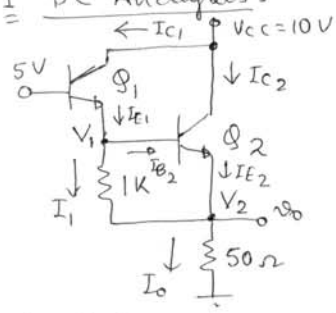
EE 210 Sol^n to HA # 10

1 DC Analysis:

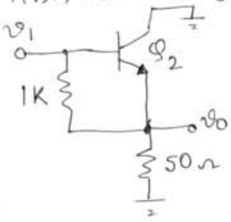


$V_1 = 5 - 0.7 = 4.3\,V$, $V_2 = V_1 - 0.7 = 3.6\,V$, $I_o = \frac{V_2}{50\Omega} = \underline{72\,mA}$, $I_1 = \frac{0.7}{1K} = 0.7\,mA$, $I_{E_2} = I_o - I_1 = 71.3\,mA$, $I_{B_2} = \frac{I_{E_2}}{\beta+1} = 0.355\,mA$, $I_{E_1} = I_{B_2} + I_1 = 1.055\,mA$, $\alpha = \frac{\beta}{\beta+1} = 0.995$, $I_{C_1} = \alpha I_{E_1} = \underline{1.05\,mA}$, $I_{C_2} = \alpha I_{E_2} = \underline{70.95\,mA}$

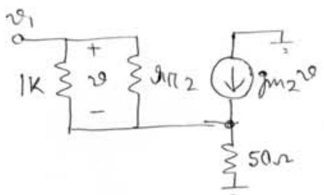
$V_{CE_1} = V_{CC} - V_1 = 10 - 4.3 = \underline{5.7V}$, $V_{CE_2} = V_{CC} - V_2 = 10 - 3.6 = \underline{6.4V}$

$\Rightarrow$ Biasing is ok $\because V_{CE} \gg 0.1V$.

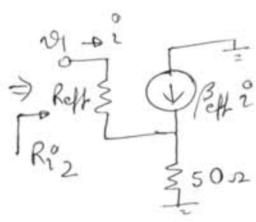
Ac Analysis: Break the prob. into 2 parts, by taking Q_2 & Q_1 separately. First, consider Q_2.



$\Rightarrow$



$\Rightarrow$



$r_{E_2} = \frac{V_T}{I_{C_2}} = 0.366\,\Omega$

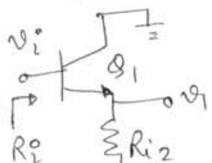
$r_{\pi_2} = \beta r_{E_2} = \underline{73.3\Omega}$

$R_{eff} = 1K \| r_{\pi 2} = \underline{68.3\Omega}$

$\beta_{eff} = g_{m2} R_{eff} = \frac{R_{eff}}{r_{E2}} = \underline{186.6}$

$\Rightarrow R_{i2} = R_{eff} + (\beta_{eff}+1) \times 50 = 68.3 + 187.6 \times 50 = \underline{9.45\,K\Omega}$

& $\frac{v_o}{v_1} = \frac{50}{50 + \frac{68.3}{186.6+1}} = \underline{0.993}$. Now, consider Q_1, which is also a CC stage, with R_{i2} as its effective load.



$r_{E_1} = \frac{V_T}{I_{C_1}} = 24.76\Omega$, $r_{\pi_1} = \beta r_{E_1} = 4.95\,K\Omega \Rightarrow R_i = r_{\pi_1} + (\beta+1) R_{i2}$

$= 4.95K + 201 \times 9.45\,K\Omega = \underline{1.9\,M\Omega}$ (Huge !).

$\frac{v_1}{v_i} = \frac{R_{i2}}{R_{i2} + r_{E1}} = \frac{9.45K}{9.45K + 24.76} = \underline{0.997} \Rightarrow \frac{v_o}{v_i} = \frac{v_o}{v_1} \times \frac{v_1}{v_i} = \underline{0.99}$

The overall o/p resistance $R_o = 50\Omega \| \left[\frac{R_{eff} + r_{E1}}{\beta_{eff}+1}\right] = 50\Omega \| \left[\frac{68.3 + 24.76}{186.6+1}\right] = 50\Omega \| 0.5\Omega = \underline{0.5\Omega}$. Thus, the stage has $R_i > M\Omega$, $A_v \sim 1$, & $R_o \sim 0 \Rightarrow$ Ideal buffer!

