)$$

At $t=0^+$



$$i(0^+) = \frac{40}{2.5\times10^3} = 16\ mA$$

At $t \geq 0$



$$V_c(t) = 40e^{(-t/(2.5 \times 10^3 \times 1 \times 10^{-6}))}\ V$$

$$= 40 \times e^{-t/(2.5 \times 10^{-3})}\ V$$

$$V_c(0^-) = V_c(0^+) = 40V$$

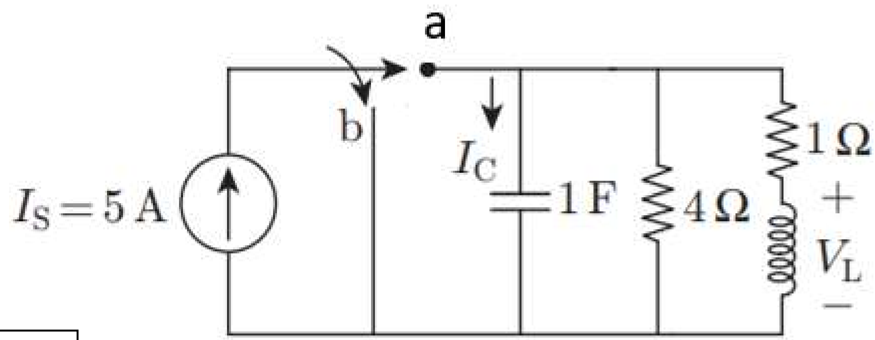
$t = 2.0$ ms

$$V_c(2ms) = 40e^{-\left(\frac{2 \times 10^{-3}}{2.5 \times 10^{-3}}\right)}$$

$$= 40e^{-2/2.5} = 17.97\ V$$

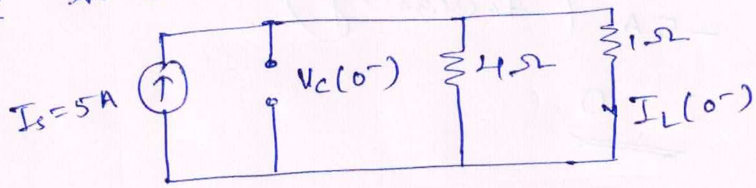
Ans = 40 V, 16 mA, 17.97V.

2. A switch, in the figure below, is in position `a' for a long time. Switch is moved to position `b' at t = 0. At t = 0$^+$, the values of I_C and V_L are -



Solution -

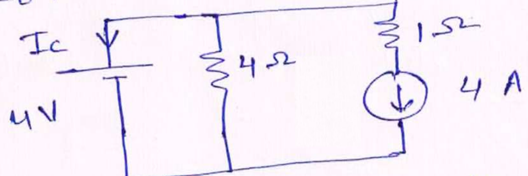
Sol^n: At $t = 0^-$



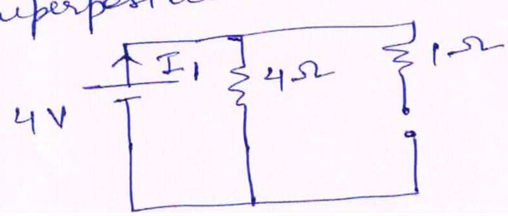
$$\frac{V_c \times 5}{4} = 5 \Rightarrow V_c = 4 \text{ V}$$

