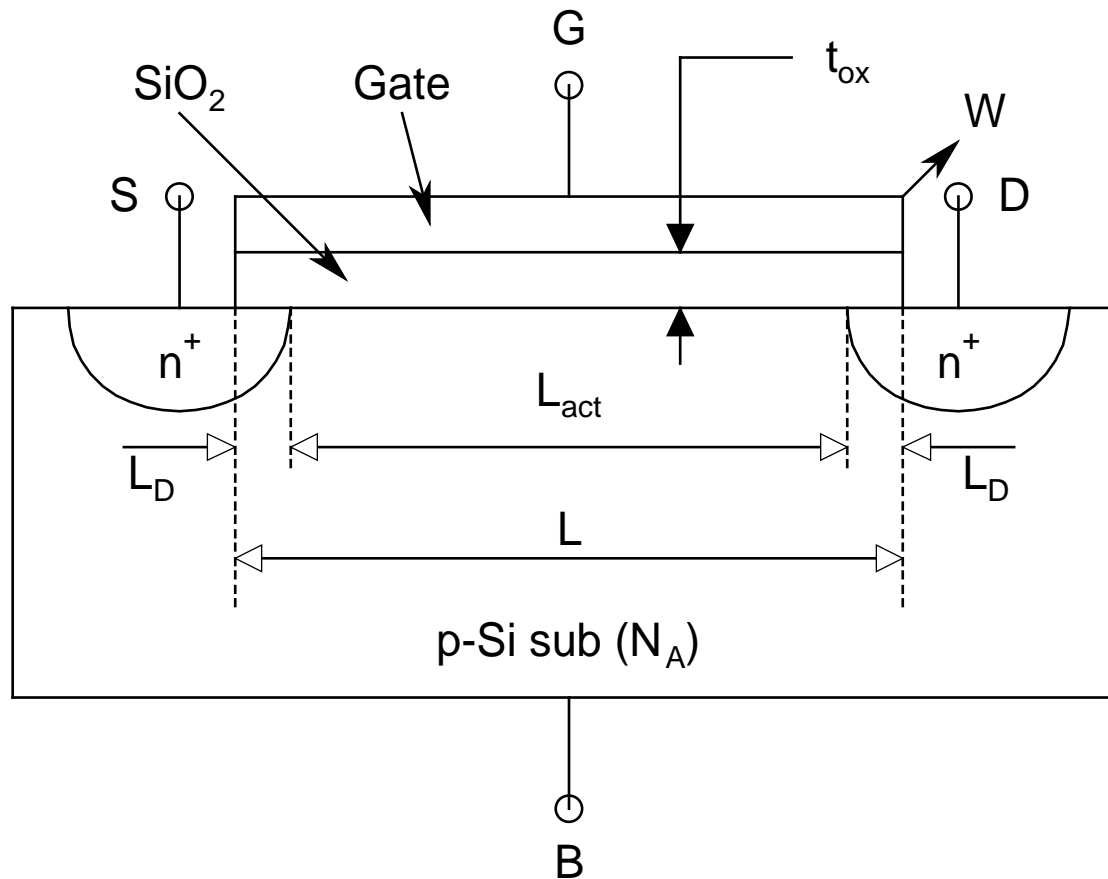


# **METAL-OXIDE- SEMICONDUCTOR FIELD-EFFECT TRANSISTOR (MOSFET)**

- *Extremely popular device* - has almost pushed BJTs out of the market
- *Three-Layer Device* (*Metal*, *Dielectric*, and *Semiconductor*)
- *Four-Terminal Device* [*Drain* (D), *Source* (S), *Gate* (G), and *Body/Substrate* (B)]
- *Current* through two terminals (D and S) can be *controlled* by the *voltages* applied at G and B
  - *Voltage Controlled Device*

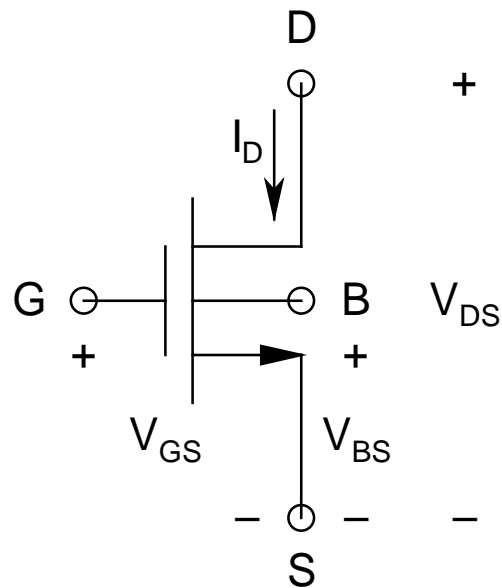
- *Unipolar device*
  - Either *electrons* or *holes* participate in *current conduction*
- *Active device*
  - Capable of producing *voltage/current/power gain*
- *Two basic usage:*
  - *Amplification* (*Analog Circuits*)
  - *Switching* (*Digital Circuits*)
- *Two Types: NMOS and PMOS*

# NMOS Structure

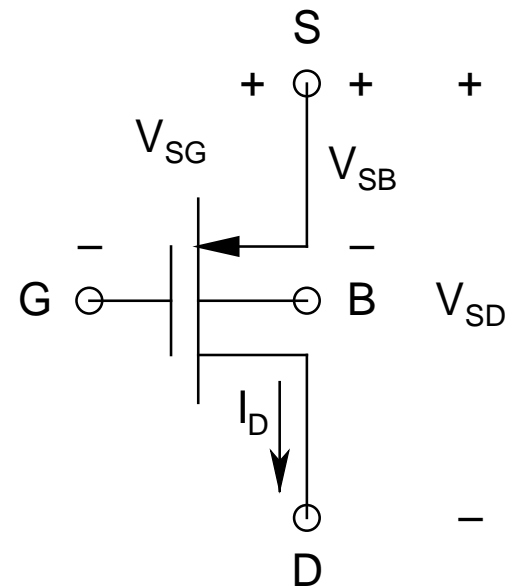


- **Technology Parameters:**
  - **Channel Length** ( $L$ )
  - **Channel Width** ( $W$ )
  - **Oxide Thickness** ( $t_{ox}$ )
  - **Substrate Doping** ( $N_A$ )
- $L_D$ : **Lateral overlap** between G and S/D
- **Actual** channel length:  $L_{act} = L - 2L_D$
- For now, we will assume  $L_D = 0$ 
  - $L_{act} = L$

