EE 200: End-Sem
Duration 2 Hrs.

Use the following format of answering:

Name: **Roll No:** **Section:**

Email: **WhatsApp no:**

*Write **only** the final answer in each question. All questions carry equal marks. Answer scripts sent after 120 minutes will be penalized with negative marks. The submission channel will be closed at completion of 120 minutes.*

1. Determine whether the up-sampler defined by the input-output relation:

$$y[n] = \begin{cases} x[\frac{n}{L}]; & n = 0, \pm L, \pm 2L, ... \\ 0; & otherwise \end{cases}$$

is a linear or a non-linear digital system. Determine whether the up-sampler is a time-invariant or a time-varying system. Justify your answer.

2. Determine the impulse response $h[n]$ of the cascade of two causal LTI digital systems with impulse responses given by $h_1[n] = 0.2\delta[n] - 0.5(0.2)^n\mu[n]$ and $h_2[n] = 0.4\delta[n] + 0.1(-0.4)^n\mu[n]$.

3. Let $y[n]$ be the convolution of $x[n]$ and $h[n]$, where $x[n]$ and $h[n]$ are finite-length sequences with their first samples at time index $n = 0$. Determine analytically by deconvolution the sequence $x[n]$ for given $y[n] = \{2, -5, -7, 27, 23, -79, -9, 36\}$,

and $h[n] = \{1, -4, 5, 2, -3\}$. Verify your results using the two identities:

$$\left(\sum_n x[n]\right)\left(\sum_n h[n]\right) = \left(\sum_n y[n]\right)$$

and

$$\left(\sum_n (-1)^n x[n]\right)\left(\sum_n (-1)^n h[n]\right) = \left(\sum_n (-1)^n y[n]\right)$$

4. Determine the DTFT of a signal

$$x[n] = (n+1)\alpha^n \mu[n], |\alpha| < 1$$

5. Derive the impluse response $h_{HT}[n]$ of the ideal Hilbert transformer given by the frequency response

$$H_{HT}(e^{j\omega}) = \begin{cases} j; & -\pi < \omega < 0 \\ -j; & 0 < \omega < \pi \end{cases}$$

6. Consider the sequence $x[n]$ from which we form the new sequences $x_p[n]$ and $x_d[n]$ by sampling and decimation, respectively, as follows $x_p[n] = \begin{cases} x[n], \ n \text{ even} \\ 0, \ n \text{ odd} \end{cases}$ and

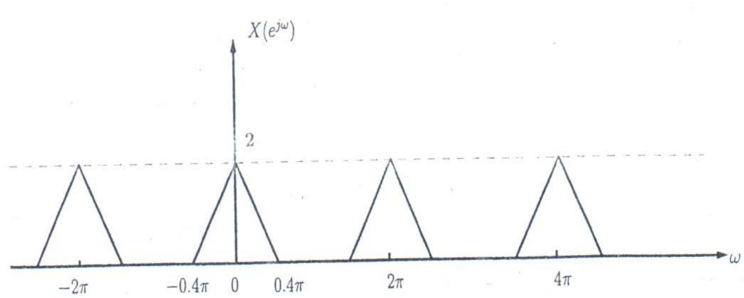

Figure 1: Diagram for Question 6

$x_d[n] = x[2n]$. Sketch the spectra $X_p(e^{j\omega})$ and $X_d(e^{jw})$ from the spectrum $X(e^{j\omega})$ of $x[n]$ as shown in Fig.1.

7. Determine the impulse response $g[n]$ of the causal LTI digital system that is the inverse of the causal LTI digital system with an impulse response

$$h[n] = 2\delta[n] + 5(0.5)^n \mu[n].$$

Use the DTFT based approach, and write the expression for $G(e^{j\omega})$ in partial-fraction expansion.

8. Determine the z-transform and the corresponding ROC of the sequence

$$x[n] = r^n \cos(\omega_0 n)\mu[n].$$

9. Let $x[n]$ be a digital signal with z-transform $X(z)$. For each of the following signals, determine the z-transform in terms of X(z).

   (a) $x_i[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

   (b) $x_d[n] = x[2n]$

10. Consider the digital filter structure shown in Fig.2. Find $H(z)$

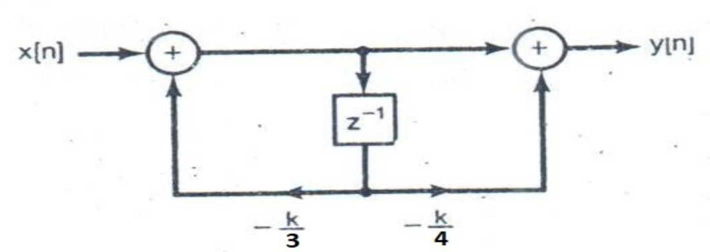

Figure 2: Diagram for Question 10

for this causal filter. Plot the pole-zero pattern and indicate the region of convergence. For what values of k is the system stable?