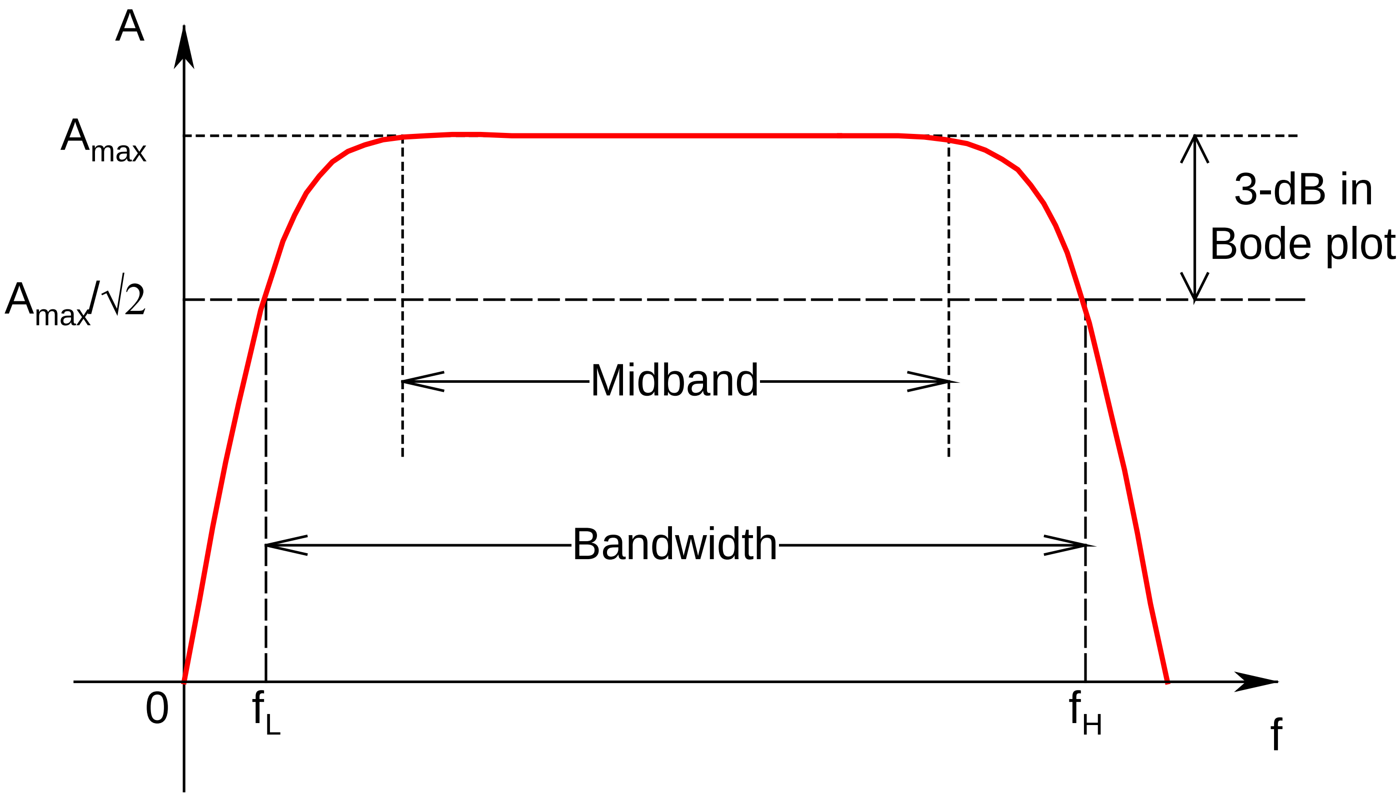
FREQUENCY RESPONSE

- So far, considered ***midband analysis***, where ***all capacitive effects were neglected***
  - ***Voltage/current gain was independent of frequency***
- In ***practical amplifier circuits***, however, the ***gain would depend on frequency***
- ***Characterized by***:
  - ***Lower Cutoff Frequency*** (f_L)
    - ***Contributed by external capacitors*** (C_E, C_B, C_C)

- ***Upper Cutoff Frequency*** (f_H)
  - ***Contributed by device capacitances*** (***for BJTs***: C_π, C_μ; ***for MOSFETs***: C_{gs}, C_{gd}, C_{sb}, C_{db})
- These capacitors create ***charge storage effects***, and ***introduce time constants*** into the circuit
- ***Discrete circuits show both f_L and f_H***
- ***IC stages show only f_H***, since most of them are ***direct coupled*** without the need for any ***external capacitors***



f_L: Lower Cutoff Frequency

f_H: Upper Cutoff Frequency

Bandwidth = $f_H - f_L$

- ***Exact analysis extremely complicated***
  - Most often, results in ***very complicated expressions***, ***completely hiding the physical feel of the phenomenon***
  - ***Makes debugging extremely difficult***
    - For example, a circuit having ***4 capacitors***, will have a ***fourth-order transfer function***, which needs to be ***solved*** to get all the ***poles and zeros*** of the system
  - However, there are ***techniques***, which make these ***analyses*** extremely ***trivial***
    - ***Not accurate***, but ***extremely simple***, and makes ***debugging easy***

- ***Techniques***:
  - ***Infinite-Value Time Constant (IVTC) Method***
    - ***Used for obtaining f_L***
  - ***Zero-Value Time Constant (ZVTC) Method***
    - ***Used for obtaining f_H***
- These techniques are ***extremely easy to apply***, and the results are ***quite close to actuals***
- However, there is ***one limitation*** of these techniques

- They give information only regarding the ***Dominant Pole*** (DP) of the circuit
- ***Completely hides information*** about ***other poles and zeros*** of the circuit [known as ***Non-Dominant Poles*** (NDP) or ***Zeros*** (NDZ)]
- Anyway, information about ***NDP and NDZ*** are ***not*** that ***critically important*** from ***practical point of view***

Low-Frequency Response

- ***The Infinite-Value Time Constant (IVTC) Technique***:
  - Used for obtaining the ***lower cutoff frequency*** (f_L)
  - If a circuit has ***n*** number of ***capacitors***, then it would have ***n*** number of ***time constants***
  - ***This technique derives the information regarding f_L from these time constants***

- ***The Algorithm***:
  - ***Null all independent sources to the circuit***
    - ***Short all independent voltage sources***
    - ***Open all independent current sources***
    - ***DO NOT TOUCH DEPENDENT SOURCES***
  - ***Name the capacitors C_i*** (***i = 1-n***)
  - ***Consider C_1 and assign infinite values to all other capacitors*** (***thus the name***!)
    - Thus, ***except C_1***, ***all other capacitors will short out***
  - ***Determine the Thevenin Resistance*** $\left(R_1^{\infty}\right)$ ***across the two terminals of C_1***

- ***Find the time constant τ_1 associated with C_1***
  $\left(\tau_1 = R_1^{\infty} C_1\right)$
- ***Calculate the corresponding frequency $f_1 = 1/(2\pi\tau_1)$***
- ***Repeat for all other capacitors***, ***taking one at a time***, and ***find all the rest of the frequencies*** $(f_2, f_3, \ldots, f_n)$
- Then the ***Lower Cutoff Frequency*** f_L can be expressed as:

$$f_L = \left[\sum_{i=1}^{n} f_i^2\right]^{1/2}$$

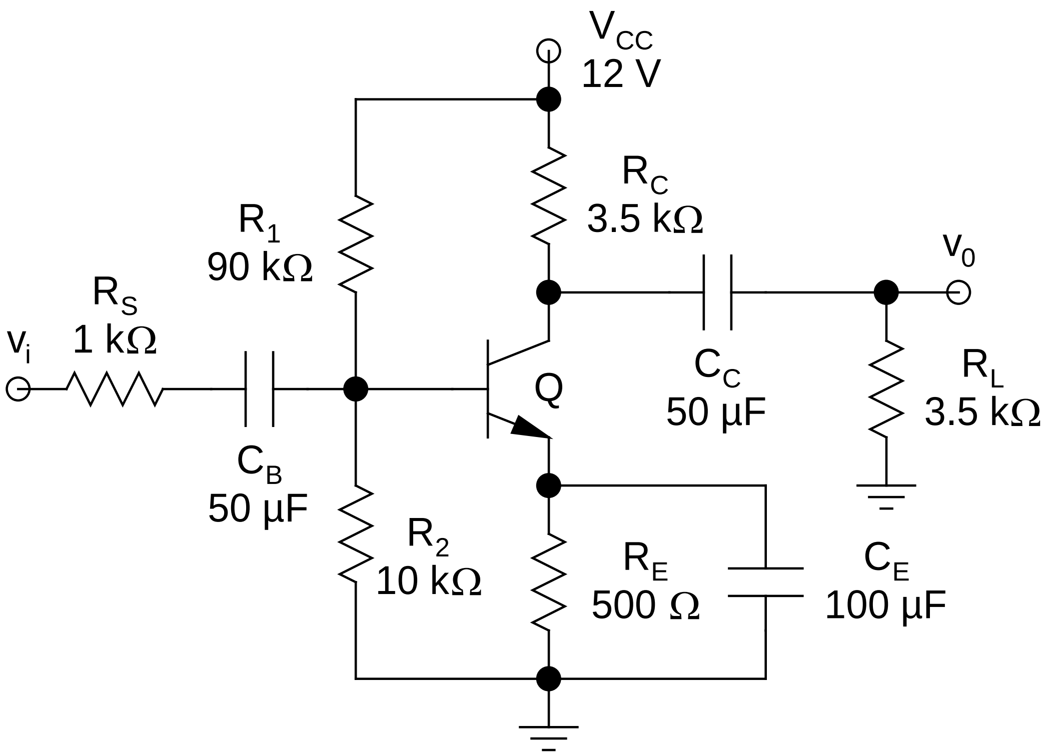
- In ***discrete circuits***, a ***major component of total cost*** is due to the ***cost of the capacitors*** (***directly proportional to the value***)
- Hence, an attempt is made to ***minimize*** the ***total capacitor requirement of the circuit***
- For this, the ***Dominant Pole*** (DP) technique is used
  - ***One of the frequencies among f_1-f_n is made dominant***
  - ***Others are made to lie at least 10 times away from it***

- For example, if ***f_d* is chosen to be the DP**, then ***all other poles are assumed to be at $f_d/10$***

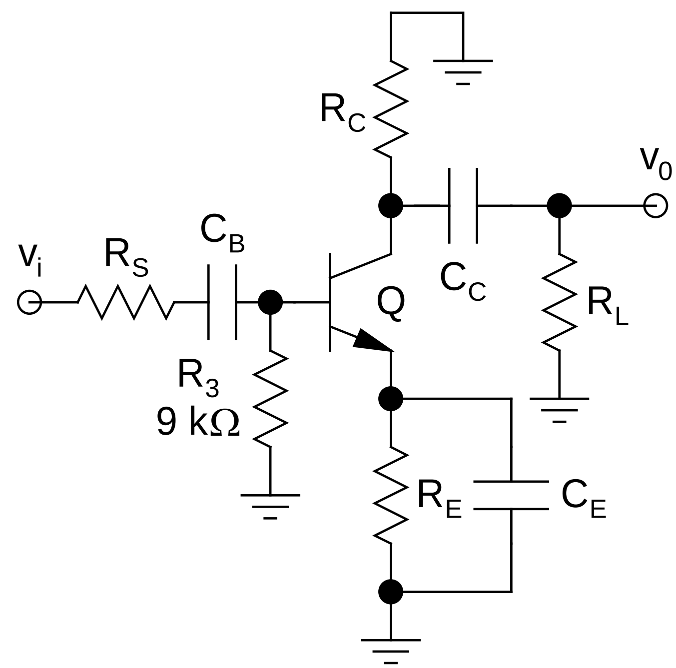
$$\Rightarrow f_L = \left[f_d^2 + \sum_{n-1} \left(\frac{f_d}{10} \right)^2 \right]^{1/2}$$

- C_d, which ***contributes*** f_d, is ***chosen*** to be that capacitor that ***sees*** the ***least Thevenin resistance*** across its terminals
- ***Reason is obvious***:
    - If ***any other capacitor*** were ***chosen*** to ***contribute*** f_d, then C_d would have been ***ten times higher***
- ***This choice is based on heuristics***

- ***Low-Frequency Response of RC-Coupled Amplifier*:**



Complete Circuit



ac Schematic

- ***DC analysis*** gives $I_C = 1$ mA and $V_{CE} = 4$ V

  $\Rightarrow r_E = 26\ \Omega$ and $r_\pi = 2.6\ k\Omega$ (assuming $\beta = 100$)
- ***Neglect Early effect***

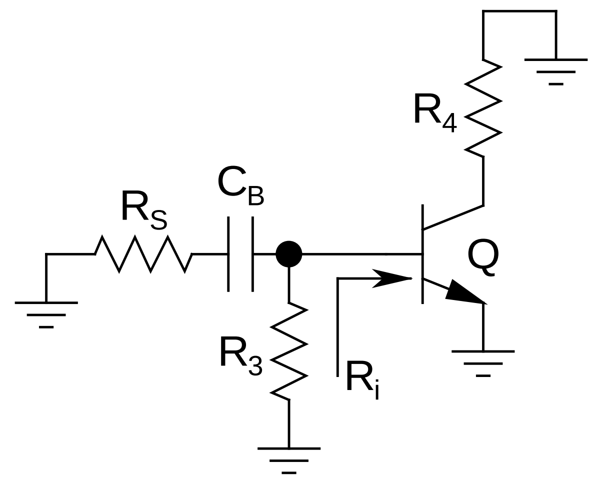
  $\Rightarrow r_0 \rightarrow \infty$
- 3 ***capacitors*** (C_B, C_E, C_C) with ***time constants*** τ_1, τ_2, τ_3, and corresponding ***cutoff frequencies*** f_1, f_2, f_3
- To apply the ***IVTC technique***, we have to take ***one capacitor at a time*** and ***treat other capacitors as short circuits***
- ***The analysis can be done by inspection***!

