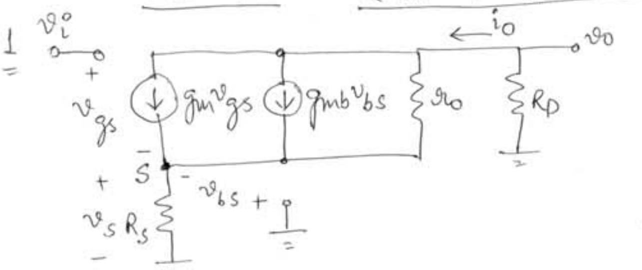
EE 210 Sol^n. to HA # 9 ①

1 

$v_o = -i_o R_D$

$v_{gs} = v_i - v_s \quad v_{bs} = -v_s \quad v_s = i_o R_s$

KCL at S: $\frac{v_s}{R_S} + \frac{v_s - v_o}{r_o} - g_m(v_i - v_s) + g_{mb} v_s = 0$

Also, $i_o = \frac{v_o - v_s}{r_o} + g_m(v_i - v_s) - g_{mb} v_s$

Combining the 3 expressions, after some algebra, it can be shown that $A_v = \frac{v_o}{v_i} = -\frac{i_o R_D}{v_i} = -\frac{g_m R_D}{1 + (g_m + g_{mb}) R_S + R_S/r_o}$.

For R_o: We short v_i to ground, remove R_D, & excite the o/p by a test source v_t.

Note: With $v_i = 0 \quad v_{gs} = v_{bs} = -v_s$.

The test current $i_t = \frac{v_t - v_s}{r_o} + (g_m + g_{mb})(-v_s)$ with $v_s = i_t R_s$.

$\Rightarrow R_o = \frac{v_t}{i_t} = R_s + r_o\left[1 + (g_m + g_{mb}) R_S\right]$. Note: if $R_s \to \infty$ $R_o \to \infty$.

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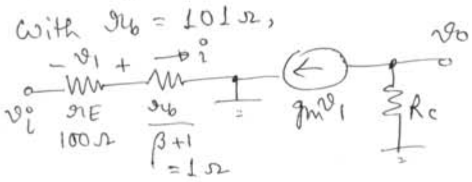
2 In the absence of base resistance r_b, $R_i = r_E$, $A_v = +\frac{R_c}{r_E}$, & $R_o = R_c$.

$I_c = 260\,\mu A \Rightarrow r_E = \frac{V_T}{I_c} = 100\,\Omega$, & $r_\pi = 10\,k\Omega$. $\Rightarrow R_i = 100\,\Omega$, $A_v = +100$, $R_o = 10\,k\Omega$.

With $r_b = 101\,\Omega$,



$v_o = -g_m v_1 R_c \quad v_1 = -i\, r_E \quad i = \frac{v_i}{r_E + \frac{r_b}{\beta+1}}$

$\Rightarrow \frac{v_o}{v_i} = +\frac{g_m R_c}{1 + \frac{r_b}{r_\pi}} = \underline{99.99}$ (very little effect)

Reason: Very small r_b as compared to r_π.

Also, R_c large compared to r_b.

Large change in A_v can be observed if R_c & r_π are low, & r_b is large.

$R_i = r_E + \frac{r_b}{\beta+1} = 101\,\Omega$. $\quad R_o = R_c = 10\,k\Omega$

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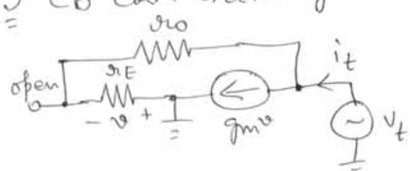
3 CB ckt. excited by an ideal current source:



$v = -\frac{v_t\, r_E}{r_E + r_o} \qquad i_t = g_m v + \frac{v_t}{r_E + r_o}$

$\Rightarrow i_t = \frac{v_t\,(1 - g_m r_E)}{r_E + r_o}$, $\Rightarrow R_o' = \frac{v_t}{i_t} = \frac{r_E + r_o}{1 - g_m r_E}$.

& if α is exactly 1, then $R_o' \to \infty$.

