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EE 210 **MID-SEM P1** **24.2.21**

The fab line data for an n-channel MOSFET: $N_A = 10^{16}$ cm^{-3}, $t_{ox} = 30$ nm, $V_{TN0} = 0.7$ V, $\mu_n = 430$ cm^2/V-sec, and (λ, C_{gs0}, C_{gd0}) $\rightarrow$ 0. Other relevant data: $V_T = 26$ mV, $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm, ε_r(Si) = 11.7, $\varepsilon_r(SiO_2) = 3.9$, $q = 1.6 \times 10^{-19}$ C, $n_i = 1.5 \times 10^{10}$ cm^{-3}.

a) Design the values (in µm) of W and L, such that with the device biased in saturation with the lowest allowed values of the gate overdrive voltage and the corresponding V_{DS}, it should have unity-gain cutoff frequency (f_T) of 5 GHz and device power dissipation of 100 nW. **8**

b) If the designed device is biased with $V_G = 3$ V, $V_D = 1.5$ V, $V_S = 1$ V, and $V_B = 0$ V, determine the drain current I_D. **4**

c) If the values of V_G and V_S are maintained as in part b), but now V_D is changed to 3 V, determine the required value of V_B that will make the device operate with a body factor (χ) of 0.1. **3**

a) $\Delta V = V_{GT} \Rightarrow \Delta V_{min} = 3V_T = 78\,mV = V_{DS,sat} = V_{DS,min}$ (80 mV also acceptable) answers will change a little

$f_T = \dfrac{g_m}{2\pi C_{gs}}$ $\because$ In sat., $C_{gd} = C_{gdo} \to 0$, as per given data.

In sat., $g_m = K_N'\left(\frac{W}{L}\right)V_{GT}$ & $C_{gs} = \frac{2}{3}WLC_{ox}' \Rightarrow f_T = \dfrac{3K_N' V_{GT}}{4\pi L^2 C_{ox}'}$

$C_{ox}' = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{3.9\epsilon_0}{30\,nm} = 1.15 \times 10^{-7}\,F/cm^2$ $K_N' = \mu_n C_{ox}' = 49.5\,\mu A/V^2$

$\Rightarrow L^2 = \dfrac{3K_N' V_{GT}}{4\pi C_{ox}' f_T} = 1.6 \times 10^{-9} \Rightarrow L = 4 \times 10^{-5}\,cm = \boxed{0.4\,\mu m}$

$P_D = 100\,nW = V_{DS,sat} \times I_D \Rightarrow I_D = 1.28\,\mu A = \dfrac{K_N'}{2}\left(\frac{W}{L}\right)V_{GT}^2$ ($\because \lambda \to 0$)

$\Rightarrow W = \dfrac{2I_D L}{K_N' V_{GT}^2} = 3.4 \times 10^{-4}\,cm = \boxed{3.4\,\mu m} \Rightarrow \boxed{(W/L) = 8.5}$

b) $V_{GS} = 2V$ $V_{DS} = 0.5\,V$ $V_{SB} = 1V$ $\phi_F = V_T \ln\frac{N_A}{n_i} = 0.35V \Rightarrow 2\phi_F = 0.7\,V$

$\gamma = \dfrac{\sqrt{2q\epsilon_S N_A}}{C_{ox}'} = 0.5\,V^{1/2} \Rightarrow V_{TN} = V_{TN0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = 0.9336\,V$

$\Rightarrow V_{GT} = 1.0664V > V_{DS} \Rightarrow$ Linear mode of operation.

$\Rightarrow I_D = K_N'\left(\frac{W}{L}\right)\left(V_{GT}V_{DS} - \frac{V_{DS}^2}{2}\right) = \boxed{171.75\,\mu A}$

c) $V_G = V_D = 3V$ & $V_S = 1V \Rightarrow V_{GS} = V_{DS} = 2V \Rightarrow$ Sat.

$\therefore$ Small-Signal Model parameters can be evaluated,

$\chi = \dfrac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = 0.1 \Rightarrow V_{SB} = 5.55V \Rightarrow \boxed{V_B = -4.55V}$

* χ independent of I_D. With change in V_B, I_D will change, but no effect of that on χ.

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All BJTs in the circuit shown are identical with (β, V_A) $\rightarrow \infty$ [*for parts a)-d)*].

