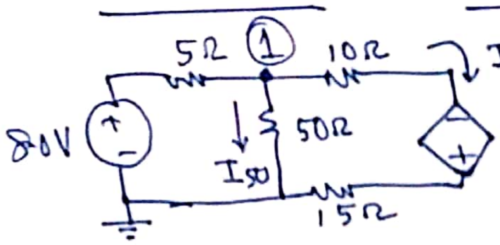
ESc 201A HA#1 Solutions. 2019-I/

1.



KCL at node 1:

$$\frac{v_1-80}{5}+\frac{v_1}{50}+\frac{v_1+75I_{50}}{10+15}=0$$

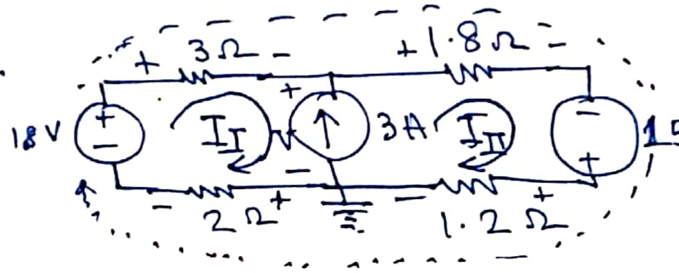
or $0.26\,v_1 + 3\frac{v_1}{50} = \frac{80}{5}$

or $0.32\,v_1 = 16$ ∴ $v_1 = 50V$. ∴ $I_{50} = \frac{50}{50} = 1A$

$I = \frac{v_1+75I_{50}}{25} = \frac{50+75}{25} = 5A$. And the power delivered by $(75I_{50})$

$P_{75I_{50}} = -75I_{50}\times I = -75\times1\times5 = -\underline{375W}$

2.



Choose mesh currents I_I & I_{II}

∴ $I_{II} - I_I = 3A$.

But voltage V is unknown.

∴ Use a supermesh of the dotted line, which gives

$18 = 3I_I + 1.8I_{II} - 15 + 1.2I_{II} + 2I_I$.

or $33 = 5I_I + 3I_{II}$ or $33 = 5I_I + 3(I_I+3)$

∴ $33-9 = 8I_I$ or $I_I = 3A$ ∴ $I_{II} = 6A$.

$V = 18 - 3I_I - 2I_I = 18 - 5I_I = 18 - 15 = 3V$.

$V_{3\Omega} + V_{2\Omega} = 18V - 3V = 15V$. ∴ $V_{3\Omega} = 9V$ & $V_{2\Omega} = 6V$. Not reqd

$P_{3\Omega} = 3\times3^2 = 27W$, $P_{2\Omega} = 2\times3^2 = 18W$, $P_{1.8\Omega} = 1.8\times6^2 = 64.8W$ and $P_{1.2\Omega} = 1.2\times6^2 = 43.2W$. Add all up and get $P_{diss} = 153W$

Cross Check: Power Developed → $P_{3A} = -3\times3 = -9$

$P_{18V} = -18\times3 = -54W$, $P_{15V} = -15\times6 = -90W$,

Total is $P_{DEV} = -54 - 90 - 9 = -153W$.

3. Apply KCL at node 1: $\frac{V_1}{50}+\frac{V_1-12}{2}+\frac{V_1-V_2}{40}=0$. or $0.545V_1 = \frac{V_2}{40}+6$

∴ $V_1 = 0.046V_2 + 11$.

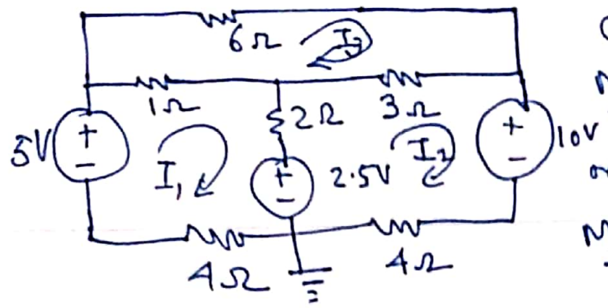
Apply KCL at node 2: $\frac{V_2-V_1}{40}+\frac{V_2}{400} - 0.5 = 0$ or $0.0275V_2 = 0.5 + \frac{V_1}{40}$

