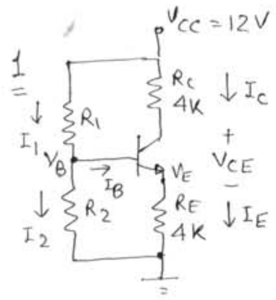
EE 210

Sol$^{\underline{n}}$ to HA#5



1. a) $I_C = 1\,mA \Rightarrow I_B = \frac{I_C}{\beta} = \frac{1\,mA}{100} = \underline{10\,\mu A}$

$I_1 = 20 I_B = \underline{200\,\mu A}$ In simple analysis, we neglect I_B.

$\therefore I_2 = I_1 = \underline{200\,\mu A}$ & $R_1 + R_2 = \frac{V_{CC}}{I_1} = \frac{12\,V}{200\,\mu A} = \underline{60\,k\Omega}$

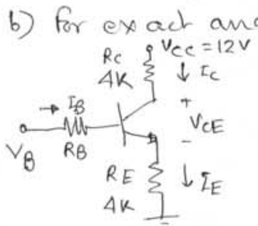
With I_B neglected, $I_E \approx I_C = 1\,mA \Rightarrow V_E = I_E R_E = \underline{4\,V}$

$\Rightarrow V_B = V_E + V_{BE} = 4 + 0.7 = \underline{4.7\,V} \Rightarrow R_1 = \frac{V_{CC} - V_B}{I_1} = \frac{12 - 4.7}{200\,\mu A}$

$= \underline{36.5\,k\Omega}$, & $R_2 = \frac{V_B}{I_2} \simeq \frac{V_B}{I_1} = \frac{4.7\,V}{200\,\mu A} = \underline{23.5\,k\Omega}$ $(R_1 + R_2 = 60\,k\Omega)$

$V_{CE} = V_{CC} - I_C(R_C + R_E) = 12 - 1\,mA \times (4K + 4K) = \underline{4\,V} = \frac{V_{CC}}{3}$ (perfect biasing)

b) For exact analysis, we open the base lead & thevenize the left part of the ckt.



$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{23.5}{60} \times 12 = \underline{4.7\,V}$ & $R_B = R_1 \| R_2 = \underline{14.3\,k\Omega}$

$\Rightarrow V_B = I_B R_B + V_{BE} + I_E R_E \Rightarrow I_B = \frac{V_B - V_{BE}}{R_B + (\beta+1) R_E} = \frac{4.7 - 0.7}{14.3K + 101 \times 4K}$

$= \underline{9.56\,\mu A} \Rightarrow I_C = \beta I_B = \underline{0.956\,mA}$ & $I_E = I_C + I_B = \underline{0.966\,mA}$

$\therefore V_{CE} = V_{CC} - I_C R_C - I_E R_E = 12 - 0.956 \times 4 - 0.966 \times 4 = \underline{4.312\,V}$

$\therefore$ % error in $I_C = \frac{1 - 0.956}{0.956} = \underline{+4.6\%}$ & % error in $V_{CE} = \frac{4 - 4.312}{4.312} = \underline{-7.24\%}$

Given the simplicity of the calculations in part a), these errors are acceptable.

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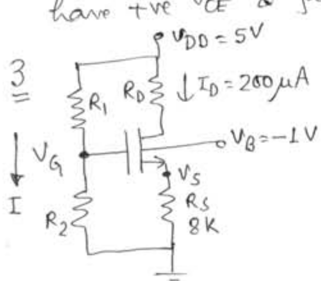
2. KVL around left most branch: $24 = 82 I_1 + 10 I_2$, with $I_2 = I_1 - I_{B_2}$ (all currents in mA) $\Rightarrow \boxed{92 I_1 - 10 I_{B_2} = 24}$ ...... (1). Now, go around the BE loop of Q_1–Q_2: $10 I_2 = 100 I_{B_2} + V_{BE_2} + V_{BE_1} + 0.1 I_{E_1}$, with $I_{E_1} = (\beta_1 + 1) I_{B_1}$, $I_{B_1} = I_{E_2}$, & $I_{E_2} = (\beta_2 + 1) I_{B_2}$. $\Rightarrow \boxed{10 I_1 - 625.1 I_{B_2} = 1.4}$ ...... (2). Solving (1) & (2), $I_1 = \underline{0.261\,mA}$, $I_{B_2} = \underline{1.937\,\mu A}$, $I_{C_2} = \beta_2 I_{B_2} = \underline{96.85\,\mu A}$, $I_{B_1} = I_{E_2} = (\beta_2 + 1) I_{B_2} = \underline{98.79\,\mu A}$, $I_{C_1} = \beta_1 I_{B_1} = \underline{9.88\,mA}$, $I_{E_1} = (\beta_1 + 1) I_{B_1} = \underline{9.98\,mA}$, $I_2 = I_1 - I_{B_2} = \underline{0.26\,mA}$.

