

- **Two-Pole System:**

- **Transfer Function:**

$$A(s) = \frac{A_0}{(1 + j\omega/\omega_{p1})(1 + j\omega/\omega_{p2})}$$

A_0 : **Low-Frequency Gain**

ω_{p1}, ω_{p2} : **Two negative real poles**, lying on the **σ axis**, with $\omega_{p2} > \omega_{p1}$

- Now, with **passive feedback** with **feedback factor** f , the **locations** of the **closed-loop poles** can be found from: $1 + fA(s) = 0$

➤ Thus:

$$s^2 + (\omega_{p1} + \omega_{p2})s + (1 + fA_0)\omega_{p1}\omega_{p2} = 0$$

➤ **Solution** gives the *locations* of the *two closed-loop poles*:

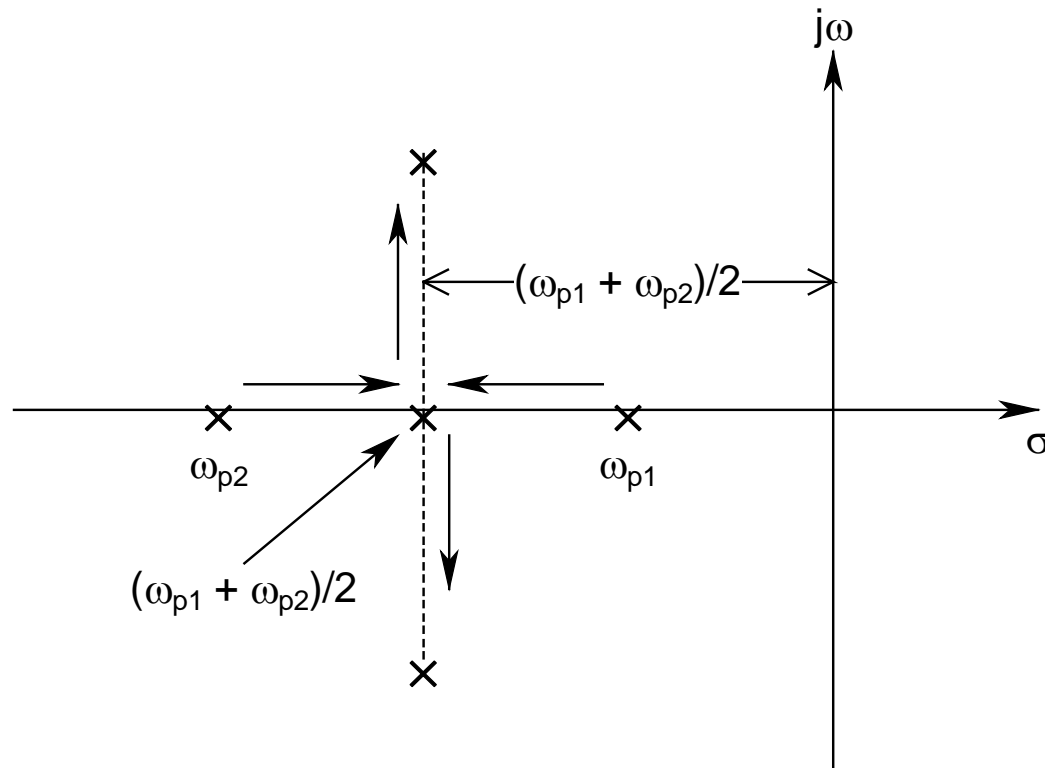
$$s_1, s_2 = -\frac{\omega_{p1} + \omega_{p2}}{2} \pm \frac{1}{2} \sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + fA_0)\omega_{p1}\omega_{p2}}$$

➤ With **increase in feedback**, the *second term reduces*

$\Rightarrow s_1$ and s_2 start to move towards each other along the σ axis

➤ Eventually, at a **particular feedback**, the *second term would vanish*

- *At this point*, the *two poles will merge* at $(\omega_{p1} + \omega_{p2})/2$
- With *further increase in feedback*, the *second term becomes imaginary*, while the *first term remains constant*
 - ⇒ *The poles remain complex conjugates*
- *Even for all the way up to unity*, *when the entire output is fed back to the input*, the *poles remain in the LHP* and *can never enter RHP*
 - ⇒ *The system remains unconditionally stable*



**Movement of the Poles for a Two-Pole System
Under Negative Feedback With Increasing D**

➤ Also, for a *two-pole system*, the *phase reaches -180° only when the frequency becomes infinite (mathematically)*

⇒ *There is no physically achievable frequency when this can happen*

⇒ *Unconditional Stability*

- *System With Three (or More) Poles:*

➤ *Actual mathematical analysis quite tedious*

➤ It can be shown that as the *amount of feedback (D) is increased:*

- *The highest frequency pole (ω_{p3}) moves outward along the $-\sigma$ -axis*

- *The other two poles (ω_{p1} and ω_{p2}) move towards each other (similar to a two-pole system)*
- *As D is increased further, these two poles eventually merge, and then start having imaginary components*
- *Their real part also keeps on changing with D , keeping the nature of complex conjugacy intact, and moves right in the s -plane*
- *The path traced out by these poles is known as the root locus*
- *For a particular value of D , this root locus intersects the imaginary axis of the s -plane at two symmetric points*

- *Under this condition, sustained sinusoidal oscillation can be achieved, since it now has a complex conjugate pair of poles without any real part (ω_{p3} will be so large that it will be inconsequential)*
- *With further increase in D , the root locus enters the RHP with the poles now having positive real part*
 - \Rightarrow *Potentially dangerous situation in terms of stability*
- *In terms of phase, the total can be -270°*
 - \Rightarrow *There exists a particular value of f , for which the phase will become -180°*