

$$\begin{aligned} i_t &= g_{m2} v_2 + (v_t + v_2)/r_{02} \\ &= v_t/r_{02} + (g_{m2} + 1/r_{02}) v_2 \simeq v_t/r_{02} + g_{m2} v_2 \end{aligned}$$

$$v_2 = -i_t R_{\text{eff}}$$

$$\Rightarrow i_t = v_t/r_{02} - g_{m2} R_{\text{eff}} i_t$$

$$\Rightarrow R_0 = v_t/i_t = r_{02}(1 + g_{m2} R_{\text{eff}})$$

➤ This is a ***Golden Equation***, which would be *used frequently*

- *Carefully note the topology that produces this result*

➤ ***Exercise***: *Reverse v_2 and show that the expression for R_0 remains invariant*

- If $r_{\pi 2} \gg R_2$, $R_0 = r_{02}(1 + g_{m2}R_2)$
- If $R_2 \gg r_{\pi 2}$, $R_0 \approx \beta_2 r_{02}$ (since $\beta = g_m r_{\pi} \gg 1$)
- *Under the second condition, the circuit produces enormously large value of $R_0 \sim 10s$ of $M\Omega$ or greater*
 - *Almost approaches a constant current source!*
- *It's good to check the relative values of R_2 and $r_{\pi 2}$ before using either of the equations*
- *This circuit does not have any MOS counterpart for obvious reasons!*

- *Cascode Current Source:*
 - *The best and most widely used*
 - *Almost universal choice for biasing IC stages*
 - *Produces extremely high R_0*
 - *Original cascode needs higher values of $V_{0,min}$*
 - *Modified cascode gets rid of this problem and pushes $V_{0,min}$ down*
 - *The topology is basically two simple CMs stacked one upon the other*
 - *Both npn and NMOS implementations exist*

- *nnp Cascode:*

- *All Q s are perfectly matched*

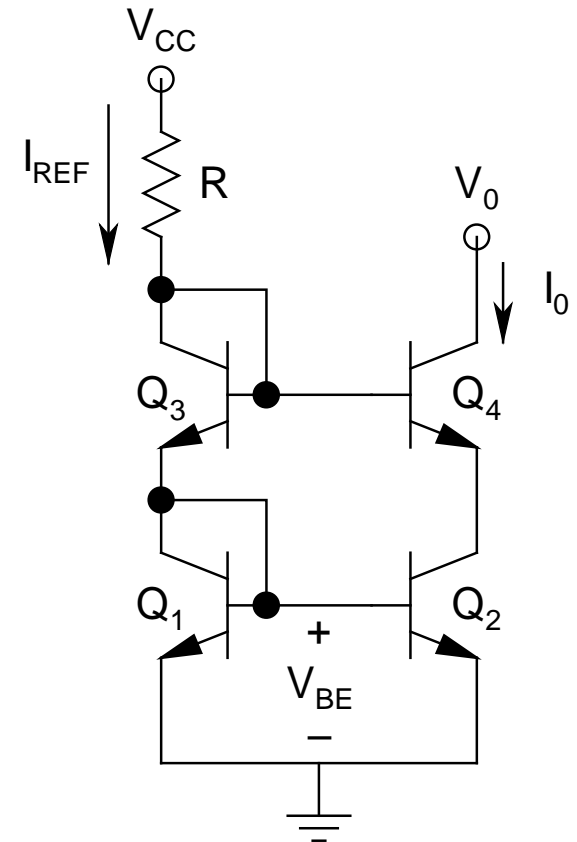
- *Neglecting I_B and V_A :*

$$I_0 = I_{REF} = (V_{CC} - 2V_{BE})/R$$

- Show that if *I_B can't be neglected*, but *all Q s have same β* :

