EE 210

Sol^n to HA # 6

1. $I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS}-V_{TN})^2 = \frac{K_n'}{2}\frac{W}{L}V_{DS,sat}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{K_n' V_{DS,sat}^2} = \frac{2\times 100\times 10^{-6}}{40\times 10^{-6}\times 0.1^2} = \underline{\underline{500}}$

$R_o = r_o = \frac{1+\lambda V_{DS}}{\lambda I_D} \simeq \frac{1}{\lambda I_D}$ ($\because \lambda V_{DS} \ll 1$) $\Rightarrow \lambda = \frac{1}{R_o I_D} = \frac{1}{10^6\times 100\times 10^{-6}} = \underline{\underline{0.01\,V^{-1}}}$

Also, $\lambda = \frac{1}{L}\frac{dX_d}{dV_{DS}} \Rightarrow L = \frac{1}{\lambda}\frac{dX_d}{dV_{DS}} = \frac{1}{0.01}\times 10\times 10^{-3} = \underline{\underline{1.0\,\mu m}}$, $W = \underline{\underline{500\,\mu m}}$ ($\because W/L = 500$)

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2. All transistors have same I_S. Q_1 & Q_2 share common base, collector, & emitter terminals $\Rightarrow$ can be clubbed to a single transistor Q_A, having $I_{SA} = 2I_S$. Similarly, Q_3, Q_4, Q_5 can be clubbed to another single transistor Q_B, with $I_{SB} = 3I_S$. $I_{REF} = \frac{V_{CC}-V_{BEA}}{R} = \frac{15-0.7}{10K} = \underline{\underline{1.43\,mA}}$. $\therefore$ Neglecting all non-idealities, $I_O = K' I_{REF}$, where $K' = \frac{I_{SB}}{I_{SA}} = \frac{3}{2} \Rightarrow I_O = \underline{\underline{2.145\,mA}}$ (ideal value)

Considering all non-idealities, except β mismatch, (also V_A same):

$$I_O = \frac{K' I_{REF}\left(1+\frac{V_{CEB}}{V_A}\right)}{\left[1+\frac{1+K'}{\beta}\right]\left(1+\frac{V_{CEA}}{V_A}\right)} = \frac{1.5\,I_{REF}\left(1+\frac{V_O}{130}\right)}{\left[1+\frac{1+1.5}{50}\right]\left(1+\frac{0.7}{130}\right)} = \underline{2.032\left(1+\frac{V_O}{130}\right)\,mA}$$

$\Rightarrow$ for $V_O = 1V$, $I_O = \underline{\underline{2.048\,mA}}$ (-4.52% change); for $V_O = 5V$, $I_O = \underline{\underline{2.11\,mA}}$ (-1.63% change); $V_O = 30V$, $I_O = \underline{\underline{2.5\,mA}}$ ($+16.55\%$ change). $\therefore$ For reasonable values of V_O, the output current tracks its ideal value closely, however, for large values of V_O ($\simeq 30V$), I_O departs significantly from its ideal value).

Neglecting all non-idealities, $R_O = \frac{V_A}{I_O} = \frac{130\,V}{2.145\,mA} = \underline{\underline{60.61\,k\Omega}}$ (ideal value).

Including all non-idealities, for $V_O = 1V$, $R_O = \underline{\underline{63.48\,k\Omega}}$ ($+4.74\%$ change); $V_O = 5V$, $R_O = \underline{\underline{61.61\,k\Omega}}$ ($+1.65\%$ change); $V_O = 30V$, $R_O = \underline{\underline{52\,k\Omega}}$ (-14.2% change).

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3. Neglecting base currents, $V_O = 0.5\,V = V_{CE2}(sat) + I_O R_2$, with $V_{CE2}(sat) = 0.2\,V$ (lowest allowed value for proper operation of current source) $\Rightarrow I_O R_2 = \underline{\underline{0.3\,V}}$

Now, assuming $r_{\pi 2} \gg R_2$ (we will verify this assumption later), the output resistance $R_O \simeq r_{o2}(1+g_{m2}R_2) = r_{o2}\left(1+\frac{I_O R_2}{V_T}\right) = r_{o2}\left(1+\frac{0.3}{0.026}\right) = 10\,M\Omega$,

$\Rightarrow r_{o2} = \underline{\underline{797.55\,k\Omega}}$. Also, $r_{o2} = \frac{V_A}{I_O} = \frac{130V}{I_O} = 797.55\,k\Omega \Rightarrow I_O = \underline{\underline{163\,\mu A}}$