# Symbols and Current-Voltage Conventions



**NMOS**



**PMOS**

- *Voltage Convention:*

- *NMOS*:  $V_{GS}$  (*gate-source voltage*),  $V_{DS}$  (*drain-source voltage*),  $V_{BS}$  (*body-source voltage*)
- *PMOS*:  $V_{SG}$  (*source-gate voltage*),  $V_{SD}$  (*source-drain voltage*),  $V_{SB}$  (*source-body voltage*)

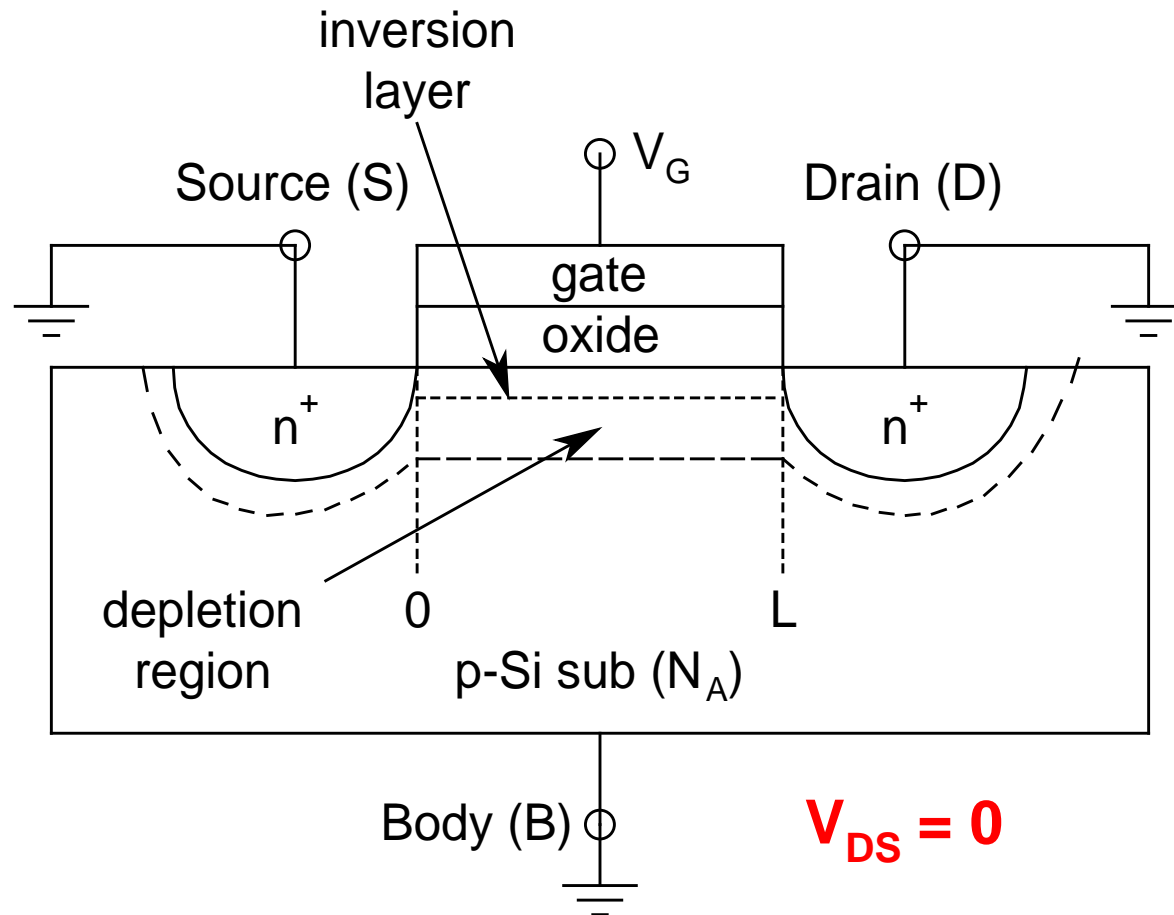
- *Current Convention:*

- *NMOS*:  $I_D$  (*drain current*) flows into the *drain terminal* and exits from the *source terminal*
- *PMOS*:  $I_D$  flows into the *source terminal* and exits from the *drain terminal*

- *Gate is DC isolated by the insulator*
  - *Gate Current  $I_G = 0$*
  - *Tremendous advantage!*
- *Same current  $I_D$  flows through the device*
- *Extremely compact device*
  - *Saves a lot of area*
- *Reversible device:*
  - *D and S terminals are determined by their bias states*



