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Routh Array

Let's solve some examples

Ex 1 $d(s) = s^4 + 2s^3 + as^2 + s + b$

Find the range of parameters in a, b for which the system is stable.

$$\boxed{\begin{array}{l} a > 0.5 \\ 0 < b < 0.5a - 0.25 \end{array}}$$

s^4	1	a	b
s^3	2	1	
s^2	$\frac{2a-1}{2}$	b	
s^1	$\left(\frac{2a-1}{2} - 2b\right) / \frac{2a-1}{2}$	0	
s^0	b		

$$\boxed{\begin{array}{l} a = 1 \\ 0 < b < 0.25 \end{array}}$$

$b > 0$

$\boxed{0 < b < 0.5a - 0.25}$

$\frac{2a-1}{2} > 0$ $\quad 2a - 1 > 0$

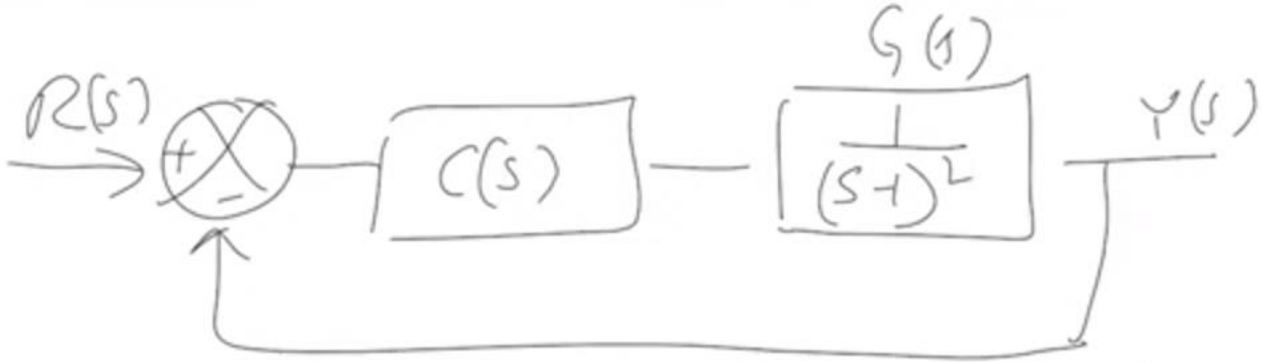
$a > 0.5$

$\frac{2a - 1 - 4b}{\cancel{2}} \times \frac{\cancel{2}}{2a-1} > 0$ $\quad \Big| \; b < 0.5a - 0.25$

$2a - 1 - 4b > 0$

$4b < 2a - 1$

Ex 2



(a) If $C(s)$ is a PI controller, will the system be stabilizable?

$$C(s) = K_p + \frac{K_I}{s},$$

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)} = \frac{K_p s + \frac{K_I}{\cancel{s}}}{(s-1)^2 + K_p s + K_I}$$

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$$d(s) = (s-1)^2 + K_p s + K_I$$

$$= s^2 - 2s + 1 + K_p s + K_I$$

$$= s^2 - 2s + K_p$$

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{\left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{s^2 - 2s + 1}\right)}{1 + \left(K_p + \frac{K_I}{s}\right)\left(\frac{1}{s^2 - 2s + 1}\right)} = \frac{\frac{K_p s + K_I}{s(s^2 - 2s + 1)}}{\frac{s(s^2 - 2s + 1) + K_p s + K_I}{s(s^2 - 2s + 1)}}$$

$$d(s) = s(s^2 - 2s + 1) + K_p s + K_I$$

$$d(s) = s^3 - 2s^2 + (k_p + 1)s + k_I$$

The necessary condition is not satisfied as one of the coeff is negative

PI Controller can't stabilize the system. $G(s) = \frac{1}{(s-1)^2}$

Unstable system.

