ESc201A Home Assignment 4 Aug. 26, 2019.

Solutions of the HA#3 will be on Brihaspati on 31/08/19.

Consider all voltage and current sources to be ideal.

1. Find the [Z] parameters for the circuit of fig. 4.1..

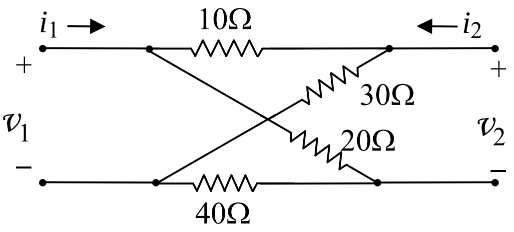

Figure 4.1

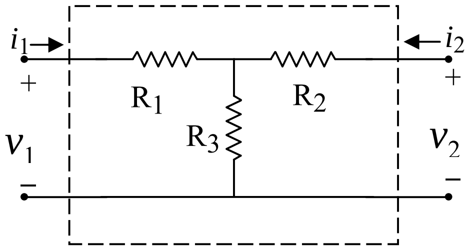

Figure 4.2

2. For the circuit shown in fig. 4.2 the [ABCD] matrix is given as : $\begin{bmatrix} 1.5 & 2.75\text{k}\Omega \\ 10^{-3}℧ & 2.5 \end{bmatrix}$. Find the values of R_1, R_2, and R_3

3. Find the [h] parameters of the 2-port network shown in fig. 4.3. Find R_2/R_1 for β=100.

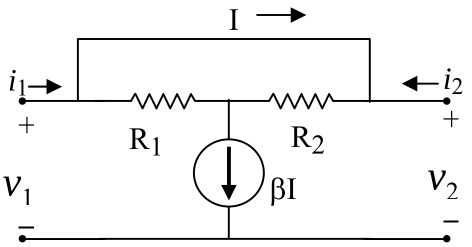

Figure 4.3

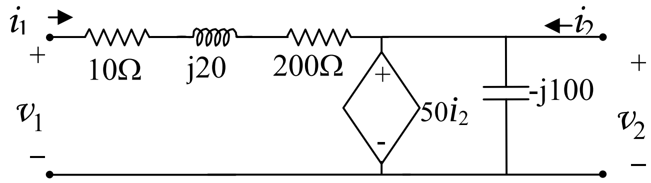

Figure 4.4

4. Find the [h] parameters of the 2-port network shown in fig. 4.4.

5. In fig. 4.5 are shown two identical amplifiers connected in cascade. Each amplifier is represented by their h-parameters, h_{11} = 1kΩ, h_{12} = 1.5x10^{-3}, h_{21} = 100, and h_{22} = 100μMhos. Given that **[a]** and **[ABCD]** parameters are exactly the same, find the voltage gain v_2/v_s.

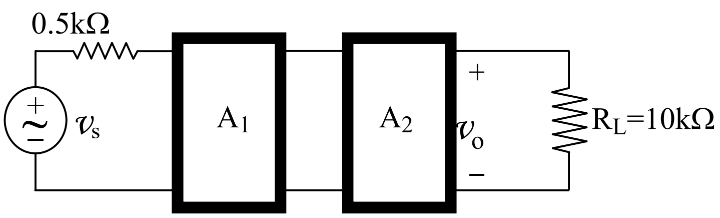

Figure 4.5

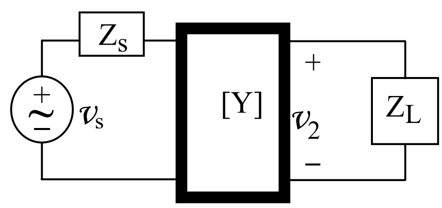

Figure 4.6

6. The [Y] parameters for a 2-port network shown in fig. 4.6 are: $\begin{bmatrix} 2\text{m}℧ & -2\mu℧ \\ 100\text{m}℧ & -50\mu℧ \end{bmatrix}$. The source impedance is Z_S = 2.5 kΩ+j0 Ω and the load impedance is Z_L = 70 kΩ+j0 Ω. The ideal voltage source is v_S=(80√2)cos(4000t) mV. Find the rms value of v_2 and the average power delivered to Z_L.