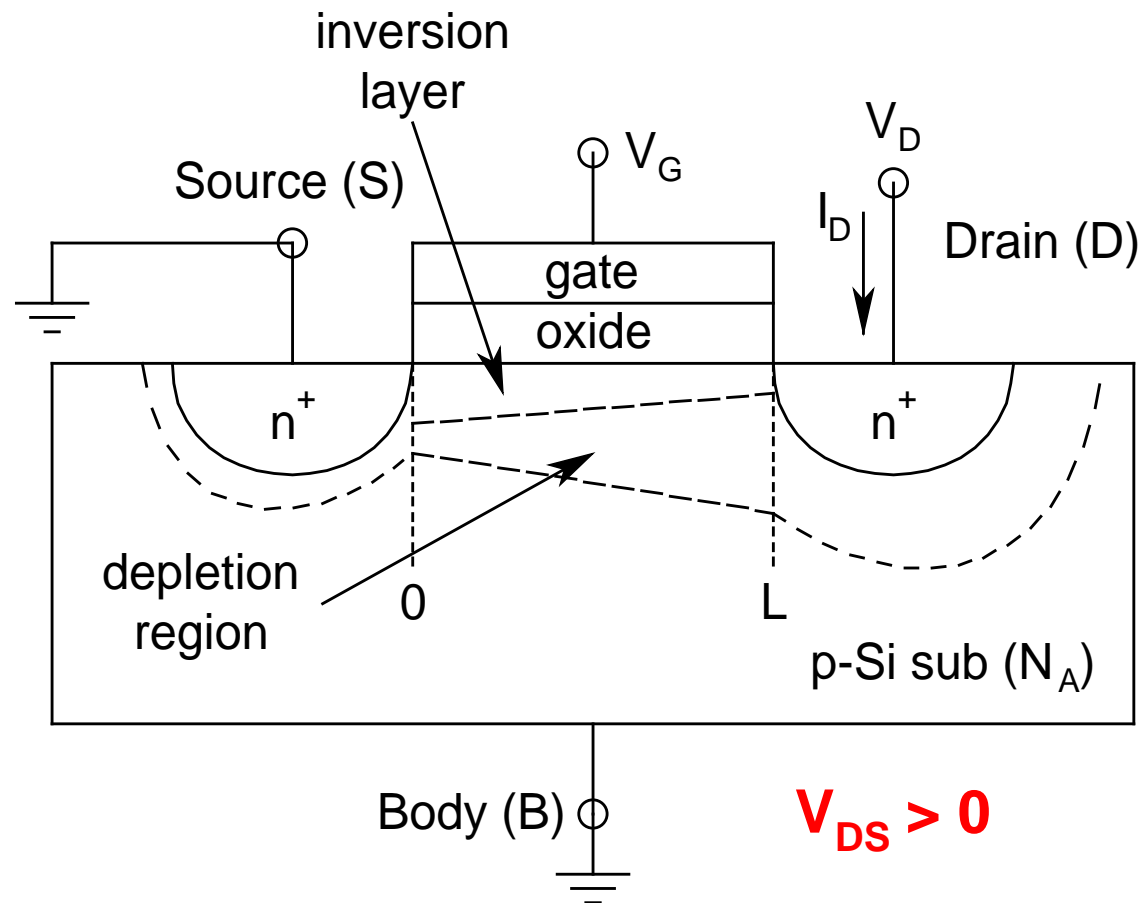
# Operation



- The structure is *similar* to an  $n^+pn^+$  *BJT*
- However, *BJT action* is *not possible* due to *large channel length* (L)
- The way to make the device *conduct* is to form a *layer of electrons* between S and D
  - Known as *Inversion Layer*
- Then, if a *bias* is applied *between S and D*, then *inversion layer electrons* will move *towards the higher potential* due to *drift*
  - A *current* would result

- Consider  $V_S = V_D = V_B = V_G = 0$ 
  - Device is *off* and *no current flows*
- Note that the *structure* is similar to a *capacitor*
- Now, as  $V_G$  is made *positive*, initially it will *repel holes from surface towards bulk*, *uncovering ionized acceptor atoms there*
  - *Formation of a depletion layer*
- There will be *depletion layers* around *SB and DB junctions* as well

- As  $V_G$  is kept on *increasing*, the *depletion charge* will keep on *increasing*
- At a *certain value* of  $V_G$  ( $= V_{GS}$ ), a *layer of electrons* will *appear at the surface*
- This *particular value* of  $V_{GS}$  is known as the *threshold voltage*  $V_{TN}$
- Still *no current* would flow, since  $V_{DS} = 0$
- *SB and DB junctions* must remain *either at zero bias or reverse bias* all the time
  - $V_{SB}$  and  $V_{DB} \geq 0$



- With  $V_{DS} > 0$ , *inversion layer electrons* will *move* towards the *higher potential*, i.e., D
  - The *drain current*  $I_D$  would *flow from D to S*
- *Note:*
  - The *depletion charge* would *increase* as we move *towards the D* (since the *DB junction* is *more reverse biased*)
    - The *inversion charge* would *decrease* as we move *towards the D*
    - For *sufficiently high*  $V_{DS}$ , it may *disappear altogether*

# Body Effect

- The *threshold voltage*  $V_{TN}$  is a *function* of the *SB voltage*  $V_{SB}$
- As  $V_{SB} \uparrow$ , the *SB junction depletion charge* would *increase*
  - For the *same*  $V_{GS}$ , *inversion charge* would *decrease* (to maintain *charge balance*)
  - Thus, to *restore* the *original level* of *inversion*,  $V_{GS}$  has to be *increased*
  - Implies that  $V_{TN}$  has *increased*

- Expressed as:

$$V_{\text{TN}} = V_{\text{TN0}} + \gamma \left( \sqrt{2\phi_{\text{F}} + V_{\text{SB}}} - \sqrt{2\phi_{\text{F}}} \right)$$

$$V_{\text{TN0}} = V_{\text{TN}} \big|_{V_{\text{SB}}=0} = \textit{Zero back-bias threshold voltage}$$

$$\gamma = \frac{\sqrt{2q\epsilon_s N_{\text{A}}}}{C'_{\text{ox}}} = \textit{Body-effect coefficient}$$

$$C'_{\text{ox}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}} = \textit{Oxide capacitance per unit area}$$

$$\phi_{\text{F}} = V_{\text{T}} \ln \left( \frac{N_{\text{A}}}{n_{\text{i}}} \right) = \textit{Bulk potential} \ (\sim 0.3 - 0.45 \text{ V})$$



# Current-Voltage Relation

- For  $V_{GS} > V_{TN}$  and *small*  $V_{DS}$ :

$$I_D = k_N \left( V_{GT} V_{DS} - V_{DS}^2 / 2 \right)$$

$$V_{GT} = V_{GS} - V_{TN} = \textit{Gate overdrive}$$

$$k_N = (W/L) k'_N$$

= *Device transconductance parameter*

$$W/L = \textit{Aspect ratio}$$

$$k'_N = \mu_n C'_{ox}$$

= *Process transconductance parameter*

$\mu_n$  = *Channel electron mobility*

- For *small*  $V_{DS}$ , the  $V_{DS}^2$  term can be *neglected*
  - $I_D$  changes *linearly* with  $V_{DS}$ 
    - *Linear* (or *Non-Saturation*) Region
- As  $V_{DS} \uparrow$ , the *restraining* effect of  $V_{DS}^2$  term  $\uparrow$ 
  - *Rate of increase* of  $I_D$  with  $V_{DS}$  *slows down*

- For *inversion channel* to exist at the *D end*,  
 $V_{GD}$  must be  $> V_{TN}$ 
  - $V_{DS}$  must be  $< V_{GT}$
- When  $V_{DS} = V_{GT}$ , the *channel* is said to be *pinched-off* at the *D end*, and  $I_D$  does not increase any more
- This value of  $V_{DS}$  is known as the *drain-to-source saturation voltage*  $V_{DS,sat}$ 
  - $V_{DS,sat} = V_{GT}$

- For  $V_{DS} > V_{DS,sat}$ , the *mode of operation* is known as *saturation*
- *Drain current in saturation:*

$$I_D = \frac{k_N}{2} V_{GT}^2$$

- *Obtained from the non-saturation  $I_D$  expression by substituting  $V_{DS} = V_{GT}$*
- *Note that  $I_D$  is independent of  $V_{DS}$*
- *Above equations are valid for  $V_{GT} > 3V_T$   
(~ 80 mV at room temperature)*

