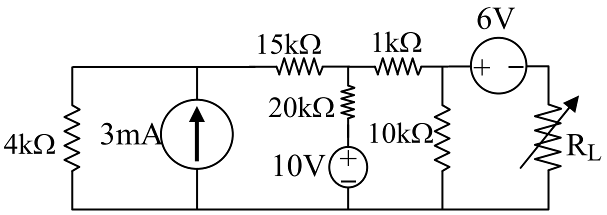
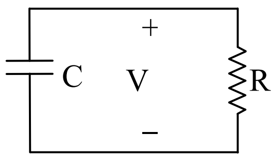
ESc201A Home Assignment 2 Aug. 11, 2019.

Solutions of the HA#2 will be on Brihaspati on 17/08/19.

Consider all voltage and current sources to be ideal.

1. In the circuit shown in fig. 2.1, R_L is an adjustable resistive load. What is the value of this load (R_L) for which maximum power is dissipated at the load? What is this maximum power dissipation at R_L?





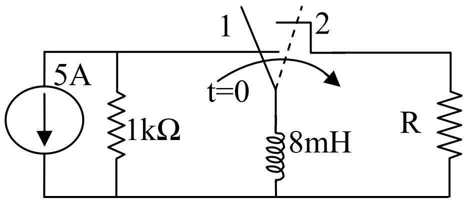
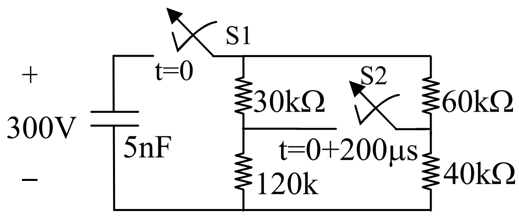


Figure 2.1 Figure 2.2 Figure 2.3

2. In the R-C circuit shown in fig. 2.2 the voltage and the current expressions are: $v(t)= 72e^{-500t}$ V, $t \geq 0$, and $i(t)=9e^{-500t}$ mA for $t \geq 0^{+}$. Find (i) R, (ii) C, (iii) τ [ms], (d) the initial energy stored in the capacitor, and (e) how many ms it takes to dissipate 75% of the initial energy stored in the capacitor?

3. The switch in the circuit seen in fig. 2.3 has been in position 1 for a long time. At t = 0 the switch moves instantaneously to position 2. Find the value of R so that half of the initial energy stored in the 8-mH inductor is dissipated in R in 10μs.

4. The capacitor in the circuit shown in fig. 2.4 has been charged to 300V. At t=0, switch S1 closes and the switch S2 closes 200μs after switch S1 has closed. Find the current through the switch S2 at a time t=300μs.



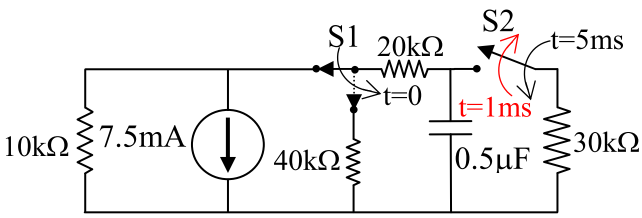


Figure 2.4 Figure 2.5

5. In the circuit of fig. 2.5, switch S1 disconnects from the 10kΩ resistance and instantaneously connects to the 40kΩ resistor at time t =0, with the switch S2 closed (connected to the 20kΩ resistor). At t = 1ms, switch S2 opens out. Again at t = 5ms, switch S2 closes.
   (a) Find the voltage across the capacitor (v_C) at all times.
   (b) What percentage of the total energy stored on the capacitor is dissipated in the 30kΩ resistor till 15ms?

6. The switch in the circuit of fig. 2.6 has been closed for a long time. The switch opens at time t = 0. Find the inductor voltage $v_L(t)$ for all $t \geq 0^{+}$.

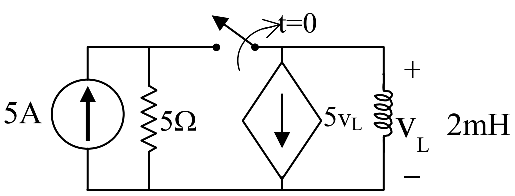


Figure 2.6

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7. The switch in fig. 2.7 has been open for a long time. At t = 0 the switch closes. Find the inductor voltage $v_L(t)$ for all $t \geq 0^+$.

8. For the 2μF capacitor circuit, shown in fig. 2.8, the switch has been closed for a long time. The capacitor is rated for a maximum voltage of 1.5kV, after which the dielectric breaks down and shorts it. If the switch opens at time t = 0, then at what time does the capacitor reach the dielectric breakdown voltage?

