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1.) $\omega = 10^6$ rad/s. $\quad 100\,pF \rightarrow X_{C_1} = \frac{1}{\omega C} = \frac{1}{10^6 \times 10^{-10}} = 10\,k\Omega$

$\quad 10\,pF \rightarrow X_{C_2} = 100\,k\Omega$.

$v_i = 2\angle 0°$.

$v_- \simeq v_+ = 0$. Let the voltage at node of 50k/20k/100pF be v_a.

$\therefore$ KCL gives $\frac{v_a}{-j10k} + \frac{v_a - v_i}{5k} + \frac{v_a - 0}{20k} + \frac{v_a - v_o}{100k} = 0$.

or $v_a\left(0.1j + \frac{1}{5} + \frac{1}{20} + \frac{1}{100}\right) - \frac{2\angle 0°}{5} = \frac{v_o}{100}$

or $v_o = v_a(10j + 20 + 5 + 1) - 40$

as $i_- \simeq 0$ $\quad \frac{v_a - 0}{20k} = \frac{0 - v_o}{-j \times 100k}$ or $v_a = -j\frac{v_o}{5}$

$\therefore v_o = -j\frac{v_o}{5}(10j + 26) - 40$

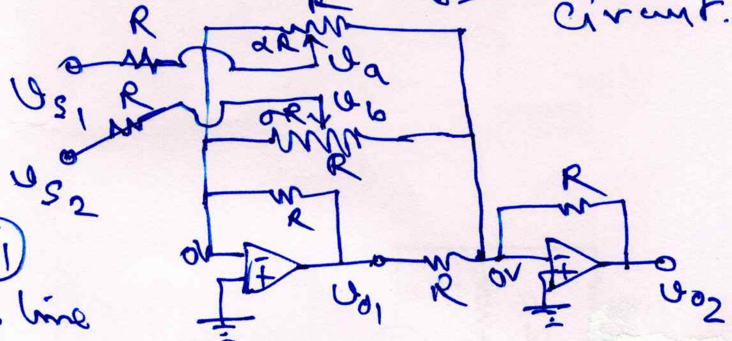
or $40 = -v_o[1 + 2 - j5.2] = -v_o(3 - j5.2)$.

$\therefore v_o = -\frac{40}{3 - j5.2} = -6.663\angle 60°$

$\therefore$ Voltage gain $\frac{v_o}{v_i} = -\frac{6.663\angle 60°}{2\angle 0°} = -3.33\angle 60°$

$\left|\frac{v_o}{v_i}\right| = |-3.33|$

2.) (a) ~~Assume~~ $v_{-1} = v_{+1} = 0$ & $v_{-2} = v_{+2} = 0$. & the ~~tap~~ point of αR potentiometer is at v_a] Check
and " " " σR " " v_b] redrawn circuit.



$\therefore$ KCL gives at left node $\Big\}$ $\frac{0 - v_{o1}}{R} + \frac{0 - v_a}{\alpha R} + \frac{0 - v_b}{\sigma R} = 0$.

or $v_{o1} = -\left(\frac{v_a}{\alpha} + \frac{v_b}{\sigma}\right)$ ....(1)

Again KCL at right vertical line node gives.

$\frac{0 - v_{o1}}{R} + \frac{0 - v_{o2}}{R} + \frac{0 - v_a}{(1-\alpha)R} + \frac{0 - v_b}{(1-\sigma)R} = 0$ or $v_{o2} = -v_{o1} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma}$

using (1) $v_{o2} = \frac{v_a}{\alpha} + \frac{v_b}{\sigma} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma} = \frac{v_a(1-2\alpha)}{\alpha(1-\alpha)} + \frac{v_b(1-2\sigma)}{\sigma(1-\sigma)}$

Again KCL at node v_a gives.