# The Complete LEVEL 0 Model

$$I_D = k_N \left( V_{GT} V_{DS} - V_{DS}^2 / 2 \right)$$

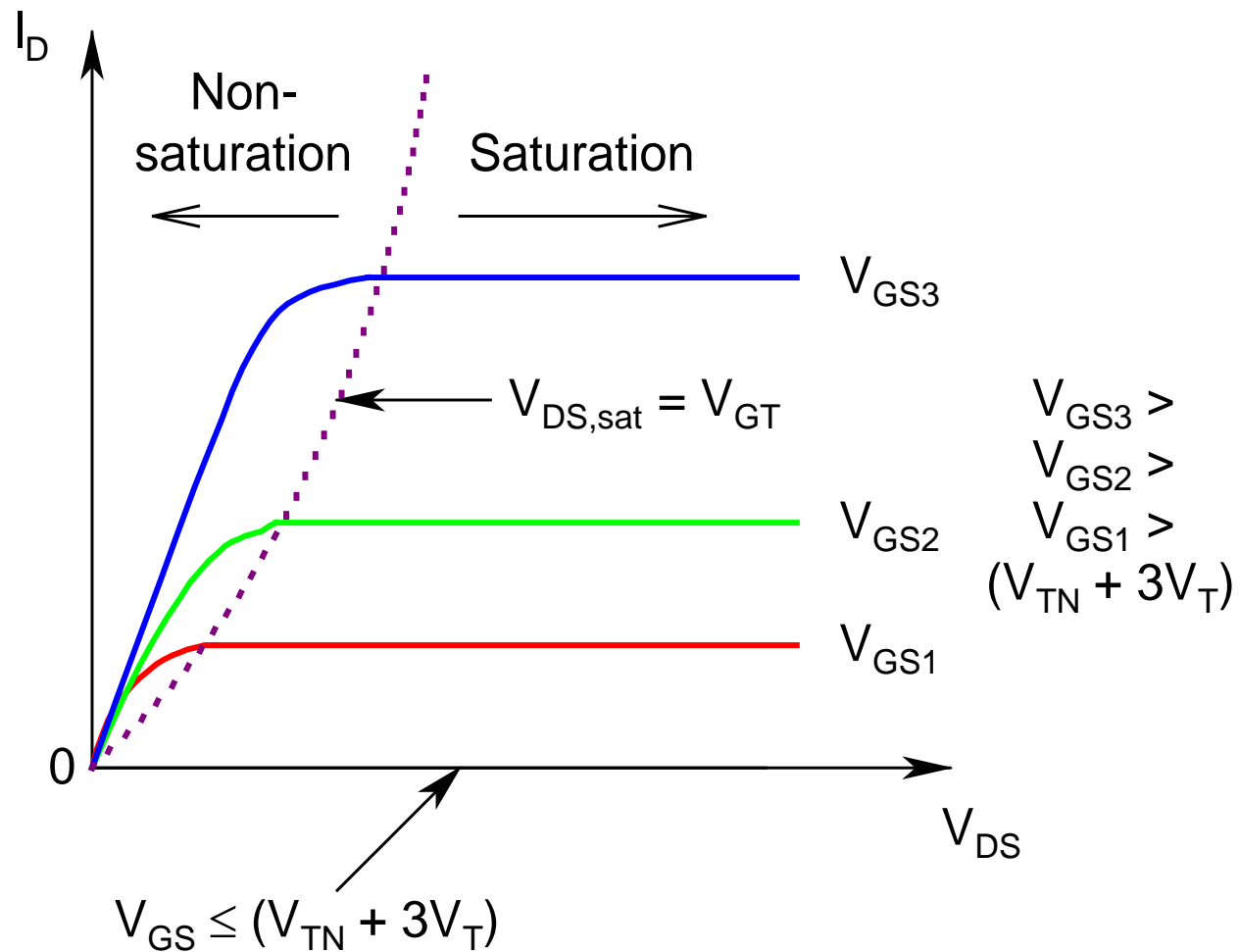
*(linear region -  $V_{GT} > 3V_T, V_{DS} < V_{GT}$ )*

