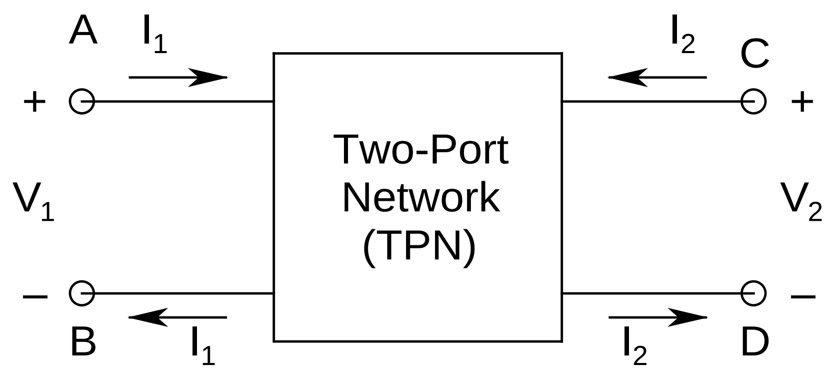
Two-Port Network (TPN)

* ***Four Variables***:

- Input voltage V_1
- Input current I_1
- Output voltage V_2
- Output current I_2



* The network within the box is known as the ***TPN***

* It must be a ***linear*** network, i.e., it should contain only linear elements, so that ***superposition*** principle can be applied

* There should be ***no connection*** between A/B and C/D

* Current ***entering*** the TPN through terminal A must ***equal*** the current ***leaving*** through terminal B
* Similarly, current ***entering*** through terminal C must ***equal*** the current ***leaving*** through terminal D
* The TPN must ***not*** contain any ***independent source***
* It may contain ***dependent sources***, however, the ***controlling parameter*** of the dependent source must be ***within*** the TPN
* The TPN is treated as a ***black box*** with the external current-voltage relations expressed by a set of TPN ***parameters***

* There are various types of ***representations*** of ***TPN parameters***

* We will discuss the ***three*** most important ones:
  - ***Admittance Parameter*** (***y-Parameter***)
  - ***Impedance Parameter*** (***z-Parameter)***
  - ***Hybrid Parameter*** (***h-Parameter***)

* Typically expressed in a ***matrix form***:

$$[x] = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}, \text{ where x is y, z, or h}$$

y-Parameters:

* I-V relations expressed as:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$V_1,\ V_2$: ***Independent*** variables

$I_1,\ I_2$: ***Dependent*** Variables

$$y_{11} = \textbf{\textit{short-circuit input admittance}} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$y_{12} = \textbf{\textit{short-circuit (output to input) transfer admittance}} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

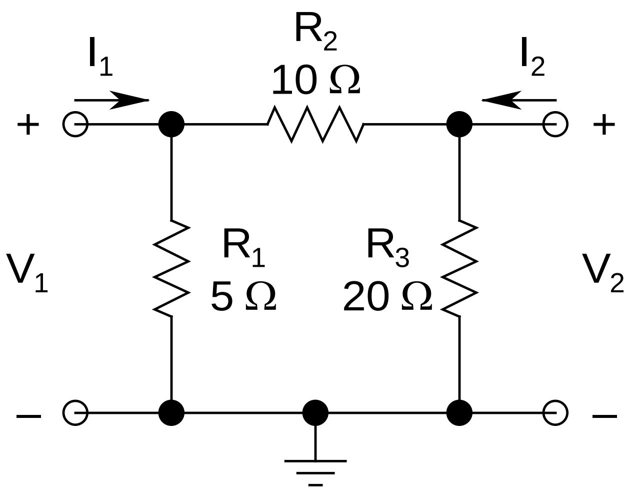
$$y_{21} = \textbf{\textit{short-circuit (input to output) transfer}}$$

$$\textbf{\textit{admittance}} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{22} = \textbf{\textit{short-circuit output admittance}} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

* ***Note***: $y_{12} = y_{21}$ is not a ***necessary*** condition
* Networks with $y_{12} = y_{21}$ are known as ***bilateral***
* Generally ***purely resistive*** networks are ***bilateral***
* Networks with $y_{12} \neq y_{21}$ are known as ***unilateral***
* Generally ***unilateral*** networks have ***active elements*** and/or ***dependent sources*** within the network

Example: To find ***y-parameters*** for the resistive network (known as the ***π-network***)



$$y_1 = 1/R_1 = 0.2\ \mho$$

$$y_2 = 1/R_2 = 0.1\ \mho$$

$$y_3 = 1/R_3 = 0.05\ \mho$$

$$I_1 = y_1 V_1 + y_2 \left(V_1 - V_2\right)$$

$$= \left(y_1 + y_2\right) V_1 - y_2 V_2$$

$$I_2 = y_2 \left(V_2 - V_1\right) + y_3 V_2 = -y_2 V_1 + \left(y_2 + y_3\right) V_2$$

Thus, $$\left[y\right] = \begin{bmatrix} y_1 + y_2 & -y_2 \\ -y_2 & y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.15 \end{bmatrix} \mho$$

z-Parameters:

* I-V relations expressed as:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$I_1,\ I_2$: ***Independent*** Variables

$V_1,\ V_2$: ***Dependent*** Variables

$$z_{11} = \textit{open-circuit input impedance} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{12} = \textit{open-circuit (output to input) transfer impedance} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$z_{21} =$ ***open-circuit (input to output) transfer impedance*** $= \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$

