QUIZ - 1

Descriptive Questions

Problem: using the function

$$f(x) = \begin{cases} 1 & \text{on } (-\pi/2, \pi/2) \\ 0 & \text{on } [-\pi, \pi] \setminus (-\pi/2, \pi/2) \end{cases}$$

to find the sum of the series

$$\sum_{n=1}^{\infty} \frac{\sin^2(n\frac{\pi}{2})}{n^2}$$

Sol (1 marks) For finding correct value of "a_0".

(NOTE THEY HAVE TO DO THE CALCULATION OF GIVE SOME JUSTIFICATION, IF THEY DIRECTLY WRITE DOWN THE VALUE OF a_0, a_n, b_n, THEN '0')

$$a_0 = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \, dx = 1/2$$

$$\longrightarrow \boxed{a_0 = 1/2}$$

1 MARKS FOR COMPUTING 'a_n'

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos(nx)\,dx$$

$$= \frac{1}{n\pi}\,\sin nx\,\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{2}{n\pi}\sin\left(n\pi/2\right)$$

$$\Rightarrow \boxed{a_n = \frac{2}{n\pi}\sin\frac{n\pi}{2}}$$

1 MARKS FOR CALCULATING b_n

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \sin nx = 0.$$

$$\boxed{b_n = 0}$$

If they say $b_n = 0$ as the integrand is odd function. Give marks.

1 marks for applying Parsevals identity

For writing the step.

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx = 2a_0^2 + \sum_{i=1}^{\infty}(a_n^2+b_n^2) \longrightarrow (*)$$

(Even if it clear that the student is trig to apply 'Parseval' identity give marks.

1 marks for finding the answer

From (*)

$$\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1\,dx = 1/2 + \sum_{1}^{\infty}\frac{4}{n^2\pi^2}\sin^2\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow 1/2 = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin^2(n\pi/2)}{n^2}$$

$$\Rightarrow \pi^2/8 = \sum_{1}^{\infty}\frac{\sin^2(n\pi/2)}{n^2}$$

for writing $\boxed{\pi^2/8}$.

If they do not write 11/8 then -0!