∴ $V_2 = \frac{0.5}{0.0275} + \frac{0.046V_2}{0.0275\times40} + \frac{11}{0.0275\times40}$ or $0.958V_2 = 28.2$

∴ $V_2 = 29.4V$, or $V_1 = 0.046V_2 + 11 = 0.046\times29.4 + 11 = 12.35V$

4. $\frac{V_0-V_1}{R} + \frac{V_0-V_2}{R} + \cdots \frac{V_0-V_n}{R} = 0$. or $V_0 = V_1+V_2+\cdots V_n$

or $V_0 = \frac{1}{n}\sum_{i=1}^{n} v_i$ which is the average.

5 

Choose mesh currents

Mesh 1: $2.5 = 1\times(I_1 - I_3) + 2\times(I_1 - I_2) + 4I_1$

or $2.5 = 7I_1 - 2I_2 - I_3$

Mesh 2: $-10 + 2.5 = 4I_2 + 2(I_2 - I_1) + 3(I_2 - I_3)$

or $-7.5 = -2I_1 + 9I_2 - 3I_3$

or $7.5 = 2I_1 - 9I_2 + 3I_3$

Mesh 3:

$0 = 1\times(I_3 - I_1) + 3(I_3 - I_2) + 6I_3$

or $0 = -I_1 - 3I_2 + 10I_3$

or $0 = I_1 + 3I_2 - 10I_3$

$$\therefore \overbrace{\begin{bmatrix} 7 & -2 & -1 \\ 2 & -9 & 3 \\ 1 & 3 & -10 \end{bmatrix}}^{A} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 7.5 \\ 0 \end{bmatrix} \quad \text{or } [A][I] = [B]$$

$\therefore \Delta = 7(90-9) + 2(-20-3) - 1(6+9) = 506.$

$a_{11} = \begin{vmatrix} -9 & 3 \\ 3 & -10 \end{vmatrix} = 90 - 9 = 81$, $a_{12} = \begin{vmatrix} 2 & 3 \\ 1 & -10 \end{vmatrix} = -20 - 3 = -23$

$a_{13} = \begin{vmatrix} 2 & -9 \\ 1 & 3 \end{vmatrix} = 6 + 9 = 15$, $a_{21} = \begin{vmatrix} -2 & -1 \\ 3 & -10 \end{vmatrix} = 20 + 3 = 23$

$a_{22} = \begin{vmatrix} 7 & -1 \\ 1 & -10 \end{vmatrix} = -70 + 1 = -69$, $a_{23} = \begin{vmatrix} 7 & -2 \\ 1 & 3 \end{vmatrix} = 21 + 2 = 23$

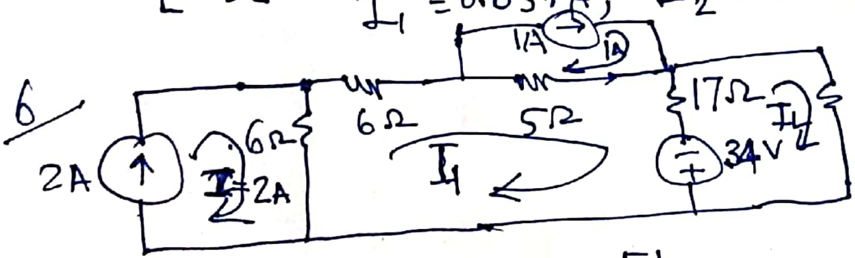
$a_{31} = \begin{vmatrix} -2 & -1 \\ -9 & 3 \end{vmatrix} = -6 - 9 = -15$, $a_{32} = \begin{vmatrix} 7 & -1 \\ 2 & 3 \end{vmatrix} = 21 + 2 = 23$

