Thursday, February 11, 2021 2:10 PM

EE 250
Tutorial sheet 4
Solution

Q1 Plot the root locus when

(a) $L(s) = \frac{1}{s^2}$

(b) $L(s) = \frac{s+1}{s+12} \frac{1}{s^2}$

(c) $L(s) = \frac{s+1}{s+9} \frac{1}{s^2}$

(d) $L(s) = \frac{s+1}{s+4} \frac{1}{s^2}$

Comment on the results

Solution 1(a)

$$L(s) = \frac{1}{s^2}$$

Characteristic polynomial

$$s^2 + K = 0$$

$$s_{1,2} = \pm j\sqrt{K}$$

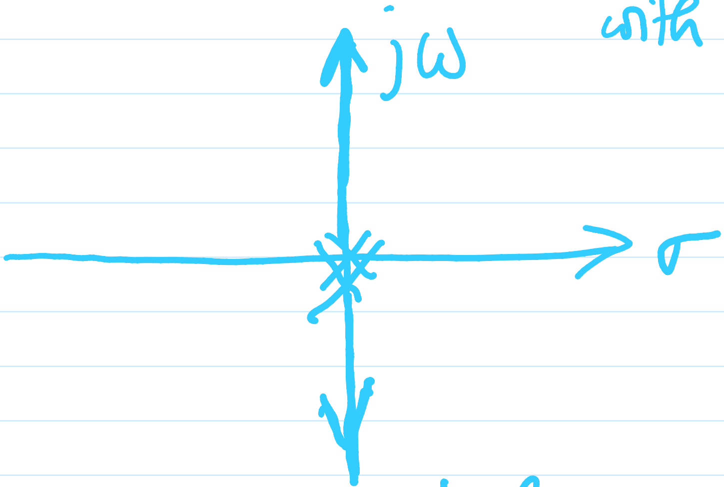
Break away pt $\frac{dK}{ds} = -2s = 0$

$$\Rightarrow s = 0$$

Two poles → Two root locus branches

→ Two asymptotes

→ Two asymptotes with angles +90, -90°



1(a) root locus plot

Solution 1(b)

$$L(s) = \frac{s+1}{s+2}\,\frac{1}{s^2}$$

1. There are three poles and one zero. Two poles will converge at ∞ and the other pole will converge at zero

2. Two root locus branches converge at ∞ ⇒ two asymptotes with angles $+90^0, -90^0$

$$\phi_l = \frac{180 + 360(l-1)}{n-m}$$

$$= 180/2, \quad \frac{180+360}{2}$$

$$= 180/2, \quad \frac{180+360}{2}$$

$$= 90^0, \quad 270^0 \ (-90^0)$$

3. Centroid of the asymptotes

$$c = \frac{\sum p_i - \sum z_i}{n-m}$$

$$= \frac{-12-(-1)}{2} = -5.5$$

4. $d(s) = s^2(s+12) + K(s+1) = 0$

$$K = -\frac{s^2(s+12)}{s+1}$$

$$\frac{dK}{ds} = -\frac{\left[(s+1)(3s^2+24s) - s^2(s+12)\right]}{(s+1)^2} = 0$$

