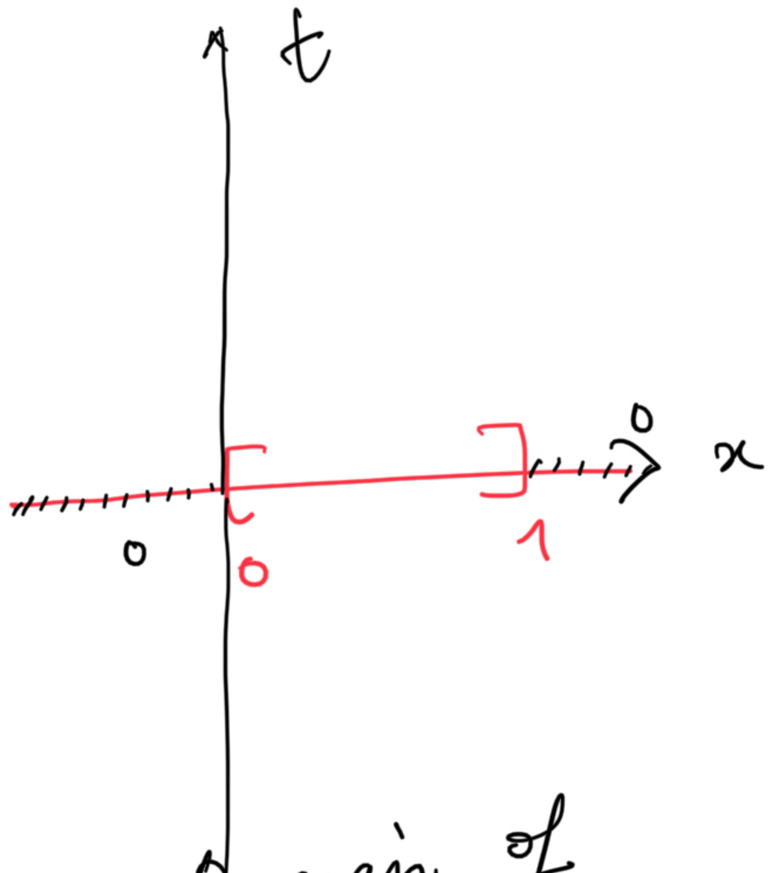
Assignment_4 Solutions

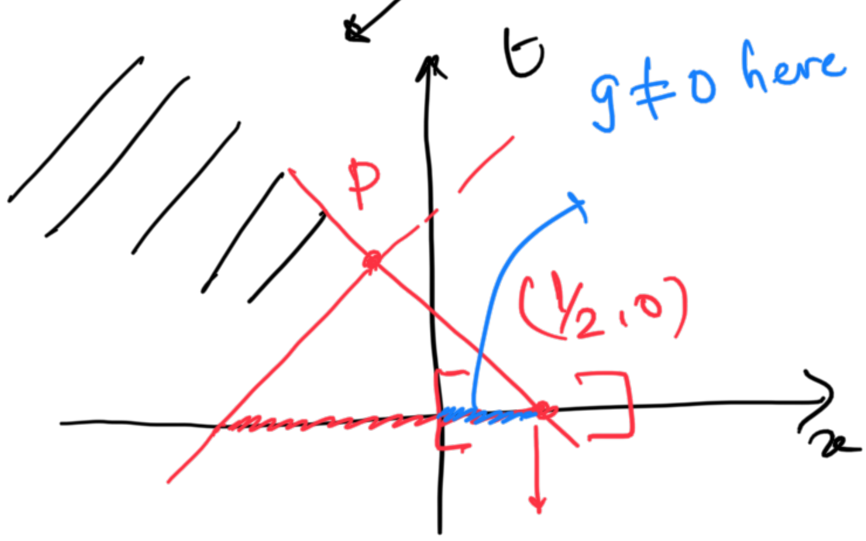
Problem 1

Given that f and g are '0' on $(-\infty, 0) \cup (1, \infty)$



We need to look at the Domain of dependence and range of influence for this problem.