& $R_2 = \frac{0.3V}{I_O} = \underline{\underline{1.84\,k\Omega}}$. Now, $r_{\pi 2} = \beta/g_{m2} = \frac{\beta V_T}{I_O} \simeq \frac{100\times 26\,mV}{163\,\mu A} = \underline{\underline{16\,k\Omega}}$, which is almost 10 times R_2. $\therefore$ The initial assumption made that $r_{\pi 2} \gg R_2$ is not too bad! Next, neglecting ΔV_{BE}, $I_{REF}R_1 = I_O R_2 \Rightarrow I_{REF} = 163\,\mu A\times\frac{1.84\,k\Omega}{1\,k\Omega} = \underline{\underline{300\,\mu A}}$, & $R' = \frac{V_{CC}-2V_{BE}}{I_{REF}} = \frac{5-1.4}{300\,\mu A} = 12\,K$, where $R' = R+R_1$. $\because R_1 = 1\,k\Omega$,

$\therefore R = \underline{\underline{11\,k\Omega}}$. Finally, $\Delta V_{BE} = V_T \ln\frac{I_{REF}}{I_O} = 26\,mV\times\ln\frac{300\,\mu A}{163\,\mu A} = 15.86\,mV$, which is way smaller than $I_O R_2$ of 300 mV.

Using the calculated value of I_{REF} $(= 300\,\mu A)$, $I_{REF} R_1 = 300$ mV. Now, the simple approximation for ratioed current mirror will break down when ΔV_{BE} equals 10% of this drop, i.e., 30 mV. Using $\Delta V_{BE} = V_T \ln \frac{I_{REF}}{I_O}$, we get $I_O = I_{REF} e^{-\Delta V_{BE}/V_T}$ $= 300\,\mu A \times e^{-30\,mV/26\,mV} = \underline{94.63\,\mu A}$, & $I_O R_2 = 270$ mV $(= I_O R_{EF} - \Delta V_{BE}) \Rightarrow R_2 = \underline{2.85\ k\Omega}$

$\therefore V_{O,min} = V_{CE_2}(sat) + I_O R_2 = 0.2 + 94.63\,\mu A \times 2.85\,k\Omega = \underline{0.47\,V}$, & $R_o = r_{o2}(1 + g_{m2} R_2)$ with $r_{o2} = \frac{V_A}{I_O} = \frac{130\,V}{94.63\,\mu A} = \underline{1.37\,M\Omega}$ $\Rightarrow R_o = 1.37\,M\Omega \times (1 + \frac{94.63\,\mu A}{26\,mV} \times 2.85k) = \underline{15.6\ M\Omega}$

$\therefore$ The ckt. performance becomes better if we push it to the limit of our approximation.

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<u>4</u> Neglecting base currents, $I_{REF} = I_{C2} = \frac{12 - 2\times 0.7}{105K + 1K} = \underline{100\,\mu A}$. Q_2-Q_3 form a ratioed current mirror, with equal resistances of 1 kΩ connected at their emitters to ground. $\therefore I_O = I_{REF} = \underline{100\,\mu A}$. Now, base voltage of Q_2, $V_2 = V_{BE_2} +$ $(1\,k\Omega) \times (100\,\mu A) = \underline{0.8\,V}$. $\therefore$ The current thru the 20 kΩ "Keep Alive" resistor, $I_{20K} = \frac{0.8\,V}{20K} = \underline{40\,\mu A}$. Neglecting base current, this is also the emitter current of Q_1, as well as its collector current. $\Rightarrow I_{C1} = \underline{40\,\mu A}$.

The actual computation of output resistance is quite tedious, without making any assumptions. However, as discussed in class, the base of Q_3 can be assumed to be at ac ground. In that case, $R_o \simeq r_{o3}(1 + g_{m3}[r_{\pi 3} \| (1k\Omega)])$. Now, $r_{o3} =$ $\frac{V_A}{I_O} = \frac{130V}{100\,\mu A} = 1.3\,M\Omega$, $r_{\pi 3} \to \infty$ ($\because$ base currents are neglected) $\Rightarrow R_o \simeq$ $r_{o3}(1 + \frac{I_O}{V_T} \times 1\,k\Omega) = 1.3\,M\Omega \times (1 + \frac{100\,\mu A}{26\,mV} \times 1\,k\Omega) = \underline{6.3\ M\Omega}$

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<u>5</u> Assuming all transistors have identical V_{BE} of 0.7V, to a first-order approximation, $I_O \simeq \frac{0.7\,V}{350\,\Omega} = \underline{2\,mA}$ (note that it is almost independent of every thing else in the ckt.). By inspection, the base of Q_2 is at a low impedance terminal, & can be taken to be at ac ground. Also, neglecting base currents, $r_{\pi 2} \to \infty$.

$\therefore R_o = r_{o2}(1 + g_{m2} \times 350) = \frac{V_A}{I_O} \times (1 + \frac{I_O}{V_T} \times 350) = \frac{130\,V}{2\,mA} \times (1 + \frac{2\,mA}{26\,mV} \times 350\,\Omega)$ $= \underline{1.8\ M\Omega}$ (the simplicity of the ckt. should be noted).

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