* Note: $g_m r_E = \alpha$

$\Rightarrow R_o' \simeq \frac{r_o}{1-\alpha}$ $\quad \because r_o \gg r_E$ (in general) & $\alpha = \frac{\beta}{\beta+1}$ $\Rightarrow R_o' \simeq r_o(\beta+1) \simeq \beta r_o$ $(\beta \gg 1)$.

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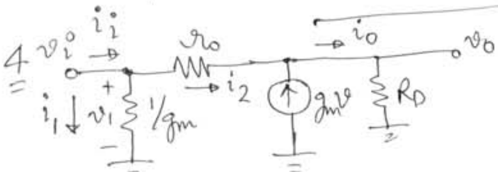
4 

$v_o = i_o R_D \quad i_o = i_2 + g_m v_1 \quad v_1 = v_i$

$i_1 = g_m v_i \quad i_i = i_1 + i_2 \quad i_2 = \frac{v_i - v_o}{r_o}$

Combine all these relations to get

$A_v = \frac{v_o}{v_i} = \frac{(1 + g_m r_o) R_D}{r_o + R_D}$. If r_o $\quad A_v \to g_m R_D$

$R_i = \frac{v_i}{i_i}$ with $i_i = i_1 + i_2$ $\quad i_1 = g_m v_i$ & $i_2 = \frac{v_i - v_o}{r_o}$ with $A_v = \frac{v_o}{v_i}$

$\Rightarrow R_i = \frac{(r_o + R_D)}{1 + g_m r_o}$. As $r_o \to \infty$, $R_i \to 1/g_m$.

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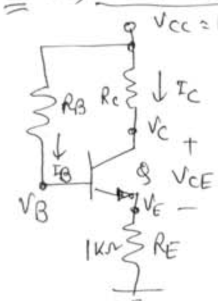
5 a) DC Analysis: Coupling & Bypass Capacitors open up.

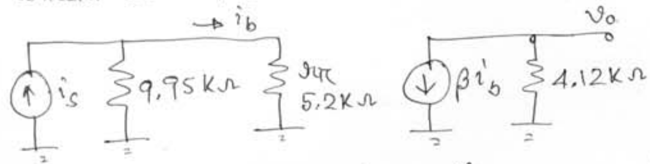


Need $I_C = 1mA$ ($\beta = 200$) $\Rightarrow I_B = \frac{I_C}{\beta} = 5\mu A \Rightarrow I_E = (\beta+1) I_B = 1.005\, mA \Rightarrow V_E = I_E R_E = 1.005\,V \Rightarrow V_B = V_E + V_{BE} = 1.705\,V$.

$\Rightarrow R_B = \frac{V_{CC} - V_B}{I_B} = \underline{\underline{2.06\,M\Omega}}$. for best biasing, $V_{CE} \simeq \frac{V_{CC}}{3} = 4V$

$\Rightarrow R_C = \frac{V_{CC} - V_{CE} - I_E R_E}{I_C} = \underline{\underline{7\,K\Omega}}$

b) AC Analysis: In midband, coupling & bypass capacitors short out. $\Rightarrow R_E$ gets shorted by C_2. R_B ($= 2.06\,M\Omega$) appears in $\|^{l}$ with $10K\Omega$, $\Rightarrow$ net res. $= \underline{9.95K\Omega}$. $I_C = 1mA \Rightarrow r_E = \underline{26\Omega}$ & $r_\pi = \underline{5.2K\Omega}$. The effective collector load of $Q = R_C \| 10K\Omega = \underline{4.12\,k\Omega}$. The midband ac eqv.:



$i_b = \frac{9.95}{9.95+5.2} i_s = 0.657\, i_s$

$v_o = (-4.12K\Omega) \times i_o$
$= -(4.12\,K\Omega) \times \beta i_b$
$= -4.12 \times 10^3 \times 200 \times 0.657\, i_s$.

