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A) $$\frac{Y(s)}{U(s)} = C.(sI-A)^{-1}.B$$

$$\therefore \frac{Y(s)}{U(s)} = \frac{-4.191}{s^2 - 1.51e^{-14}s - 29.98} \sim \frac{-4.191}{s^2 - 29.98}$$

B) a) $$M_p = e^{\frac{-\eta\pi}{\sqrt{1-\eta^2}}} \Rightarrow \frac{\ln M_p}{\pi} = \frac{-\eta}{\sqrt{1-\eta^2}}$$

$$\Rightarrow \boxed{\eta = \sqrt{\frac{(\ln M_p)^2}{(\pi^2 + (\ln M_p)^2)}}}$$

So, for $M_p = 0.2$, $\boxed{\eta = 0.4559}$

and settling time = 1 sec $\Rightarrow \frac{4}{\eta\omega_n} = 1 \Rightarrow \boxed{\omega_n = 8.7729}$

So dominant poles are $-\eta\omega_n \pm \omega_n\sqrt{1-\eta^2}$

$$\Rightarrow -4.0 \pm 7.8079j$$

b) The open loop system is unstable so we need a Lead ~~comp~~ compensator to ~~increase~~ improve the transient behaviour.

angle contributed = $-219.8095°$ (from MATLAB)

So, we need a lead of $39.8095°$.

~~After trial and error let us put compensator zero at s=-3, then using angle criterion, we get the compensator pole at s=~~

let the choosen compensator zero be cz.

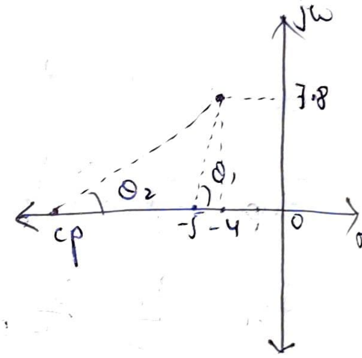
if $cz = -1 \Rightarrow$ system is unstable ~~can't find pole~~

if $cz = -2 \Rightarrow$ system is unstable ~~can't find pole~~

if $cz = -3 \Rightarrow$ System is stable but it has sluggish growth as the third pole is nearer to origin, hence dominant.

if $cz = -5 \Rightarrow$ System is stable
Third pole is farther than the desired dominant poles from the origin

to find compensator pole cp,



So, $\theta_1 - \theta_2 = 39.8095°$

$\Rightarrow \theta_2 = (\theta_1 - 39.8095°)$

$\therefore \dfrac{7.8}{-4-cp} = \tan\theta_2 = \tan(\theta_1 - 39.8095°)$

$\Rightarrow cp = -4 - \dfrac{7.8}{\tan\theta_2} \Rightarrow cp = -12.4047$ (MATLAB)

So, $\boxed{c(s) = \dfrac{K(s+5)}{(s+12.4047)}}$ and $L(s) = c(s) * g(s)$

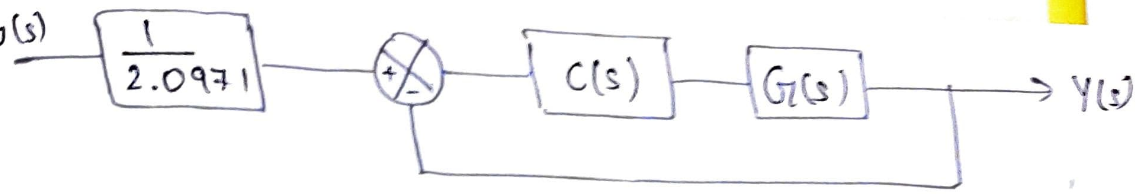
where $g(s) \sim \dfrac{-4.191}{s^2 - 29.98}$

$\therefore K = \left.\dfrac{-1}{L(s)}\right|_{s=-4+7.8079j} = -33.9263$

$\therefore c(s) = \dfrac{-33.9263(s+5)}{s+12.4047}$

Also, dc. gain $= \lim\limits_{s \to 0} \dfrac{K\,L(s)}{1+KL(s)} \sim 2.0971$

(c)



The compensated system, $M(s) = \dfrac{K \cdot L(s)}{1 + K \cdot L(s)}$

has roots at $-4 \pm 7.8079j$ and -4.4047

d) $M(s) = \dfrac{K \cdot L(s)}{1 + K \cdot L(s)} = \dfrac{142.2s + 710.9}{s^3 + 12.4s^2 + 112.2s + 339}$

final transfer function $= \dfrac{1}{2.0971} \cdot M(s)$

e) Step response is plotted as figure 1.

Peak overshoot = 14.646%

Settling Time = 0.9548 s

f) Root Locus is plotted as figure 2

Asymptote angles = $90°, -90°$

Centroid = -3.7023

for breakaway point,

$$\frac{d\left(\frac{1}{L(s)}\right)}{d(s)} = 0 \Rightarrow \left\{\frac{d}{ds}\left(s^3 + 12.4s^2 - 29.98s - 371.9\right)\right\}(s+5) - \left\{\frac{d}{ds}(s+5)\right\}\left(s^3 + 12.4s^2 - 29.98s - 371.9\right) = 0$$

$$\Rightarrow (s + 7.2749)(s^2 + 6.4256s + 15.258) = 0$$

$$\Rightarrow \boxed{s = -7.2749}$$ (can be verified from figure 2)

for crossing of imaginary axis,

the root locus crosses the imaginary axis at $s = 0$

for $K = -17.7$

c)

a) $\dot{x} = Ax + Bu$

$\Rightarrow \dot{x} = (A - BK)x$

The state trajectories are plotted in figure 3

b) When h is increased to 1, the plots lose their smoothness and run a risk of becoming unbounded (figure 4)

figure 5 validates the claim at higher values of the sampling rate (here, $h = 5$)

c) figure 6 shows the state trajectories for the modified initial conditions.