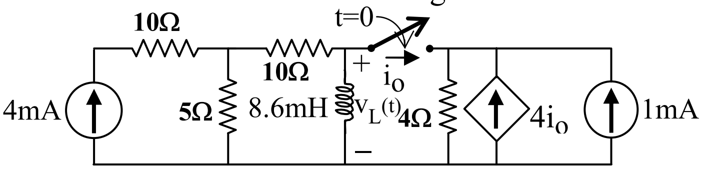


Figure 2.7

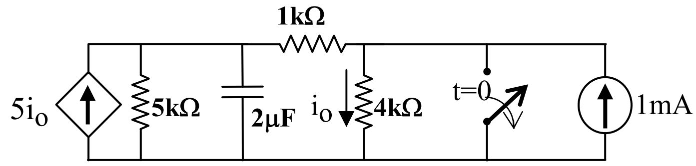
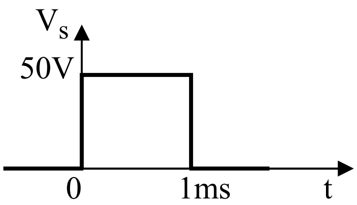
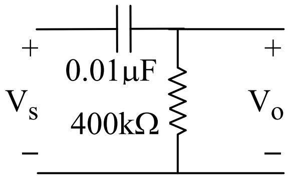
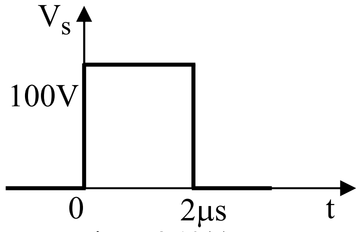


Figure 2.8

9. The voltage waveform in Fig. 9(a) is applied to the circuit in Fig. 9(b). If the initial voltage on the capacitor is zero, (a) Calculate $v_o(t)$, and (b) Sketch $v_o(t)$ versus t.







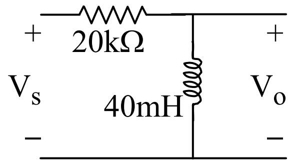


Figure 2.9(a) Figure 2.9(b) Figure 2.10(a) Figure 2.10(b)

10. The voltage waveform shown in Fig. 10(a) is applied to the circuit of Fig. 10(b). If the initial current in the inductor is zero, then (i) calculate $v_o(t)$ and (ii) sketch $v_o(t)$ versus t.

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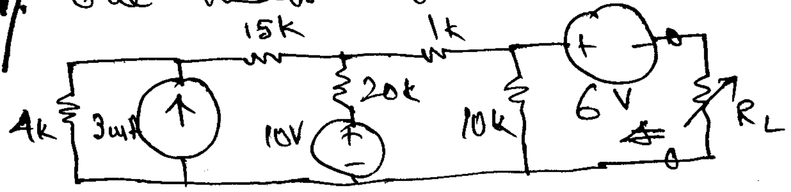
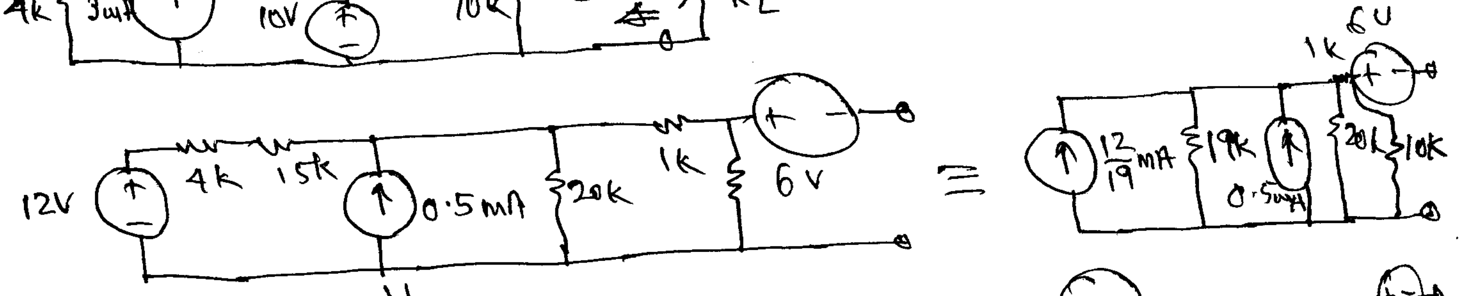
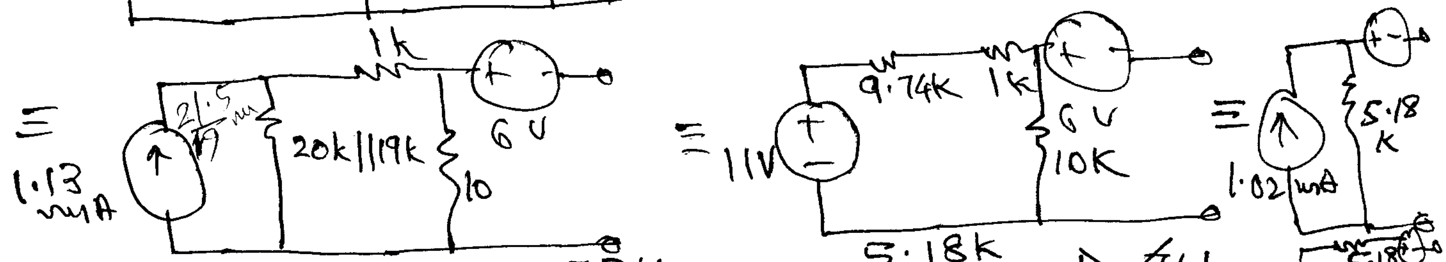
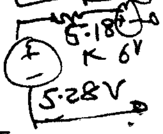
1/ One needs to find the Thevenin Equivalent across R_L.