$\Rightarrow$ Midband transresistance $\frac{v_o}{i_s} = \underline{\underline{-5.4 \times 10^5\, V}}$

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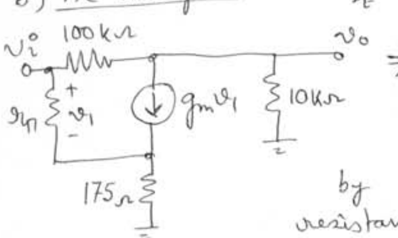
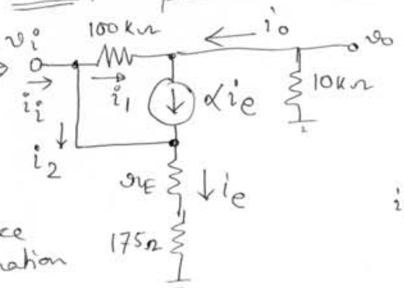
6 a) DC Analysis: open capacitors C_1 & C_2. Then, $I_E = 1mA = I_C + I_B = (\beta+1) I_B = 101\, I_B \Rightarrow I_B = \underline{9.9\mu A}$. $\therefore I_C = \underline{\underline{0.99\,mA}}$. $V_C = I_E \times 175 + 0.7 + I_B \times 100\,k\Omega$, & $V_E = I_E \times 175 \Rightarrow V_{CE} = V_C - V_E = 0.7 + (9.9\mu A) \times (100\,K\Omega) = \underline{\underline{1.69V}}$, which is much larger than the value of $V_{CE}(sat)$ of $0.1V$ $\Rightarrow$ The biasing is okay.

b) AC analysis: $r_E = \frac{V_T}{I_E} = \underline{26\Omega}$, $\alpha = \frac{\beta}{\beta+1} = 0.99$. Refer to the ac midband eqv.:



by resistance transformation



$i_e = \frac{v_i}{r_E + 175} = \underline{\underline{4.975 \times 10^{-3} v_i}}$

$i_o = \alpha i_e + \frac{(v_o - v_i)}{100K\Omega}$

& $v_o = -(10K\Omega) \times i_o$

$i_o = 0.99 \times 4.975 \times 10^{-3} v_i + \frac{v_o}{100K} - \frac{v_i}{100K}$

$\Rightarrow \frac{v_o}{100K} + \frac{v_o}{10K} = \frac{v_i}{100K} - 0.99 \times 4.975 \times 10^{-3} v_i$

$\Rightarrow \frac{v_o}{v_i} = -\frac{4.915 \times 10^{-3}}{1.1 \times 10^{-4}} = \underline{\underline{-44.68}}$

$R_i = \frac{v_i}{i_i}$ with $i_i = i_1 + i_2$ $\quad i_2 = (1-\alpha) i_e = (1 - 0.99) \times 4.975 \times 10^{-3} v_i = 4.975 \times 10^{-5} v_i$

& $i_1 = \frac{v_i - v_o}{100K}$ with $v_o = -44.68\, v_i \Rightarrow i_1 = \frac{45.68}{100K} v_i = 4.568 \times 10^{-4} v_i$

$\Rightarrow i_1 + i_2 = 5.065 \times 10^{-4} v_i \Rightarrow R_i = \frac{v_i}{i_i} = \underline{\underline{1.974\,k\Omega}}$.

By inspection $R_o = 100\,K\Omega$ (note: v_i is shorted under this cond$^{\underline{n}}$).

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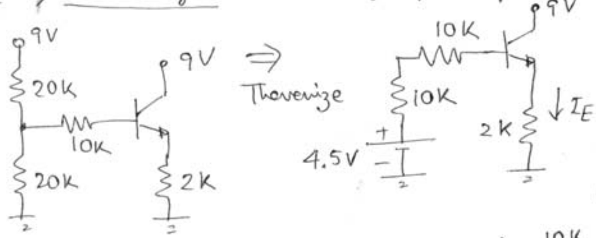
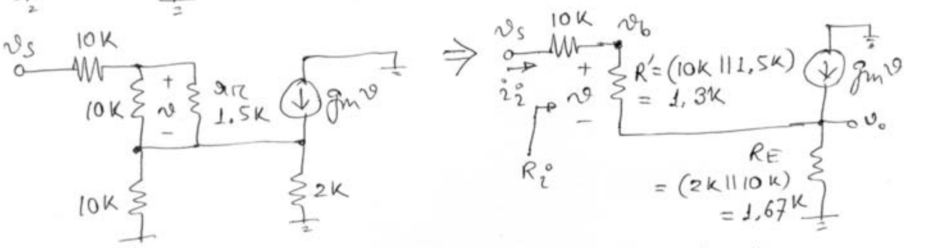
7 a) DC Analysis: C_1 & C_2 open up.



$$I_E = \frac{4.5-0.7}{2K + \frac{10K+10K}{101}} = 1.73\,mA$$

$I_C = \alpha I_E = 1.71\,mA$, $r_E = 15\Omega$, $r_\pi = 1.5\,k\Omega$, $g_m = 65.8\,m℧$.

b) AC Analysis: Both C_1 & C_2 shorted.



