EE 250: Control Systems Analysis
Module III: s-plane analysis
Lecture 16: Time Response – An Example

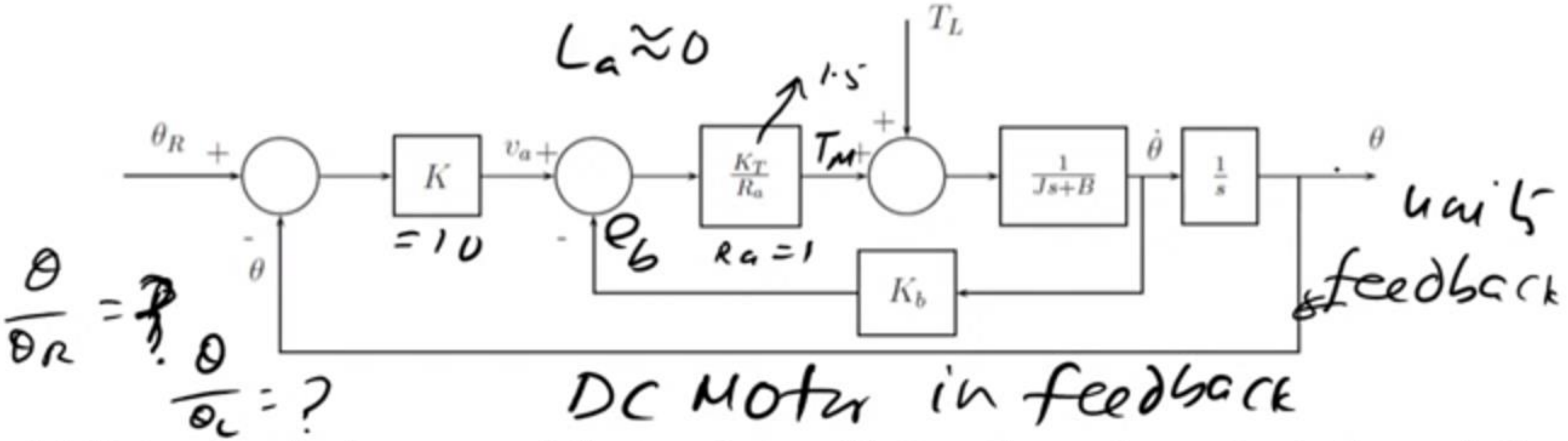
Dr Laxmidhar Behera

Professor, Department of EE, IIT Kanpur

Lecture 13 - PowerPoint (Product Activation Failed)

SLIDE 23 OF 28

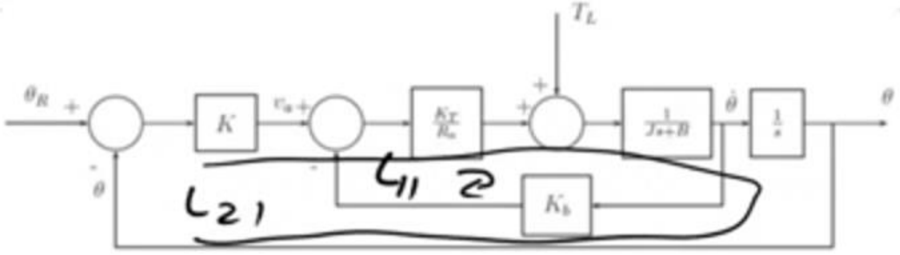
Example 5. Consider the following position control system where $K = 10$, $R_a = 1\Omega$, $K_T = 1.5N-m/amp$, $J = 5Kg-m^2$ and $B = 1.75N-ms/rad$.



(a) Determine the frequency of the transient oscillation, the peak overshoot, the peak time, settling time and the steady state error when the command signal θ_R is a unity step.

(b) Determine the steady state error when the command signal $\theta_R = t$, a ramp signal.

(c) Determine the steady state error when a step disturbance torque T_L is applied.



The forward gain between $\theta(s)$ and $\theta_R(s)$ is

$$M_1 = \frac{15}{s(5s+1.75)}$$

The forward gain between $\theta(s)$ and $T_L(s)$ is

$$M_2 = \frac{1}{s(5s+1.75)}$$

The loop gains are as follows :

