EE 200: Solution Set 8

1.
$$y[n] = x_u[n] + \frac{1}{2}x_u[n+1] + \frac{1}{2}x_u[n-1]$$

Taking the DTFT of both sides, we get

$$\begin{aligned} Y(e^{j\omega}) &= X_u(e^{j\omega}) + \frac{1}{2}e^{j\omega}X_u(e^{j\omega}) + \frac{1}{2}e^{-j\omega}X_u(e^{j\omega}) \\ &= (1 + \frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega})X_u(e^{j\omega}) \end{aligned}$$

Hence, the frequency response

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X_u(e^{j\omega})} = 1 + \cos(\omega)$$

which has zero phase and the magnitude given by

$$|H(e^{j\omega})| = 1 + \cos(\omega)$$

Note that $H(e^{j0}) = 2$, $H(e^{j\pi/2}) = 1$, and $H(e^{j\pi}) = 0$; the magnitude is a monotonically decreasing function of ω, lowpass characteristic. Hence the interpolator is a low-pass filter.

2. $y[n]-1.0148y[n-1]+0.7265y[n-2] = 0.1367x[n]-0.1367x[n-2]$
Take the z-transform of both sides and simplifying, we get the transfer function

$$H(z) = \frac{0.1367 - 0.1367z^{-2}}{1 - 1.0148z^{-1} + 0.7265z^{-2}}$$

By setting $z^{-1} = e^{-j\omega}$, we get the frequency response

$$H(e^{j\omega}) = \frac{0.1367 - 0.1367e^{-j2\omega}}{1 - 1.0148e^{-j\omega} + 0.7265e^{-j2\omega}}$$

The magnitude and gain responses are shown in Fig.1. The response has a bandpass characteristics, and it is a bandpass filter with a center frequency at 0.3π

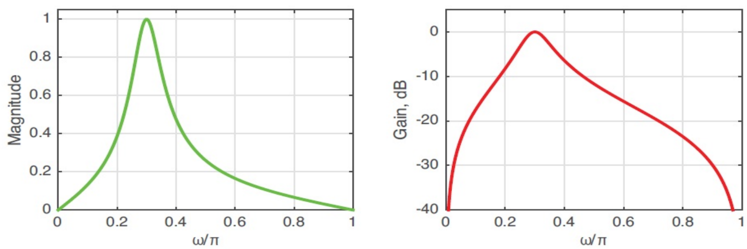

Figure 1: Diagram for solution 2

3. $y[n] - 1.0148y[n-1] + 0.7265y[n-2] = 0.8633x[n] - 1.0148x[n-1] + 0.8633x[n-2]$
   Take the z-transform of both sides, simplifying, and setting $z^{-1} = e^{-j\omega}$, we get the frequency response

$$H(e^{j\omega}) = \frac{0.8633 - 1.0148e^{-j\omega} + 0.8633e^{-j2\omega}}{1 - 1.0148e^{-j\omega} + 0.7265e^{-j2\omega}}$$

The magnitude and gain responses are shown in Fig.2. The response has a bandstop characteristics, and it is a notch filter with a center frequency at 0.3π

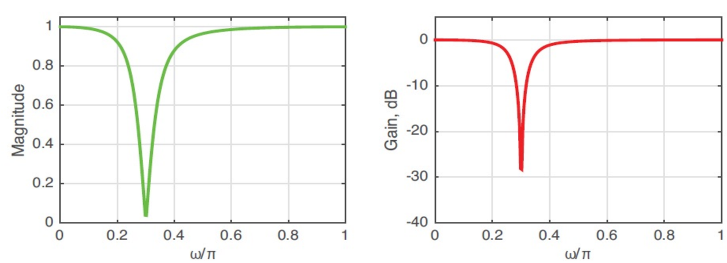

Figure 2: Diagram for solution 3

4. The frequency response is

$$H(e^{j\omega}) = \frac{b + ce^{-j\omega}}{1 + ae^{-j\omega}}$$

The magnitude square function is given by

$$|H(e^{j\omega})|^2 = H(e^{j\omega})H(e^{-j\omega}) = \Big(\frac{b+ce^{-j\omega}}{1+ae^{-j\omega}}\Big)\Big(\frac{b+ce^{j\omega}}{1+ae^{j\omega}}\Big)$$
$$= \frac{b^2+c^2+2bc\cos(\omega)}{1+a^2+2a\cos(\omega)}$$

Hence $|H(e^{j\omega})|^2$ will be constant for all values of ω if $\{b = k$ and $c = ka\}$ or $\{b = ka$ and $c = k\}$. For $b = k$ and $c = ka$, $|H(e^{j\omega})|^2 = k^2$ implying $H(e^{j\omega}) = \pm k$, a trivial solution. For $b = ka$ and $c = k$ $H(e^{j\omega}) = \pm k\frac{a+e^{-j\omega}}{1+ae^{j\omega}}$

5.