Case I Let us look at the set $(-\infty, 0) \times (0, \infty)$



non zero values of f and g

choose a Point 'P' such that the st line with gradient "-1" intersects x-axis at $(1/2, 0)$.

Then clearly from D-alembert's formula

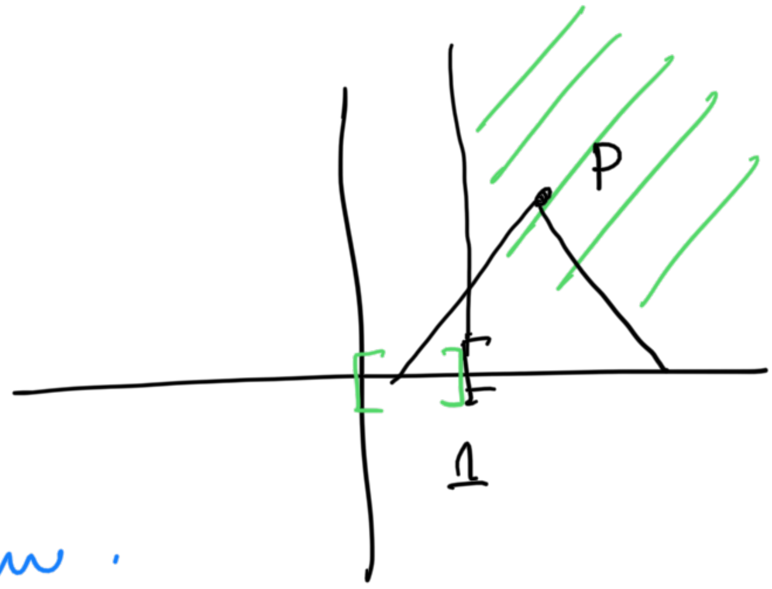
$u(P)$ may be non zero

$\therefore f(1/2) \neq 0$.

(a) not true.

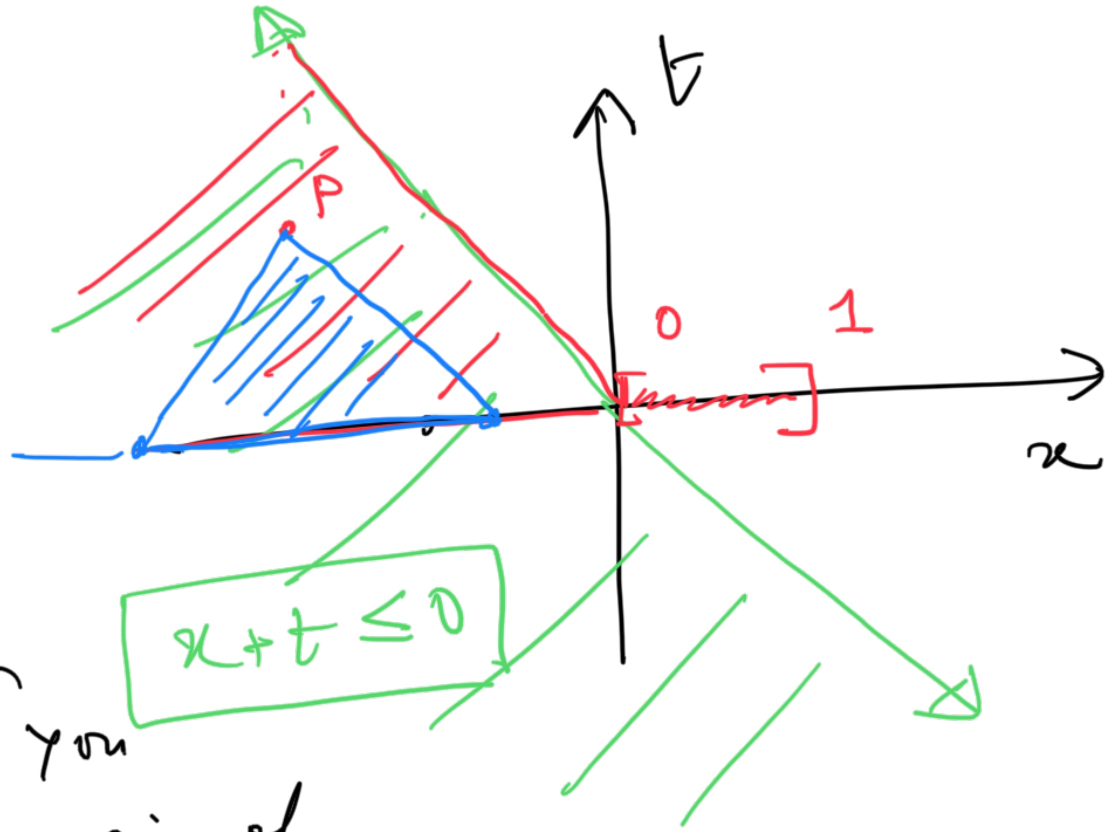
(b)