- ***C_B***:
  - ***Short C_C and C_E***
  - $R_3 = R_1||R_2 = 9\ k\Omega$
  - $R_4 = R_C||R_L = 1.75\ k\Omega$
  - $R_i = r_\pi = 2.6\ k\Omega$
  - By inspection, the ***Thevenin resistance*** seen by C_B:

    $$R_B^\infty = R_S + \left(R_3 \,||\, R_i\right) = 3\ k\Omega$$

    $$\Rightarrow \tau_1 = R_B^\infty C_B = 150\ ms$$

    $$\Rightarrow f_1 = 1/(2\pi\tau_1) = 1.06\ Hz$$

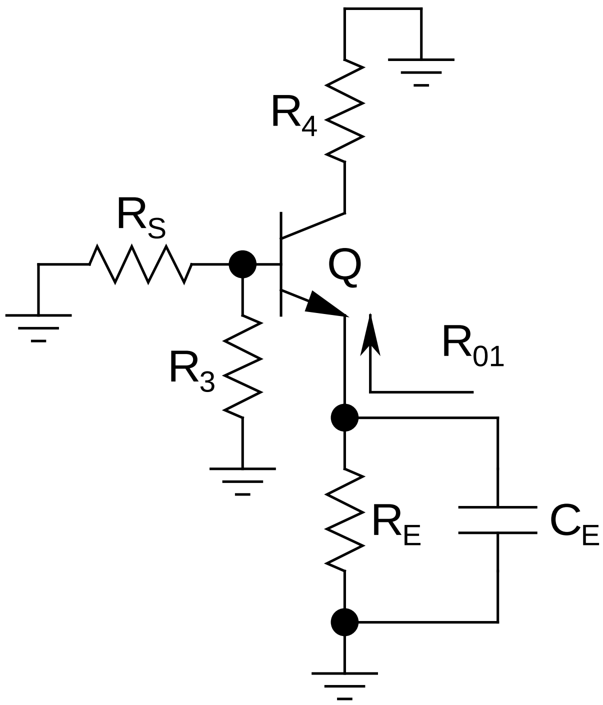


- ***C_E*:**
  - ***Short C_C and C_B***
  - $R_{01} = r_E + (R_S \| R_3)/(\beta + 1)$
    $= 34.9\ \Omega$
  - By inspection, the ***Thevenin resistance*** seen by C_E:

$$R_E^{\infty} = R_E \,\|\, R_{01} = 32.6\ \Omega$$

$$\Rightarrow \tau_2 = R_E^{\infty} C_E = 3.26\ \text{ms}$$

$$\Rightarrow f_2 = 1/(2\pi\tau_2) = 48.8\ \text{Hz}$$



- ***C_C***:
  - ***Short C_E and C_B***
  - By inspection, the ***Thevenin resistance*** seen by C_C:

    $$R_C^\infty = R_C + R_L = 7\ \text{k}\Omega$$

    $$\Rightarrow \tau_3 = R_C^\infty C_C = 350\ \text{ms}$$

    $$\Rightarrow f_3 = 1/(2\pi\tau_3) = 0.45\ \text{Hz}$$

  - Thus, the ***lower cutoff frequency*** of the circuit:

    $$f_L = \left[f_1^2 + f_2^2 + f_3^2\right]^{1/2} = 48.8\ \text{Hz}$$

- Note that ***f_L is equal to f_2*** (***contributed by C_E***)
- Now let's attempt to ***minimize*** the ***total capacitance requirement*** of the circuit

➢ *Minimization of the Total Capacitance*:

- From the previous analysis, we note that ***C_E sees the least Thevenin resistance across its two terminals***

  $\Rightarrow$ ***Let's choose C_E to contribute the DP f_d***, and let ***C_C and C_B each contribute poles at $f_d/10$***

  $$\Rightarrow 48.8 = \sqrt{f_d^2 + 2\left(f_d/10\right)^2}$$

  $$\Rightarrow f_d = 48.3 \text{ Hz and } f_d/10 = 4.83 \text{ Hz}$$

- Thus:

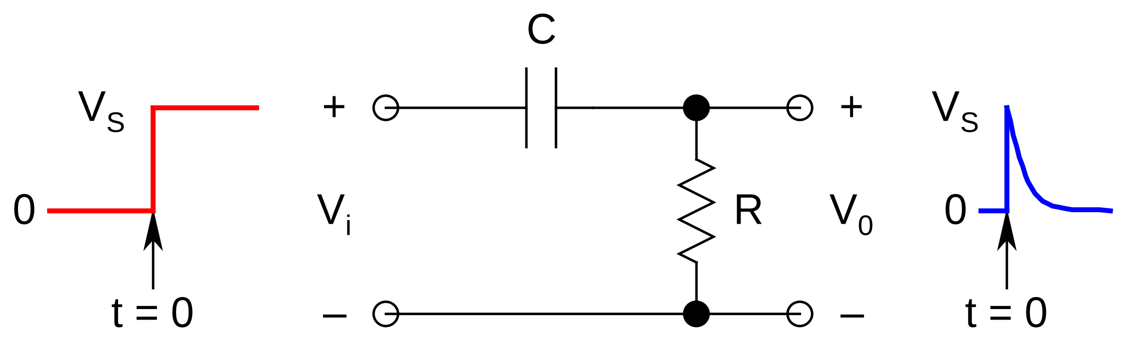
  $$C_E = 1\Big/\left(2\pi f_d R_E^\infty\right) = 101.1\ \mu\text{F}$$

  $$C_B = 1\Big/\left[2\pi\left(f_d/10\right)R_B^\infty\right] = 11\ \mu\text{F}$$

  $$C_C = 1\Big/\left[2\pi\left(f_d/10\right)R_C^\infty\right] = 4.7\ \mu\text{F}$$

- Thus, the ***total capacitance*** requirement comes out to be ***116.8 μF***, for the ***same f_L of 48.8 Hz***
- The ***original circuit*** had a ***total capacitance*** of ***200 μF***
- Thus, this approach gave a ***cost saving*** of almost ***42%*** in terms of the ***capacitors***
- As an ***exercise***, you can pick ***either C_C or C_B*** to ***contribute f_d***, and find the ***total capacitance*** requirement for each case
- Finally, after all, this is a ***heuristic***
- To get the ***absolute minimum value*** of the ***total capacitance***, we need to ***formulate the problem***, and ***find the minima of the function mathematically***

- ***Tilt/Sag***:
  - For ***pulse/square wave excitation***, f_L dictates the amount of ***tilt/sag*** present in the ***output***
  - ***Due to f_L***, the circuit effectively behaves like a ***HPF***, represented by a simple ***RC circuit***
  - Under ***step input***, the ***output*** would be a ***spike***



- Thus:

$$V_0 = V_S \exp(-t/\tau_L)$$

$$\tau_L = RC = 1/\omega_L \quad (\omega_L = 2\pi f_L)$$

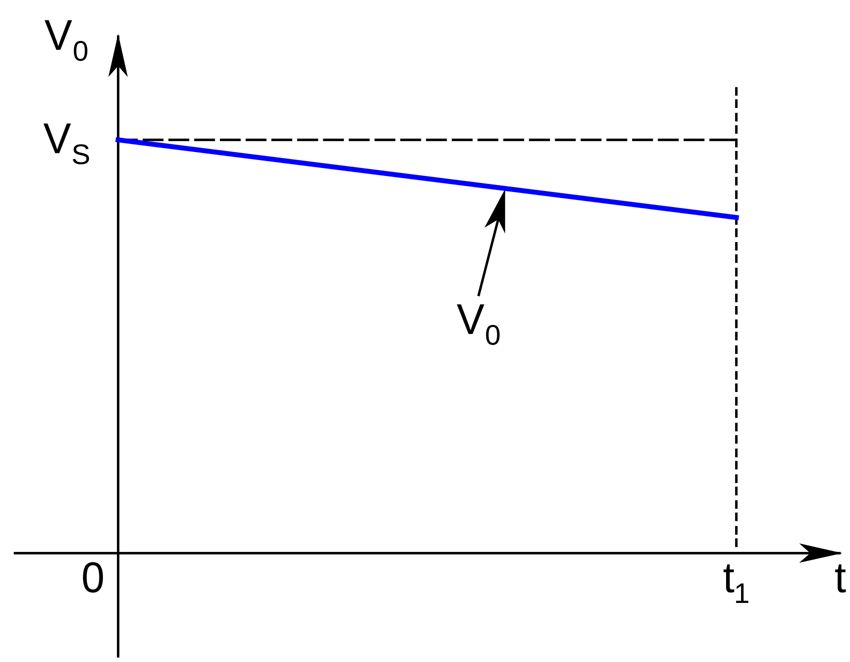
- For $t << \tau_L$:

$$V_0 \approx V_S(1 - t/\tau_L)$$

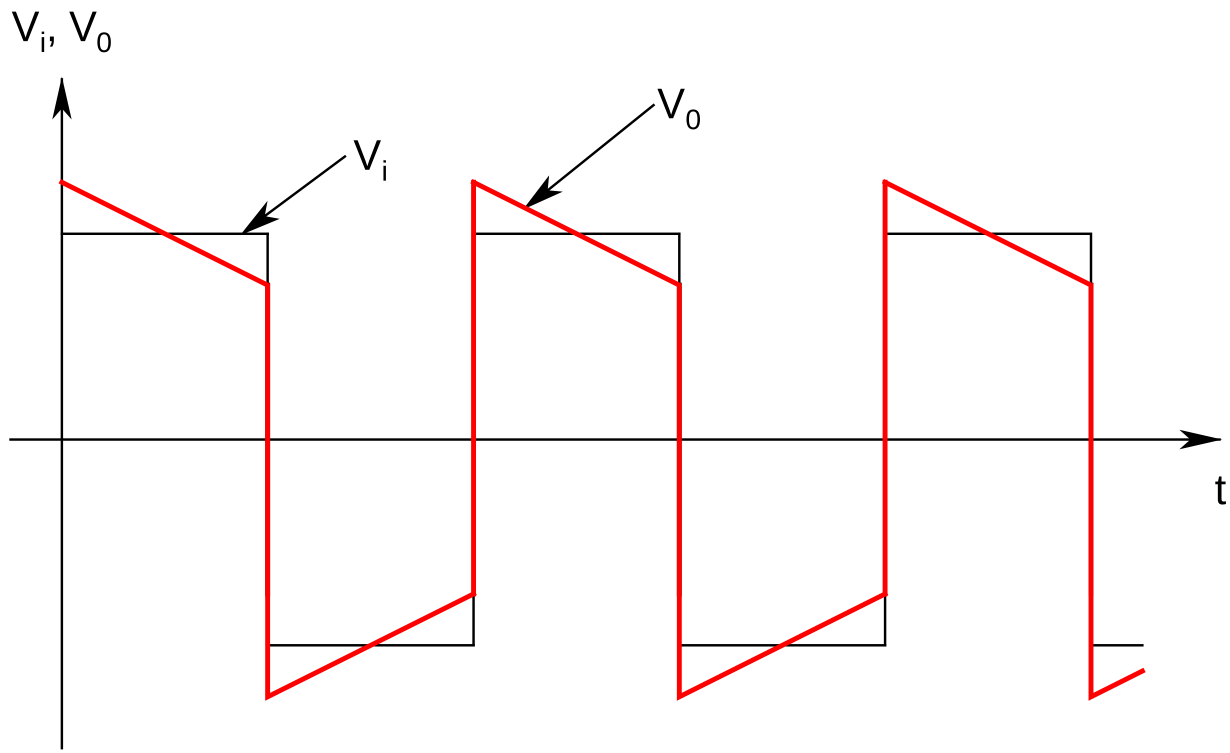
$$= V_S(1 - \omega_L t)$$

$$= V_S(1 - 2\pi f_L t)$$

- Thus, ***V_0 drops linearly with time***
- ***Quantified by percent tilt/sag*** (P)



- $P = [(V_S - V_0)/V_S] \times 100\%$
  $= (t_1/\tau_L) \times 100\%$
  t_1 = ***Time at which the tilt is measured***
- For ***square wave input***, $t_1 = T/2$ (T = ***period*** $= 1/f$, f = ***cycle frequency***)
  $\Rightarrow P = [T/(2\tau_L)] \times 100\% = [\omega_L/(2f)] \times 100\%$
  $= (\pi f_L/f) \times 100\%$
- ***Note***: ***P is directly proportional to f_L*** and ***inversely proportional to f***
  $\Rightarrow$ ***Circuits having low f_L, will show significant amount of tilt/sag at low frequencies***



