1.a. Let X be a random variable with pdf

$$f(x) = \begin{cases} c(x+4) & \text{if} -4 < x < 5, \\ 0 & \text{otherwise.} \end{cases}$$

Find c.
Define $Y = X|X|$. Find the df of Y, and its pdf. $[1 + 4 = 5]$

1.b. (i) Let $\beta > 0$. Find the median and the mode of the following pdf:

$$f(x) = \begin{cases} 2\beta x e^{-\beta x^2} & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

(ii) Let X be a random variable with mgf

$$M_X(t) = \frac{e^{-t}}{8} + \frac{1}{4} + \frac{5e^{2t}}{8} \text{ for } t \in \mathbb{R}.$$

Find the distribution of X. Use it to compute $Var(X)$. $[4 + 2 = 6]$