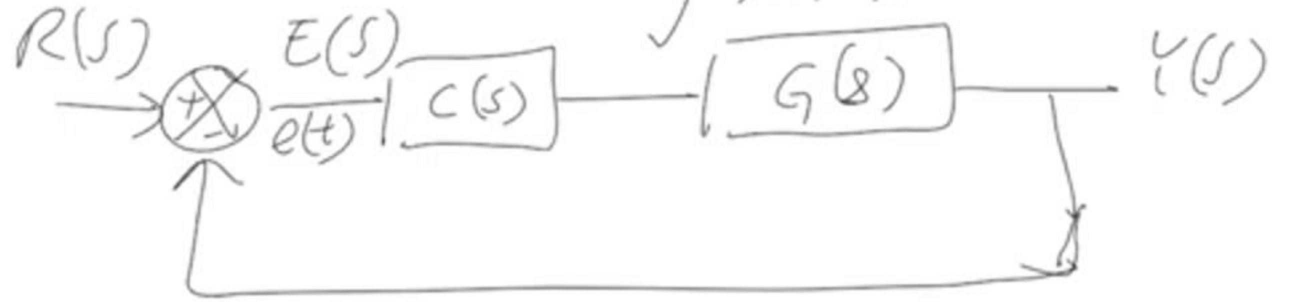
Steady state error Computation



$$\frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

$$E(s) = R(s) - Y(s) = R(s) - C(s)\,G(s)\,E(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + C(s)\,G(s)}$$

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$$E(s) = \frac{1}{1 + C(s)G(s)} \quad R(s)$$

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)}$$

Type 0 System $\qquad G(s) = \dfrac{1}{s^0(s+1)}, \quad \dfrac{1}{s^2+2s+1}$

Type 1 Syst $\qquad G(s) = \dfrac{1}{s(s+1)}, \quad \dfrac{1}{s(s^2+2s+1)}$

Type 2 " $\qquad G(s) = \dfrac{1}{s^2(s+1)}, \quad \dfrac{1}{s^2(s^2+2s+1)}$

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$$C(s) = 1, \quad G(s) = \frac{1}{s+2}$$

$$E(s) = \frac{R(s)}{1 + C(s)\,G(s)}, \quad R(s) = 1/s$$

$$= \frac{1/s}{1 + \frac{1}{s+2}}$$

$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{1}{1 + \frac{1}{s+2}} = \frac{1}{1 + \frac{1}{2}} = 2/3$$

$$\boxed{K_p = \lim_{s \to 0} C(s)\,G(s)} = 1/2, \quad e_{ss} = \frac{1}{1 + K_p}$$

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Type 1

$$C(s) = 1, \quad G(s) = \frac{1}{s(s+2)}$$

$$R(s) = 1/s^2$$

$$E_s = \frac{R(s)}{1 + C(s)\,G(s)}$$

$$e_{ss} = \lim_{s \to 0} s\,E_s = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + \frac{1}{s(s+2)}}$$

$$= \lim_{s \to 0} \frac{1/s}{1 + \frac{1}{s(s+2)}}$$

$$= \lim_{s \to 0} \frac{s+2}{s^2 + 2s + 1} = 2$$

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$$K_v = \lim_{s \to 0} s\, C(s)\, G(s)$$

for type 1, response to $1/s^2$

$$e_{ss} = \frac{1}{K_v}$$

$$K_a = \lim_{s \to 0} s^2\, C(s)\, G(s)$$

$\rightarrow$ Type 2 system $\rightarrow$ to $\left(t^2/2\right)$

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e_{ss}

Input

	$1/s$	$1/s^2$	$1/s^3$ (Parabola)
type 0	$\frac{1}{1+K_p}$	∞	∞
type 1	0	$\frac{1}{K_v}$	∞
type 2	0	0	$\frac{1}{K_a}$

