# FEEDBACK, STABILITY, & COMPENSATION

# Feedback

- ***Connection between input and output*** - ***either directly through a wire***, ***or through some circuit elements***
  - ⇒ ***Input and output gets coupled***
  - ⇒ ***Any change in either of them***, ***affects the overall behavior***
- ***2 Types***:
  - ➢ ***Negative***
  - ➢ ***Positive***

- ***Negative Feedback***:
  - ➢ ***Output fed back to input in such a way that it reduces net input***

    $\Rightarrow$ ***Causes a reduction in the output***
  - ➢ Known as ***Degenerative Feedback***
- ***Positive Feedback***:
  - ➢ ***Output fed back to input in such a way that it increases net input***

    $\Rightarrow$ ***Causes an increase in the output***
  - ➢ Known as ***Regenerative Feedback***

- ***Properties of Negative Feedback***:
  - ➢ *Reduction in gain*

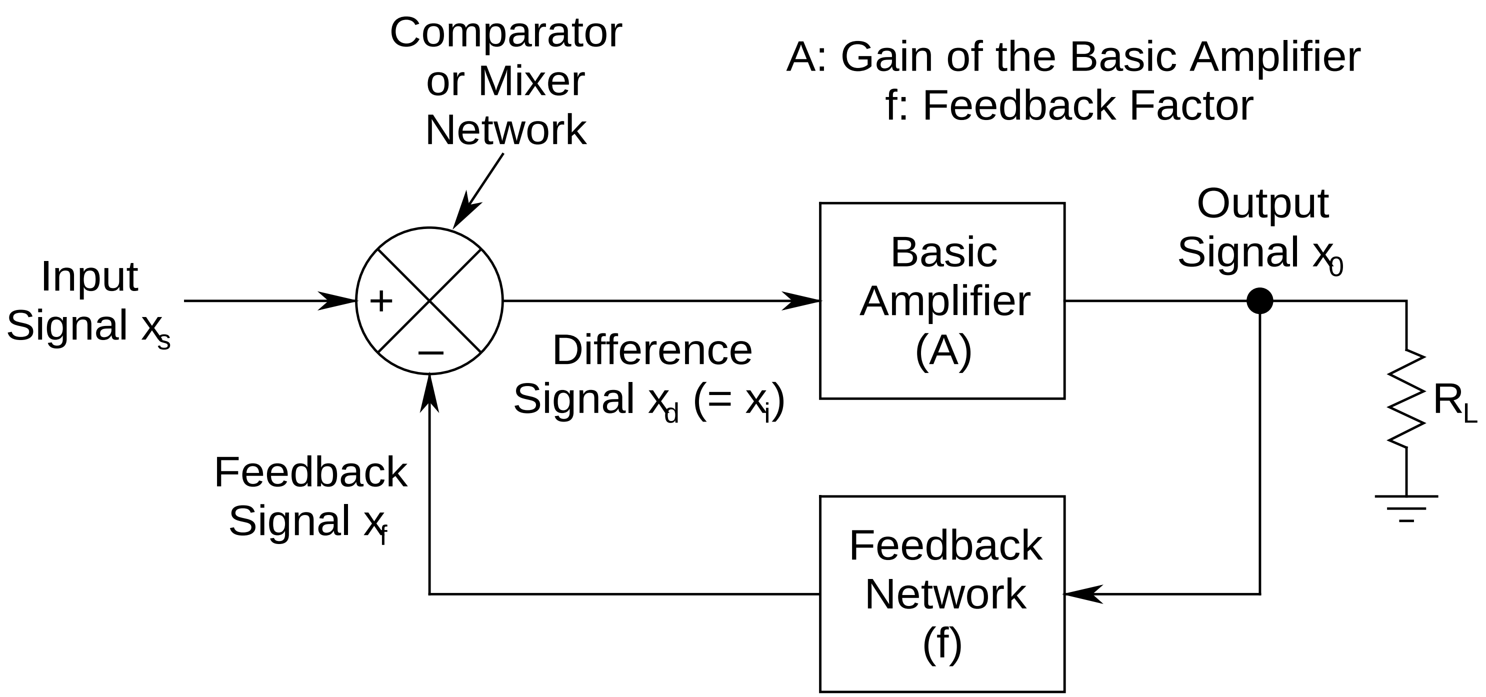
    $\Rightarrow$ *Improvement in bandwidth*

    (*Due to constant GBP*)
  - ➢ *Tailoring of input and output resistances*
  - ➢ *Desensitization of gain*
    - *Gain becomes almost independent of the properties of the active device*
  - ➢ *Minimization of frequency and phase distortion*

- ***Reduction in nonlinear distortion***
  - ***By suppression of harmonics present in the output***
- ***Reduction of noise***
- ***If not properly designed**, **can have problem of stability***

- ***Properties of Positive Feedback***:
  - ***Inherently unstable***
    - ***Due to its regenerative nature***
    - ***This property can be effectively utilized in the design of oscillators**, **which do not need any input***

- ***Mathematical Foundation of Negative Feedback***:




**Block Schematic of a Negative Feedback System**

- ***3 Main Blocks***:
  - ***The Basic Amplifier*** (***Gain A***)
  - ***The Feedback Network*** (***Feedback Factor f*** )
  - ***The Mixer*** (***note the negative sign***)
- ***Defining Relations***:
  - ***Input Signal*** $x_s$
  - ***Output Signal*** $x_0 = Ax_i$
  - ***Feedback Signal*** $x_f = fx_0$
  - ***Difference Signal*** $x_d = x_i = x_s - x_f$
- ***Gain with feedback***: $A_f = x_0/x_s$

- Thus:

$$A_f = \frac{x_0}{x_s} = \frac{x_0}{x_i}\frac{x_i}{x_s} = A\frac{x_s - x_f}{x_s} = A\left(1 - \frac{x_f}{x_s}\right)$$

$$= A\left(1 - \frac{x_f}{x_0}\frac{x_0}{x_s}\right) = A\left(1 - fA_f\right)$$

- Gives the ***fundamental expression*** for ***negative feedback***:

$$A_f = \frac{A}{1 + fA}$$

- ***Some Definitions***:
  - ***Loop Gain*** (L) = fA
  - ***Return Difference*** (D) = 1 + L
  - ***Amount of Feedback*** (N) = 20 $\log_{10}$D (dB)
- ***Positive Feedback***:
  - ***Output fed back to the input through the mixer***, ***but now with a positive sign***

    $\Rightarrow$ ***Feedback signal gets added to the input signal***

- ***Show that under this condition***:

$$A_f(j\omega) = \frac{A(j\omega)}{1 - f(j\omega)A(j\omega)} = \frac{A(j\omega)}{1 - L(j\omega)}$$

- This is a ***general expression***, taking both A and f as ***frequency dependent***
- Note: As $L \rightarrow 1$, $A_f \rightarrow \infty$
  - ***Implies that output is possible even without any input***
  - This is the ***basic principle of oscillation***

- ***Conditions for Oscillation***:

  ***Barkhausen's Criteria***:

  ➢ ***L becoming unity implies that the signal has completely regenerated itself while traversing once through the loop***

  $\Rightarrow$ ***There is no need for any input any more***, ***since the loop has become self-sustained***!

  ➢ ***Since A and f are frequency dependent***, hence, ***there may exist a frequency*** $\omega_0$, at which:

  $$L(j\omega_0) = f(j\omega_0)A(j\omega_0) = 1$$

- Since $\omega_0$ is a ***particular frequency***, for which ***this condition holds***, hence, the output will be a ***pure sinusoid*** of ***this frequency***
    - Similar to ***picking out*** $f_0$ only from a ***Fourier Spectrum***
    - This phenomenon is known as ***Sinusoidal Oscillation***
- German physicist ***Heinrich Georg Barkhausen*** summed this up by ***two conditions***, came to be known as the ***Barkhausen's Criteria***:

1. $|L(j\omega_0)| = 1$ and
2. $\angle L(j\omega_0) = 0°$

- ***Barkhausen's Criteria in words***:

  ***For a feedback system to oscillate, the magnitude of the loop gain must at least be unity, and the total phase shift around the loop should be 0° or 360°***

- ***If these criteria are satisfied exactly***, then the ***oscillations would go on forever***, and ***can be stopped only by shutting the power off for the system***

- However, for ***practical circuits***, the ***exact conditions for oscillations*** are ***very difficult to achieve***

- If $|L|$ becomes ***slightly less than 1***, but $\angle L$ is ***exactly 0°***, then with ***each pass around the loop***, the ***amplitude of oscillation*** would keep on ***going down***, and eventually, it will ***die down*** on its own
  - Thus, ***under this condition***, ***sustained sinusoidal oscillation won't be achieved***
- On the other hand, if $|L|$ becomes ***slightly larger than unity***, but $\angle L$ is ***exactly 0°***, then with ***each pass around the loop***, the ***amplitude*** of the signal will ***keep on growing***
  - Will eventually ***get limited*** by the ***nonlinearities*** present in the circuit

# Stability

- ***2 Types of Systems***:
  - ***Stable***
  - ***Unstable***
- ***Stable System***:
  - ***Any transient disturbance would result in a response that will die down with time***
  - ***The system will be able to get rid of the disturbance on its own***

