EE 250: Control Systems Analysis
Module III: s-plane analysis
Lecture 14: Poles and Zeros

Dr Laxmidhar Behera

Professor, Department of EE, IIT Kanpur

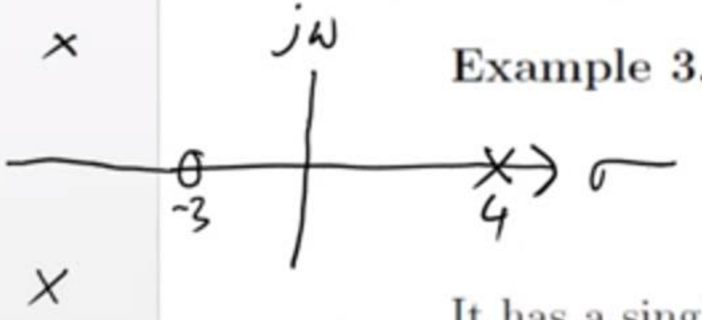
A generic transfer function of a plant is given as

$$G(s) = \frac{n(s)}{d(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1}s + b_m}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n} \tag{3}$$

where $m \leq n$. All engineering systems are causal

Definition 1 (Poles). Poles are given by the roots of denominator polynomial $d(s)$, i.e., $d(s) = 0$.

Definition 2 (Zeros). Zeros are given by roots of polynomial $n(s)$, i.e., $n(s) = 0$. Zeros are the points in s-plane where the transfer function becomes zero.



Example 3. Consider the following transfer function

$$G(s) = \frac{s+3}{\underbrace{(s-4)(s^2+12s+52)}} = 0$$

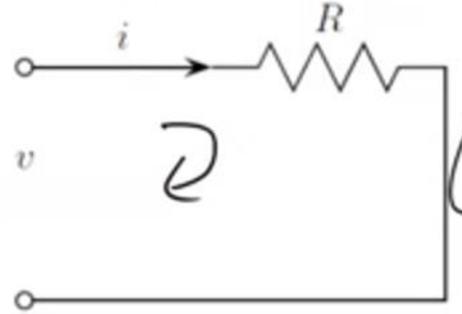
$n(s) = s+3 = 0 \Rightarrow s = -3$

It has a single zero, $s = -3$ and three poles, $s = 4$, $s = -6 \pm 4j$.

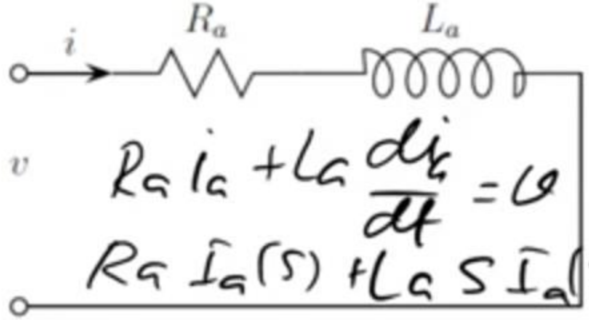
$s = \sigma + j\omega$

Origin of Poles:

Poles represent internal dynamics of a system.



$v = iR$



$R_a i_a + L_a \frac{di_a}{dt} = v$

$R_a I_a(s) + L_a s I_a(s) = V$

$\frac{I}{V} = \frac{1}{R}$. Thus, there is no dynamics.

$\frac{I}{V}(s) = \frac{1}{R_a+L_a s}$. It has a single pole at $s = -\frac{R_a}{L_a}$.

$s = -\frac{R_a}{L_a}$

Consider a first order system $G(s) = \frac{1}{1+\tau s}$. It has a pole at $s = -\frac{1}{\tau}$, then its impulse response is given as $g(t) = e^{-t/\tau}$.

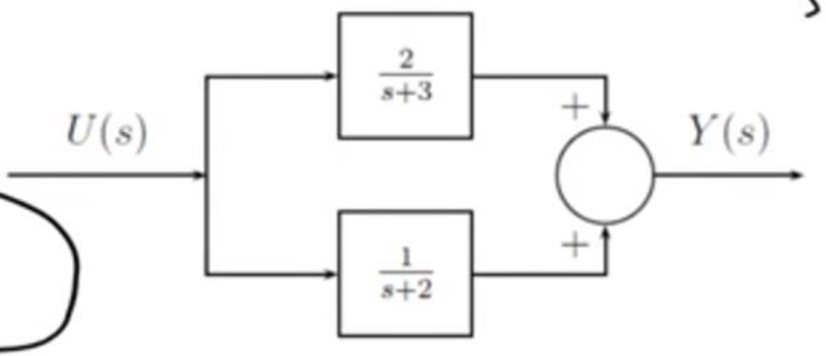
$s = -1/\tau$

$g(t) = e^{-t/\tau}$

Origin of Zero:

$Y_1(s) = \frac{2}{s+3} U(s)$

$+ \frac{1}{s+2} U(s)$



$3s = -7$

$\boxed{s = -7/3}$

$\frac{Y(s)}{U(s)} = \frac{2}{s+3} + \frac{1}{s+2}$

The transfer function between $Y(s)$ an $U(s)$ is given by

$2(s+2) + s + 3$

$3s + 7$

$$\frac{Y(s)}{U(s)} = \frac{s+1 \quad 3}{(s+2)(s+3)}$$

$$\frac{Y(s)}{U(s)} = \frac{3s+7}{(s+2)(s+3)}$$

Zero arises ~~due~~ due to interaction

Zero can affect input Signal

Let us consider a transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s(s+3)}$$

Let $U(s) = \frac{1}{s+2}$ i.e., input signal has a pole at the 'zero' of $G(s)$. Then $Y(s) = \frac{1}{s(s+3)}$, i.e., input signal does not appear at the output. Here is another example to make this point very clear.

$$\boxed{Y(s) = \frac{1}{s(s+3)}}$$

$$\frac{\cancel{s+2}}{s(s+3)} \times \frac{1}{\cancel{s+2}}$$

Example 4 (Electrical Circuit). The transfer function is

$$G(s) = \frac{V_o}{V_i} = \frac{s+1}{(s+2)(s+3)}$$

Let $V_i = \frac{10}{s}$, then

$V_i = \frac{1}{s}$ $\quad \frac{1}{s}$

$$\frac{-2}{-3(-1)}$$

$$V_o(s) = \frac{s+1}{s(s+2)(s+3)} = \frac{1}{6s} + \frac{1}{2(s+2)} - \frac{2}{3(s+3)}$$

$$\Rightarrow v_o(t) = \frac{1}{6}(1 + 3e^{-2t} - 4e^{-3t}), \quad t > 0$$

2 3

$$\frac{(s+2)(s+3) + 3s(s+3)}{6s(s+2)(s+3)}$$

$-2(s(s+2))$

$2s^2 + 10s + 6$

$s^2 + 5s + 6 + 3s^2 + 9s$

$-2s^2 - 4s$

Thus, the output voltage contains three components

1. due to input term $\frac{1}{s}$.
2. due to pole at $s = -2$ and
3. due to pole at $s = -3$.

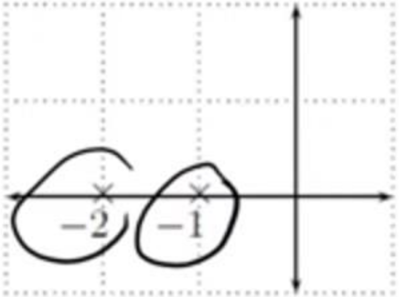
Take $v_i(t) = 10e^{-t}$, i.e., $V_i(s) = \frac{10}{s+1}$. Now, the output is given by

$$V_o(s) = \frac{10}{(s+2)(s+3)} = \frac{10}{s+2} - \frac{10}{s+3}$$

$$\Rightarrow v_o(t) = 10e^{-2t} - 10e^{-3t}$$

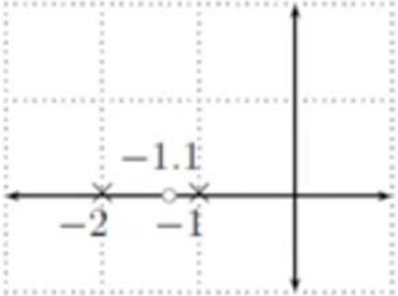
Thus, the input signal $v_i(t) = 10e^{-t}$ appear to be lost inside the circuit. Hence, a zero can block the transmission of a signal.

Zero can reduce the effect of a Pole



zero at +1

$$\frac{2}{s+2}$$



$$\begin{aligned} H_1(s) &= \frac{2}{(s+1)(s+2)} \\ &= \frac{2}{s+1} - \frac{2}{s+2} \\ or,\ h_1(t) &= 2e^{-t} - 2e^{-2t} \end{aligned}$$

$$\begin{aligned} H_2(s) &= \frac{2(s+1.1)}{1.1(s+1)(s+2)} \\ &= \frac{0.18}{s+1} + \frac{1.64}{s+2} \\ or,\ h_2(t) &= 0.18e^{-t} + 1.64e^{-2t} \end{aligned}$$

Conclusion:

1. Zero at $s = -1.1$ reduces the effect of the pole near to it ($s = -1$) drastically
2. We can use a zero to cancel a pole completely if the pole is in the left half of the s-plane.