EE 200: Solution for Problem Set 5

1. Determine the total energy and average power of the following sequences:
   (a) $x_1[n] = \mu[n]$
   (b) $x_2[n] = n\mu[n]$
   (c) $x_3[n] = 4\sin(0.5\pi n)$
   Solution 1: (a) $E_{x_1} = \sum_{n=-\infty}^{\infty} |x_1[n]|^2 = \sum_{n=0}^{\infty} 1 = \infty$

$$P_{x_1} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_1[n]|^2$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N\to\infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

(b) $E_{x_2} = \sum_{n=-\infty}^{\infty} |x_2[n]|^2 = \sum_{n=1}^{\infty} n^2 = \infty$

$$P_{x_2} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_2[n]|^2$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=1}^{N} n^2$$
$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{N(N+1)(2N+1)}{6}\right)$$

(c) $x_3[n] = 4\sin(0.5\pi n) = 4\sin(\omega_0 n)$ is a periodic sequence with the fundamental period $N_0 = \frac{2\pi}{\omega_0} = 4$

$$E_{x_3} = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} |4\sin(0.5\pi n)|^2 = \infty$$

$$P_{x_3} = \frac{1}{N_0} \sum_{n=0}^{N_0-1} |x_3[n]|^2 = \frac{1}{4} \sum_{n=0}^{4} |\sin(0.5\pi n)|^2 = 0.5$$

2. Determine at least two other sinusoidal sequences having the fundamental period as that of the sinusoidal sequence $\tilde{s}[n] = \cos(0.35\pi n)$
Solution 2: Let the fundamental period of $\hat{s}[n]$ be N_0. Now, $\frac{2\pi}{0.35\pi} = \frac{200}{35} = \frac{40}{7} = \frac{N_0}{r}$ which yields $N_0 = 40$.
For other angular frequencies with the same fundamental period $N_0 = 40$, we write $\frac{2\pi}{\omega_0} = \frac{40}{r}$ or $\omega_0 = \frac{2\pi r}{40}$.
For $r = 3$, we get $\omega_0 = \frac{2\pi\times 3}{40} = 0.15\pi$ and for $r = 9$, we get $\omega_0 = \frac{2\pi\times 9}{40} = 0.45\pi$.

3. A two-sided sequence is given as $\{r[n]\} = \{-3, 4, 0, -2, 5, 4\}$, $-1 \leq n \leq 4$. The sample values of the above sequence outside the specified range of time index n are zeros.
(a) Express the above sequence as a linear weighted combination of delayed and advanced unit sample sequences.
(b) Express the above sequence as a linear weighted combination of delayed and advanced unit step sequences.
(c) Determine the even and odd parts of the above sequence.
Solution 3:
(a) $r[n] = -3\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 5\delta[n-3] + 4\delta[n-4]$

(b) $r[n] = -3(\mu[n+1] - \mu[n]) + 4(\mu[n] - \mu[n-1]) - 2(\mu[n-2] - \mu[n-3]) + 5(\mu[n-3] - \mu[n-4]) + 4(\mu[n-4] - \mu[n-5])$
$= -3\mu[n+1] + 7\mu[n] - 4\mu[n-1] - 2\mu[n-2] + 7\mu[n-3] - \mu[n-4] - 4\mu[n-5]$

(c)$\{r[n] = \{0, 0, 0, -3, 4, 0, -2, 5, 4\}; -4 \leq n \leq 4$
$\{r[-n]\} = \{4, 5, -2, 0, 4, -3, 0, 0, 0\}$

$\{r_{ev}[n]\} = \frac{1}{2}(r[n]+r[-n]) = \{2, 2.5, -1, -1.5, 4, -1.5, -1, 2.5, 2\}$

$\{r_{od}[n]\} = \frac{1}{2}(r[n]-r[-n]) = \{-2, -2.5, 1, -1.5, 0, +1.5, -1, 2.5, 2\}$;
$-4 \leq n \leq 4$

4. Determine the output sequence y[n] of Fig.1 for an input sequence $x[n] = 2n\mu[n]$
Solution 4:
Denote the output of the up-sampler of $x[n] = 2n\mu[n]$ as $x_\mu[n]$. Then,