$$I_L = 5 \times \frac{4}{4+1} = 4 \text{ A}$$

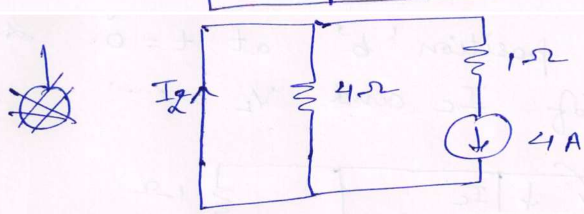
At $t = 0^+$



By superposition theorem



$\underline{I_1 = 1A}$



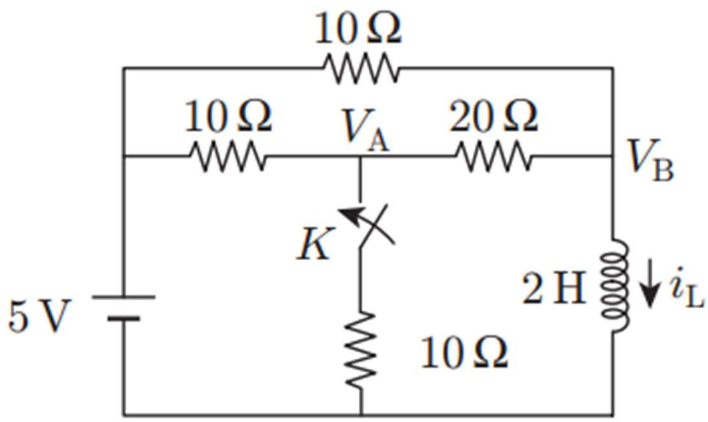
$I_2 = 4$ A

$I_c = I_1 + I_2 = 5$ A

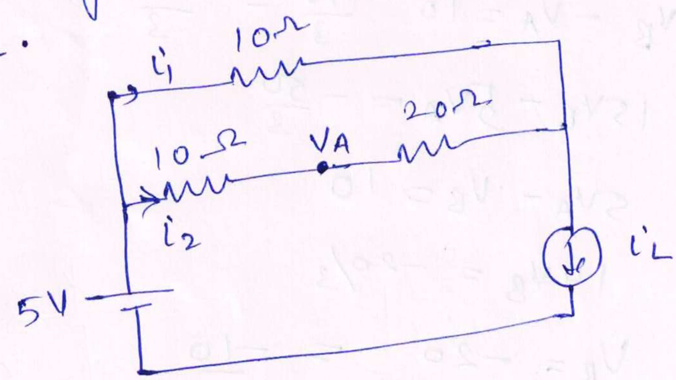
$\underline{V_L = 0}$

(Voltage across inductor is zero as inductor is short circuited)

3. In the network shown in the figure below, a steady state is reached with the switch K open. At t = 0, the switch K is closed. For the element values given, determine the values of $V_A(0^-)$ and $V_A(0^+)$.



Sol^n:
The steady state is reached with switch 'K' open.

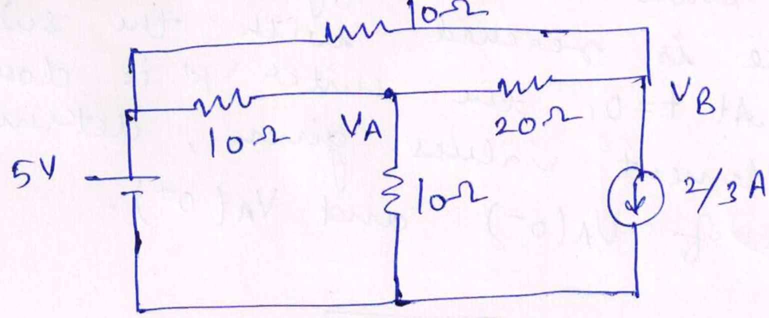


$$i_L' = \frac{5}{30} \times 4 = \frac{2}{3} A$$

$$\therefore V_A(0^-) = i_2' \times 20 = \frac{10}{40} \times \frac{2}{3} \times 20 = \frac{10}{3} V$$

$$i_L'(0^-) = i_L'(0^+)$$

At $t=0$, K is closed



$$\frac{V_A-5}{10}+\frac{V_A}{10}+\frac{V_A-V_B}{20}=0$$

$$\frac{V_B-5}{10}+\frac{V_B-V_A}{20}+i_L=0$$

$$2(V_A-5+V_A)+V_A-V_B=0$$

$$5V_A-V_B=10$$

$$2(V_B-5)+V_B-V_A=-\tfrac{2}{3}\times 20$$

$$3V_B-V_A=10-\frac{40}{3}=\frac{-10}{3}$$

$$15V_B-5V_A=-\frac{50}{3}$$

$$5V_A-V_B=10$$

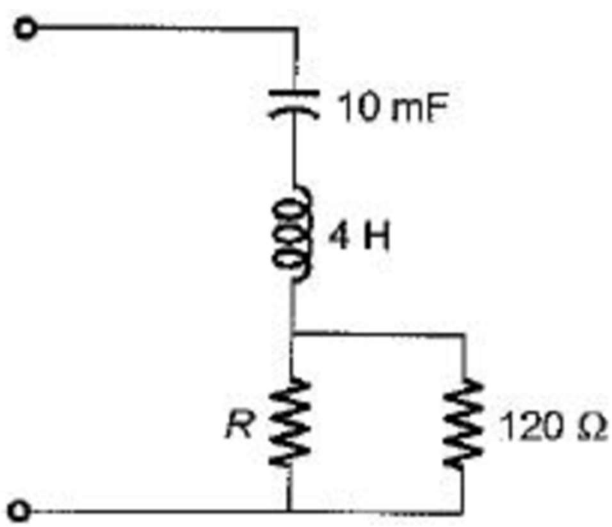
$$14V_B=-20/3$$

$$V_B=\frac{-20}{3\times 14}=\frac{-10}{21}$$

$$5V_A=10-\frac{10}{21}=\frac{200}{21}\Rightarrow V_A(0+)=\frac{40}{21}\text{ V.}$$

Ans. $\left(\frac{10}{3}\text{ V},\ \frac{40}{21}\text{ V}\right)$.

4. **If the circuit, shown below, is critically damped, find the value of R (in ohms) -**



Solution –

Given circuit is series RLC circuit.