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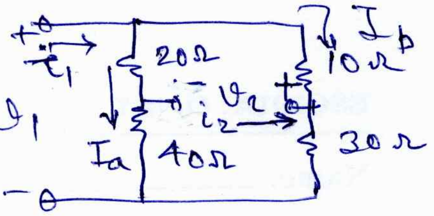
1/ The given circuit is redrawn as:



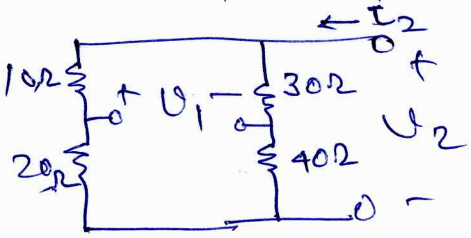
$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{(20+40)(10+30)}{20+40+10+30} = 24\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} \qquad v_2^+ = \frac{i_1 \times 24}{(10+30)} \times 30$$

$$v_2^- = \frac{i_1 \times 24}{(20+40)} \times 40 \qquad \therefore v_2 = v_2^+ - v_2^- = 18 i_1 - 16 i_1 = 2 i_1$$

$$= \frac{2 i_1}{i_1} = 2\,\Omega$$

For $i_1 = 0$ Need to redraw the circuit.



$$v_1^+ = \frac{i_2 \times \frac{30 \times 70}{100}}{10+20} \times 20 = i_2 14\,V$$

$$v_1^- = \frac{i_2 \times \frac{30\times 70}{100}}{30+40} \times 40 = i_2 12\,V$$

$$z_{12} = \frac{v_1}{i_2}\bigg|_{i_1=0} = \frac{2 i_2}{i_2} = 2\,\Omega \qquad v_1 = v_1^+ - v_1^- = (14-12) i_2 = 2 i_2$$

$$Z_{22} = \frac{v_2}{i_2}\bigg|_{i_1=0} = \frac{i_2 \times \frac{30\times 70}{100}}{i_2} = 21\,\Omega$$

2/ ABCD $\Rightarrow$ $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

$$A = \frac{v_1}{v_2}\bigg|_{i_2=0}, \quad v_2 = \frac{v_1 R_3}{R_1+R_3}\bigg|_{i_2=0}.$$

$$= \frac{R_1 + R_3}{R_3}$$

$$= 1 + \frac{R_1}{R_3} = 1.5 \quad \therefore \frac{R_1}{R_3} = 0.5 \quad \therefore R_3 = 2R_1$$

$$C = \frac{i_1}{v_2}\bigg|_{i_2=0} \quad \bigg| \quad i_1 = \frac{v_1}{R_1+R_3}\bigg|_{i_2=0}$$ and from above $v_2 = \frac{v_1 R_3}{R_1+R_3}$ or $v_1 = \frac{R_1+R_3}{R_3} v_2$

$$= \frac{(R_1+R_3) v_2}{R_3(R_1+R_3)}$$

$$= \frac{1}{R_3} = 10^{-3}\,\mho$$

or $R_3 = 1\,k\Omega$

$\therefore R_1 = 0.5\,k\Omega$

For B & D short the v_2 voltage terminal i.e. $R_2 \| R_3$, current flowing in from port ① is $\frac{v_1}{R_1 + R_2\|R_3} = i_1$

$$B = -\frac{v_1}{i_2}\bigg|_{v_2=0}, \quad \therefore -i_2 \text{ (the current in } R_3) = \frac{i_1 \cancel{R_3} R_2}{(R_2+R_3)} \frac{1}{\cancel{R_2}} = \frac{i_1}{1+\frac{R_3}{R_2}}$$

$$\therefore \frac{v_1}{R_1 + R_2\|R_3} = -i_2\left(1 + \frac{R_3}{R_2}\right) \quad \therefore B = \frac{v_1}{-i_2} = \left(R_1 + \frac{R_2 R_3}{R_2+R_3}\right)\left(1 + \frac{R_3}{R_2}\right)$$ troubled

$$D = -\frac{i_1}{i_2}\bigg|_{v_2=0} = 1 + \frac{R_3}{R_2} = 2.5 \quad \therefore \frac{R_3}{R_2} = 1.5 \text{ or } R_2 = 1.5 R_3 = 1.5\,k\Omega$$

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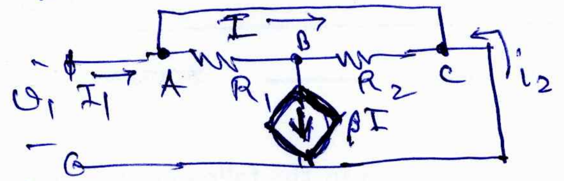
3/ $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$ $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ & $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$.



