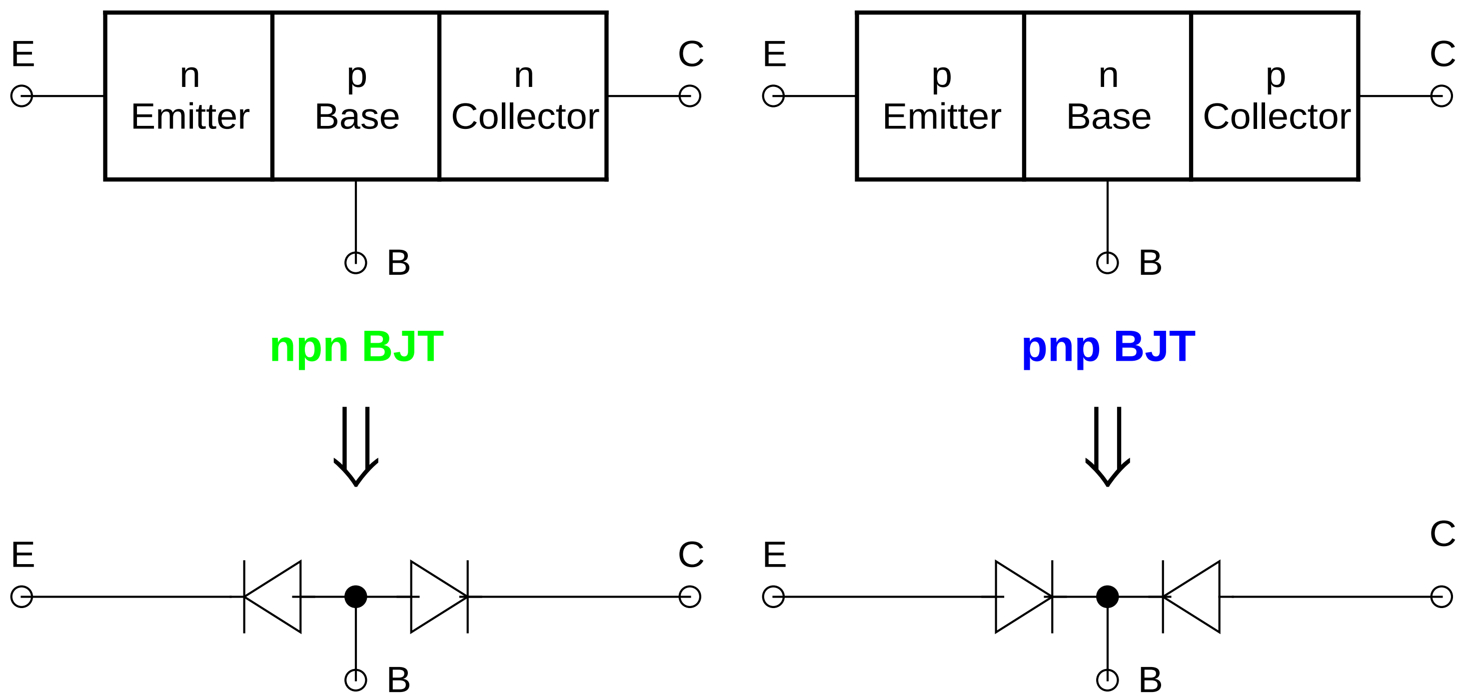
BIPOLAR JUNCTION TRANSISTOR (BJT)

- Basically two ***back-to-back diodes***

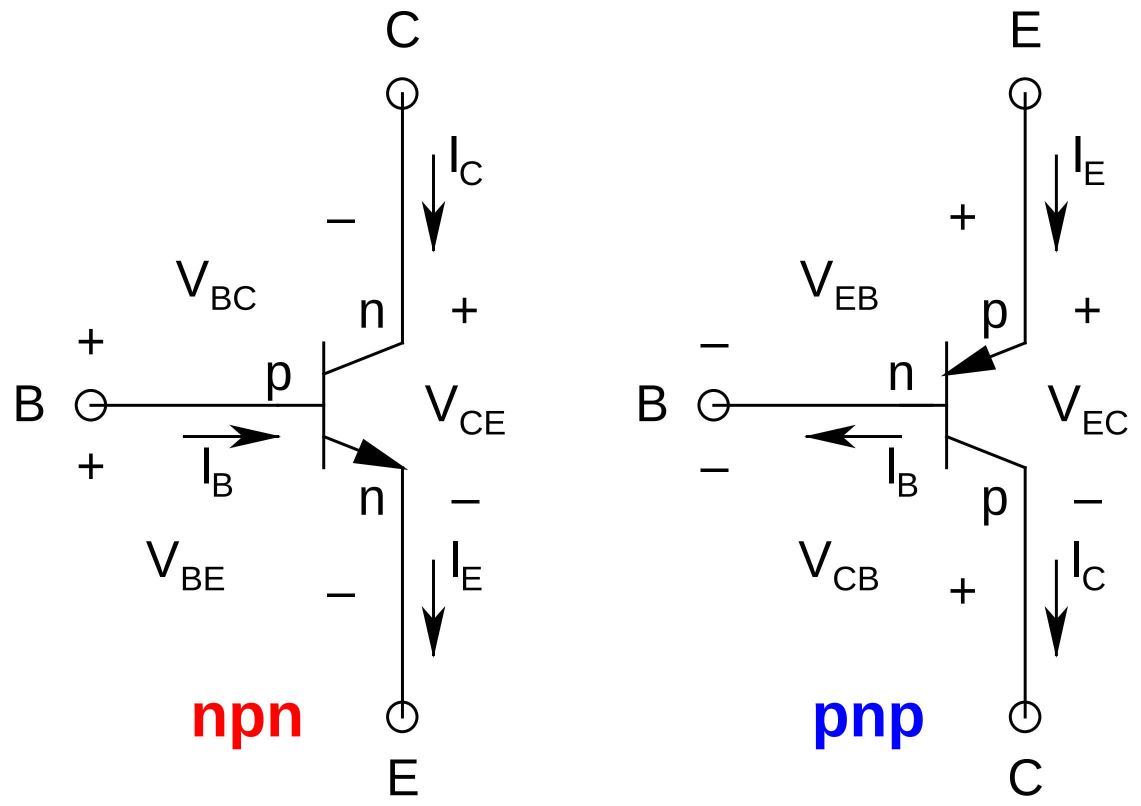


- Name originates from ***transfer of resistor***

- ***Three-Layer***/***Terminal*** [***Emitter*** (E), ***Base*** (B), ***Collector*** (C)] ***Two-Junction*** [***Base-Emitter*** (BE), ***Base-Collector*** (BC)] device
- Current through ***two terminals*** (E and C) can be ***controlled*** by the current through the ***third terminal*** (B)
  - ***Current controlled device***
- ***Bipolar device***
  - ***Both electrons and holes*** participate in ***current conduction***

- ***Active device***
  - Capable of producing ***voltage***/***current***/***power gain***
- ***Two basic usage***:
  - **Amplification** (***Analog Circuits***)
  - **Switching** (***Digital Circuits***)
- ***Two Types:***
  - ***npn***
  - ***pnp***
- ***Immensely important device***