$$\Rightarrow 3s^3 + 3s^2 + 24s^2 + 24s - s^3 - 12s^2 = 0$$

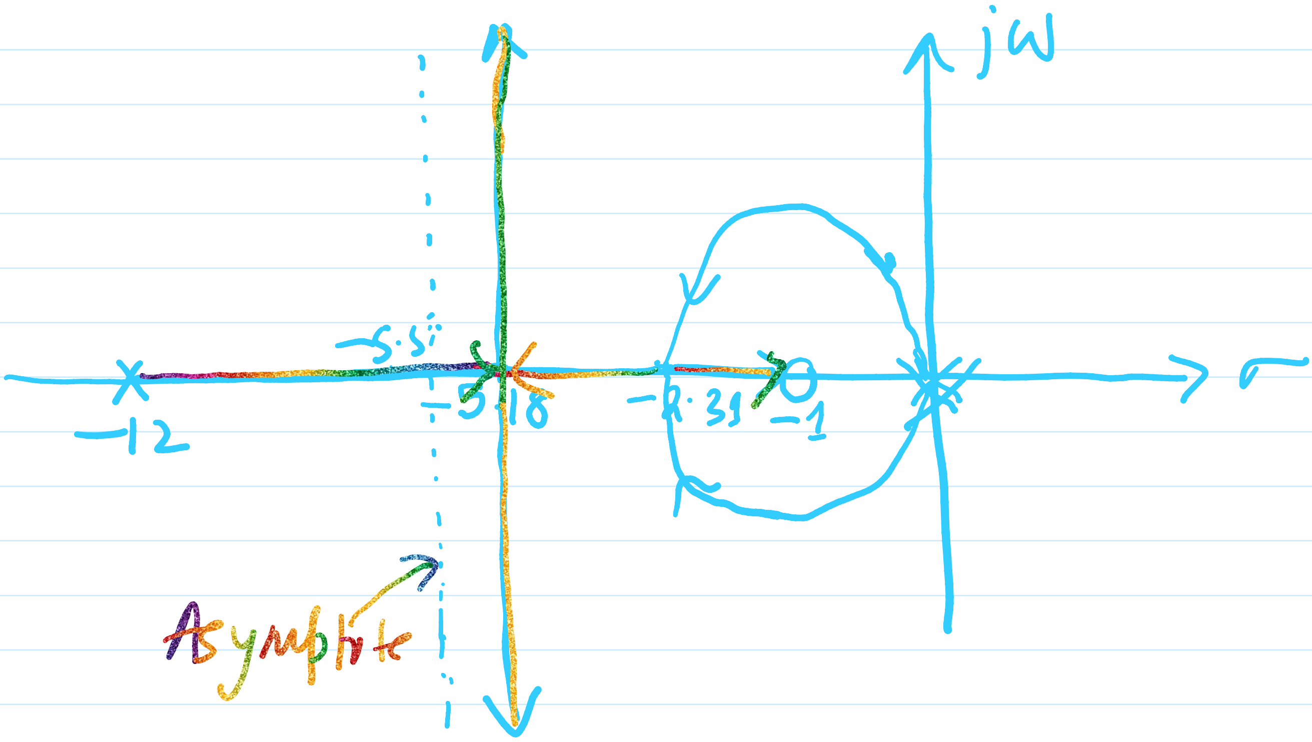
$$\Rightarrow 2s^3 + 15s^2 + 24s = 0$$

$$\Rightarrow s(2s^2 + 15s + 24) = 0$$

roots are $s_1 = 0, s_2 = -2.31, s_3 = -5.18$

There are three breakaway points

point



1(b) root locus

Two poles break away at s=0 and break in on the real axis at s = -2.31. One branch converges at zero (s = -1) and other branch moves towards the other branch that is coming from s = -12. These two branches break away at s = -5.18 and follow the asymptotes at $\pm 90^\circ$.

1.C $L(s) = \dfrac{s+1}{s+9} \quad \dfrac{1}{s^2}$

1. 3 poles & one zero $\Rightarrow$ two branches converges to infinity along the asymptotes at $\pm 90^\circ$.