$V_{O1} = 24 - 1 \times I_{C_1} = \underline{14.12\,V}$, $V_{O2} = 0.1\,I_{E_1} \simeq \underline{1\,V} \Rightarrow V_{CE_1} = V_{O1} - V_{O2} = \underline{13.12\,V}$.

$V_{CE_2} = 24 - V_{B_1} = 24 - (V_{E_1} + 0.7) = 24 - 0.7 - V_{O2} = \underline{22.3\,V}$. Thus, both transistors have +ve V_{CE} & far away from 0.7V. However, Q_2 is biased too close to cutoff.



3. $V_S = I_D R_S = \underline{1.6\,V} \Rightarrow V_{BS} = V_B - V_S = \underline{-2.6\,V}$

$V_{TN} = V_{TN0} + \gamma(\sqrt{2\phi_F - V_{BS}} - \sqrt{2\phi_F}) = 1 + 0.4(\sqrt{0.6 + 2.6} - \sqrt{0.6})$

$= \underline{1.406\,V}$. $V_{GS} = V_G - V_S = V_G - 1.6$.

$I_D = \frac{K_n'}{2} \frac{W}{L} (V_{GS} - V_{TN})^2$ (in saturation, & neglecting CLM)

$\Rightarrow \frac{40 \times 10^{-6}}{2} \times 10 \times (V_G - 1.6 - 1.406)^2 = 200 \times 10^{-6}$

Sol$^{\underline{n}}$ gives $V_G = \underline{4\,V}$ or $2\,V$, with corresponding $V_{GS} = 2.4\,V$ or $\underline{0.4\,V}$

$V_{GS} = 0.4V$ is clearly unacceptable, $\because$ it is less than $V_{TN} \Rightarrow \boxed{V_{GS} = 2.4V}$. ②

$\therefore V_G = 4V = \frac{R_2}{R_1+R_2} V_{DD}$. P_{DC} not to exceed $2\,mW \Rightarrow I_{supply} = \frac{2\,mW}{5V} = \underline{\underline{400\,\mu A}}$

$\therefore I = I_{supply} - I_D = \underline{200\,\mu A} \Rightarrow R_1 + R_2 = \frac{V_{DD}}{I} = \frac{5V}{200\,\mu A} = \underline{\underline{25\,k\Omega}}$.

$\Rightarrow R_2 = \frac{4V \times 25\,k\Omega}{5V} = \underline{\underline{20\,k\Omega}}$ & $R_1 = \underline{\underline{5\,k\Omega}}$. Now, for best biasing, $V_{DS} \sim \frac{V_{DD}}{3}$ $= \underline{\underline{1.67V}}$ (3-element output branch). $\Rightarrow R_D = \frac{V_{DD} - V_{DS}}{I_D} - R_S = \frac{5-1.67}{200\,\mu A} - 8\,k\Omega$ $= \underline{\underline{8.67\,k\Omega}}$. As a check, we note $V_{GS} - V_{TN} = 2.4 - 1.406 = \underline{\underline{0.994V}}$ & $V_{DS} =$ $\underline{\underline{1.67V}} \Rightarrow V_{DS} > (V_{GS} - V_{TN})$, & the device is indeed saturated.