& $a_{33} = \begin{vmatrix} 7 & -2 \\ 2 & -9 \end{vmatrix} = -63 + 4 = -59.$

$$C = \begin{bmatrix} +a_{11} & -a_{12} & +a_{13} \\ -a_{21} & +a_{22} & -a_{23} \\ +a_{31} & -a_{32} & +a_{33} \end{bmatrix} \Longrightarrow \begin{bmatrix} 81 & 23 & 15 \\ -23 & -69 & -23 \\ -15 & -23 & -59 \end{bmatrix}, \quad A^{-1} = \frac{1}{506}\begin{bmatrix} 81 & -23 & -15 \\ 23 & -69 & -23 \\ 15 & -23 & -59 \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{506}\begin{bmatrix} 81 & -23 & -15 \\ 23 & -69 & -23 \\ 15 & -23 & -59 \end{bmatrix}\begin{bmatrix} 2.5 \\ 7.5 \\ 0 \end{bmatrix} = \frac{1}{506}\begin{bmatrix} 30 \\ -460 \\ -135 \end{bmatrix} = \begin{bmatrix} 0.059 \\ -0.91 \\ -0.266 \end{bmatrix} A$$

$I_1 = 0.059A$, $I_2 = -0.91A$ & $I_3 = -0.266A$

6 

In the central mesh:

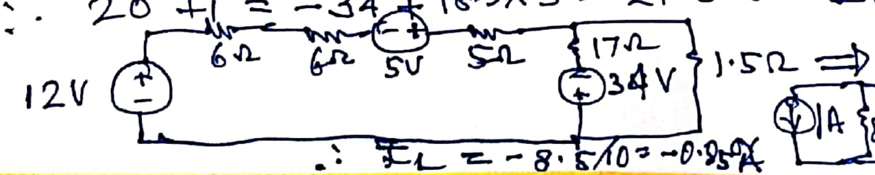
$6(I_1 - 2) + 6I_1 + 5(I_1 - 1) + 17(I_1 - I_L) - 34 = 0$

or $34I_1 - 17I_L = 51 \quad \therefore 2I_1 - I_L = 3$ or $I_L = 2I_1 - 3$

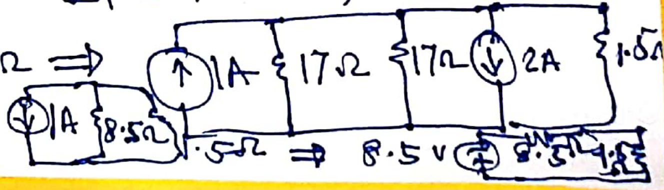
In the rightmost mesh: $17(I_L - I_1) + 1.5I_L + 34 = 0$.

giving $-17I_1 + 18.5I_L = -34$ or $-17I_1 + 18.5(2I_1 - 3) = -34$

$\therefore 20I_1 = -34 + 18.5\times 3 = 21.5$ or $I_1 = 1.075A$, $I_L = -0.85A$



$\Rightarrow$



$\therefore I_L = -8.5/10 = -0.85A$

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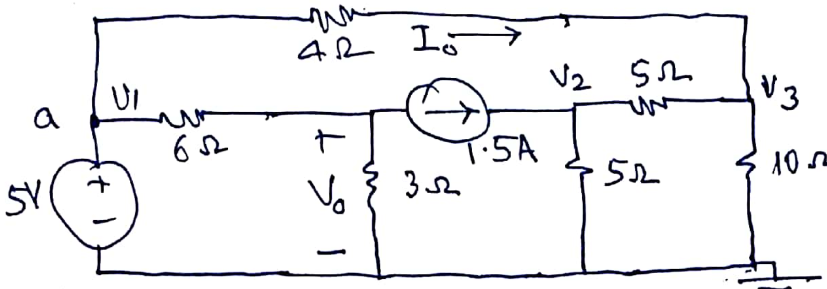
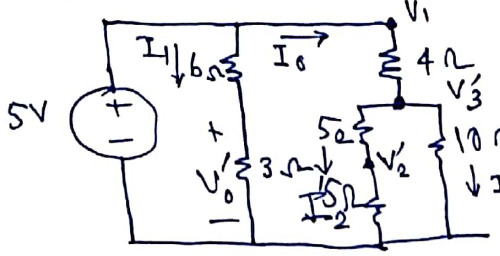


By superposition

(i) Open 1.5A current source:



$I_1' = \frac{5}{9}$ A $\quad V_o' = \frac{15}{9}$ A

$V_1' = 5V$

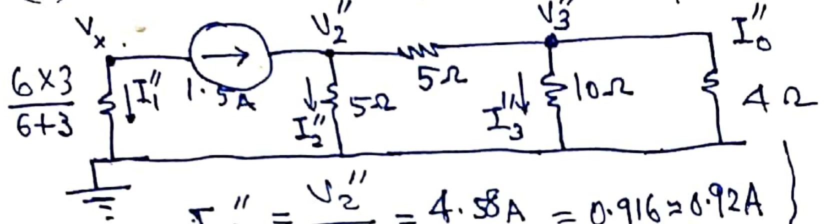
$V_2' = \frac{5 \times 5}{18}$ V.

