ESc201A Home Assignment 3 Aug. 19, 2019.

Solutions of the HA#3 will be on Brihaspati on 26/08/19.

Consider all voltage and current sources to be ideal.

1. The frequency of the sinusoidal voltage source in the circuit of fig. 3.1 is adjusted until the amplitude of the sinusoidal output voltage (v_o) is maximum. The maximum amplitude of the voltage source (v_i) is 600V. (a) Find ω_s, (b) Amplitude of V_o at ω_s, (c) The bandwidth of the circuit, (d) The Q of the circuit, and the frequencies at which the amplitude of V_o is $0.707\{V_o\}_{max}$.

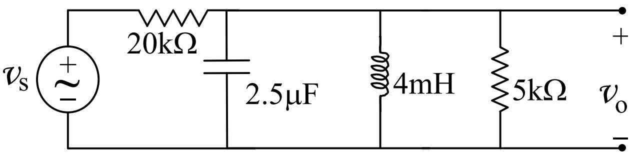


Figure 3.1

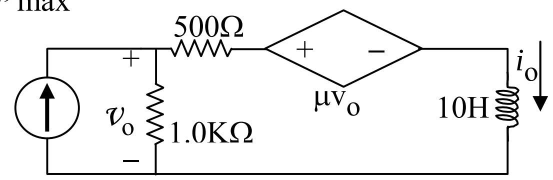


Figure 3.2

2. Find the transfer function i_o/i_g as a function of μ for the circuit shown in fig. 3.2. Hence find i_o at μ=1 and μ=2.5.
3. Find R, L, and C of a series RLC resonant circuit such that it resonates at f_o=500MHz and has a bandwidth of Δf=500kHz. The constraints are that the maximum current through R is 17.32mA and the maximum power dissipation is 62.84mW.
4. The peak amplitude of the sinusoidal voltage source in the circuit shown in fig. 3.3 is $100\sqrt{2}$ V and its period is 200π μs. The load resistor can be varied from 0 to 300Ω, and the load capacitor can be varied from 1 to 4μF. Determine the settings of R_L and C_L that will result in the most average power being transferred to R_L. Calculate the average power dissipated in R_L.

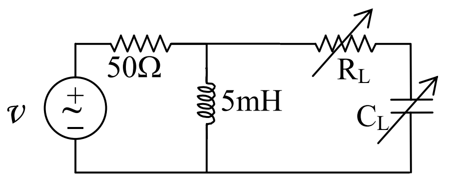


Figure 3.3

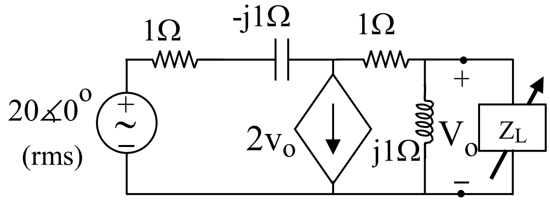


Figure 3.4

5. In fig. 3.4 the load impedance is varied till maximum average power is delivered to Z_L. What is this maximum average power? What percentage of the total power developed in the circuit is delivered to Z_L?
6. A machine is running at a load of 1200kW at a power factor (p.f.) of 0.8 lag. This p.f. needs to be improved to a p.f. of 0.96 lagging by adding an extra load of 300kW to the real power load.
(a) Find the Reactive Power of the added load. (b) What is its p.f.? (c) If the input voltage to the machine is always maintained at 3kV, then what is the rms current drawn by the machine, before and after addition of the 300kW load?
7. Find the numerical expression for the transfer function $H(j\omega) = v_o/v_i$ for the circuit shown in fig. 3.5.
Give the numerical value of each pole and zero of $H(j\omega)$.

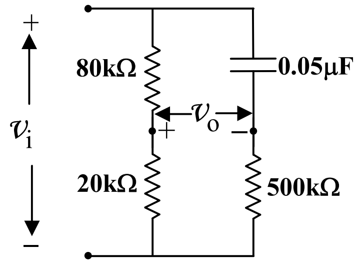


Figure 3.5

ESc201A Home Assignment 3 Aug. 19, 2019.

8. The numerical expression for a transfer function is $H(j\omega) = \left\{10^5(j\omega + 5)\right\} \Big/ \left[(j\omega + 100)(j\omega + 500)\right]$.