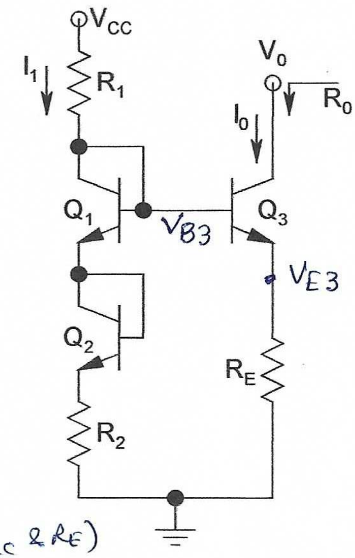
a) Show that I_0 is a function only of V_{CC} and R_E, if $R_1 = R_2$. **4**

b) If $I_0 = I_1$, how is R_E related to R_1 (or R_2, since $R_1 = R_2$)? **2**

c) If $V_{CC} = 5$ V, determine R_1 (= R_2) and R_E to give $I_0 = 1$ mA. **2**

d) What is $V_{0,min}$? Is the value acceptable to you? Comment. **2**

e) *Only for this part*, assuming $\beta = 100$, $V_A = 100$ V, I_0 and I_1 remain at 1 mA, and using the values of R_1 (= R_2) and R_E calculated in part c), estimate the output resistance R_0. **5**



a) $\beta \to \infty \Rightarrow$ Base Current Neglected.

$\Rightarrow V_{B3} = 2V_{BE} + I_1 R_2$ & $I_1 = \dfrac{V_{CC} - 2V_{BE}}{R_1 + R_2}$

$\Rightarrow V_{B3} = \dfrac{V_{CC}}{2} + V_{BE}$ ($\because R_1 = R_2$)

$\Rightarrow V_{E3} = V_{B3} - V_{BE} = \dfrac{V_{CC}}{2} \Rightarrow \boxed{I_0 = \dfrac{V_{E3}}{R_E} = \dfrac{V_{CC}}{2R_E}}$ (fn. only of V_{CC} & R_E)

b) $I_0 = I_1 \Rightarrow \dfrac{V_{CC}}{2R_E} = \dfrac{V_{CC} - 2V_{BE}}{2R_1} \Rightarrow \boxed{R_1 = R_2 = R_E\left[1 - \dfrac{2V_{BE}}{V_{CC}}\right]}$

c) $R_E = \dfrac{V_{CC}}{2I_0} = \dfrac{5\text{ V}}{2 \times 1\text{ mA}} = \boxed{2.5\text{ k}\Omega}$ & $R_1 = R_2 = 2.5\text{K} \times \left[1 - \dfrac{2 \times 0.7}{5}\right] = \boxed{1.8\text{ k}\Omega}$

d) $V_{o,min} = V_{CE3}(SS) + I_0 R_E = 0.2 + 1\text{ mA} \times 2.5\text{K} = \boxed{2.7\text{ V}}$

Horrendous value, $\because$ it's more than 50% of V_{CC}!

e) To a first-order, base of Q_3 is at ac gnd.

$\therefore$ By inspection: $R_o = r_{o3}[1 + g_{m3} R_{eff}]$ with $R_{eff} = r_{\pi 3} \| R_E$

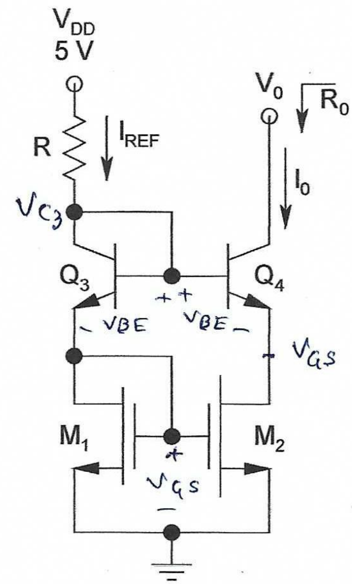
$r_{E_3} = \dfrac{V_T}{I_0} = 26\,\Omega$ $\quad g_{m_3} = \dfrac{1}{r_{E3}} = \dfrac{1}{26}$ ℧ $\quad r_{\pi 3} = \beta r_{E_3} = 2.6\text{K}$ $\quad R_{eff} = 2.6\text{K} \| 2.5\text{K} = 1.275\text{K}$

$r_{o3} = \dfrac{V_A}{I_0} = 100\text{K}$

$\Rightarrow R_o = 100\text{K} \times \left(1 + \dfrac{1}{26} \times 1.275\text{K}\right) = \boxed{5\text{ M}\Omega}$

Excellent Value!