For critically damped condition, we have

$$\boxed{\alpha = \omega_o}$$

$\Rightarrow \alpha = \frac{R_{eq}}{2L}$; $\omega_o = \frac{1}{\sqrt{LC}}$

$\Rightarrow$ Here $R_{eq} = R \| 120 = \frac{120 \times R}{120 + R}$

$\Rightarrow \frac{R_{eq}}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow R_{eq} = \sqrt{\frac{4L}{C}}$

$R_{eq} = \sqrt{\frac{4 \times 4}{10 \times 10^{-3}}} = 40\ \Omega$

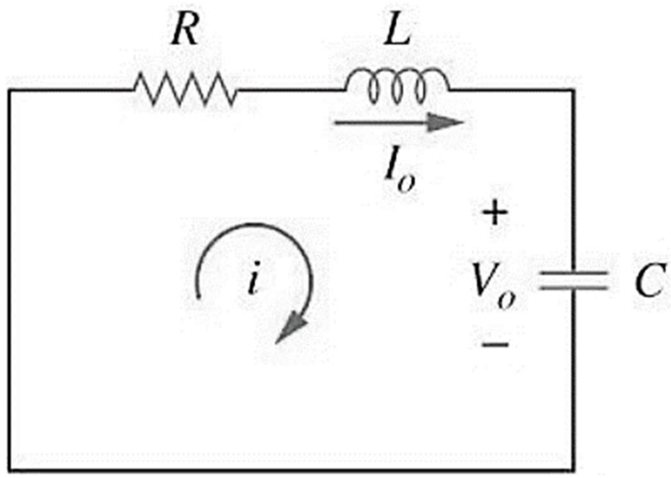
$\Rightarrow \frac{120R}{120 + R} = 40$

$120R = 4800 + 40R$

$80R = 4800$

$$\boxed{R = 60\ \Omega}$$

5. **In the given circuit, If L = 2H, C = 1 mF, R = 20Ω, the value of loop current i(t) is -**



Solution -

Given circuit is series RLC. Also, $R = 20\,\Omega$, $C = 1\,mF$, $L = 2\,H$

$$\Rightarrow \quad \alpha = \frac{R}{2L} = \frac{20}{2 \times 2} = 5 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 10^{-3}}} = 22.36 \text{ rad/s}$$

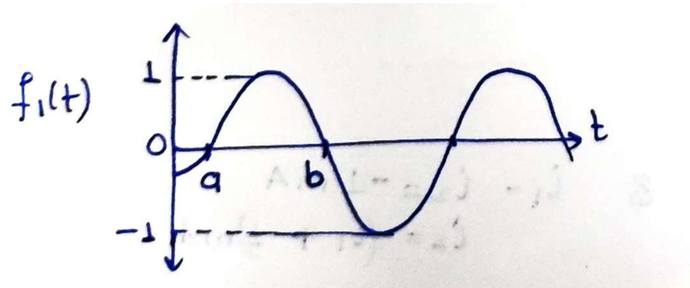
→ Since $\alpha < \omega_0$, $i(t)$ has underdamped response of the form

$$i(t) = e^{-\alpha t}\left(A_1 \cos\omega_d t + A_2 \sin\omega_d t\right);$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 21.8$

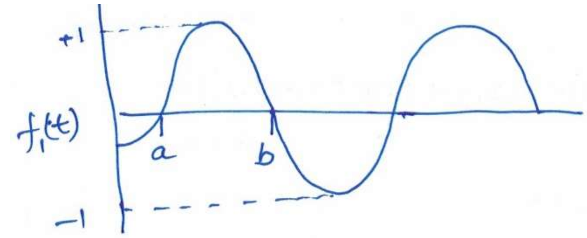
$$\therefore \boxed{i(t) = e^{-5t}\left(A_1 \cos 21.8t + A_2 \sin 21.8t\right)}$$

6. Find the Laplace transform of the following function f1(t) –



$$\boxed{\omega = \frac{\pi}{b-a}}$$

Solution –



$$\omega = \frac{\pi}{b-a}$$

Here,

$$\mathcal{L}(f_1(t)) = \int_{0^-}^{\infty} f_1(t)\, e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} \sin(\omega t - \omega a)\, e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} (\sin \omega t \cos \omega a - \cos \omega t \sin \omega a)\, e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} \cos \omega a \sin \omega t\, e^{-st}\, dt - \int_{0^-}^{\infty} \sin \omega a \cos \omega t\, e^{-st}\, dt$$

$$= \cos \omega a \int_{0^-}^{\infty} \sin \omega t\, e^{-st}\, dt - \sin \omega a \int_{0^-}^{\infty} \cos \omega t\, e^{-st}\, dt$$

$$= \frac{\cos \omega a \cdot \omega}{s^2 + \omega^2} - \frac{\sin \omega a \cdot s}{s^2 + \omega^2}$$

$$= \frac{\omega \cos \omega a - s \cdot \sin \omega a}{(s^2 + \omega^2)}$$