- ***Unstable System***:
  - ***Any transient disturbance would result in a response that will persist or even blow up with time***
    - ***Eventually gets limited by the nonlinearities of the system***
  - ***Positive feedback systems are inherently unstable***
    - ***They are designed as such***, e.g., ***oscillators***
  - ***Negative feedback systems are inherently stable***

- ***However, there may be situations when they may become unstable and break out into spontaneous oscillations***
- ***Potentially dangerous situation***, and the ***system should be protected against it***

- ***How does a negative feedback system become unstable***?
  - Write the ***loop gain*** expression in ***polar form***:

$$L(j\omega) = f(j\omega)A(j\omega) = |f(j\omega)A(j\omega)|\exp[j\phi(\omega)]$$

$\phi(\omega)$: ***Frequency dependent phase of the system***

- Consider a ***particular frequency*** $\omega_x$, at which $\phi(\omega_x) = 180°$
- At $\omega_x$, L would be a ***real number*** with ***negative sign***

  $\Rightarrow$ ***The feedback turns positive at this frequency***
- ***3 conditions may arise at $\omega_x$:***
  - $|L| < 1$:
    - $A_f(j\omega_x) > A(j\omega_x)$, but the ***system will be stable***
  - $|L| = 1$:
    - $A_f(j\omega_x) \rightarrow \infty$, and ***output will appear without any input*** $\Rightarrow$ ***Oscillator***

- $|L| > 1$:
  - $A_f(j\omega_x) < A(j\omega_x)$, but the ***output will oscillate with gradually increasing amplitude***, and will ***eventually get limited by the nonlinearities present in the system***

- Thus, for a ***negative feedback system*** to turn into a ***positive feedback one***, the ***loop gain*** ($L = fA$) being ***equal to or less than –1*** is a ***sufficient and necessary condition***
- ***For this to happen***, the ***magnitude of the loop gain*** ($L$) ***should be equal to or greater than unity***, and the ***total phase around the loop should be 180°***

# The Complex Frequency & The s-Plane

- Needed to understand the concept of ***stability*** of a system
- ***s-Plane***: ***Complex Frequency Plane***
- Consider a ***sinusoidal signal*** with an ***exponential envelope***:

$$v(t) = V_M[\cos(\omega t + \phi)]\exp(\sigma t)$$

$V_M$: ***Amplitude***

$\phi$: ***Phase***

$\sigma$: ***Coefficient*** of the ***exponential enevelope***, having ***unit*** of ***time inverse*** (similar to ***frequency***)

- For ***positive*** $\sigma$, the signal will ***keep on growing with time***
- For ***negative*** $\sigma$, the signal would ***decay exponentially all the way to zero***

- ***3 interesting scenarios***:

1. $\sigma = \omega = 0$:

   $\Rightarrow v(t) = V_M \cos\phi$

   $\Rightarrow$ ***DC signal*** (***constant***)

2. $\sigma = 0$:

   $v(t) = V_M[\cos(\omega t + \phi)]$

   $\Rightarrow$ ***Normal ac signal***

3. $\omega = 0$:

   $v(t) = [V_M \cos\phi]\exp(\sigma t)$

   $\Rightarrow$ ***Exponential signal*** (***increasing/decreasing with time*** for ***positive/negative $\sigma$***)

- A normal ***sinusoidal voltage*** v(t), having ***angular frequency*** $\omega$ and ***phase*** $\phi$, can be represented in ***polar form*** as:

$$v(t) = V_M \exp[j(\omega t + \phi)] \qquad (1)$$

- Comparing Eq.(1) with ***scenario 3***, we note that their ***functional forms*** are the ***same***

$\Rightarrow$ $\sigma$ can be thought of as a ***frequency***, and is referred to as the ***neper frequency***, with unit of ***nepers/sec***

➢ However, this definition is ***not much used***

- Note also that the ***comparison*** of the two expressions show that $\sigma$ is actually an ***imaginary number***
  - This needs further ***exploration***
- Eq.(1) can be written in ***polar form*** as:

$$
\begin{aligned}
v(t) &= (V_M/2)[\exp\{j(\omega t + \phi)\} + \exp\{-j(\omega t + \phi)\}] \\
&= [(V_M/2)\exp(j\phi)]\exp(j\omega t) \\
&\quad + [(V_M/2)\exp(-j\phi)]\exp(-j\omega t) \\
&= A\exp(s_1 t) + B\exp(s_2 t) \qquad (2)
\end{aligned}
$$

$s_1 = j\omega$ and $s_2 = -j\omega$

$A = (V_M/2)\exp(j\phi)$ and $B = (V_M/2)\exp(-j\phi)$

$\Rightarrow$ ***$s_1$ and $s_2$ as well as A and B are complex conjugates***

$\Rightarrow$ The 2 terms of Eq.(2) are also ***complex conjugates***, with their ***sum*** being a ***real number***

- Similarly, a ***sinusoidal signal*** with an ***exponential envelope*** can be expressed by:

$$v(t) = [(V_M/2)\exp(j\phi)]\exp[(\sigma + j\omega)t] + [(V_M/2)\exp(-j\phi)]\exp[(\sigma - j\omega)t]$$

$$= A\exp(s_1 t) + B\exp(s_2 t) \qquad (3)$$

- ***Matching coefficients*** of Eq.(3), we get a ***complex pair of frequencies***:

  $s_1 = (\sigma + j\omega)$ and $s_2 = (\sigma - j\omega)$

  which are also ***complex conjugates***
- Thus, ***sinusoidal signals*** having ***complex envelopes***, can be expressed in terms of a ***complex frequency*** s
- s has ***both real and imaginary parts*** (***$\sigma$ and $j\omega$ respectively***)

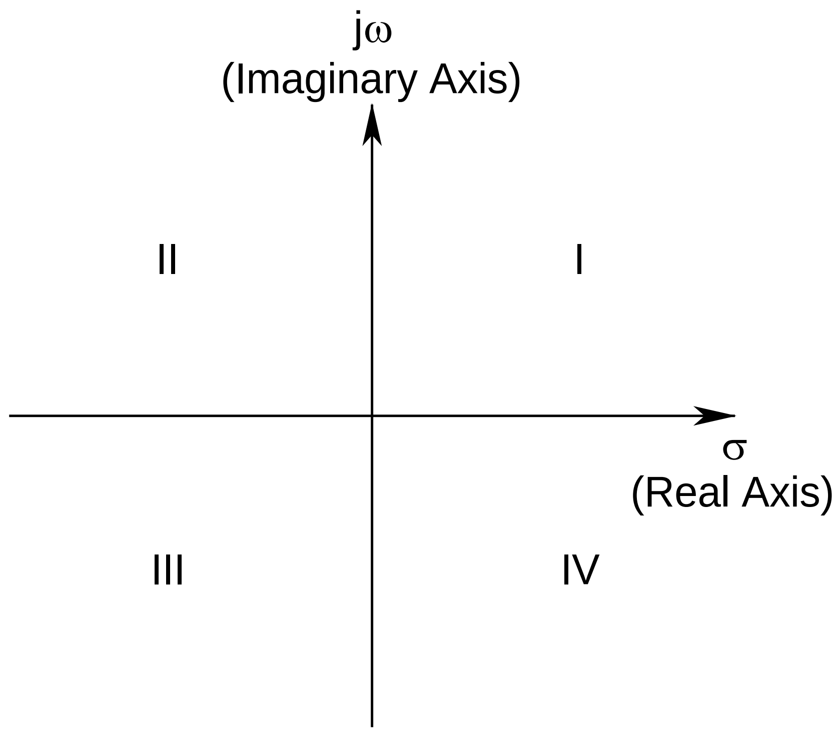
- s is defined by:

  $s = \sigma \pm j\omega$

  $\sigma$: ***Real part*** - dictates the ***exponential rise/fall*** of the signal

  $\omega$: ***Imaginary part*** - ***actual angular frequency***, describes the ***sinusoidal variation*** of the signal

- s is represented in a ***graphical form*** as a 2D plane, with ***$\sigma$ plotted along the x-axis*** (known as the ***real axis***), and ***$\omega$ plotted along the y-axis*** (known as the ***imaginary axis***)