$\frac{v_a - v_{s1}}{R} + \frac{v_a - 0}{\alpha R} + \frac{v_a - 0}{(1-\alpha)R} = 0$. or $v_a\left(1 + \frac{1}{\alpha} + \frac{1}{1-\alpha}\right) = v_{s1}$

Similarly at v_b, $\quad v_b\left(1 + \frac{1}{\sigma} + \frac{1}{1-\sigma}\right) = v_{s2}$

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2) Contd. or $v_a = \dfrac{v_{s_1}\alpha(1-\alpha)}{\alpha(1-\alpha)+1-\alpha+\alpha}$ & $v_b = \dfrac{v_{s_2}\sigma(1-\sigma)}{\sigma-\sigma^2+1}$

$= \dfrac{v_{s_1}\alpha(1-\alpha)}{\alpha-\alpha^2+1}$.

or $v_{o_2} = v_{s_1}\dfrac{\cancel{\alpha(1-\alpha)}(1-2\alpha)}{\cancel{\alpha(1-\alpha)}(\alpha-\alpha^2+1)} + v_{s_2}\dfrac{\cancel{\sigma(1-\sigma)}(1-2\sigma)}{\cancel{\sigma(1-\sigma)}(\sigma-\sigma^2+1)}$

$= \dfrac{1-2\alpha}{\alpha-\alpha^2+1}v_{s_1} + \dfrac{1-2\sigma}{\sigma-\sigma^2+1}v_{s_2}$.

(b) $\alpha = \sigma = 1$. $v_{o_2} = -(v_{s_1}+v_{s_2}) \Rightarrow$ Inverted Summing Amplifier.

(c) $\alpha = \sigma = 0$., $v_{o_2} = v_{s_1}+v_{s_2} \Rightarrow$ Non-inverted Summing Amplifier.

3) $v_+ = 4.02 \times \dfrac{99k}{10.1k + 99k} = 3.65V$

$v_- \cong 3.65V \therefore \dfrac{v_1 - 3.65}{9.9k} = \dfrac{3.65 - v_o}{101k}$ as $i_- \simeq 0$.

or $v_o = 3.65 - (v_1 - 3.65)\times\dfrac{101}{9.9}$

$= 3.65 - (3.98 - 3.65)\times\dfrac{101}{9.9} = 3.65 - 3.37$

$= +0.283\ V$

Ideally $v_o^{ideal} = -10(v_1 - v_2) = -10(3.98 - 4.02)$

$= +0.4\ V$ (when the resistors are matched to $100k\Omega$ & $10k\Omega$)

$\therefore$ Error $0.4 - 0.283 = \underline{0.117\ V}$

4) Ideal OpAmp $\therefore v_- = v_+$ for all.

$\therefore v_b = v_1 = 4.99V$ and $v_c = v_2 = 5.01V$ | & Same current from v_a node to v_d node

$\therefore v_a = v_b + 4.9k \times \dfrac{v_b - v_c}{200}$

$= 4.99 + 24.5(4.99 - 5.01)$

$= 4.5V$

and $v_d = v_c - 4.9k \times \dfrac{v_b - v_c}{200} = 5.01 - 24.5(4.99 - 5.01)$

$= 5.5V$

$\therefore v_f = v_d \times \dfrac{9.99k}{9.99k + 10.01k} = 2.748V \cong v_e$

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4. Contd.) $\therefore \dfrac{v_a - v_e}{9.99k} = \dfrac{v_e - v_o}{10.01k}$

or $\dfrac{4.5 - 2.748}{9.99} = \dfrac{2.748 - v_o}{10.01}$ or $v_0 = 2.748 - 1.755$

or $v_o = 0.992 \simeq 1V$.

N.B. No gain in the 1st Stage

$v_{cm} = \dfrac{v_a + v_d}{2} = \dfrac{4.5 + 5.5}{2} = 5V$

$v_{dm} = v_a - v_d = 4.5 - 5.5 = -1V$.

$\therefore \pm \dfrac{v_{dm}}{2}$ are the two inputs

$\therefore v_a = -0.5V$ & $v_d = +0.5V$

Again $v_e \simeq v_f = 0.5 \times \dfrac{9.99k}{10.01k + 9.99k} = 0.24975V$

& $\dfrac{v_a - v_e}{9.99k} = \dfrac{v_e - v_{o_{dm}}}{10.01k}$ or $\dfrac{-0.5 - 0.24975}{9.99} = \dfrac{0.24975 - v_{o_{dm}}}{10.01}$

or $v_{o_{dm}} = 0.24975 + 0.74975 \times \dfrac{10.01}{9.99} = 1.001 V$

for $v_{cm} \Rightarrow v_f = v_e = 5 \times \dfrac{9.99k}{10.01k + 9.99k} = 2.4975 V$.