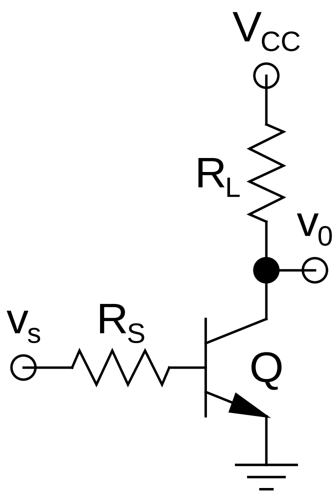
Tilt/Sag

High-Frequency Response

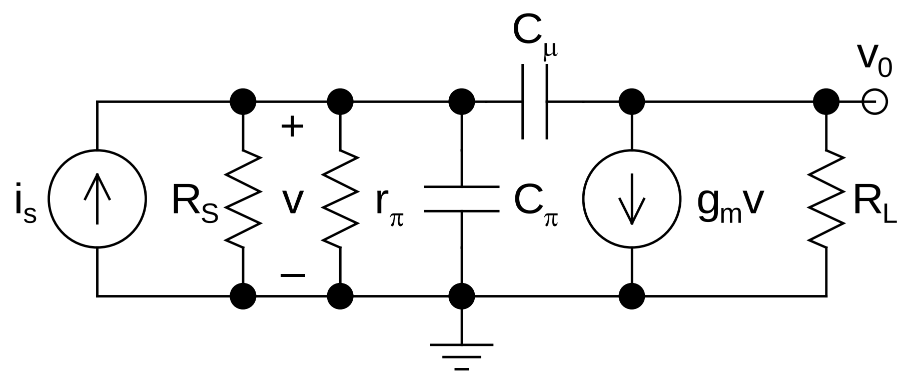
- Will consider ***3 methods***:
  - ***Exact Analysis***:
    - The ***most accurate*** and the ***most rigorous***
    - ***Gives information about all poles and zeros of the system***
  - ***Miller Effect Approximation***:
    - ***One level of approximation***
    - ***Gives information about the Dominant Pole (DP) and one Non-Dominant Pole (NDP)***

- ***Zero-Value Time Constant (ZVTC) Technique***:
  - ***The easiest one***
  - ***Information regarding only the DP***
  - ***Suppresses information about all other poles and zeros of the system***
  - ***Reasonable accuracy***
  - ***Underestimates f_H slightly*** (***better than overestimating and not achieving it***!)
  - ***Based on heuristic***
  - ***Similar to the IVTC technique***, ***based on an algorithm***

- ***Exact Analysis of a CE Stage*:**



ac Schematic



High-Frequency Equivalent

- ***Biasing circuits omitted for simplicity***
- ***Converted input v_s to its Norton equivalent***

- ***KCL at input node*** (using ***Laplace operator*** s $= j\omega$ and $R = R_S \| r_\pi$):

$$i_s = v/R + sC_\pi v + sC_\mu(v - v_0)$$
$$= [1/R + s(C_\pi + C_\mu)]v - sC_\mu v_0$$

- ***KCL at output node***:

$$sC_\mu(v_0 - v) + g_m v + v_0/R_L = 0$$

$$\Rightarrow v = -\frac{1/R_L + sC_\mu}{g_m - sC_\mu} v_0$$

$$\Rightarrow \frac{v_0}{i_s}(s) = -\frac{R_L R\left(g_m - sC_\mu\right)}{1 + s\left(R_L C_\mu + RC_\mu + RC_\pi + g_m R_L RC_\mu\right) + s^2 R_L RC_\pi C_\mu}$$

- Thus, the ***voltage gain***:

$$A_v(s) = \frac{v_0}{v_s} = -\frac{g_m R_L R}{R_S} \frac{\left(1 - sC_\mu / g_m\right)}{1 + sa + s^2 R_L R C_\pi C_\mu} \qquad (1)$$

$$a = R_L C_\mu + R(C_\pi + C_\mu) + g_m R_L R C_\mu$$

- Hence, the circuit has ***one zero*** and ***two poles***

$$\Rightarrow A_v(s) = A_{v0} \frac{\left(1 - s/z_1\right)}{\left(1 - s/p_1\right)\left(1 - s/p_2\right)} \qquad (2)$$

A_{v0} = ***midband gain*** $= -\, g_m R_L R/R_S$

$= -\, g_m R_L r_\pi /(r_\pi + R_S)$

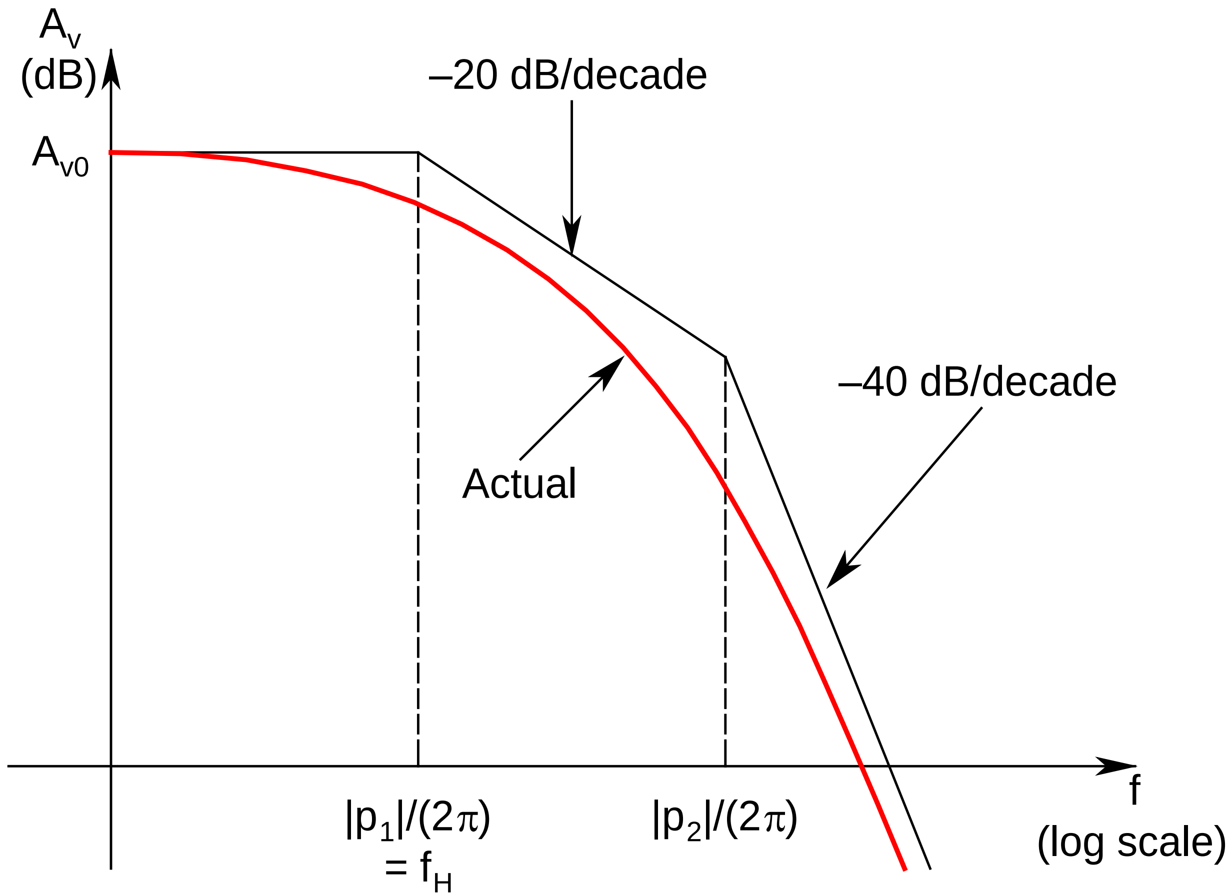
- z_1 (= g_m/C_μ): ***positive real zero***
- The ***frequency*** corresponding to z_1 occurs at $z_1/(2\pi)$, which is ***extremely high***, and generally, is ***not of much consequence***
- ***Computation of the two poles p_1 and p_2 is slightly more tricky***
- From Eqs.(1) and (2), it is obvious that ***both p_1 and p_2 are real and negative***

- To find these, ***write the denominator*** of Eq.(1) as:

$$D(s) = (1 - s/p_1)(1 - s/p_2)$$
$$= 1 - s(1/p_1 + 1/p_2) + s^2/(p_1 p_2) \qquad (3)$$

- ***Matching coefficients*** with Eq.(2), we can get p_1 and p_2, however, the ***resulting algebra*** will become ***extremely tedious***
- Hence, we invoke the ***Dominant Pole Approximation*** (DPA)

- ***DPA***:
  - The ***smallest pole*** [***Dominant Pole*** (DP)] is ***at least 10 times away from its nearest pole***
  - This is an ***excellent approximation for practical analog circuits***
- ***Apply this approximation*** and ***assume p_1 to be the DP*** and ***at least 10 times away from p_2*** [***Non-Dominant Pole*** (NDP)]
- The ***pole frequencies*** are $|p_1|/(2\pi)$ and $|p_2|/(2\pi)$
- ***Note***: $|p_1|/(2\pi)$ is the ***Upper Cutoff Frequency*** (f_H)



Bode Plot of the Frequency Response of a 2-Pole System

- ***2-pole system***
- ***For frequencies till the first pole p_1, gain remains constant at its midband value of $20\log_{10}A_{v0}$***
- ***Beyond this***, the ***gain rolls off at –20 dB/decade till the second pole p_2 is encountered***
- ***After this***, the ***gain rolls off at –40 dB/decade***, and ***eventually crosses zero***
- ***Beyond this***, the circuit actually ***attenuates the input signal instead of amplifying it*** (***gain magnitude drops below unity***)

- ***It's assumed that z_1 is >> $|p_2|$***
- ***Task remains to find p_1 and p_2***
- ***Under DPA***, Eq.(2) can be simplified as:

  $D(s) \approx 1 - s/p_1 + s^2/p_1p_2$ (4)

- ***Comparing*** Eq.(4) with the ***denominator*** of Eq.(1):

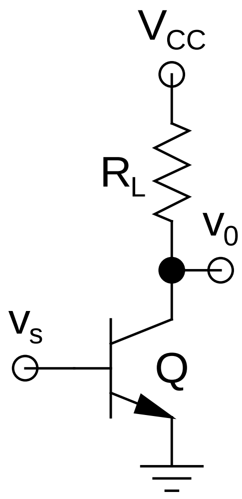
$$p_1 = -\frac{1}{(R_S \| r_\pi)C_\pi + \left[(R_S \| r_\pi) + R_L + g_m (R_S \| r_\pi) R_L\right] C_\mu}$$

$$p_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{(R_S \| r_\pi) C_\pi} + \frac{1}{R_L C_\pi} + \frac{g_m}{C_\pi}\right)$$

- In general, $|p_2| >> |p_1|$
- ***Ex.***: I_C = 1 mA, β = 200, R_S = 1 kΩ, R_L = 2 kΩ, C_π = 10 pF, C_μ = 0.5 pF

  $\Rightarrow$ ***DPF*** = 3.8 MHz, ***NDPF*** = 798.8 MHz, ***ZF*** = 12.3 GHz, and ***f_H = DPF*** = 3.8 MHz
- ***Note***: Even for a ***simple CE circuit***, the ***analysis is so cumbersome***, and the ***results are so complicated***
- ***Definitely not acceptable for routine application***, ***particularly for circuits having more than one active device***

- ***Miller Effect Approximation***:
  - ***Technique by which an input-output coupled circuit can be decoupled by removing the coupling element***
  - This ***removal*** is done by ***splitting*** it into ***two components*** - putting ***one in the input circuit***, and the ***other in the output circuit***
  - We take the ***same example*** as the ***CE circuit*** discussed earlier, but now ***without*** R_S



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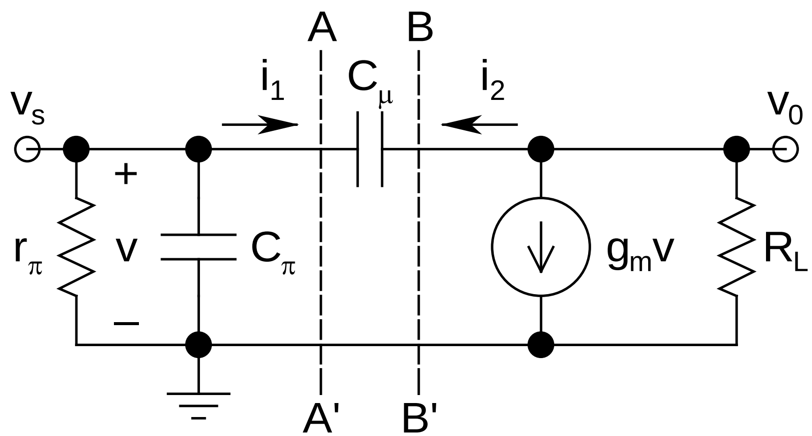


High-Frequency Equivalent

- ***Identify C_μ as the input-output coupling element***
- After ***application*** of the ***technique***, this ***coupling element*** will be ***removed*** by ***splitting*** it into ***two parts*** - ***one at input***, ***other at output***

- These ***two parts*** can be found by ***evaluating*** the ***impedances*** looking into the ***planes*** AA′ and BB′
- ***KCL at output node***:

$$g_m v + v_0/R_L + sC_\mu(v_0 - v) = 0$$

- Noting that $v = v_s$, the ***voltage gain***:

$$A_v(s) = v_0/v_s = -\,g_m R_L(1 - sC_\mu/g_m)/(1 + sR_L C_\mu)$$

$\Rightarrow$ ***Midband or low-frequency gain***:

$$A_v(0) = -\,g_m R_L$$

This result can also be written from inspection

- ***Current entering plane AA′***:

  $$i_1 = sC_\mu(v - v_0) = sC_\mu[1 - A_v(s)]v$$

- Hence, the ***admittance*** looking into the ***plane*** AA′:

  $$y\big|_{AA'} = i_1/v = sC_\mu\left[1 - A_v(s)\right]$$

- This ***admittance*** is ***capacitive*** in nature, and is known as the ***Miller Capacitance*** C_M:

  $$C_M = C_\mu[1 - A_v(s)]$$

- Now, since ***$A_v(s)$ is a function of frequency***, ***so would C_M*** $\Rightarrow$ ***Problem***!