$\therefore V_{oc} = \cancel{11} - 6 = \cancel{5}\,V$ $5.28 - 6 = -0.72\,V$, $R_{Th} = \cancel{9.74+1} = \cancel{10.74}k$ $5.18k$

$\therefore$ Maximum power transfer when $R_L = \cancel{10.74}k\Omega$ $5.18k$

$v_{R_L}\big|_{R_L=R_{Th}} = 2.5$ V $\therefore P_{max_{R_L}} = \frac{(V_{R_L})^2}{R_L} = \frac{(0.7)^2}{\cancel{10.74}k \; 5.18K} = 99.5\,mW$

2/ $v(t) = 72e^{-500t}$ V, $i(t) = 9e^{-500t}$ mA $v = Ae^{-t/\tau}$.

$\therefore \frac{1}{\tau} = 500$ or $RC = \frac{1}{500} = 2\times10^{-3} = 2\,ms$

$R = \frac{v(t)}{i(t)} = \frac{72}{9\times10^{-3}} = 8k\Omega$ $\therefore C = \frac{2\times10^{-3}}{8\times10^{3}} = 0.25\,\mu F$

$v_c(\infty) = 0$, $\therefore v_c(0^+) = 72V$ $\therefore W_{c\,initial}(0^+) = \frac{1}{2}Cv(0^+)^2$

$= 0.5\times0.25\mu\times72^2$

$= 648\,\mu J$

$\cancel{W_c(t) = v(t)\cdot i(t)}$

$\cancel{= 72\times9\times e^{-1000t}}$

$\cancel{= 0.75\times648\times\mu J}$

$\cancel{\ln\left(\frac{0.75\times648\times10^{-6}}{72\times9}\right) = -1000t}$ $\cancel{\text{or } t = \frac{14.1}{1000} = 14.1\,ms}$.

At time "t" energy stored in the capacitor is

$W_c(t) = \frac{1}{2}Cv^2(t) = 0.5\times0.25\,\mu\times72^2e^{-1000t} = 0.25\times648\,\mu J$

or $1000t = -\ln(0.25) = 1.39$

$\therefore t = 1.39\,ms$.

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3/ $i_L(0^-) = -5A$ as the $110\,\Omega$ resistor is shorted.

$\therefore i_L(0^+) = -5A$. $\therefore W_L(0^+) = \frac{1}{2} L i_L^2(0^+) = 0.5 \times 8 \times 10^{-3} \times 25 = 100\,mJ$.

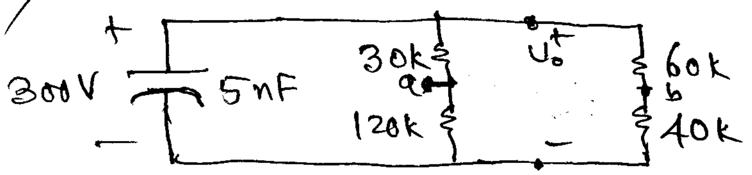
$i_L(\infty) = 0$ $\therefore i_L(t) = 0 + [-5 - 0] e^{-t/\tau_{LR}}$

where $\tau_{LR} = \frac{L}{R}$ $\Rightarrow$ or $\frac{1}{2} L \times 25 e^{-2t/(L/R)} = 50\,mJ$. and $t = 10\,\mu s$.

$\therefore \frac{50 \times 2}{8 \times 25} = e^{-2 \times 10^{-5}/(\frac{L}{R})}$ or $\ln(0.5) = -\frac{2 \times 10^{-5}}{\frac{L}{R}}$.

or $0.693 = \frac{2 \times 10^{-5}}{8 \times 10^{-3}} R$ or $R = 277.2\,\Omega$.