$$I_0 = \frac{I_{REF}}{1 + (4\beta + 2)/\beta^2}$$

- Thus, *β immunity is not that pronounced*



➤ *All Q s operate with the same V_{BE}*

$$\Rightarrow V_{B1} = V_{B2} = V_{BE}, V_{B3} = V_{B4} = 2V_{BE}$$

$$\Rightarrow V_{E4} = V_{C2} = V_{BE}$$

$$\Rightarrow V_{BE2} = V_{CE2}$$

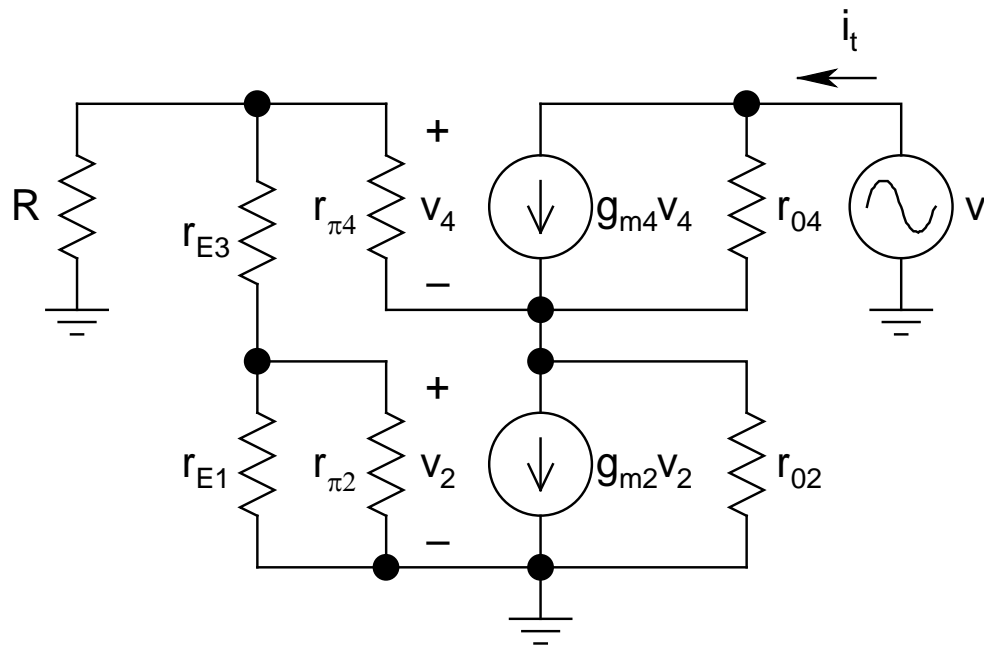
$$\Rightarrow Q_2 \text{ can never saturate, but } Q_4 \text{ can!}$$

$$\Rightarrow V_{0,\min} = V_{BE} + V_{CE4}(\text{SS}) = 0.7 + 0.2 = 0.9 \text{ V}$$

➤ *The output voltage swing is sacrificed quite a bit!*

➤ *However, the main advantage of this circuit is enormously large R_0*

➤ *Calculation of R_o :*



Exact Equivalent

- *Q_1 and Q_3 diode-connected* $\Rightarrow r_{E1}$ and r_{E3}

- Note that to a *first-order estimate*, *bases of Q_1 - Q_2 and Q_3 - Q_4* can be considered to be at a *fixed DC potential*, and thus, *ac ground*

$$\Rightarrow v_2 = 0 \Rightarrow g_{m2}v_2 = 0$$

\Rightarrow Leads to the *simplified equivalent* (looks familiar?)

- *By inspection:*

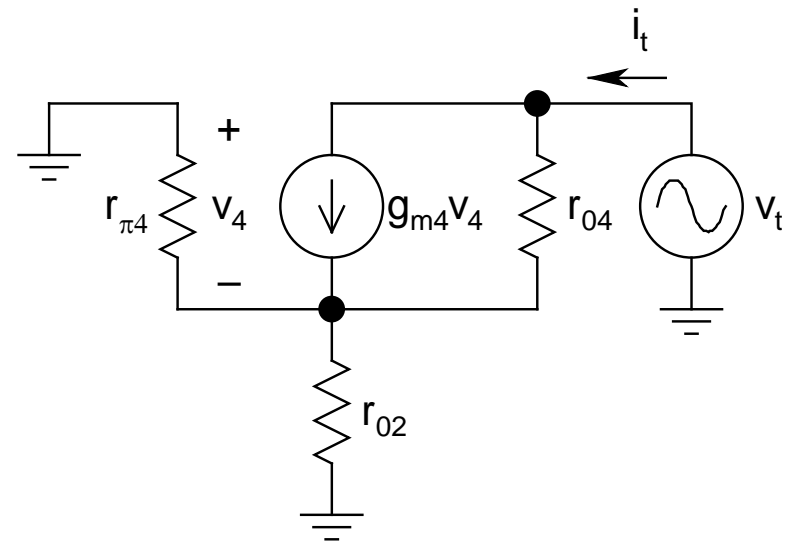
$$R_0 \approx r_{o4}(1 + g_{m4}r_{\pi4})$$

$$\approx \beta_4 r_{o4}$$

(*assuming $r_{o2} \gg r_{\pi4}$*)

- *Actual analysis gives:*

$$R_0 = \beta_4 r_{o4}/2 \text{ (large error!)}$$



Simplified Equivalent