Indian Institute of Technology Kanpur
Department of Electrical Engineering
EE 250 Control Systems Analysis
MATLAB Test - I
Max Marks - 15, Duration - 90 minutes

21 March 2021

Instructions:

1. All MATLAB codes must be published and a copy of the published PDF must be submitted for evaluation. Pasting the code in a separate document with the outputs will NOT BE ACCEPTED.
2. All procedural steps (hand written) for each question must be submitted as a separate PDF.
3. All programs will be strictly checked for plagiarism. Codes that match another fellow student will be considered as a case of malpractice and penalties will be levied on both candidates.

Question 1.

Consider the Linearized state-space model of an inverted pendulum mounted on a cart, given below:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3g}{4L-3maL} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-3mag}{4-3ma} & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{3a}{3maL-4L} \\ 0 \\ \frac{4a}{4-3ma} \end{bmatrix} \mathbf{u}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}$$

The parameters of the system are given as m = 0.23, M = 0.5, g = 9.8, L = 0.321 and a = $\frac{1}{m+M}$.

A). Derive the transfer function $\frac{Y(s)}{U(s)}$.

B). Design a suitable compensator for the above transfer function such that the peak overshoot $M_p = 20\%$ and the settling time $t_s = 1$ sec for 5% steady state error, are met. Explicitly answer the following questions as you develop your design:

   (a) What are desired dominant pole pairs?

   (b) Justify the selection of your compensator (Lead, Lag or Lead-lag?). Provide all design steps.

   (c) After the design, verify that the compensated system has desired dominant pole pairs. Where are other poles located?

   (d) Compute the closed loop transfer function.

   (e) Plot the step response and check if the desired specifications are met.

   (f) Draw the root locus of the compensated system. Find out asympote angles, centroid, breakaway points, and crossing of imaginary axis.

C). Consider the state space model of the linearized model of the inverted pendulum. The input is given as $\mathbf{u} = -\mathbf{K}\mathbf{x}$, where

$$\mathbf{K} = \begin{bmatrix} -9.5818 & -1.2973 & -0.0974 & -0.2435 \end{bmatrix}$$

Using Runge-kutta 4th order method find the state response. Consider the sampling rate $h = 0.1$ and the initial conditions as $\mathbf{x_0} = [1\ 0\ 1\ 0]^T$.

(a) Plot the state trajectories with the given initial conditions.

(b) What changes do you observe if the value of the sampling rate is changed to $h = 1$. Can the higher value of the sampling rate make the state response to go unbounded? Validate your answer.

(c) Is it possible to make the modified system unstable by changing the initial conditions? Try taking another initial condition $\mathbf{x_0} = [-5\ 2\ 10\ \ -3]^T$. Give an explanation for the plot you obtained.