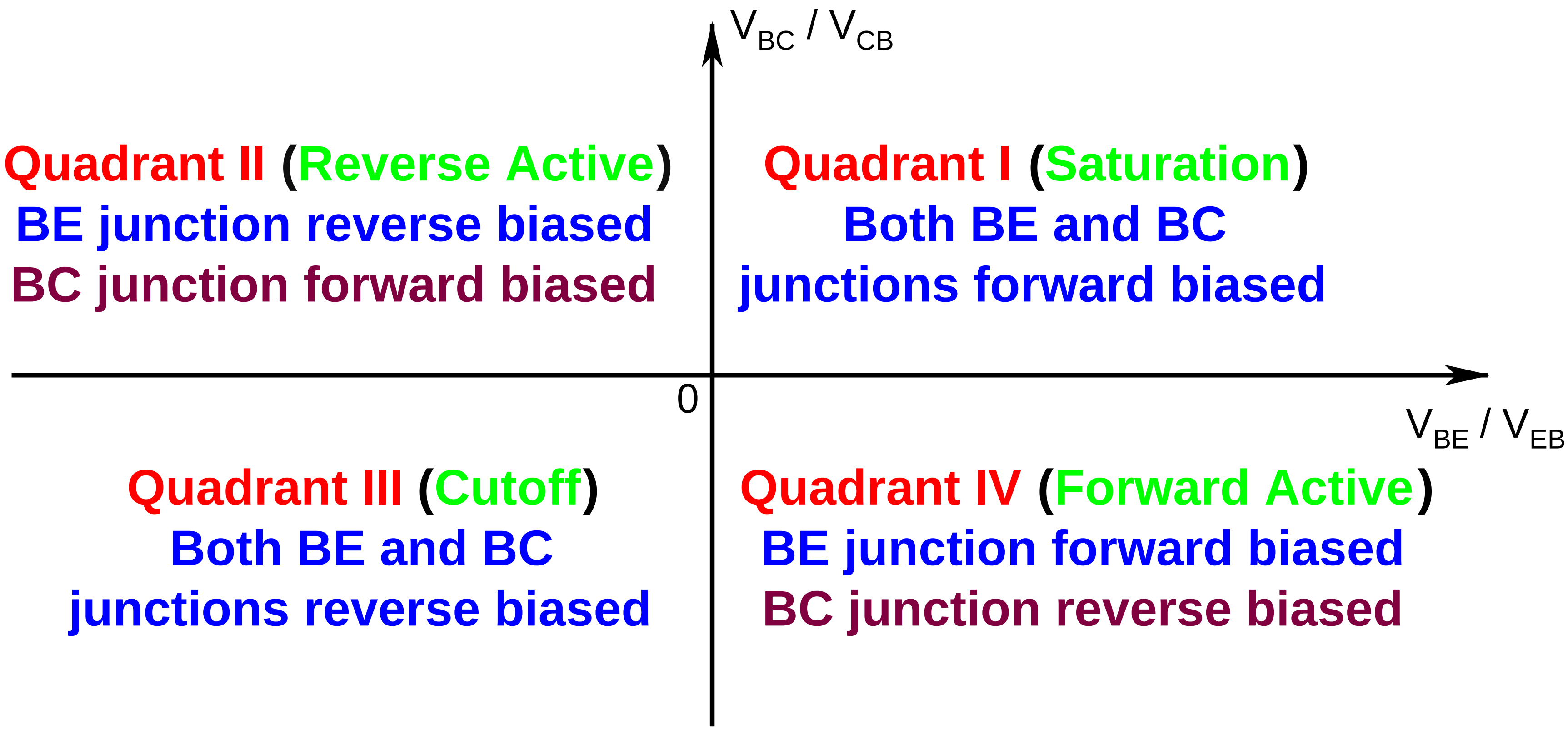
Symbols and Current-Voltage Conventions



- ***Voltage Convention***:
  - ***p-side first***:
    - ***Sign of the voltage*** immediately lets us know the ***biasing state*** (***forward or reverse***)
  - ***npn***: V_{BE} (***base-emitter voltage***), V_{BC} (***base-collector voltage***), and V_{CE} (***collector-emitter voltage***)
  - ***pnp***: V_{EB} (***emitter-base voltage***), V_{CB} (***collector-base voltage***), and V_{EC} (***emitter-collector voltage***)
  - **Note**: ***Collector-Emitter is NOT a junction***

- ***Current Convention***:
  - ***npn***:
    - ***I_C and I_B flow in***, ***I_E flows out***
  - ***pnp***:
    - ***I_E flows in***, ***I_C and I_B flow out***
- ***Applying KCL***, treating the whole BJT as a ***big node***, for ***both npn and pnp***:
  - $I_E = I_C + I_B$
  - ***Extremely important KCL for the BJT***

Modes of Operation

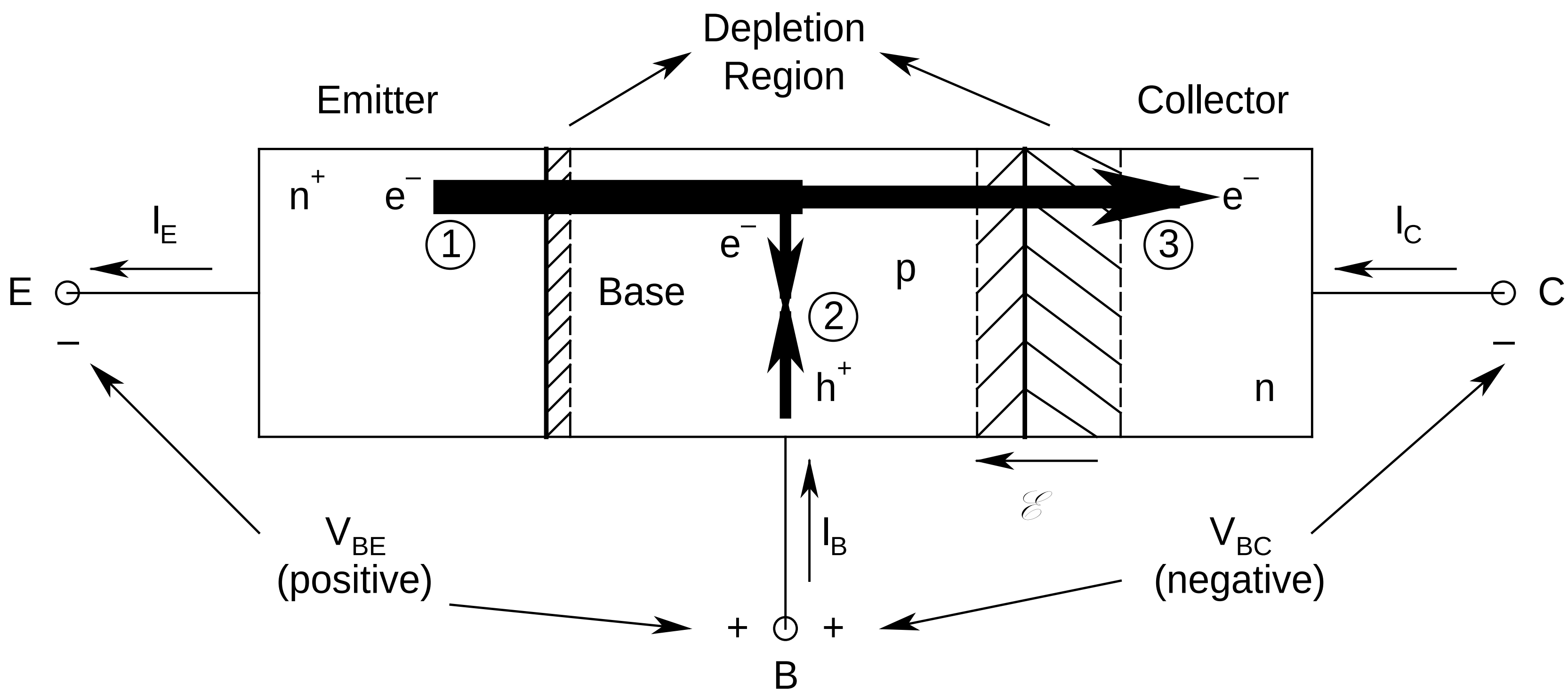


Quadrants III and IV: Analog Domain

Quadrants I and III: Digital Domain

Quadrant II: Finds use only in TTL circuits

Operation in the Forward Active (FA) Mode



1 Injection Component, 2 Recombination Component, 3 Collection Component

- *BE junction forward biased*, *BC junction reverse biased*
- *Emitter injects electrons to base*
  - *Supplied by the external terminal to maintain charge neutrality in emitter*
    - *Emitter current (I_E) flows out of the emitter terminal*
- *Base injects holes to emitter*
  - *This component is reduced as much as possible by doping emitter very heavily*

- Injected electrons ***diffuse*** through the base due to ***concentration gradient***
  - At the same time, some of them ***recombine*** with the ***holes*** in the ***base***
  - Supplied by the ***external terminal*** to maintain ***charge neutrality*** in ***base***
    - ***Base current*** (I_B) ***flows into the base terminal***
- Electrons that ***survived recombination*** will reach the ***base edge*** of the ***BC depletion region***

- Note the ***direction*** of the ***electric field*** (E) present in the ***BC depletion region***
- This ***field*** will ***sweep*** the ***survived electrons*** to the ***collector***
  - ➢ These ***electrons*** will ***flow out*** of the ***collector terminal***
    - ***Collector current*** (I_C) ***flows into the collector terminal***
- ***Base Control***:
  - ➢ A ***small change*** in I_B can cause a ***large change*** in $I_C \Rightarrow$ ***Transistor action***

- For a ***good transistor***, the ***ratio*** I_C/I_B should be ***as large as possible***
- Can be ***achieved*** by ***reducing*** the ***chances of recombination*** in the ***base***
- ***Two ways***:
  - ***Reduce base doping*** $\Rightarrow$ ***Limits supply of holes*** $\Rightarrow$ ***Reduces recombination***
  - ***Reduce base width*** $\Rightarrow$ ***Reduces amount of time electrons spend in base*** $\Rightarrow$ ***Reduces recombination***