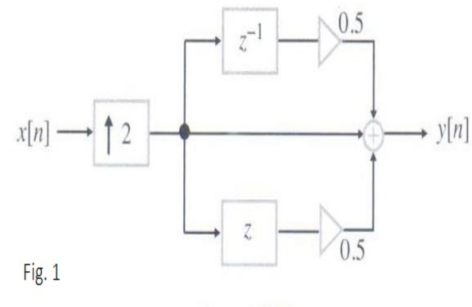

Fig. 1

$$x_\mu[n] = \begin{cases} x[n/2], n = 0, 2, 4, 6, \cdots \\ 0, \text{otherwise} \end{cases}$$
$$y[n] = x_\mu[n] + 0.5(x_\mu[n+1] + x_\mu[n-1])$$
$$\{x[n] = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, \cdots\}$$
$$\{x_\mu[n] = \{0, 0, 2, 0, 4, 0, 6, 0, 8, 0, 10, \cdots\}$$
$$\{y[n] = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots\} = n\mu[n] = \frac{1}{2}x[n]$$

5. Determine the digital signal p[n] generated by sampling uniformly the analog signal:

$$\begin{aligned} p_a(t) = & 3\cos(20\pi t/sec) - 7\sin(25\pi t/sec) + 5\cos(80\pi t/sec) + 4\cos(125\pi t/sec) \\ & -6\cos(220\pi t/sec) \end{aligned}$$

at a sampling rate of 50Hz.
Solution 5:
$p_a(t) = 3\cos(20\pi t/sec) - 7\sin(25\pi t/sec) + 5\cos(80\pi t/sec) + 4\cos(125\pi t/sec) - 6\cos(220\pi t/sec)$
$T = \frac{1}{50} = 0.02sec$. Hence,

$$\begin{aligned} p[n] &= 3\cos(20 \times 0.02\pi n) - 7\sin(25 \times 0.02\pi n) + 5\cos(80 \times 0.02\pi n) \\ &\quad + 4\cos(125 \times 0.02\pi n) - 6\cos(220 \times 0.02\pi n) \\ &= 3\cos(0.4\pi n) - 7\sin(0.5\pi n) + 5\cos(1.6\pi n) + 4\cos(2.5\pi n) - 6\cos(4.4\pi n) \end{aligned}$$

Using trigonometric identity, we note:

$$\cos(1.6\pi n) = \cos(2\pi n - 0.4\pi n) = \cos(0.4\pi n)$$
$$\cos(2.5\pi n) = \cos(2\pi n + 0.5\pi n) = \cos(0.5\pi n)$$
$$\cos(4.4\pi n) = \cos(4\pi n + 0.4\pi n) = \cos(0.4\pi n)$$

Thus,

$$\begin{aligned} p[n] &= 3\cos(0.4\pi n) - 7\sin(0.5\pi n) + 5\cos(0.4\pi n) + 4\cos(0.5\pi n) - 6\cos(0.4\pi n) \\ &= 2\cos(0.4\pi n) + 4\cos(0.5\pi n) - 7\sin(0.5\pi n) \end{aligned}$$

6. Write a MATLAB code to generate the sinusoidal sequence

$$\tilde{x}[n] = \sin(2\pi F_0 n)$$

for a range of time index $0 \leq n \leq 40$. Using this code generate the sinusoidal sequences for the following cyclic frequencies: (a) $F_0 = 0.05Hz$, (b) $F_0 = 0.1Hz$, (c) $F_0 = 0.4Hz$, (d) $F_0 = 0.45Hz$.
Determine analytically the fundamental periods of each of the above sinusoidal sequences and verify your results from the plots of the sequences generated using your code.

Solution 6: CODE:

$$n = 0 : 40;$$
$$\text{omega} = 2 * pi * 0.05;$$
$$x = \sin(\text{omega} * n);$$
$$\text{stem} = (n, x);$$
$$\text{xlabel}(\text{'}n\text{'});$$
$$\text{ylabel}(\text{'Amplitude'});$$
$$\text{title}(\text{'}F_0 = 0.05\text{'});$$

The fundamental periods of the 4 sinusoidal sequences are: (a)$\frac{1}{F_0} = \frac{1}{0.05} = \frac{100}{5} = \frac{20}{1} = \frac{N_0}{r}$. Hence, $N_0 = 20$.

(b)$\frac{1}{F_0} = \frac{1}{0.1} = \frac{10}{1} = \frac{N_0}{r}$. Hence, $N_0 = 10$.

(c)$\frac{1}{F_0} = \frac{1}{0.4} = \frac{10}{4} = \frac{5}{2} = \frac{N_0}{r}$. Hence, $N_0 = 10$.

(d)$\frac{1}{F_0} = \frac{1}{0.45} = \frac{100}{45} = \frac{20}{9} = \frac{N_0}{r}$. Hence, $N_0 = 20$.