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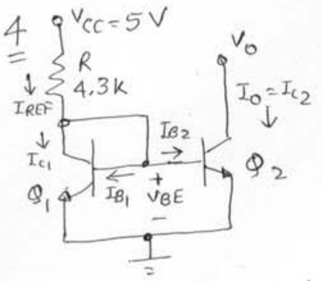

$I_{REF} = \frac{V_{CC} - V_{BE}}{R} = \frac{5 - V_{BE}}{4.3K}$. Also, $I_{REF} = I_{C_1} + I_{B_1} + I_{B_2}$. Now, $I_{S_1} = I_{S_2} = 10^{-16} A$, & $\beta_1 = \beta_2 = 50 \Rightarrow$ thus, Q_1 & Q_2 are perfectly matched $\Rightarrow I_{C_1} = I_{C_2}$, & $I_{B_1} = I_{B_2} = \frac{I_{C_1}}{\beta}$. $\Rightarrow I_{REF} = I_{C_1}\left(1 + \frac{2}{\beta}\right)$ $= 1.04\, I_{C_1}$. Also, neglecting base width modulation effect, $I_{C_1} = I_{S_1} e^{V_{BE}/V_T} = 10^{-16} e^{V_{BE}/26mV}$. This set of eqns. is transcendental, & has to be solved by iterations. Choosing $V_{BE} = 0.7V$ as the initial guess, the iterations converged rapidly at $V_{BE} = \underline{\underline{0.777V}}$, with $I_{C_1} = \underline{\underline{0.944\,mA}}$, & $I_{REF} = \underline{\underline{0.982\,mA}}$. $\therefore$ Output current $I_o = I_{C_2} = \underline{\underline{0.944\,mA}}$ ($\because I_{C_2} = I_{C_1}$).

With $V_{A_2} = 100\,V$, $I_o = 0.944\,mA \times \left(1 + \frac{V_o}{100}\right)$ $\because V_o = V_{CE2}$. If $V_o \ll 100\,V$, $I_o \approx \underline{\underline{0.944\,mA}}$.

Output resistance $R_o = r_{o2} = \frac{V_{A2}}{I_o} = \frac{100\,V}{0.944\,mA} = \underline{\underline{105.93\,k\Omega}}$.

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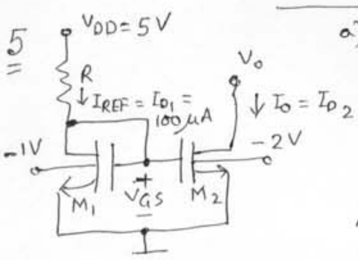

a) $V_{o,min} = 0.1V \Rightarrow V_{DS,sat} = V_{GS} - V_{TNO} = 0.1V$ (w/o body effect).

$\Rightarrow \left(\frac{W}{L}\right)_2 = \frac{2 I_o}{K_n' (V_{GS} - V_{TNO})^2} = \frac{2 \times 50 \times 10^{-6}}{40 \times 10^{-6} \times 0.1^2} = \underline{\underline{250}}$ ($I_o = 50\,\mu A$)

$\because I_{REF} = 100\,\mu A \therefore \left(\frac{W}{L}\right)_1 = 2\left(\frac{W}{L}\right)_2 = \underline{\underline{500}}$.

Also, with $V_{TNO} = 0.7V$, we have $V_{GS} = \underline{\underline{0.8V}}$.

$\therefore R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{5 - 0.8}{100\,\mu A} = \underline{\underline{42\,k\Omega}}$

b) Now, with body effect, $V_{BS_1} = -1V$ & $V_{BS_2} = -2V \Rightarrow V_{TN1} = \underline{\underline{0.896V}}$, & $V_{TN2} = \underline{\underline{1.035V}}$.

Thus, $V_{GS} - V_{TN2} = V_{DS,sat2} = V_o = 0.1V$ for $I_o = I_{D2} = 50\,\mu A$.

$\Rightarrow \left(\frac{W}{L}\right)_2 = \frac{2 \times 50 \times 10^{-6}}{40 \times 10^{-6} \times 0.1^2} = \underline{\underline{250}}$ (Thus, body effect is immaterial for M_2).

However, $V_{GS} = 0.1 + V_{TN2} = \underline{\underline{1.135V}}$. $\therefore$ for $I_{REF} = 100\,\mu A$, we have

$\left(\frac{W}{L}\right)_1 = \frac{2 \times 100 \times 10^{-6}}{40 \times 10^{-6} \times (1.135 - 0.896)^2} = \underline{\underline{87.53}}$ (Significant reduction from $\underline{\underline{500}}$)

Here, body effect of M_1 works to our advantage.

$R = \frac{V_{DD} - V_{GS}}{I_{REF}} = \frac{5 - 1.135}{100\,\mu A} = \underline{\underline{38.65\,k\Omega}}$ (slight reduction from $\underline{\underline{42\,k\Omega}}$)