2. Centroid of the asymptotes

$$C = \frac{\sum p_i - \sum z_i}{n-m} = \frac{-9-(-1)}{2}$$

$$= -4$$

3. Break away points

$$\frac{dK}{ds} = 0,$$

Char. poly $d(s) = s^2(s+9) + K(s+1) = 0$

$$K = -\frac{s^2(s+9)}{s+1}$$

$$\frac{dK}{ds} = -\frac{(s+1)(3s^2+18s) - s^2(s+9)}{(s+1)^2} = 0$$

$$\Rightarrow \quad 3s^3 + 3s^2 + 18s^2 + 18s - s^3 - 9s^2 = 0$$

$$2s^3 + 12s^2 + 18s = 0$$

$$s(2s^2 + 12s + 18) = 0$$

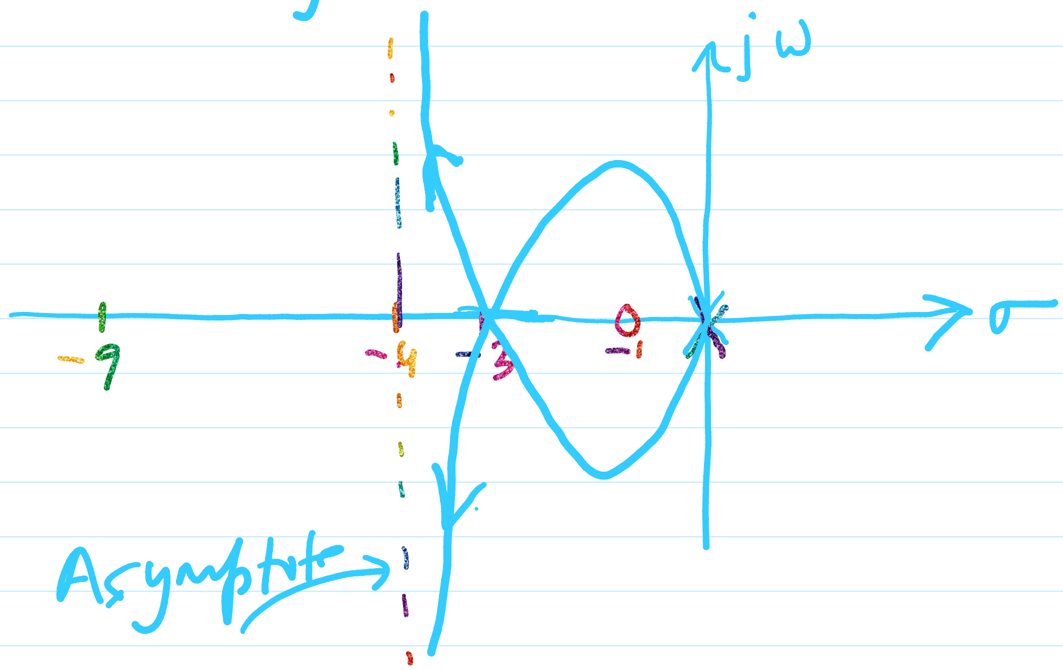
$$s(s^2 + 6s + 9) = 0$$

$$s(s+3)^2 = 0$$

break away pts are $s=0$, $s_1 = -3$, $s_2 = -3$

$s(s+3) \quad = 0$

break away pts are $s_1=0, s_2=-3, s_3=-3$



1.c root locus plot

1d. $L(s) = \frac{s+1}{s+4}\frac{1}{s^2}$

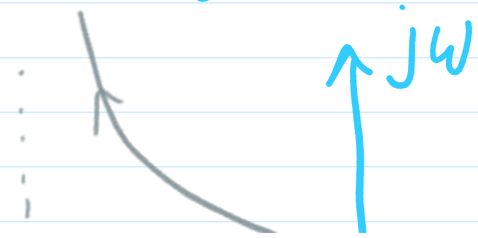
Centroid $c = \frac{-4-(-1)}{2} = -1.5$

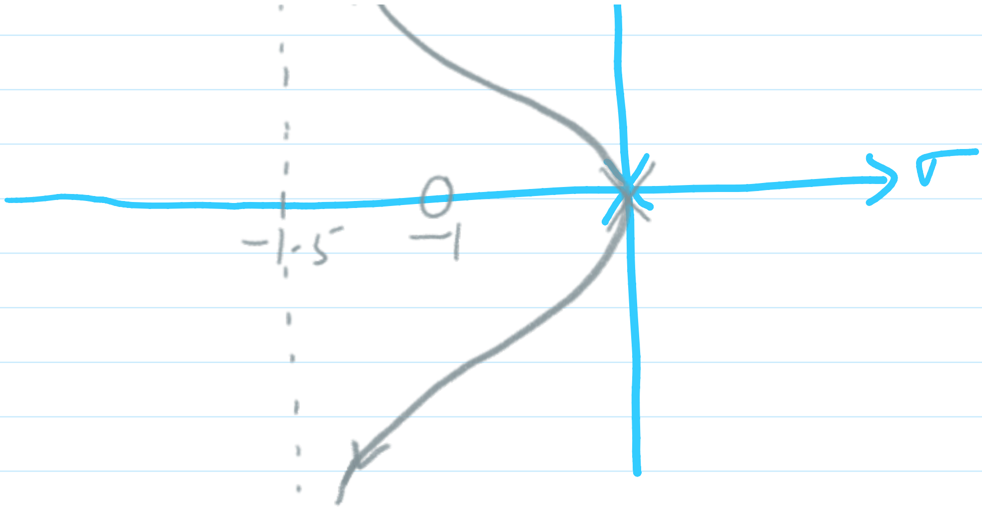
Break away pts $\frac{dk}{ds} = 0$

$2s^3 + 7s^2 + 8s = 0$

$s_1 = 0, \; s_{1,2} = -1.75 \pm j0.9$

$s_{1,2}$ do not lie on the root locus.





Comments on results:

$\frac{s+1}{s+12}$, $\frac{s+1}{s+5}$, $\frac{s+1}{s+4}$ are all valid compensators. Desired specifications will determine which one is a better one.

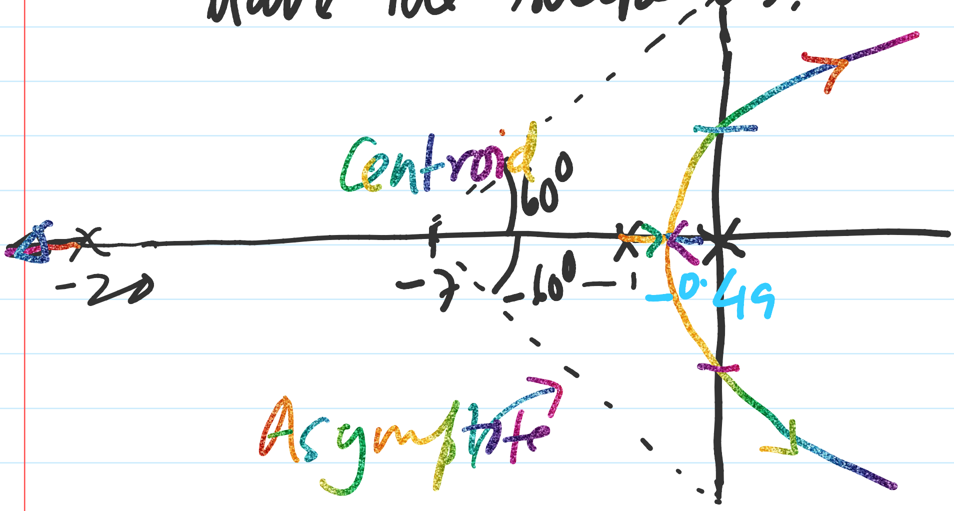
Q2 Draw the root locus of the system with $L(s) = \frac{K}{s(s+1)(s+20)}$.

