2.a. Let $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of random variables with $\mathbb{E}(X_i) = 0$ for all $i \in \mathbb{N}$. Suppose that $Var(X_i) \leq 2$ for all $i \in \mathbb{N}$ and $\left|Cov(X_i, X_j)\right| = 0.48^{|i-j|}$ for $i \neq j$.

(i) Find an upper bound of $\sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{Cov}(X_i, X_j)$, which is linear in n.

(ii) Examine whether $\frac{S_n}{n} := \frac{1}{n} \sum_{i=1}^{n} X_i$ converges in probability to a limit, or not. Give clear arguments.

[2+2]

2.b. A randomly selected product from a factory can belong to any one of the three distinct categories I, II and III with probabilities $\theta^2, 2\theta(1-\theta)$ and $(1-\theta)^2$, respectively, where $0 < \theta < 1$. Suppose in a random sample of 100 products, we obtain 60, 10 and 30 products belonging to categories I, II and III, respectively.
Find the maximum likelihood estimator of θ. [2]