- Here, we invoke the ***Miller Effect Approximation*** (MEA)
  - ***$A_v(s)$ is replaced by $A_v(0)$***, i.e., by its ***midband value***, which is a ***constant***
  - Thus, ***C_M becomes a constant*** with a value of

    $C_M = [1 - A_v(0)]C_\mu = (1 + g_m R_L)C_\mu$
- Thus, ***$C_M >> C_\mu$***, since, in general, ***$g_m R_L >> 1$***
- This effect is known as the ***Miller Effect Multiplication***
- ***Care***: ***The gain that multiplies C_μ is across its two ends***

- Similarly, ***current entering plane BB′***:

  $i_2 = sC_\mu(v_0 - v) = sC_\mu[1 - 1/A_v(s)]v_0$

- Hence, the ***admittance*** looking into the ***plane*** BB′:

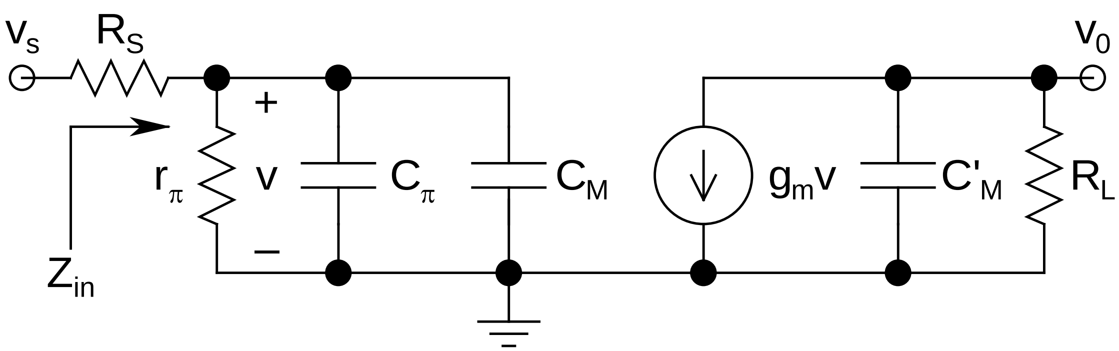
  $$y'\big|_{BB'} = i_2/v_0 = sC_\mu\left[1 - 1/A_v(s)\right]$$

- Again ***replacing*** $A_v(s)$ by $A_v(0)$, we get:

  $$C'_M = \left[1 - 1/A_v(0)\right]C_\mu = \left[1 + 1/(g_m R_L)\right]C_\mu$$

- In general, $g_m R_L >> 1 \Rightarrow \; C'_M \simeq C_\mu$

- C_μ can now be ***removed*** as the ***coupling element***, ***split into 2 parts*** C_M and C'_M, with C_M appearing in the ***input circuit*** and C'_M appearing in the ***output circuit***
- Now, ***include R_S*** - note that the circuit is ***completely decoupled*** now



Complete Circuit Including R_S

- $Z_{in} = r_\pi \| [1/(sC_T) = r_\pi/(1 + sr_\pi C_T)$

  $C_T = C_\pi + C_M$

  $$\Rightarrow v = \frac{Z_{in}}{Z_{in} + R_S} v_s$$

  $$= \frac{r_\pi}{\left(R_S + r_\pi\right)\left[1 + sr_\pi R_S C_T \big/ \left(R_S + r_\pi\right)\right]} v_s$$

  $$v_0 = -g_m \left(R_L \parallel \frac{1}{sC'_M} \right) v = -\frac{g_m R_L}{1 + sR_L C'_M} v$$

- Thus:

$$A_v(s) = \frac{v_0}{v_s}$$

$$= -g_m R_L \frac{r_\pi}{R_S + r_\pi} \frac{1}{\left[1 + sR_S r_\pi C_T / \left(R_S + r_\pi\right)\right]\left(1 + sR_L C'_M\right)}$$

- ***Comparing*** this expression with

$$A_v(s) = \frac{A_{v0}}{\left(1 - s/p_1\right)\left(1 - s/p_2\right)}$$

we note that the ***denominator*** is already in a ***factorized form***

- A_{v0} = ***midband gain*** = $- g_m R_L r_\pi/(R_S + r_\pi)$
- The ***transfer function*** shows that the system has ***two negative real poles*** and ***no zero***

  $\Rightarrow$ ***Information regarding the zero is suppressed by this technique***

- Also, the ***two poles*** obtained by ***this technique*** are ***not identical*** to those obtained from the ***exact analysis***

- Pole ***p*** $_1$ (***p*** $_2$) is referred to as the pole of the ***input*** (***output***) circuit
- Also, $|p_1| << |p_2|$

  $\Rightarrow$ ***p*** $_1$ (***p*** $_2$) is the ***DP*** (***NDP***) of the system
- ***Matching coefficients***:

$$p_1 = -\frac{R_S + r_\pi}{R_S r_\pi}\frac{1}{C_T} = -\frac{1}{\left(R_S \parallel r_\pi\right)\left[C_\pi + \left(1 + g_m R_L\right)C_\mu\right]}$$

$$p_2 = -\frac{1}{R_L C'_M}$$

- Obviously, $|p_1| << |p_2|$

- Thus, ***using DPA***: $f_H = |p_1|/(2\pi)$
- Applying ***this technique*** to the ***previous example***, ***f_H = 3.9 MHz*** and ***NDP frequency = 156 MHz***
  - ***Error of only 2.6% in f_H***, but the ***ease of solution is much more***
- Thus, ***this technique*** is ***quite popular*** in getting a ***quick estimate*** of f_H, even though the ***solution*** may not be ***exact***
- ***Care***: ***The gain in the multiplicative factor is that between the input and output terminals of the capacitor***

- ***The Zero-Value Time Constant (ZVTC) Technique***:
  - ***Gives information only about the DP of the system***
  - ***Suppresses all information regarding other poles and zeros***
  - ***The ease of application of this technique is mind-boggling***

- ***Slightly less accurate***
- ***The maximum error can be as high as 22%***
- ***Underestimates f_H***
  - ***Far better than overestimation and eventually not achieving it***
- ***Applicable only for circuits that have a DP***
  - ***Fortunately, almost all analog circuits of interest do have a DP***

- ***The Algorithm***:
  - ***Null all independent sources to the circuit***
    - ***Short all independent voltage sources***
    - ***Open all independent current sources***
    - ***DO NOT TOUCH DEPENDENT SOURCES***
  - ***Name the capacitors C_i*** (***$i = 1$-n***)
  - ***Consider C_1 and assign zero values to all other capacitors*** (***thus the name***!)
    - Thus, ***except C_1***, ***all other capacitors will open out***
  - ***Determine the Thevenin Resistance*** $\left(R_1^0\right)$ ***across the two terminals of C_1***

- ***Find the time constant τ_1 associated with C_1*** $\left(\tau_1 = R_1^0 C_1\right)$
- ***Repeat for all other capacitors***, ***taking one at a time***, and ***find all the rest of the time constants*** (τ_2, τ_3, …, τ_n)
- Determine the ***net time constant*** τ_{net} by ***summing up*** all the ***individual time constants***

$$\Rightarrow \tau_{net} = \sum_{i=1}^{n} \tau_i$$

- Then the ***Upper Cutoff Frequency*** f_H is simply given by: $f_H = 1/(2\pi\tau_{net})$

- ***Note***: ***The capacitor contributing the largest time constant, in effect, determines f_H***
- ***The technique suppresses all information regarding other poles and zeros***
- Will present ***several examples*** to understand the ***application*** of this ***technique***
- Some ***topologies*** will be appearing ***frequently***, known as ***Standard Forms***, which can be treated as ***individual modules***, and the ***results can be used freely***

- ***CE***:
  - Refer to the ***high-frequency equivalent*** given in the ***exact analysis***
  - ***2 capacitors***: C_π and C_μ

    $\Rightarrow$ ***2 time constants***: τ_1 and τ_2
  - $\boldsymbol{C_\pi}$:
    - ***C_μ opens up***
    - ***By inspection***:

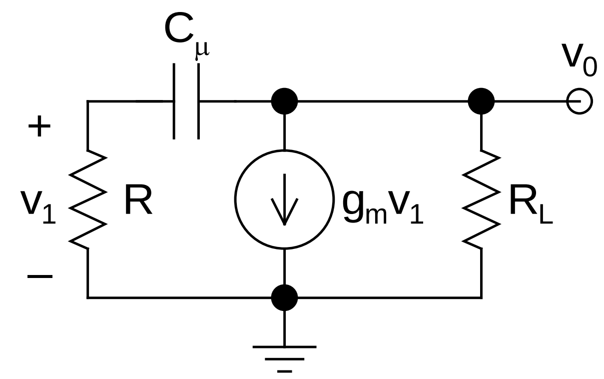
$$R_\pi^0 = R_S \,||\, r_\pi$$

$$\Rightarrow \tau_1 = R_\pi^0 C_\pi$$

- ***C_μ*:**
  - ***C_π* opens up**
  - This is one ***Standard Form***, known as the ***Three-Legged Creature***
  - Show that:

$$R_\mu^0 = R + R_L + g_m R_L R \quad (R = R_S \parallel r_\pi)$$

$$\Rightarrow \tau_2 = R_\mu^0 C_\mu$$



- Thus, $\tau_{net} = \tau_1 + \tau_2$, and $f_H = 1/(2\pi\tau_{net})$
- Note the ***amazing simplicity*** of the analysis

- Putting ***values*** of our previous ***example***:

$$R_\pi^0 = 838.7\ \Omega,\ \ \tau_1 = 8.4\ \text{ns}$$

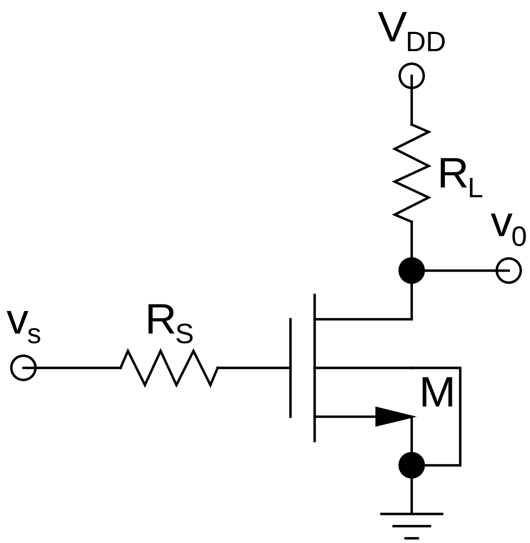
$$R_\mu^0 = 67.4\ \text{k}\Omega,\ \ \tau_2 = 33.7\ \text{ns}$$

$$\Rightarrow \tau_{net} = 42.1\ \text{ns}\ \text{ and }\ f_H = 3.8\ \text{MHz}$$

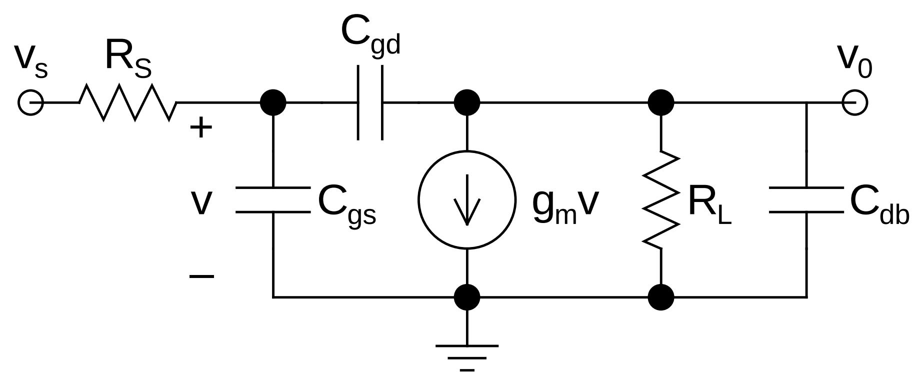
- This is ***identical*** to the ***result*** obtained from the ***exact analysis***, however, at a ***fraction*** of the ***effort***!
- Also, ***τ_2 is the dominant time constant***

  $\Rightarrow$ ***f_H is primarily dictated by C_μ***

- ***CS***:



ac Schematic



High-Frequency Equivalent

- ***C_{sb} absent*** (*Why*?)