**The Complex Frequency Plane (s-Plane)**

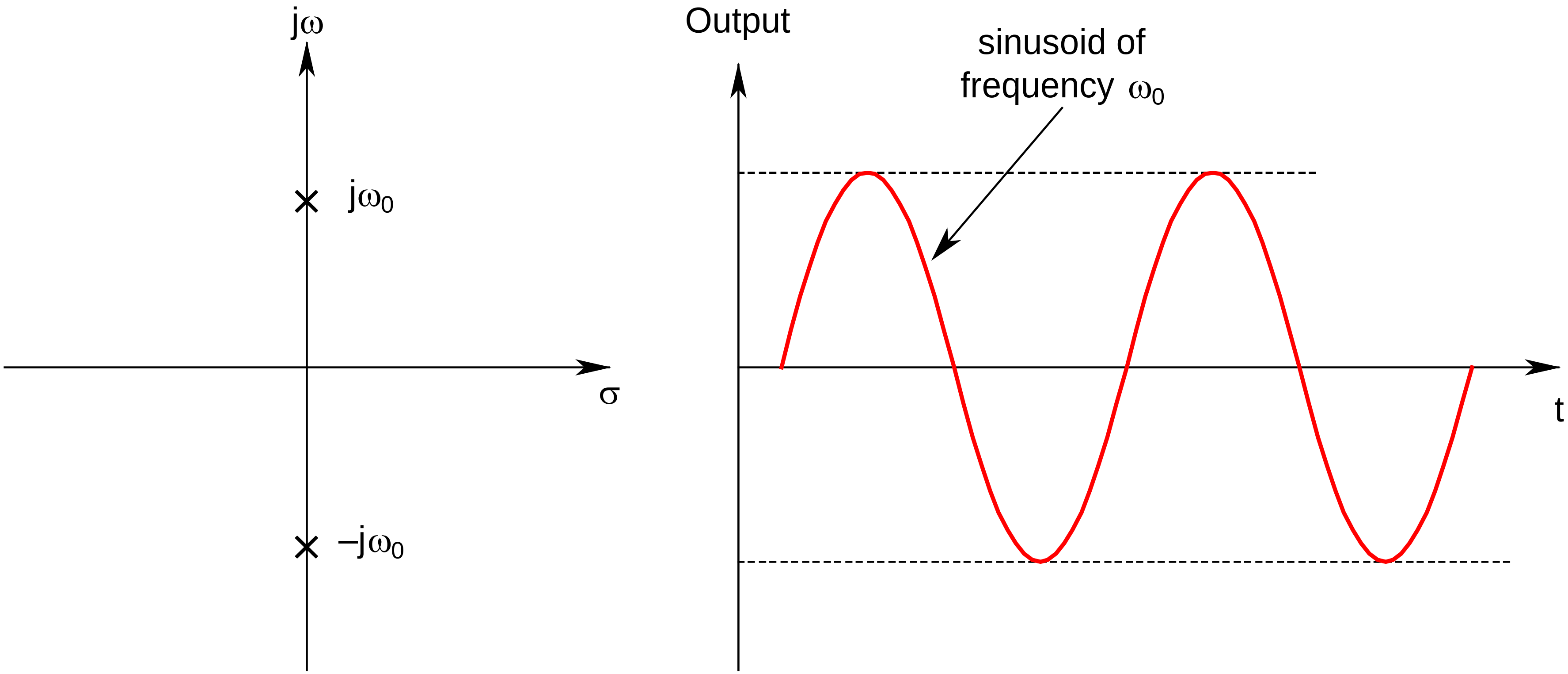
- ***Poles of a transfer function can lie anywhere on this plane***
- If $\sigma = 0$, ***poles lie on the*** $j\omega$ ***axis***

  $\Rightarrow$ ***Perfect sinusoidal response***
- If $\omega = 0$, ***poles lie on the*** $\sigma$ ***axis***

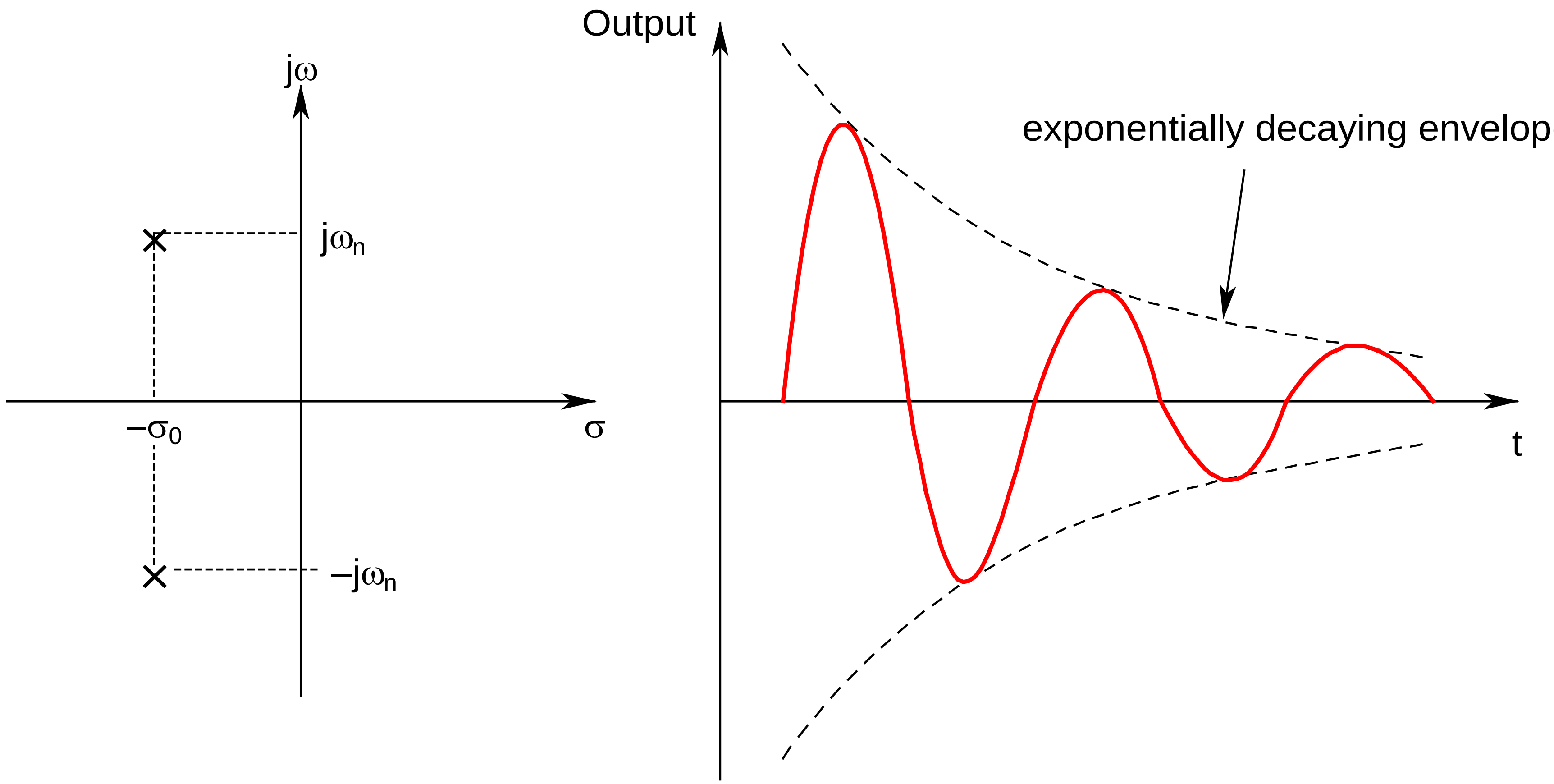
  $\Rightarrow$ ***Pure exponential response***
- If a pole has ***both real and imaginary parts***, then the ***response*** would be either an ***exponentially increasing or decreasing sinusoid***

- ***Pole Location & Stability***:
  - ***Locations of the poles in the s-plane governs the stability of the system***
  - We will consider ***3 cases***:
    - ***Complex conjugate poles without any real part***
    - ***Complex conjugate poles with negative real part***
    - ***Complex conjugate poles with positive real part***

- ***Complex conjugate poles $s_1$ (= $j\omega_0$) and $s_2$ (= $-j\omega_0$), without any real part*:**
  - $\Rightarrow$ ***Undamped sinusoidal response***
  - $\Rightarrow$ ***Perfectly stable system***