   On the basis of a straight-line approximation of $H(j\omega)$ versus ω, estimate (a) the maximum $|H(j\omega)|$ in dB and (b) the value of $\omega > 0$ where the $|H(j\omega)|$ equals unity.

9. In the circuit shown in fig. 3.6 find the transfer function $H(j\omega) = v_o/v_i$. Sketch $\measuredangle H(j\omega)$ versus ω.

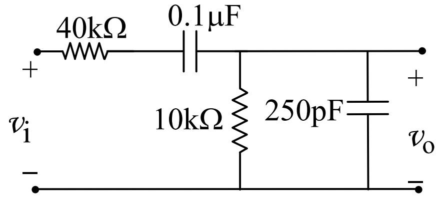


Figure 3.6

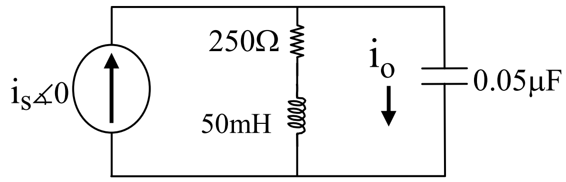


Figure 3.7

10. Derive the transfer function $H(j\omega) = i_o/i_s$ of the circuit shown in fig. 3.7. Sketch the asymptotic $|H(j\omega)|$ as a function of ω and find the bandwidth of the circuit.

ESc102A HA# 3 Soln 2019 — I

1/ (a) This is a parallel resonant circuit.

$\therefore \omega_0^2 = \frac{1}{LC} = \frac{1}{4\times10^{-3} \times 2.5\times10^{-6}} = \frac{1}{10\times10^{-9}} = 10^8$, $\omega_0 = 10^4$ rad/s.

$\omega_s = \omega_0$ when v_o is maximum, as the $Z_{eq} = Z_C || Z_L \Big|_{\omega_s=\omega_0} \to \infty$

$\therefore$ All current sent by v_s goes through the 5kΩ resistance.

$\therefore \omega_s = 10^4$ rad/s. & $f_s = \frac{10^4}{2\pi} = 1.59$ kHz.

(b) At $\omega_s = \omega_0$ (Resonance of L||C) the load to v_s is completely resistive. If $v_s = V_{s_m} e^{j\omega_0 t}$ & $v_o = V_{o_m} e^{j\omega_0 t}$

Then $V_{om} = V_{sm} \times \frac{5k}{20k+5k}$ & $V_{sm} = 600$V given.

$\therefore V_{om} = \frac{600\times5}{25} = 120$V.

(c) $\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)^2}$, $\omega_2 = +\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)^2}$ at $0.707V_m$

$\therefore \Delta\omega = \omega_2 - \omega_1 = \frac{1}{RC}$, But, Here there is also a 20 kΩ other than the 5kΩ parallel to the L||C combination.

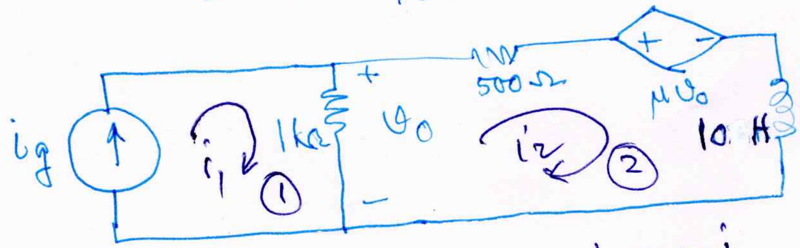
$\therefore$ One need to consider the R_{eq} across L||C combination

$\therefore R_{eq} = 20k\Omega || 5k\Omega = \frac{20\times5k}{25} = 4k\Omega$

$\therefore \Delta\omega = \frac{1}{4k\Omega \times 2.5\mu} = \frac{1000}{10} = 100$ rad/s } Let's call this the Bandwidth $= \frac{100}{2\pi} \simeq 16$ Hz

should give a very high Q

(d) $Q = \frac{\omega_0}{\Delta\omega} = \frac{10^4}{100} = 10^2 = 100$

2/ 

KVL in loops ① & ②

① $v_o = (i_1 - i_2)10^3$, $\begin{cases} i_g = i_1 \\ i_2 = i_o \end{cases}$
$= (i_g - i_o)10^3$