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$$C(s) = K_P + K_D s \triangleq \text{PD Controller}$$

$$\frac{Y(s)}{R(s)} = \frac{(K_P + K_D s)\dfrac{1}{s^2 - 2s + 1}}{1 + \dfrac{(K_P + K_D s)}{s^2 - 2s + 1}}$$

$$= \frac{K_P + K_D s}{s^2 - 2s + 1 + K_P + K_D s}$$

$$d(s) = s^2 - 2s + K_D s + (K_P + 1) = s^2 + (K_D - 2)s + K_P + 1$$

$$K_D - 2 > 0$$
$$K_D > 2$$
$$K_P + 1 > 0$$
$$K_P > -1$$

OL System is unstable

(open loop) Go for PD controller

(OL) " PID "

OL System is stable

Go for PI controller

that will improve steady state performance

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PD → improves transient performance

PI → improves steady state performance

PID → improves both transient & steady state performance

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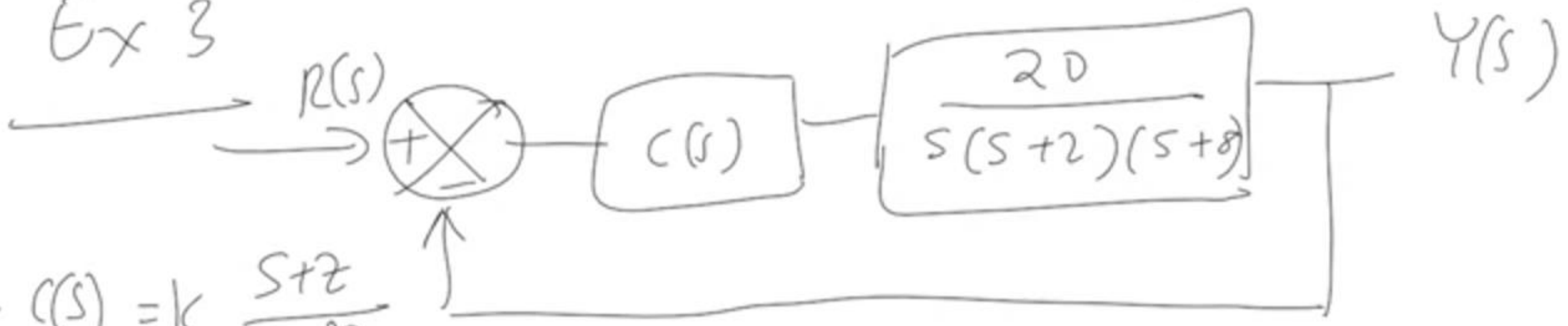
Ex 3



Let $C(s) = k\,\frac{s+z}{s+p}$

Let $\boxed{p = 1}$

Find k, z, p that are stabilizing

PI ⟷ Lag

PD ⟷ Lead

$20k = K$

$$\frac{Y(s)}{R(s)} = \frac{20k(s+z)}{s(s+2)(s+8)(s+1)+20k(s+z)}$$

$$d(s) = s^4 + 11s^3 + 26s^2 + (K+16)s + Kz = 0$$

$K+16>0$ $\quad$ $Kz>0$

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$$d(s) = s^4 + 11s^3 + 26s^2 + (k+16)s + kz$$

$k + 16 > 0$

$k > -16.$

s^4	1	26	kz
s^3	11	$k+16$	
s^2	$\frac{286-k-16}{11}$	kz	
s^1	$\frac{(270-k)(k+16)-121kz}{270k}$	0	
s^0	kz		

$k = 1$

$-16 < k < 270$

$$\frac{\left(\frac{270-k}{11}\right)(k+16) - 11kz}{\frac{270-k}{11}}$$

$k^2 - 254k - 4320 < -121kz$

$\frac{270-k}{11} > 0$

$k < 270$

If $k = 1$, $z < \frac{4573}{121}$, $p = 2$

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Routh Array

1. Necessary condition for stability.

2. Sufficient condition for stability.

3. Special cases
   - → One element in 1st column becomes zero
   - → One row (odd) becomes all zero
     - → Aux eqn

4. Stabilizing Controller