$$H(e^{j\omega}) = 3 - 5e^{-j\omega} + ae^{-j2\omega} + be^{-j3\omega}$$
$$= e^{-j3\omega/2}\Big(3e^{j3\omega/2} - 5e^{j\omega/2} + ae^{-j\omega/2} + be^{-j3\omega/2}\Big)$$
$$= e^{-j3\omega/2}\Big((3e^{j3\omega/2} + be^{-j3\omega/2}) - (5e^{j\omega/2} - ae^{-j\omega/2})\Big)$$

Thus, for $b = 3$ and $a = -5$, we have

$$H(e^{j\omega}) = e^{-j3\omega/2}\Big(6\cos(3\omega/2) - 10\cos(\omega/2)\Big)$$

which has a linear phase characteristic.

6. $H(e^{j\omega}) = \frac{2-0.8e^{-j\omega}}{1+0.9e^{-j\omega}}$ and $\tilde{x}[n] = 5\cos(0.2\pi n + 0.4) + 3\cos(0.8\pi n + 0.8)$.
Thus the steady-state output response $\tilde{y}[n]$ is of the form

$$\tilde{y}[n] = 5|H(e^{j0.2\pi})|\cos(0.2\pi n+\theta_1+0.4)+3|H(e^{j0.8\pi})|\cos(0.8\pi n+\theta_2+0.8)$$

where $\theta_1 = \{\arg H(e^{j0.2\pi})\}$, $\theta_2 = \{\arg H(e^{j0.8\pi})\}$
Now, $H(e^{j0.2\pi}) = \frac{2-0.8e^{-j0.2\pi)}}{1+0.9e^{-j0.2\pi)}} = \frac{1.3528+j0.4702}{1.7281-j0.5290} = 0.6369 + j0.4679$
Thus, $|H(e^{j0.2\pi})| = 0.7925, \theta_1 = 0.6316$.

$$H(e^{j0.8\pi}) = \frac{2-0.8e^{-j0.8\pi}}{1+0.9e^{-j0.8\pi}} = \frac{2.6472+j0.4702}{0.2719-j0.5290} = 1.3313 + j4.3199$$

Thus, $|H(e^{j0.8\pi})| = 4.5204, \theta_2 = 1.2718$. Therefore, the steady-state response is

$$\tilde{y}[n] = 3.9625\cos(0.2\pi n + 1.0316) + 13.5612\cos(0.8\pi n + 2.0718)$$

7.

$$H_{HP}(e^{j\omega}) = \begin{cases} 0; & |\omega| < \omega_c \\ 1; & \omega_c \le |\omega| \le \pi \end{cases}$$

$$\begin{aligned} h_{HP}[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{HP}(e^{j\omega})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{-\omega_c} + \frac{1}{2\pi}\frac{e^{j\omega n}}{jn}\Big|_{\omega_c}^{\pi} \\ &= \frac{1}{2\pi}\Big(\frac{e^{-j\omega_c n}}{jn} - \frac{e^{-j\pi n}}{jn} + \frac{e^{j\pi n}}{jn} - \frac{e^{j\omega_c n}}{jn}\Big) \\ &= \frac{\sin(\pi n)}{\pi n} - \frac{\sin(\omega_c n)}{\pi n} \end{aligned}$$

Using the L'Hospital's rule, we obtain $h_{HP}[0] = 1 - \frac{\omega_c}{\pi}$ and $h_{HP}[n] = \frac{\sin(\omega_c n)}{\pi n}$ for $|n| \ge 0$.

8. Note that the magnitude response of $G(-z)G(z^2)$ is the same as that of $G(z^2)$. Further, the magnitude response of $G(z)G(-z^2)$ is the same as that of $G(z)G(z^2)$.

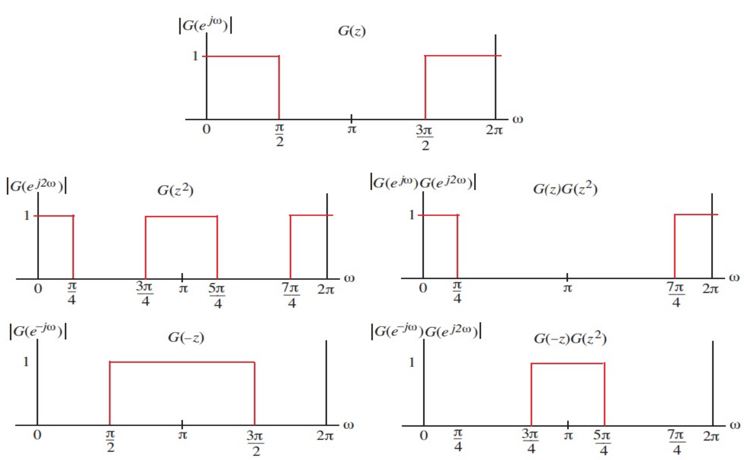

Figure 3: Diagram for solution 8