

Descriptive Solutions

1.

$$\begin{cases} y'' + \lambda y = 0 \\ y(5\pi) = 0 = y(6\pi) \end{cases}$$

1 marks

For showing $\lambda < 0$
cannot be an Eigen value.

STUDENTS MAY OBTAIN THIS BY "Two" WAYS.

Way 1: They can directly say that this is Regular Sturm-Liouville eigen value problem (RSLEVP). Since we know " $\lambda < 0$ " not possible.

Way 2 Writing down the Auxiliary Eqn $m^2 + \lambda = 0$
and assuming $\lambda = -\delta^2$ (< 0) (if possible) (for $\delta > 0$)
 $m = \pm \delta$

\Rightarrow General soln
 $y(x) = Ae^{\delta x} + Be^{-\delta x}$

$$y(5\pi) = y(6\pi) = 0$$

$$\Rightarrow \begin{cases} A e^{5\pi} + B e^{-5\pi} = 0 \\ A e^{6\pi} + B e^{-6\pi} = 0 \end{cases}$$

Here $\begin{vmatrix} e^{5\pi} & e^{-5\pi} \\ e^{6\pi} & e^{-6\pi} \end{vmatrix} = e^{5\pi} e^{-6\pi} - e^{-5\pi} e^{6\pi} = e^{-\pi} - e^{11\pi} \neq 0$

$$\Rightarrow A = B = 0$$

(only if $\gamma = 0$).

$\Rightarrow \gamma \equiv 0$ is the sol.

Contradiction

1 marks

To rule out that $\lambda = 0$ is an eigenvalue.

AGAIN STUDENTS MAY ACHIEVE IN
TWO WAYS :

Way 1

$\lambda = 0$ cannot be E.V as this is RSLEVP.

Way 2

Auxiliary Eqn $m^2 = 0$.

$\Rightarrow \gamma(x) = Ax + B$ is General sol.

$$\Rightarrow \gamma(5\pi) = \gamma(6\pi) = 0 \Rightarrow \gamma \equiv 0$$

(Contradiction).

1 marks :- For the case $\lambda > 0$,
writing down the general
solution correctly.

Auxiliary Eqn. is
 $m^2 + \lambda^2 = 0$

$$\Rightarrow y(x) = A \cos(\sqrt{\lambda} x) + B \sin(\sqrt{\lambda} x) \quad \text{--- } (*)$$

Now they have to put the Boundary
Conditions

$$y(5\pi) = 0 = y(6\pi)$$

$$\Rightarrow \begin{cases} A \cos(5\sqrt{\lambda}\pi) + B \sin(5\sqrt{\lambda}\pi) = 0 \\ A \cos(6\sqrt{\lambda}\pi) + B \sin(6\sqrt{\lambda}\pi) = 0 \end{cases}$$

1 marks

For getting non trivial soln of
 A, B , we want

$$\begin{vmatrix} \cos(5\sqrt{\lambda}\pi) & \sin(5\sqrt{\lambda}\pi) \\ \cos(6\sqrt{\lambda}\pi) & \sin(6\sqrt{\lambda}\pi) \end{vmatrix} = \sin(6\pi) = 0$$

1 marks

$$\Rightarrow \sin(k\pi) = \sin(n\pi) \quad n \in \mathbb{Z}$$

$$\Rightarrow k = n$$

$$\lambda = k^2 = n^2, \quad n \in \mathbb{N}$$

1 mark for finding out the
Eigen functions corresponding
to the Eigen value " n^2 ".

$$y_n(x) = A_n \cos(nx) + B_n \sin(nx)$$

from (*)

$$y_n(5\pi) = 0 \Rightarrow A_n = 0$$

$$\Rightarrow y_n(x) = B_n \sin(nx)$$

is the required
Eigen function.

They
may put
 6π also

ANOTHER SMART WAY

IF SOME ONE SAYS,

we already know that is an E.F

$$y_n(t) = A_n \sin(nt)$$

of the problem,

$$\begin{cases} y'' + xy = 0 & \text{on } (0, \pi) \\ y(0) = y(\pi) = 0 \end{cases}$$

" n^2 "

Corresponding to the

PROBLEM

IN THE GIVEN

IS SHIFTED.

JUST THE DOMAIN

Hence

and the E.F. will be the same.

$$\begin{aligned} y_n(t) &= y_n(t - 5\pi) \\ &= A_n \sin n(t - 5\pi) \\ &= -A_n \sin(nt). \end{aligned}$$

(Done)

