MSO201A: Probability and Statistics

2021 (2nd Semester)

Assignment-VIII

1. In 1000 tosses of a coin, 560 heads and 440 tails appear. Is it reasonable to assume that the coin is fair? Justify your answer.

2. In a city it is assumed that the number of automobile accidents in a given year follows a Poisson distribution. In past years the average number of accidents per year was 15 and this year it was 10. Is it justified to claim that the accident rate has dropped?

3. For a random sample $X_1, \ldots, X_n$ of Bernoulli(p) variables, it is desired to test

$$H_0 : p = .49 \quad \text{versus} \quad H_1 : p = .51.$$

   Use the Central Limit Theorem to determine, approximately, the sample size needed so that the two probabilities of error are both about .01. Use a test function that rejects H_0 if $\sum_{i=1}^{n} X_i$ is large.

4. Show that for a random sample $X_1, \ldots, X_n$ from a $N(0, \sigma^2)$ population, the most powerful test of $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma = \sigma_1$, where $\sigma_1 > \sigma_0$ is given by

$$\phi\left(\Sigma X_i^2\right) = \begin{cases} 1 & \text{if } \Sigma X_i^2 > c \\ 0 & \text{if } \Sigma X_i^2 \leq c. \end{cases}$$

   Show how the value of c is explicitly determined for a level α test. Given $\sigma_0 = 2$, $\alpha = 0.10$ and $n = 15$, compute c (use the table appended below).

5. The random variable X has pdf $f(x) = e^{-x}$ for $x > 0$. One observation is obtained on the random variable $Y = X^\theta$, and a test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$ needs to be constructed. Find the UMP level $\alpha = .10$ test and compute the Type II Error probability.

6. Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x \mid H_0)$	.01	.01	.01	.01	.01	.01	.94
$f(x \mid H_1)$	.06	.05	.04	.03	.02	.01	.79

Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = .04$. Compute the probability of Type II Error for this test.

7. Let $X_1, \ldots, X_{10}$ be i.i.d. Bernoulli(p). Find the most powerful test of size $\alpha = .0547$ of the hypotheses $H_0 : p = \frac{1}{2}$ versus $H_1 : p = \frac{1}{4}$. Find the power of this test.

8. Suppose X is one observation from a population with Beta$(\theta, 1)$ distribution. Find the most powerful level α test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$.

Tables of distributions

Normal CDF at z			**χ^2-distribution quantiles**			**t-distribution quantiles**			
z	+0.00	+0.05	n=d.f	$\chi^2_n(0.10)$	$\chi^2_n(0.90)$	n=d.f	$t_n(0.10)$	$t_n(0.05)$	$t_n(0.025)$
0	0.5	0.51994	1	2.7055	0.015791	1	3.0777	6.3138	12.706
0.1	0.53983	0.55962	2	4.6052	0.21072	2	1.8856	2.92	4.3027
0.2	0.57926	0.59871	3	6.2514	0.58437	3	1.6377	2.3534	3.1824
0.3	0.61791	0.63683	4	7.7794	1.0636	4	1.5332	2.1318	2.7764
0.4	0.65542	0.67364	5	9.2364	1.6103	5	1.4759	2.015	2.5706
0.5	0.69146	0.70884	6	10.645	2.2041	6	1.4398	1.9432	2.4469
0.6	0.72575	0.74215	7	12.017	2.8331	7	1.4149	1.8946	2.3646
0.7	0.75804	0.77337	8	13.362	3.4895	8	1.3968	1.8595	2.306
0.8	0.78814	0.80234	9	14.684	4.1682	9	1.383	1.8331	2.2622
0.9	0.81594	0.82894	10	15.987	4.8652	10	1.3722	1.8125	2.2281
1	0.84134	0.85314	11	17.275	5.5778	11	1.3634	1.7959	2.201
1.1	0.86433	0.87493	12	18.549	6.3038	12	1.3562	1.7823	2.1788
1.2	0.88493	0.89435	13	19.812	7.0415	13	1.3502	1.7709	2.1604
1.3	0.9032	0.91149	14	21.064	7.7895	14	1.345	1.7613	2.1448
1.4	0.91924	0.92647	15	22.307	8.5468	15	1.3406	1.7531	2.1314
1.5	0.93319	0.93943	16	23.542	9.3122	16	1.3368	1.7459	2.1199
1.6	0.9452	0.95053	17	24.769	10.085	17	1.3334	1.7396	2.1098
1.7	0.95543	0.95994	18	25.989	10.865	18	1.3304	1.7341	2.1009
1.8	0.96407	0.96784	19	27.204	11.651	19	1.3277	1.7291	2.093
1.9	0.97128	0.97441	20	28.412	12.443	20	1.3253	1.7247	2.086
2	0.97725	0.97982	21	29.615	13.24	21	1.3232	1.7207	2.0796
2.1	0.98214	0.98422	22	30.813	14.041	22	1.3212	1.7171	2.0739
2.2	0.9861	0.98778	23	32.007	14.848	23	1.3195	1.7139	2.0687
2.3	0.98928	0.99061	24	33.196	15.659	24	1.3178	1.7109	2.0639
2.4	0.9918	0.99286	25	34.382	16.473	25	1.3163	1.7081	2.0595
2.5	0.99379	0.99461	26	35.563	17.292	26	1.315	1.7056	2.0555
2.6	0.99534	0.99598	27	36.741	18.114	27	1.3137	1.7033	2.0518
2.7	0.99653	0.99702	28	37.916	18.939	28	1.3125	1.7011	2.0484
2.8	0.99744	0.99781	29	39.087	19.768	29	1.3114	1.6991	2.0452
2.9	0.99813	0.99841	30	40.256	20.599	30	1.3104	1.6973	2.0423