1.a. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from $N(0, 5\tau^2)$ with $\tau > 0$. Find the asymptotic distribution of $T_n := \frac{1}{n} \sum_{i=1}^{n} X_i^2$. Further, find a function g and constant $\beta > 0$ (independent of τ) such that

$$\sqrt{n}[g(T_n) - g(k\tau^2)] \xrightarrow{D} N(0, \beta^2) \text{ as } n \to \infty.$$

[1+2]

1.b. A random sample $X_1, X_2, \ldots, X_n$ is obtained from the probability density function:

$$f(x|\theta) = \begin{cases} \dfrac{x^{19}}{\theta^2} \exp\left(-\dfrac{x^{20}}{20\theta^2}\right) & x > 0, \\ 0 & \text{else.} \end{cases}$$

Find the maximum likelihood estimate and the method of moments estimate of θ. [2+1]