$$C(s)\,G(s) = \frac{1}{s(s+2)}$$

$$e_{ss} = \lim_{s \to 0} s \, \frac{1/s}{1 + s(s+2)} = \frac{1}{1+0} = 1$$

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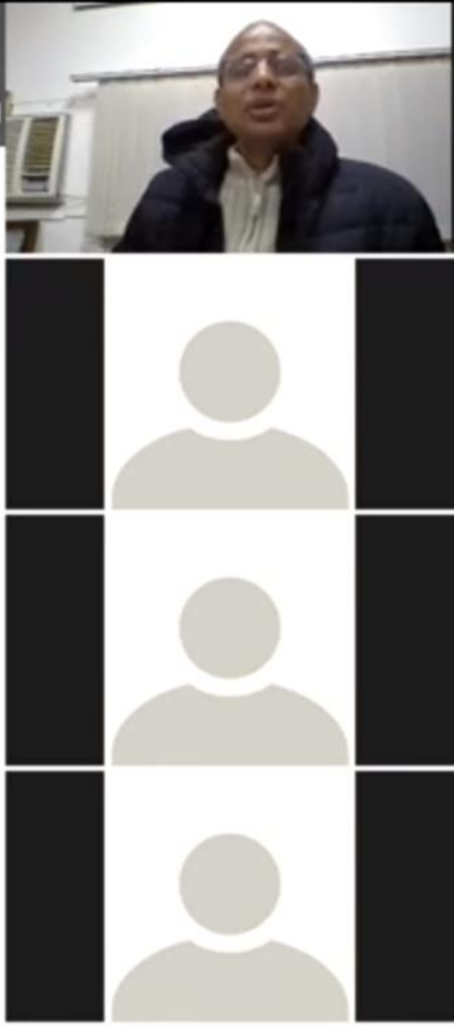


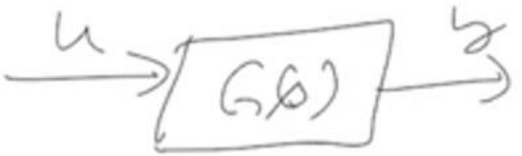
$$\frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

$$= \frac{\cancel{8}\,\frac{1}{s(s+2)}}{1 + \frac{1}{s(s+2)}} = \frac{1}{s(s+2)+1}$$

$$Y(s) = \frac{1}{s}\;\frac{1}{s(s+2)+1}$$

$$y_{ss} = \lim_{s \to 0} s\,Y(s) = \underline{1}\;,\; e_s$$

Stability



BIBO stable

stable

unstable

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$$G(s) = \frac{\beta_0 s^m + \beta_1 s^{m-1} + \cdots + \beta_m}{s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n}$$

$$= \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$G(t) = \mathcal{L}^{-1} G(s) = \mathcal{L}^{-1}\left[\frac{a_1}{s+p_1} + \frac{a_2}{s+p_2} + \cdots\right]$$

$$= a_1 e^{-p_1 t} + a_2 e^{-p_2 t} + \cdots$$

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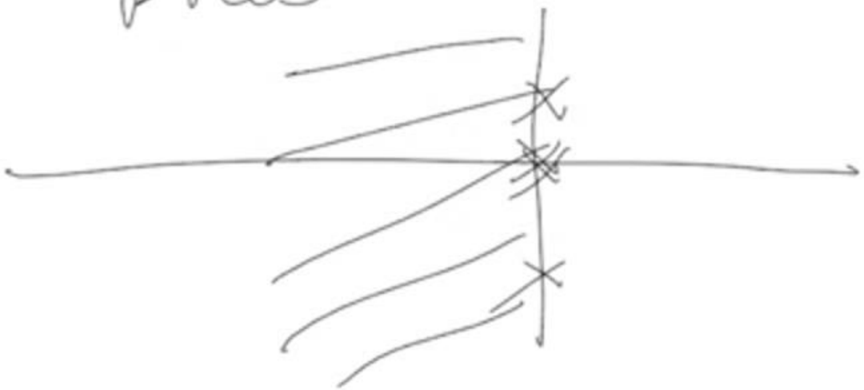
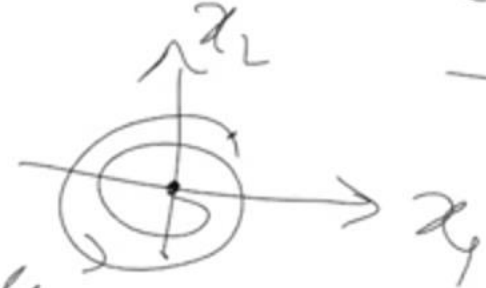
poles



$\frac{1}{s^2} \rightarrow t$

LHS



All stable, marginally stable, unstable, ~~stable~~

All poles in RHS

two poles on the imaginary

if one pole in RHS

Stability → poles in LHS

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

roots of $\quad 1 + C(s)G(s) = 0$

$$s^n + a_1 s^{n-1} + \cdots + a_n = 0$$

Routh array