$$= (k_N / 2) V_{GT}^2$$

*(saturation region -  $V_{GT} > 3V_T, V_{DS} \geq V_{GT}$ )*

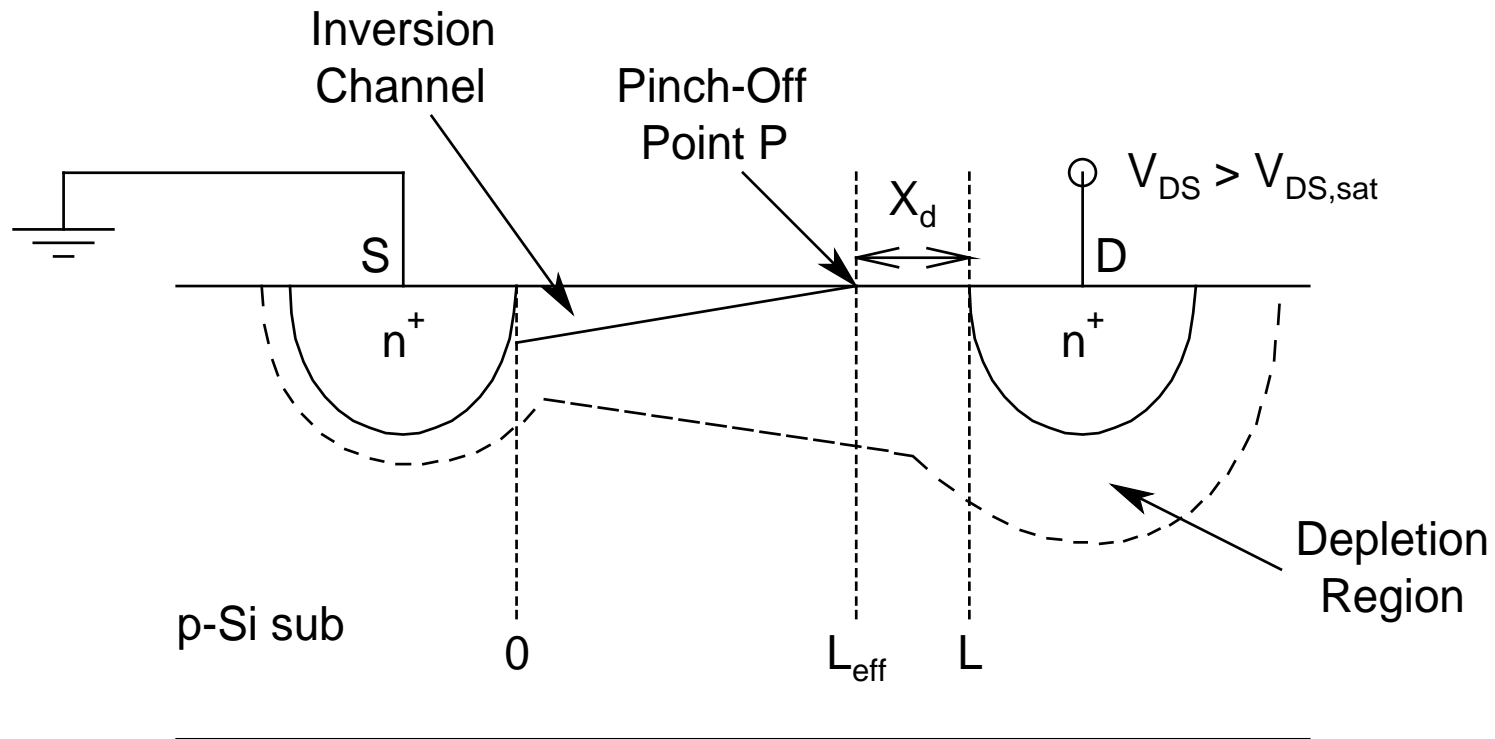
$$= 0$$

*(cutoff region -  $V_{GT} \leq 3V_T, \text{ any } V_{DS}$ )*



## $I_D$ - $V_{DS}$ Characteristics

# Channel Length Modulation (CLM)



**$X_d \rightarrow$  length of the pinched-off region**

- For  $V_{DS} = V_{DS,sat}$ , *pinch-off point P at D end*
- For  $V_{DS} > V_{DS,sat}$ , *P moves towards source*
- *Effective channel length reduces* from L to  $L_{eff} = L - X_d$ 
  - $X_d =$  *pinch-off region/drain region/saturation region length*
- *Excess voltage*  $(V_{DS} - V_{DS,sat})$  *drops across*  $X_d$



- *Reduction of effective channel length causes an increase in current*

- *Channel length modulation*

- With  $V_{DS} \uparrow$ ,  $X_d \uparrow$ ,  $L_{eff} \downarrow$ , and  $I_D \uparrow$

- *No real current saturation*

- Thus, *saturated drain current*:

$$\begin{aligned} I_{D,sat} &= (k'_N/2)(W/L_{eff}) V_{GT}^2 \\ &= (k_N/2) V_{GT}^2 (1 + \lambda V_{DS}) \end{aligned}$$

- $\lambda = \text{Channel length modulation parameter}$   

$$= \frac{1}{L} \frac{dX_d}{dV_{DS}}$$
  - *Function of  $L$  and  $N_A$*
  - *Higher  $L$  and  $N_A \Rightarrow \text{Lower } \lambda$*
  - *Typical values of  $\lambda$  may range from close to 0 to as high as 0.1-0.3  $V^{-1}$*
  - *Very similar to  $V_A$  for BJTs*

- This gives **LEVEL 1 model** (also known as **Shichman-Hodges model**) for MOSFETs:

$$I_D = k_N \left[ V_{GT} V_{DS} - V_{DS}^2 / 2 \right] (1 + \lambda V_{DS})$$

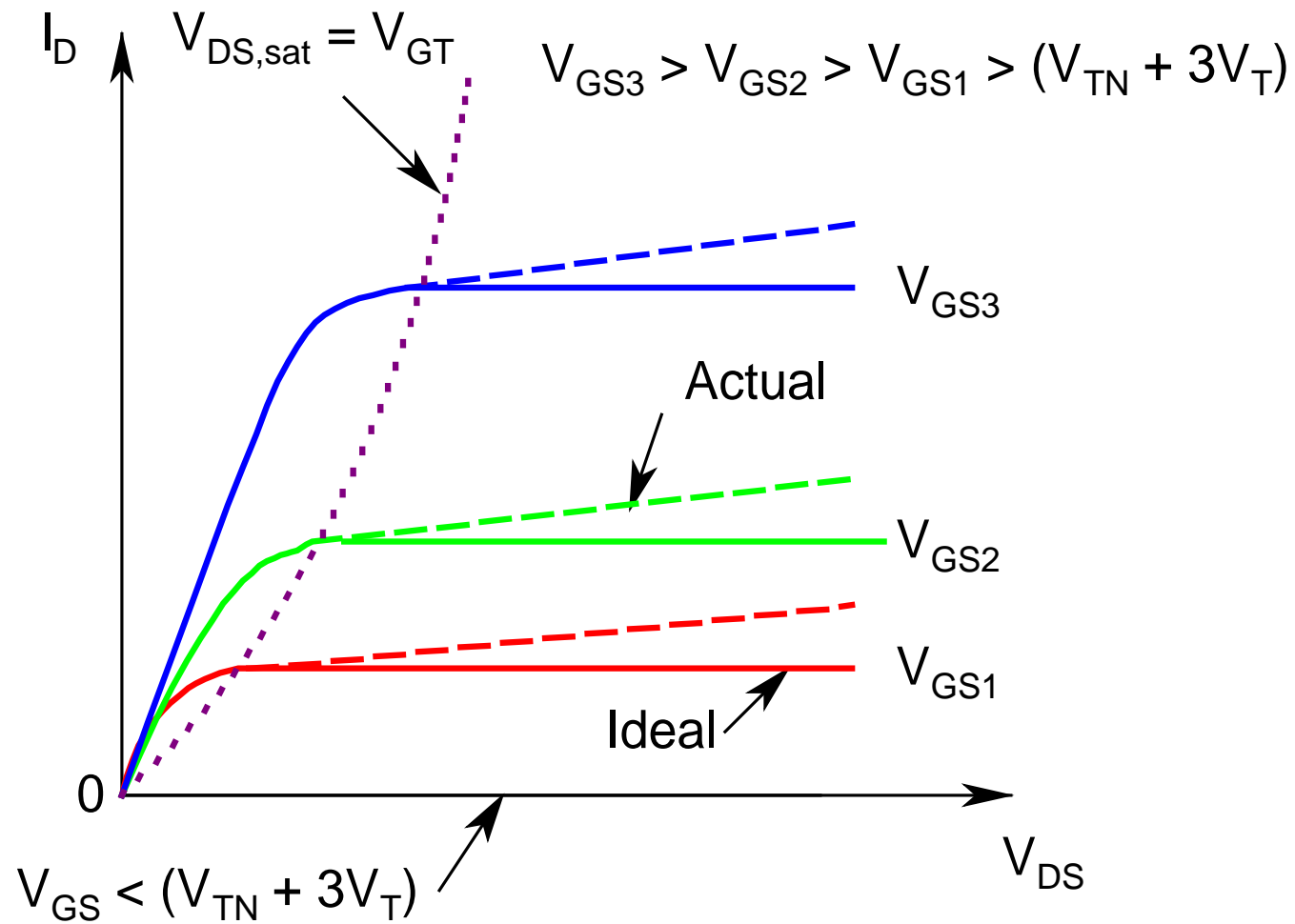
*(linear region -  $V_{GT} > 3V_T$ ,  $V_{DS} < V_{GT}$ )*

