Date 10/02/2021

Tutorial Session 3

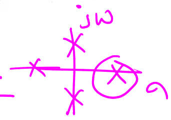
10 Feb 2021

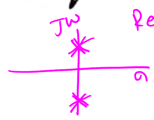
EE 250 - Control System Analysis

Q1 Determine the stability of the system with characteristic eqn

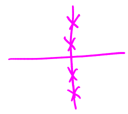
$d(s) = s^4 - 1$

$s = \pm 1, \pm 1j$

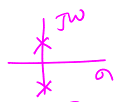




Repeated pole on imaginary axis → unstable



Non-repeated or one pair of poles on imaginary axis – marginally stable



Solution $d(s) = s^4 + 0.s^3 + 0.s^2 + 0.s + 1$

Routh Array

marginally stable

$s^4 + 0.s^2 - 1$

s^4	1	0	-1
s^3	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$
s^3	4	0	$\cancel{0}$
s^2	0	-1	
s^2	$\epsilon \to 0+ve$	-1	
s^1	$\frac{4}{\epsilon}$		0
s^0	-1		

$\frac{4\times 0 - 1\times 0}{4}$

$\frac{d}{ds}(s^4 - 1) = 4s^3 + 0s^0$

Auxiliary eqn approach

For complete zero row in routh table – Equation formed by above row gives poles of T.F.

Elements of first column are

		Sign
s^4	1	+
		] NC
s^3	4	+
		] NC
s^2	ϵ	+
		] NC
s^1	$4/\epsilon$	+
		] change
s^0	-1	−

$d(s) = s^4 - 1 = 0$

$(s^2+1)(s^2-1) = 0$

poles are $\pm 1, \pm j1$

one pole in RHS.

There is one ~~negative element &~~ one sign change. One pole in RHS plane. System is unstable.

Q2 Determine the stability of the system with $d(s) = s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50$ using Routh Array.

Soln

$2s^4 + 48s^2 - 50$

s^5	1	24	-25
s^4	2	48	-50
s^3	0	0	0
s^3	8	96	
s^2	24	-50	
s^1	112.66		
s^0	-50		

$$\frac{d}{ds}\left(2s^4 + 48s^2 - 50\right) = 0$$

$$8s^3 + 96s = 0$$

There is one element negative & one sign change. Hence the system is unstable with one pole in RHS plane.

Can you validate? Hint (Aux eqn)

Aux eqn $2s^4 + 48s^2 - 50 = 0$

$$s^4 + 24s^2 - 25 = 0$$

$$(s^2 + a^2)(s^2 - b^2) = 0$$

$$\Rightarrow a = 5, \; b = 1$$

These are also poles of original system.

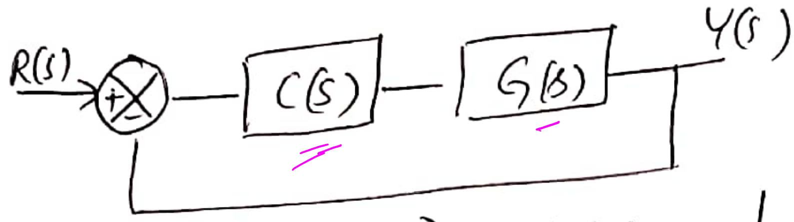
$\rightarrow \pm 5, \; \boxed{\pm j1}$ $\pm j1$

$s = -2$

We can notice that $(s^4 + 24s^2 - 25)(s+2) \leftarrow d(s)$

Thus there is one pole in RHS plane which is at $s = 5$

Q 3



$$C(s) = \frac{K(s+z)}{s+p}, \quad G(s) = \frac{1}{s(s-1)}$$

Find K, ~~z~~, p such that the closed loop system is stable.

Soln

$$\frac{Y(s)}{R(s)} = \frac{K(s+z)}{s^3 + (p-1)s^2 + (K-p)s + Kz}$$

$$d(s) = s^3 + (p-1)s^2 + (K-p)s + Kz$$

For $d(s)$ to have all poles in LHS plane,

$(p-1) > 0$, $(K-p) > 0$, $Kz > 0$.

