# EE 210 Sol$^{n}$ to HA #11

**1** $I_{d1} + I_{d2} = I_{SS}$, & $I_{d1} - I_{d2} = \Delta I_d \Rightarrow I_{d1} = \dfrac{I_{SS} + \Delta I_d}{2}$ & $I_{d2} = \dfrac{I_{SS} - \Delta I_d}{2}$

Define $a = \dfrac{K_n'}{2}\left(\dfrac{W}{L}\right)$. Then, $V_{id} = \dfrac{\sqrt{I_{d1}} - \sqrt{I_{d2}}}{\sqrt{a}} \Rightarrow V_{id}^2 = \dfrac{1}{a}\left(I_{d1} + I_{d2} - 2\sqrt{I_{d1} I_{d2}}\right)$

$$= \frac{1}{a}\left[I_{SS} - 2\sqrt{\frac{(I_{SS} + \Delta I_d)(I_{SS} - \Delta I_d)}{4}}\right] = \frac{1}{a}\left[I_{SS} - \sqrt{I_{SS}^2 - \Delta I_d^2}\right]$$

$\Rightarrow \sqrt{I_{SS}^2 - \Delta I_d^2} = I_{SS} - aV_{id}^2$. Squaring both sides & solving for $\Delta I_d$, we get

$\Delta I_d = aV_{id}\left[\sqrt{\dfrac{2I_{SS}}{a} - V_{id}^2}\right]$. Substituting the expression for $a$, & putting the result back in the expressions for $I_{d1}$ & $I_{d2}$, we get the desired relations.

Now, range of $V_{id}$ over which the devices stay in saturation can be found by putting either $I_{d1}$ or $I_{d2}$ in the expression for $V_{id}$, equal to $I_{SS}$.

Thus, $V_{id} = \pm\sqrt{\dfrac{2I_{SS}}{K_n'(W/L)}} = \pm\sqrt{2}\left[\sqrt{\dfrac{2I_{d1}}{K_n'(W/L)}}\right]\Bigg|_{V_{id}=0}$, as desired.

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**2** $I_{SS} = 2I_D \Rightarrow g_m = \sqrt{2K_n'\left(\frac{W}{L}\right)I_D} = \sqrt{K_n'\left(\frac{W}{L}\right)I_{SS}} = 1\ \text{m℧} \Rightarrow K_n'\left(\frac{W}{L}\right) = \dfrac{10^{-6}}{I_{SS}}$

Also, $\Delta I_d = \dfrac{K_n'}{2}\left(\dfrac{W}{L}\right)V_{id}\sqrt{\dfrac{4I_{SS}}{K_n'(W/L)} - V_{id}^2} = 0.85\,I_{SS}$ for $V_{id} = 0.2$ V.

Substituting $K_n'\left(\frac{W}{L}\right)$ $(= 10^{-6}/I_{SS})$ in the expression for $\Delta I_d$, we get a quadratic in terms of $I_{SS}$: $7.225 \times 10^{13} I_{SS}^4 - 4 \times 10^{6} I_{SS}^2 + 0.04 = 0 \Rightarrow I_{SS}^2 = 4.226 \times 10^{-8}\ A^2$ or $1.31 \times 10^{-8}\ A^2 \Rightarrow I_{SS} = \underline{205.6\ \mu A}$ or $\underline{114.5\ \mu A}$. Now, both these values are possible, but may not be practical. Consider $I_{SS} = 205.6\ \mu A \Rightarrow \left(\frac{W}{L}\right) = 25.07$ $\Rightarrow \Delta V = \underline{0.206 V}$. The linear range of $V_{id} = \sqrt{2} \times \Delta V = 0.291$ V, & the applied $V_{id}$ of 0.2 V lies well within this range. $\therefore$ $I_{SS} = \underline{205.6\ \mu A}$ is a physically correct value. Now, consider $I_{SS} = 114.5\ \mu A \Rightarrow \left(\frac{W}{L}\right) = 45.02 \Rightarrow \Delta V = 0.115 \Rightarrow$ linear range of $V_{id} = 0.162$ V, which is less than the applied $V_{id}$ of 0.2 V. $\Rightarrow$ One of the transistor would leave their saturation mode of operation, & the pair will act like a switch. $\therefore$ $I_{SS} = \underline{114.5\ \mu A}$ is $\underline{\text{NOT}}$ a physically correct value. Final answers: $I_{SS} = 205.6\ \mu A$, $W/L = 25.07$, & with $L = 1\ \mu m$, $W = 25.07\ \mu m$.