4/ $0 \le t \le 200\,\mu s$



$v_o(0^-) = 300V$

$R_{eq} = (30 + 120) \| (60 + 40)$ k Ω

$\therefore v_o(\infty) = 0\,V$ discharging by R_{eq}.

$R_{eq} = 60\,k\Omega$ & $\tau_{RC} = 60 \times 10^3 \times 5 \times 10^{-9} = 0.3\,ms$.

$\therefore V_o(t) = 300 e^{-t/0.3 \times 10^{-3}}$ for $0 \le t \le 200\,\mu s$.

at $t = 200\,\mu s$ $v_o(200\,\mu s) = 300 e^{-200/300} = 154\,V$.

$t \ge 200\,\mu s$ $v_o(\infty) = 0$, $v_o(0^-) = 154V$. and point a & b are shorted.

Now $R_{eq} = (30 \| 60) + (120 \| 40)$ kΩ $= 50\,k\Omega$

$\tau_{RC} = 5 \times 10^{-9} \times 50 \times 10^3 = 250 \times 10^{-6}\,s$, $\frac{1}{\tau} = 4000$

$v_o(t \ge 200\,\mu s) = 154 e^{-4000(t - 200 \times 10^{-6})}$

a & b are shorted $\therefore v_a = v_b = \frac{154}{50} \times (120 \| 40) e^{-4000(t - 200 \times 10^{-6})}$

$= 92.4 e^{-4000(t - 200 \times 10^{-6})}$

$i_{30k} = \frac{v_o(t \ge 200\,\mu s) - v_a(t \ge 200\,\mu s)}{30k} = \left(\frac{154 - 92.4}{30}\right) e^{-4000(t - 200 \times 10^{-6})}$ mA

$\therefore i_{30k}(300\,\mu s) = 2.05 e^{-4000 \times 100 \times 10^{-6}} = 2.05 \times 0.67 = 1.374$ mA

similarly

$i_{60k}(300\,\mu s) = \frac{154 - 92.4}{60} \times 0.67 = 0.688$ mA

$i_{120k}(300\,\mu s) = \frac{92.4 \times 0.67}{120k} = 0.516$ mA

$i_{40k}(300\,\mu s) = \frac{92.4}{40} \times 0.67 = 1.55$ mA

In the left branch current reduces from 1.374 mA to 0.516 mA & in the right branch current increases from 0.688 mA to 2.31 mA.

$\therefore i_{a \to b} = \underbrace{1.374 - 0.516}_{0.858} = \underbrace{1.55 - 0.688}_{0.862} = 0.86\,mA$

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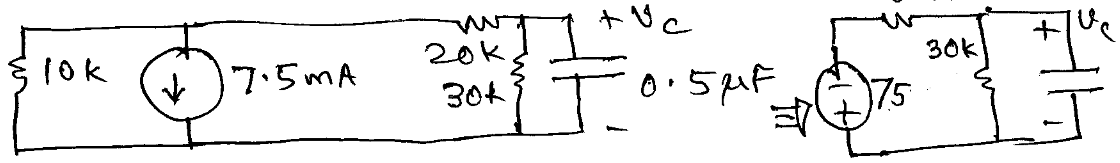
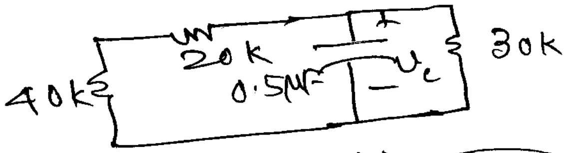
5/ $t < 0$



$v_c(0^-) = -3.75V = v_c(0^+)$

$0 \le t \le 1ms$



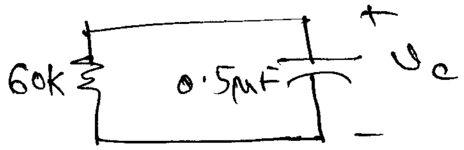
$R_{Th} = 30k \| 60k$
$= 20k\Omega$

$\tau = 0.5\mu \times 20k = 10ms$.

$v_c(0 \le t \le 1ms) = -3.75 e^{-100t}$, $v(\infty) = 0$

$v_c(t = 1ms) = -3.75 e^{-0.1} = -3.39 V$

$1ms \le t \le 5ms$



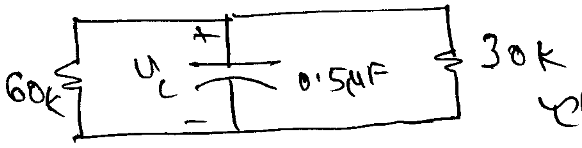
$\tau_{1-5ms} = 60k \times 0.5\mu$
$= 30ms$.

$v_c(1ms) = -3.39$ & $v_c(\infty) = 0$.

$\therefore v_c(1ms \le t \le 5ms) = -3.39 e^{-\frac{(t-1ms)}{30ms}}$.