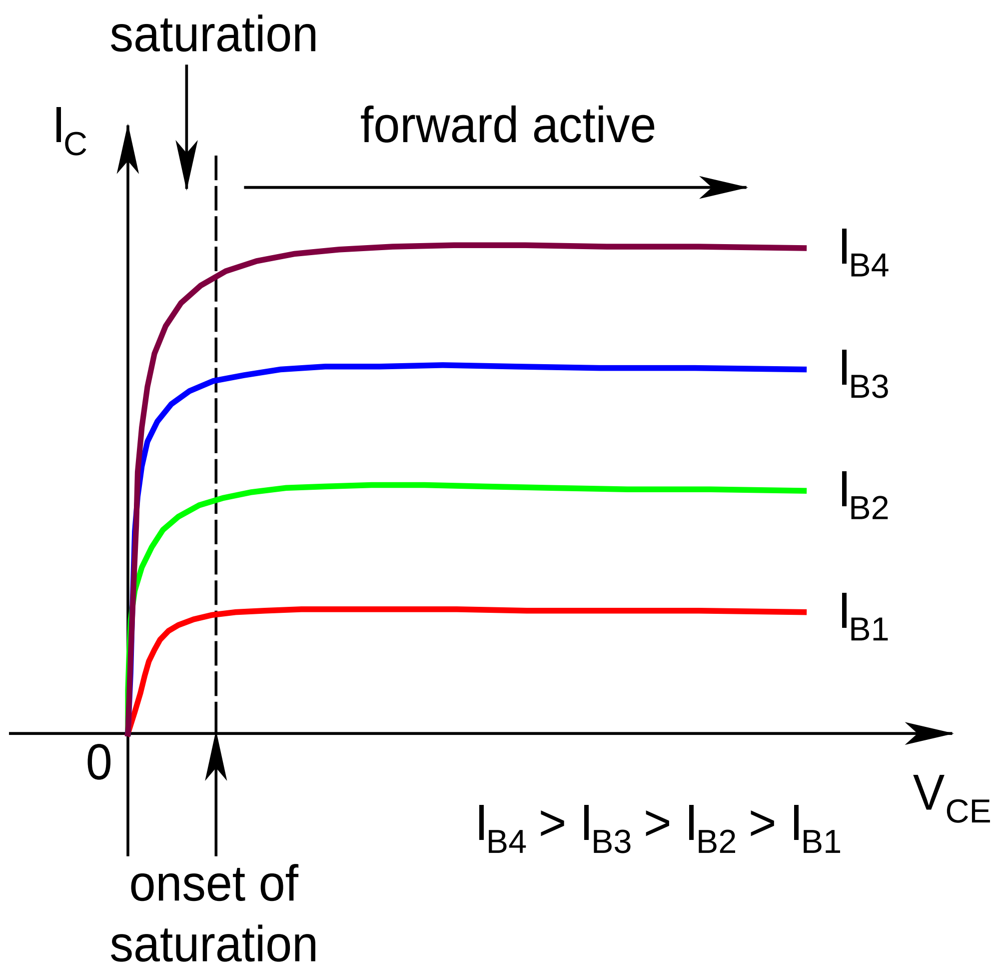
Current Gain

- ***Common-Emitter (CE) Current Gain***:
  - $\beta = I_C/I_B$ (***Higher the better***!)
- ***Common-Base (CB) Current Gain***:
  - $\alpha = I_C/I_E$ (≤ 1: ***closer to 1, better it is***!)
- Also, $I_E = I_C + I_B$
  - $\alpha = \beta/(\beta + 1)$ and $\beta = \alpha/(1 - \alpha)$
- ***Note***: As $\alpha \to 1$, $\beta \to \infty$
- ***Typical values***: $\beta \sim 100$-5000, $\alpha \sim 0.99$-0.9998

Current-Voltage Relation

- ***BE junction*** basically a ***diode***
  - $I_E = I_{ES}\exp(V_{BE}/V_T)$ $(V_{BE} > 4V_T)$
    - I_{ES}: ***Reverse Saturation Current of BE junction***
- A ***fraction*** α of I_E reaches ***collector***
  - $I_C = \alpha I_E = I_S\exp(V_{BE}/V_T)$
    - I_S $(= \alpha I_{ES})$: ***Saturation current of the BJT***
- The ***difference*** between I_E and I_C is I_B
  - $I_B = I_E - I_C$

Output Characteristic



- ***Quick Estimate***:
  - Under ***forward bias***, $V_{BE} \sim 0.7$ V
  - ***Justification***:
    - $V_\gamma \sim 0.6$ V
    - 0.7 V is 100 mV above $V_\gamma \Rightarrow$ ***junction sufficiently forward biased***
    - I_C-V_{BE} relation ***exponential*** $\Rightarrow$ A ***little change*** in V_{BE} can cause a ***large change*** in I_C
  - ***Heuristic estimate***: ***not accurate***, however, ***extremely useful*** for a ***quick hand-calculation***
- $V_{CE} = V_{BE} - V_{BC}$ (***applying chain rule***)

- Thus, for $V_{CE} > 0.7$ V, V_{BC} ***negative***
  - BC junction ***reverse biased*** and FA operation is ***maintained***
- As $V_{CE} \rightarrow 0.7$ V, $V_{BC} \rightarrow 0$
  - BC junction ***losing its reverse bias***
- At $V_{CE} = 0.7$ V, $V_{BC} = 0$
  - BC junction ***under zero bias***
- For $V_{CE} < 0.7$ V, V_{BC} turns ***positive***
  - Both BE and BC junctions become ***forward biased*** $\Rightarrow$ ***saturation***

- V_{CE} = 0.7 V is known as ***onset of saturation*** (OS)
- ***Saturation***:
  - For V_{CE} < 0.7 V
  - CB junction becomes ***forward biased***
  - Collector also starts to ***inject*** electrons to base
  - ***Two effects***:
    - ***Net electrons reaching collector***$\downarrow \Rightarrow I_C\downarrow$
    - ***Base gets flooded with electrons***

      $\Rightarrow$ ***Recombination increases manyfold*** $\Rightarrow I_B\uparrow$
    - Thus, $\beta\downarrow \Rightarrow$ Defined as β_{sat} (= $I_{C,sat}/I_{B,sat}$)

- Noting that $V_\gamma = 0.6$ V, for $V_{BC} \le 0.5$ V, ***injection*** of electrons from ***collector to base*** will be ***negligible***
  - It can be ***assumed*** that ***FA operation*** is ***maintained*** till this point, with β ***retaining*** its ***nominal (FA) value***
  - $V_{CE} = 0.2$ V at this point, and is known as the ***point of soft saturation*** (SS)
- Beyond this point, BJT enters the ***operating domain*** known as ***hard saturation*** (HS)

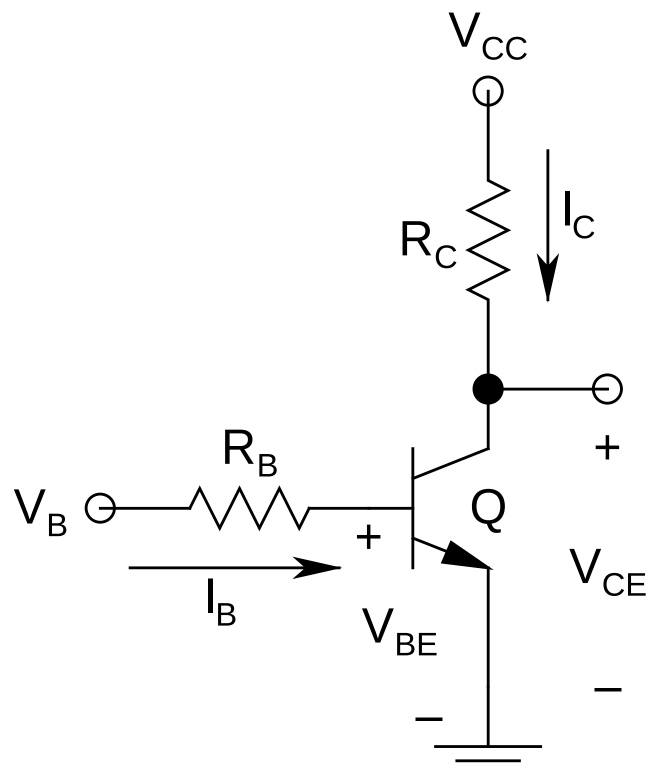
- In ***hard saturation***, $V_{BC} \approx 0.7$ V, and collector ***injects*** electrons ***vigorously*** into the base
- To ***counter*** this effect, V_{BE} automatically ***increases*** to about 0.8 V
- At this point, $V_{CE} = 0.1$ V, and is known as the ***point of hard saturation*** (HS)
- Note that all these numbers are for ***quick estimates***, and ***actual values*** can be a ***little different*** from these