- ***3 capacitors***: C_{gs}, C_{gd}, and C_{db}

  $\Rightarrow$ ***3 time constants***: τ_1, τ_2, and τ_3
- ***C_{gs}***:
  - ***C_{gd} and C_{db} open up***
  - ***By inspection***:

    $$R_{gs}^0 = R_S$$

    $$\Rightarrow \tau_1 = R_{gs}^0 C_{gs}$$
- ***C_{gd}***:
  - ***C_{gs} and C_{db} open up***
  - ***By inspection***, it can be ***identified*** as a ***Three-Legged Creature***

- Thus:

$$R_{gd}^{0} = R_S + R_L + g_m R_S R_L$$

$$\Rightarrow \tau_2 = R_{gd}^{0} C_{gd}$$

- ***C_{db}***:
  - ***C_{gs} and C_{gd} open up***
  - ***By inspection***:

$$R_{db}^{0} = R_L$$

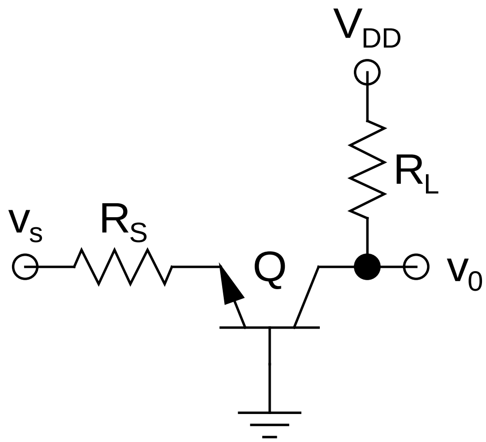
$$\Rightarrow \tau_3 = R_{db}^{0} C_{db}$$

- Thus:

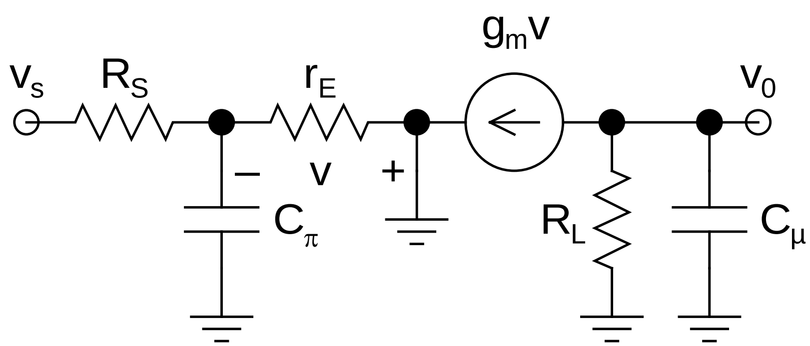
$$\tau_{net} = \tau_1 + \tau_2 + \tau_3, \text{ and } f_H = 1/(2\pi\tau_{net})$$

- ***Mind-bogglingly simple*** - isn't it?

- ***CB***:



ac Schematic



High-Frequency Equivalent

➢ ***Note that there is no input-output coupling capacitor present in this circuit***

$\Rightarrow$ ***Miller effect will be absent***, and the ***circuit will have very high f_H***

- ***C_π***:

  $$R_\pi^0 = R_S \,||\, r_E \quad \text{and} \quad \tau_1 = R_\pi^0 C_\pi$$

- ***C_μ***:

  $$R_\mu^0 = R_L \quad \text{and} \quad \tau_2 = R_\mu^0 C_\mu$$

- Taking the ***values*** of our previous ***example***:

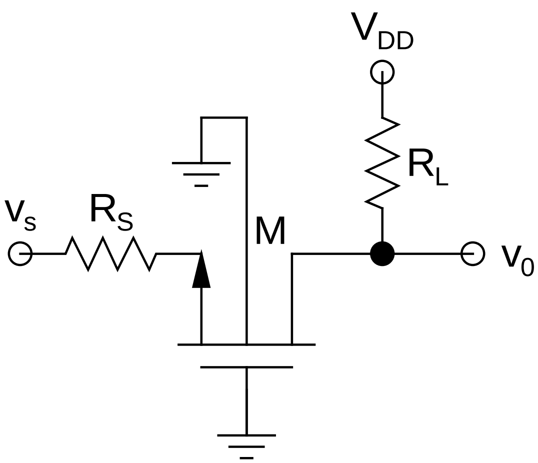
  $$R_\pi^0 = 25.34\ \Omega, \ \ \tau_1 = 0.253\ \text{ns}$$

  $$R_\mu^0 = 2\ \text{k}\Omega, \ \ \tau_2 = 1\ \text{ns}$$

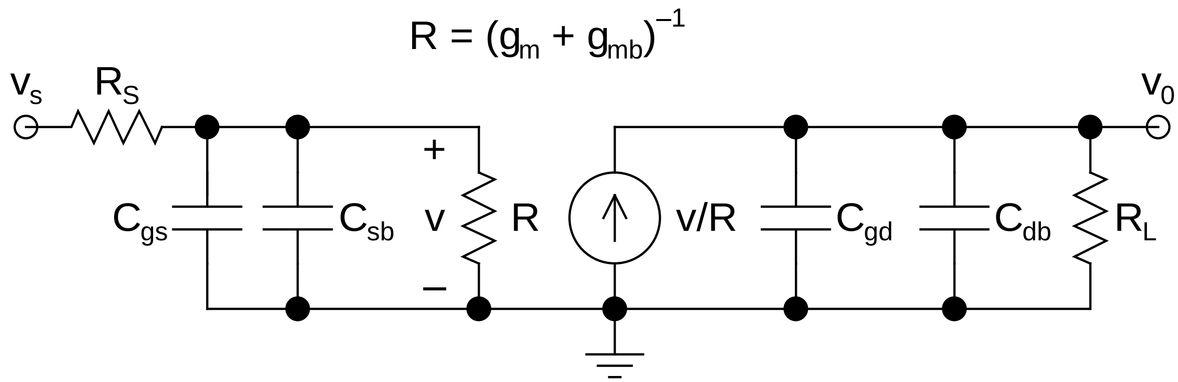
  $$\Rightarrow \tau_{net} = 1.25\ \text{ns} \ \ \text{and} \ \ f_H = 127.3\ \text{MHz}$$

- ***Note the enormous increase of f_H from about 4 MHz for a CE amplifier***

- ***CG***:



ac Schematic

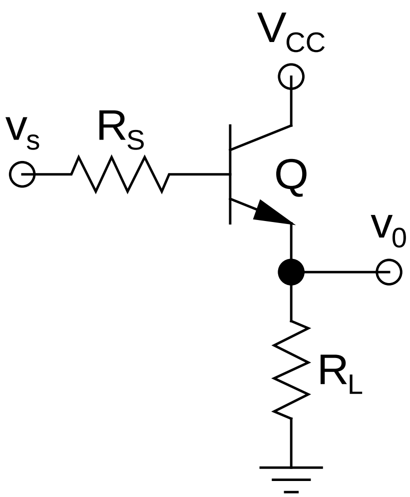


High-Frequency Equivalent

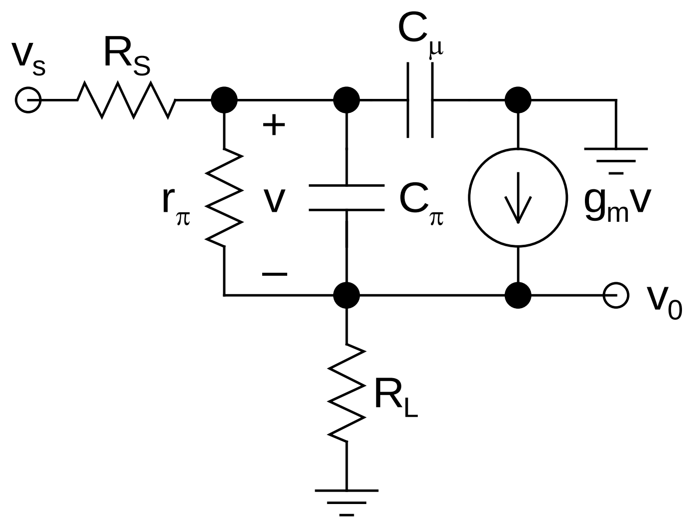
➢ ***Note that all 4 capacitors would be present*** and ***none could be eliminated***

- ***C_{gs} and C_{sb} are in parallel***
  - $\Rightarrow$ ***Can be clubbed to a single capacitor*** $C_1 = C_{gs} + C_{sb}$, with time constant τ_1
- Also, ***C_{gd} and C_{db} can be clubbed to another single capacitor*** $C_2 = C_{gd} + C_{db}$, with time constant τ_2
- ***Again note the absence of any input-output coupling capacitor***
  - $\Rightarrow$ ***This circuit should also have very high f_H***
- ***C_1***: $R_1^0 = R_S \| R$ and $\tau_1 = R_1^0 C_1$
- ***C_2***: $R_2^0 = R_L$ and $\tau_2 = R_2^0 C_2$

- ***CC*:**



ac Schematic



High-Frequency Equivalent

- ***This circuit is slightly more involved* - *can't be done by inspection***
- ***But we will have some other Standard Forms***

- This circuit has a ***peculiar frequency response***
  - ***At midband***:

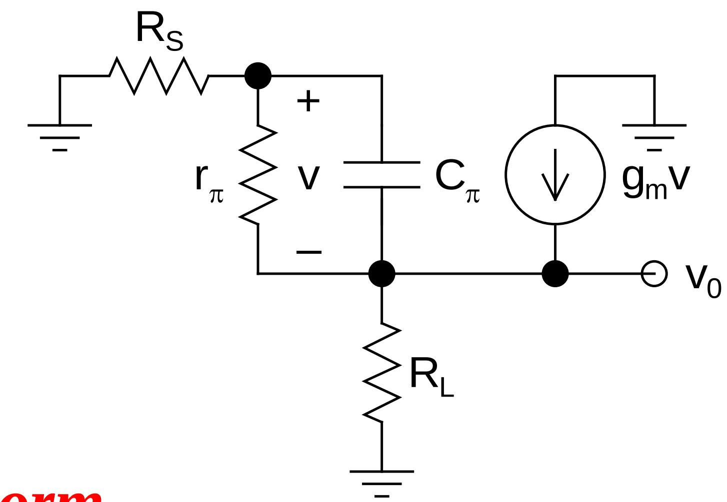
    $A_v = v_0/v_s = [R_L/(R_L + r_E)] \times [R_i/(R_i + R_S)]$

    $R_i = r_\pi + (\beta + 1)R_L$
  - ***Beyond f_H, as $f\uparrow$, reactance of $C_\pi\downarrow$ earlier than that of C_μ*** (***since, in general, $C_\pi >> C_\mu$***)
  - Eventually, ***reactance of C_π would approach zero***, thus ***shorting out r_π***
  - Under this condition, circuit behaves like a ***simple voltage divider*** with a ***gain*** of $R_L/(R_L + R_S)$
  - ***If $f\uparrow$ further***, then ***eventually C_μ also will short out***, and ***v_0 would go to zero***

- Thus, the ***frequency response*** of this circuit looks like a ***staircase***, having ***two steps***

➢ ***C_π***:

- R_π^0 ***can't be obtained by inspection***
- ***Analyze the circuit and show that***:



$$R_\pi^0 = r_\pi \,||\, \left(\frac{R_S + R_L}{1 + g_m R_L} \right)$$

$$\Rightarrow \tau_1 = R_\pi^0 C_\pi$$

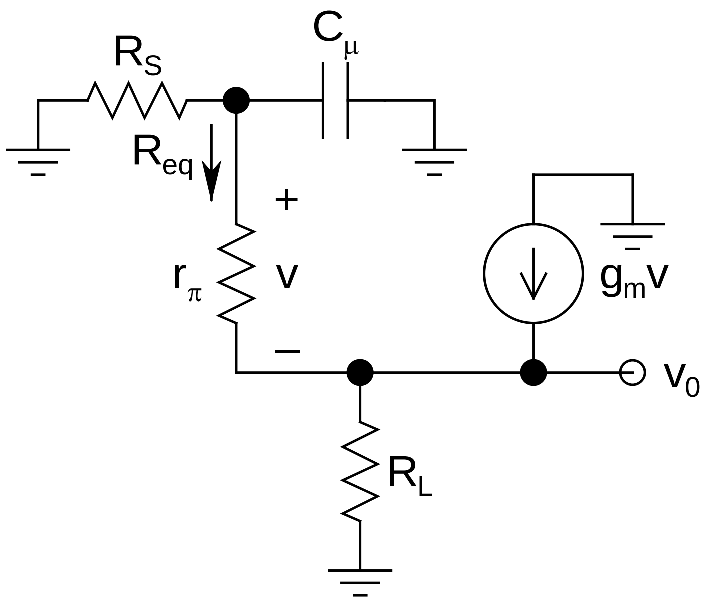
- This is another ***Standard Form*** and the ***topology should be carefully noted***