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**3** __DC Analysis:__ $I_{C1} = I_{REF} = I_{SN}\left[\exp\left(V_{BE1}/V_T\right)\right]\left(1 + V_{CE1}/V_{AN}\right)$ with $V_{BE1} = V_I$, $V_{CE1} = V_O$, & $I_{REF} = 1$ mA. For best biasing, $V_O = V_{CC}/2 = \underline{2.5 V}$. Thus,

$V_I = V_T \ln\left[\dfrac{I_{REF}}{I_{SN}\left(1 + V_O/V_{AN}\right)}\right] = \underline{0.78 V}$. Now, $I_{C2} = I_{C1} = I_{REF} = I_{S2}\, e^{|V_{BE2}|/V_T}\left(1 + \dfrac{|V_{CE2}|}{V_{AP}}\right)$

with $|V_{CE2}| = V_{CC}/2 = \underline{2.5 V} \Rightarrow |V_{BE2}| = 0.74 V \Rightarrow V_{EB2} = \underline{0.74 V}$ (EB voltage is +ve for pnp)

AC Analysis: $A_v = -g_{m1}R_o$, $g_{m1} = \frac{I_{C1}}{V_T} = \frac{1}{26}$ ℧, $R_o = r_{o1} \| r_{o2}$,

$r_{o1} = \frac{V_{AN}}{I_{C1}} = 130\ k\Omega$, $r_{o2} = \frac{V_{AP}}{I_{C2}} = 52\ k\Omega \Rightarrow R_o = \underline{\underline{37.14\ k\Omega}} \Rightarrow A_v = \underline{\underline{-1428.57}}$ (large!)

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4 $5 - V_{o,max} = 0.6 + 0.2\left(\sqrt{0.6 + V_{o,max}} - \sqrt{0.6}\right) \Rightarrow V_{o,max}^2 - 9.15\ V_{o,max} + 20.724 = 0$

$\Rightarrow V_{o,max} = \underline{\underline{5.03\ V}}$ or $\underline{\underline{4.12\ V}}$, 5.03V is unphysical (does not satisfy the eqn., as well as is greater than $V_{DD}$ of 5V) $\Rightarrow V_{o,max} = \underline{\underline{4.12\ V}}$. $V_{TN2} = \underline{\underline{0.77\ V}}$. (at best biasing $V_o = \frac{V_{o,max}}{2} = \underline{\underline{2.06V}}$) $\Rightarrow I_{D2} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN2})^2 \times (1 + \lambda V_{DS2})$

$= \frac{40}{2} \times 1 \times (5 - 2.06 - 0.77)^2 \times (1 + 0.2 \times 2.94) = \underline{\underline{149.55\ \mu A}}$ $\left(\begin{array}{l} V_{GS} = V_{DD} - V_{OQ} \\ V_{DS} = V_{DD} - V_{OQ} \end{array}\right)$

Also, $I_{D2} = I_{D1} = 149.55\ \mu A = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN1})^2 \times (1 + \lambda V_{DS1})$

$\Rightarrow \frac{40}{2} \times 25 \times (V_I - 0.6)^2 \times (1 + 0.2 \times 2.06) = 149.55 \Rightarrow V_I = \underline{\underline{1.06\ V}}$

$\chi_2 = \frac{\gamma}{2\sqrt{2\phi_F + V_{OQ}}} = \frac{0.2}{2\sqrt{0.6 + 2.06}} = \underline{\underline{0.06}}$ $\quad g_{m1} = \sqrt{2k_n'\left(\frac{W}{L}\right)_1 I_{D1}(1 + \lambda V_{DS1})}$

$$\boxed{\begin{array}{l} V_{DS1} = V_{OQ} = 2.06V \\ V_{DS2} = V_{DD} - V_{OQ} \\ \quad = 2.94V \end{array}}$$

$= \sqrt{2 \times 40 \times 10^{-6} \times 25 \times 149.55 \times 10^{-6} \times (1 + 0.2 \times 2.06)} = \underline{\underline{649.87\ \mu A/V}}$