A & C are shorted & for $V_2 = 0$.

$v_1 = C$

$\therefore V_1 = 0$. $\therefore \underline{h_{11} = 0}$.

$i_1 = I + \frac{V_{AB}}{R_1}$ & $i_2 = -I + \frac{V_{CB}}{R_2}$

$= -I - \frac{(-V_{AB})}{R_2}$

$= -I + \frac{V_{AB}}{R_2}$

But $v_1 = 0$ & $v_2 = 0$.

$\therefore V_{BA} = V_{BC} = -V_{CB}$

Again $i_2 + I = I_{R_{2C\to B}}$

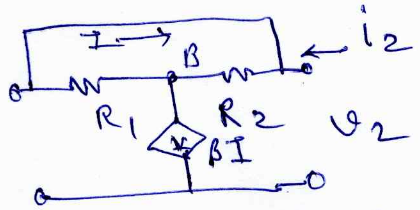
and $i_1 - I = I_{R_{1A\to B}}$.

and $I_{R_{1A\to B}} + I_{R_{2C\to B}} = \beta I = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2}$

$\therefore \left.\frac{i_2}{i_1}\right|_{V_2=0} = \frac{-I + \frac{V_{AB}}{R_2}}{I + \frac{V_{AB}}{R_1}} = \frac{-\frac{V_{AB}}{\beta R_1} - \frac{V_{AB}}{\beta R_2} + \frac{V_{AB}}{R_2}}{\frac{V_{AB}}{\beta R_1} + \frac{V_{AB}}{\beta R_2} + \frac{V_{AB}}{R_1}}$

$h_{21} = \frac{\frac{\beta}{R_2} - \frac{1}{R_1} - \frac{1}{R_2}}{\frac{\beta}{R_1} + \frac{1}{R_1} + \frac{1}{R_2}}$

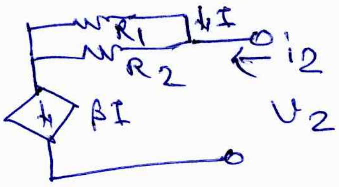
for $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$



At node (B) $\beta I = -I + i_2 + I = i_2$

for h_{12} Do we really need to calculate? It is shorted by the line carrying current I $\therefore v_1 = v_2$ & $\underline{h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0} = 1}$

Redraw :



If $i_2 = \beta I$

$\therefore I$ (the current in the short has to be equal to zero)

$\therefore \beta I = 0 = i_2$

$\therefore i_2 = 0$.

$\therefore h_{22} = \left.\frac{I_2}{v_2}\right|_{i_1=0} = 0$

Since this is a reciprocal circuit i.e. either end could be called (1) & (2) ports. $\Delta h=1$

$\therefore |h_{12} h_{21}| = 1$ or $h_{21} = 1$ also.

$\therefore \beta/R_1 + \frac{1}{R_1} + \frac{1}{R_2} = \frac{\beta}{R_2} - \frac{1}{R_1} - \frac{1}{R_2}$

which gives $\frac{R_2}{R_1} = \frac{\beta - 2}{\beta + 2} = \frac{100-2}{100+2} = \frac{98}{102} = \underline{0.96}$

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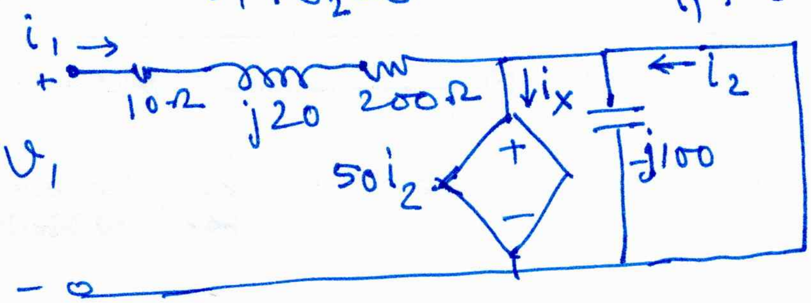
4/ $h_{11} = \frac{v_1}{i_1}\Big|_{v_2=0}$ $h_{21} = \frac{i_2}{i_1}\Big|_{v_2=0}$.



