2.a. (i) Let $X_1, \ldots, X_n$ be i.i.d. $N(0,1)$, $\bar{X}_n$ is the sample mean and S_n^2 is the sample variance. Fix $n = 5$. Find the value of $E(\bar{X}_5/S_5)$ and $Var(\bar{X}_5/S_5)$.

(ii) Let $X_1, \ldots, X_n$ be i.i.d. Unif$[0,1]$. Prove that $X_{(1)}X_{(n)} \xrightarrow{P} c$ as $n \to \infty$. Find the limit c. $[3+3=6]$

2.b. Let $(Z_1, Z_2) \sim N_2((0,0), \Sigma)$ with $\Sigma = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$. Find

(i) $E(\max\{Z_1, Z_2\})$.
(ii) $E(e^{tZ_1 Z_2})$ with $t \in \mathbb{R}$.

Hint for (ii): $Z_2|Z_1 \sim$? $[2+3=5]$