Descriptive Solutions

1.
$$\begin{cases} y'' + \lambda y = 0 \\ y(\pi) = 0 = y(2\pi). \end{cases}$$

1 marks For showing $\lambda < 0$ cannot be an Eigen value.

STUDENTS MAY OBTAIN THIS BY "TWO" WAYS.

Way 1: They can directly say that this is Regular sturm Liouville eigen value problem (RSLEVP) since we know "$\lambda < 0$" not possible.

Way 2 writing down the Auxilliary eqn $m^2 + \lambda = 0$

and assuming $\lambda = -r^2$ (<0) (if possible) (for $r > 0$)

$m = \pm r$

$\Rightarrow$ General sol^n

$y(x) = Ae^{rx} + Be^{-rx}$.

$\because$ $y(\pi) = y(2\pi) = 0$

$$\Rightarrow \begin{cases} A e^{\sigma\pi} + B e^{-\sigma\pi} = 0 \\ A e^{\sigma 2\pi} + B e^{-2\pi\sigma} = 0 \end{cases}$$

Here $\begin{vmatrix} e^{\sigma\pi} & e^{-\sigma\pi} \\ e^{2\sigma\pi} & e^{-2\sigma\pi} \end{vmatrix} = e^{-\sigma\pi} - e^{\sigma\pi}$

$= 0$ (only if $\sigma = 0$).

$\Rightarrow A = B = 0$

$\Rightarrow y \equiv 0$ is the soln. Contradiction

$\boxed{1 \text{ marks}}$ To rule out that $\boxed{\lambda = 0}$ is an eigenvalue.

∴ AGAIN STUDENTS MAY ACHEIVE IN TWO WAYS :-

Way 1 $\lambda = 0$ cannot be E.V as this is RSLEVP.

Way 2 Auxilliary Eqn $m^2 = 0$.

$\Rightarrow y(x) = Ax + B$ is General Soln

$\Rightarrow y(0) = y(2\pi) = 0 \Rightarrow y \equiv 0$.

(Contradiction).

1 marks :- For the case $\lambda > 0$, writing down the general Solution correctly..

, Auxilliary Eqn. is

$m^2 + \delta^2 = 0$.

$$\Rightarrow \boxed{y(x) = A\cos(\delta x) + B\sin(\delta x)} \rightarrow (*)$$

Now they have to put the Boundary conditions

$$y(\pi) = 0 = y(2\pi)$$

$$\Rightarrow \begin{cases} A\cos(\delta\pi) + B\sin(\delta\pi) = 0 \\ A\cos(2\pi\delta) + B\sin(2\pi\delta) = 0 \end{cases}$$

1 marks

For getting nontrivial soln. of A, B, we want

$$\begin{vmatrix} \cos(\delta\pi) & \sin(\delta\pi) \\ \cos(2\delta\pi) & \sin(2\delta\pi) \end{vmatrix} = \sin(\delta\pi) = 0$$

1 marks

$\Rightarrow \sin(r\pi) = \sin(n\pi) \quad n \in \mathbb{Z}$

$\Rightarrow r = n$

$$\boxed{\lambda = r^2 = n^2 \quad , n \in \mathbb{N}}$$

1 mark for finding out the Eigen functions corresponding to the Eigen value "n^2".

$y_n(x) = A_n \cos(nx) + B_n \sin(nx)$ from ⊛

$y_n(\pi) = 0 \Rightarrow A_n = 0$

$$\Rightarrow \boxed{y_n(x) = B_n \sin(nx)}$$

is the required Eigen function.

ANOTHER SMART WAY

IF SOME ONE SAYS,

we already know that

$y_n(t) = A_n \sin(nt)$ is an E.F

of the problem,

$$\begin{cases} y'' + \lambda y = 0 \quad \text{on } [0, \pi) \\ y(0) = y(\pi) \end{cases}$$

corresponding to the E.V "n^2."

IN THE GIVEN PROBLEM

JUST THE DOMAIN IS SHIFTED.

Hence "EV" will be the same.

and the EF will just be shifted

given by

$$y_n(t) = y_n(t-\pi)$$

$$= A_n \sin n(t-\pi)$$

$$= -A_n \sin(nt).$$

(Done)