$\therefore h_{11} = 210 + j20$

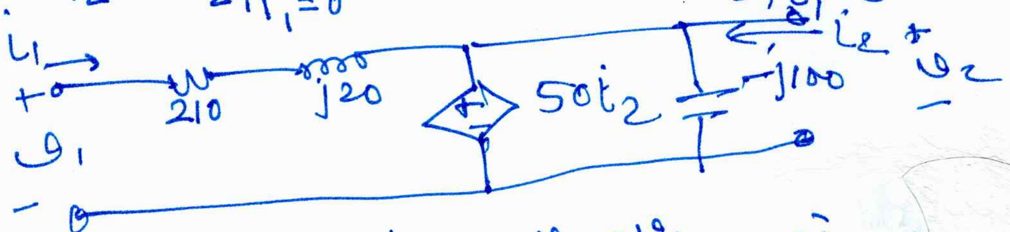
No current flows ~~through~~ the capacitor. The controlled voltage source is also shorted.

$\therefore v_1 = i_1(210 + j20)$

As the controlled voltage source has zero volts across its terminals, it cannot maintain the voltage across it.

$\therefore i_x = 0$. $\therefore i_1 = -i_2$ $\therefore h_{21} = -1$

$h_{12} = \frac{v_1}{v_2}\Big|_{i_1=0}$ & $h_{22} = \frac{i_2}{v_2}\Big|_{i_1=0}$



$v_2 = 50\, i_2$, $i_1 = \frac{v_1 - v_2}{210 + j20} = 0$ $\therefore$

$\therefore v_1 = v_2$

or $\frac{i_2}{v_2} = h_{22}$ $\therefore \underline{h_{12} = 1}$ $\frac{i_2}{v_2} = 50^{-1} = \underline{h_{22} = 0.02}$

$[h] = \begin{bmatrix} 210 + j20 & -1 \\ 1 & 0.02 \end{bmatrix}$ $h_{12} h_{21} \neq 1$??

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6/ $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ But $i_2 = -\dfrac{v_2}{Z_L}$ & $v_1 = v_s - i_1 Z_s$

$\therefore i_1 = y_{11}(v_s - i_1 Z_s) + y_{12} v_2$ or $i_1(1 + y_{11} Z_s) = y_{11} v_s + y_{12} v_2$

$$\therefore i_1 = \frac{y_{11} v_s + y_{12} v_2}{1 + y_{11} Z_s}$$

Again $i_2 = -\dfrac{v_2}{Z_L} = y_{21}(v_s - i_1 Z_s) + y_{22} v_2$

$$= y_{21} v_s - y_{21} Z_s \frac{y_{11} v_s + y_{12} v_2}{1 + y_{11} Z_s} + y_{22} v_2$$

or $v_2\left(-\dfrac{1}{Z_L} + \dfrac{y_{21} y_{12} Z_s}{1 + y_{11} Z_s} - y_{22}\right)$

$$= v_s\left(y_{21} - \frac{y_{21} y_{11} Z_s}{1 + y_{11} Z_s}\right)$$

$$\therefore v_2 = \frac{y_{21} - \dfrac{y_{21} y_{11} Z_s}{1 + y_{11} Z_s}}{\dfrac{y_{21} y_{12} Z_s}{1 + y_{11} Z_s} - \dfrac{1}{Z_L} - y_{22}}$$ Multiplying by $[(1 + y_{11} Z_s) Z_L]$

$$\frac{v_2}{v_s} = \frac{y_{21}(1 + y_{11} Z_s) Z_L - y_{21} y_{11} Z_s Z_L}{y_{21} y_{12} Z_s Z_L - (1 + y_{11} Z_s) - y_{22}(1 + y_{11} Z_s) Z_L}$$

$$= \frac{y_{21} Z_L}{y_{21} y_{12} Z_s Z_L - (1 + y_{11} Z_s)(1 + y_{22} Z_L)}$$

$y_{21} Z_L = 0.1 \times 70k = 7k$, $y_{21} y_{12} Z_s Z_L = 0.1 \times (-2 \times 10^{-6}) \times 2.5 \times 70 \times 10^6$
$= -35$