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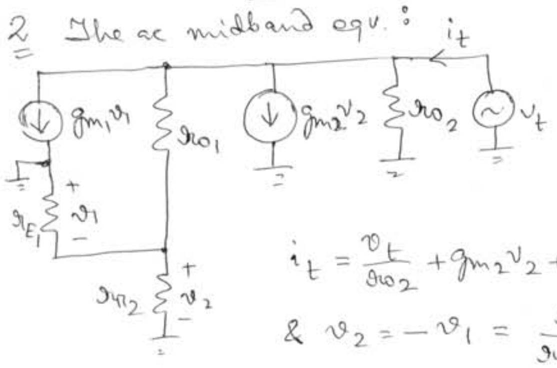
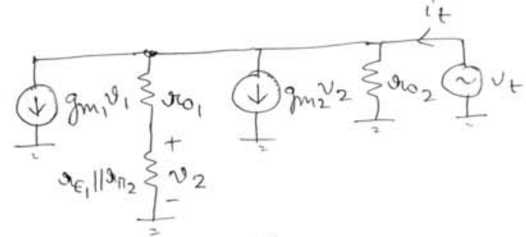
2 The ac midband eqv.:



$\Rightarrow$



$$i_t = \frac{v_t}{r_{o2}} + g_{m2} v_2 + \frac{v_t}{r_{o1} + r_{E_1} \| r_{\pi 2}} + g_{m1} v_1$$

$$\& \; v_2 = -v_1 = \frac{r_{E_1} \| r_{\pi 2}}{r_{o1} + r_{E_1} \| r_{\pi 2}} v_t \simeq \frac{r_{E1} \| r_{\pi 2}}{r_{o1}} v_t \quad \left[\because r_{o1} \gg r_{E_1} \| r_{\pi 2}\right]$$

Solving: $R_o = \frac{v_t}{i_t} = \left[\frac{1}{r_{o2}} + \frac{1}{r_{o1}}\left\{1 + (g_{m2} - g_{m1})(r_{E_1} \| r_{\pi 2})\right\}\right]^{-1}$

i) $I_{C_2} = 1\,mA$, $I_{BIAS} = 1\,mA \Rightarrow$ neglecting I_{B_2}, $I_{C_1} \simeq I_{C_2} \Rightarrow g_{m1} = g_{m2} \Rightarrow r_{o1} = r_{o2} = r_o$.

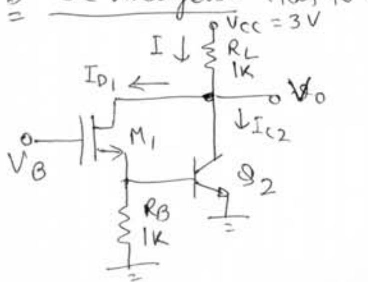
Thus, $\underline{R_o = \frac{r_o}{2}}$.

ii) $I_{C_2} = 1\,mA$, $I_{BIAS} = 0$, $I_{C_1} \simeq \frac{I_{C_2}}{\beta}$, $r_{E_1} = r_{\pi_2}$, $g_{m_2} \gg g_{m_1}$, $r_{o_1} = \beta r_{o_2}$.

$$\Rightarrow R_o = \left[\frac{1}{r_{o_2}} + \frac{1}{r_{o_1}}\left(1 + \frac{g_{m_2} r_{\pi_2}}{2}\right)\right]^{-1} = \left[\frac{1}{r_{o_2}} + \frac{1}{r_{o_1}}\left(1 + \frac{\beta}{2}\right)\right]^{-1} \simeq \left[\frac{1}{r_{o_2}} + \frac{\beta}{2 r_{o_1}}\right]^{-1}$$

$$= \left[\frac{1}{r_{o_2}} + \frac{1}{2 r_{o_2}}\right]^{-1} = \frac{2}{3} r_{o_2}.$$