② $v_o = 0.5\times10^3 i_o + \mu v_o + j\omega\times10\times i_o = i_g\times\cancel{10^3} - \cancel{10^3} i_o$.

or $0.5\times\cancel{10^3} i_o + \frac{\mu v_o}{10^3} + \frac{j\omega 10 i_o}{10^2}$

or $i_o + 0.5 i_o + 0.01 j\omega i_o + \cancel{10^{-3}}\mu(i_g - i_o)\cancel{10^3} = i_g$

$(1.5 - \mu + 0.01 j\omega) i_o = i_g(1-\mu)$

$\therefore \frac{i_o}{i_g} = \frac{1-\mu}{1.5 - \mu + 0.01 j\omega}$

$\therefore i_o\big|_{\mu=1} = 0$, & $i_o\big|_{\mu=2.5} = \frac{-1.5}{-1 + 0.01 j\omega}$

$= \frac{1.5}{1 - 0.01 j\omega} =$

$i_o\big|_{\mu=2.5} = \frac{1.5\angle\tan^{-1}0.01\omega}{\sqrt{1+10^{-4}\omega^2}}$

ESc 201A HA#3 Soln 2019 – I

3/ Series Resonant circuit. $\therefore \omega_0 = \frac{1}{\sqrt{LC}}$, $Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{\Delta\omega}$

$f_0 = 500\,MHz$ $\therefore \omega_0 = 2\pi f_0 = 3.142\,GRad/s$

$\Delta f = 500\,kHz$ $\therefore \Delta\omega = 2\pi\Delta f = 3.142\,MRad/s$, If the voltage across R is supposed to be Max at resonance the Imag. through R is used.

$P_{R_{D max}} = I_{R_{max}}^2 \times R$ or $R = \frac{62.84\,mW}{(17.32)^2 \mu(A^2)} = 209.5\,\Omega$

$Q = \frac{3.142 \times 10^9}{3.142 \times 10^6} = 10^3$ (very high) $= \frac{3.142 \times 10^9 \times L}{209.5}$

Obvious from Δf being very narrow.

$\therefore L = \frac{10^3 \times 209.5}{3.142 \times 10^9} = 66.7\,\mu H$

$C = 1/\omega_0^2 L = \frac{1}{(3.142)^2 \times 10^{18} \times 66.7 \times 10^{-6}} = 1.5 \times 10^{-15} = 1.5\,fF$

$\therefore$ R, L, & C found.

4/ $T = 200\pi\,\mu s$ $\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{200\pi \times 10^{-6}} = \frac{1}{10^{-4}} = 10^4\,rad/s$.

Impedance of L is $j\omega L = j \times 10^4 \times 5 \times 10^{-3} = j50\,\Omega$

The voltage drop across the $R_L + \frac{1}{j\omega C_L}$ is divided across the two components, but for maximum drop across R_L choose $C_{L_{max}} = 4\mu F$

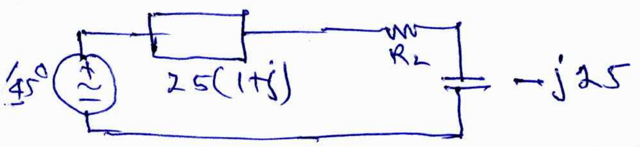
$\therefore -jX_{R_L} = \frac{1}{j\omega C_L} = \frac{-j}{10^4 \times 4 \times 10^{-6}} = -j25\,\Omega$.

Compute the Thevenin equivalent of all components seen by R_L, C_L combination. $\therefore Z_{Th} = \frac{50 \times (j50)}{50 + j50} = \frac{j50}{1+j} = \frac{j50(1-j)}{(\sqrt{2})^2} = \frac{50(1+j)}{2}$

$= 25(1+j)$

$V_{Th} = \frac{v \times (j50)}{50 + j50} = v \cdot \frac{j(1-j)}{2} = \frac{v}{2}(1+j)$

Using max of v, $V_{Th_{max}} = \frac{100\sqrt{2}}{2}(1+j) = \frac{100\sqrt{2}\sqrt{2}\angle 45^\circ}{2} = 100\angle 45^\circ$



