Descriptive Solutions

1. $\begin{cases} y'' + \lambda y = 0 \\ y(3\pi) = 0 = y(4\pi). \end{cases}$

$\boxed{1 \text{ marks}}$ For showing $\lambda < 0$ cannot be an Eigen value.

STUDENTS MAY OBTAIN THIS BY "TWO" WAYS.

Way 1: They can directly say that this is Regular sturm Liouville eigen value problem (RSLEVP) since we know "$\lambda < 0$" not possible.

Way. 2 writing down the Auxilliary eqn: $m^2 + \lambda = 0$

and assuming $\lambda = -\gamma^2$ (<0) (if possible) (for $\gamma > 0$)

$m = \pm \gamma$

$\Rightarrow$ General soln:

$y(x) = Ae^{\gamma x} + Be^{-\gamma x}$.

As $y(3\pi) = y(4\pi) = 0$

$$\Rightarrow \begin{cases} Ae^{\sigma 3\pi} + Be^{-3\sigma\pi} = 0 \\ Ae^{\sigma 4\pi} + Be^{-4\pi\sigma} = 0 \end{cases}$$

Here $\begin{vmatrix} e^{3\sigma\pi} & e^{-3\sigma\pi} \\ e^{4\sigma\pi} & e^{-4\sigma\pi} \end{vmatrix} = e^{-\sigma\pi} - e^{\sigma\pi}$

$= 0$ (only if $\sigma = 0$).

$\Rightarrow A = B = 0$

$\Rightarrow y \equiv 0$ is the soln.

Contradiction

$\boxed{1 \text{ marks}}$ To rule out that $\lambda = 0$ is an eigenvalue.

AGAIN STUDENTS MAY ACHEIVE IN TWO WAYS :-

Way 1 $\lambda = 0$ cannot be E.V as this is RSLEVP..

Way 2 Auxilliary Eqn $m^2 = 0$.

$\Rightarrow y(x) = Ax + B$ is General soln

$\Rightarrow y(3\pi) = y(4\pi) = 0 \Rightarrow y \equiv 0$.

(Contradiction).

1 marks :- For the case $\lambda > 0$, writing down the general Solution correctly..

, Auxilliary Eqn. is

$m^2 + \sigma^2 = 0$.

$$\Rightarrow \boxed{y(x) = A\cos(\sigma x) + B\sin(\sigma x)} \rightarrow (*)$$

Now they have to put the Boundary conditions

$$y(3\pi) = 0 = y(4\pi)$$

$$\Rightarrow \begin{cases} A\cos(3\sigma\pi) + B\sin(3\sigma\pi) = 0 \\ A\cos(4\pi\sigma) + B\sin(4\pi\sigma) = 0 \end{cases}$$

$\boxed{1 \text{ marks}}$

For getting non trivial Sol. of A, B, we want

$$\begin{vmatrix} \cos(3\sigma\pi) & \sin(3\sigma\pi) \\ \cos(4\sigma\pi) & \sin(4\sigma\pi) \end{vmatrix} = \sin(\sigma\pi) = 0$$

$\boxed{1 \text{ marks}}$

$\Rightarrow \quad \sin(r\pi) = \sin(n\pi) \quad n \in \mathbb{Z}$

$\Rightarrow \quad r = n$

$$\boxed{\lambda = r^2 = n^2 \quad , n \in \mathbb{N}}$$

1 mark for finding out the Eigen functions corresponding to the Eigen value "n^2".

$y_n(x) = A_n \cos(nx) + B_n \sin(nx)$ from ⊛

$y_n(3\pi) = 0 \Rightarrow A_n = 0$

$$\Rightarrow \boxed{y_n(x) = B_n \sin(nx)}$$

is the required Eigen function.

Another Smart Way

If some one says:
we already know that
$y_n(t) = A_n \sin(nt)$ is an E.F
of the problem,

$$\begin{cases} y'' + \lambda y = 0 \quad \text{on } [0,\pi] \\ y(0) = y(\pi) \end{cases}$$

corresponding to the E.V "n^2."

In the given problem
Just the domain is shifted.
Hence " E V " will be the same.
and the E.F. will just be shifted
given by

$$y_n(t) = y_n(t - 3\pi)$$
$$= A_n \sin n(t - 3\pi)$$
$$= -A_n \sin(nt).$$

(Done)