In the BiMOS (combination of BJT and MOS) cascode current source shown in the figure, M_1-M_2 is a *perfectly matched pair*, and so is Q_3-Q_4. Neglect DC base current, and assume that $\lambda V_{DS} \ll 1$.
Data: for M_1-M_2: $V_{TN0} = 0.7$ V, $k'_N = 40\ \mu A/V^2$, $\gamma = 0.4\ V^{1/2}$, $2\phi_F = 0.6$ V;
for Q_3-Q_4: $\beta = 100$, $V_A = 100$ V.



a) Show that $R_O \approx \beta r_{O4}$. Clearly highlight all the assumptions made in arriving at this result. **5**

b) Choose the values of I_{REF}, R, and (W/L) of M_1-M_2, in order to have R_O and $V_{O,min}$ of 1 GΩ and 1 V respectively. **7**

c) What is the most critical parameter and what should be its value for the assumption made in the derivation of R_O [part a)] to hold? An error band of 5% is acceptable. **3**

a) Base of Q_4 and Gate of M_2 at ac gnd.

$\Rightarrow$ By inspection: $R_O = r_{o4}(1 + g_{m4} R_{eff})$

$R_{eff} = r_{\pi 4} \| r_{o2}$. Now, if $r_{o2} \gg r_{\pi 4} \Rightarrow R_{eff} \simeq r_{\pi 4}$

$\Rightarrow R_O \simeq r_{o4}(1 + g_{m4} r_{\pi 4}) = r_{o4}(1+\beta)$ $(\because \beta = g_{m4} r_{\pi 4})$

$\Rightarrow R_O \simeq \beta r_{o4}$ (If $\beta \gg 1$)

b) $R_O = \beta r_{o4} = 1\ G\Omega \Rightarrow r_{o4} = 10\ M\Omega = \dfrac{V_A}{I_O} \Rightarrow \boxed{I_O = I_{REF} = 10\,\mu A}$ (Current Mirror)

$V_{O,min} = V_{CE4}(ss) + V_{DS2} = V_{CE4}(ss) + V_{D2} = V_{CE4}(ss) + V_{GS} = \underline{1V}$

$\Rightarrow V_{GS} = 1 - V_{CE4}(ss) = 1 - 0.2 = \underline{0.8V}$

$V_{TN} = V_{TN0}$ ($\because$ M_1 & M_2 has bodies connected to gnd.) $= \underline{0.7V} \Rightarrow V_{GT} = \underline{0.1V}$

$\Rightarrow I_{REF} = \dfrac{k'_N}{2}\dfrac{W}{L}V_{GT}^2$ $(\because \lambda V_{DS} \ll 1) \Rightarrow \boxed{\left(\frac{W}{L}\right) = 50}$

$V_{C3} = V_{BE} + V_{GS} = \underline{1.5V} \Rightarrow R = \dfrac{V_{DD} - V_{C3}}{I_{REF}} = \boxed{350\ k\Omega}$

c) Most imp. assumption: $r_{o2} \gg r_{\pi 4}$ $\Rightarrow$ Most imp. parameter $\Rightarrow \boxed{\lambda_2}$

For 5% error, $r_{o2} \approx 20 r_{\pi 4} = 20\beta r_{E4} = 20 \times 100 \times \dfrac{26\ mV}{10\,\mu A} = \boxed{5.2\ M\Omega}$

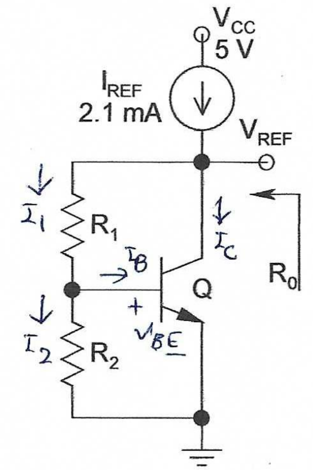
$r_{o2} \gtrsim \dfrac{1}{\lambda_2 I_O} \Rightarrow \boxed{\lambda_2 \lesssim \dfrac{1}{r_{o2} I_O} \lesssim 0.02\ V^{-1}}$ $\boxed{\lambda_2 V_{DS} = 0.016 \ll 1}$