Obviously resonance happens at $\omega = 10^4\,rad/s$ as $j25 + (-j25) = 0$

$\therefore I_{R_{L max}} = \frac{100\angle 45^\circ}{25 + R_L}$

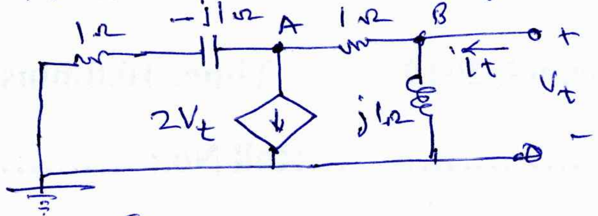
Again to have bigger drop across R_L as compared to C_L choose the max. $R_L = 300\,\Omega$.

$= \frac{100\angle 45^\circ}{25 + 300} = 0.31\angle 45^\circ$

$P_{R_{AV}} = R_L \frac{I_{R_{L max}}^2}{2} = \frac{R_L}{2} I_{R_{L max}} \times I_{R_{L max}}^* = \frac{300}{2} \times 0.31^2 = 14.2\,W$

ESc 201A HA#3 Soln 2019-I

5/ Find the Thevenin equivalent across Z_L. Eqv. ckt. is



KCL at (A). $\frac{V_A}{1-j} + 2V_t + \frac{V_A - V_t}{1} = 0$

or $V_A + 2(1-j)V_t - (1-j)V_t + (1-j)V_A = 0$

or $(2-j)V_A = -(1-j)V_t$

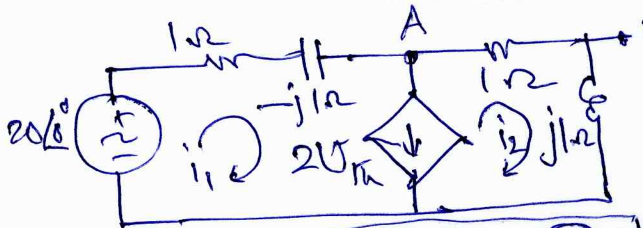
$\therefore V_A = -\frac{(1-j)}{2-j}V_t$

At (B)

$\frac{V_t - V_A}{1} + \frac{V_t}{j} - i_t = 0$. or $V_t + \frac{V_t}{j} + \frac{(1-j)}{2-j}V_t = i_t$

$\therefore V_t\left(1-j+\frac{(1-j)}{2-j}\right) = i_t$ $\therefore \frac{V_t}{i_t} = \frac{2-j}{1-3j+1-j} = \frac{2-j}{2-4j}$

$= \frac{\sqrt{4+1}}{\sqrt{4+16}} \angle \tan^{-1}2 - \tan^{-1}0.5$



$Z_{Th} = \frac{1}{2}\angle 36.9^\circ = 0.4 + j0.3$

For max Power Transfer $Z_L = Z_{Th}^* = 0.4 - j0.3$

KVL in the loops (1) & (2)

loop (2) $\frac{v_{Th}}{j1} = i_2$, $20\angle 0^\circ = 20 = i_1 - ji_1 + V_A$, $i_1 - i_2 = 2v_{Th}$

$V_A = v_{Th} + 1\Omega \times i_2$ $\therefore 20 = (1-j)i_1 + v_{Th} + i_2$

$i_1 - \frac{v_{Th}}{j} = 2v_{Th}$ & $20 = (1-j)i_1 + v_{Th} + \frac{v_{Th}}{j}$

or $i_1 = (2-j)v_{Th}$. or $20 = (1-j)(2-j)v_{Th} + (1-j)v_{Th}$

$= (1-3j+1-j)v_{Th}$

$= (2-4j)v_{Th}$.

$\therefore v_{Th} = \frac{20}{2(1-2j)} = \frac{10}{\sqrt{5}}\angle 63.4^\circ$

$= 1.97 + j4.0$ (rms) V.