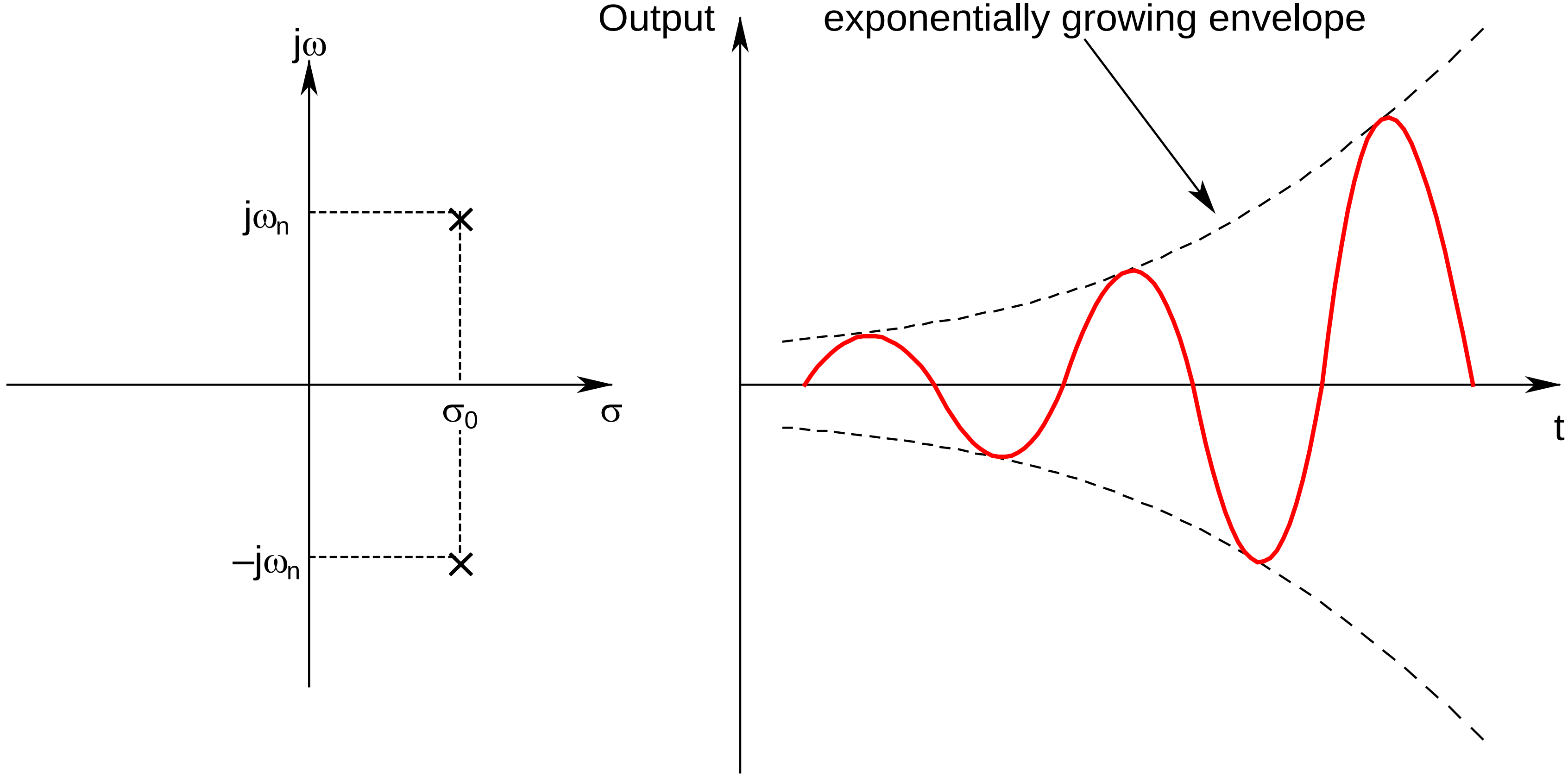


- ***Complex conjugate poles [$s_1 = (\sigma_0 + j\omega_n)$ and $s_2 = (\sigma_0 - j\omega_n)$], with negative real part ($\sigma_0$ negative)***:
  - ***Poles lie in the left-half plane*** (***LHP*** - ***Quadrants II and III***)
  - ***Response to any transient disturbance will be sinusoidal***, but ***with an exponentially decaying envelope***
  - ***Such systems also are stable or well-behaved***




**Response to Transient Disturbance of a System Having Poles in the LHP (Stable System)**

- ***Complex conjugate poles [$s_1 = (\sigma_0 + j\omega_n)$ and $s_2 = (\sigma_0 - j\omega_n)$], with positive real part ($\sigma_0$ positive)***:
  - ***Poles lie in the right-half plane*** (***RHP*** - ***Quadrants I and IV***)
  - ***Response to any transient disturbance will still be sinusoidal***, but now ***with an exponentially rising envelope***
  - ***The system now is NOT well-behaved***, rather ***ill-behaved***, and an ***unstable system***




**Response to Transient Disturbance of a System Having Poles in the RHP (Unstable System)**

# Transfer Function & Stability

- There is a ***strong correlation*** between the ***transfer function*** and ***stability*** of a system
- ***Single-Pole System***:
  - ***Transfer function*** with a ***negative real pole*** at $\omega_p$:

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_p}$$

$A_0$: ***Low-Frequency Gain***

- Now, assume that the ***system*** is ***connected*** in a ***feedback loop***, with the ***feedback network*** having ***feedback factor*** f

  $\Rightarrow$ The ***closed-loop transfer function***:

$$A_f = \frac{A_{f0}}{1 + j\omega/\omega_{pf}}$$

$$A_{f0} = A_0/(1 + fA_0) \text{ and } \omega_{pf} = \omega_p(1 + fA_0)$$

- The ***gain with feedback reduces by the same amount as the bandwidth gets increased***, ***keeping the GBP constant***

- Thus, the ***new pole frequency*** is D (the ***return difference***) times the ***old pole frequency***

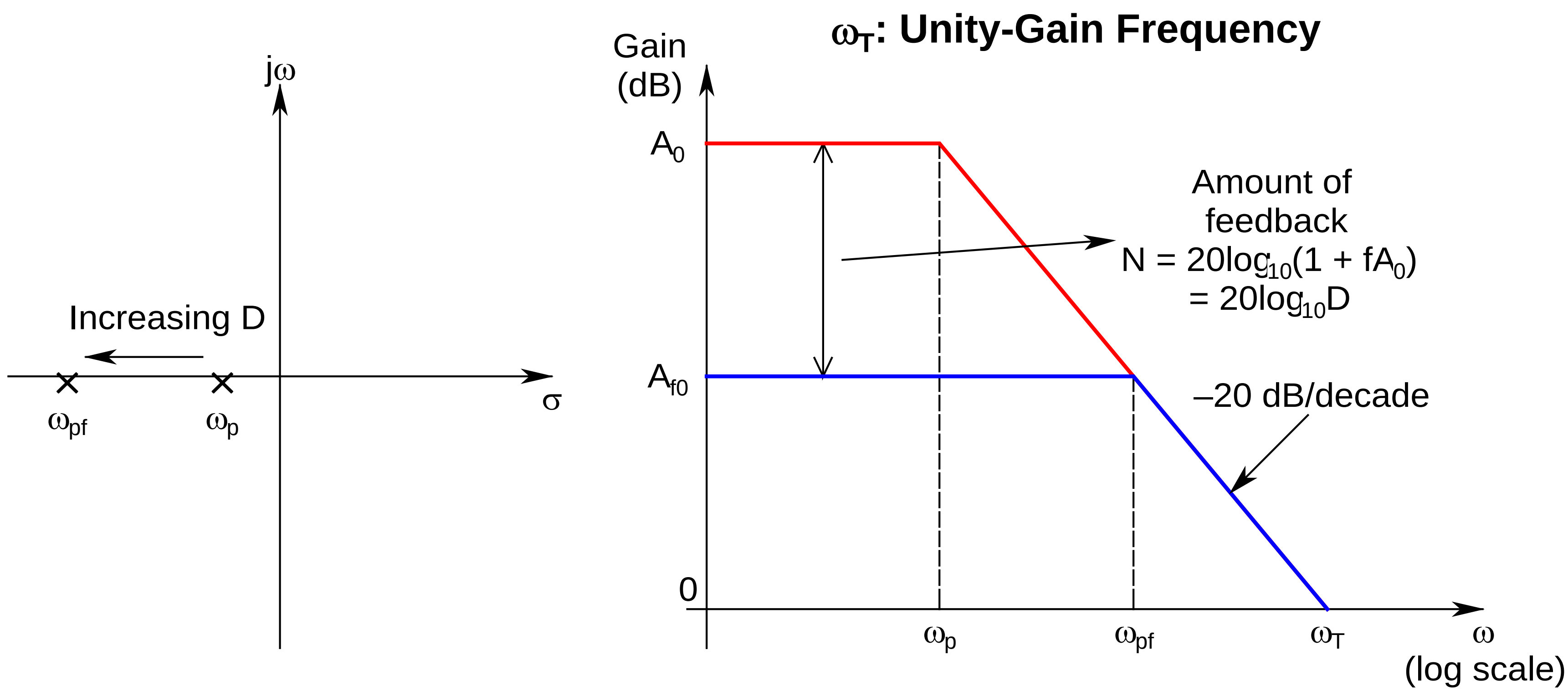
  $\Rightarrow$ It shifts ***left*** along the $\boldsymbol{\sigma}$ ***axis*** in the ***s-plane***, and ***remains on the LHP without any imaginary component***

  $\Rightarrow$ ***The system remains stable even with feedback***

- Also, the ***phase*** of the system ***can never fall below –90°***

- Here, of course we are assuming a ***passive feedback network***, i.e., ***f is a real number***

- Thus, ***f does not add any phase to the system***
- Hence, ***Barkhausen's criteria can never be satisfied for this case***
- Also, the ***pole can never enter the RHP***
- Thus, we ***conclude***:
  - A system with ***single-pole transfer function*** is ***Unconditionally Stable***, i.e., it will ***remain stable*** for ***values of f*** all the way ***up to unity*** (i.e., ***the entire output fed back to the input***)




**Movement of the Pole for a Single-Pole System Under Negative Feedback and the Bode Plot of the Gain**

- ***Two-Pole System***:
  - ***Transfer Function***:

$$A(s) = \frac{A_0}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

$A_0$: ***Low-Frequency Gain***

$\omega_{p1}$, $\omega_{p2}$: ***Two negative real poles***, ***lying on the $\sigma$ axis***, ***with $\omega_{p2} > \omega_{p1}$***

  - Now, with ***passive feedback*** with ***feedback factor*** f, the ***locations*** of the ***closed-loop poles*** can be found from: $1 + fA(s) = 0$