With similar logic as in part (a) one can show (b) is not true.



(c)

(c) is true



because if you draw the domain of dependence from any point (arbitrary) in the region $x+t \le 0$ (in green).

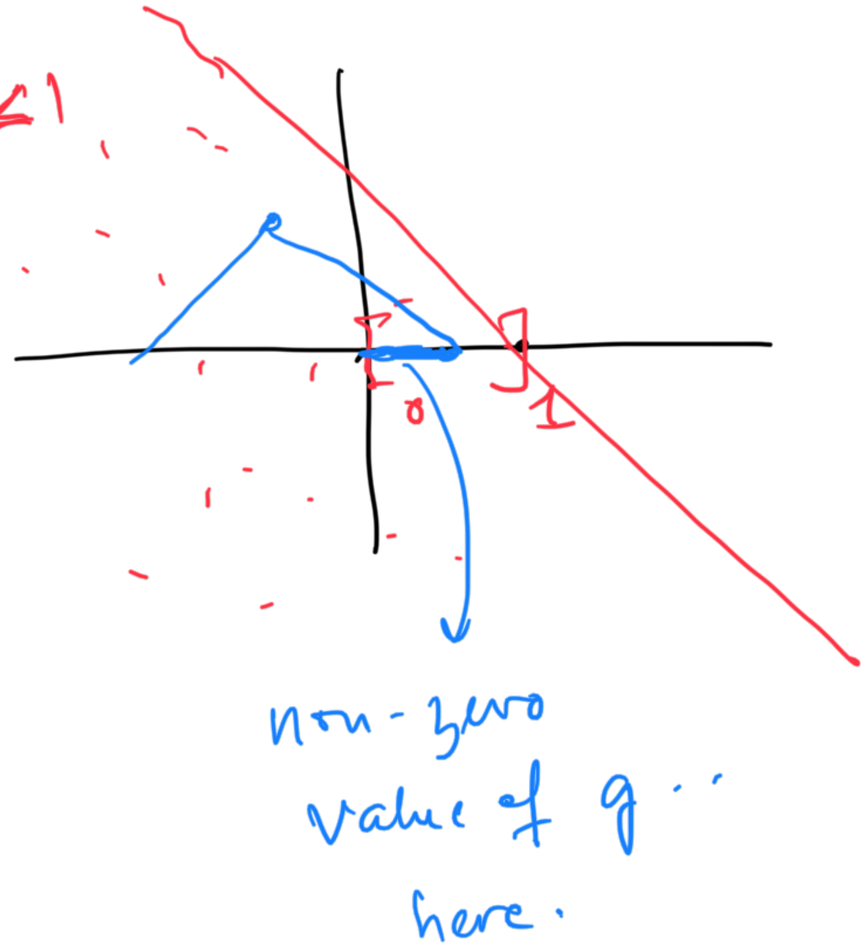
Note that $t \ge 0$ is always for us,

$P = (x,t) \in \{x+t<0\} \cap \{t \geq 0\} = A$

$u(P) = 0$ [by D'Alembert's formula]

$\because f, g \equiv 0$ on the domain of $(x,t) \in A$, dependence ...

