
List of Common Continuous Distributions

beta

$$\alpha > 0$$

$$\beta > 0$$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha+\beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

$$m(t) = 1 + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{i-1} \frac{\alpha+j}{\alpha+\beta+j} \right) \frac{t^i}{i!}, \quad -\infty < t < \infty$$

Cauchy

$$f(x) = \frac{1}{\pi} \frac{1}{x^2+1}, \quad -\infty < x < \infty$$

Neither the mean nor the variance exists.

The mgf does not exist.

Chi-squared, $\chi^2(r)$

$$r > 0$$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{(r/2)-1} e^{-x/2}, \quad x > 0$$

$$\mu = r, \quad \sigma^2 = 2r$$

$$m(t) = (1-2t)^{-r/2}, \quad t < \frac{1}{2}$$

$$\chi^2(r) \Leftrightarrow \Gamma(r/2, 2)$$

r is called the degrees of freedom.

Exponential

$$\lambda > 0$$

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$

$$\mu = \frac{1}{\lambda}, \quad \sigma^2 = \frac{1}{\lambda^2}$$

$$m(t) = [1 - (t/\lambda)]^{-1}, \quad t < \lambda$$

$$\text{Exponential}(\lambda) \Leftrightarrow \Gamma(1, 1/\lambda)$$

 $F, F(r_1, r_2)$

$$r_1 > 0$$

$$r_2 > 0 > 0$$

$$f(x) = \frac{\Gamma[(r_1+r_2)/2] (r_1/r_2)^{r_1/2}}{\Gamma(r_1/2)\Gamma(r_2/2)} \frac{(x)^{r_1/2-1}}{(1+r_1x/r_2)^{(r_1+r_2)/2}}, \quad x > 0$$

$$\text{If } r_2 > 2, \mu = \frac{r_2}{r_2-2}. \quad \text{If } r_2 > 4, \sigma^2 = 2 \left(\frac{r_2}{r_2-2} \right)^2 \frac{r_1+r_2-2}{r_1(r_2-4)}.$$

The mgf does not exist.

r_1 is called the numerator degrees of freedom.

r_2 is called the denominator degrees of freedom.

Gamma, $\Gamma(\alpha, \beta)$

$$\alpha > 0$$

$$\beta > 0$$

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad x > 0$$

$$\mu = \alpha\beta, \quad \sigma^2 = \alpha\beta^2$$

$$m(t) = (1 - \beta t)^{-\alpha}, \quad t < \frac{1}{\beta}$$

Continuous Distributions, Continued

Laplace (2.2.1)
 $-\infty < \theta < \infty$
 $f(x) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty$
 $\mu = \theta, \quad \sigma^2 = 2$
 $m(t) = e^{t\theta} \frac{1}{1-t^2}, \quad -1 < t < 1$

Logistic (6.1.8)
 $-\infty < \theta < \infty$
 $f(x) = \frac{\exp\{-(x-\theta)\}}{(1+\exp\{-(x-\theta)\})^2}, \quad -\infty < x < \infty$
 $\mu = \theta, \quad \sigma^2 = \frac{\pi^2}{3}$
 $m(t) = e^{t\theta} \Gamma(1-t) \Gamma(1+t), \quad -1 < t < 1$

Normal, $N(\mu, \sigma^2)$ (3.4.6)
 $-\infty < \mu < \infty$
 $\sigma > 0$
 $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad -\infty < x < \infty$
 $\mu = \mu, \quad \sigma^2 = \sigma^2$
 $m(t) = \exp\{\mu t + (1/2)\sigma^2 t^2\}, \quad -\infty < t < \infty$

$t, t(r)$ (3.6.1)
 $r > 0$
 $f(x) = \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2)} \frac{1}{(1+x^2/r)^{(r+1)/2}}, \quad -\infty < x < \infty$
 If $r > 1, \mu = 0$. If $r > 2, \sigma^2 = \frac{r}{r-2}$.
 The mgf does not exist.
 The parameter r is called the degrees of freedom.

Uniform (1.7.4)
 $-\infty < a < b < \infty$
 $f(x) = \frac{1}{b-a}, \quad a < x < b$
 $\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$
 $m(t) = \frac{e^{bt} - e^{at}}{(b-a)t}, \quad -\infty < t < \infty$