- Thus:

$$s^2 + (\omega_{p1} + \omega_{p2})s + (1 + fA_0)\omega_{p1}\omega_{p2} = 0$$

- ***Solution*** gives the ***locations*** of the ***two closed-loop poles***:

$$s_1,\ s_2 = -\frac{\omega_{p1} + \omega_{p2}}{2} \pm \frac{1}{2}\sqrt{\left(\omega_{p1} + \omega_{p2}\right)^2 - 4\left(1 + fA_0\right)\omega_{p1}\omega_{p2}}$$

- With ***increase in feedback***, the ***second term reduces***

  $\Rightarrow$ ***$s_1$ and $s_2$ start to move towards each other along the $\sigma$ axis***

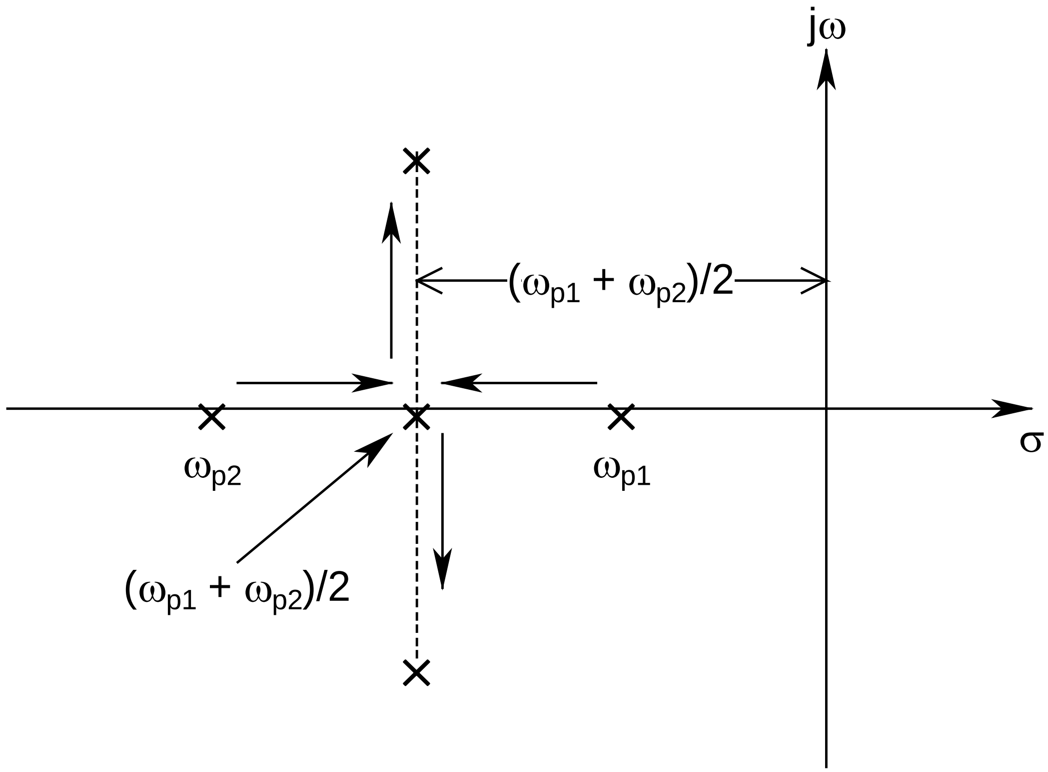
- Eventually, at a ***particular feedback***, the ***second term would vanish***

- ***At this point***, the ***two poles will merge*** at $(\omega_{p1} + \omega_{p2})/2$
- With ***further increase in feedback***, the ***second term becomes imaginary***, while the ***first term remains constant***

  $\Rightarrow$ ***The poles remain complex conjugates***

- ***Even for f all the way up to unity***, ***when the entire output is fed back to the input***, the ***poles remain in the LHP*** and ***can never enter RHP***

  $\Rightarrow$ ***The system remains unconditionally stable***




**Movement of the Poles for a Two-Pole System Under Negative Feedback With Increasing D**

- Also, for a ***two-pole system***, the ***phase reaches –180°*** ***only when the frequency becomes infinite*** (***mathematically***)
  - ⇒ ***There is no physically achievable frequency when this can happen***
  - ⇒ ***Unconditional Stability***

- ***System With Three (or More) Poles***:
  - ***Actual mathematical analysis quite tedious***
  - It can be shown that as the ***amount of feedback (D) is increased***:
    - ***The highest frequency pole ($\omega_{p3}$) moves outward along the $-\sigma$-axis***

- ***The other two poles ($\omega_{p1}$ and $\omega_{p2}$) move towards each other*** (***similar to a two-pole system***)
- As ***D is increased further***, these ***two poles eventually merge***, and then start having ***imaginary components***
- Their ***real part*** also ***keeps on changing with D***, ***keeping the nature of complex conjugacy intact***, and ***moves right in the s-plane***
- ***The path traced out by these poles is known as the root locus***
- ***For a particular value of D***, this ***root locus intersects the imaginary axis of the s-plane at two symmetric points***

- ***Under this condition***, ***sustained sinusoidal oscillation can be achieved***, ***since it now has a complex conjugate pair of poles without any real part*** ($\omega_{p3}$ ***will be so large that it will be inconsequential*** )
- ***With further increase in D***, the ***root locus enters the RHP*** with the ***poles now having positive real part***

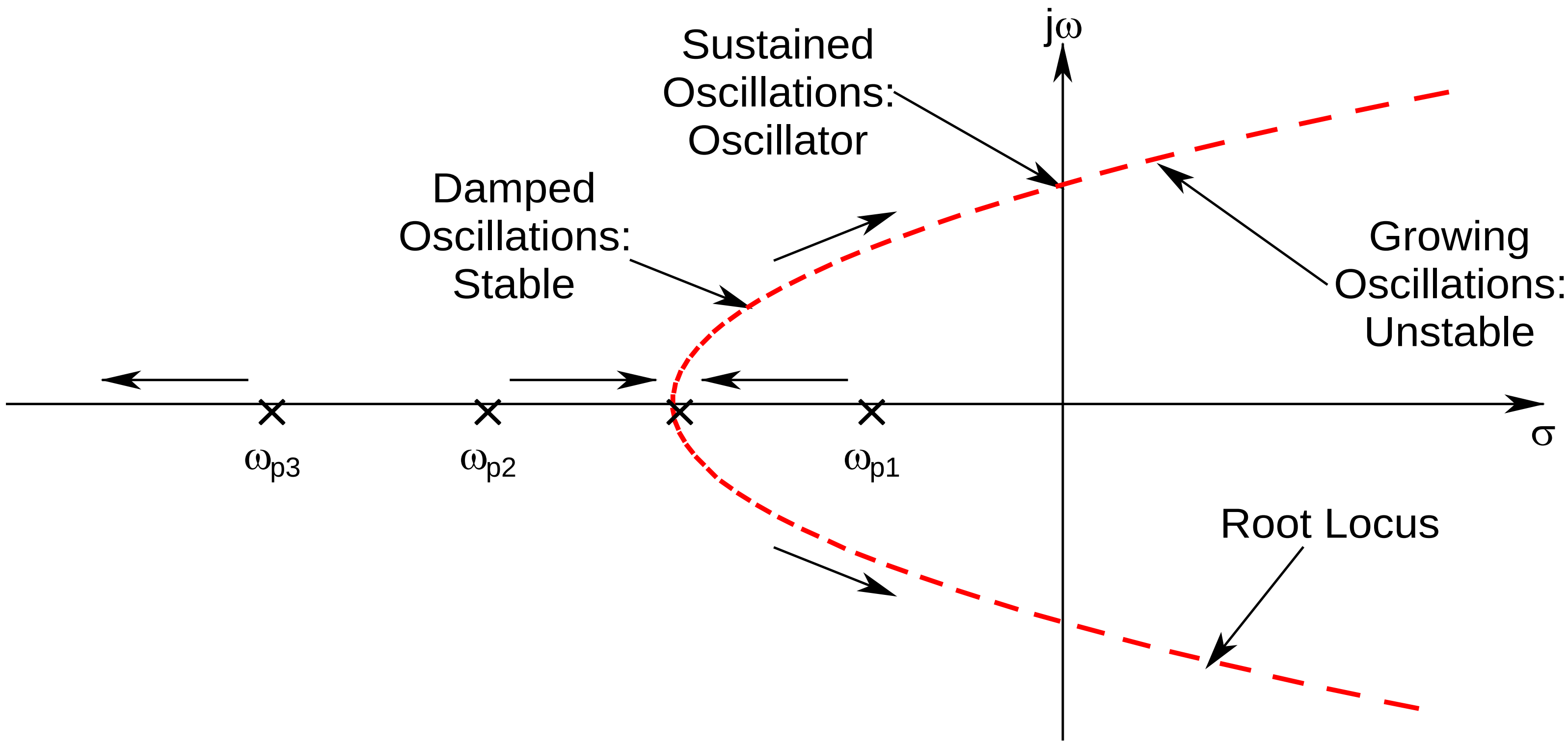
  $\Rightarrow$ ***Potentially dangerous situation in terms of stability***
- ***In terms of phase***, the ***total can be –270°***

  $\Rightarrow$ ***There exists a particular value of f***, for which the ***phase will become –180°***