- ***C_μ***:
  - This is ***relatively straightforward***
  - ***By inspection***:

$$R_{eq} = r_\pi + (\beta + 1) R_L$$

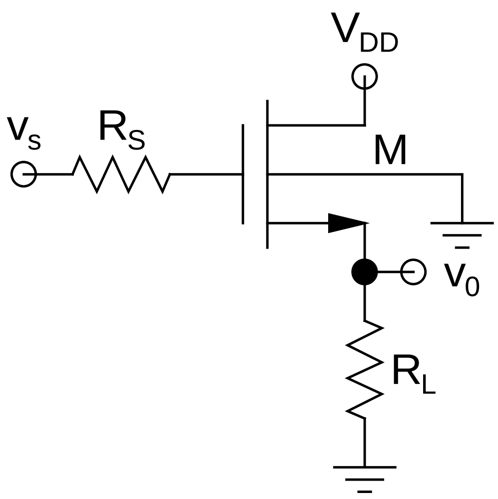
$$R_\mu^0 = R_S \,||\, R_{eq}$$

$$\Rightarrow \tau_2 = R_\mu^0 C_\mu$$

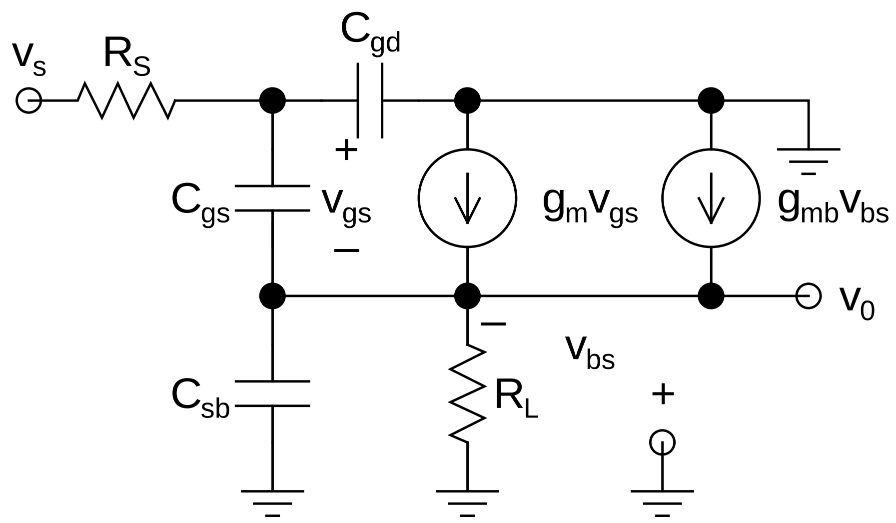


- ***This circuit also has reasonably good frequency response***

- ***CD***:



ac Schematic



High-Frequency Equivalent

- ***C_{db} absent due to obvious reason***

- $v_{bs} = -v_0$

  $\Rightarrow$ ***$g_{mb}v_{bs}$ is simple a conductance g_{mb}, in parallel with R_L***

  $\Rightarrow$ ***Club them to R*** $[= R_L||(1/g_{mb})]$

- ***C_{gs}***:
  - ***Standard Form sans r_π*** (***CC***)

    $$\Rightarrow R_{gs}^0 = \frac{R_S + R}{1 + g_m R}$$

    $$\Rightarrow \tau_1 = R_{gs}^0 C_{gs}$$

- ***C**$_{gd}$*:
  - ***By inspection***:

$$R_{gd}^{0} = R_S$$

$$\Rightarrow \tau_2 = R_{gd}^{0} C_{gd}$$

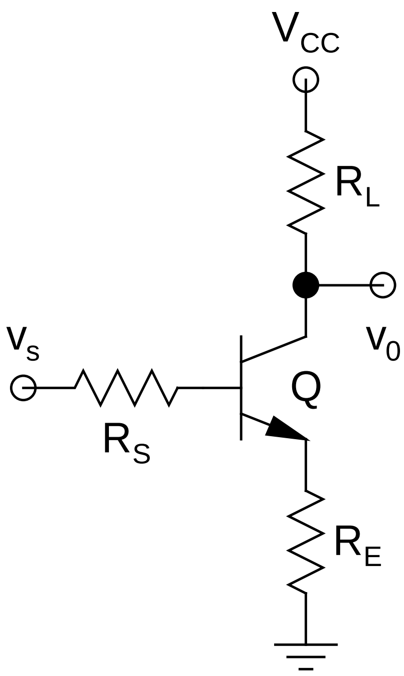
- ***C**$_{sb}$*:
  - ***By inspection***:

$$R_{sb}^{0} = R \,||\, \left(1/g_m\right)$$

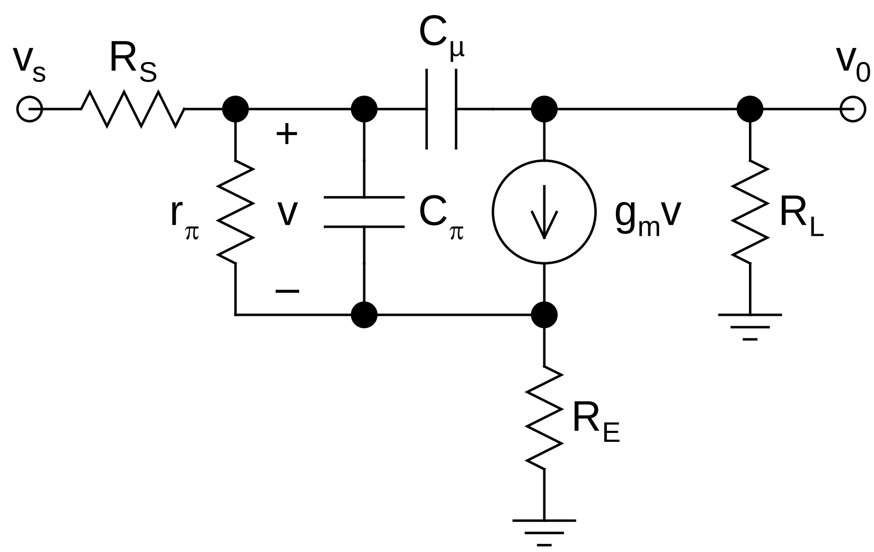
$$\Rightarrow \tau_3 = R_{sb}^{0} C_{sb}$$

- ***Loving it***? :)

- ***CE(D)*:**



ac Schematic



High-Frequency Equivalent

- ***C_π*:**
  - ***Standard Form*** (***similar to CC***, with ***R_L replaced by R_E***)

$$\Rightarrow R_\pi^0 = r_\pi \,||\, \left(\frac{R_S + R_E}{1 + g_m R_E}\right)$$

$$\Rightarrow \tau_1 = R_\pi^0 C_\pi$$

- ***C_μ*:**
  - ***Slightly more complicated***
  - ***Remove C_π*** and ***look across 2 terminals of C_μ***
  - ***Can be represented by a 2-port network***

- ***Show that***:

  $R_{eq} = R_S \| R_\pi$

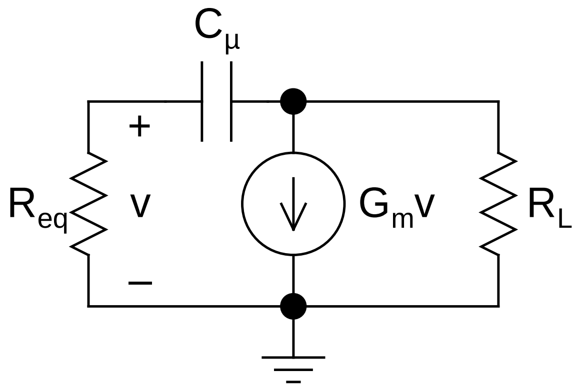
  with $R_\pi = r_\pi(1 + g_m R_E)$

  $G_m = g_m/(1 + g_m R_E)$

- This can be ***easily identified*** as a ***Three-Legged Creature***

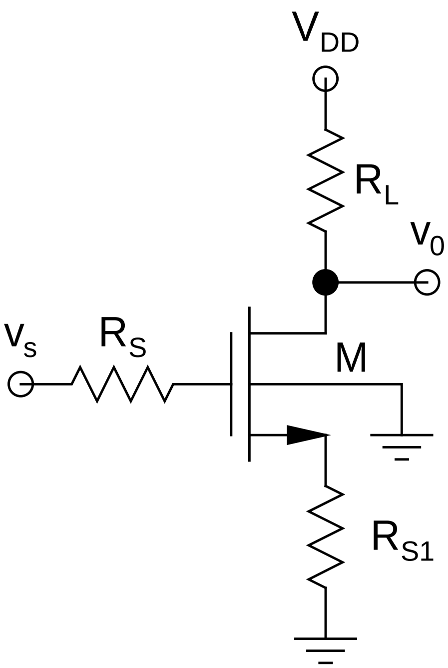
  $$\Rightarrow R_\mu^0 = R_{eq} + R_L + G_m R_{eq} R_L$$

  $$\Rightarrow \tau_2 = R_\mu^0 C_\mu$$

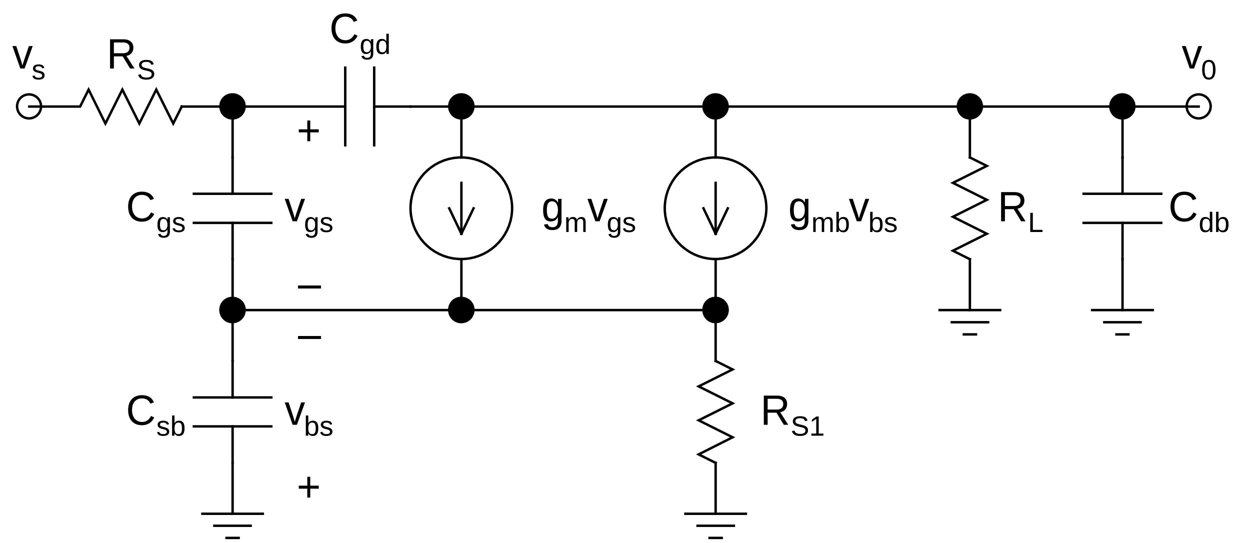


2-Port Representation of a CE(D) Stage

- ***CS(D)*:**

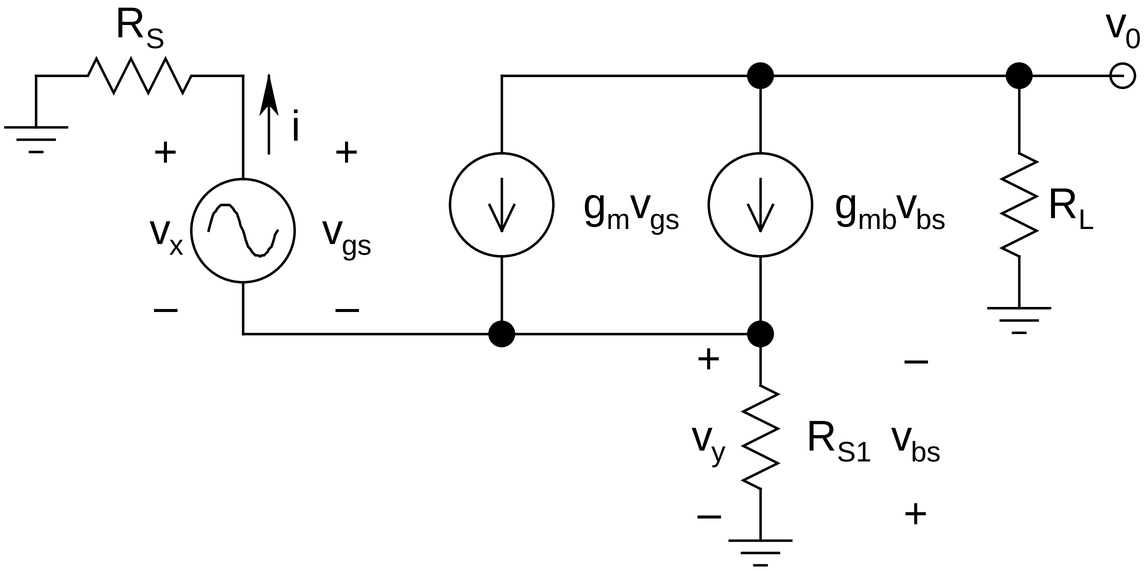


ac Schematic



High-Frequency Equivalent

- ***Inarguably, the most complex module***
  - ***All the capacitors will be present***
  - ***None will have Standard Form***
  - ***Detailed analysis needed for each of them***
- C_{gs}:



- ***Open all other capacitors***
- ***Replace C_{gs} by a voltage source v_x***
- $v_{gs} = v_x$ and $v_{bs} = -v_y$
- $i = (v_x + v_y)/R_S$

$$= g_m v_{gs} + g_{mb} v_{bs} - v_y/R_{S1}$$

$$= g_m v_x - g_{mb} v_y - v_y/R_{S1}$$

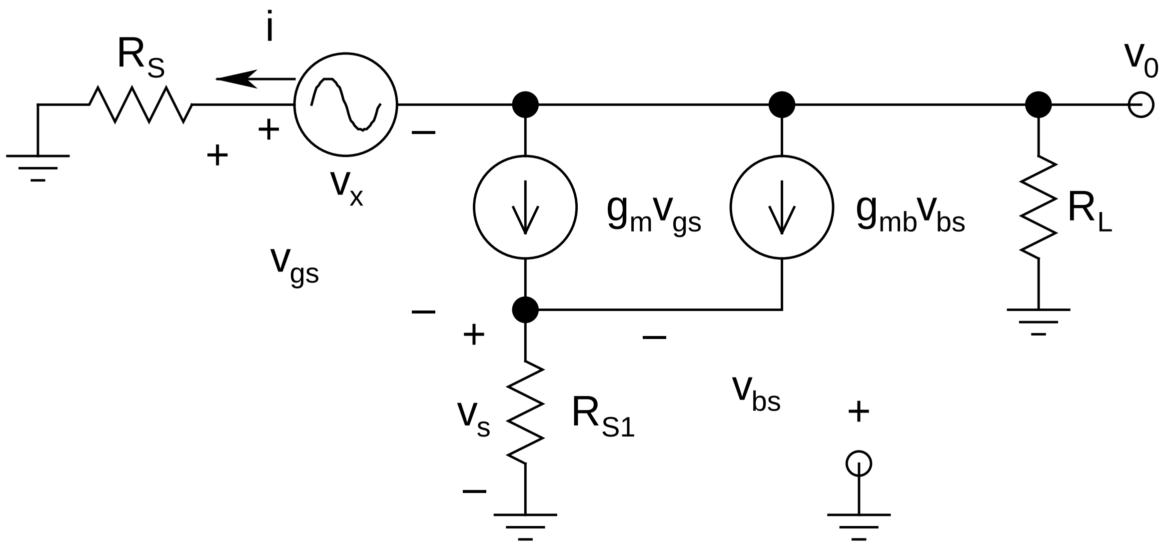
$$\Rightarrow v_y = [R_{S1}(g_m R_S - 1)]v_x/(R_{S1} + R_S + g_{mb} R_S R_{S1})$$

$$\Rightarrow R_{gs}^0 = \frac{v_x}{i} = \frac{R_S + R_{S1} + g_{mb} R_S R_{S1}}{1 + (g_m + g_{mb}) R_{S1}}$$

$$\Rightarrow \tau_1 = R_{gs}^0 C_{gs}$$

- Note that ***if body effect is neglected*** (***g_{mb} ignored***), then it becomes ***identical to that of a CD stage***

➢ C_{gd}:



- *Open all other capacitors*
- *Replace C_{gd} by v_x*
- $v_{gs} = (v_0 + v_x - v_s)$ and $v_{bs} = -v_s$

- $i = (v_0 + v_x)/R_S$
- $v_s = (g_m v_{gs} + g_{mb} v_{bs}) R_{S1}$

  $\Rightarrow v_s = g_m R_{S1}(v_0 + v_x)/[1 + (g_m + g_{mb})R_{S1}]$
- ***KCL at output node***:

  $i + g_m v_{gs} + g_{mb} v_{bs} + v_0/R_L = 0$
- ***The rest of the process involves huge amount of algebra***!
- ***Finally, if done right*** (***check***!)

$$R_{gd}^0 = \frac{v_x}{i} = R_L \left[1 + g_m R_S + \frac{R_S}{R_L} - \frac{(g_m + g_{mb}) g_m R_S R_{S1}}{1 + (g_m + g_{mb}) R_{S1}} \right]$$

$$\Rightarrow \tau_2 = R_{gd}^0 C_{gd}$$

- ***This is by far the most complicated calculation/expression***
- However, an ***exact analysis*** would have yielded a ***4th-order transfer function*** in ω, which had to be ***solved*** to get the ***individual poles***
- ***This is still simpler than that*** :)

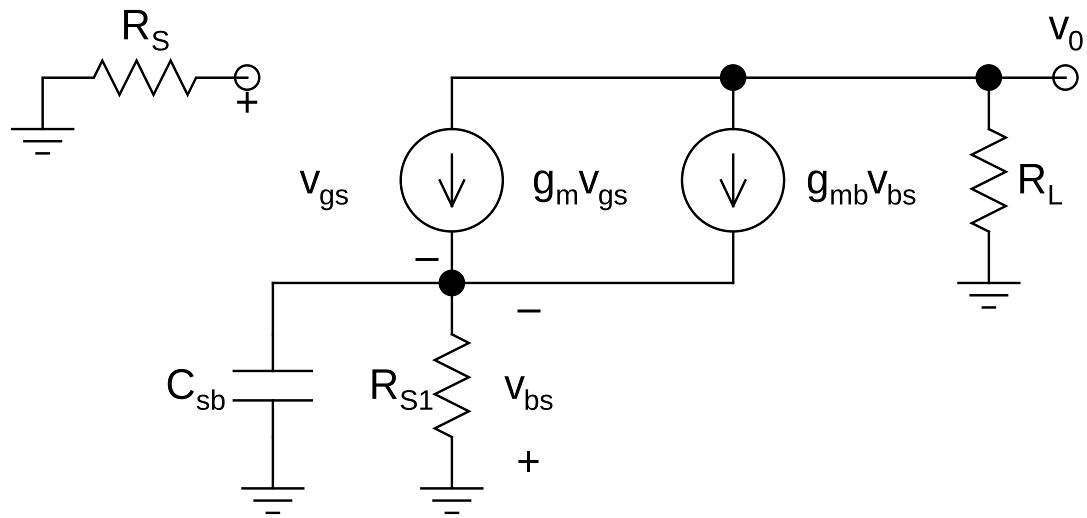
➢ ***C_{db}***:

- ***The easiest of the lot***
- ***By inspection***:

$$R_{db}^{0} = R_{L}$$

$$\Rightarrow \tau_3 = R_{db}^{0} C_{db}$$

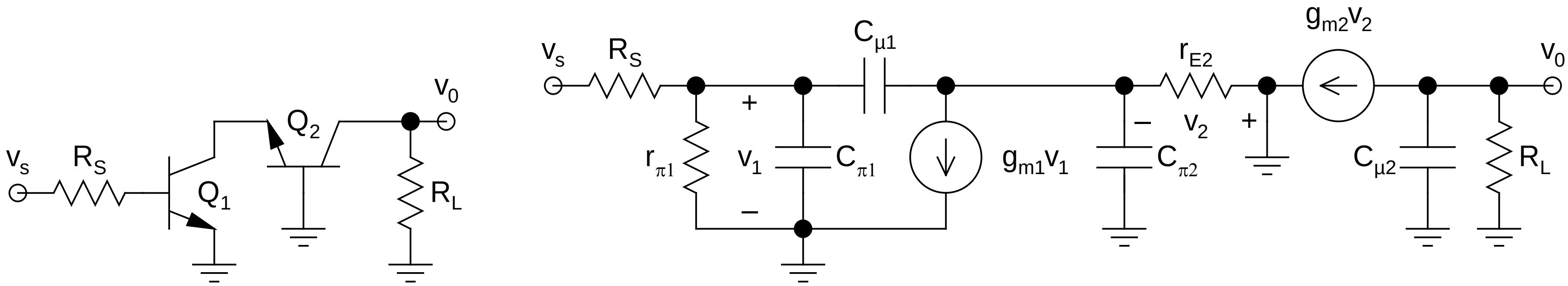
➢ C_{sb}:



- *Analysis of this circuit is pretty straightforward*

$$R_{sb}^{0} = \frac{R_{S1}}{1+\left(g_m + g_{mb}\right)R_{S1}} \quad \Rightarrow \quad \tau_4 = R_{sb}^{0} C_{sb}$$

- ***npn Cascode***:



ac Schematic　　High-Frequency Equivalent

- ***Looks intimidating***, but ***extremely easy to solve*** (***just by inspection***)
- Also known as ***Wideband*** (or ***Broadband***) ***Amplifier*** due to its ***superb frequency response***

- ***Reason***:
  - ***The circuit does have an input-output coupling capacitor*** ($C_{\mu 1}$)
  - ***Miller Effect Multiplication Factor*** (MEMF) of $C_{\mu 1} = (1 - A_{v1})$

    A_{v1} = ***voltage gain of*** $\boldsymbol{Q_1}$ $= - r_{E2}/r_{E1} = -1$

    (***since*** $\boldsymbol{Q_1}$ ***and*** $\boldsymbol{Q_2}$ ***are biased with the same*** $\boldsymbol{I_C}$)

    $\Rightarrow$ Thus, the ***MEMF of*** $\boldsymbol{C_{\mu 1}}$ ***is only 2***
  - For ***NMOS Cascode*** stage, the ***MEMF of*** $\boldsymbol{C_{gd1}}$ of M_1 (***CS stage***) will be $[1 + 1/(1 + \chi_2)]$ (***verify this expression***), which is ***even less than 2***

- ***$C_{\pi 1}$***:
  - ***By inspection***:

$$R_{\pi 1}^{0} = R_S \,||\, r_{\pi 1} \quad \Rightarrow \tau_1 = R_{\pi 1}^{0} C_{\pi 1}$$

- ***$C_{\mu 1}$***:
  - Can be easily identified as the ***Three-Legged Creature***

$$\Rightarrow R_{\mu 1}^{0} = R_{\pi 1}^{0} + r_{E2} + g_{m1} R_{\pi 1}^{0} r_{E2}$$

$$\Rightarrow \tau_2 = R_{\mu 1}^{0} C_{\mu 1}$$

- ***$C_{\pi 2}$***:
  - ***By inspection***:

$$R_{\pi 2}^{0} = r_{E2} \quad \Rightarrow \tau_3 = R_{\pi 2}^{0} C_{\pi 2}$$

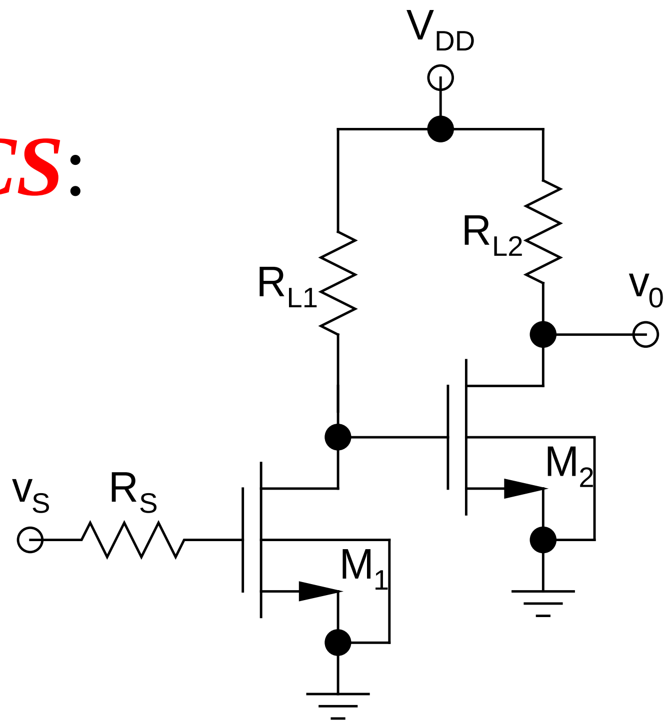
- ***$C_{\mu 2}$***:
  - ***By inspection***:

$$R_{\mu 2}^{0} = R_L \quad \Rightarrow \tau_4 = R_{\mu 2}^{0} C_{\mu 2}$$

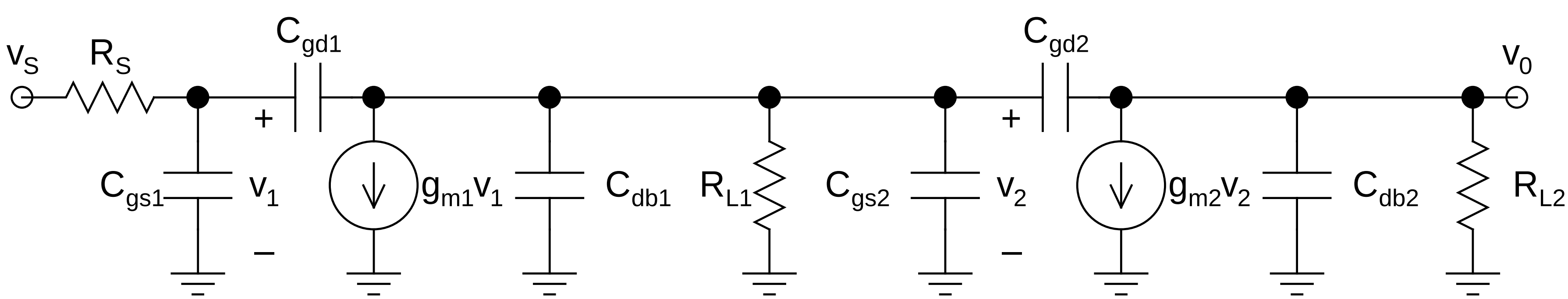
- Generally, for this circuit, $C_{\pi 1}$ is the ***dominant capacitor*** that determines f_H, since it sees the ***largest resistance***
- The ***resistance seen by $C_{\mu 1}$***, which is the ***largest for CE stage***, ***becomes quite small here***, ***due to the low gain of Q_1***
- Note how ***simple*** it is to use this ***technique*** even for ***multi-stage amplifiers***!

