- Note also that the ***comparison*** of the two expressions show that σ is actually an ***imaginary number***
  - This needs further ***exploration***
- Eq.(1) can be written in ***polar form*** as:

$$
\begin{aligned}
v(t) &= (V_M/2)[\exp\{j(\omega t + \phi)\} + \exp\{-j(\omega t + \phi)\}] \\
&= [(V_M/2)\exp(j\phi)]\exp(j\omega t) \\
&\quad + [(V_M/2)\exp(-j\phi)]\exp(-j\omega t) \\
&= A\exp(s_1 t) + B\exp(s_2 t) \qquad (2)
\end{aligned}
$$

$s_1 = j\omega$ and $s_2 = -j\omega$

$A = (V_M/2)\exp(j\phi)$ and $B = (V_M/2)\exp(-j\phi)$

$\Rightarrow$ ***s_1 and s_2 as well as A and B are complex conjugates***

$\Rightarrow$ The 2 terms of Eq.(2) are also ***complex conjugates***, with their ***sum*** being a ***real number***

- Similarly, a ***sinusoidal signal*** with an ***exponential envelope*** can be expressed by:

$$v(t) = [(V_M/2)\exp(j\phi)]\exp[(\sigma + j\omega)t] + [(V_M/2)\exp(-j\phi)]\exp[(\sigma - j\omega)t] = A\exp(s_1 t) + B\exp(s_2 t) \qquad (3)$$

- ***Matching coefficients*** of Eq.(3), we get a ***complex pair of frequencies***:

  $s_1 = (\sigma + j\omega)$ and $s_2 = (\sigma - j\omega)$

  which are also ***complex conjugates***
- Thus, ***sinusoidal signals*** having ***complex envelopes***, can be expressed in terms of a ***complex frequency*** s
- s has ***both real and imaginary parts*** (***σ and $j\omega$ respectively***)

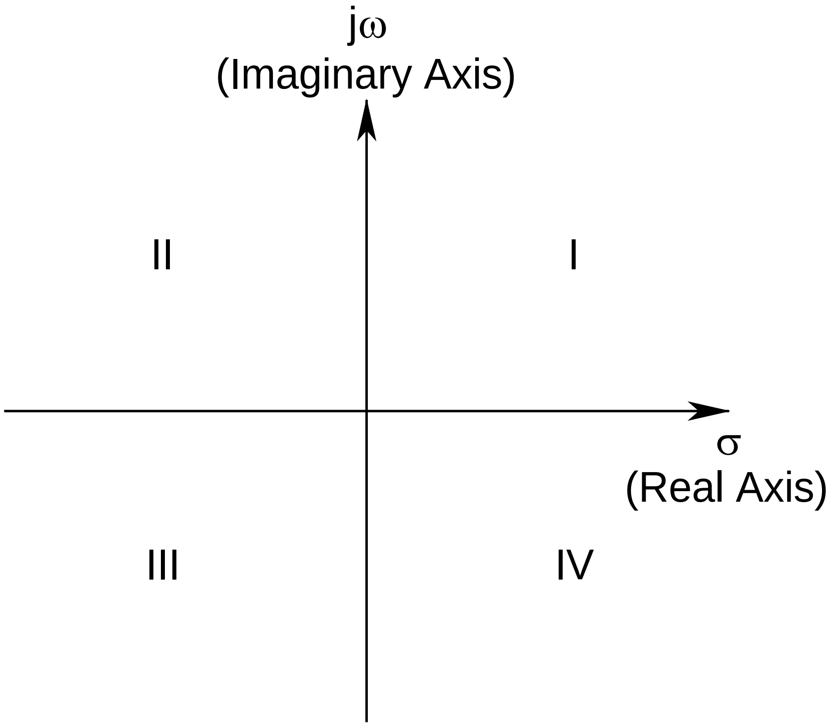
- s is defined by:

  $s = \sigma \pm j\omega$

  σ: ***Real part*** - dictates the ***exponential rise/fall*** of the signal

  ω: ***Imaginary part*** - ***actual angular frequency***, describes the ***sinusoidal variation*** of the signal

- s is represented in a ***graphical form*** as a 2D plane, with ***σ plotted along the x-axis*** (known as the ***real axis***), and ***ω plotted along the y-axis*** (known as the ***imaginary axis***)



The Complex Frequency Plane (s-Plane)

- ***Poles of a transfer function can lie anywhere on this plane***
- If $\sigma = 0$, ***poles lie on the*** $j\omega$ ***axis***