$\therefore I_{Load} = \frac{10}{\sqrt{5}}\angle 63.4^\circ / 0.8 = 5.6\angle 63.4^\circ$

$P_{load}\Big|_{\text{at max power transfer}} = |I_{load}|^2 \times 0.4 = 12.54$ W

ESc 201A HA#3 Soln 2019—I

6/ a) Complex Power $S = P + jQ$

$$= \frac{V_m I_m}{2}\cos\theta + j\frac{V_m I_m}{2}\sin\theta$$

$$= 1200k + j\frac{1200k}{0.8} \times \sin 36.87^\circ \text{ VA}$$

$$= 1200k + j\,900k \text{ VA}$$

After improving p.f.

$$S_i = (1200 + 300)k + j\frac{1500k}{0.96} \times \sin(16.26^\circ)$$

$$= 1500k + j\frac{1500k}{0.96} \times 0.28 \text{ VA} = (1500 + j437.5) \text{ kVA}$$

$$\therefore Q_{load} = (437.5 - 900) \text{ KVAR} = -462.5 \text{ kVAR}$$

b) $S_{load} = 300k - j462.5k = 551.1k\angle -57^\circ$ VA.

$\text{p.f.}_{load} = \cos 57^\circ = 0.545$ leading

c) $3k \times I^*_{machine} = 1200k + j900k$

or $I^*_{machine} = 400 + j300$ or $I_{machine} = 400 - j300$ A $= 500\angle -36.9^\circ$

$\therefore I_{machine}\big|_{rms} = 500$ A.

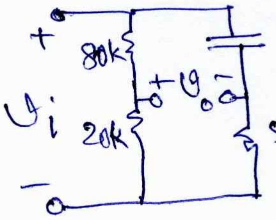
d) $3k \times I'^*_{machine} = 1500k + j437.5k$

$I'^*_{machine} = 500 + j145.8$

$I'_{ma} = 500 - j145.8 = 521\angle -16.26^\circ$

$I'_{rms} = 521$ A.

7/ a)



$$v_{o+} = \frac{v_i \times 20}{80 + 20} = 0.2 v_i$$

$$v_{o-} = \frac{v_i \times 500k}{\frac{10^6}{j\omega \times 0.05} + 500k} = \frac{v_i}{1 + \frac{10^6}{j\omega 0.05 \times 500k}} = \frac{v_i}{1 - j\frac{40}{\omega}}$$

$$v_o = v_{o+} - v_{o-} = 0.2 v_i - \frac{v_i}{1 - j\frac{40}{\omega}}$$

$$\frac{v_o}{v_i} = 0.2 - \frac{1}{1 - j\frac{40}{\omega}} = \frac{0.2 - j\frac{8}{\omega} - 1}{1 - j\frac{40}{\omega}} = \frac{-0.8 - j\frac{8}{\omega}}{1 - j\frac{40}{\omega}}$$

$$H(\omega) = \frac{-0.8 \times \left(1 + j\frac{10}{\omega}\right)}{\left(1 - j\frac{40}{\omega}\right)} = \frac{-0.8\,(j\omega - 10)}{(j\omega + 40)} \quad \therefore H(s) = \frac{-0.8(s - 10)}{(s + 40)}$$

b) $\therefore s = 10$ for $H(s) = 0$ $\therefore$ Zero is at 10 rad/s.

$s = -40$ for $H(s) = \infty$ $\therefore$ Pole is at -40 rad/s.

where $s = j\omega$

ESc 201A HA#3 Soln 2019-I

8/ $H(j\omega) = \frac{10^5 (j\omega + 5)}{(j\omega+100)(j\omega+500)} = \frac{10^5 \times 5}{100 \times 500} \times \frac{(1+j\frac{\omega}{5})}{(1+j\frac{\omega}{100})(1+j\frac{\omega}{500})}$

$|H(\omega)|_{dB} = 20\log 10 + 20\log\left|(1+j\frac{\omega}{5})\right| - 20\log\left|(1+j\frac{\omega}{100})\right| - 20\log\left|(1+j\frac{\omega}{500})\right|$

$20\log 10 = 20dB$. So for $\omega \ll 5 \ll 100 \ll 500$ the $|H(\omega)|_{dB} = 20dB$

At $\omega = 5$ there would be a corner frequency and the asymptotic line would increase at +20dB/decade.

At $\omega = 100$ corner frequency is at 100 and it starts decreasing at −20dB/decade.

At $\omega = 500$ corner frequency is at 500 and it goes downward at −20dB/decade.