$$= (k_N / 2) V_{GT}^2 (1 + \lambda V_{DS})$$

*(saturation region -  $V_{GT} > 3V_T$ ,  $V_{DS} \geq V_{GT}$ )*

$$= 0$$

*(cutoff region -  $V_{GT} \leq 3V_T$ , any  $V_{DS}$ )*



## $I_D$ - $V_{DS}$ Characteristics in presence of CLM

# DC Bias Point Calculation

- *To find  $R_D$  for BB*

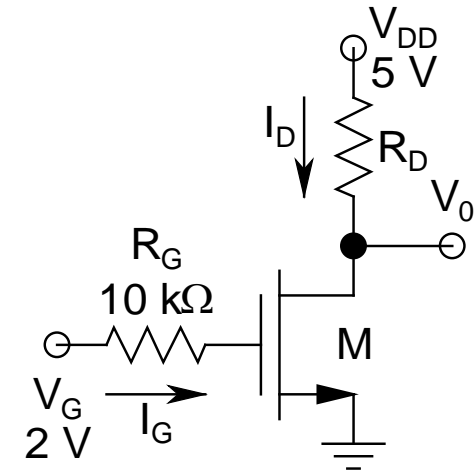
- $V_{TN0} = 1 \text{ V}$ ,  $k'_N = 40 \mu\text{A/V}^2$ ,  
 $W/L = 10$

- *Body terminal not shown*

➤ *Implies that it is connected  
to the most negative potential available in the  
circuit (**ground in this case**)*

$$\Rightarrow V_{SB} = 0 \Rightarrow V_{TN} = V_{TN0}$$

- $I_G = 0 \Rightarrow V_{GS} = V_G = 2 \text{ V}$



- $V_{GT} = V_{GS} - V_{TN} = 1 \text{ V}$
- *Assuming saturation mode of operation and neglecting CLM:*  

$$I_D = (k_N/2) V_{GT}^2 = 200 \text{ } \mu\text{A}$$
- *For BB,  $V_{DS} = V_{DD}/2 = 2.5 \text{ V}$  (2-element output branch):*  

$$R_D = (V_{DD} - V_{DS})/I_D = 12.5 \text{ k}\Omega$$
- $V_{DS} > V_{GT} \Rightarrow$  *Assumption of saturation mode of operation validated*
- $P_D = V_{DS} \times I_D = 0.5 \text{ mW}$

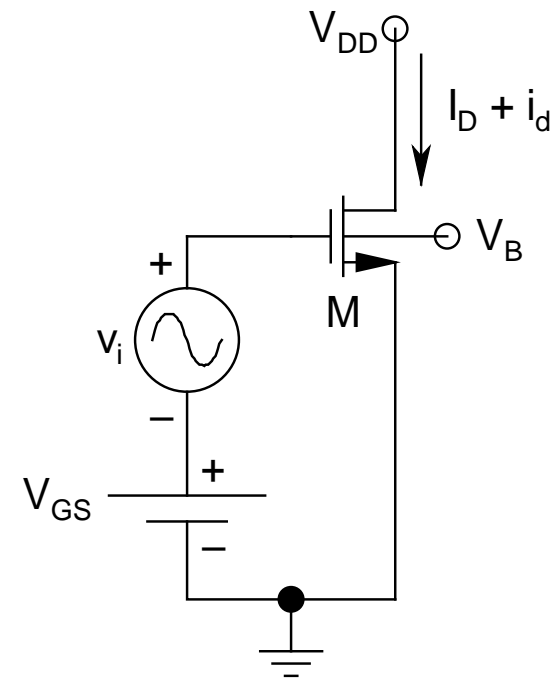
# Small-Signal Model

- The *electrical equivalent* of the MOSFET at the *DC bias point*
- *Must be biased in saturation*
  - *Resembles a constant current source*
- *DC analysis must precede*, since *need the information regarding the Q-point* ( $I_D$ ,  $V_{DS}$ )
- *This model for NMOS and PMOS is the same (incremental model)*

# Validity of the Small-Signal Model

- The *instantaneous current* (assuming  $\lambda V_{DS} < 0.1$ ):

$$\begin{aligned} I_d &= \frac{k_N}{2} (V_{GT} + v_i)^2 \\ &= I_D + \frac{k_N}{2} [2V_{GT}v_i + v_i^2] \\ \Rightarrow i_d &= k_N V_{GT} v_i \left[ 1 + \frac{v_i}{2V_{GT}} \right] \end{aligned}$$





- Thus, for *linear relationship* between  $i_d$  and  $v_i$ ,  *$v_i$  must be  $\ll V_{GT}$*
- Note that  $V_{GT} (\text{minimum}) = 3V_T$
- Hence,  *$v_i$  should be at least ten times less than  $3V_T$*
- Recall in *BJT*, for *linear relationship* between  $i_c$  and  $v_i$ ,  *$v_i$  has to be  $\ll V_T$* 
  - *Three times less than that for MOSFET*
  - *MOSFETs are inherently more linear device than BJTs (compare quadratic with exponential)*

# Small-Signal Model Parameters

- **Transconductance** ( $g_m$ ):

$$g_m \triangleq \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} \text{ and } V_{SB} \text{ constant}}$$
$$= k_N V_{GT} (1 + \lambda V_{DS}) = \sqrt{2k_N I_D (1 + \lambda V_{DS})}$$

➤ *If  $\lambda V_{DS} < 0.1$ :*

$$g_m \approx k_N V_{GT} \approx \sqrt{2k_N I_D}$$

- An important *Figure of Merit* is *transconductance to current ratio*
  - For *MOSFETs*:  $g_m/I_D = 2/V_{GT}$
  - For *BJTs*:  $g_m/I_C = 1/V_T$
  - Thus, *BJTs produce more  $g_m$  per unit current*
- As we will see later, a *high value of  $g_m$  is highly desirable*, since it *dictates the gain*
- *$g_m/I_D$  can be changed by changing the bias current and/or aspect ratio*
- *$g_m/I_C$  is a function only of temperature*

- **Body Transconductance** ( $g_{mb}$ ):

$$g_{mb} \triangleq \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V_{GS} \text{ and } V_{DS} \text{ constant}} = \chi g_m$$

$$\chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = \textit{Body factor} \quad (\sim 0.1-0.3)$$

- *Note: As  $V_{SB} \uparrow$ ,  $V_{TN} \uparrow \Rightarrow I_D \downarrow$*
- *$\partial I_D / \partial V_{SB}$  would have yielded negative  $g_{mb}$*
- *If both B and S are tied to fixed DC potentials (including ground),  $g_{mb}$  won't matter!*