- ***Degree of Saturation*** (DoS):
  - DoS = β/β_{sat} (≥ 1)
  - Portrays how ***deeply*** the BJT is driven into ***saturation***
- ***Commonly used values*** of ***parameters*** for ***quick estimate***:
  - V_{BE}(FA) = V_{BE}(SS) = 0.7 V
  - V_{BE}(HS) = 0.8 V
  - V_{CE}(OS) = 0.7 V, V_{CE}(SS) = 0.2 V
  - DoS(FA,OS,SS) = 1, DoS(HS) > 1

- BJTs in ***analog circuits*** are used as ***amplifiers***, and should ***never*** be pushed to ***hard saturation*** (***β drops significantly***)
  - ***Lowest limit*** of V_{CE} = 0.2 V (***soft saturation***)
- On the other hand, BJTs used in ***digital circuits***, while ***on***, are always pushed to ***hard saturation***, since they act basically as ***switches***
  - V_{CE} = 0.1 V (***hard saturation***)

Finding the Operating Point: Load Line Analysis

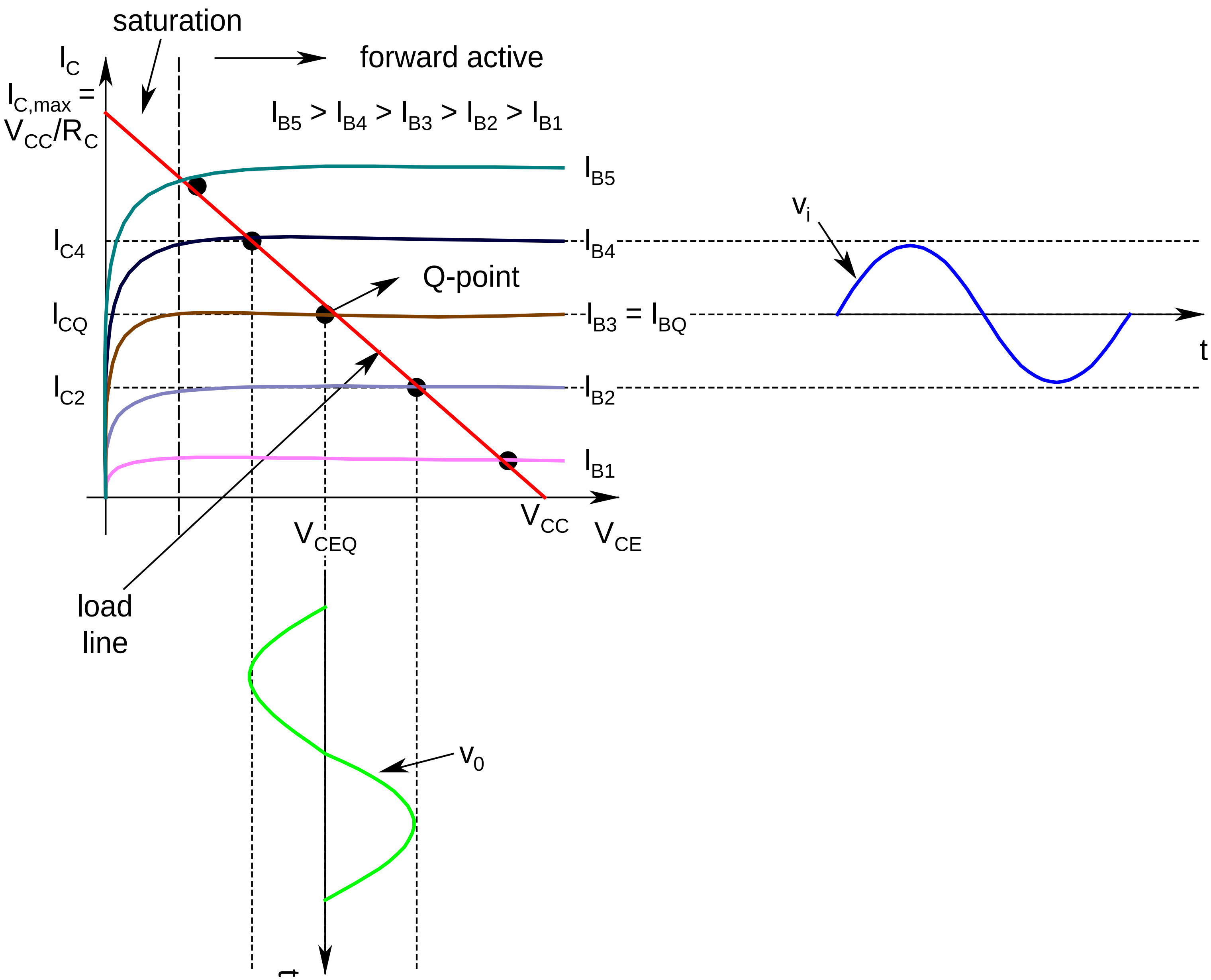
- ***Quick estimate*** in ***FA mode***:
  - $I_B = (V_B - V_{BE})/R_B$
  - $V_{BE} = 0.7\ V$
  - $I_C = \beta I_B$
  - ***Independent*** of R_C, so long as ***FA operation*** is ***maintained***



- For ***continuous variation*** of V_B, ***continuous variation*** of I_C and I_B
  - The ***output characteristics*** will ***fill up*** the ***entire quadrant***
- The ***operating point*** (***Q-point***) can ***lie anywhere*** in this ***quadrant***
- To find the ***unique*** Q-point, need to ***draw*** the ***load line***
- ***Load line equation***:
  - $I_C = (V_{CC} - V_{CE})/R_C$

- ***2 boundary points***:
  - For $I_C = 0$, $V_{CE} = V_{CC}$
  - For $V_{CE} = 0$, $I_C = V_{CC}/R_C$
- ***Joining*** these ***2 points*** by a ***straight line*** gives the ***load line***
- The ***intersection point*** of the ***load line*** with the ***output characteristic*** gives the ***Q-point***
- Gives ***infinite number*** of ***choices*** for ***possible Q-point***

- The ***best choice*** for the ***Q-point*** is ***right at the center*** of the ***load line***
  - $V_{CEQ}(\text{best}) = V_{CC}/2$ and $I_{CQ}(\text{best}) = V_{CC}/(2R_C)$
- ***Permits the maximum possible signal swing in both directions***
- If $V_{CEQ} > V_{CC}/2$, it's ***biased more towards cutoff***
- If $V_{CEQ} < V_{CC}/2$, it's ***biased more towards saturation***
- ***Either way***, we will get a ***distorted output***



saturation
forward active
I_C
$I_{C,max}$ =
V_{CC}/R_C
$I_{B5} > I_{B4} > I_{B3} > I_{B2} > I_{B1}$
I_{B5}
I_{C4}
I_{B4}
Q-point
I_{CQ}
$I_{B3} = I_{BQ}$
I_{C2}
I_{B2}
I_{B1}
V_{CC}
V_{CE}
V_{CEQ}
load
line
v_0
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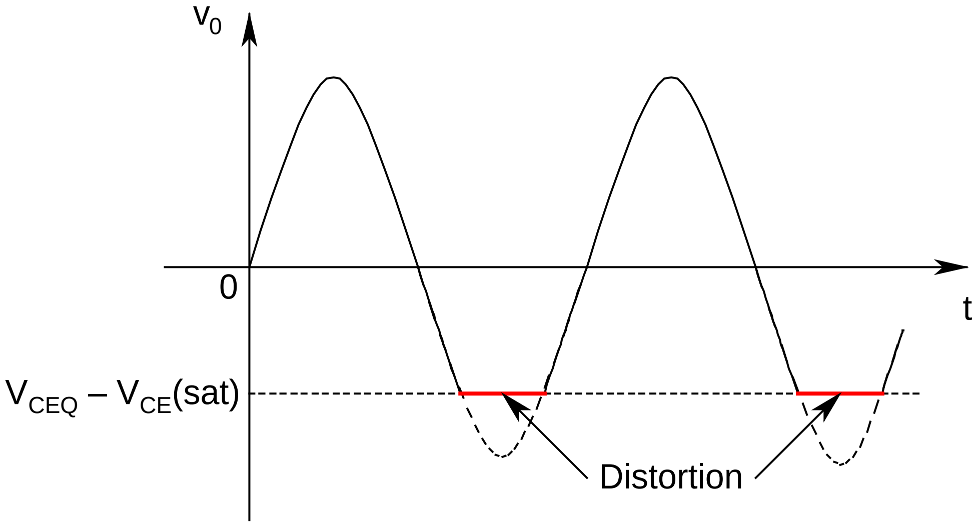
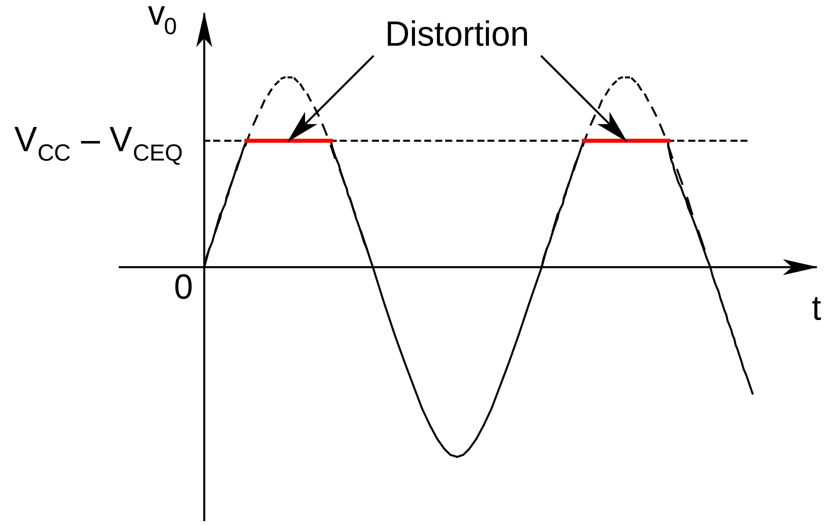


