MSO202A-Complex Analysis

Quiz

30 September 2020

Instructions:

- Use the bubble sheet provided along (or the one sent for trial test) to give your answers.
- Please sign in the Name field, of the bubble sheet.
- There are multiple versions of the paper and the version sent to you is on record, so don't try to share your paper with anyone. If your response sheet is different from the version sent to you, action will be taken appropriately.
- Because of the above, it is very important to answer exactly only those question numbers appearing in your question paper.
- If you mark some other question by mistake, remember to strike through that entire row.
- You have time until 9.50am to upload your bubble sheet on Gradescope.
- There is an option on gradescope to resubmit until 9.50am so it is recommended that you don't wait till the last minute. Submit latest by 9.45am and resubmit later any updated version, if you want to.
- Since the final submission is time-bound, I suggest that if you can afford it, you keep gradescope login open on multiple devices with different sources of connectivity. (Using your parent's phone or some such alternatives).
- Note again that, the time constraint makes it important that you do not wait till last minute for submission.
- When shading, make sure to shade atleast 3/4th of the circle.

Notation:

- z^w **refers to the principal value.**
- **Log**(z) **is the principal logarithm.**
- $\mathbb{R}^{<0} = \{x \in \mathbb{R} : x < 0\}$.

NOTE:-

1. Download the question paper since the course page on Piazza will be frozen after 9.15am.
2. There can be multiple correct choices for each question below. Each correct choice scores 1 point. If a wrong choice is chosen the entire question is awarded 0 marks.
3. Be careful with question number when shading bubble sheet.

I. Choose the correct statements in the following.:

3. Let $I = \displaystyle\int_\gamma \frac{dz}{(z-a)(z-b)}$ where $\gamma(t) = e^{2\pi i t}$, $0 \le t \le 1$. Then

   (a) $I = 0$ if $|a| < 1$, $|b| < 1$.
   (b) $I = 0$ if $|a| < 1 < |b|$.
   (c) $I = 0$ if $|b| < 1 < |a|$.
   (d) $I = 0$ if $1 < |a|$, $1 < |b|$.
   (e) none of the above

8. Let $\{z_n\}_{n=0}^\infty$ be a sequence of complex numbers. Then

   (a) Let $|z| = 1$. Then $\lim\limits_{n\to\infty} z^n = l$ such that $|l| = 1$.
   (b) $\displaystyle\sum_{n=0}^\infty |z_n| = |z| \Rightarrow \sum_{n=0}^\infty z_n = z$.
   (c) $\displaystyle\lim_{n\to\infty} \frac{z_n}{n} = 0 \Rightarrow \{z_n\}_{n=0}^\infty$ is convergent.
   (d) $\displaystyle\lim_{n\to\infty} z_n = 1 \Rightarrow \lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^n z_k = 1$.
   (e) none of the above

16. Let $f : \mathbb{C} \to \mathbb{C}$ such that $f(x+iy) = u(x,y) + iv(x,y)$ where $u, v : \mathbb{R}^2 \to \mathbb{R}$. Then

   (a) f is differentiable at $z_0 = x_0 + iy_0 \Rightarrow v$ is a harmonic conjugate of u around (x_0, y_0).
   (b) if f is analytic at z and $|f(z)| = 1$ $\forall z \in \mathbb{C} \Rightarrow f$ is constant on $\mathbb{C}$.
   (c) $\sin(f(z))$ is analytic wherever $f(z)$ is analytic.
   (d) $\mathrm{Log}(f(z))$ is analytic wherever $f(z)$ is analytic.
   (e) none of the above

24.

(a) $\sin(z)$ is analytic and unbounded on $\mathbb{C}$.

(b) $\sin(z)$ has no zeroes in $\mathbb{C} \setminus \{x+iy : x, y \in \mathbb{R} \text{ and } y=0\}$.

(c) $n\text{Log}(z) = z^n \ \forall z \in \mathbb{C}^*$ where n is an integer.

(d) $(e^{iw})^z = e^{iwz} \ \forall w, z \in \mathbb{C}$.

(e) none of the above

28.

(a) e^z does not take values in $\mathbb{R}^{<0}$.

(b) e^z is not an injection from $\mathbb{C}$ to $\mathbb{C}^*$.

(c) $\sin(z) = 2$ has no solution in $\mathbb{C}$.

(d) $\text{Log}(\exp(z)) = z \ \forall z \in \mathbb{C}$.

(e) none of the above

34. Let $f : \mathbb{C} \to \mathbb{C}$.

(a) Then $f(z)$ satisfies the CR equation at $z_0 \Rightarrow [f(z)]^n$ also does for $z = z_0$ and all $n > 0$.

(b) Let $f(z) = \begin{cases} \frac{\bar{z}^3}{z^2} & \text{if } z \neq 0, \\ 0 & \text{if } z = 0 \end{cases}$ then f is continuous on $\mathbb{C}$.

(c) Let $f(z) = \begin{cases} \frac{\bar{z}^3}{z^2} & \text{if } z \neq 0, \\ 0 & \text{if } z = 0 \end{cases}$ then f is differentiable on $\mathbb{C} \setminus \{0\}$.

(d) If $f(z)$ and $\overline{f(z)}$ are analytic on $\mathbb{C}$ then f is a constant function.

(e) none of the above

II.

40. Let $f : \mathbb{C} \to \mathbb{C} \setminus \{0\}$ be a non-constant analytic function on $\mathbb{C}$. Let $f(x+iy) = u(x,y) + iv(x,y)$ where $u, v : \mathbb{R}^2 \to \mathbb{R}$. Then which of the following are **not analytic** on $\mathbb{C}$:

(a) $g = v + iu$.

(b) $h = \frac{v-iu}{u^2+v^2}$.

(c) $\text{Log}(u+iv)$.

(d) e^{v-iu}.

(e) none of the above

56. Radius of convergence is 1 for which of the following series:

(a) $\displaystyle\sum_{n=0}^{\infty} (2 + (-1)^n) z^n$.

(b) $\displaystyle\sum_{n=0}^{\infty} 2^n z^{n^2}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{i^n}{n} z^n$.

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{n!}{n^n} z^n$.

(e) none of the above

59. Let f be analytic everywhere on $\mathbb{C}$ except the following points:

- $z_1 = \displaystyle\lim_{z \to 1} \frac{2\mathrm{Log}(z)}{z-1}$.
- $z_2 = f(2)$ where $f(z) = \displaystyle\sum_{n=0}^{\infty} \left[2 + \left(\frac{-1}{2}\right)^n\right] z^n$.

The Taylor series of f at 0 converges to f in a disk of radius r around 0. Then the maximum value of r is equal to:

(a) $\frac{6}{4}$

(b) $\frac{8}{4}$

(c) $\frac{21}{4}$

(d) $\frac{27}{4}$

(e) none of the above

65. $\mathrm{Log}\left(\frac{i+z}{i-z}\right)$ is analytic at which of the following values of z:

(a) 0

(b) $-i$

(c) 1

(d) -1

(e) none of the above

67. Choose the points of discontinuity for $\mathrm{Log}(z^3 + 1)$ from the following:

(a) 0

(b) -1

(c) $1 + i\sqrt{3}$

(d) $1 - i\sqrt{3}$

(e) none of the above