$(1 + y_{11} Z_s) = 1 + 2 \times 10^{-3} \times 2.5 \times 10^3$, $(1 + y_{22} Z_L) = 1 + (-50) \times 10^{-6} \times 70 \times 10^3$
$= 1 + 5 = 6$ $\quad = 1 - 3.5 = -2.5$

$$\therefore v_2 = \frac{7000}{-35 - 6 \times (-2.5)}$$

$= -350\, v_s = -350 \times 80\sqrt{2} \angle 0^\circ$

$\therefore v_2|_{rms} = -350 \times 80 \angle 0^\circ \times 10^{-3} = 350 \times 80 \angle 180^\circ \times 10^{-3} = 28 \angle 180^\circ$ V

$$P = \frac{|v_{2rms}|^2}{70K} = 11.2 \text{ mW}$$

5/ [h] parameters are not cascadeable
∴ one needs to convert it to [ABCD] parameters.

$$A_1 = -\frac{\Delta h}{h_{21}},\quad B_1 = -\frac{h_{11}}{h_{21}},\quad C_1 = -\frac{h_{22}}{h_{21}} \text{ and } D_1 = -\frac{1}{h_{21}}$$

$$\Delta h = h_{11}h_{22} - h_{21}h_{12} = 1k \times 100 \times 10^{-6} - 100 \times 1.5 \times 10^{-3}$$
$$= 0.1 - 0.15 = -0.05$$

$$\therefore A_1 = \frac{0.05}{10^2} = 5\times10^{-2}\times10^{-2} = 5\times10^{-4}$$

$$B_1 = -\frac{10^3}{100} = -10\,\Omega,\quad C_1 = -\frac{10^{-4}}{100} = -10^{-6}\,\mho,\quad D_1 = -\frac{1}{100} = -10^{-2}$$

$$[ABCD] = [ABCD]_1 \times [ABCD]_2$$

$$= \begin{bmatrix} 5\times10^{-4} & -10 \\ -10^{-6} & -10^{-2} \end{bmatrix} \times \begin{bmatrix} 5\times10^{-4} & -10 \\ -10^{-6} & -10^{-2} \end{bmatrix} = \begin{bmatrix} 25\times10^{-8} + 10^{-5} & -5\times10^{-3} + 10^{-1} \\ -5\times10^{-10} + 10^{-8} & 10^{-5} + 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 10.25\times10^{-6} & 0.095 \\ +95\times10^{-10} & 1.1\times10^{-4} \end{bmatrix},\quad \Delta[ABCD] = 11.275\times10^{-10} - 9.025\times10^{-10} = 2.25\times10^{-10}$$

Instead of doing the calculations again we can use the derivation done in Q6 by converting the [ABCD] to [y].

$$y_{11} = \frac{D}{B},\quad y_{12} = -\frac{\Delta(ABCD)}{B},\quad y_{21} = -\frac{1}{B},\quad y_{22} = \frac{A}{B}$$

$$y_{11} = \frac{1.1\times10^{-4}}{0.095} = 11.58\times10^{-4}\,\mho \qquad y_{12} = -\frac{2.25\times10^{-10}}{0.095} = -0.0024\times10^{-6}\,\mho \qquad y_{21} = -10.53\,\mho,\quad y_{22} = 1.08\times10^{-4}\,\mho$$

$$\frac{v_2}{v_s} = \frac{y_{21}Z_L}{y_{21}y_{12}Z_sZ_L - (1+y_{11}Z_s)(1+y_{22}Z_L)}$$

$$y_{21}Z_L = -10.53\times10^4 = -10.53\times10^4$$

$$y_{21}y_{12}Z_sZ_L = (-10.53)(-0.0024\times\cancel{10^{-6}})\times0.5\times\cancel{10^3}\times10\times\cancel{10^3}$$
$$= 0.126$$

$$1 + y_{11}Z_s = 1 + 11.58\times10^{-4}\times0.5\times10^3 = 1.58$$

$$1 + y_{22}Z_L = 1 + 1.08\times10^{-4}\times10^4 = 2.08$$

$$\therefore \frac{v_2}{v_s} = \frac{-10.53\times10^4}{0.126 - 1.58\times2.08} = 3.318\times10^4$$
$$= 33{,}318$$