

ac Schematic

**High-Frequency Equivalent** 

- ► Identify  $C_{\mu}$  as the input-output coupling element
- After application of the technique, this coupling element will be removed by splitting it into two parts one at input, other at output

- These *two parts* can be found by *evaluating* the *impedances* looking into the *planes* AA' and BB'
- > KCL at output node:

$$g_{\rm m}v + v_0/R_{\rm L} + sC_{\mu}(v_0 - v) = 0$$

 $\triangleright$  Noting that  $v = v_s$ , the *voltage gain*:

$$A_{v}(s) = v_{0}/v_{s} = -g_{m}R_{L}(1 - sC_{\mu}/g_{m})/(1 + sR_{L}C_{\mu})$$

⇒ Midband or low-frequency gain:

$$A_{v}(0) = -g_{m}R_{L}$$

This result can also be written from inspection

> Current entering plane AA':

$$i_1 = sC_{\mu}(v - v_0) = sC_{\mu}[1 - A_{\nu}(s)]v$$

➤ Hence, the *admittance* looking into the *plane* AA':

$$y|_{AA'} = i_1/v = sC_{\mu}[1-A_{\nu}(s)]$$

This *admittance* is *capacitive* in nature, and is known as the *Miller Capacitance*  $C_M$ :

$$C_{\mathrm{M}} = C_{\mu}[1 - A_{\mathrm{v}}(\mathrm{s})]$$

Now, since  $A_{\nu}(s)$  is a function of frequency, so would  $C_M \Rightarrow Problem!$ 

- ➤ Here, we invoke the *Miller Effect Approximation* (MEA)
  - $A_{\nu}(s)$  is replaced by  $A_{\nu}(0)$ , i.e., by its midband value, which is a constant
  - Thus,  $C_M$  becomes a constant with a value of  $C_M = [1 A_v(0)]C_{\mu} = (1 + g_m R_L)C_{\mu}$
- $\gt$  Thus,  $C_M >> C_\mu$ , since, in general,  $g_m R_L >> 1$
- This effect is known as the *Miller Effect Multiplication*
- $\succ$  Care: The gain that multiplies  $C_{\mu}$  is across its two ends

> Similarly, *current entering plane BB* ':

$$i_2 = sC_{\mu}(v_0 - v) = sC_{\mu}[1 - 1/A_{\nu}(s)]v_0$$

➤ Hence, the *admittance* looking into the *plane* BB':

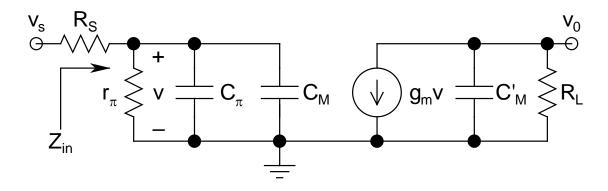
$$y'|_{BB'} = i_2/v_0 = sC_{\mu}[1-1/A_{\nu}(s)]$$

Again *replacing*  $A_v(s)$  by  $A_v(0)$ , we get:

$$C'_{M} = [1-1/A_{v}(0)]C_{\mu} = [1+1/(g_{m}R_{L})]C_{\mu}$$

$$ightharpoonup$$
In general,  $g_m R_L >> 1 \implies C'_M \simeq C_\mu$ 

- $ightharpoonup C_{\mu}$  can now be *removed* as the *coupling* element, split into 2 parts  $C_M$  and  $C_M'$ , with  $C_M$  appearing in the *input circuit* and  $C_M'$  appearing in the *output circuit*
- Now, include  $R_S$  note that the circuit is completely decoupled now



## Complete Circuit Including R<sub>s</sub>

$$\begin{split} & > Z_{in} = r_{\pi} || [1/(sC_{T}) = r_{\pi}/(1 + sr_{\pi}C_{T}) \\ & C_{T} = C_{\pi} + C_{M} \\ & \Rightarrow v = \frac{Z_{in}}{Z_{in} + R_{S}} v_{s} \\ & = \frac{r_{\pi}}{\left(R_{S} + r_{\pi}\right) \left[1 + sr_{\pi}R_{S}C_{T}/\left(R_{S} + r_{\pi}\right)\right]} v_{s} \\ & v_{0} = -g_{m} \left(R_{L} || \frac{1}{sC_{M}'}\right) v = -\frac{g_{m}R_{L}}{1 + sR_{L}C_{M}'} v \end{split}$$

> Thus:

$$\mathbf{A}_{\mathbf{v}}\left(\mathbf{s}\right) = \frac{\mathbf{v}_{0}}{\mathbf{v}_{\mathbf{s}}}$$

$$= -g_{m}R_{L}\frac{r_{\pi}}{R_{S} + r_{\pi}}\frac{1}{\left[1 + sR_{S}r_{\pi}C_{T}/(R_{S} + r_{\pi})\right](1 + sR_{L}C_{M}')}$$

> Comparing this expression with

$$A_{v}(s) = \frac{A_{v0}}{(1-s/p_{1})(1-s/p_{2})}$$

we note that the *denominator* is already in a *factorized form* 

- >  $A_{v0} = midband gain = -g_m R_L r_{\pi}/(R_S + r_{\pi})$
- The *transfer function* shows that the system has *two negative real poles* and *no zero* 
  - ⇒ Information regarding the zero is suppressed by this technique
- Also, the *two poles* obtained by *this technique* are *not identical* to those obtained from the *exact analysis*