# EE 200: Solution Assignment 1

1. Prove de Moivre's formula: $(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$.

   Using the above formula, find the rectangular form and polar form representations of the following complex numbers:

   (a) $\eta_1 = (1 + j\sqrt{3})^{10}$ (b) $\eta_2 = (\sqrt{2} - j\sqrt{2})^{1/5}$

   **Solution**:
   Using Euler's formula: $e^{j\theta} = \cos(\theta) + j\sin\theta$, we have $(\cos(\theta) + j\sin\theta)^n = (e^{j\theta})^n = e^{jn\theta} = \cos(n\theta) + j\sin n\theta$.

   (a) $\cos(\pi/3) = 1/2$ and $\sin(\pi/3) = \sqrt{3}/2$
   Hence, we can write

$$
\begin{aligned}
\eta_1 &= (2[\cos(\pi/3) + j\sin(\pi/3)])^{10} \\
&= 2^{10}\cos(10\pi/3) + j2^{10}\sin(10\pi/3) \\
&= -512 - j886.81 \quad \text{(Rectangular form)}
\end{aligned}
\tag{1}
$$

   $|\eta_1| = \sqrt{(-512)^2 + (-886.81)^2} = 1024$
   $\arg\{\eta_1\} = \tan^{-1}(886.81/512) = 2.0944$

   Hence, $\eta_1 = 1024e^{j2.0944}$ (Polar form)

   (b) $\cos(\pi/4) = \sqrt{2}/2$ and $\sin(\pi/4) = \sqrt{2}/2$
   Hence, we can write

$$
\begin{aligned}
\eta_2 &= (2[\cos(\pi/4) + j\sin(\pi/4)])^{1/5} \\
&= 2^{1/5}\cos(\pi/20) - j2^{1/5}\sin(\pi/20) \\
&= 1.1346 - j0.1797 \quad \text{(Rectangular form)}
\end{aligned}
\tag{2}
$$

$$|\eta_2| = \sqrt{(1.1346)^2 + (-0.1797)^2} = 1.1487$$
$$\arg\{\eta_2\} = \tan^{-1}(-0.1797/1.1346) = -0.1571$$

Hence, $\eta_1 = 1.1487e^{-j0.1571}$ (Polar form)

2. Show that: $\dfrac{1}{1-e^{-j\omega}} = \frac{1}{2}(1 - j\cot^{(\omega/2)})$
   **Solution:**

$$\frac{1}{1-e^{-j\omega}} = \frac{1-e^{j\omega}}{(1-e^{-j\omega})(1-e^{j\omega})} = \frac{1-\cos\omega - j\sin\omega}{2(1-\cos\omega)}$$
$$= \frac{1}{2}\left(1 - j\frac{sin\omega}{1-\cos\omega}\right) = \frac{1}{2}\left[1 - j\frac{2sin(\omega/2)\cos(\omega/2)}{1-(1-2sin^2(\omega/2))}\right]$$
$$= \frac{1}{2}\left(1 - j\frac{\cos(\omega/2)}{\sin(\omega/2)}\right) = \frac{1}{2}\left(1 - j\cot(\omega/2)\right) \tag{3}$$

3. Prove that $\sum_{n=0}^{\infty}\alpha^n = \dfrac{1}{1-\alpha}$, $|\alpha| < 1$, where $\alpha$ is a complex number. Thus, the geometric series is convergent. Show that for $|\alpha| \geq 1$, the above series is divergent.
   **Solution:** Consider,

   $(1 + \alpha + \alpha^2 + \cdots + \alpha^N)(1-\alpha) = (1 + \alpha + \alpha^2 + \cdots + \alpha^N) - \alpha(1 + \alpha + \alpha^2 + \cdots + \alpha^N) = (1 - \alpha^{N+1})$.

   If $|\alpha| < 1$, then $\lim_{N\to\infty}\alpha^{N+1} \to 0$ and $(1-\alpha)\sum_{n=0}^{\infty}\alpha^n = 1$,

   Hence, $S = \frac{1}{1-\alpha}$.

   If $|\alpha| \geq 1$, then $\lim_{N\to\infty}\alpha^{N+1}$ is not finite, and the series does not converge.

4. Prove that

$$\sum_{n=0}^{N-1}\alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & ; \quad \alpha \neq 1 \\ N & ; \quad \alpha = 1 \end{cases} \tag{4}$$

**Solution**: We rewrite the finite sum as:

$$\sum_{n=0}^{N-1} \alpha^n = \sum_{n=0}^{\infty} \alpha^n - \sum_{n=N}^{\infty} \alpha^n = \sum_{n=0}^{\infty} \alpha^n - \alpha^N \sum_{n=0}^{\infty} \alpha^n$$
$$= S - \alpha^N S = S(1-\alpha^N) = \frac{1-\alpha^N}{1-\alpha}, \alpha \neq 1$$
$$\text{For} \quad \alpha = 1, \quad \sum_{n=0}^{N-1} \alpha^n = \sum_{n=0}^{N-1} 1 = N \tag{5}$$

5. Write a MATLAB function *polar2rect* to convert a complex number represented in polar form to its rectangular form. Using this function find the rectangular form of $\eta = 4e^{\frac{j2\pi}{3}}$.
**Solution:**
function [RE, IM]= polar2rect(data)
RE= real(data);
IM= img(data);
Substituting data= $4 \times \exp(2 \times \pi \times j/3)$ yields:
RE= -2.5 and IM= 4.3301.

6. Write a MATLAB function *rect2polar* to convert a complex number represented in rectangular form to its polar form. Using this function find the polar form of $\eta = 2.2 - j6.4$.

   **Solution:**
function [mag, phase]= rect2polar(data)
mag=abs(data);
phase=angle(data);
Substituting data= $2.2 - 6.4j$ yields:
mag= 6.7676 and phase= -1.2397.

7. Write a MATLAB script *gcd2* to determine the GCD of three integers. The input data is the set of integers and the output data is their GCD. Using this code find the GCD of the set of integers $\{12, 32, 96\}$.
**Solution:**
N= input('Enter three integers=');
d= gcd(N(1), N(2));
d= gcd(d, N(3));
disp('GCD=')

```
disp(d);
```
Output: GCD=4