$\Rightarrow$ ***Perfect sinusoidal response***

- If $\omega = 0$, ***poles lie on the*** σ ***axis***

$\Rightarrow$ ***Pure exponential response***

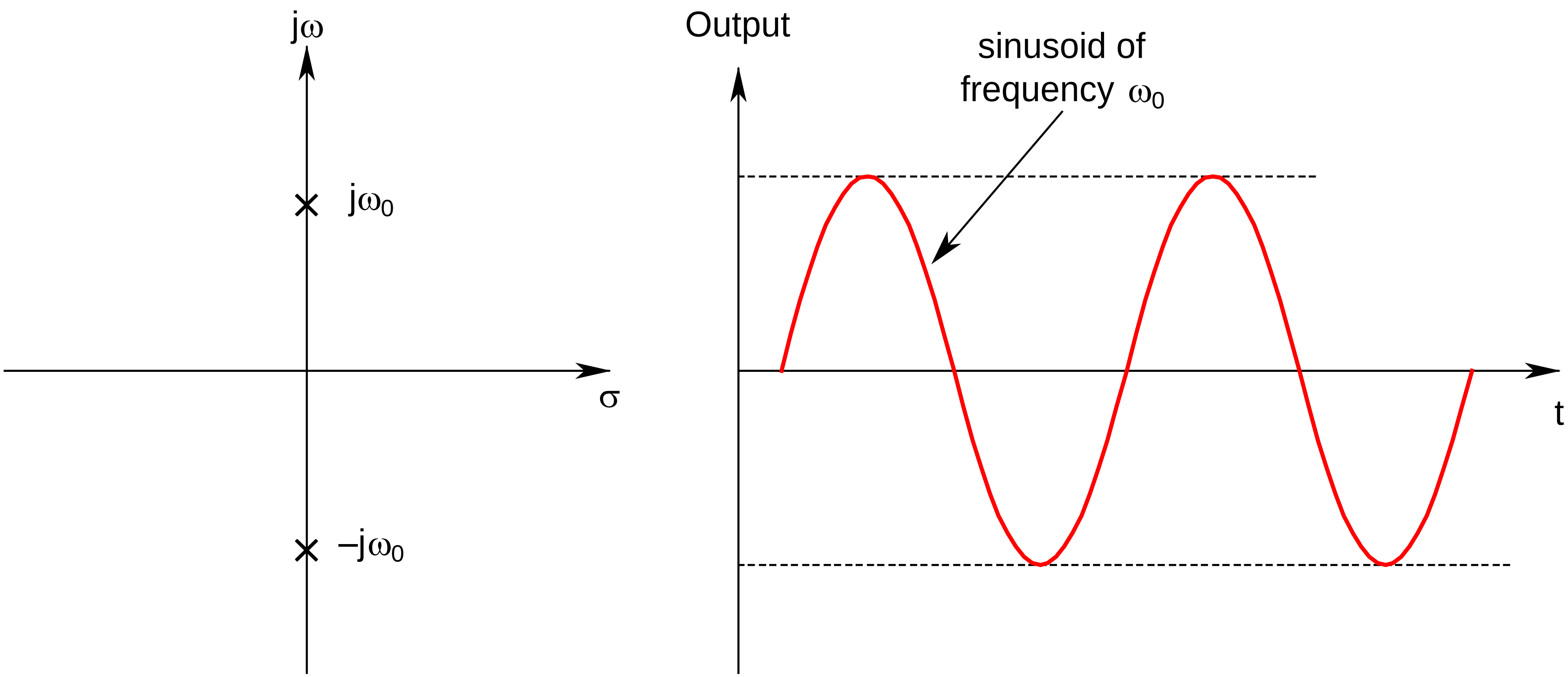
- If a pole has ***both real and imaginary parts***, then the ***response*** would be either an ***exponentially increasing or decreasing sinusoid***

- ***Pole Location & Stability***:
  - ***Locations of the poles in the s-plane governs the stability of the system***
  - We will consider ***3 cases***:
    - ***Complex conjugate poles without any real part***
    - ***Complex conjugate poles with negative real part***
    - ***Complex conjugate poles with positive real part***

