Date 03|02|2021

EE250: Tutorial 2 Solution Set

3rd Feb 2021

Answer 1

1. A Signal Flow Graph is shown in Figure 1. Find the transfer function between the input node and the output node, using Mason's gain formula.

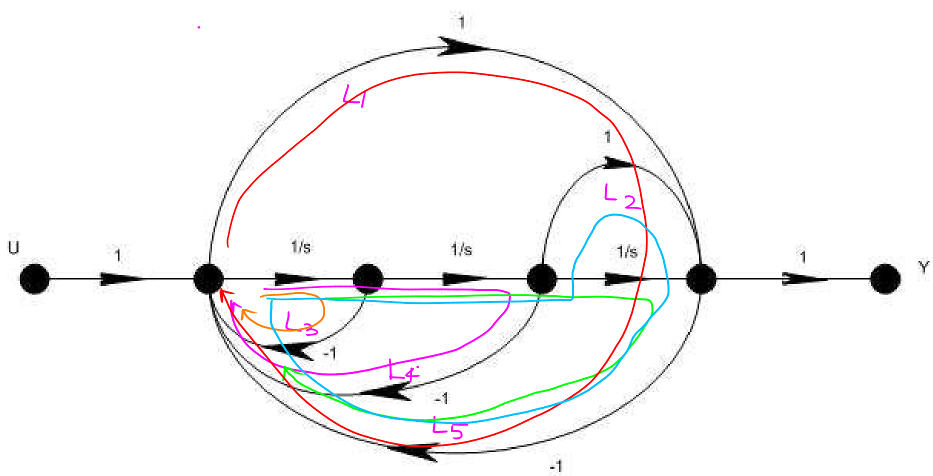


Figure 1: Problem 1

3 forward paths:

$M_1 = 1$

$M_2 = \frac{1}{s^2}$

$M_2 = \frac{1}{s^3}$

5 loops:

$L_{11} = \frac{-1}{s}$

$L_{21} = \frac{-1}{s^2}$

$L_{31} = \frac{-1}{s^3}$

$L_{41} = -1$

$L_{51} = -\frac{1}{s^2}$

There are no non-touching loops.

4 Mason's Gain Formula

Given an SFG with N forward paths and K loops, the gain between the input node y_{in} and output node y_{out} is

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \triangle_K}{\triangle} \qquad (19)$$

where
y_{in} = input-node variable
y_{out} = Output-node variable
M = gain between y_{in} and y_{out}.
N = total number of forward paths between y_{in} and y_{out}.
M_k = gain of the kth forward path between y_{in} and y_{out}

$$\frac{Y(s)}{U(s)} = \frac{M_1 + M_2 + M_3}{1-(L_1 + L_2 + L_3 + L_4 + L_5)}$$

$$\triangle = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \ldots$$

L_{mr} = gain product of mth$(m = i, j, k, \ldots)$ possible combinations of r nontouching loops $(1 \le r < K)$.
or
$\triangle$ = 1-(sum of the gains of **all individual loops**)+(sum $= 0$ of products of gains of all possible combinations of **two** nontouching loops)-(sum of products of gains of all possible combinations of **three** nontouching loops)+ …
$\triangle_k$ = the $\triangle$ for that part of the SFG that is nontouching with the kth forward path.

$$T(s) = \frac{\frac{1}{s^3} + \frac{1}{s^2} + 1}{1 + \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + 1 + \frac{1}{s^2}} = \frac{s^3 + s + 1}{2s^3 + s^2 + 2s + 1}$$

2. A low-pass filter is shown in Figure 2. $R = 1k\Omega$, $C = 1\mu F$
Construct a signal flow graph connecting input $V_1(s)$ and output $V_{\cancel{2}3}(s)$ and showing internal signals $I_1(s)$, $I_2(s)$, and $V_2(s)$.
Find $T(s) = \frac{V_3(s)}{V_1(s)}$