$V_3' = 10 \times \frac{5}{18}$

$I_o' = \frac{5}{4 + (5+5)\|10} = \frac{5}{4+5} = \frac{5}{9}$ A

$\therefore I_2' = I_3' = \frac{5}{18}$ A

(ii) Short 5V voltage Source: point (a) is grounded i.e. $V_1'' = 0$



$V_o'' = V_x = -1.5 \times \frac{12}{9}$ V

$V_2'' = 1.5 \times [5 \| (5 + 10\|4)]$

$= 1.5 \times [5 \| (5 + \frac{40}{14})]$

$= 1.5 \times (5 \| 7.86) = 4.58$ V.

$I_2'' = \frac{V_2''}{5} = \frac{4.58}{5}$ A $= 0.916 \approx 0.92$ A

$\therefore I_{5\Omega}'' = 1.5 - 0.92 = 0.584$ A

$V_3'' = 0.584 \times \frac{40}{14} = 1.67$ V

$I_o'' = \frac{1.67}{4} = 0.42$ A $\quad \therefore I_3'' = \frac{1.67}{10} = 0.167$ A

$\therefore V_o = V_o' + V_o'' = \frac{15}{9} + (-\frac{1.5 \times 12}{9}) = 1.67 - 2 = -0.33$ V

$I_o = I_o' + I_o'' = \frac{5}{9} + 0.42 = 0.98$ A

Need not calculate all node voltages & currents only V_o & I_o needed!

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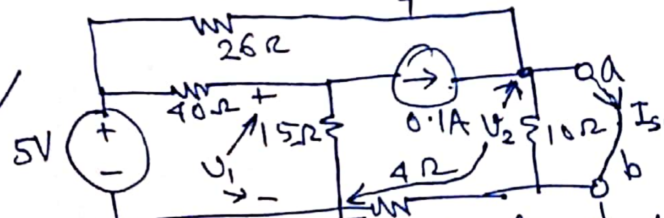
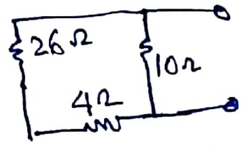


Null the independent sources.



$R_{Th} = 30 \| 10 = 7.5\Omega$

I_{sc} or V_{oc} needed to be found, & as there is a current source easier to find I_{sc} → Nodal Equations!

$\frac{V_2 - 5}{26} + \frac{V_2}{14} - 0.1 = 0 \longrightarrow 0.11 V_2 = 0.29 \rightarrow V_2 = 2.65$ V.

$V_{Th} = \frac{V_2}{14} \times 10 = 1.9$ V. Just for the sake of completeness find I_{sc}.

again nodal equations $\frac{V_2' - 5}{26} + \frac{V_2'}{4} - 0.1 = 0$ or $0.29 V_2' = 0.29$

or $V_2' = 1$ V. $\therefore I_{sc} = \frac{V_2'}{4} = 0.25$ A or $R_{Th} = \frac{1.9}{0.25} = 7.6\Omega$

R_{Th} very close to the direct method. The difference is due to rounding off errors.

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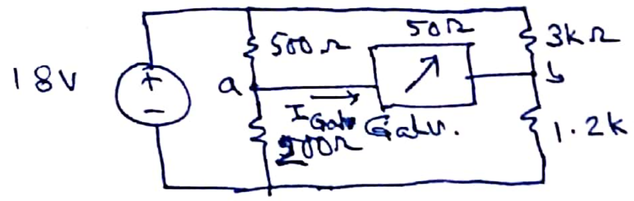

At null $V_{ab} = 0$. $\therefore I_{50\Omega} = 0$.

$\therefore \frac{500}{200} = \frac{3k\Omega}{1.2k\Omega}$ Bridge is balanced.