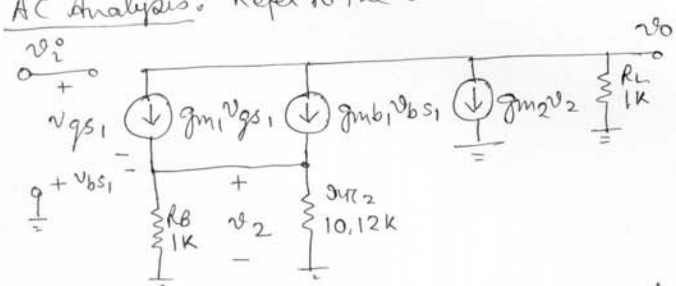
<u>3</u> <u>DC Analysis</u>: Has to be done self-consistently. V_{BE_2} can't be assumed to equal exactly 0.7 V. $V_o(DC) = 2\,V \Rightarrow I R_L = 1\,V$; with $I = I_{D_1} + I_{C_2} = 1\,mA$. As an initial guess, assume $V_{BE_2} = 0.7\,V$. Neglecting I_{B_2}, $I_{D_1} \simeq V_{BE_2}/R_B = 0.7\,mA \Rightarrow I_{C_2} = 0.3\,mA$. Corresponding $V_{BE_2} = V_T \ln \frac{I_{C_2}}{I_S} = 0.747\,V$, which is off from our initial guess. Repeat the iterations, which converge to $V_{BE_2} = \underline{0.743\,V}$, $I_{D_1} = \underline{0.743\,mA}$, $I_{C_2} = \underline{0.257\,mA}$.



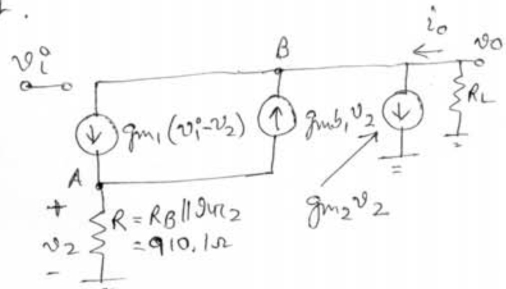
$g_{m_1} = \sqrt{2 K_n' \left(\frac{W}{L}\right)_1 I_{D_1}} = \underline{1.724\,m\mho}$. $V_{SB} = V_{BE_2} = \underline{0.743\,V}$. $\Rightarrow \chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} = \underline{0.108}$

$\Rightarrow g_{mb_1} = \chi g_{m_1} = \underline{0.186\,m\mho}$. $r_{E_2} = \underline{101.17\,\Omega}$, $g_{m_2} = \underline{9.885\,m\mho}$, $r_{\pi_2} = \underline{10.12\,k\Omega}$.

<u>AC Analysis</u>: Refer to the ac midband eqv. ckt.



$\Rightarrow$



$v_{gs_1} = v_i - v_2$ $\quad v_{bs_1} = -v_2$. KCL at node A:

$$g_{m_1}(v_i - v_2) = g_{mb_1} v_2 + \frac{v_2}{R} \Rightarrow v_2 = \frac{g_{m_1} R}{1 + (g_{m_1} + g_{mb_1}) R} v_i$$

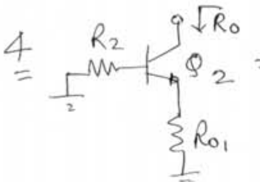
$$\Rightarrow v_i - v_2 = \left[\frac{1 + g_{mb_1} R}{1 + (g_{m_1} + g_{mb_1}) R}\right] v_i$$

KCL at node B: $i_o = g_{m_2} v_2 - g_{mb_1} v_2 + g_{m_1}(v_i - v_2)$

$$= \left[\frac{(g_{m_2} - g_{mb_1}) g_{m_1} R}{1 + (g_{m_1} + g_{mb_1}) R} + \frac{g_{m_1}(1 + g_{mb_1} R)}{1 + (g_{m_1} + g_{mb_1}) R}\right] v_i \Rightarrow i_o = \frac{g_{m_1}(1 + g_{m_2} R)}{1 + (g_{m_1} + g_{mb_1}) R} v_i$$

$$\& \ v_o = -i_o R_L \Rightarrow A_v = \frac{v_o}{v_i} = -\frac{g_{m_1} R_L (1 + g_{m_2} R)}{1 + (g_{m_1} + g_{mb_1}) R} = \underline{\underline{-6.29}}$$