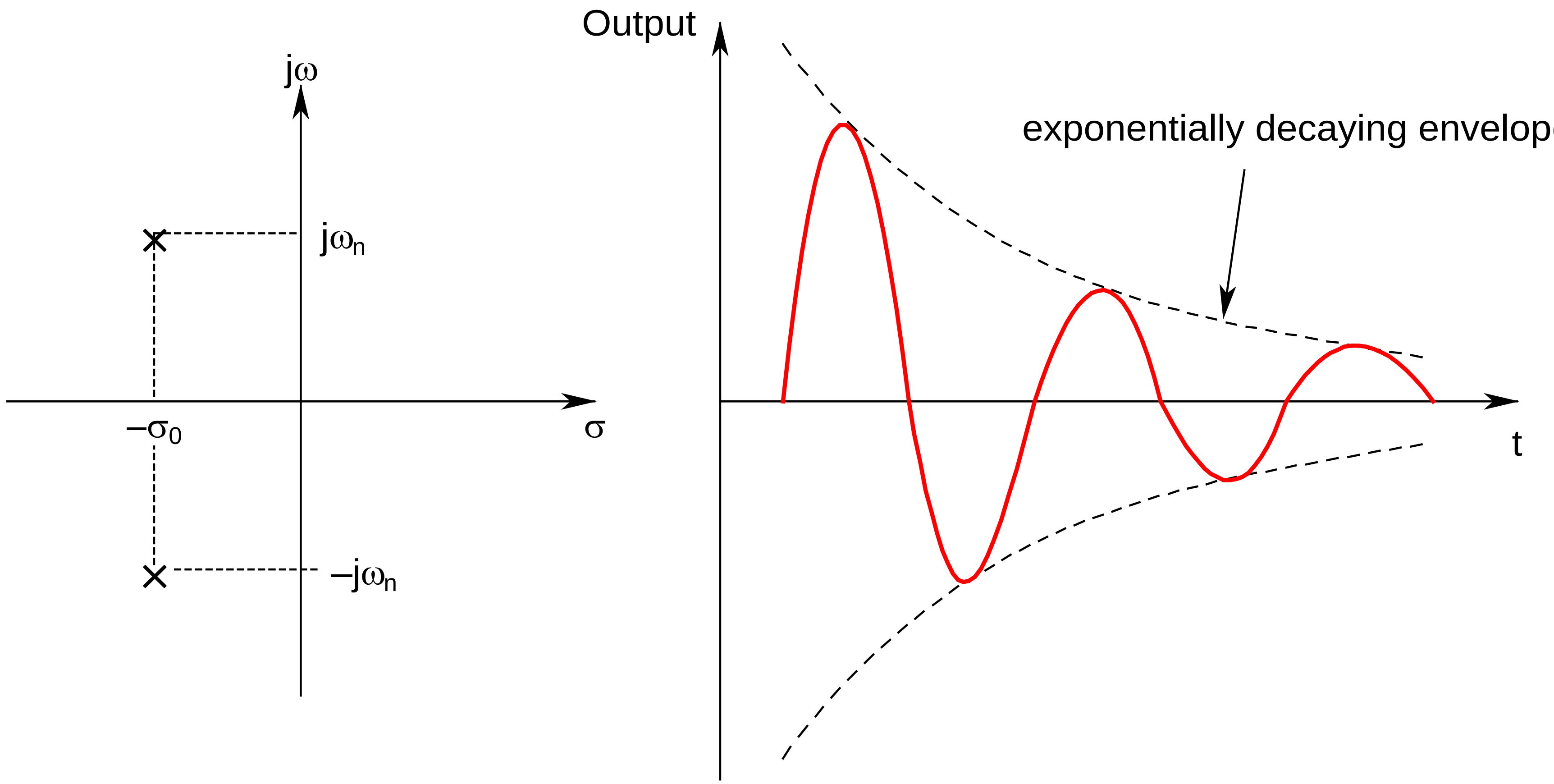
- ***Complex conjugate poles s_1 (= $j\omega_0$) and s_2 (= $-j\omega_0$), without any real part*:**