(a) **Negative Clipping** **:**
Saturation Induced



(b) **Positive Clipping** **:**
Cutoff Induced

- Under ***application*** of an ***ac signal*** (v_i), the ***dynamic operating point*** (DOP) will ***move along the load line***
- For ***positive*** v_i, the DOP ***will move*** Q ***towards saturation*** ($V_{CE} \rightarrow 0$, $I_C \rightarrow I_{C,max}$)
  - The ***output signal*** (v_0) will be in its ***negative excursion***
  - If Q enters ***saturation***, ***negative peak*** of v_0 will get ***clipped***
  - ***Distorted output***

- For ***negative*** v_i, the DOP ***will move*** Q ***towards cutoff*** ($V_{CE} \rightarrow V_{CC}$, $I_C \rightarrow 0$)
  - The ***output signal*** (v_0) will be in its ***positive excursion***
  - If Q ***cuts off***, ***positive peak*** of v_0 will get ***clipped***
  - ***Distorted output***
- ***Golden rule of thumb for BJT biasing***:
  - ***To get maximum undistorted peak-to-peak swing of v_0, Q-point must be chosen to be at the middle of the load line***

- ***Role of R_C***:
  - Under ***FA mode***, ***R_C does not control I_C***, however, it ***changes V_{CE}*** ($= V_{CC} - I_C R_C$)
  - If $R_C\uparrow$, $V_{CE}\downarrow \Rightarrow$ Q moves ***towards saturation***
  - If $R_C\downarrow$, $V_{CE}\uparrow \Rightarrow$ Q moves ***towards cutoff***
  - Thus, ***different values of R_C*** can produce ***different Q-points*** (***in terms of V_{CE}***)
- ***DC Power Dissipation***:
  - $$P_D = V_{BEQ} \times I_{BQ} + V_{CEQ} \times I_{CQ}$$
    $$\approx V_{CEQ} \times I_{CQ}$$ (***under FA mode***)

Some Observations

- Q should be ***biased*** such that it is in the ***FA*** mode of operation
  - Behaves like a ***constant and ideal current source with infinite output resistance***, since I_C is ***independent*** of V_{CE}
  - ***Ideal region to bias a BJT***
- For ***very high*** R_C, $I_{C,max}$ ***very small***
  - ***Load line may not have any intersection point in the FA region at all***

- ***Q-point moves to saturation region***
- ***Ceases to become a constant current source***, since in ***saturation***, I_C becomes a ***strong function*** of V_{CE}
- ***Disastrous way of biasing a BJT***

• ***Similar situation*** will arise if R_C is ***very small***

- $I_{C,max}$ will become ***very large*** and ***Q-point will move towards cutoff***
- ***Another disastrous way of biasing a BJT***

- **Example**: Let $V_{CC} = V_B = 5$ V, $R_B = 430$ kΩ, and $\beta = 100$
  - ➢ $I_B = (V_B - V_{BE})/R_B = (5 - 0.7)/(430\ k\Omega) = 10$ μA (assuming ***FA*** mode of operation with $V_{BE} = 0.7$ V)
  - ➢ $I_C = \beta I_B = 1$ mA
  - ➢ V_{CE} will ***depend*** on our ***choice*** of R_C
  - ➢ R_C for ***best biasing*** (BB) ($V_{CE}(BB) = V_{CC}/2$):
    - $R_C(BB) = V_{CC}/(2I_C) = 2.5$ kΩ
  - ➢ R_C that puts Q at OS (VCE(OS) = 0.7 V):
    - $R_C(OS) = [V_{CC} - V_{CE}(OS)]/I_C = 4.3$ kΩ

- ***Any value of R_C higher than 4.3 kΩ would push Q in saturation***
- Choose R_C = 20 kΩ:
  - Assuming ***FA*** operation is maintained, V_{CE} comes out to be –15 V!
  - ***Golden rule***:
    - ***Potential at any point in a circuit can never go beyond the positive and negative extremes of the power supply voltages, unless there is a power source within the circuit***
  - Thus, V_{CE} = –15 V is ***absurd***!
  - Hence, Q is ***no more*** in the ***FA*** mode of operation, rather it has been pushed into ***saturation***

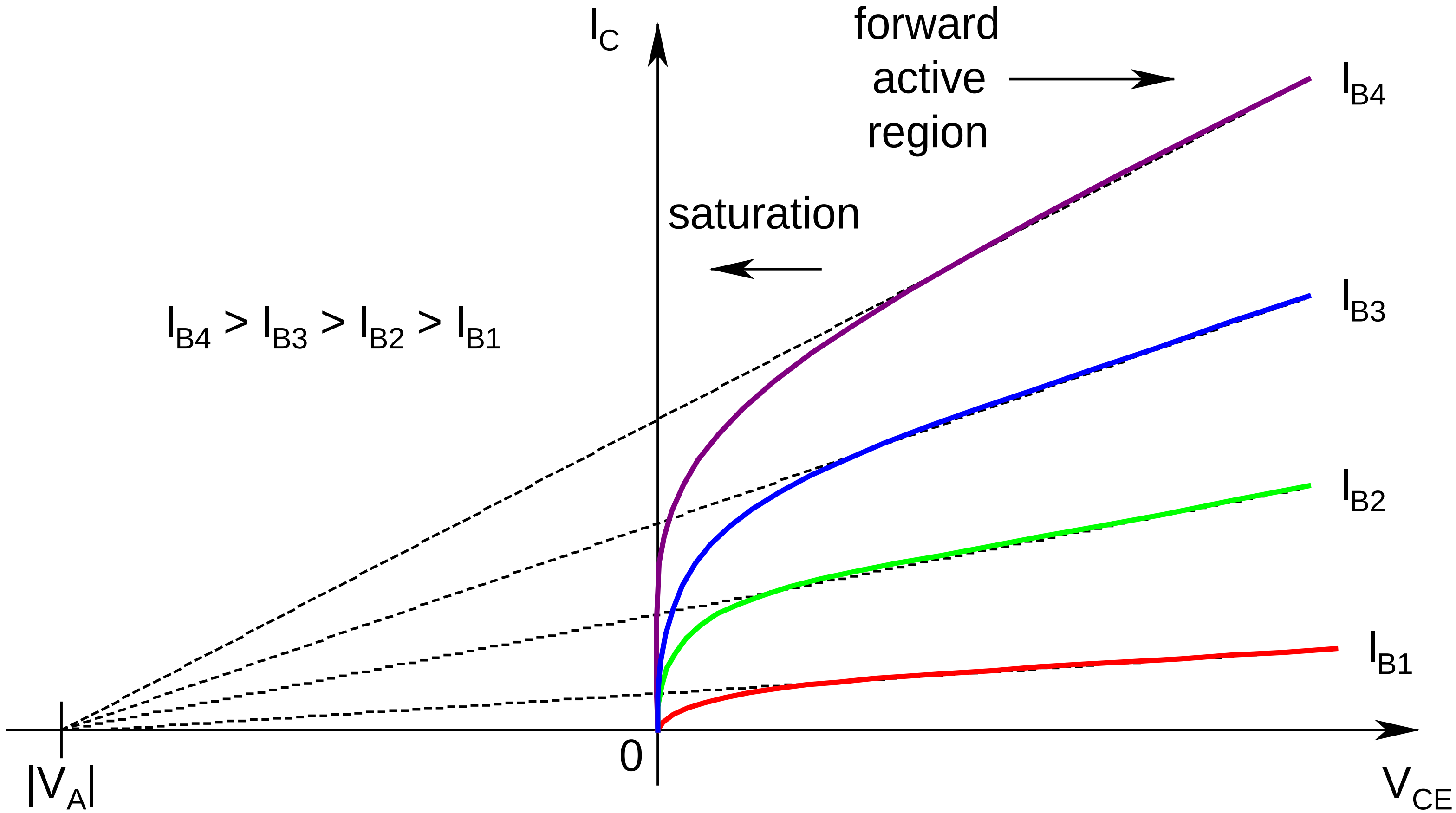
- Whether it is in ***soft saturation*** (SS) or ***hard saturation*** (HS), would ***depend*** on the ***degree of saturation*** (DoS)
- ***For HS, DoS must be ≥ 2*** ($\beta_{sat} \leq \beta/2$)
- Assume ***HS***: V_{BE}(HS) = 0.8 V, V_{CE}(HS) = 0.1 V
  - ❖ $I_{B,sat} = [V_{CC} - V_{BE}(HS)]/R_B = (5 - 0.8)/(430\ k\Omega) = 9.77\ \mu A$
  - ❖ $I_{C,sat} = [V_{CC} - V_{CE}(HS)]/R_C = (5 - 0.1)/(20\ k\Omega) = 245\ \mu A$
  - ❖ $\beta_{sat} = I_{C,sat}/I_{B,sat} = 245/9.77 = 25$
  - ❖ DoS = $\beta/\beta_{sat} = 4\ (> 2)$
  - ❖ ***Assumption verified, and analysis is correct***!