$v_o = \left(g_m v + \frac{v}{R'}\right) R_E = 111.17\, v$

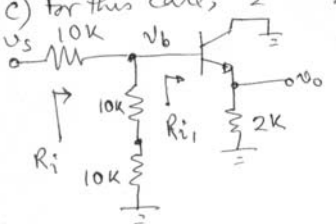
$i_i = \frac{v}{R'} = 7.69\times10^{-4}\, v$

$v_b = v_o + v = 112.17\, v$

$\Rightarrow R_i = \frac{v_b}{i_i} = \frac{112.17\, v}{7.69\times10^{-4}\, v} = 145.86\,K\Omega$ (very large).

$\frac{v_o}{v_s} = \frac{v_o}{v_b}\times\frac{v_b}{v_s} = \frac{111.17}{112.17}\times\frac{R_i}{R_i+10K} = \frac{111.17}{112.17}\times\frac{145.86}{145.86+10} = 0.923 \Rightarrow$ a perfect buffer (almost!)

c) For this case, C_2 is open ckted, however, C_1 remains shorted.



$R_{i_1} = r_\pi + (\beta+1)\times 2K = 1.5K + 101\times 2K = 203.5K$

(compare with 145.86K of part b))

& $R_i = R_{i_1} \| 20K = 18.21K$

$\frac{v_o}{v_s} = \frac{v_o}{v_b}\times\frac{v_b}{v_s} = \frac{2K}{2K+r_E}\times\frac{R_i}{R_i+10K} = \frac{2K}{2K+15}\times\frac{18.21}{18.21+10} = 0.641$

(compare with 0.923 of part b)). Thus, the advantages of bootstrapping is obvious!

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8 a) All capacitors open up. Neglecting base current, $V_{B1} = V_{B2} = \frac{R_2 V_{CC}}{R_1+R_2} = 4.8V \Rightarrow V_{E1} = V_{E2} = 4.1V \Rightarrow I_{C1} = I_{C2} \simeq \frac{V_{E1}}{R_{E1}} = 1.05\,mA$. $V_{CE1} = V_{CE2} = V_{CC} - I_{C1}(R_{C1} + R_{E1}) = 3.77V$

Biasing is okay ∵ $V_{CE} \gg 0.1V$, however, not the best (∵ V_{CE} (best) $\simeq \frac{V_{CC}}{3} = 5V$).

b) $g_{m1} = g_{m2} = 40.38\,mA/V$, $r_E \simeq 1/g_m = 24.76\Omega$, $r_\pi = \beta r_E = 2.48\,k\Omega$.

In midband, all coupling & bypass capacitors short out. The entire problem can be done by simple inspection.

i) $R_{i_1} = R_1 \| R_2 \| r_\pi = 2.3\,k\Omega$, $\frac{v_{b1}}{v_s} = \frac{R_{i_1}}{R_{i_1}+R_s} = 0.315$.

ii) $R_{i_2} = R_{i_1} = 2.3\,k\Omega$. Load of Q_1, $R_{L1} = R_{C1} \| R_{i_2} = 1.72\,k\Omega \Rightarrow \frac{v_{b2}}{v_{b1}} = -\frac{R_{L1}}{r_E} = -69.47$.

iii) Load of Q_2, $R_{L2} = R_{C2} \| R_L = 1.55\,k\Omega \Rightarrow \frac{v_o}{v_{b2}} = -\frac{R_{L2}}{r_E} = -62.6$.

iv) Overall $A_v = \frac{v_o}{v_s} = \frac{v_o}{v_{b2}}\times\frac{v_{b2}}{v_{b1}}\times\frac{v_{b1}}{v_s} = +1370$ (huge!)

Thus, cascading of 2 CE stages is capable of producing very large voltage gain.

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