$z_{22} =$ ***open-circuit output impedance*** $= \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$

* Again, only for a ***bilateral*** network, $z_{12} = z_{21}$, otherwise not

* Also, ***caution*** that for a given network $z_{ij} \neq 1/y_{ij}$

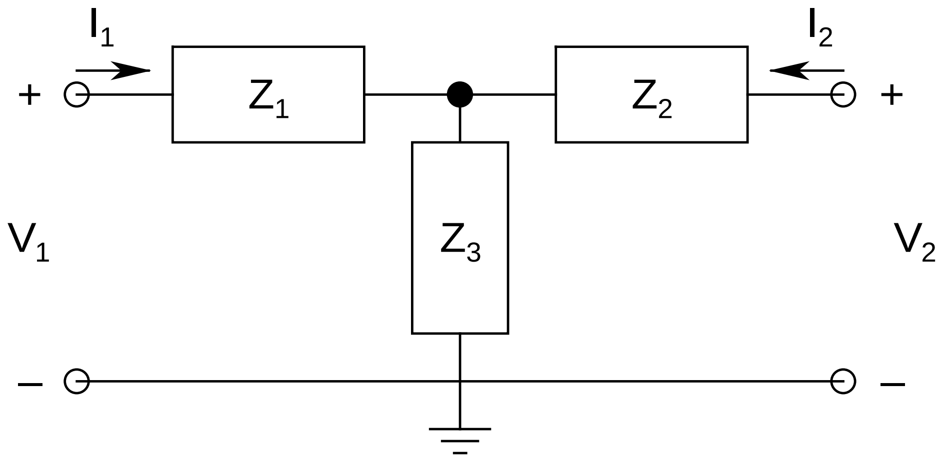
Example: To find ***z-parameters*** for the network (known as the ***T-network***)

$$V_1 = Z_1 I_1 + Z_3 \left(I_1 + I_2 \right)$$

$$= \left(Z_1 + Z_3 \right) I_1 + Z_3 I_2$$

$$V_2 = Z_2 I_2 + Z_3 \left(I_1 + I_2 \right)$$

$$= Z_3 I_1 + \left(Z_2 + Z_3 \right) I_2$$



Thus, $\left[z \right] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$

All T-networks are bilateral

Transformation Relations Between y- and z-Parameters:

* $y_{11} = \dfrac{z_{22}}{\Delta z},\ y_{12} = -\dfrac{z_{12}}{\Delta z},\ y_{21} = -\dfrac{z_{21}}{\Delta z},\ y_{22} = \dfrac{z_{11}}{\Delta z}$

  with $\Delta z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}$

* $z_{11} = \dfrac{y_{22}}{\Delta y},\ z_{12} = -\dfrac{y_{12}}{\Delta y},\ z_{21} = -\dfrac{y_{21}}{\Delta y},\ z_{22} = \dfrac{y_{11}}{\Delta y}$

  with $\Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$

* ***Note***: While ***y-parameter*** representation is preferred for ***π-networks***, ***z-parameter*** representation is the choice for ***T-networks***

h-Parameters:

* I-V relations expressed as:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$I_1,\ V_2$: ***Independent*** Variables

$V_1,\ I_2$: ***Dependent*** Variables

$$h_{11} = \textbf{\textit{short-circuit input impedance}} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

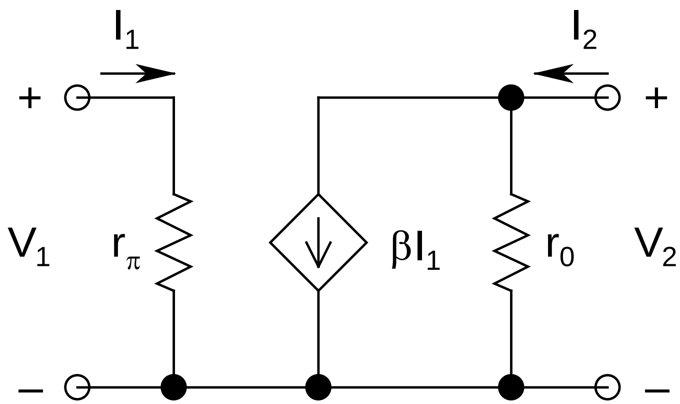
$$h_{12} = \textbf{\textit{open-circuit reverse voltage gain}} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \textbf{\textit{short-circuit forward current gain}} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{22} = \textbf{\textit{open-circuit output admittance}} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

* These are known as ***hybrid*** parameters, since they involve both ***voltages and currents***, with h_{12} and h_{21} being ***dimensionless*** quantities, while h_{11} and h_{12} have units of $\boldsymbol{\Omega}$ and $\boldsymbol{\mho}$ respectively
* In contrast, note that all ***y-parameters*** are expressed in $\boldsymbol{\mho}$, while all ***z-parameters*** are expressed in $\boldsymbol{\Omega}$

Example: To find ***h-parameters*** for the ***transistor*** (***BJT***) equivalent circuit (***low-frequency hybrid-π***):



r_π: ***Input Resistance***

r_0: ***Output Resistance***

β: ***Current Gain***

$$V_1 = r_\pi I_1$$

$$I_2 = \beta I_1 + g_0 V_2$$

$g_0 = 1/r_0$ = ***Output Conductance***

$$\text{Thus, } [h] = \begin{bmatrix} r_\pi & 0 \\ \beta & g_0 \end{bmatrix}$$