Answer 2

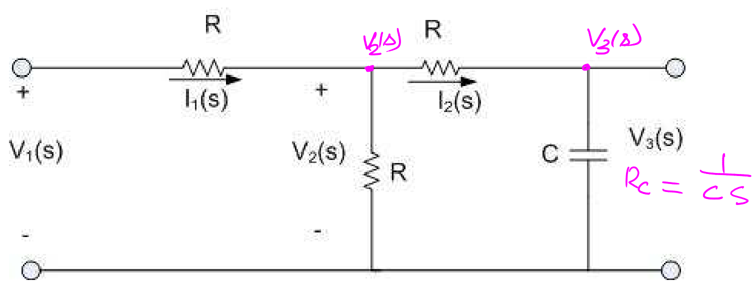


Figure 2: Problem 2

Direct method:

$$V_3 = \frac{V_2 \cdot \frac{1}{cs}}{R + \frac{1}{cs}} = \frac{V_2}{1 + RCs}$$

$$\Rightarrow V_2 = V_3(1 + RCs)$$

$$I_1(s) = \frac{V_1 - V_2}{R}$$

$$I_2(s) = \frac{V_2 - V_3}{R}$$

$$V_2(s) = (I_1 - I_2)R$$

$$V_3(s) = \frac{I_2(s)}{Cs} \qquad [since \quad C\frac{dV_s}{dt} = i_2]$$

$$V_1 = RI_1 + (I_1 - I_2)R$$
$$= 2RI_1 - I_2 R.$$
$$= R(2I_1 - I_2)$$
$$= \cancel{R}\left(2\left(\frac{V_1 - V_2}{\cancel{R}}\right) - \frac{V_2 - V_3}{\cancel{R}}\right)$$
$$= 2V_1 - 2V_2 - V_2 + V_3$$
$$\Rightarrow V_1 = 3V_2 - V_3$$
$$= 3(V_3(1 + RCs)) - V_3$$
$$\Rightarrow V_1 = V_3(2 + 3RCs)$$
$$\Rightarrow \frac{V_3}{V_1} = \frac{1}{2 + 3RC}$$

The signal flow graph can be drawn as shown in Figure 3.

Only one forward path: $M = \frac{1}{CsR}$

Loops:

$L_{11} = -1$

$L_{21} = -1$

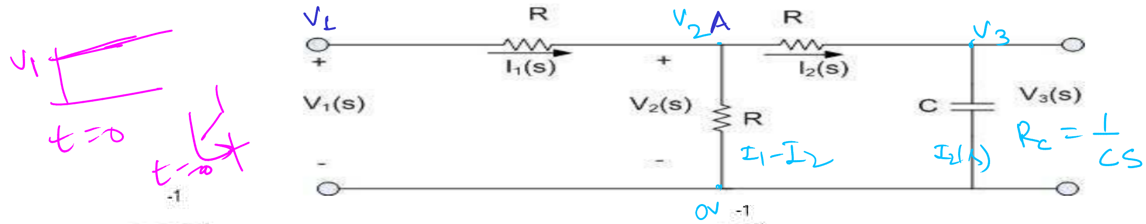
$L_{31} = \frac{-1}{CsR}$

① $I_1 = \frac{V_1 - V_2}{R}$ — ①

② $V_2 = (I_1 - I_2)R$ — ②

③ $V_2 - V_3 = I_2 R$ — ③

④ $V_3 = \frac{1}{Cs} \cdot I_2(s)$ — ④



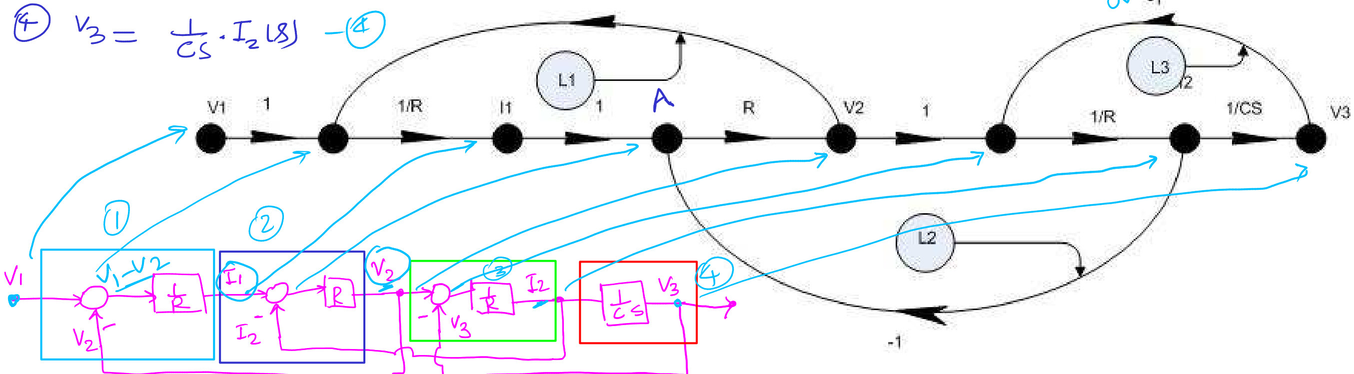


Figure 3: Signal flow graph

$$T(s) = \frac{\frac{1}{CsR}}{1+1+1+\frac{1}{CsR}+L_1L_3}$$

$$\boxed{T(s) = \frac{1}{3CsR+1+1} = \frac{1}{2+3RCs}}$$

$$= \frac{333.35}{s+666.7}$$

$$\boxed{DCgain = \frac{333.35}{s+666.7}\Big|_{s=0} = 0.5}$$

$\frac{V_3(s)}{V_1(s)} = \frac{333.35}{s+666.7}$ step input

Final value theorem.

$V_3(s) = \frac{333.35}{s+666.7} \cdot \frac{1}{s}$

$e_{ss} = \lim_{s\to 0} V_3(s) \cdot s$

$= \lim_{s\to 0} s \cdot \frac{333.35}{s+666.7} \cdot \frac{1}{s}$ $e_{ss} = \lim_{s\to 0} s f(s)$

$= \frac{333.35}{666.7} = 0.5$

3. The block diagram of a feedback control system is shown in Fig. 3.

(a) Apply the SFG formula to the block diagram to find the transfer functions

$$\frac{Y(s)}{R(s)}\Big|_{N=0} \qquad \frac{Y(s)}{N(s)}\Big|_{R=0}$$

Express $Y(s)$ in terms of $R(s)$ and $N(s)$ when both inputs are applied simultaneously.

(b) Find the desired relation among the transfer functions $G_1(s)$, $G_2(s)$, $G_3(s)$, $G_4(s)$, $H_1(s)$ and $H_2(s)$ so that the output $Y(s)$ is not affected by the disturbance signal $N(s)$ at all.

Answer 3

Forward path gains:

$M_1 = G_1G_2G_3$

$M_2 = G_4$

Loop gains:

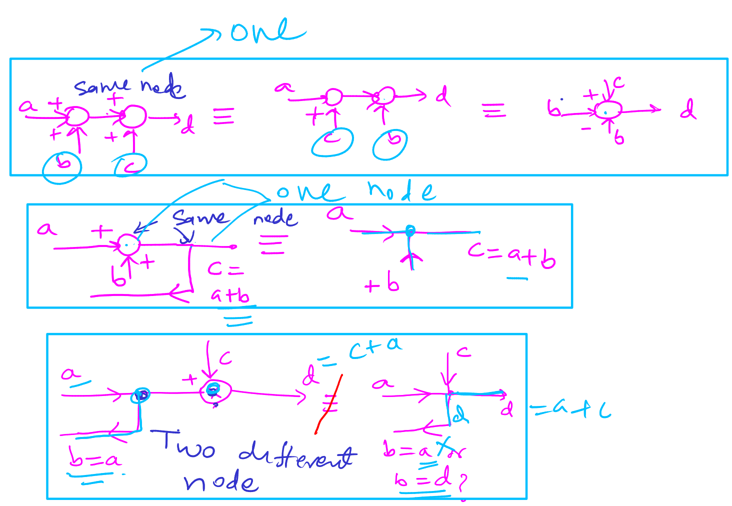
$L_{11} = -H_1G_1G_2$

$L_{21} = -G_1G_2G_3$

$L_{31} = -G_2G_3H_2$

$L_{41} = -G_4$

$L_{51} = G_4H_2G_2H_1$



$\triangle_1 = 1$

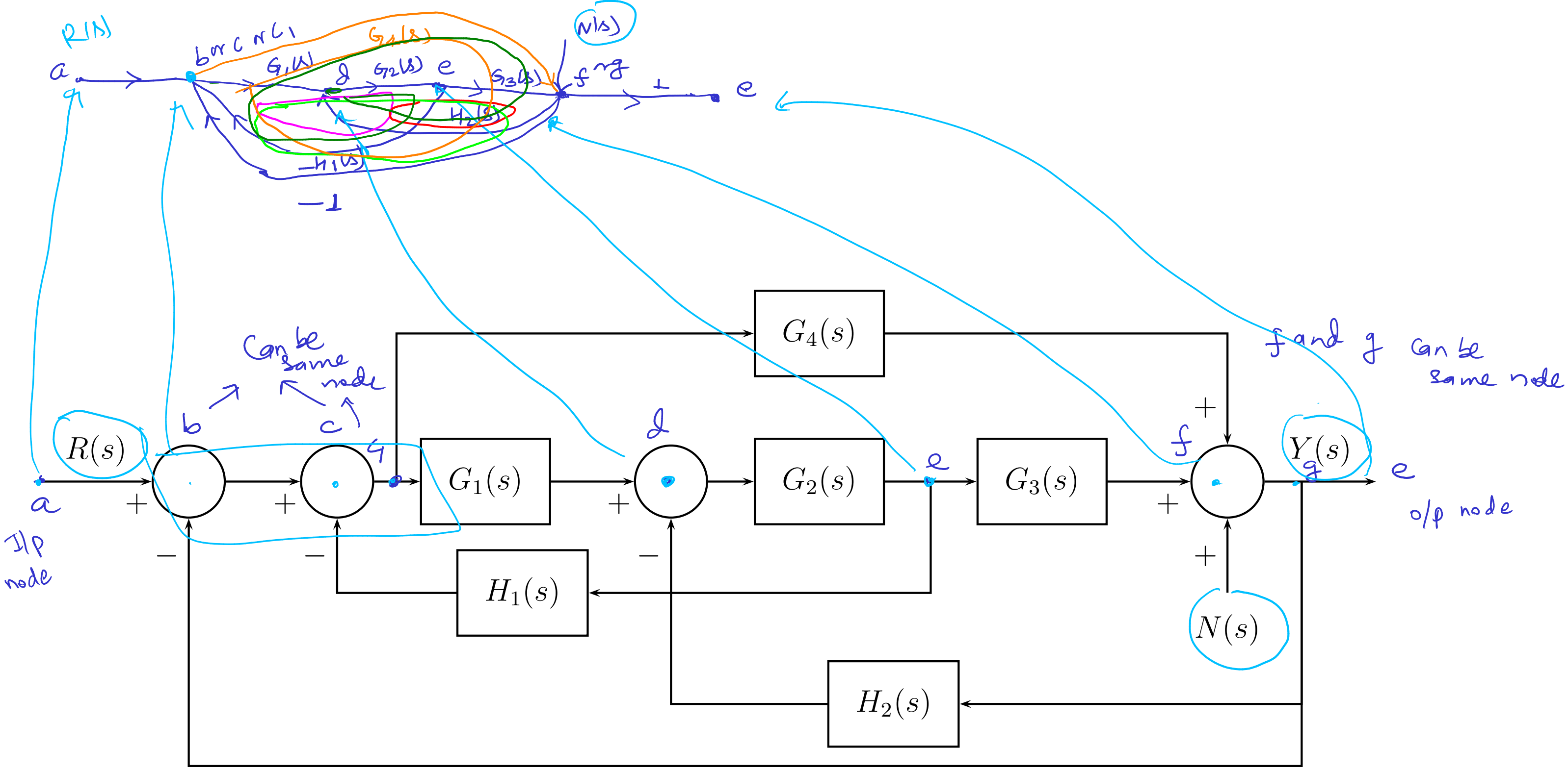


Figure 4: Problem 3

$\triangle_2 = 1$

$\triangle = 1 + G_1G_2H_1 + G_1G_2G_3 + G_2G_3H_2 + G_4 - H_1H_2G_2G_4$

$$\frac{Y(s)}{R(s)}|_{N(s)=0} = \frac{M_1\triangle_1 + M_2\triangle_2}{\triangle} = \frac{G_1G_2G_3 + G_4}{1 + G_1G_2H_1 + G_1G_2G_3 + G_2G_3H_2 + G_4 - H_1H_2G_2G_4}$$

