MSO201A
Hints for solution of Q1

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Q1. P(a random biscuit made by machine A) $= p_1$,
P(a random biscuit made by machine B) $= p_2$, and
P(a random biscuit made by machine C) $= p_3 := 1 - p_1 - p_2$.
P(broken biscuit| made by A) $= a_1$,
P(broken biscuit| made by B) $= a_2$, and
P(broken biscuit| made by C) $= a_3$.

We have to find P(not made by A| broken biscuit) $= 1 - P$(made by A| broken biscuit). Using Bayes' theorem, we get

$$\begin{aligned}&P(\text{made by A}|\text{ broken biscuit})\\ =&\frac{P(\{\text{made by A}\}\cap\{\text{ broken biscuit}\})}{P(\text{ broken biscuit})}.\end{aligned}$$

Here,

$$\begin{aligned}P(\{\text{made by A}\}\cap\{\text{ broken biscuit}\}) =&P(\text{made by A})P(\text{ broken biscuit}|\text{made by A})\\ =&p_1a_1.\end{aligned}$$

$$\begin{aligned}P(\text{ broken biscuit}) =&P(\text{made by A})P(\text{ broken biscuit}|\text{made by A})\\ &+P(\text{made by B})P(\text{ broken biscuit}|\text{made by B})\\ &+P(\text{made by C})P(\text{ broken biscuit}|\text{made by C})\\ =&p_1a_1+p_2a_2+p_3a_3.\end{aligned}$$

Thus, the required probability is

$$P(\text{not made by A}|\text{ broken biscuit}) = 1 - \frac{p_1a_1}{p_1a_1+p_2a_2+p_3a_3} = \frac{p_2a_2+p_3a_3}{p_1a_1+p_2a_2+p_3a_3}.$$

Q2. Use enumeration.

Q3. Note the following facts:

$$\text{Events } A \text{ and } B \text{ are independent} \iff P(A\cap B) = P(A)P(B).$$

$$\begin{aligned}P(A\cap B) =&P(A)+P(B)-P(A\cup B); P(A^c\cap B) = P(B)-P(A\cap B);\ \text{and}\\ P(A^c|B) =&\frac{P(A^c\cap B)}{P(B)}.\end{aligned}$$

Q4. Total number of triangles that can be formed is $\binom{n}{3}$. For a regular polygon with even number of vertices, longest diagonal is diameter of the circumcircle of the polygon. Note that the angle of semicircle is right angle. So, the number of favourable cases is $\frac{n-2}{2} \times n$.

Q5. Favorable event is $\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$ and the probability of this event is $2(\frac{1}{6}+x)(\frac{1}{6}-x)+4\times\frac{1}{36}$.

Q6. Observe the following:

$$\begin{aligned}
P(B|A) =& \frac{P(B\cap A)}{P(A)} = \frac{P(B)+P(A)-P(A\cup B)}{P(A)} \\
\geq& \frac{P(B)+P(A)-1}{P(A)} = 1+\frac{P(B)}{P(A)}-\frac{1}{P(A)}.
\end{aligned}$$

Q7. Use enumeration.

Q8. Favourable cases, and their probabilities are as follows:

Biased	Unbiased	Probability
HT	TT	$x(1-x)\times\frac{1}{2}\times\frac{1}{2}$
TH	TT	$x(1-x)\times\frac{1}{2}\times\frac{1}{2}$
TT	HT	$(1-x)(1-x)\times\frac{1}{2}\times\frac{1}{2}$
TT	TH	$(1-x)(1-x)\times\frac{1}{2}\times\frac{1}{2}$