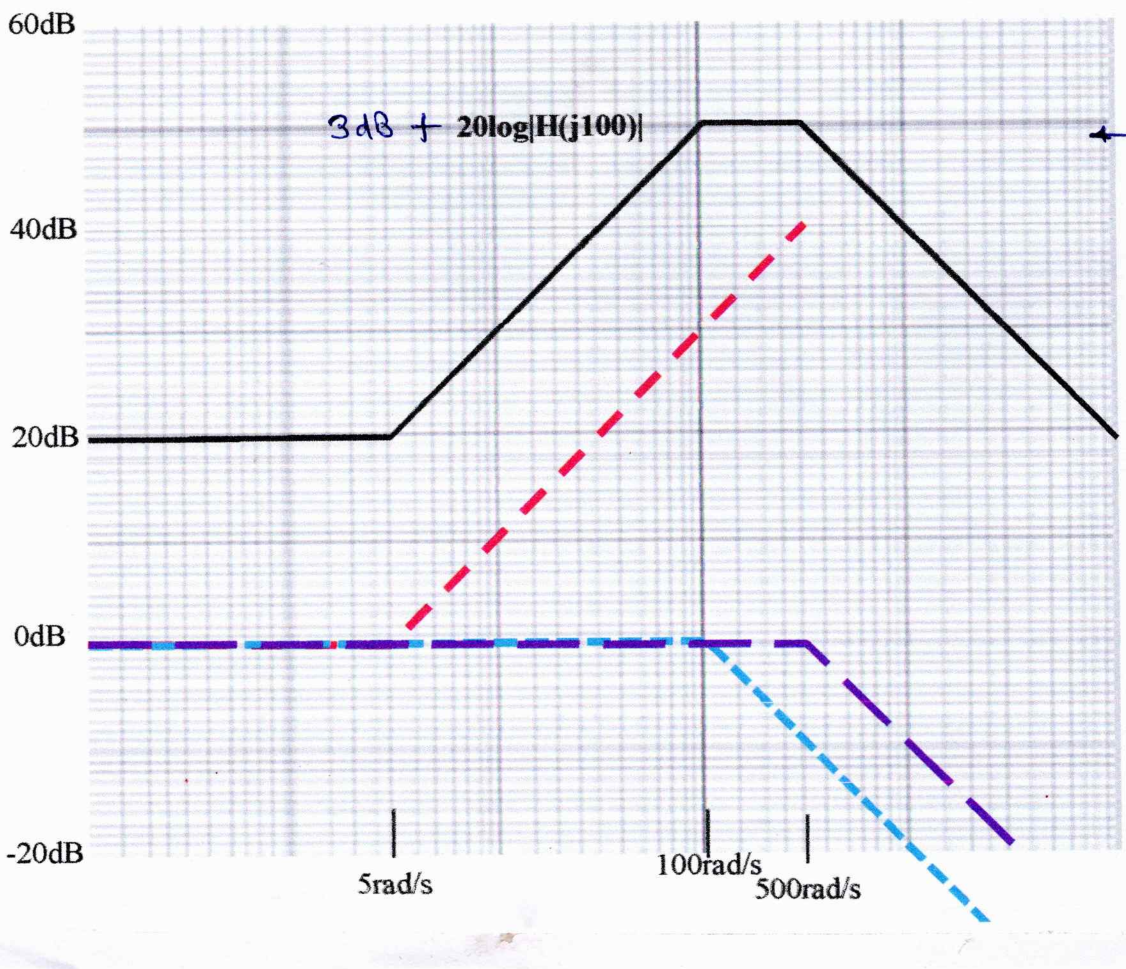
$H(j100) = \frac{10(1+j\frac{100}{5})}{(1+j)(1+j\frac{100}{500})} = \frac{10(1+j20)}{(1+j)(1+0.2j)} \simeq \frac{10 \times 20\angle 87.14}{\sqrt{2}\angle 45^0 \, 1.02\angle 11.3^0}$

$= 138.65\angle 30.84^0$

$20\log|H(j100)| = 20\log 138.65 = 42.8 dB$.

$20\log|H(j500)| \simeq 20\log\left|\frac{10 \times (100)}{(1+j5)(1+j)}\right| \simeq 20\log\left|\frac{10 \times (100)}{5.1 \times \sqrt{2}}\right| = 42.84 dB$.