$\therefore \dfrac{v_a - v_e}{9.99k} = \dfrac{v_e - v_{o_{cm}}}{10.01k} \Rightarrow \dfrac{5 - 2.4975}{9.99} = \dfrac{2.4975 - v_{o_{cm}}}{10.01}$

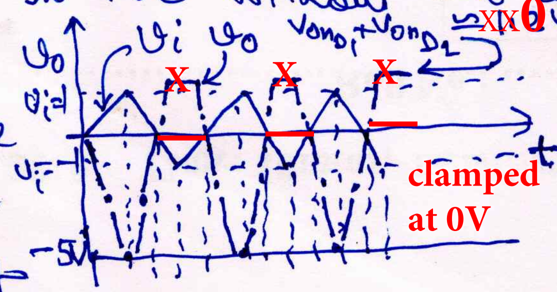
or $v_{o_{cm}} = 2.4975 - (5 - 2.4975) \times \dfrac{10.01}{9.99} = -0.0100$

$\therefore A_{dm} = \dfrac{1.001}{4.99 - 5.01} = -\dfrac{1.001}{0.02} = -50.05 \simeq -50$

$A_{cm} = \dfrac{-0.01}{5} = -0.002$

$\therefore CMRR = \left| \dfrac{A_{dm}}{A_{cm}} \right| = \dfrac{-50}{-0.002} = 2.5 \times 10^4$

5) As the input voltage v_i is +ve without the diodes D_1, v_o is −ve $\therefore D_2$ is off.
$\therefore$ If D_1 is put in it will also be off. $\therefore v_o$ is −ve and continue to go down to −5V when v_i is = 1V. Again when v_i is +ve but reduces to 0V, v_o is negative and reaches 0V.



For v_i −ve $v_o \to$ +ve but the diodes can conduct only when v_o reaches $V_{on} + V_{on}$ given to diode D_2

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6.) Assuming that the zener diodes each have a forward bias cutin voltage $V_{on} \cong 0.6$ V and each breaks down at 4.3 V in the reverse bias. V_o is within $\pm$ (4.3 + 0.6) V irrespective of V_{CC} & V_{EE} (both > |(4.3+0.6)| V. Output clamped. (Note: the 220 Ω resistor is necessary only to limit current through the zeners or current flow through the zeners to supply to any suitable load, if connected).

When v_o is clamped at the upper level $V_o = 4.9$ V.

$$\therefore V_+|_{High} = 4.9 \times \frac{4.3k}{39k + 4.3k} = 0.4866 \simeq \underline{0.487\ V}\ (V_{IH})$$

Similarly when v_o is clamped at the lower level $V_o = -4.9$ V

$$\text{or } V_+|_{low} = -4.9 \times \frac{4.3}{39+4.3} = -0.487\ V.\ (V_{IL}).$$

$$\therefore \text{Hysteresis Width} = 0.487 - (-0.487) = 0.974\ V.$$

7) By superposition: For 0101

$$v_o = (0) \times \left[-\frac{R}{2R} V_{REF}\right] + (1) \times \left[-\frac{R}{4R} V_{REF}\right] + (0) \times \left[-\frac{R}{8R} V_{REF}\right] + (1) \times \left[\frac{R}{16R} V_{REF}\right]$$

$$= 3 \times \left\{0 \times \left(-\frac{1}{2}\right) + 1 \times \left(-\frac{1}{4}\right) + 0 \times \left(-\frac{1}{8}\right) + 1 \times \left(-\frac{1}{16}\right)\right\}$$

$$= -3 \times \left(\frac{1}{4} + \frac{1}{16}\right)$$

$$= -0.9375\ V.$$

possible digital combinations are

b_1	b_2	b_3	b_4	v_o (V)
0	0	0	0	0
0	0	0	1	−0.0625×3
0	0	1	0	−0.125×3
0	0	1	1	−0.1875×3
0	1	0	0	−0.25×3
0	1	0	1	−0.9375
0	1	1	0	−0.375×3
0	1	1	1	−0.4375×3
1	0	0	0	−0.5×3
1	0	0	1	−0.5625×3
1	0	1	0	−0.625×3
1	0	1	1	−0.6875×3
1	1	0	0	−0.75×3
1	1	0	1	−0.8125×3
1	1	1	0	−0.875×3
1	1	1	1	−0.9375×3

The value of the digital quantization is the smallest increment that the DAC can make and a smoother conversion to analog is not possible.

