Q1. The circuit given in Figure 1 has two inputs v_1, *and* v_2. The voltage across the capacitor is v_C, the response (output) of the circuit. The voltage across the inductor is v_L. Given R=L=C=1 unit.

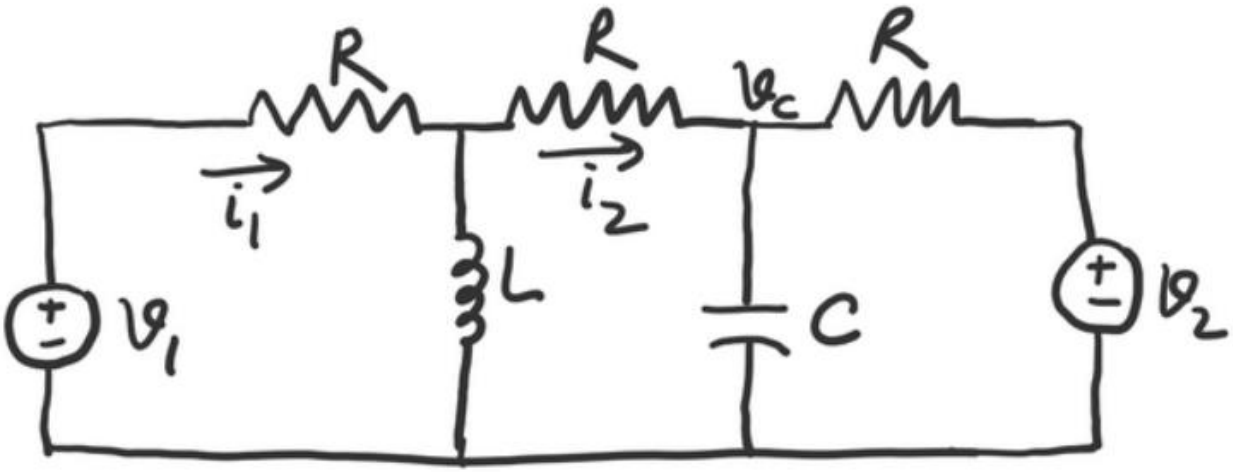

Figure 1: An Electrical Circuit

i. Write down relations among variables v_1, i_1, v_L, i_2, v_C, *and* v_2. [3 marks]

ii. Draw the signal flow graph where v_1, *and* v_2 are inputs and v_C is the output. i_1, v_L, i_2 are other node variables. [5 marks]

iii. Find $\frac{V_C(s)}{V_1(s)}$ and $\frac{V_C(s)}{V_2(s)}$ [7 marks]

Q2. The circuit diagram for the speed control of the DC servo motor is given in Figure 2. Parameters are given as:

$$K_T = 2.5\ \text{Nm/Amp};\ R_a = 1\Omega;\ B = 2\ \text{Nm/(rad/sec)};\ L_a = 0.25\text{H},\ J = 10\text{kgm}^2.$$

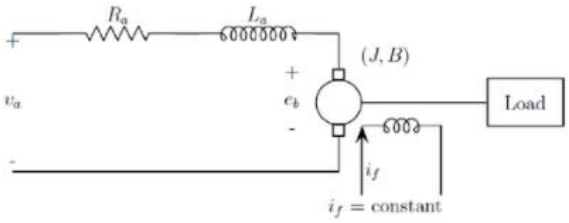

Figure 2: Separately excited DC motor

a. Compute the transfer function $\frac{\omega(s)}{V_a(s)}$ using block-diagram approach where ω is the speed (output) of the motor while v_a is the armature voltage (input). [4 marks]

b. The motor is given 100 volt sudden input. Compute the speed response $\omega(t)$ of the motor. Assume all initial conditions to be zero. Estimate the overshoot and settling time. [5 marks]

c. Derive the state space model taking states as $x_1 = \omega; x_2 = i_a$; input $u = v_a$ and output $y = x_1$. [2 marks]

d. Compute the state transition matrix e^{At}. [4 marks]

Q3. a. Determine the stability of the system with characteristic polynomial $s^4 + 1 = 0$ using a Routh array. Find out how many poles are in the right half of the s-plane. [4 marks]

b. Given $G(s) = \frac{1}{(s-2)^2}$ and $C(s) = K_P + \frac{K_I}{s} + K_D s$, find out the characteristic polynomial of the closed loop system in the unity feedback configuration. Which gain is vital to ensure stability? If $K_I = 8$, find the parametric space for which the system is stable. If the dominant pole pairs are given as $-1 \pm j1$, find K_P and K_D. [6 marks]

c. Find the roots of the polynomial
$s^7 + 2s^6 - 4s^5 - 8s^4 - 25s^3 - 50s^2 + 100s + 200$ using a Routh array. Validate your answer by interpreting the first column of the Routh array. [5 marks]

Q4. The open loop transfer function of a system in a unity-feedback configuration is given as

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}.$$

a. Make a rough sketch of the root locus plot of the system explicitly identifying the angle of asymptotes, the centroid of asymptotes, the breakaway points, and angles of departure from the complex conjugate poles. [6 marks]

b. Compute the points of intersection of the root locus with the imaginary axis, if any. Determine K corresponding to these cross-over points. [3 marks]

c. A zero is added at $s = -1.0$. Repeat part (a). [6 marks]