$g_{m2} = \sqrt{2 \times 40 \times 10^{-6} \times 1 \times 149.55 \times 10^{-6} \times (1 + 0.2 \times 2.94)} = \underline{\underline{137.84\ \mu A/V}}$

$\Rightarrow g_{mb2} = \chi_2 g_{m2} = \underline{\underline{8.27\ \mu A/V}}$. $\quad g_{o1} = \frac{\lambda I_{D1}}{1 + \lambda V_{DS1}} = \frac{0.2 \times 149.55}{1 + 0.2 \times 2.06} = \underline{\underline{21.18\ \mu A/V}}$

$g_{o2} = \frac{\lambda I_{D2}}{1 + \lambda V_{DS2}} = \frac{0.2 \times 149.55}{1 + 0.2 \times 2.94} = \underline{\underline{18.84\ \mu A/V}} \Rightarrow R_o = (g_{m2} + g_{mb2} + g_{o1} + g_{o2})^{-1}$

$= (137.84 + 8.27 + 21.18 + 18.84)^{-1}\ M\Omega = \underline{\underline{5.37\ k\Omega}}$ (Note how low it is!)

& $A_v = -g_{m1}R_o = -649.87 \times 10^{-6} \times 5.37 \times 10^{3} = \underline{\underline{-3.49}}$ (Note: Ideal max. gain $= \underline{\underline{-5}}$).

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5 As $V_o$ swings from 0 to $V_{DD}$ (= 5V), the max. change in $V_{TN}$ for $M_2$ $= 0.2\left(\sqrt{0.6 + 5} - \sqrt{0.6}\right) = \underline{\underline{0.318\ V}}$. To keep a cushion of 78 mV in $V_{TN2}$, its threshold voltage $V_{TD0}$ should be $-0.318 - 0.078 = \underline{\underline{-0.396\ V}}$. For best biasing, $V_{OQ} = \frac{V_{DD}}{2} = \underline{\underline{2.5V}}$, corresponding $V_{TD} = -0.396 + 0.2\left(\sqrt{0.6 + 2.5} - \sqrt{0.6}\right) = \underline{\underline{-0.2V}}$

$\Rightarrow I_{D2} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (-V_{TD})^2 (1 + \lambda V_{DS2}) = \frac{40}{2} \times 1 \times (0.2)^2 \times (1 + 0.2 \times 2.5) = \underline{\underline{1.2\ \mu A}}$

$\Rightarrow I_{D1} = I_{D2} = 1.2\ \mu A = \frac{40}{2} \times 25 \times (V_I - 0.6)^2 \times (1 + 0.2 \times 2.5) \Rightarrow V_I = \underline{\underline{0.64\ V}}$.

$g_{m1} = \sqrt{2 \times 40 \times 25 \times 1.2 \times 10^{-12} \times (1 + 0.2 \times 2.5)} = \underline{\underline{60\ \mu A/V}}$ $\quad \chi_2 = \frac{\gamma}{2\sqrt{2\phi_F + V_o}} = \underline{\underline{0.057}}$

$g_{m2} = \frac{g_{m1}}{5} = \underline{\underline{12\ \mu A/V}} \Rightarrow g_{mb2} = \chi_2 g_{m2} = \underline{\underline{684\ nA/V}}$ $\quad g_{o1} = g_{o2} = \frac{\lambda I_D}{1 + \lambda V_{DS}}$

$= \frac{0.2 \times 1.2}{1 + 0.2 \times 2.5} = \underline{\underline{160\ nA/V}} \Rightarrow R_o = \frac{1}{g_{mb2} + g_{o1} + g_{o2}} = (684 \times 10^{-9} + 2 \times 160 \times 10^{-9})^{-1}$

$= \underline{\underline{1\ M\Omega}}$ (note the vast improvement over Prob. 4). & $A_v = -g_{m1}R_o = \underline{\underline{-59.76}}$

If $M_2$ in its well, $R_o = \underline{\underline{3.125\ M\Omega}}$ & $A_v = \underline{\underline{-187.5}}$ (again remarkable improvement).