- **Output Conductance** ( $g_0$ )/

**Output Resistance** ( $r_0$ ):

$$g_0 = r_0^{-1} \triangleq \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS} \text{ and } V_{SB} \text{ constant}} = \frac{\lambda I_D}{1 + \lambda V_{DS}}$$

➤ *If  $\lambda V_{DS} < 0.1$ :*

$$g_0 = 1/r_0 \approx \lambda I_D$$

➤  *$\lambda$  has a very wide range  $\sim 0.01$ - $0.5 \text{ V}^{-1}$*

➤ *When  $\lambda \rightarrow 0$ ,  $g_0 \rightarrow 0$ , and  $r_0 \rightarrow \infty$*

- *Device starts to behave like a constant current source*

- *Gate-Source and Gate-Drain Capacitance*

( $C_{gs}$  and  $C_{gd}$ ):

- Each has *two components*: *intrinsic* (i) and *technological* (t)
- *Total intrinsic gate-body capacitance*:  
$$C_{gbi} = C'_{ox} WL$$
- Using *Meyer's model*, *intrinsic component*:
  - In *linear region*:  $C_{gsi} = C_{gdi} = C_{gbi}/2$
  - In *saturation region*:  $C_{gsi} = (2/3)C_{gbi}$ ,  $C_{gdi} = 0$
- *Technology component arises due to gate-source and gate-drain overlap* ( $L_D$ )

➤ ***Technology components:***

$$C_{gst} = C_{gdt} = C'_{gs0} W = C'_{gd0} W$$

***Gate-Source/Drain Overlap Capacitance***

***per unit width:  $C'_{gs0} = C'_{gd0} = C'_{ox} L_D$***

➤ Thus, ***total capacitance in saturation:***

$$C_{gs} = (2/3) C'_{ox} WL + C'_{gs0} W$$

$$C_{gd} = C'_{gd0} W$$

➤  **$C_{gs} \gg C_{gd}$**

- ***Source-Body and Drain-Body Capacitance***

( $C_{sb}$  and  $C_{db}$ ):

➤ ***Both reverse-biased  $n^+p$  junctions***

$$C_{sb} = \frac{C_{sb0}}{(1 + V_{SB}/V_0)^m} \quad \text{and} \quad C_{db} = \frac{C_{db0}}{(1 + V_{DB}/V_0)^m}$$

$$C_{sb0} = C_{sb}|_{V_{SB}=0} \quad \text{and} \quad C_{db0} = C_{db}|_{V_{DB}=0}$$

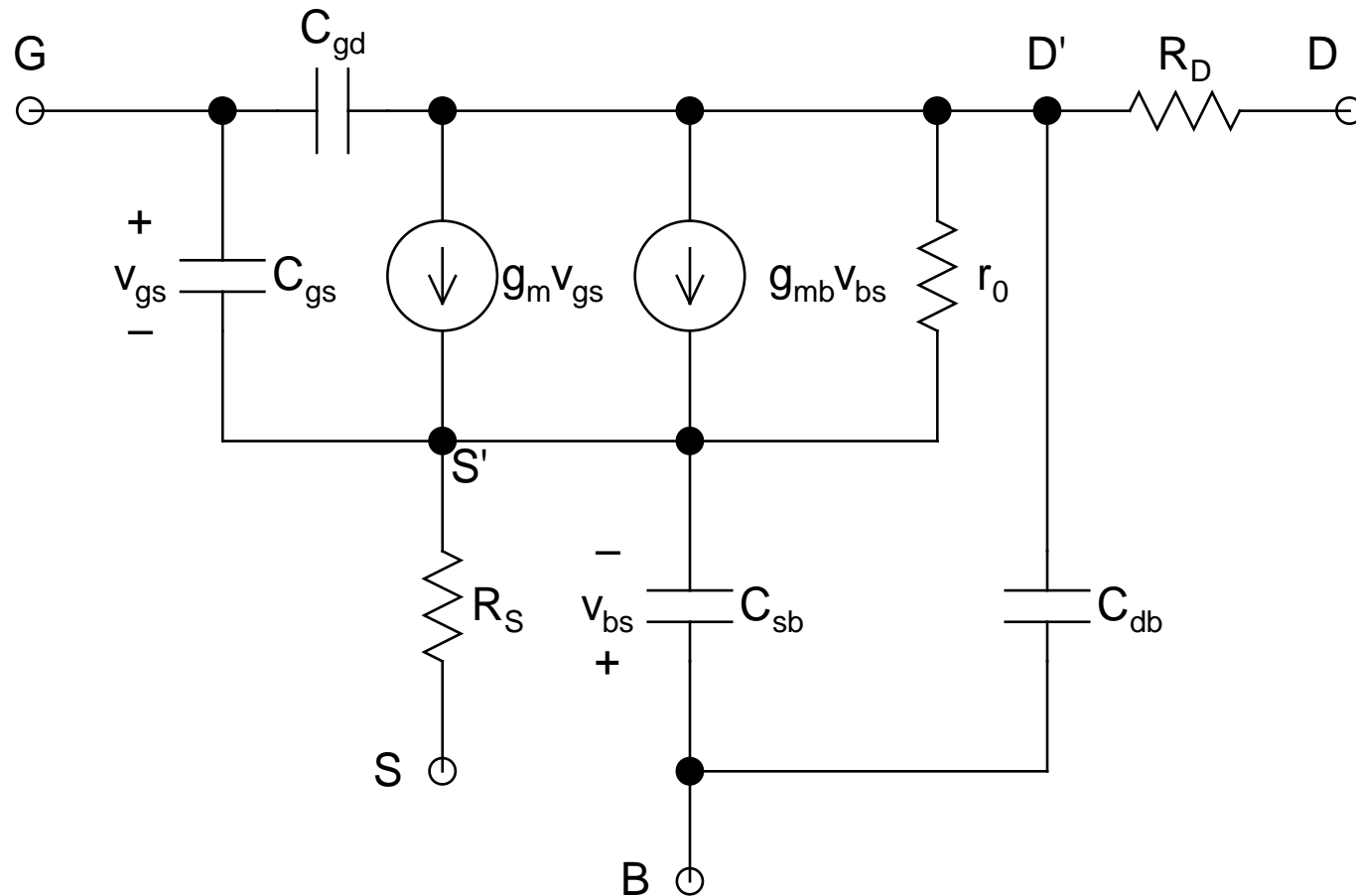
$$V_{SB} \text{ and } V_{DB} \geq 0$$

- ***Drain/Source Series Resistance*** ( $R_S$  and  $R_D$ ):