(i) Determine K such that dominant poles have $\zeta = 0.5$

(ii) Find the second order approximation of the closed loop system for part (i).

(iii) Find range of K for which the system is stable.

Sol^n We have solved the similar problem in the class. The root locus will similarly ... class. The root locus will have the shape as:



Break away pt

Char poly $s^3+21s^2+20s+K=0$

$$K = -(s^3+21s^2+20s)$$

$$\frac{dK}{ds} = 0 \Rightarrow 3s^2+42s+20=0$$

$$s_1 = -13.5, \; s_2 = -0.49$$

Break away pt must lie between 0 & -1. Hence $s_2 = -0.49$ is the breakaway pt

Crossing of $j\omega$ axis

$$s^3+21s^2+20s+K=0$$

s^3	1	20
s^2	21	K
s^1	$\frac{21\times 20-K}{21}$	0

$K = 420$ when $j\omega$ line

21 0 $K = 420$ when jw line is crossed

s^0 K

At this point,

Aux eqn $21s^2 + 420 = 0$

$s^2 + 20 = 0$

$s = \pm j\sqrt{20}$

$= \pm j4.47$

Range of K for which the system is stable

(iii) $0 < K < 420$

(i) Draw the root locus on a graph sheet.

The asymptote will cross the jw axis at ± 12.12

The root locus will be parallel to the asymptote.

Then draw the line of ζ

$\theta = \cos^{-1}\zeta = \cos^{-1}0.5 = 60^0$

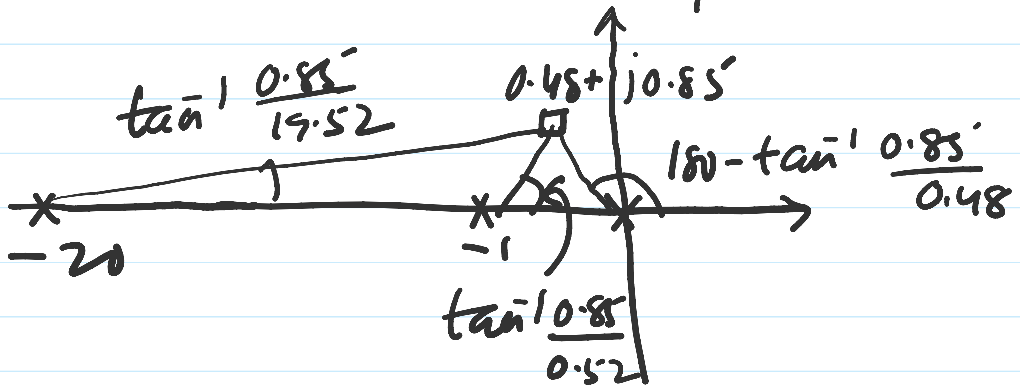
This will meet the root locus at $s_1 = -0.475 \pm 0.85j$

(tutors can demonstrate using MATLAB)

One can also verify if it is

One can also verify if it is correct by angle criteria

$$\angle G(s)\Big|_{s=s_1} = -180^0$$



$$-(180-60.54)-58^0-2.5^0 \cong 180^0$$

$$|KG(s)| = 1$$

$$K = |s_1|\,|s_1+1|\,|s_1+20|$$

$$= 0.97 \times 0.996 \times 19.5$$

$$\simeq 18.95$$

(ii) Dominant pole pairs are

$$s_{1,2} = -0.48 \pm j0.85$$

Ideal 2nd order char poly

$$(s-s_1)(s-s_2)$$

$$= s^2+0.96s+0.95$$

$$\text{Actual TF} = \frac{K}{s^3+21s^2+20s+K}$$

$$= \frac{18.95}{\quad}$$

$$= \frac{18.95}{s^3+21s^2+20s+18.95}$$

$$= \frac{18.95}{(s^2+0.96s+0.95)(s+20.05)}$$

$$\simeq \frac{18.95/20.05}{s^2+0.96s+0.95}$$

$$\simeq \frac{0.95}{s^2+0.96s+0.95}$$

DC gain for both $= 1$