➢ **Ex.**: Find the values of R_C that would put Q at the edge of: i) HS, and ii) SS

Base Width Modulation Effect

- In ***FA*** mode, as $|V_{BC}|\uparrow$, BC ***depletion region width***$\uparrow$ $\Rightarrow$ ***neutral base width***$\downarrow$
  - ***Electrons spend less time in base*** $\Rightarrow$ ***chance of recombination***$\downarrow$
  - ***More electrons make it to the collector*** $\Rightarrow$ $I_C\uparrow$ as $V_{CE}\uparrow$
  - Known as the ***Base Width Modulation Effect*** (or ***Early Effect***)

- The ***current-voltage characteristic***, including ***Early Effect***, is modeled as:
  - $I_C = I_S[\exp(V_{BE}/V_T)](1 + V_{CE}/V_A)$
  - V_A: ***Early Voltage*** (~ 130 V for ***npn***, and ~ 52 V for ***pnp***)
  - V_A is a ***negative number***, but taken to be a ***positive quantity***
- Imparts a ***positive slope*** in the ***output characteristics*** in the ***FA region***
  - Introduces an ***output resistance***, and makes the current source ***non-ideal***!



All characteristics merge at $|V_A|$ in the negative V_{CE} axis

Note: If $V_A \to \infty$, all characteristics become horizontal in the FA region

IEEE Notational Convention

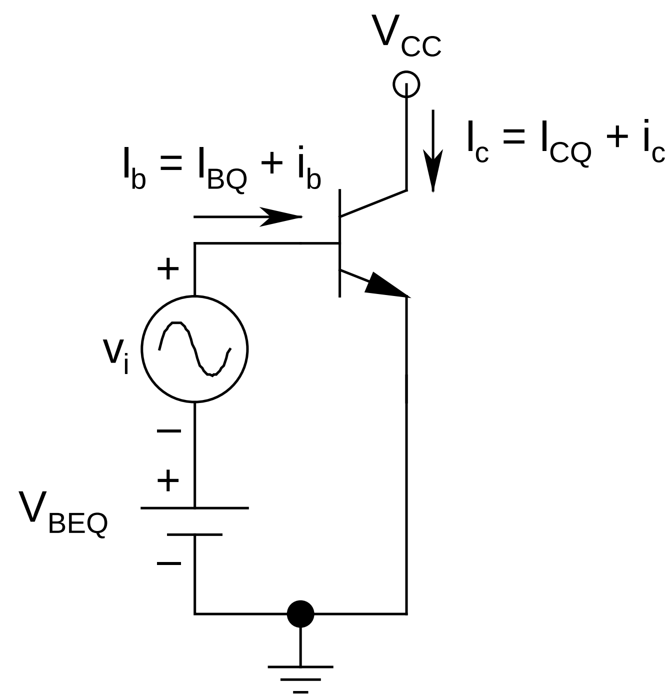
- ***Pure DC quantities***:
  - ***Capital letter with capital subscript*** (e.g., V_{BE})
- ***Pure ac quantities***:
  - ***Small-case letter with small-case subscript*** (e.g., v_{be})
- ***Instantaneous (DC + ac) quantities***:
  - Either ***capital letter with small-case subscript*** (e.g., V_{be}) or ***small-case letter with capital subscript*** (e.g., v_{BE})

Small-Signal Model

- The ***electrical equivalent*** of the BJT at the ***DC bias point***
- Basically an ***electrical network***, having ***passive and active elements***
- To obtain this model, ***DC analysis*** is needed, since the ***information*** regarding the ***Q-point*** (I_C, V_{CE}) is necessary
- ***This model for npn and pnp BJT is same***

Validity of the Small-Signal Model

- Basically ***linearization*** of the ***operating region*** around the ***Q-point***
- ***This linearization should not contain any higher-order terms***



- To start with, assume $V_A \rightarrow \infty$
  - $I_{CQ} = I_S \exp(V_{BEQ}/V_T)$
- Thus:

$$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right) = I_S \exp\left(\frac{V_{BEQ} + v_i}{V_T}\right)$$

$$= I_S \exp\left(\frac{V_{BEQ}}{V_T}\right) \exp\left(\frac{v_i}{V_T}\right) = I_{CQ} \exp\left(\frac{v_i}{V_T}\right)$$

- Expand the ***exponential term*** in series:

- $$I_c = I_{CQ}\left[1 + \frac{v_i}{V_T} + \frac{1}{2!}\left(\frac{v_i}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_i}{V_T}\right)^3 + \cdots\right]$$

- Thus:

- $$i_c = I_c - I_{CQ} = I_{CQ}\left[\frac{v_i}{V_T} + \frac{1}{2!}\left(\frac{v_i}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_i}{V_T}\right)^3 + \cdots\right]$$

- ***True linearization*** of i_c-v_i relation ***can be achieved*** only if ***all higher-order terms*** can be ***neglected*** $\Rightarrow$ v_i should be $<< V_T$

Small-Signal Model Parameters

- ***Incremental Emitter Resistance*** (r_E):

$$r_E = \frac{i_e}{v_i} = \frac{\Delta I_E}{\Delta V_{BE}} \equiv \left.\frac{dI_E}{dV_{BE}}\right|_{V_{CE}\ \text{constant}} = \frac{V_T}{I_E}$$

- ***Transconductance*** (g_m):

$$g_m = \frac{i_c}{v_i} = \frac{\Delta I_C}{\Delta V_{BE}} \equiv \left.\frac{dI_C}{dV_{BE}}\right|_{V_{CE}\ \text{constant}} = \frac{I_C}{V_T}$$

- Thus, $g_m r_E = I_C/I_E = \alpha \approx 1$
- ***A frequently used approximation***:
  - $g_m = 1/r_E$
- For $I_C = 1$ mA:
  - $r_E = 26\ \Omega$ and $g_m = 1/26$ A/V
- As $I_C\uparrow$:
  - $g_m\uparrow$ and $r_E\downarrow$
  - Also $P_D\uparrow$
- Gain = $f(g_m)$

  $\Rightarrow$ For ***higher gain***, the circuit has to be fed ***more power***

- ***Base-Emitter Resistance*** (r_π):

$$r_\pi = \frac{v_i}{i_b} = \frac{\Delta V_{BE}}{\Delta I_B} \equiv \frac{dV_{BE}}{dI_C}\frac{dI_C}{dI_B}\bigg|_{V_{CE}\ \text{constant}} = \frac{\beta}{g_m} \simeq \beta r_E$$

➢For I_C = 1 mA and β =100: r_π = 2.6 kΩ

- ***Output Resistance*** (r_0):

$$r_0 = \frac{v_{ce}}{i_c} = \left[\frac{dI_C}{dV_{CE}}\right]^{-1}\Bigg|_{V_{BE}\ \text{constant}} = \frac{V_A}{I_C} = \frac{V_A}{V_T}\frac{V_T}{I_C} = \frac{1}{\eta g_m}$$

- For $I_C = 1$ mA, $V_{AN} = 130$ V, and $V_{AP} = 52$ V: r_0(npn) = 130 kΩ and r_0(pnp) = 52 kΩ
- η (= V_T/V_A): 2×10^{-4} (npn) and 5×10^{-4} (pnp)
- $g_m r_0 = \eta^{-1}$

- ***Collector-Base Resistance*** (r_μ):

$$r_\mu = \frac{v_{ce}}{i_b} = \frac{\Delta V_{CE}}{\Delta I_B}\Bigg|_{V_{BE}\ \text{constant}} = \frac{dV_{CE}}{dI_C}\frac{dI_C}{dI_B} = \beta r_0$$

- ***Oversimplification*** – ***actual value much higher*** (~ 5-10βr_0) > 100s of MΩ

- ***Emitter-Base Capacitance*** (C_π):

  $C_\pi = C_{je} + C_b$

  - C_{je}: ***Emitter-base depletion capacitance***

    $\approx 2C_{je0}$

    - C_{je0}: ***Emitter-base depletion capacitance at zero bias***

  - C_b: ***Emitter-base diffusion capacitance*** (known as ***base charging capacitance***)

    $= \tau_F g_m \quad (>> C_{je})$

    - τ_F: ***Base transit time***

  - $C_\pi \uparrow$ as $g_m \uparrow$ (***Problem***!)