  $\Rightarrow$ ***Undamped sinusoidal response***

  $\Rightarrow$ ***Perfectly stable system***

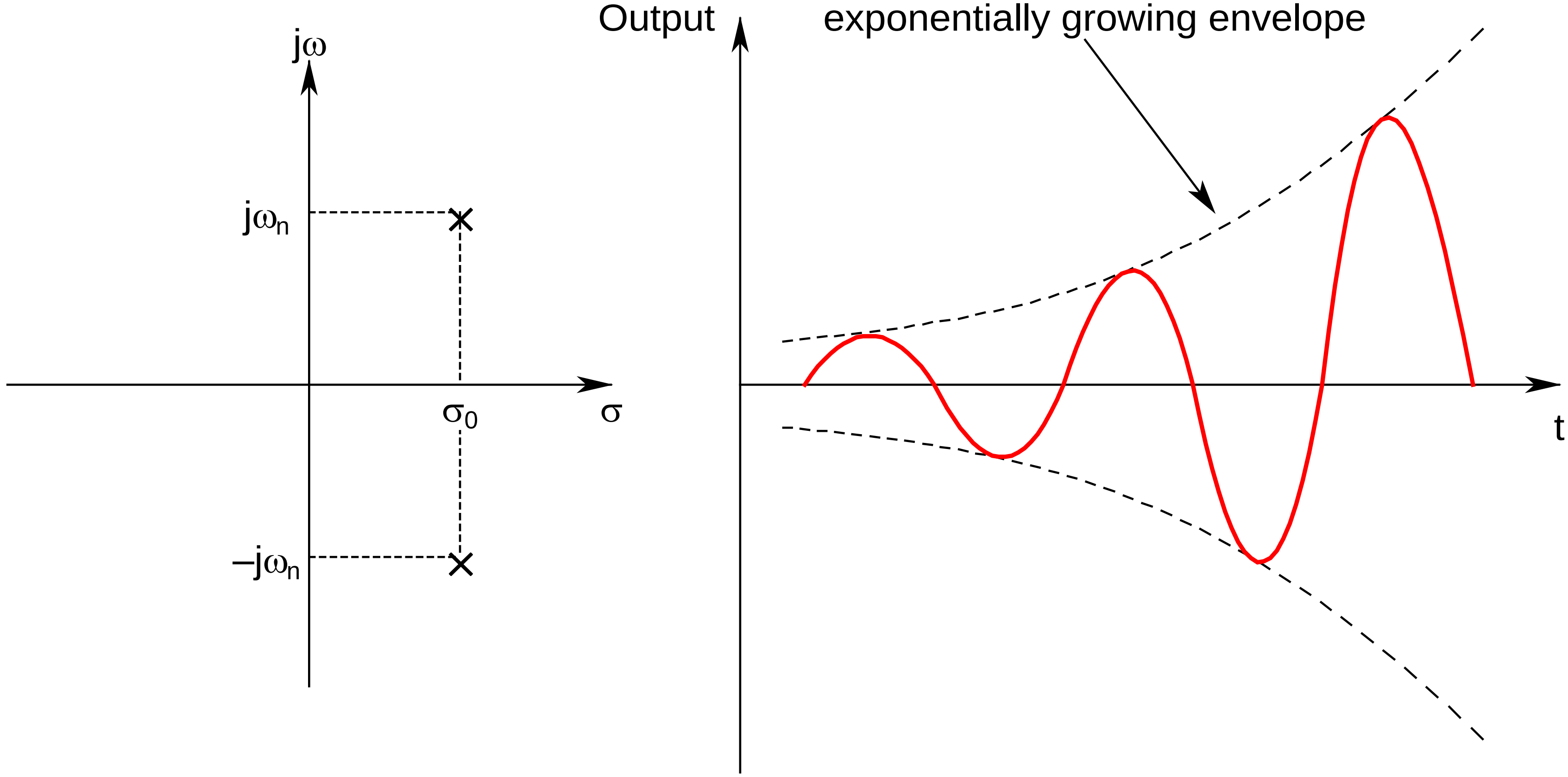


- ***Complex conjugate poles [$s_1 = (\sigma_0 + j\omega_n)$ and $s_2 = (\sigma_0 - j\omega_n)$], with negative real part (σ_0 negative)***:
  - ***Poles lie in the left-half plane*** (***LHP*** - ***Quadrants II and III***)
  - ***Response to any transient disturbance will be sinusoidal***, but ***with an exponentially decaying envelope***
  - ***Such systems also are stable or well-behaved***



Response to Transient Disturbance of a System Having Poles in the LHP (Stable System)

- ***Complex conjugate poles [$s_1 = (\sigma_0 + j\omega_n)$ and $s_2 = (\sigma_0 - j\omega_n)$], with positive real part (σ_0 positive)***:
  - ***Poles lie in the right-half plane*** (***RHP*** - ***Quadrants I and IV***)
  - ***Response to any transient disturbance will still be sinusoidal***, but now ***with an exponentially rising envelope***
  - ***The system now is NOT well-behaved***, rather ***ill-behaved***, and an ***unstable system***



Response to Transient Disturbance of a System Having Poles in the RHP (Unstable System)