At t=5ms $v_c(5ms) = -3.39 e^{-4/30} = -2.97 V$.

$5ms \le t \cancel{<} \infty)$



$R_{Th} = \frac{60 \times 30}{90} = 20k$

$\tau(t \ge 5ms) = 0.5 \times 10^{-6} \times 20 \times 10^{3} = 10ms$

Again $v(\infty) = 0$.

$\therefore v_c(t \ge 5ms) = -2.97 e^{-(t-5ms)/10ms}$.

$v_c(t = 15ms) = -2.97 e^{-1} = -1.1V$

Energy stored at $t = 0^+ \longrightarrow w_c = \frac{1}{2} C v^2(0^+) = 0.5 \times 0.5 \times 10^{-6} \times 3.75^2$
$= 352\ \mu J$.

Dissipation:

$p_{30k} = \frac{v_c^2(t \le 1ms)}{30k} = \frac{3.75^2 e^{-200t}}{30k}$ or $w_{30k}(0 \le t \le 1ms) = 0.47 \times 10^{-3} \int_0^{1ms} e^{-200t} dt$

$\therefore w_{30k}(0 \le t \le 1ms) = -\frac{0.47 \times 10^{-3}}{200} \left[e^{-200t}\right]_0^{1ms} = -2.35\mu \times \left[e^{-0.2} - 1\right] \mu J$
$= 42.6 \mu J$.

$p_{30k}(1ms \le t \le 5ms) = 0$.

$p_{30k}(5ms \le t \le 15ms) = \frac{2.97^2}{30k} e^{-2(t-5ms)/10ms} = \frac{2.97^2}{30k} e^{-2t/10ms} . e^{+1}$

$w_{30k} = +29.4m \times \int_{5ms}^{15ms} e^{-200(t-5ms)} dt = -\frac{29.4}{200} m \left[e^{-200(t-5)m}\right]_5^{15}$

$= -1.47\mu \times \left[e^{-200 \times 10 \times 10^{-3}} - 1\right] = -1.47\mu \times \left[e^{-2} - 1\right] = 127 \mu J$.

$\sum w_{30k} = (42.6 + 0 + 127) \simeq 170 \mu J$. % to $\frac{170}{352} \times 100 = 48.3\%$ of $w_c(0)$

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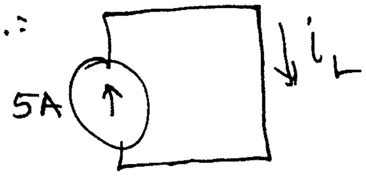
6/ At $t<0$ The inductor acts as a short ∴

$i_L(0^-) = 5A$ and $v_L(0^-) = 0$. =

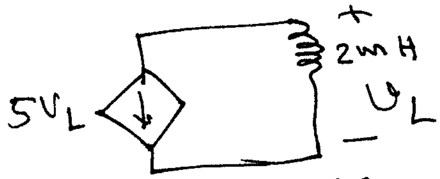


$t>0$ v_L should decrease with time as there are no independent sources connected

∴ $i_L(\infty) = 0$ $v_L \to$, $5v_L \to 0$.

∴ $i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)] e^{-t/\tau} = 5e^{-t/\tau}$.

To find τ. for $t>0$



∴ $v_{oc} = v_L(t)$

$R_{Th} = \frac{v_t}{i_t} = \frac{v_L(t)}{5v_L(t)} = 0.2\Omega$ ∴ $\tau = \frac{2mH}{0.2\Omega} = 10$ ms.

∴ $i_L(t)\Big|_{t>0} = 5e^{-t/10m} = 5e^{-100t}$ A

Again $v_L(t)\Big|_{t>0} = -i_L(t)\Big|_{t>0} \times R_{Th} = -5\times0.2\,e^{-100t} = -e^{-100t}$

Alternately $v_L(t)\Big|_{t>0} = L\frac{di_L(t)|_{t>0}}{dt} = 2\times10^{-3}\times(-100)\times5e^{-100t}$

$= -e^{-100t}$

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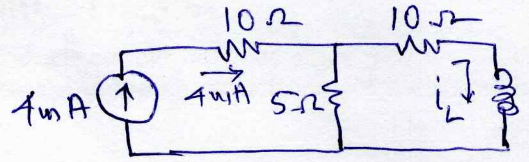
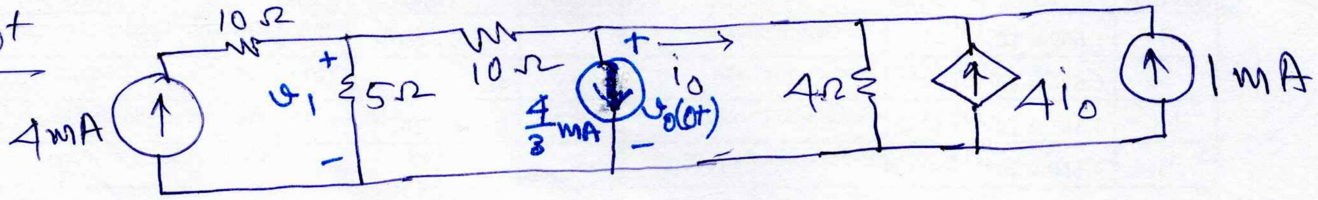
7/