(Necessary Condition)

Routh Array

s^3	1	$K-p$
s^2	$p-1$	Kz
s^1	$\frac{(p-1)(K-p)-Kz}{p-1}$	0
s^0	Kz	

1.Thenecessary(but not sufficient) condition for stability is that all the coefficients of thecharacteristic polynomiald(s) be positive.

2. Sufficient condition for stability:
A system is stable if and only if all theelements in the firstcolumn of the Routh array are positive

Sufficient conditions are

$$p-1 > 0$$
$$(p-1)(K-p) - Kz > 0$$
$$Kz > 0$$

These conditions can be summarized as

$$\left\{\begin{array}{l} P > 1 \\ K > P \\ Kz > 0 \\ Kz < (P-1)(K-P) \end{array}\right. \qquad \left.\begin{array}{l} Kz > 0 \\ Kz < (P-1)(K-P) \end{array}\right\} \Rightarrow 0 < Kz < (P-1)(K-P)$$

Let's select $P = 2$.

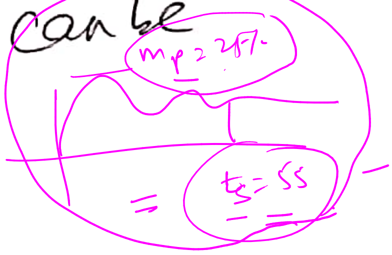
Then $\quad 0 < Kz < K-2$

$$\Rightarrow 0 < z < 1 - \frac{2}{K}.$$

If we select $K = 10$,

then $\quad 0 < z < 0.8$

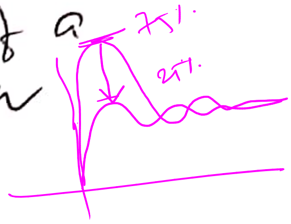
~~The~~ One of the possible stabilizing controller can be



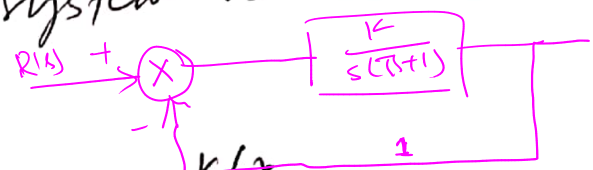
$$C(s) = 10\,\frac{s+0.5}{s+2}$$

We need to consider more time domain specifications such as M_p and t_s, to select a proper controller.

Q4 The open loop transfer fn of a unity feedback system is given by $G(s) = \frac{K}{s(\tau s+1)}$; $K>0, \tau>0$.



By what factor K be reduced so that peak overshoot of unity step response of the system is reduced from 75% to 25%?



Soln $T(s) = \frac{n(s)}{d(s)} = \frac{K}{\tau s^2 + s + K} = \frac{K/\tau}{s^2+\frac{1}{\tau}s+\frac{K}{\tau}}$.

$$d(s) = s^2 + \frac{1}{\tau}s + \frac{K}{\tau}$$

$$= s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$2\zeta\omega_n = \frac{1}{\tau}, \quad \omega_n^2 = \frac{K}{\tau}, \quad \omega_n = \sqrt{\frac{K}{\tau}}$$

$$2\zeta = \frac{1/\tau}{\sqrt{K/\tau}} = \frac{1}{\sqrt{K\tau}}$$

$$2\zeta_1 = \frac{1}{\sqrt{K_1\tau}}, \quad 2\zeta_2 = \frac{1}{\sqrt{K_2\tau}}.$$

$$\boxed{\frac{\zeta_2^2}{\zeta_1^2} = \frac{K_1}{K_2}} \quad — (1)$$

For ζ_1, $M_{p_1} = 0.75 \Rightarrow e^{-\frac{\pi\zeta_1}{\sqrt{1-\zeta_1^2}}} = 0.75$

