Department of Electrical Engineering, IIT Kanpur
EE250: Control Systems Analysis
Tutorial 1

Question 1

We found out the dynamics of a servo-motor in the class as

$$\frac{dx_1}{dt} = -\frac{B}{J}x_1 + \frac{K_T}{J}x_2$$
$$\frac{dx_2}{dt} = -\frac{K_b}{L_a}x_1 - \frac{R_a}{L_a}x_2 + \frac{1}{L_a}u$$

where

x_1 = speed (ω) of the motor
x_2 = Armature current I_a
u = Armature voltage e_a

Parameters are:

$B = 0.25$ N-m/(rad/sec)
$R_a = 5\ \Omega$
$L = 0.1$ H
$J = 2$ N-M/(rad/sec)
$K_b = 1$ volt/(rad/sec)

(i) Given output $y = x_1$, find $\frac{Y(s)}{U(s)}$
(ii) Given $u(t) = 100$ volt (sudden), find $y(t)$. Compute y_{ss}
(iii) Find state transition matrix e^{At}
(iv) Find $y(t)$ using e^{At}

Question 2

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 2y(t) = 6\frac{du(t)}{dt} + u(t)$$

Find $\frac{Y(s)}{U(s)}$

Question 3

$$\frac{d^3y(t)}{dt^3} + 10\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) + 2\int_0^t y(\tau)d\tau = \frac{du(t)}{dt} + 2u(t)$$

Find $\frac{Y(s)}{U(s)}$

Question 4

Solve the following ordinary differential equation using Laplace transform

$$\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0; y(0) = 1, \dot{y}(0) = 2$$