∴ Corners would be +3dB above this value $\simeq$ 46 dB

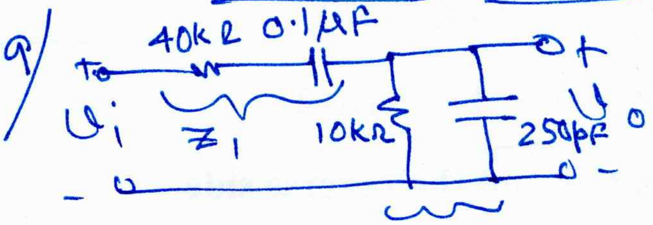


Should have been 46dB

But due to bad graph paper, looks more like 50dB

ESc 201A HA#3 Soln 2019-I

q/



Looks like the interstage coupling circuit of an amplifier.

$$Z_1 = 40k - \frac{j \times 10^6}{0.1\omega}$$

$$= -j\frac{10^7}{\omega} + 4\times10^4$$

$$= 4\times10^4\left(1 - j\frac{250}{\omega}\right)$$

$$(s = j\omega) \Rightarrow 4\times10^4\left(1 + \frac{250}{s}\right)$$

$$Z_2 = \frac{-j10k / 250\times10^{-12}\times\omega}{10k - j\frac{1}{250\times10^{-12}\omega}}$$

$$= \frac{10^{12}/250\omega}{j + \frac{10^{12}}{250\times10^4\omega}}$$

$$= \frac{4\times10^9}{4\times10^5 + j\omega}$$

$$\Rightarrow \frac{4\times10^9}{s\left(1 + \frac{4\times10^5}{s}\right)}$$

$$\frac{v_0}{v_i} = H(s) = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{\frac{4\times10^9}{s\left(1+\frac{4\times10^5}{s}\right)}}{4\times10^4\left(1+\frac{250}{s}\right) + \frac{4\times10^9}{s\left(1+\frac{4\times10^5}{s}\right)}}$$

$$= \frac{\frac{10^5 s}{(s + 4\times10^5)}}{(s+250) + \frac{10^5 s}{(s+4\times10^5)}}$$

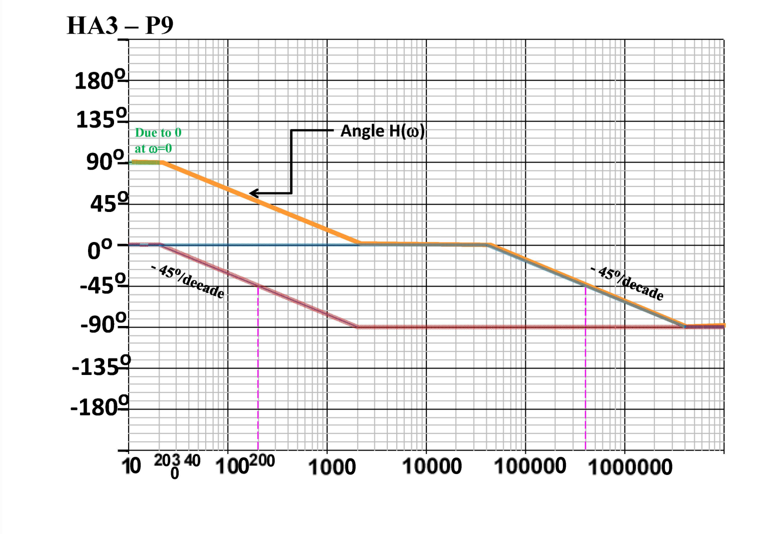
$$= \frac{10^5 s}{s^2 + 4\times250\times10^5 + 4\times10^5 s + 250s + 10^5 s}$$

$$= \frac{10^5 s}{s^2 + (5\times10^5 + 250)s + 10^8}$$

$$= \frac{10^5 s}{(s + 199.98)(s + 500050)}$$

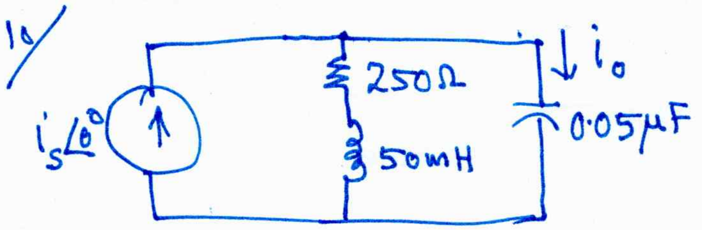
$\therefore$ Poles at $\omega_1 = -199.98$ rad/s
and $\omega_2 = -500{,}050$ rad/s