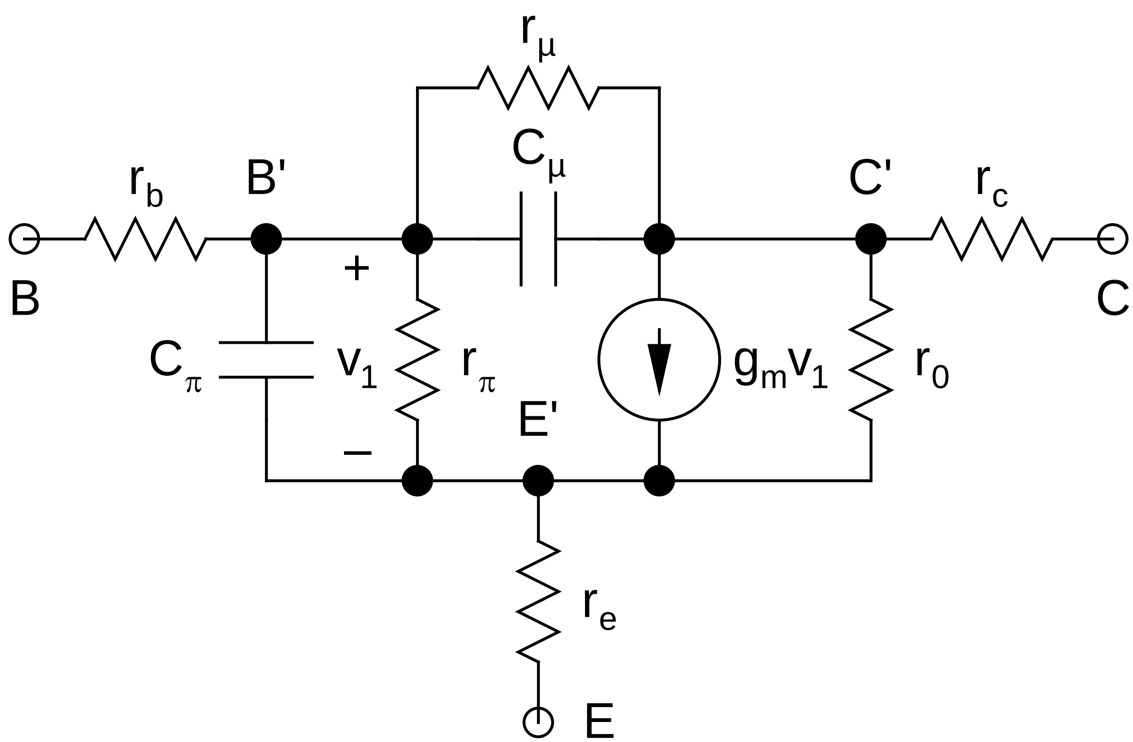
- ***Collector-Base Capacitance*** (C_μ):

$$C_\mu = \frac{C_{\mu 0}}{\left(1 - \frac{V_{BC}}{V_{0,BC}}\right)^m}$$

  - $C_{\mu 0}$: ***Collector-base depletion capacitance at zero bias***
  - $V_{0,BC}$: ***Built-in voltage of collector-base junction***
  - m: ***Grading coefficient*** (1/2 for ***abrupt step junction***, 1/3 for ***linearly graded junction***)

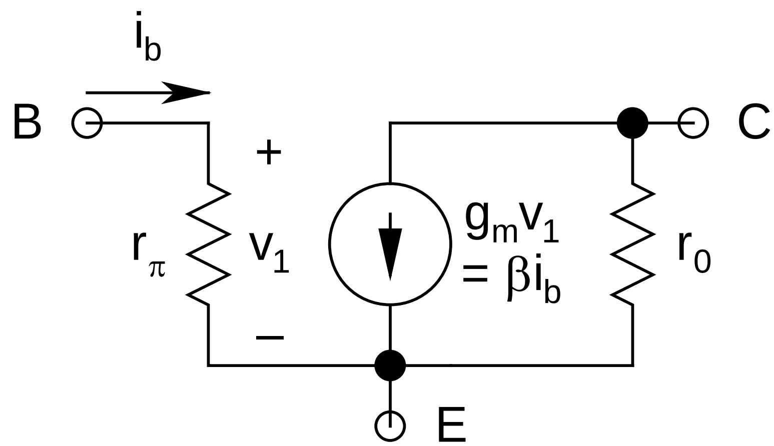
- ***Quasi-Neutral Emitter, Base, and Collector Resistances*** (r_e, r_b, and r_c):
  - In ***IC BJT***, ***emitter highest doped***, ***followed by base***, with ***collector being least doped***
  - Thus, $r_c > r_b > r_e$
  - ***Typical values***:
    - $r_e \sim$ 5-10 Ω
    - $r_b \sim$ 100-200 Ω
    - $r_c \sim$ can be as high as kΩ
  - ***Become important only at very high frequencies***

The Hybrid-π Model



E,B,C: Extrinsic Terminals
E',B',C': Intrinsic Terminals

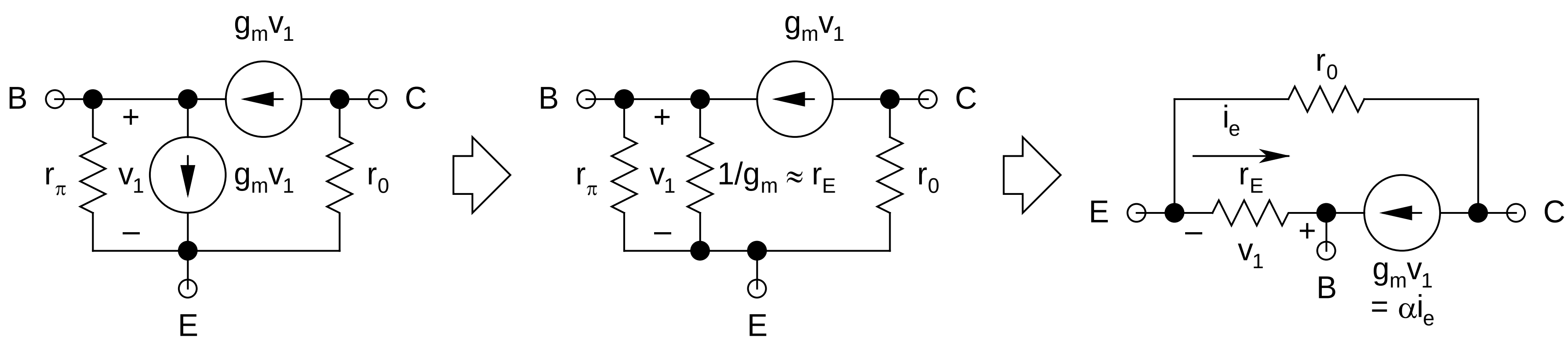
- ***Simplifications***:
  - r_e, r_b, r_c can be ***safely neglected*** under ***low to moderate frequencies*** of operation
  - r_μ can be ***neglected***, since it's ***extremely large***
  - At ***low to moderate frequencies***, the ***capacitive reactances*** of C_π and C_μ will be ***extremely large*** $\Rightarrow$ can be ***neglected***
  - Leads to the ***Low-Frequency T-Model***, having only ***three components***: r_π, $g_m v_1$, and r_0
  - ***Simplest possible equivalent results if r_0 is also neglected***!