A DC reference voltage (V_{REF}) generator circuit is shown in the figure. For Q, assume $\beta = 100$, and neglect Early effect. The DC current source I_{REF} is ideal.

a) Choosing the base current of Q to be 20% of the current flowing through R_1, design the values of R_1 and R_2 to produce $V_{REF} = 2$ V. *Caution: Do not use equations blindly.* **5**

b) *Quantitatively* prove that the DC power of the circuit is a conserved quantity, i.e., the DC power supplied by V_{CC} is completely dissipated in the circuit. **4**

c) What is the ac small-signal resistance (R_0) of the designed voltage reference? Is it acceptable? Why or why not? **6**



a) $I_{REF} = I_C + I_1 = \underline{2.1\,mA}$ $\quad I_C = \beta I_B = \underline{100\, I_B}$ $\quad I_B = \underline{0.2 I_1}$

$\Rightarrow I_C = \underline{20 I_1} \Rightarrow \boxed{I_1 = 100\,\mu A}$ $\boxed{I_C = 2\,mA}$ $\boxed{I_B = 20\,\mu A}$

$I_2 = I_1 - I_B = \underline{80\,\mu A}$ $\quad V_{BE} = I_2 R_2 \Rightarrow R_2 = \dfrac{V_{BE}}{I_2} = \dfrac{0.7}{80\,\mu A} = \boxed{8.75\,K\Omega}$

$R_1 = \dfrac{V_{REF} - V_{BE}}{I_1} = \dfrac{2 - 0.7}{100\,\mu A} = \boxed{13\,K\Omega}$

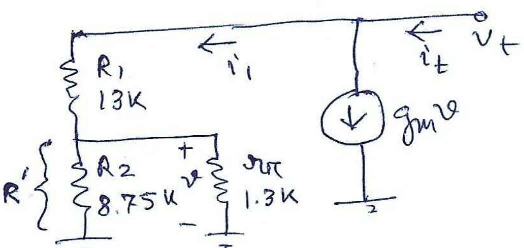
b) Power supplied by $V_{CC} = V_{CC} \times I_{REF} = 5 \times 2.1 = \boxed{10.5\,mW}$

Power dissipated $= P_{I_{REF}} + P_Q + P_{R_1} + P_{R_2} = (V_{CC} - V_{REF}) \times I_{REF} + V_{CE} \times I_C + V_{BE} \times I_B + I_1^2 R_1 + I_2^2 R_2$

$= (5-2) \times 2.1\,mA + 2 \times 2\,mA + 0.7 \times 20\,\mu A + (100\,\mu A)^2 \times 13K + (80\,\mu A)^2 \times 8.75K$

$= 6.3\,mW + 4\,mW + 14\,\mu W + 130\,\mu W + 56\,\mu W = \boxed{10.5\,mW} \Rightarrow$ Conserved!

c) $g_m = \underline{\dfrac{1}{13}\,\mho}$ $\quad r_E = \underline{13\,\Omega}$ $\quad r_\pi = \underline{1.3\,K}$ $\quad I_{REF}$ opens up.



$R' = 8.75K \,\|\, 1.3K = \underline{1.13K}$

$i_1 = \dfrac{v_t}{R_1 + R'} = \underline{7.08 \times 10^{-5}\, v_t}$

$v = \dfrac{R'}{R_1 + R'} v_t = 0.08\, v_t$

$g_m v = \dfrac{1}{13} \times 0.08\, v_t = \underline{6.15 \times 10^{-3}\, v_t}$

$i_t = i_1 + g_m v = (7.08 \times 10^{-5} + 6.15 \times 10^{-3})\, v_t = 6.22 \times 10^{-3}\, v_t$

$\Rightarrow R_0 = \dfrac{v_t}{i_t} = \boxed{160.75\,\Omega}$

Excellent value, $\because$ it's a voltage reference, it's R_0 should be small, & a number less than 200Ω is not bad at all!

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