$$L_{11} = \frac{-2.25}{(5s+1.75)}$$

$$L_{21} = \frac{-15}{s(5s+1.75)}$$

$$\frac{M_1 \times 1}{1 - L_{11} - L_{21}}$$

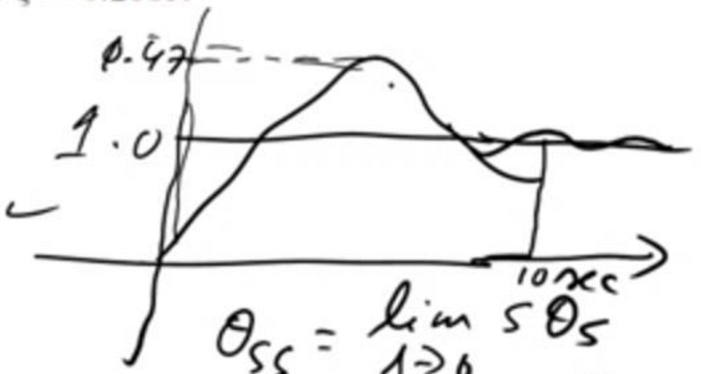
$$\frac{15/s(5s+1.75)}{1 + \frac{2.25}{5s+1.75} + \frac{15}{s(5s+1.75)}}$$

$$\frac{\theta(s)}{\theta_R(s)} = \frac{15}{(5s^2+4s+15)} = \frac{3}{s^2+0.8s+3}$$

$$\frac{\theta(s)}{T_L(s)} = \frac{1}{(5s^2+4s+15)}$$

$$\frac{\theta(s)}{\theta_R(s)} = \frac{3}{s^2 + 0.8s + 3}$$
$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where the parameter $\omega_n = \sqrt{3} = 1.732$ and $2\zeta\omega_n = 0.8$ which gives $\zeta = 0.2309$.



(a) The frequency of transient oscillation

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 1.6852$$

The peak time $t_p = \frac{\pi}{\omega_d} = 1.8645$, the peak overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.4744$$

$$\theta_{ss} = \lim_{s\to 0} s\theta_s = \lim_{s\to 0} s\frac{3}{s(s^2+0.8s+3)}$$

Final value Thm

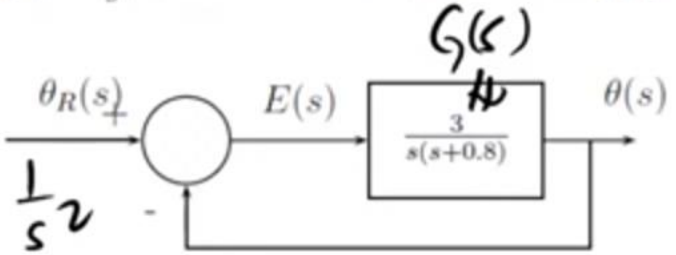
The steady state value of the output is given to be $\theta_{ss} = \lim_{s\to 0} s\frac{3}{s^2+0.8s+3} = 1$ and thus the steady state error is $e_{ss} = 1 - \theta_{ss} = 0$.

$$\textit{The settling time } t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.2309 * 1.732} = 10sec$$

Lecture 13 - PowerPoint (Product Activation Failed)

(b) $\theta_R(s) = \frac{1}{s^2}$. Please see that you can't apply the same procedure.

$\theta_R = \frac{1}{s^2}$



$e = \theta_R - \theta$

$$E = \theta_R - \frac{3}{s(s+0.8)}E$$

$$\text{or, } E(s) = \frac{1}{1+\frac{3}{s(s+0.8)}}\theta_R(s)$$

$$= \frac{1}{\left(1+\frac{3}{s(s+0.8)}\right)s^2} \times 1$$

$$= \frac{1}{s^2 + \frac{3s^2}{s(s+0.8)}}$$

where $\theta_R(s) = \frac{1}{s^2}$. Therefore, the steady state error

$$\frac{\theta_s}{\theta_R} = \frac{3}{s^2+0.8s+3} = \frac{G(s)}{1+G(s)}$$

$$E_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{1}{s+\frac{3}{s+0.8}} = \frac{0.8}{3}$$

$$= 0.266\ radian$$

$$sE(s) = \frac{1}{s+\frac{3}{s+0.8}}$$

SLIDE 27 OF 28 ENGLISH (INDIA) 70%

(c) The transfer function between $\theta(s)$ and $T_L(s)$ is given by

$$\boxed{T_L = \frac{1}{s}}$$

$$\frac{\theta(s)}{T_L(s)} = \frac{1}{5s^2 + 4s + 15}$$

For a unit step disturbance, the steady state value of output is given as

$$\theta_{ss} = \lim_{s \to 0} s \frac{1}{5s^2 + 4s + 15} \frac{1}{s} = \frac{1}{15} = \underline{0.067}$$

$$\theta(s) = \frac{1}{s(5s^2 + 4s + 15)}$$

$$\theta_{ss} = \lim_{s \to 0} s\,\theta_s = \lim_{s \to 0} \frac{1}{5s^2 + 4s + 15} = \frac{1}{15} = 0.067$$