$For \quad \frac{Y(s)}{N(s)}|_{R(s)=0}$

$M_1 = 1$

$L_{11} = -H_1G_1G_2$

$L_{21} = -G_1G_2G_3$

$L_{31} = -G_2G_3H_2$

$L_{41} = -G_4$

$L_{51} = G_4H_2G_2H_1$

$\triangle_1 = 1 + H_1G_1G_2$

$\triangle = 1 + G_1G_2H_1 + G_1G_2G_3 + G_2G_3H_2 + G_4 - H_1H_2G_2G_4$

so,

$$\frac{Y(s)}{N(s)}|_{R(s)=0} = \frac{1 + H_1G_1G_2}{1 + G_1G_2H_1 + G_1G_2H_3 + G_2G_3H_2 + G_4 - H_1H_2G_2G_4}$$

Answer 3b

For the output Y(s) not to be effected by the noise N(s)

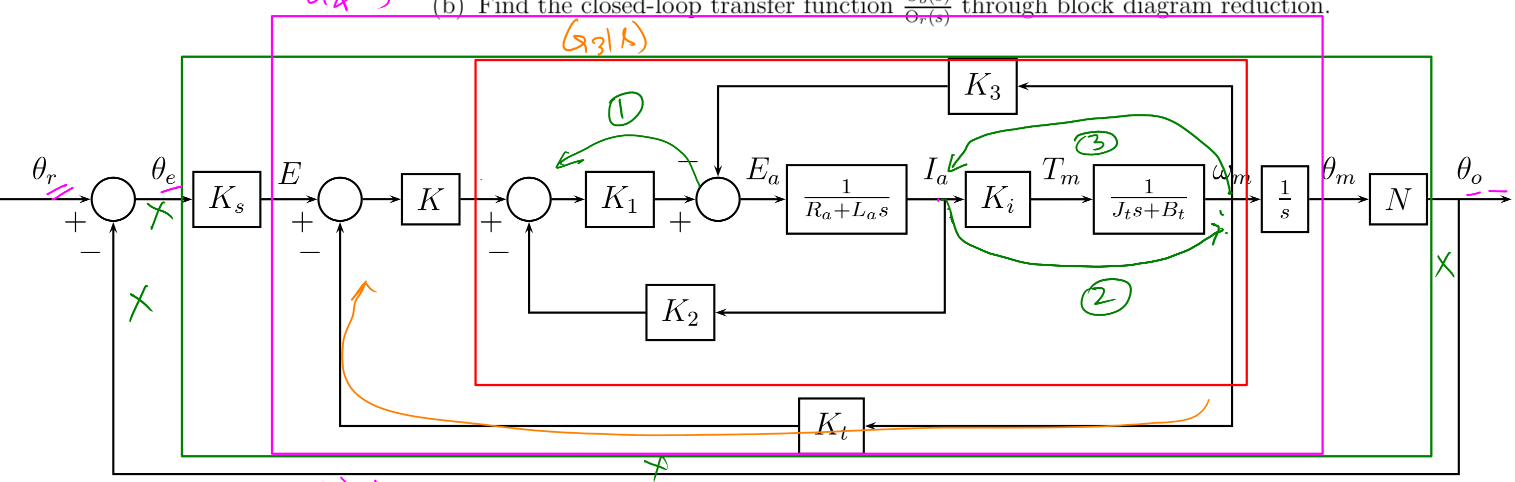
$\frac{Y(s)}{N(s)}|_{R(s)=0} = 0$ $<< 1$