$$\frac{\pi\zeta_1}{\sqrt{1-\zeta_1^2}} = \ln 0.75 \simeq 0.29$$

Similarly

$$\frac{\pi k_2}{\sqrt{1-k_2^2}} = \ln 0.25 = 1.38$$

$$(\pi^2 + (1.38)^2)\, k_2^2 = (1.38)^2.$$

$$(\pi^2 + (0.29)^2)\, k_1^2 = (0.29)^2$$

$$k_2^2 = (1.38)^2 / 11.77$$

$$k_1^2 = (0.29)^2 / 9.95$$

$$\frac{k_2^2}{k_1^2} \simeq 19.14 = \frac{K_1}{K_2} = 19.14 \Rightarrow K_2 = \frac{K_1}{19.14} \quad \frac{1}{19.14}$$

mp = 25% ; mp (%) = 75%

The gain needs to be reduced by a factor of 19.14

Q5 A servomechanism Controls the angular position θ of a load of moment of inertia $J = 2.5\ kg\text{-}m^2$. The damping torque (friction) coeff referred to the load shaft is B 40 N-m/(rad/sec). The motor develops a torque at the load at the rate of 1000 n-m/rad of error

P-controller or Proportional controller

θ error $\boxed{1000}$ T

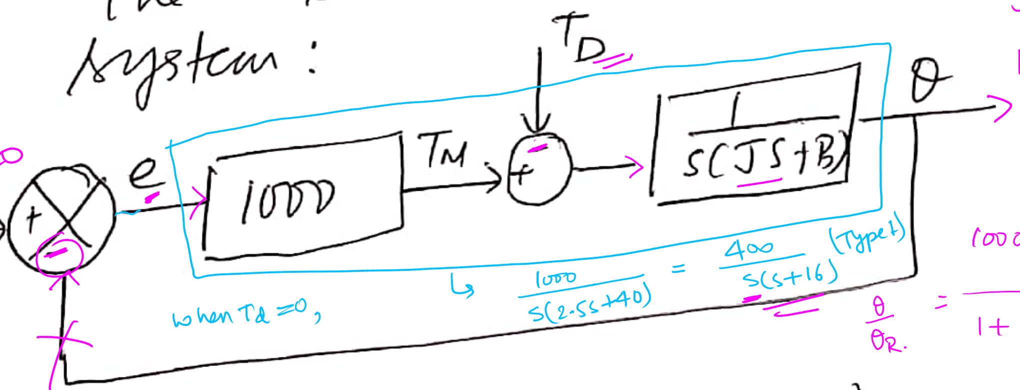
(a) Determine the frequency of transient oscillation ω_d, the peak overshoot, the time to peak and the steady state error due to unit step input of 1 rad.

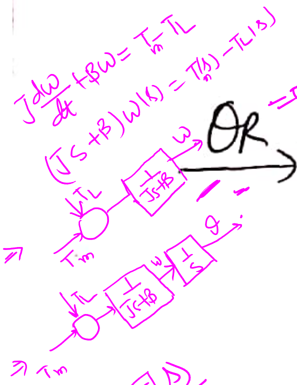
(b) Determine the steady state error when the command signal is 1 rev/min

(c) Determine the steady state error when a steady torque of 10 N-m is applied at the load shaft.

Solⁿ The block diagram of the system:

Given
$J = 2.5\ kg.m^2$
$B = 40\ N\text{-}m/(rad/sec)$





when $T_d = 0$, $\hookrightarrow \frac{1000}{s(2.5s+40)} = \frac{400}{s(s+16)}$ (Type 1)

$$\frac{\theta}{\theta_R} = \frac{1000 \times \frac{1}{s(Js+B)}}{1 + \frac{1000 \times 1}{s(Js+B)}}$$

$$\frac{\theta}{\theta_R} = \frac{1000}{Js^2 + Bs + 1000} = \frac{1000}{2.5s^2 + 40s + 1000}$$

$$= \frac{400}{s^2 + 16s + 400}$$

$E(s)$ $T_D(s)$

$$\frac{E(s)}{T_d(s)} = \frac{\frac{1}{s(Js+B)}}{1 - \left(1000 \times \frac{1}{s(Js+B)} \times -1\right)} = \frac{1}{Js^2 + Bs + 1000}$$