Zero at $\omega = 0$. $\rightarrow \angle +90^\circ$



ESc 201A HA#3 Soln 2019-I

1)



Parallel ∴ use Admittance.

$$Y_s = \frac{1}{R+j\omega L} + j\omega C = \frac{R}{R^2+(\omega L)^2} - \frac{j\omega L}{R^2+(\omega L)^2} + j\omega C$$

At resonance $\frac{\omega_0 L}{R^2+(\omega_0 L)^2} = \omega_0 C$

or $L = R^2C + \omega_0^2 L^2 C$ or $1 = \frac{R^2C}{L} + \omega_0^2 LC$

$\therefore \omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$ $\therefore \omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2} = \sqrt{4\times10^8 - (5\times10^3)^2} = 1.94\times10^4 \frac{rad}{s}$

Let $j\omega = s$.

$\therefore \frac{i_o}{sC} = i_s \times \frac{R+sL}{R+sL+\frac{1}{sC}}$ or $H(\omega) = \frac{250+j\omega\times0.05}{250+j\omega\times0.05 - j\frac{10^6}{\omega\times0.05}}$

$$\therefore |H(\omega)| = \frac{\sqrt{250^2+\omega^2\times0.05^2}}{\sqrt{250^2+\left[(\omega\times0.05)-\left(\frac{10^6}{\omega\times0.05}\right)\right]^2}}$$

$$\therefore |H(\omega_0)| = \frac{\sqrt{250^2+1.94^2\times10^8\times0.05^2}}{\sqrt{250^2+\left[(1.94\times0.05\times10^4)-\left(\frac{10^6}{1.94\times0.05\times10^4}\right)\right]^2}}$$

$$= \frac{\sqrt{250^2+940900}}{\sqrt{250^2+[970-1031]^2}} = \frac{1001.7}{257.33} \cong 3.9.$$

At ω_1 & ω_2 $|H(\omega_{1,2})| = 0.707|H(\omega_0)| = 2.76$

$$\therefore 2.76 = \left|\frac{250+j\omega_{1,2}\times0.05}{250+j\omega_{1,2}\times0.05 - j\frac{2\times10^7}{\omega_{1,2}}}\right|$$

or $$2.76 = \left|\frac{250\omega_{1,2}+j\omega_{1,2}^2\times0.05}{250\omega_{1,2}+j\omega_{1,2}^2 - j2\times10^7}\right| = \frac{\sqrt{250^2\omega_{1,2}^2+\omega_{1,2}^4\times0.05^2}}{\sqrt{250^2\omega_{1,2}^2+(\omega_{1,2}^2-2\times10^7)^2}}$$

$$= \frac{\sqrt{250^2+\omega_{1,2}^2\times0.05^2}}{\sqrt{250^2+\left(1-\frac{2\times10^7}{\omega_{1,2}^2}\right)^2}}$$ or $250^2+\omega_{1,2}^2\times0.05^2 = 2.76^2\times250^2 + 2.76^2\left(1-\frac{2\times10^7}{\omega_{1,2}^2}\right)^2$

or $41.36\times10^4 = 25\times10^{-4}\omega_{1,2}^2 - \frac{30.5\times10^{14}}{\omega_{1,2}^4} + \frac{30.5\times10^{14}}{\omega_{1,2}^2}$

or $41.36\omega_{1,2}^4 = 25\times10^{-8}\omega_{1,2}^6 - 30.5\times10^{10} + 30.5\times10^{10}\omega_{1,2}^2$

or $\omega_{1,2}^4 - 7.37\omega_{1,2}^2\times10^9 + 7.37\times10^{10} \cong 0$

Too lengthy a problem → Solve for $\omega_{1,2}^2$ & then ω_1 & ω_2.

That would give the asymptotic plot