- ***Under this condition***, ***if the magnitude of the loop gain is exactly unity***, ***then the system will break out into spontaneous oscillation***, however, ***the amplitude will be controlled***

  $\Rightarrow$ ***Sustained sinusoidal oscillation***

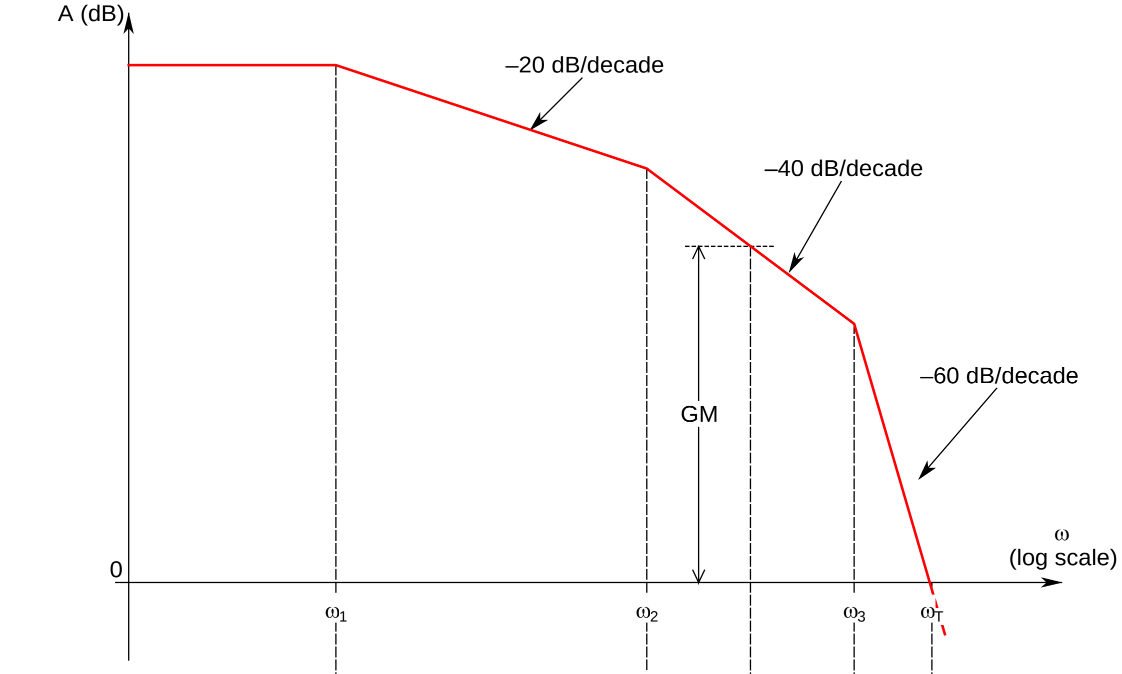
- ***This particular value of f is known as the critical feedback factor ($f_{crit}$) for oscillation***
  - ❖ For $f < f_{crit}$, the ***system will be stable***
  - ❖ For $f > f_{crit}$, the ***system will be unstable***
- Thus, the system is ***NOT Unconditionally Stable***, but ***stable only till a specific value of f***
  - ❖ Known as ***Conditionally Stable System***



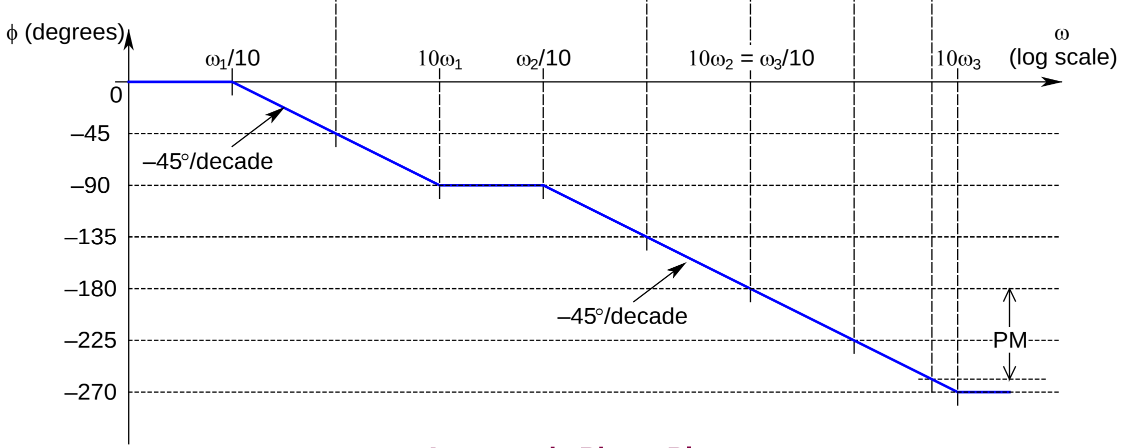

**Root Locus of the Poles of a Three-Pole System as D is Increased**

# Stability Study Using Bode Plot

- The ***most convenient*** and the ***most useful***
- ***Recall***: ***Single- and Two-Pole Systems are unconditionally stable***
- Consider a ***Three-Pole System***, with the ***pole frequencies*** at $\omega_1$, $\omega_2$, and $\omega_3$, with $\omega_3 = 100\omega_2$, and $\omega_2 > 100\omega_1$
- ***Note***: ***$A = L$ if $f = 1$*** (***100% feedback***)
- Refer to the next slide (***Bode Plot***)




**Asymptotic Gain Plot**




**Asymptotic Phase Plot**

- ***Profile of A*:**
  - ***Remains constant at its low-frequency value for $\omega \leq \omega_1$***
  - ***Then drops @ 20 dB/decade till $\omega_2$***
  - ***Followed by a drop @ 40 dB/decade till $\omega_3$***
  - ***Then drops @ 60 dB/decade***
  - ***Finally crosses 0 dB at $\omega$ (= $\omega_T$ : unity-gain cutoff frequency) slightly less than $10\omega_3$***
- ***Profile of $\phi$*:**
  - ***Remains zero till $\omega_1/10$***
  - ***Then drops @ 45°/decade***

- ***Reaches $-90°$ at $10\omega_1$***
- ***Stays constant at $-90°$ till $\omega_2/10$***
- ***Then starts to drop again @ $45°$/decade till $10\omega_3$***
- ***Reaches $-180°$ at $10\omega_2$ ($= \omega_3/10$) and $-270°$ at $10\omega_3$***

- ***Gain Margin*** (***GM***) and ***Phase Margin*** (***PM***):
  - ***Extremely important terms with regard to stability of a system***
  - ***From the sign and magnitude of these terms***, ***the stability of the system can be predicted***

- $GM = A\ (dB)$ (*when* $\phi = -180°$)
- $PM = 180° - |\phi|$ (*when* $A = 0$ *dB*)
- In our example, ***GM is positive*** (as shown in the figure)
- This is ***potentially a dangerous situation***, and characterizes a ***highly unstable system***
  - For ***positive GM***, ***with each pass around the loop***, the ***output amplitude will keep on growing***
- On the contrary, if ***GM were negative***, ***with each pass around the loop***, the ***output amplitude would have decreased***

- *The system would have come out of any unwanted oscillations*

- ***The GM dictates the maximum amount of feedback that can be allowed for the system to remain stable***