Now assume 3kΩ is off by 0.1% or 3kΩ → 3.003 Ω

$V_{ab} = \frac{18}{700} \times 200 - \frac{18 \times 1200}{4203}$

$= 5.143 - 5.139$ (keeping significant decimal digit here is necessary)

$= 0.00367\,V = 3.67\,mV$

$R_{Th} = 500 \| 1200 + 3.003\,k\Omega \| 1.2\,k\Omega = 1{,}000.245\,\Omega$

$\therefore I_{Galv.} = \frac{3.67\,mV}{1{,}000.245 + 50} = 0.00349\,mA \cong 3.5\,\mu A$.

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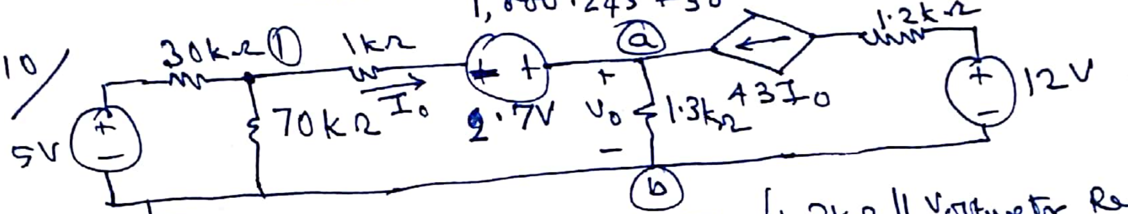
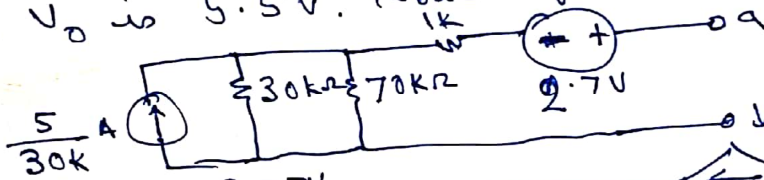


Does one Need to find the $V_{Th_{ab}}$ & $R_{Th_{ab}}$??

Note that the current through the (1.3kΩ ‖ Voltmeter Resistance) is $44 I_0$ and V_0 is 5.5V. Node equation at ① gives simplifying the circuit.



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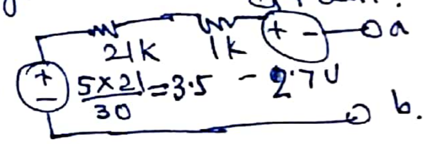

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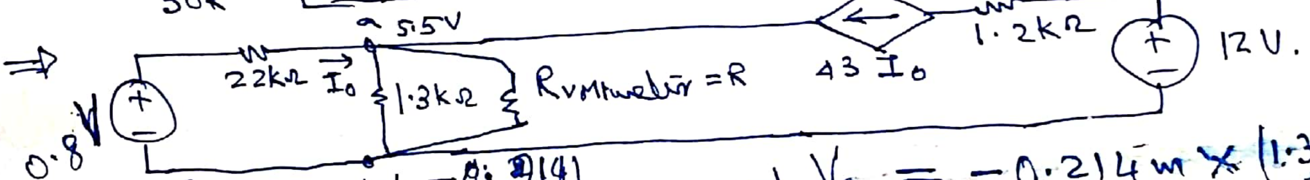

$|I_0| = \left|\frac{0.8 - 5.5}{22k}\right| = -0.214\,mA$ and $V_a = -0.214\,m \times (1.3k \| R)$ → voltage is negative with respect to (b)

$\therefore$ Problem is incorrect. Choose 2.7V to be "4.7V then $I_0 = 0.123\,mA$ & $R_{Galv} = 4.7\,k\Omega$"

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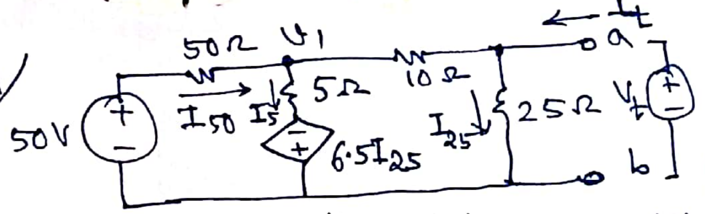

