Transfer Function & Stability

- There is a ***strong correlation*** between the ***transfer function*** and ***stability*** of a system
- ***Single-Pole System***:
  - ***Transfer function*** with a ***negative real pole*** at ω_p:

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_p}$$

A_0: ***Low-Frequency Gain***

- Now, assume that the ***system*** is ***connected*** in a ***feedback loop***, with the ***feedback network*** having ***feedback factor*** f

  $\Rightarrow$ The ***closed-loop transfer function***:

$$A_f = \frac{A_{f0}}{1 + j\omega/\omega_{pf}}$$

$$A_{f0} = A_0/(1 + fA_0) \text{ and } \omega_{pf} = \omega_p(1 + fA_0)$$

- The ***gain with feedback reduces by the same amount as the bandwidth gets increased***, ***keeping the GBP constant***

- Thus, the ***new pole frequency*** is D (the ***return difference***) times the ***old pole frequency***

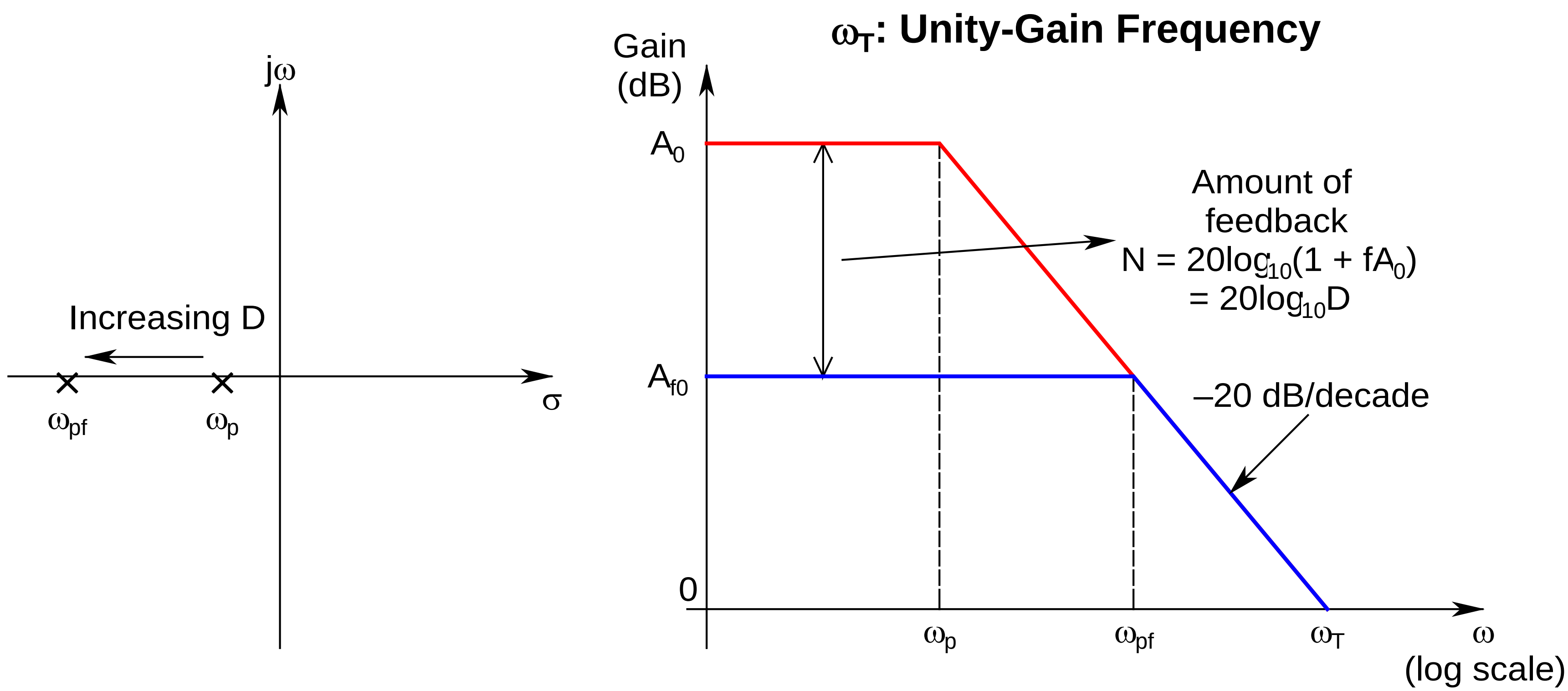
  $\Rightarrow$ It shifts ***left*** along the σ ***axis*** in the ***s-plane***, and ***remains on the LHP without any imaginary component***

  $\Rightarrow$ ***The system remains stable even with feedback***

- Also, the ***phase*** of the system ***can never fall below*** $-90°$

- Here, of course we are assuming a ***passive feedback network***, i.e., ***f is a real number***

- Thus, ***f does not add any phase to the system***
- Hence, ***Barkhausen's criteria can never be satisfied for this case***
- Also, the ***pole can never enter the RHP***
- Thus, we ***conclude***:
  - A system with ***single-pole transfer function*** is ***Unconditionally Stable***, i.e., it will ***remain stable*** for ***values of f*** all the way ***up to unity*** (i.e., ***the entire output fed back to the input***)



Movement of the Pole for a Single-Pole System Under Negative Feedback and the Bode Plot of the Gain

- ***Two-Pole System***:
  - ***Transfer Function***:

$$A(s) = \frac{A_0}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

A_0: ***Low-Frequency Gain***

ω_{p1}, ω_{p2}: ***Two negative real poles***, ***lying on the σ axis***, ***with $\omega_{p2} > \omega_{p1}$***

  - Now, with ***passive feedback*** with ***feedback factor*** f, the ***locations*** of the ***closed-loop poles*** can be found from: $1 + fA(s) = 0$