Low-Frequency T-Model

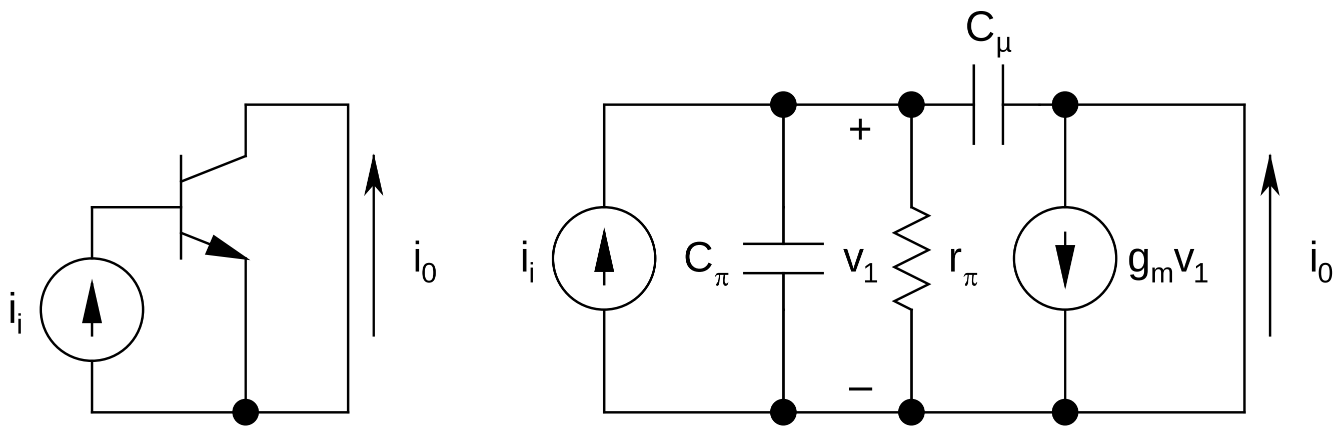
- **Note**: The ***output circuit*** resembles a ***non-ideal current source*** of ***magnitude*** $g_m v_1$ (or equivalently, βi_b) with ***output resistance*** r_0

- ***This model is appropriate when the ac input is applied to base***
- ***When the ac input is applied to the emitter, then need to draw this circuit in a slightly different way***



Frequency Specifications of BJTs

- ***Four important characteristic frequencies***:
  - ➢ ***Beta Cutoff Frequency*** (f_β)
  - ➢ ***Unity Gain Cutoff Frequency*** (f_T)
  - ➢ ***Alpha Cutoff Frequency*** (f_α)
  - ➢ ***Maximum Operable Frequency*** (f_{max})



- $i_0 \approx g_m v_1$ (***neglecting reverse transmission through C_μ***)
- $v_1 = i_i Z_{eq}$

$$Z_{eq} = \frac{r_\pi}{1 + s r_\pi \left(C_\pi + C_\mu\right)} \qquad \left(s = j\omega\right)$$

- Thus:

$$\beta(j\omega) = \frac{i_0(j\omega)}{i_i(j\omega)} = \frac{\beta_0}{1 + j\omega/\omega_\beta}$$

β_0 (= $g_m r_\pi$): ***Low-frequency short-circuit common-emitter current gain***

$$\omega_\beta = \frac{g_m}{\beta_0 \left(C_\pi + C_\mu\right)}$$

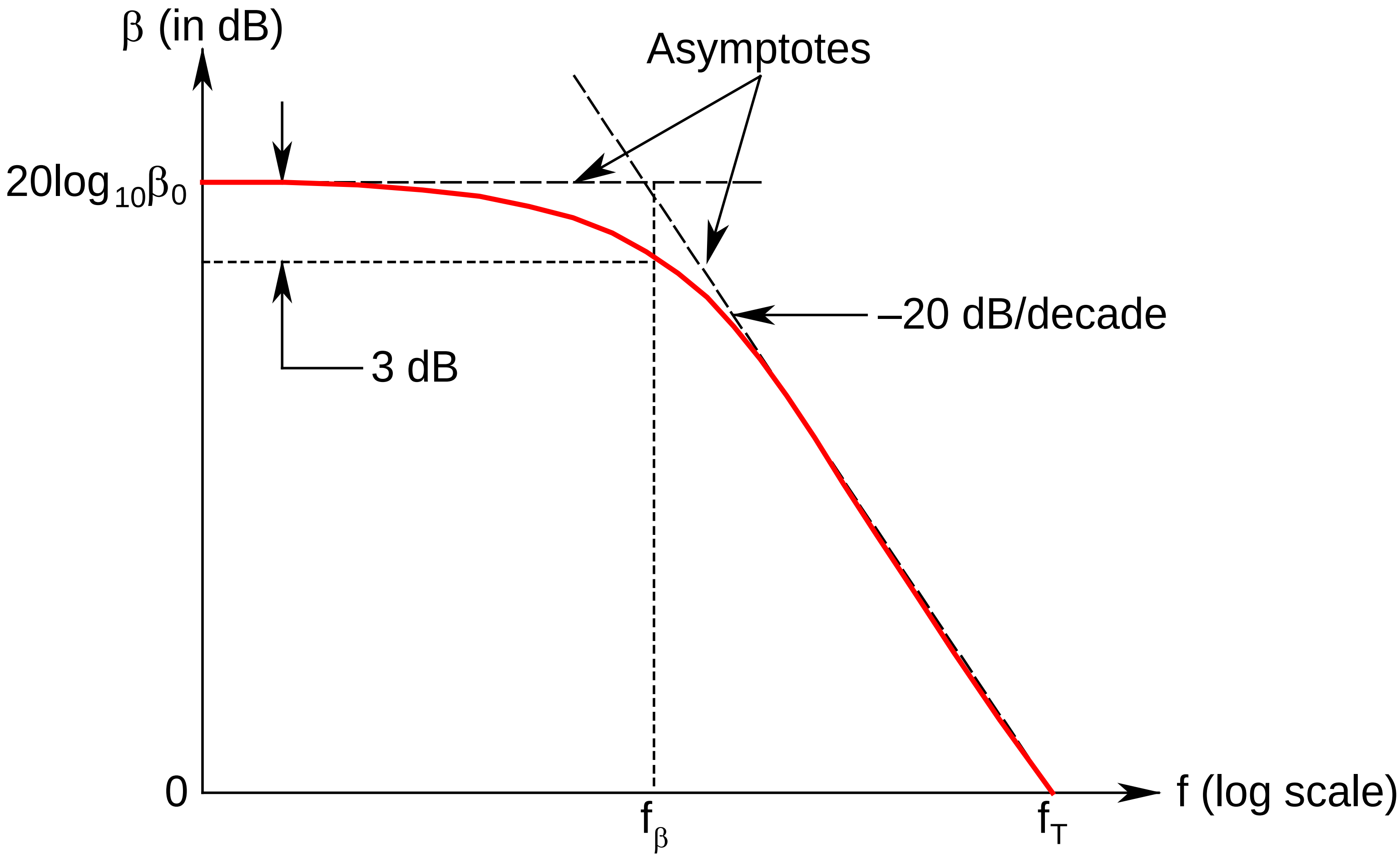
- f_β [= $\omega_\beta/(2\pi)$]: ***Beta Cutoff Frequency***
- At $f = f_\beta$, $\beta = \beta_0/\sqrt{2}$

- For $f \gg f_\beta$:

$$\beta(j\omega) \simeq \frac{g_m}{j\omega\left(C_\pi + C_\mu\right)}$$

- At $\omega = \omega_T = g_m/(C_\pi + C_\mu)$, $|\beta| = 1$
- f_T [$= \omega_T/(2\pi)$]: ***Unity Gain Cutoff Frequency*** (also known as ***Unity Gain Bandwidth***)

- **Note**: $f_T = \beta_0 f_\beta$
- ***$f_T > f_\beta$, and their spacing depends on β_0***


β (in dB)
Asymptotes
20log₁₀β₀
–20 dB/decade
3 dB
0
f (log scale)
fβ
fT

- ***Actual measurement of f_T difficult*** - ***measured indirectly***
- ***Measurement done at $f_x >> f_\beta$***, where ***β has dropped to about 5-10***
- Then, $f_T = \beta(f_x)f_x$
- Using $\alpha = \beta/(\beta + 1)$:

$$\alpha(j\omega) = \frac{\beta(j\omega)}{1+\beta(j\omega)} = \frac{\alpha_0}{1+j\omega/\omega_\alpha}$$

α_0 [$= \beta_0/(\beta_0 + 1)$]: ***Low-frequency short-circuit common-base current gain***

$\omega_\alpha = (\beta_0 + 1)\omega_\beta$

- f_α [$= \omega_\alpha/(2\pi)$]: ***Alpha Cutoff Frequency***
- At $f = f_\alpha$, $\alpha = \alpha_0/\sqrt{2}$

- **Note**: ***f_α and f_T extremely close to each other***, with ***f_α marginally higher than f_T***, with ***both being much larger than f_β***
- ***Maximum Operable frequency***:

$$f_{max} = f_T\Big|_{max} = \frac{1}{2\pi\tau_F}$$

➢ Known as the ***Transit Time Model***