➤ ***Due to neutral  $n^+$  source/drain regions***

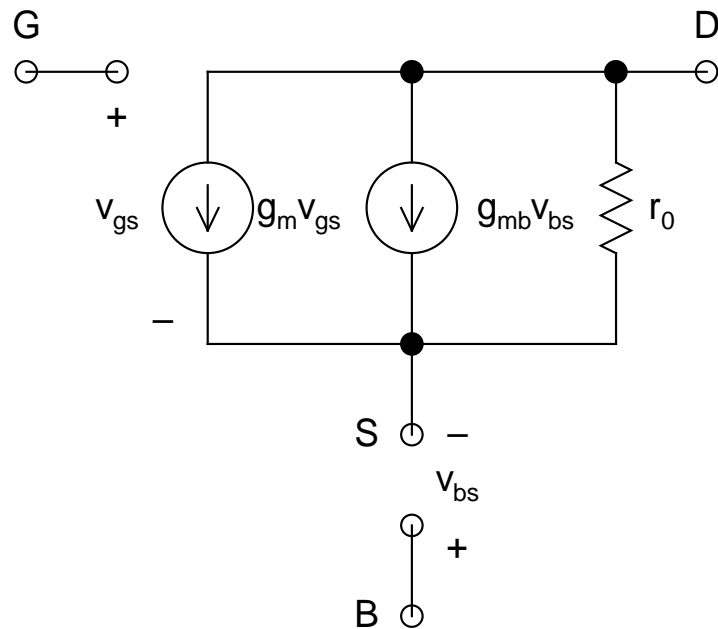


# The Hybrid- $\pi$ Model

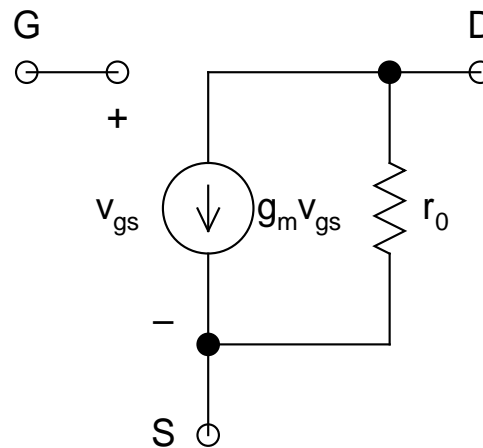


- *Simplifications:*

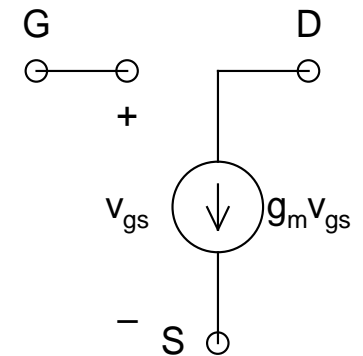
- $R_S$  and  $R_D$  can be safely neglected
- For *low to moderate frequencies*, the *capacitive reactances* of all the capacitances will be *extremely large*  $\Rightarrow$  *can be neglected*
- *If both B and S are connected to fixed DC potentials*, *current source  $g_{mb}v_{bs}$  disappears*
- Leads to the *Low-Frequency T-Model*, having only *two components*:  $g_m v_{gs}$  and  $r_o$
- *Simplest possible equivalent results if  $r_o$  is also neglected (ideal current source!)*



**Low-Frequency T-Model  
With Body Effect**



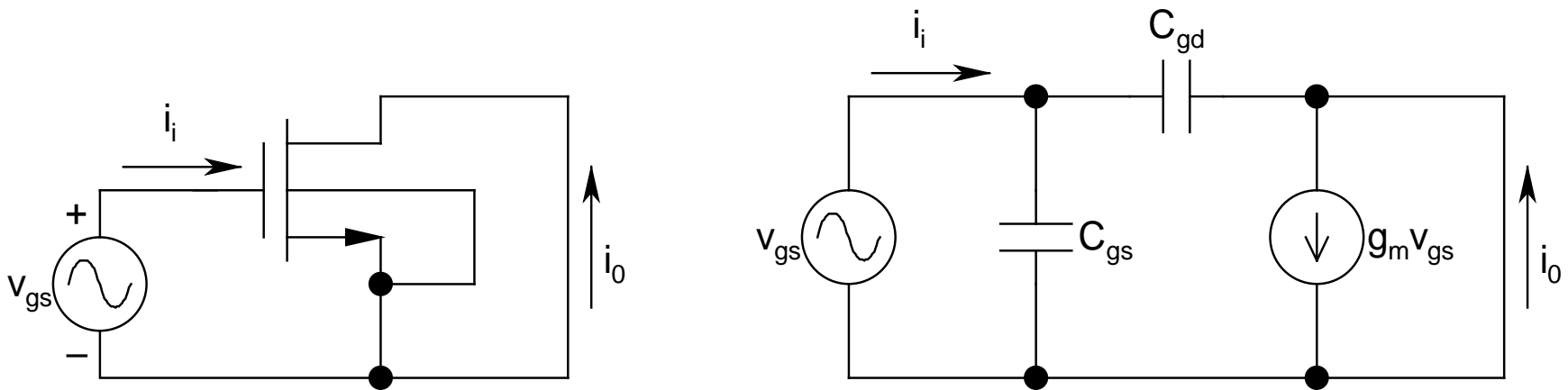
**Low-Frequency T-Model  
Without Body Effect**



**Without CLM**

# Frequency Specification of MOSFETs

- Only *Unity-Gain Frequency* ( $f_T$ )



- $i_0 \approx g_m v_{gs}$  (neglecting *reverse transmission* through  $C_{gd}$ )
- $i_i = j\omega(C_{gs} + C_{gd})$

$$\Rightarrow \frac{i_0(j\omega)}{i_i(j\omega)} = \frac{g_m}{j\omega(C_{gs} + C_{gd})}$$

$$\text{At } f = f_T, |i_0/i_i| = 1$$

$$\Rightarrow f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

- *Remarkable similarity with that for BJT*

- **Maximum Operable Frequency** ( $f_{\max}$ ):

- *Maximum possible  $f_T$*

- Noting that  $C_{gs} \gg C_{gd}$ , **neglecting  $C_{gst}$** , and *substituting the expressions for  $C_{gsi}$  and  $g_m$* :

$$f_{\max} = f_T \Big|_{\max} = \frac{3\mu_n V_{GT}}{4\pi L^2}$$

- $f_{\max} \propto 1/L^2$

- *Thrust towards making  $L$  as small as possible*

- $f_{\max} \propto V_{GT}$

- ***Making  $V_{GT}$  large may be detrimental!***