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6 $I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_N (V_{GSN}-V_{TN0})^2 (1+\lambda_n V_{DSn})$ No body effect. $V_{GSN} = V_I$

$\Rightarrow I_{D1} = I_{REF} = 100\mu A = \frac{40}{2}\times 10 \times (V_I - 0.6)^2 \times (1+0.2\times 2.5)$ $V_O = 2.5V$ for best biasing

$\Rightarrow V_I = 1.177V$. Also, $I_{D2} = 100\mu A = \frac{20}{2}\times 20 \times (|V_{GSP}|-|V_{TP0}|)^2(1+\lambda|V_{DSP}|)$

$\Rightarrow 100 = 200(|V_{GSP}|-0.7)^2 \times (1+0.15\times 2.5) \Rightarrow |V_{GSP}| = 1.3V$ $\boxed{V_{GSP}=-1.3V \quad V_{SGP}=+1.3V}$

$g_{m1} = \sqrt{2K_n'\left(\frac{W}{L}\right)_n I_{D1}(1+\lambda_n V_{DS1})} = \sqrt{2\times 40\times 10^{-6}\times 10\times 100\times 10^{-6}\times(1+0.2\times 2.5)} = 346.4\ \mu A/V$

$g_{o1} = \frac{\lambda_n I_{D1}}{1+\lambda_n V_{DS1}} = \frac{0.2\times 100\times 10^{-6}}{1+0.2\times 2.5} = 13.33\mu A/V$ $g_{o2} = \frac{0.15\times 100\times 10^{-6}}{1+0.15\times 2.5} = 10.91\mu A/V$

$\Rightarrow R_o = (g_{o1}+g_{o2})^{-1} = (13.33\times 10^{-6}+10.91\times 10^{-6})^{-1} = 41.25K\Omega$

& $A_V = -g_{m1}R_o = -346.4\times 10^{-6}\times 41.25\times 10^3 = -14.29$ (not that high)

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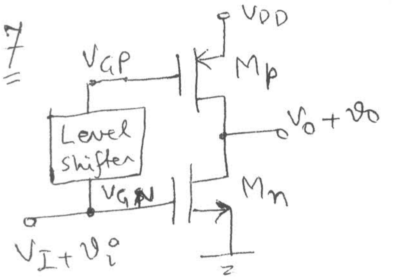


$I_{DN} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_n (V_{GSn}-V_{Tn})^2(1+\lambda_n V_{DSn})$

$V_{GSn} = V_I = 2V$ $V_{Tn} = V_{TN0} = 0.6V$ $V_{DSn} = 2.5V$

$\therefore I_{Dn} = \frac{40}{2}\times 10\times(2-0.6)^2(1+0.2\times 2.5) = 588\mu A$

Also, $|I_{Dp}| = I_{Dn} = \frac{20}{2}\times 20\times(|V_{GSP}|-0.7)^2(1+0.15\times 2.5) = 588$

$\Rightarrow |V_{GSP}| = 2.16V \Rightarrow V_{GSP} = -2.16V \Rightarrow V_{GP} - V_{SP} = -2.16V \Rightarrow V_{GP} - V_{DD} = -2.16$

$\Rightarrow V_{GP} = -2.16 + 5 = 2.84V \Rightarrow$ Amount of level shift $= V_{GP} - V_{GN} = +0.84V$.

$g_{mn} = \sqrt{2K_n'\left(\frac{W}{L}\right)_n I_{Dn}(1+\lambda_n V_{DSn})} = \sqrt{2\times 40\times 10^{-6}\times 10\times 588\times 10^{-6}\times(1+0.2\times 2.5)} = 840\ \mu A/V$

$g_{mp} = \sqrt{2\times 20\times 10^{-6}\times 20\times 588\times 10^{-6}\times(1+0.15\times 2.5)} = 804.24\ \mu A/V$

$g_{on} = \frac{\lambda_n I_{Dn}}{1+\lambda_n V_{DSn}} = \frac{0.2\times 588}{1+0.2\times 2.5} = 78.4\mu A/V$ $g_{op} = \frac{0.15\times 588}{1+0.15\times 2.5} = 64.15\mu A/V$

$\Rightarrow R_o = (g_{on}+g_{op})^{-1} = (78.4+64.15)^{-1} = 7K\Omega$ (really low!)

& $A_v = -(g_{mn}+g_{mp})R_o = -(840+804.24)\times 10^{-6}\times 7\times 10^3 = -11.53$

Getting high gain from CMOS Amplifier stages is not an easy task!

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