- ***For an unconditionally stable system, GM must be negative***

  $\Rightarrow$ ***A must be negative when $\phi = -180°$***

- ***With regard to phase, when A crossed 0 dB, $\phi$ is close to $-270°$***

  $\Rightarrow$ ***PM is negative, with a value of ~ $-90°$***

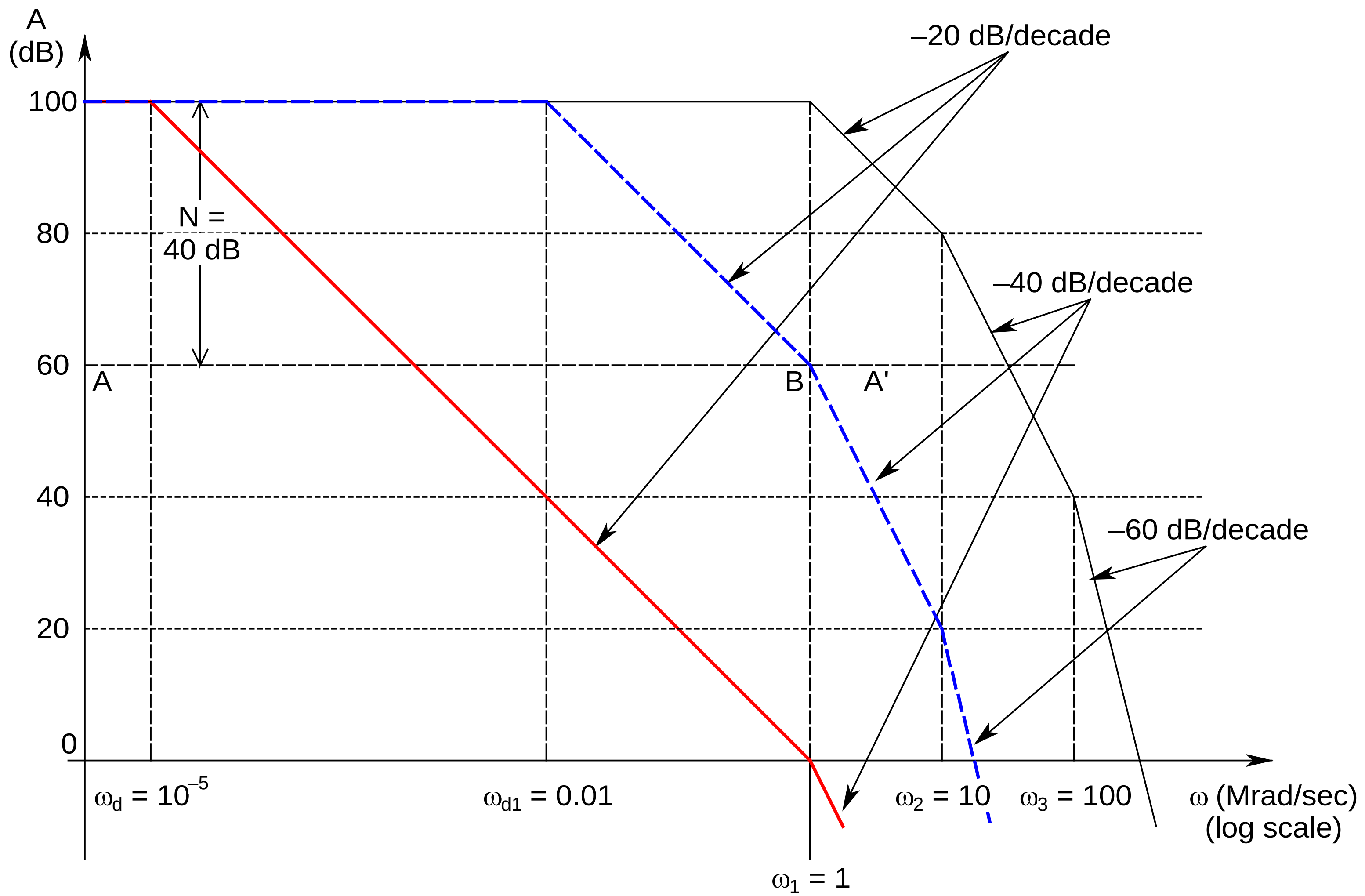
- ***This also implies that when $\phi$ crossed –180°***, ***A of the system was greater than unity*** (***0 dB***)
  - ***A potentially dangerous situation in terms of stability***
- Therefore, ***for an unconditionally stable system***, ***PM must be positive***
- ***The two conditions with regard to GM and PM are actually correlated***
- ***Rule of Thumb***:
  - ***For a stable system***, ***GM ~ –10 dB and PM ~ 45° are generally good enough***

# Compensation

- ***Basic Idea***:
  - To ***tailor*** the ***gain characteristic*** of a system, having ***three or more poles***, such that it would be ***stable*** for ***any value*** of the ***feedback factor*** f, all the way ***up to unity*** (referred to as the ***unity feedback system***, where the ***entire output*** is ***fed back*** to the ***input***)
  - ***After compensation***, the system will become ***either conditionally (f < 1) or unconditionally stable (f = 1)***

- ***Two widely used methods***:
  - ***Dominant Pole Compensation*** (***DPC***)
  - ***Pole Zero Compensation*** (***PZC***)
- ***Dominant Pole Compensation (DPC)***:
  - This technique introduces a ***dominant pole*** (***DP***) into the system
  - Also known as ***Miller Compensation Scheme***
  - This ***DP*** is chosen such that the ***compensated gain characteristic*** meets the ***first pole*** of the ***uncompensated system*** at ***0 dB***, with a ***slope*** of ***–20 dB/decade***

- This will make the system ***unconditionally stable***, i.e., the ***stability*** of the system will be ***independent*** of the ***amount of feedback***
- ***Example***:
  - Assume A = $10^5$ (100 dB), $\omega_1$ = 1 Mrad/sec, $\omega_2$ = 10 Mrad/sec, $\omega_3$ = 100 Mrad/sec
  - Refer to the slide on the next page
  - For ***unconditional stability***:
    - Refer to the ***red line***
    - The ***compensated transfer function*** should meet the ***first pole*** ($\omega_1$) of the ***uncompensated system*** at ***A = 0 dB*** with a ***slope*** of ***–20 dB/decade***




**Normal Line: Open-loop system**
**Red Line: Compensated system for unconditional stability**
**Blue Line: Compensated system with conditional stability**
**(till a feedback of 40 dB)**

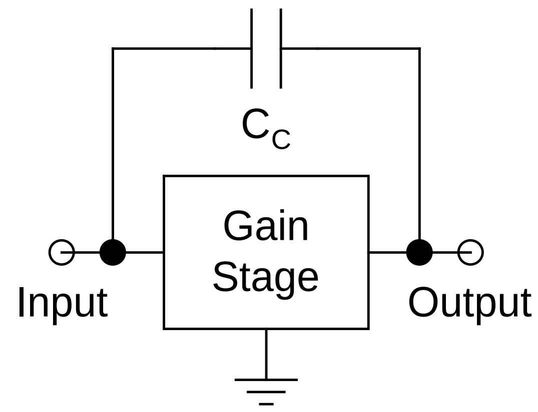
- To construct the ***compensation characteristic***, ***start at $\omega_1$*** and ***go back 5 decades*** (= 100/20)
- ***Ends up at the DP frequency ($\omega_d$) of 10 rad/sec***
- ***Note that in between $\omega_d$ and $\omega_1$, the system behaves as if it has a single-pole transfer function***
- The ***total phase*** of the system at $\omega_1$ will be ***–135°*** [***–90° due to the pole at $\omega_d$***, and ***–45° due to the pole at $\omega_1$*** (***since $\omega_2$ is ten times away from $\omega_1$***, the ***phase due to $\omega_2$ is yet to start at this point***)]
- Thus, the ***PM of the compensated system will be 45°***
- This implies a ***stable system***, since the ***PM is positive***
- Note that ***if $\omega_2$ and $\omega_1$ were closer than 10 times***, the ***PM would have been less than 45°***, but ***still positive***, and thus, ***would have retained the stable nature of the system***

- Note that in order to achieve ***unconditional stability*** of the system, the ***bandwidth*** has ***reduced drastically*** from ***1 Mrad/sec*** to only ***10 rad/sec***!
- ***This is the most severe limitation of the DPC technique***

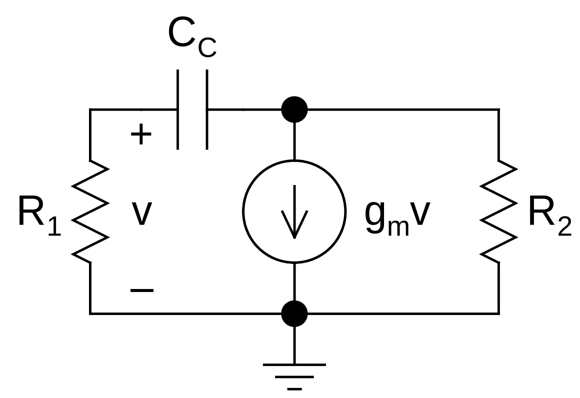
- ***For conditional stability***:
    - The ***previous compensation scheme*** ensured ***system stability*** for f all the way ***up to unity*** (corresponding to the ***amount of feedback*** of ***100 dB***, i.e., the ***entire output is fed back to the input***)
    - In some cases, it may be an ***overkill***, if it is known ***a priori*** that the ***entire output*** will ***NOT*** be ***fed back*** to the ***input***, ***rather only a part of it***
    - This is what is known as ***conditional stability***
    - Suppose that the ***maximum amount of feedback*** that the system would have is ***40 dB***

- For this system to be ***stable***, the ***DP frequency need not be at $\omega_d$, but at a higher value***
- To construct the ***compensation characteristic*** of this system, draw a ***horizontal line*** AA′, corresponding to the ***amount of feedback*** (***40 dB in our example*** $\Rightarrow$ $A_f = 60$ ***dB***)
- From the ***intersection point*** (B) of this ***line*** with the ***first pole*** ($\omega_1$), ***go back 2 decades*** (40/20), to get the ***new dominant pole*** $\omega_{d1}$ at ***10 krad/sec*** (shown by the ***blue line***)
- This ***compensation scheme protects the system from any stability issues only till a maximum feedback of 40 dB***, ***by ensuring*** that ***from 0 to 40 dB of feedback***, ***no other pole will be encountered***, ***apart from $\omega_{d1}$***
- Note the ***tremendous bandwidth improvement*** of ***1000 times*** (***from 10 rad/sec*** for ***unconditional stability to 10 krad/sec*** for ***conditional stability till a feedback of 40 dB***)

- ***Technique***:
  - ***Simplest way***: ***Attach a capacitor between the input and output of the gain stage*** (***similar to Miller Capacitor***)
  - ***This capacitor is labeled as the Compensation Capacitor*** ($C_C$)