Find $V_{ab} = V_{oc}$

$I_{25} = \frac{V_1 - V_{oc}}{10}$ as point 'a' is open. $I_{10} = I_{25}$

Also $I_{25} = \frac{V_{oc}}{25}$

$\therefore V_1 = V_{oc}\left(\frac{1}{10} + \frac{1}{25}\right) \times 10 = 1.4\,V_{oc}$

$I_{50} = \frac{50 - V_1}{50} = 1 - \frac{V_1}{50} = 1 - \frac{1.4\,V_{oc}}{50}$ and $I_5 = I_{50} - I_{25}$

$= 1 - \frac{1.4\,V_{oc}}{50} - \frac{V_{oc}}{25}$

$= 1 - 0.068\,V_{oc}$

$5 I_5 - 6.5 I_{25} = V_1 = 1.4\,V_{oc}$

$5 \times (1 - 0.068\,V_{oc}) - 6.5\frac{V_{oc}}{25} = 1.4\,V_{oc}$ Solve to get $V_{oc} = 2.87\,V$.

R_{Th} is easier to be found by applying a test voltage V_t at a, b. and nulling all independent sources. and node analysis at V_1

$\therefore I_t = \frac{V_t}{25} + \frac{V_t - V_1}{10} = 0.14\,V_t - 0.1\,V_1$

so $\frac{V_1 - 0}{50} + \frac{V_1 + 6.5(V_t/25)}{5} + \frac{V_1 - V_t}{10} = 0$ or $0.32\,V_1 - 0.048\,V_t = 0$ or $V_1 = \frac{0.048\,V_t}{0.32}$

$\therefore I_t = 0.14\,V_t - 0.1 \times \frac{0.048\,V_t}{0.32} = \frac{V_t}{8}$ $\therefore R_{Th} = \frac{V_t}{I_t} = 8\,\Omega$.

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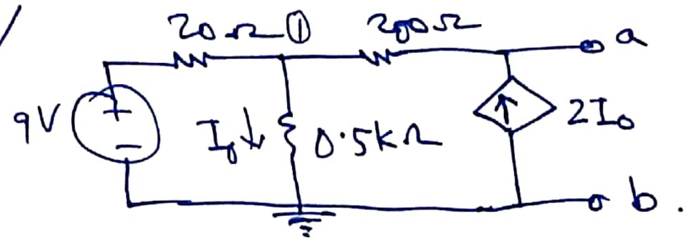

KCL at node ① gives.

$$\frac{v_1 - 9}{20} + I_o + \frac{v_1 - V_{oc}}{200} = 0.$$

$$I_o = \frac{V_1}{500}$$

Note $V_{oc} = V_{ab}$

and $\dfrac{V_1 - V_{oc}}{200} = -2I_0 = -\dfrac{V_1}{250}$

$\therefore V_{oc} = 200 \times 0.009 V_1 = 1.8 V_1$

The KCL Eqn. can now be written as $\dfrac{V_1}{20} + \dfrac{V_1}{500} + \dfrac{V_1}{200} - \dfrac{1.8 V_1}{200} = \dfrac{9}{20}$

or $(0.05 + 0.002 + 0.005 - 0.009) V_1 = 0.45$ or $V_1 = 9.375$ V

$\therefore V_{oc} = 1.8 \times 9.375 = \underline{16.875}$ V.

To find R_{Th} use V_t & I_t technique after shorting the 9V supply and at "a" KCL is

$$\frac{v_t - v_1}{200} - 2I_o = I_t \quad \text{where} \quad I_o = \frac{v_1}{500} \text{ from node ①}$$

$$\therefore I_t = \frac{v_t}{200} - v_1\left(\frac{1}{200} + \frac{1}{250}\right) = 0.005V_t - V_1(0.005 + 0.004)$$

$$= 0.005V_t - V_1(0.009)$$

KCL at node ① gives.

$$\frac{v_1}{20 \| 500} + \frac{v_1 - v_t}{200} = 0 \text{ or } 200(0.052 + 0.005)V_1 = V_t$$

$$\therefore V_1 = \frac{V_t}{11.4} \qquad \therefore I_t = 0.005V_t - \frac{0.009}{11.4}V_t$$

$$= 0.00421 \times V_t$$

$$\therefore R_{Th} = \frac{V_t}{I_t} = \underline{\underline{237.53\Omega}}$$