(d)



$\boxed{\text{(d) is not true}}$

By similar logic as in (a), (b)

(4) Since the problem is set on $\mathbb{R}\times[0,\infty)$ we have to solve it by assuming solution of the form

$$u(x,t) = \varphi(x+2t) + \psi(x-2t).$$

and then proceeding as in lecture video.

(3) Proceed exactly as in lecture video.

Advanced problem

(5)

$$K(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_t^2\,dx \quad , \quad P(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_x^2\,dx .$$

Let $f(t) = K(t) + P(t)$

$$f'(t) = (K' + P')(t)$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} 2u_t u_{tt} + \frac{1}{2}\int_{-\infty}^{\infty} 2u_x u_{tx}$$

$$= \int_{-\infty}^{\infty} u_t u_{xx} + \int_{-\infty}^{\infty} u_x u_{tx}\,dx.$$

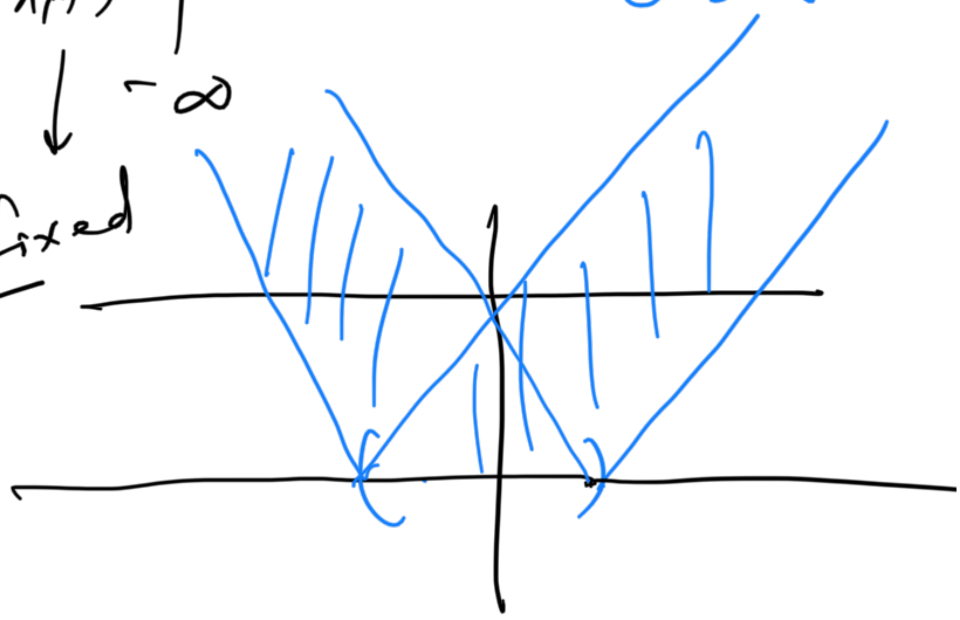
Integration by parts

$$= -\int_{-\infty}^{\infty} \cancel{u_{tx} u_x} + u_t u_x(x,t)\Big|_{-\infty}^{\infty} + \cancel{\int_{-\infty}^{\infty} u_x u_{tx}}$$

$$= u_t u_x(x,t)\Big|_{-\infty}^{\infty} \rightarrow (*)$$

- t is fixed

grad 1 line



$\because$ f, g takes non zero values in $(-1,1)$.

non zero value of 'u' may happen only in the region drawn.

$\Rightarrow$ (*) is '0'

$\Rightarrow \quad f'(t) = 0$

$\Rightarrow$ Kinetic + Potential Energy is conserved for all time

(Conservation of Energy).