$-\frac{E}{T_D}$

$$= \frac{\theta}{T_D} = -\frac{1}{2.5s^2+40s+1000}$$

(a) $d(s) = s^2 + 16s + 400$

$\omega_n^2 = 400, \quad \omega_n = 20 \text{ rad/sec}.$

$2\zeta\omega_n = 16$

$$\zeta = \frac{16}{2\times 20} = 0.8$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$
$$= 20\sqrt{1-0.64} = 12 \text{ rad/sec}.$$

Peak overshoot $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$
$$= e^{-\frac{2.513}{0.6}}$$
$$= e^{-4.18} = e^{-4.18}$$
$$= 0.015 \cong 1.5\%$$

Time to Peak $t_p = \pi/\omega_d$
$$= \pi/12 = 0.26 \text{ sec}.$$

$\frac{E(s)}{\theta(s)} = \frac{1}{1+\frac{1000}{s(Js+B)}} = \frac{s(Js+B)}{Js^2+Bs+1000}$

$\theta(s) = \frac{1}{s}$;

$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{s(Js+B)}{Js^2+Bs+1000}\cdot\frac{1}{s} \to 0$

$e_{ss}\big|_{\text{step input}} = 0$ → position input

→ type I system.

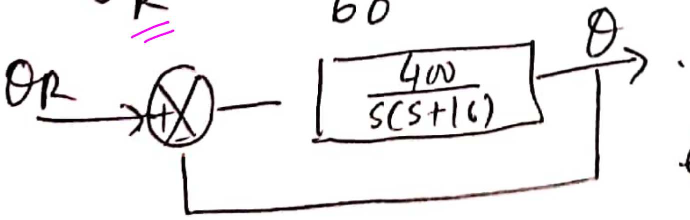
[Integrator or $\frac{1}{s}$ is inside the plant].

$e_{ss} = \frac{1}{1+K_p} \to 0$, $K_p \to \infty$ position error constant

$K_p = \lim_{s\to 0} \frac{400}{s(s+16)}$

(b) Input $= 1$ rev/min $= \frac{2\pi}{60}$ rad/sec. ← velocity input or Ramp input.

$$\theta_R = \frac{2\pi}{60}t \rightarrow \theta_R(s) = \frac{2\pi/60}{s^2}$$



It is a type 1 System.

$$e_{ss}\big|_{\text{ramp}} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} s\frac{400}{s(s+16)} = \frac{400}{16}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \cancel{s}\cdot \frac{\cancel{s}(Js+B)}{Js^2+Bs+1000}\cdot\frac{\frac{2\pi}{60}}{\cancel{s^2}}$$

$$= \frac{B}{1000}\times\frac{2\pi}{60} = \frac{40\times 2\pi}{1000\times 60}$$

$$= 0.00418\,\text{rad}$$

$$K_v = \frac{400}{16}.$$

$$e_{ss}\Big|_{\text{unity ramp}} = \frac{16}{400} = 0.04.$$

$$e_{ss}\Big|_{\frac{2\pi}{60}\cdot t} = 0.04\times\frac{2\pi}{60}\ \text{rad}.$$

$$= 0.00418\ \text{rad}.$$

?

(c) $$\theta_{ss}\Big|_{\frac{10}{s}} = \lim_{s\to 0} s\cdot\frac{-1}{2.5s^2+40s+1000}\times\frac{10}{s}$$

$$\frac{E(s)}{T_d(s)} = \frac{-\theta(s)}{T_d(s)}$$

$$= \frac{1}{2.5s^2+40s+1000}$$

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot\frac{1}{2.5s^2+40s+1000}\times\frac{10}{\cancel{s}}$$

$$= \frac{-10}{1000}$$

$$= -0.01\ \text{rad}.$$

$$= \frac{10}{1000} = 0.01$$