Laplace Transform

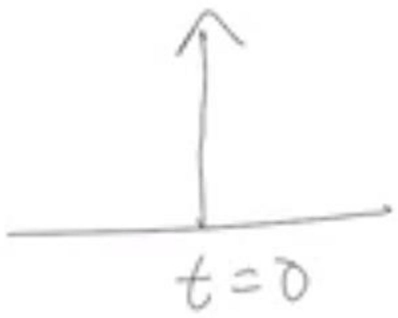
Def^n Given $f(t)$ which follows $\int_0^\infty |f(t)\, e^{-\sigma(t)}|\, dt < 0$

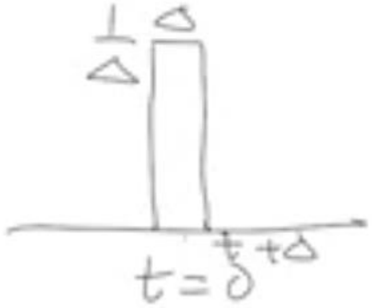
$$\mathcal{L}\, f(t) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

Standard Contrl Signals

1. impulse ~~fn~~ signal

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

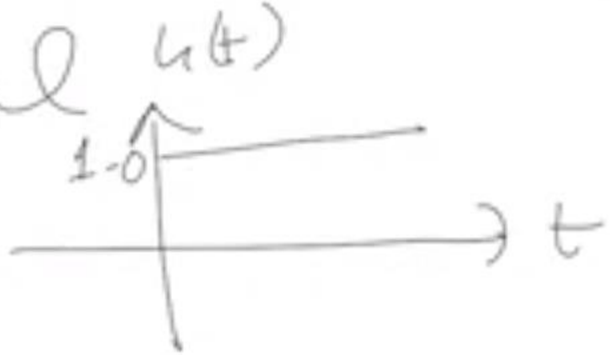




2. unity step signal u(t)

u(t)

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



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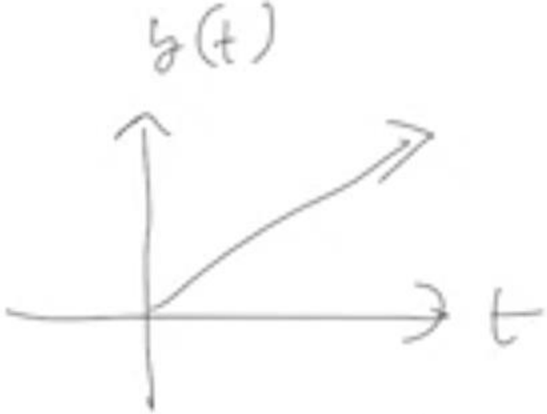
3. Ramp signal

$$y(t) = t\,u(t)$$



4. exponentially decaying signal

$$y(t) = e^{-at}\,u(t)$$

$$\frac{dy}{dt} + ay = u(t)$$

$$h(t) = e^{-at}$$

Laplace Transform

Impulse signal

$$\mathcal{L}\,\delta(t) = \int_0^\infty \delta(t)\, e^{-st}\, dt$$

$$= \lim_{\Delta \to 0} \int_0^{\Delta} \frac{1}{\Delta}\, e^{-st}\, dt = \lim_{\Delta \to 0} \frac{1}{\Delta}\left(-\frac{1}{s}\right)\left[e^{-st}\right]_0^{\Delta}$$

$$= \lim_{\Delta \to 0} -\frac{1}{\Delta s}\left(e^{-s\Delta} - 1\right) = \lim_{\Delta \to 0} \frac{1 - e^{-\Delta s}}{\Delta s}$$

Apply L Hospital's rule

$$\lim_{\Delta \to 0} \frac{s\,e^{-\Delta s}}{s} = \frac{s}{s} = 1$$

$$\mathcal{L}\,\delta(t) = 1$$

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Unit Step

$$\mathcal{L}u(t) = \int_0^\infty u(t)\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-st}\, dt = -\frac{1}{s}\left[e^{-st}\right]_0^\infty$$

$$= -\frac{1}{s}\left[0-1\right] = \frac{1}{s}$$

$$\mathcal{L}u(t) = \frac{1}{s}$$

Ramp signal :

$f(t) \Longleftrightarrow F(s)$

$y(t) = t\,u(t)$

$$\mathcal{L}\, t f(t) = -\frac{d}{ds} F(s)$$

← one rule

$y(t) = t\,u(t)\,, \quad u(t) \Longleftrightarrow \frac{1}{s}$

$$\mathcal{L}\, y(t) = -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

$$y(t) = e^{-at}\, u(t)$$

$$\mathcal{L}\, y(t) = \int_0^\infty e^{-at}\, e^{-st}\, dt$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= -\frac{1}{s+a}\left[e^{-(s+a)t}\right]_0^\infty$$

$$= -\frac{1}{s+a}\left[\lim_{t\to\infty} e^{-(s+a)t} - 1\right]$$

$$\lim_{t \to \infty} e^{-(s+a)t} \qquad s = \sigma + j\omega$$

$$\lim_{t \to \infty} e^{-(\sigma+a)t} \cdot e^{-j\omega t}$$

$$\lim_{t \to \infty} e^{-(\sigma+a)t} = 0 \qquad \sigma + a > 0$$

$$\sigma > -a$$

a > 0

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$$\mathcal{L}\, e^{-at} = -\frac{1}{s+a}\left[-1\right]$$

$$= \frac{1}{s+a}$$

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$1/s$
$t\,u(t)$	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$