**Schematic**




**Equivalent Circuit**

- ***By inspection***, the equivalent circuit can be identified as a ***Three-Legged Creature***:

$$\Rightarrow R_C^0 = R_1 + R_2 + g_m R_1 R_2$$

$R_1$ = ***Effective total resistance on the left of $C_C$***

$R_2$ = ***Effective total resistance on the right of $C_C$***

$g_m$ = ***Transconductance of the gain stage***

- Thus:

$$\omega_d = 1 \big/ \left( R_C^0 C_C \right)$$

- ***From a knowledge of $\omega_d$, we can find $C_C$***

- ***Pole Zero Compensation (PZC)***:
  - In the ***DPC technique***, we observed a ***drastic reduction*** in ***bandwidth*** after ***compensation***
  - ***PZC technique alleviates this problem to some extent***
  - ***Novelty of this technique***:
    - ***It adds both a pole and a zero to the open-loop transfer function***, ***with the added zero canceling the first pole of the uncompensated system***

- Consider a ***three-pole uncompensated transfer function***:

$$A(s)\Big|_{\text{uncompensated}} = \frac{A_0}{\left(1+s/\omega_1\right)\left(1+s/\omega_2\right)\left(1+s/\omega_3\right)}$$

$A_0$: ***Low-Frequency Gain***

$\omega_1$, $\omega_2$, $\omega_3$: ***Pole Frequencies*** ($\omega_3 > \omega_2 > \omega_1$)

- ***After adding the network for PZC***, the ***compensated transfer function*** will be:

$$A(s)\Big|_{\text{compensated}} = \frac{A_0\left(1+s/\omega_z\right)}{\left(1+s/\omega_p\right)\left(1+s/\omega_1\right)\left(1+s/\omega_2\right)\left(1+s/\omega_3\right)}$$

$\omega_z$: ***added zero***, and $\omega_p$: ***added pole***

- ***By design***, ***$\omega_z$ is made equal to $\omega_1$***

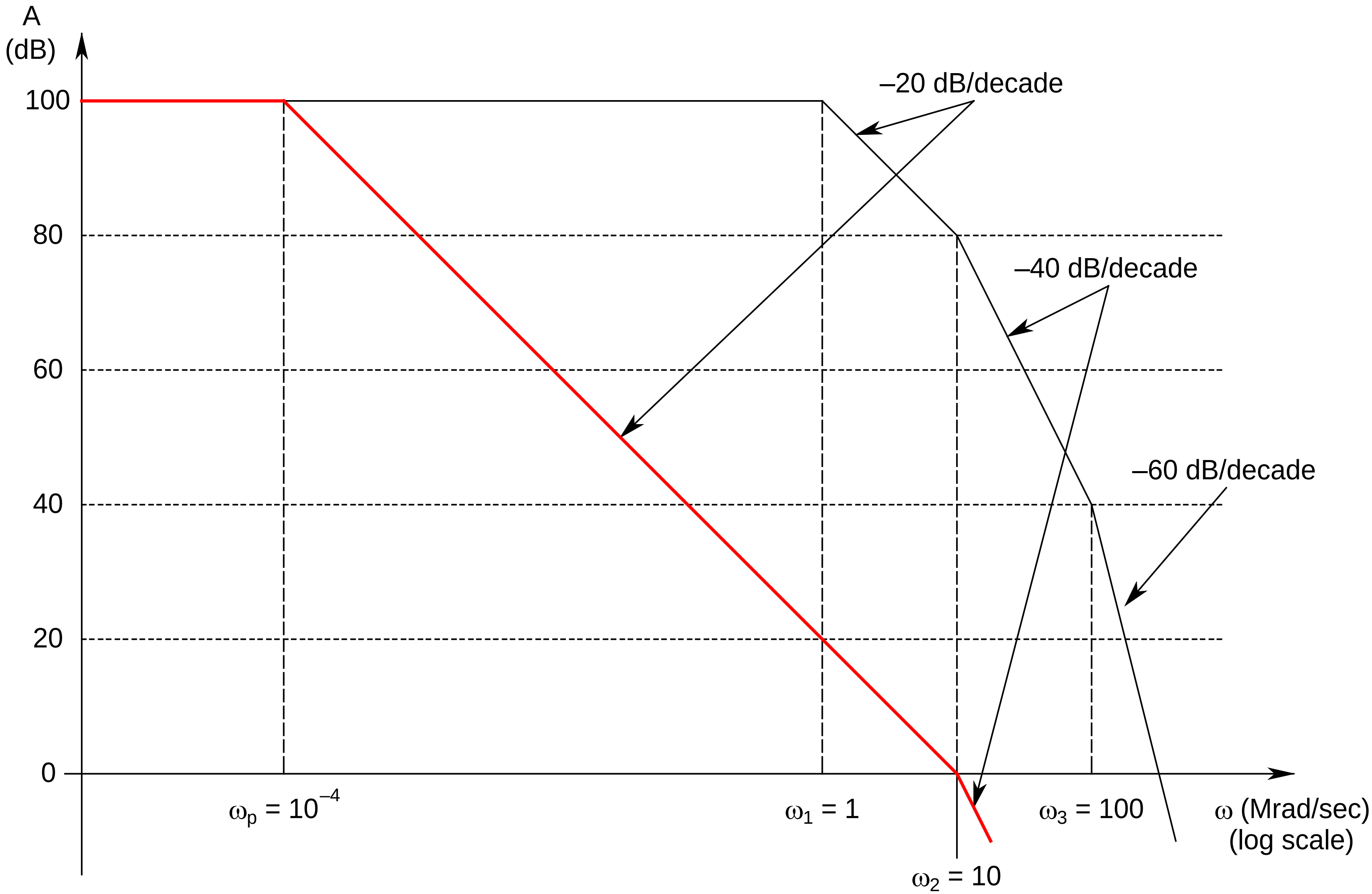
  $\Rightarrow$ ***They cancel each other***

- Thus, the ***compensated transfer function*** still has ***three poles***, but the ***first pole gets shifted from $\omega_1$ to $\omega_p$***

- ***The procedure for finding $\omega_p$ is the same as that for the DPC technique***

- We take the ***same example*** as that considered for the ***DPC technique***

- Refer to the next slide



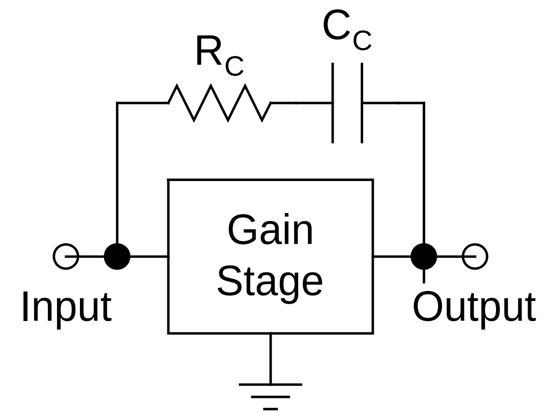

**Normal Line: Open-loop system**

**Red Line: Compensated system for unconditional stability**

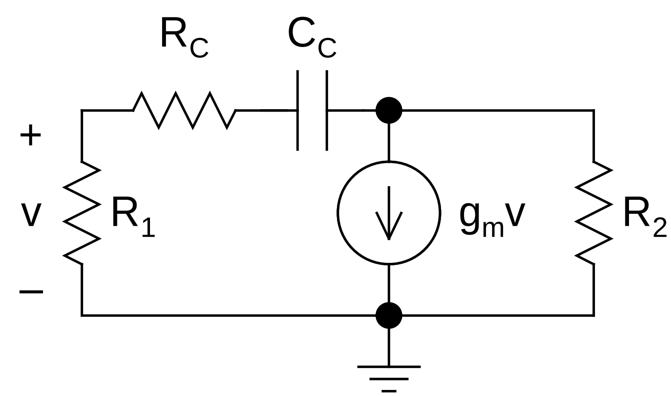
- Here, the ***added zero*** ($\omega_z$) ***cancels*** the ***first pole*** ($\omega_1$)
- Thus, we now ***start from $\omega_2$*** and ***go back 5 decades*** to find $\omega_p$, which comes out to be ***100 rad/sec*** (refer to the ***red line***)
- The ***compensated system*** will be ***unconditionally stable*** with ***PM of 45°*** (***since $\omega_3$ is ten times away from $\omega_2$***)
- The ***increase in bandwidth***, ***as compared to DPC***, is ***10 times*** (***from 10 rad/sec to 100 rad/sec***: ***equal to the ratio of $\omega_2$ and $\omega_1$***)

- ***Technique***:
  - ***Just attach a resistor $R_C$ with the compensation capacitor $C_C$***
  - ***Put this $R_C$-$C_C$ network between the input and output of the gain stage***
  - Show that the ***transfer function*** of the ***compensated system*** is of the form:

$$A(s)\Big|_{\text{compensated}} \propto \frac{1+s\left(R_C - 1/g_m\right)C_C}{1+s\left(R_C + R_2\right)C_C}$$




**Schematic**




**Equivalent Circuit**

- Here

  $\omega_z = 1/[(R_C - 1/g_m)C_C]$

  $\omega_p = 1/[(R_C + R_2)C_C]$
- ***Choose $R_C$ and $C_C$ such that***
  - ***$\omega_z$ is equal to $\omega_1$*** (***the first pole of the uncompensated system***)
  - ***$\omega_p$ is as found from the example given***