$t<0$



$i_L(0^-) = \frac{4m\times 5}{15} = \frac{4}{3}$ mA

$t=0^+$



$v_o(0^+)$ node KCL

$$\frac{4}{3}mA + \frac{v_o(0^+)-v_1}{10} + \frac{v_o(0^+)}{4} - 4i_o - 1mA = 0.$$

or $v_o(0^+)\times\frac{14}{40} = 4i_o + \frac{v_1}{10} - \frac{1}{3}m \Rightarrow 14v_o(0^+) = 160i_o + 4v_1 - \frac{40}{3}m$ ... (1)

At node for v_1 KCL: $\frac{v_1}{5} + \frac{v_1 - v_o(0^+)}{10} - 4mA = 0.$

or $\frac{v_o(0^+)}{10} = \frac{3}{10}v_1 - 4m \Rightarrow v_o(0^+) = 3v_1 - 40m.$

or $v_1 = \frac{v_o(0^+)}{3} - \frac{40}{3}m$ ... (2)

KCL at right node

$$i_o = \frac{v_o(0^+)}{30} - \frac{v_o(0^+)}{10} - \frac{4}{3}m - \frac{4}{3}m = -\frac{1}{15}v_o(0^+) - \frac{8}{3}m \quad \text{... (3)}$$

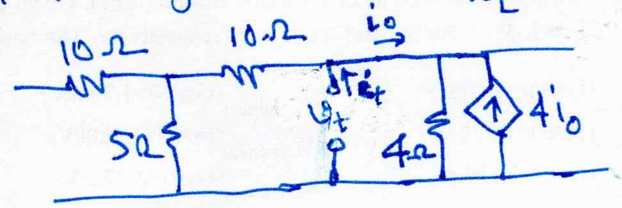
∴ using (2) & (3) in (1)

$$14v_o(0^+) = -160\left(\frac{v_o(0^+)}{15} + \frac{8}{3}m\right) + \frac{4}{3}\left(v_o(0^+) - 40m\right) - \frac{40}{3}m$$

or $-1.33\,v_o(0^+) = -493.33\,m$ ∴ $v_o(0^+) = 371$ mV.

$v(\infty)$ has to be $v(\infty)=0$ as parallel resistances are dissipative.

Next is to find the R_{Th_L} whose equivalent circuit is:



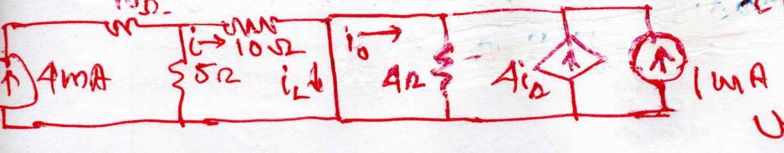
∴ $i_t = \frac{v_t}{15} + \frac{v_t}{4} - 4i_o$

& $i_o = \frac{v_t}{4} - 4i_o$ or $i_o = \frac{v_t}{20}$

∴ $i_t = \frac{v_t}{15} + \frac{v_t}{4} - \frac{v_t}{5} = 0.1167\,v_t$ or $R_{Th} = \frac{v_t}{i_t} = 8.57 \simeq 8.6\,\Omega$

∴ $\tau_L = \frac{L}{R_{Th}} = \frac{8.6m}{8.6} = 1$ ms.

At $t=\infty$: $i = \frac{4m\times 5}{15}$, $i_o = -4i_o - 1mA \rightarrow i_o = -0.2$ mA



$i(\infty) - i_o(\infty) = i_L(\infty) = 1.533$ mA

∴ $i_L(t\geq 0) = 1.533 + [1.333 - 1.533]e^{-1000t}$

$= 1.533 - 0.2e^{-1000t}$

$v_L(t\geq 0) = L\frac{di_L}{dt} = 1.72\,e^{-1000t}$

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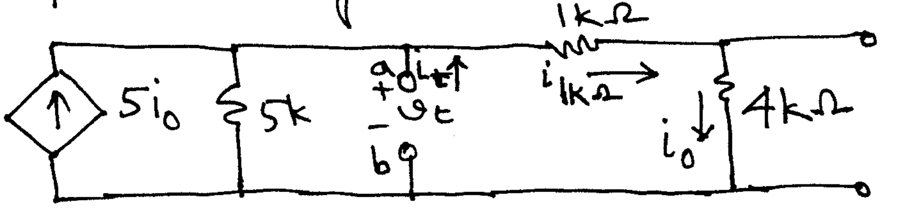
8/ For $t>0$ find the Thevenin eqv. at the terminals a, b, after nulling the independent sources. The eqv. circuit is



