3.a. Let $X_1, \ldots, X_n$ be i.i.d. Unif$[\theta - a, \theta + a]$, where $\theta \in \mathbb{R}$ and $a > 0$ are unknown parameters.
(i) Find the MLE $(\hat{\theta}, \hat{a})$ of (θ, a).
(ii) Is $\hat{\theta}$ an unbiased estimate of θ?
(iii) Is $\hat{a}$ an unbiased estimate of a? $[2 + 2 + 2 = 6]$

3.b. If $X_1, \ldots, X_n$ be i.i.d. f, where $f(x) = \theta x^{\theta - 1}$, $0 < x < 1$, zero elsewhere. Using the NP lemma, find the critical region for testing

$$H_0 : \theta = 1 \text{ against } H_1 : \theta = 2$$

at $\alpha \in (0, 1)$ level of significance.
Hint: Under H_0, $-2 \log_e X_1 \sim$?
Compute the cut-off for $n = 10$ and $\alpha = 0.10$. $[3 + 1 = 4]$