$\implies \quad 1 + H_1G_1G_2 = 0$

$\implies \quad H_1G_1G_2 = -1$

Answer 4

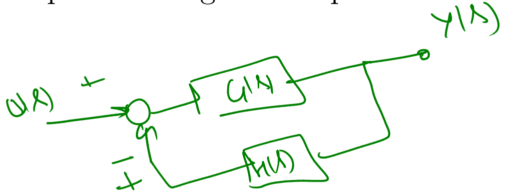
4. The block diagram of the position-control system of the electronic word processor is shown in Fig. 4.
   (a) Find the loop transfer function $\frac{\Theta_0(s)}{\Theta_e(s)}$ through block diagram reduction (the outer feedback path is open)
   (b) Find the closed-loop transfer function $\frac{\Theta_o(s)}{\Theta_r(s)}$ through block diagram reduction.



$\frac{\theta_o(s)}{\theta_e(s)} = K_s \cdot G_4(s) N$

Figure 5: Problem 4

After rearranging, the signal I_a which was passing through K_2 , is now taken from ω_m with appropriate change in the path.



$\frac{Y(s)}{U(s)} = \frac{G(s)}{1+G(s)H(s)}$

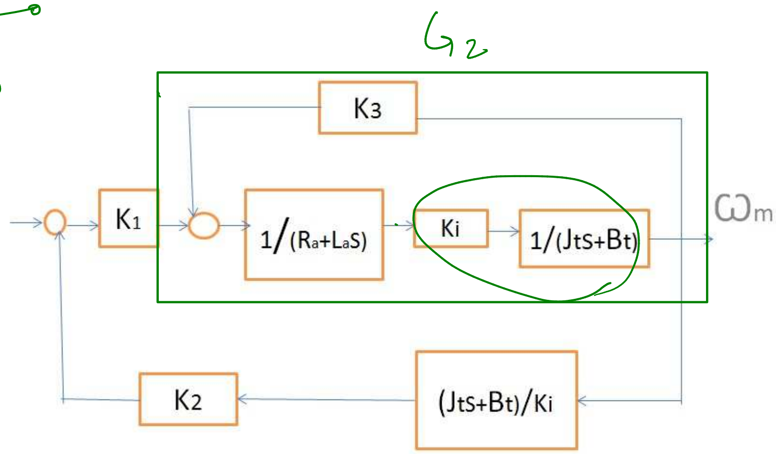


Figure 6: Transfer function G_1

Transfer function G_1 can be simplified as shown in Figure 7.