Check and see that each step is $0.0625 \times 3 = \underline{0.1875\ V}$

This is the quantization error.

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8.) a) Let's assume that the circuit had been switched off for a long time $\therefore v_c(0^-) = 0$ and $v_o(0^-) = 0$.

Note: The $+V_{cc} = 12V$, and $-V_{EE} = 0\,V$.

$\therefore v_o$ can only swing between +12 and 0 V.

When the circuit has been switched on, v_c cannot change instantaneously and so does v_o.

$$\therefore \text{for } v_o = 0,\ v_+ = \frac{12 \times (6.8k \| 24k)}{6.8k + (6.8k \| 24k)} = \frac{12 \times 5.3}{6.8 + 5.3} = 5.255V$$

Now when the capacitor slowly gets charged, at one point in time v_- will reach the v_+ voltage, when the output will swing to $\simeq +12V$. To find v_+ now, use superposition for $v_o = 12V$.

$$\therefore v_+ = \underbrace{5.255}_{\text{due to the +12V supply at the input.}} + \frac{12 \times (6.8k \| 6.8k)}{24k + (6.8k \| 6.8k)} \Bigg| \leftarrow \text{due to } v_o \text{ at } +12V.$$

$$= 5.255 + \frac{12 \times 3.4k}{24k + 3.4k} = 6.744\,V \simeq 6.74\,V.$$

(If you approximate earlier you would get 6.75V.)

$$\therefore v_c(t) = v_c(\infty) - (v_c(\infty) - v_c(0^+))e^{-t/\tau}$$

where $\tau = RC = 6.2k \times 0.33\,\mu F = 2.05\,ms$

For $v_+ = 5.255 \longrightarrow v_c(\infty) = 0,\ v_c(0^+) = 6.74\,V$.

$$\therefore 5.255 = 0 - (0 - 6.74)e^{-t/2.05ms}$$

$$\text{or } t_1 = -2.05m \times \ln\left(\frac{5.255}{6.74}\right) = 0.51\,ms.$$

Similarly for $v_+ = 6.74V$, $v_c(\infty) = 12V$, $v_c(0^+) = 5.255V$.

$$\text{or } 6.74 = 12 - (12 - 5.255)e^{-t_2/2.05ms}$$

$$\therefore t_2 = 2.05m \times \ln\left(\frac{12 - 6.74}{12 - 5.255}\right) = 0.5098\,ms \simeq 0.51\,ms.$$

$\therefore$ switching times are $t_1 = T/2 = t_2 = 0.51\,ms$.

$$\therefore \text{frequency of oscillation} = f_o = \frac{1}{t_1 + t_2} = \underline{980.4\,Hz}$$

b) Here when v_o is at 0 at switch on $v_+ = 0$.

$\therefore v_o$ does not get a chance to change state. $\therefore$ The circuit does not oscillate

$$\therefore f_o = 0.$$

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9.) For the closed loop gain to be real (i), as required for oscillation, the closed loop gain has to be equal to be unity (ii) for stability in the oscillator output.

$\therefore$ (i) $\omega_0 L = \frac{1}{\omega_0 C}$ or $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\mu \times 10m}} = 15.915kHz$

At resonance Z of $(Z_L || Z_C) >> R$ (tank circuit).

$\therefore Z_L || Z_C >> 15k\Omega$. Let $(Z_L || Z_C)_{real} \cong 10 \times 15k\Omega$ = much larger.

Note: that the feedback is at the non-inverting terminal.

$\therefore$ loop gain is $\left(1 + \frac{100K}{5K}\right) = 21$.

Now the feedback is $\frac{[(1-\sigma)R] || [Z_L || Z_C]_{real}}{[(1-\sigma)R] || [(Z_L || Z_C)]_{real} + \sigma R} \simeq \frac{(1-\sigma)R}{(1-\sigma)R + \sigma R}$

$\simeq \frac{(1-\sigma)R}{R}$ since $(Z_L || Z_C)_{real} >> R$.

$= x$ where $x = 1 - \sigma$

$\therefore$ For oscillations to sustain

$\therefore$ Closed loop gain $\cong 21 \times x \geq 1$ for oscillation to sustain.

or $x|_{min} = 1/21 = 0.04762$

$\therefore R_{min} \cong 15k \times 0.04762 = 0.714k\Omega$

$= 714.3\ \Omega$