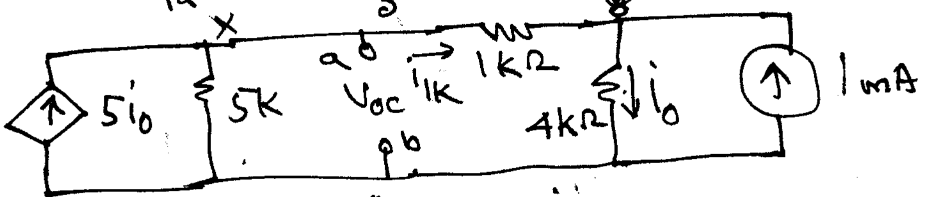
$i_0 = i_{1k\Omega} = \dfrac{v_t}{5k}$

KCL at a gives:

$$i_t = \frac{v_t}{5k} - 5i_0 + \frac{v_t}{5k} = \frac{2v_t}{5k} - 5i_0 = \frac{2v_t - v_t}{5k}$$

$$= -\frac{3v_t}{5k} \quad \text{or} \quad R_{Th} = -\frac{5k}{3} = -1.67\,k\Omega$$

To find v_{oc}



At node y, $i_{1k} = i_0 - 1m$ or $i_{1k} + 1m = i_0$

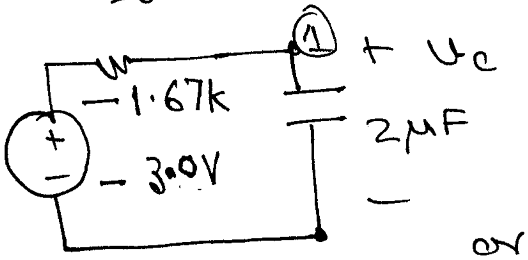
$v_{oc} = 1k \times i_{1k} + i_0 \times 4k = 5k\, i_{1k} + 4$

At node x, $i_{1k} - 5i_0 + \dfrac{v_{oc}}{5k} = 0$ or $\dfrac{i_{1k}}{5} + \dfrac{v_{oc}}{25k} = i_0 = i_{1k} + 1m$

$$\therefore \frac{4}{5} i_{1k} = \frac{v_{oc}}{25k} - 1m \quad \text{or} \quad i_{1k} = \frac{1}{20k} v_{oc} - \frac{5}{4}m$$

$$\therefore v_{oc} = 5k \times \left(\frac{v_{oc}}{20k}\right) - 5k \times \frac{5}{4}m + 4 = \frac{5}{20} v_{oc} - \frac{25}{4} + 4$$

$$\text{or} \quad \frac{15}{20} v_{oc} = -\frac{9}{4} \quad \text{or} \quad v_{oc} = -\frac{\cancel{9}^{3} \times \cancel{20}^{4}}{4 \times \cancel{15}_{3}} = -3.0 \simeq -3\ V$$



KCL gives at 1

$$c\frac{dv_c}{dt} + \frac{v_c - (-3.0)}{-1.67k} = 0.$$

$$\text{or} \quad 2 \times 10^{-6} \frac{dv_c}{dt} - \frac{v_c + 3.0}{1.67K} = 0.$$

$$\text{or} \quad \frac{dv_c}{dt} = 299.4\,(v_c + 3.0)$$

Let $x = v_c + 3.0$

$$\therefore \frac{dx}{dt} = \frac{dv_c}{dt}$$

$$\text{or} \quad \frac{dx}{dt} = 299.4\,x.$$

$$\therefore \int \frac{dx}{x} = \int 299.4\,dt \quad \text{or} \quad \ln x = 299.4t$$

$$\therefore x = e^{299.4t}$$

$$\therefore v_c = -3.0 + e^{299.4t} = 1500\ V$$ for capacitor breakdown.

$$\text{or} \quad 299.4\,t_B = \ln 1503.0$$

$$\text{and} \quad t_B = \frac{\ln(1503.0)}{299.4} = 0.0244\ s \cong 24.4\ ms.$$

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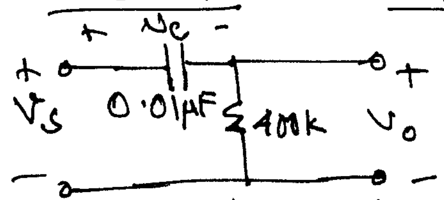