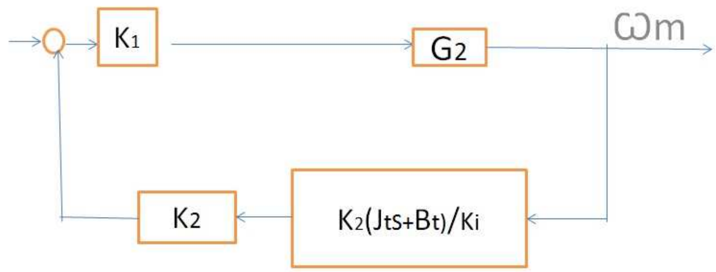


Figure 7: Transfer function G_1

$$
\begin{aligned}
G_2 &= \frac{\frac{K_i}{(R_a+sL_a)+(sJ_t+B_t)}}{1+\frac{K_sK_i}{(R_a+sL_a)(sJ_t+B_t)}} \\
&= \frac{K_i}{(R_a+sL_a)(sJ_t+B_t)+K_sK_i} \\
G_1 &= \frac{K_1G_2}{1+\frac{K_1G_2K_2(J_ts+B_t)}{K_i}} \\
&= \frac{K_1K_i}{[(R_a+sL_a)(sJ_t+B_t)+K_3K_i]+K_1K_2(J_ts+B_t)}
\end{aligned} \tag{1}
$$

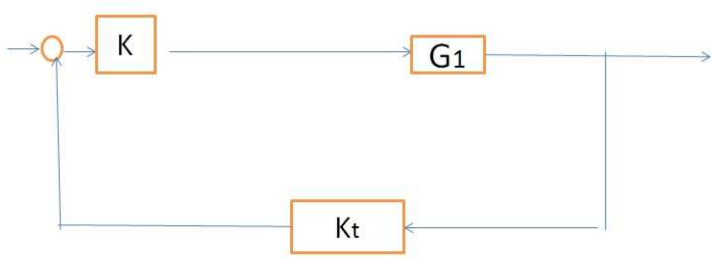


Figure 8: Transfer function G_3

$$
\begin{aligned}
G_3 &= \frac{KG_1}{1+K_tKG_1} \\
&= \frac{KK_1K_i}{[(R_a+sL_a)(sJ_T+B_t)+K_iK_3]+K_1K_2(J_ts+B_t)+K_tKK_1K_i}
\end{aligned}
$$

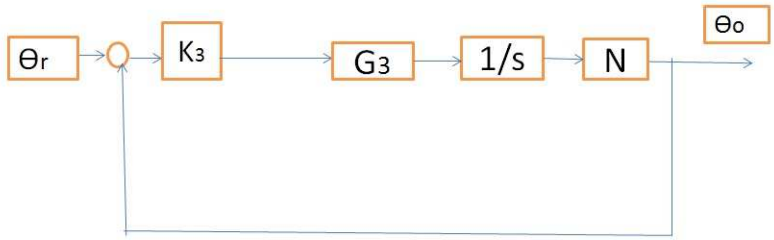


Figure 9: Transfer function G_4

$$G_4 = \frac{\theta_o(s)}{\theta_r(s)}$$

$$and \qquad \frac{\theta_o(s)}{\theta_l(s)}|_{outer \quad feedback \quad open} = K_3 G_3 \frac{1}{s} N = answer(a)$$

$$answer(a) = \frac{K_3 K K_1 K_i N}{s[(R_a + sL_a)(sJ_t + B_t) + K_i K_3] + sK_1 K_2 (J_t s + B_t) + K_t K K_1 K_i s}$$

$$\begin{aligned}
answer(b) &= G_4 \\
&= \frac{K_s G_3 \frac{1}{s} N}{1 + \frac{K_s G_3 N}{s}} \\
&= \frac{\frac{K_s K K_1 K_i N}{s[(R_t+sL_t)(R_a+sL_a)+K_i K_3]+sK_1 K_2 (J_t s+B_t)+K_f K K_1 K_i s}}{1 + \frac{K_s K K_1 K_i N}{s[(R_t+sL_t)(R_a+sL_a)+K_i K_3]+sK_1 K_2 (J_t s+B_t)+K_f K K_1 K_i s}} \\
&= \frac{K_s K K_1 K_i N}{s[(R_a + sL_a)(J_t s + B_t) + K_i K_3] + sK_1 K_2 (J_t s + B_t) + sK_t K K_1 K_i + K_s K K_i K_1 N}
\end{aligned}$$