- ***NMOS 2-Stage Cascaded CS***:
  - *Except C_{sb}, all other capacitors will be present for both devices*
  - *6 capacitors*

    $\Rightarrow$ *6 time constants*



ac Schematic



High-Frequency Equivalent

- ***Note***: An ***exact analysis*** would have required ***solving*** a ***6th-order equation in*** $\boldsymbol{\omega}$!
- Let's perform a ***quantitative analysis*** of this circuit
- ***Data***: g_{m1} = 3 mA/V, g_{m2} = 6 mA/V, C_{gs1} = 5 pF, C_{gs2} = 10 pF, $C_{gd1} = C_{gd2}$ = 1 pF, $C_{db1} = C_{db2}$ = 2 pF, R_S = 10 kΩ, R_{L1} = 10 kΩ, and R_{L2} = 5 kΩ.
  - ***C_{gs1}***:

$$R_{gs1}^0 = R_S = 10\ \text{k}\Omega \quad \Rightarrow \tau_1 = R_{gs1}^0 C_{gs1} = 50\ \text{ns}$$

- ***C_{gd1}***:

$$R_{gd1}^{0} = R_S + R_{L1} + g_{m1}R_SR_{L1} = 320\ \text{k}\Omega$$

$$\Rightarrow \tau_2 = R_{gd1}^{0}C_{gd1} = 320\ \text{ns}$$

- ***C_{db1} and C_{gs2} in parallel***

$\Rightarrow$ ***Club them to a single capacitor*** $C_3 = C_{db1} + C_{gs2} = 12\ \text{pF}$

$$R_3^0 = R_{L1} = 10\ \text{k}\Omega$$

$$\Rightarrow \tau_3 = R_3^0C_3 = 120\ \text{ns}$$

- ***C_{gd2}***:

$$R^0_{gd2} = R_{L1} + R_{L2} + g_{m2}R_{L1}R_{L2} = 315\ k\Omega$$

$$\Rightarrow \tau_4 = R^0_{gd2}C_{gd2} = 315\ ns$$

- ***C_{db2}***:

$$R^0_{db2} = R_{L2} = 5\ k\Omega$$

$$\Rightarrow \tau_5 = R^0_{db2}C_{db2} = 10\ ns$$

- Thus:

τ_{net} = 815 ns and f_H = 195.3 kHz

- Such a ***low value*** of f_H is the ***result*** of the ***presence*** of a ***large number of capacitors*** in the circuit

- ***Limitations of the ZVTC Technique***:
  - ***One obvious limitation is the suppression of information of all other poles and zeros of the system except the DP***
    - ***This limitation is not that acute since we are actually interested in only the DP, which gives the information about f_H***
  - ***The other limitation is the error, which can reach as high as 22%***
  - However, ***this error is negative***, i.e., ***underestimation*** (far better than ***overestimation***)

- The ***maximum error*** of 22% occurs if the ***actual circuit*** has ***two overlapping poles***
- In ***real situations***, this is ***highly unlikely***, due to the effect of ***pole splitting*** caused by ***compensation*** (***to be discussed in the next chapter***)
- ***The resulting circuit after compensation would have a single DP***

➢ *Proof that the maximum error is 22%*

- Consider a circuit having ***2 negative real poles*** at the ***same angular frequency*** ω_x
- The ***Transfer Function***:

  $A(j\omega) = A_0/(1 + j\omega/\omega_x)^2$

  A_0: ***Midband gain***

  $\Rightarrow |A(j\omega)| = A_0/[1 + (\omega/\omega_x)^2]$
- At the ***upper cutoff frequency*** ω_H, the ***gain would drop to 1/√2 of its maximum value***

  $\Rightarrow 1 + (\omega_H/\omega_x)^2 = \sqrt{2}$

  $\Rightarrow \omega_H = [\sqrt{(\sqrt{2} - 1)}]\omega_x = 0.64\omega_x$

- Now, using the ***ZVTC technique***, the ***net time constant***

$$\tau_{net} = \sum_{i=1}^{n}(-1/p_i)$$

i = ***number of poles***

p_i = ***individual poles***

- For the ***given problem***, $i = 2$ and $p_i = -\omega_x$ (***for both***)
- Thus:

$\tau_{net} = 2/\omega_x$ and $\omega_H = 1/\tau_{net} = 0.5\omega_x$

- Therefore, the ***maximum error is about –22%***
- This being an ***underestimation***, is ***not that dangerous*** :)

- ***Rise/Fall Time***:
  - ***Recall***: ***f_L caused tilt/sag in the output for square-wave input***
  - ***On the other side of the frequency spectrum***, ***f_H causes rise/fall time of the output for square-wave input***
  - ***These two phenomena can be thought of as an interlinking between the analog and digital domains***
  - Assume that a circuit has some ω_H, with the ***corresponding pole*** at p_1 ($= -\omega_H$)

- The ***Transfer Function*** is ***single-pole***:

  $v_0(s)/v_i(s) = A_0/(1 - s/p_1)$

  A_0: ***Midband gain***

- Now, consider v_i to be ***step input*** of ***amplitude*** V_A ($\Rightarrow v_i = V_A/s$)

  $$\Rightarrow v_0(s) = (A_0V_A/s)/(1 - s/p_1)$$
  $$= A_0V_A[1/s - 1/(s - p_1)]$$

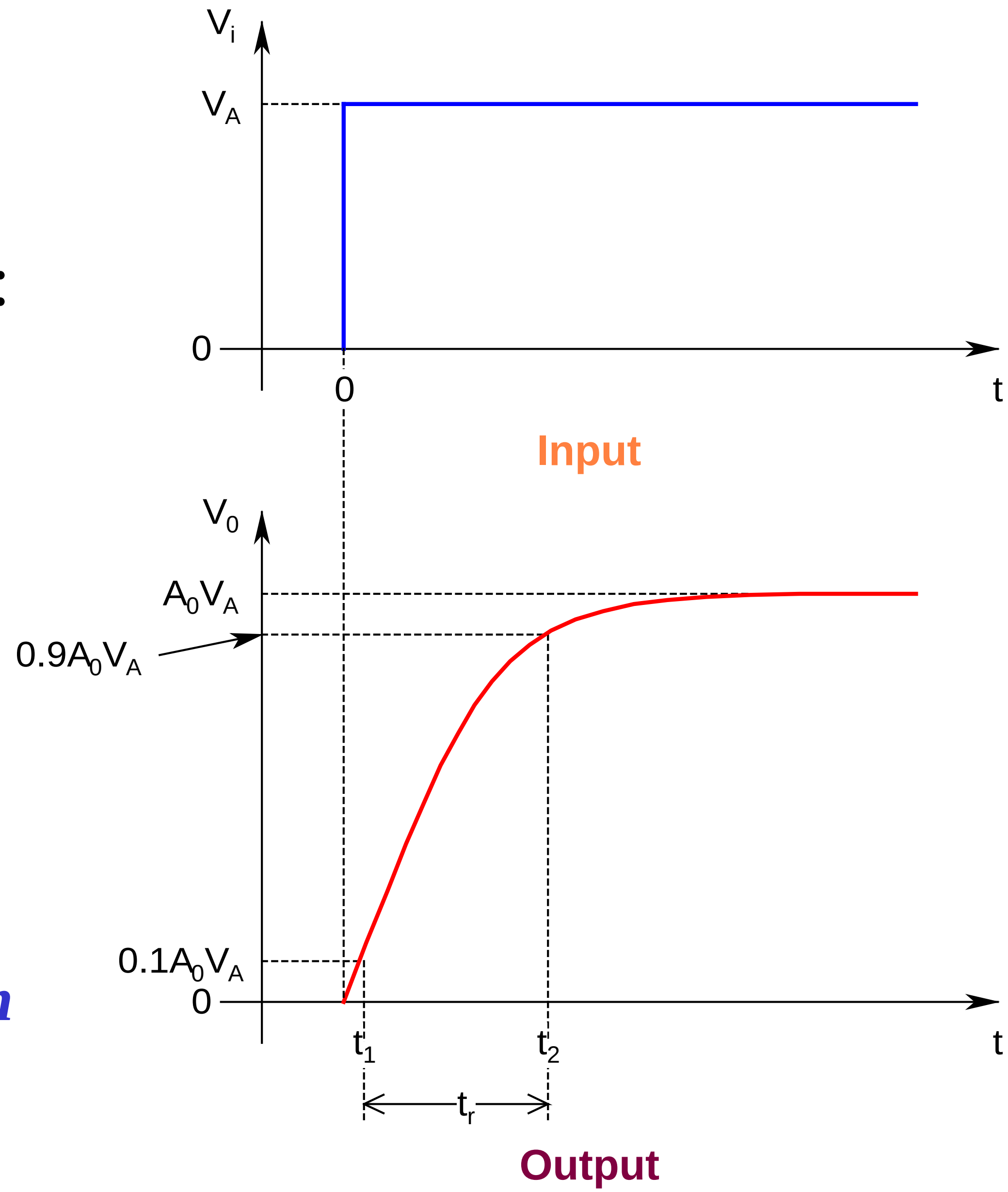
- Taking ***inverse Laplace Transform***:

  $v_0(t) = A_0V_A[1 - \exp(p_1t)]$

  $\Rightarrow$ ***Output approaches its maximum value of A_0V_A with a time constant $1/|p_1|$*** (***p_1 negative***)

➢ *Calculation of Rise/Fall Time*:

- *Time taken for the output to rise (fall) from 10% (90%) to 90% (10%)*
- *Can be calculated from the figure*



- ***At t = t_1***:

  $0.1A_0V_A = A_0V_A[1 - \exp(p_1t_1)]$

  $\Rightarrow t_1 = \ln(0.9)/p_1$

- ***At t = t_2***:

  $0.9A_0V_A = A_0V_A[1 - \exp(p_1t_2)]$

  $\Rightarrow t_2 = \ln(0.1)/p_1$

- Thus, the ***rise time***:

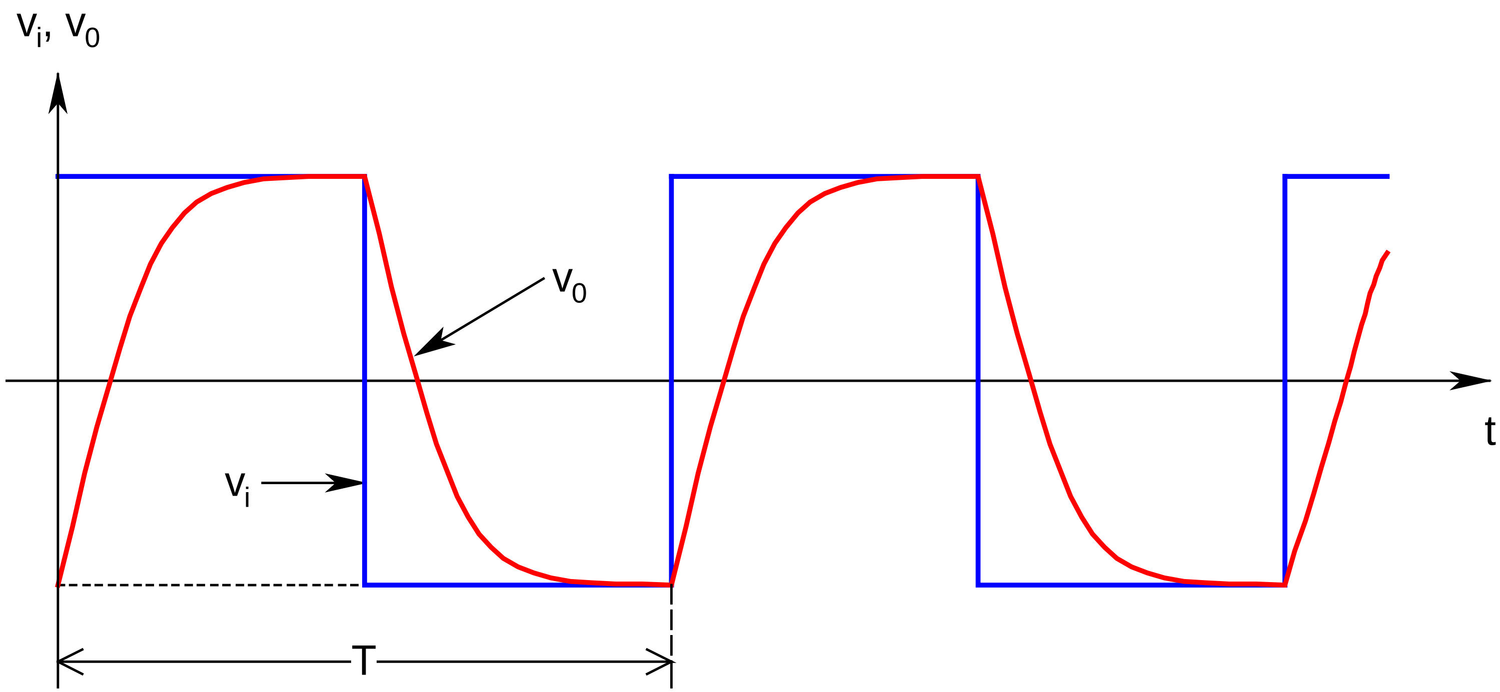
  $t_r = t_2 - t_1 = -2.2/p_1 = 2.2/\omega_H = 0.35/f_H$

- Hence, ***higher the f_H***, ***smaller the t_r***

- ***The same expression holds for the fall time (t_f) as well***

- Thus, circuits having ***high bandwidth*** under ***sinusoidal excitation*** (***analog domain***), will also have ***superb switching characteristics*** under ***square-wave excitation*** (***digital domain***)
- Under ***square-wave excitation***, ***due to*** t_r/t_f, ***enough time should be provided for the transient in the output to get completed***
- ***Rule of Thumb***:
  - ***At least 5 time constants should be allowed for each rising and falling transient***
  - This determines the ***maximum allowable frequency*** of the ***input pulse train***:

$$f_{max} = 1/T_{min} \approx 1/(10\tau) = \omega_H/10 = |p_1|/10$$



Effect of t_r/t_f on the Output for Square-Wave Excitation

- ***As f↑, T↓, V_0 first starts to become trianglular*** (***incomplete transient***), ***then the amplitude starts to drop, and eventually drops to zero*** (***no output at all***!)