$v_c(0^-) = 0, \therefore v_c(0^+) = 0.$

$v_o = v_s - v_c$

$\underline{0 \leq t \leq 1ms}$ $\quad v_c(0^+) = 0,\ v_c(\infty) = 50V,\ \tau = 0.01 \times 10^{-6} \times 400 \times 10^3 = 4ms.$

$\therefore v_c(0 \leq t \leq 1ms) = 50 - 50e^{-t/4ms} = 50(1 - e^{-250t}),\ \therefore v_o = 50e^{-250t}$ ------ (1)

$\underline{1ms \leq t \leq \infty}$ $\quad v_s = 0V.$

$v_c(1ms) = 50(1 - e^{-250 \times 10^{-3}}) = 11.06V$ and $v_c(\infty) = 0.$

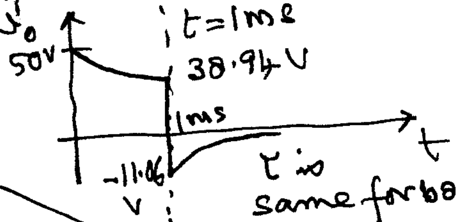
$v_c(1ms \leq t \leq \infty) = 0 + (11.06 - 0)e^{-250(t-0.001)}$

$= 11.06e^{-250(t-0.001)} = 14.2e^{-250t}$ --- (2)

$v_o(t = 1ms)\Big|_{\text{from (1)}} = 50 - 11.06 = 38.94V.$

$v_o(t = 1ms)\Big|_{\text{from (2)}} = 0 - 14.2e^{-0.25} = -11.06. = \underline{38.94 - 50}$

This calculation need not be done as when v_s goes from $50 \to 0$ the voltage across the capacitor (38.94V) cannot change. $\therefore$ the change must occur across the $400k\Omega$ resistance. which is $38.94 - 50 = -11.06V.$



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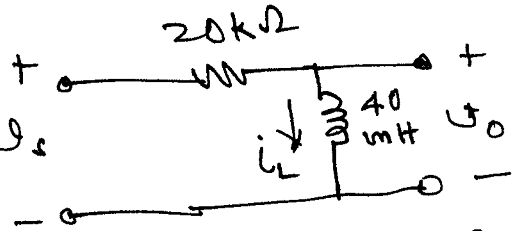


$\underline{0 \leq t \leq 2\mu s}$

$v_s(0) = 100V.$

$\therefore i_L(\infty) = \dfrac{100}{20k} = 5mA$

$\tau = \dfrac{L}{R} = \dfrac{40mH}{20k} = 2\mu s.$ Since $v_s = 0$ at $t = 0^-$, $i_L(0^-) = i_L(0^+) = 0$

$\therefore i_L = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-5\times10^5 t} = (5m) - (5m)e^{-5\times10^5 t}$

$= 5\times10^{-3}[1 - e^{-5\times10^5 t}]$

$\therefore v_o = v_s - 20k \times i_L = 100 - 20\times10^3 \times 5\times10^{-3}[1 - e^{-5\times10^5 t}]$ — (1)

$= 100e^{-5\times10^5 t}$ (can also be found from $L\dfrac{di_L}{dt}$)

$v_o(t = 2\mu s) = 36.79V$

$\underline{2\mu s \leq t \leq \infty}$: $\quad i_L(2\mu s) = 5m[1 - e^{-1}] = 3.16mA$

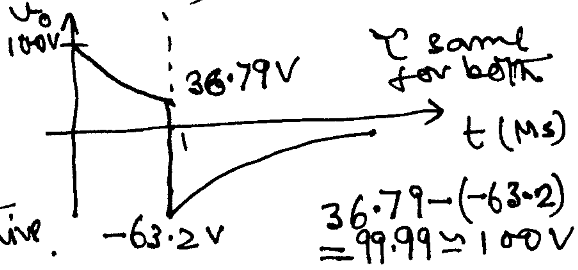
again $i_L(\infty) = 0$ & $i_L(t \geq 2\mu s) = 0 + (3.16m - 0)e^{-5\times10^5(t - 2\times10^{-6})}$

$= 3.16e^1 e^{-5\times10^5 t}$ mA $= 8.59e^{-5\times10^5 t}$ mA

At $t = 2\mu s$ $\quad v_o = L\dfrac{di_L}{dt} = 40m \times 3.16m \times (-5\times10^5)e^{-5\times10^5(t - 2\mu s)}$

$\therefore v_o(t = 2\mu s) = -63.2V$

Again the argument here is that since the current cannot change instantaneously in the inductor at 2μs is dropped across R and when $v_s \to 0$ at 2μs the voltage across the inductor is negative.



$36.79 - (-63.2)$
$= 99.99 \simeq 100V$