<u>4</u>



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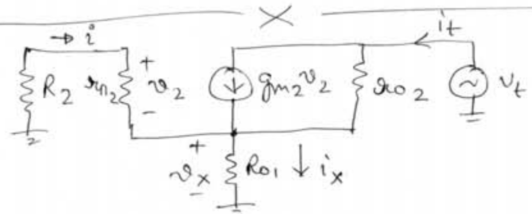

$R_{o1} = r_{o1}\left[1 + g_{m_1}(R_1 \| r_{\pi_1})\right]$

$i_x = i + i_t$, $i = -\frac{v_x}{R_2 + r_{\pi_2}}$

$v_2 = i\, r_{\pi_2}$, $v_x = i_x R_{o1}$

$= (i + i_t) R_{o1} = -\frac{v_x R_{o1}}{R_2 + r_{\pi_2}} + i_t R_{o1}$.

$$\Rightarrow v_x = \frac{R_{o1}(R_2 + r_{\pi_2})}{R_2 + r_{\pi_2} + R_{o1}} i_t$$

Now, $i_t = g_{m2} v_2 + \dfrac{v_t - v_x}{r_{o2}} = -\dfrac{\beta}{R_2 + r_{\pi 2}} v_x + \dfrac{v_t}{r_{o2}} - \dfrac{v_x}{r_{o2}} = \dfrac{v_t}{r_{o2}} - \left(\dfrac{1}{r_{o2}} + \dfrac{\beta}{R_2 + r_{\pi 2}}\right) v_x$

$$\Rightarrow i_t = \frac{v_t}{r_{o2}} - \left(\frac{\beta r_{o2} + R_2 + r_{\pi 2}}{r_{o2}(R_2 + r_{\pi 2})}\right)\left(\frac{R_{o1}(R_2 + r_{\pi 2})}{R_2 + r_{\pi 2} + R_{o1}}\right) i_t$$

$$\Rightarrow R_o = \frac{v_t}{i_t} = r_{o2}\left[1 + \frac{R_{o1}(\beta r_{o2} + R_2 + r_{\pi 2})}{r_{o2}(R_2 + r_{\pi 2} + R_{o1})}\right]$$ (complicated analysis!)

Simplification: $\beta r_{o2} \gg R_2 + r_{\pi 2}$ (in general) $\Rightarrow R_o \simeq r_{o2}\left[1 + \dfrac{\beta R_{o1}}{R_2 + r_{\pi 2} + R_{o1}}\right]$

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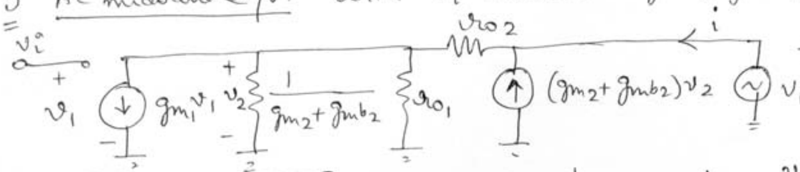
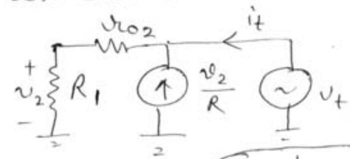
5 AC midband equ: With v_i shorted to gnd., for o/p res. calculation.



$\Rightarrow$



$\boxed{R_1 = \dfrac{1}{g_{o1} + g_{m2} + g_{mb2}}}$ with $g_{o1} = \dfrac{1}{r_{o1}}$. $i_t = \dfrac{v_t - v_2}{r_{o2}} - \dfrac{v_2}{R}$ $\boxed{R = \dfrac{1}{g_{m2} + g_{mb2}}}$

$\Rightarrow i_t = g_{o2} v_t - (g_{o2} + g_{m2} + g_{mb2}) v_2$ where $g_{o2} = \dfrac{1}{r_{o2}}$.

Also, $v_2 = \dfrac{R_1}{R_1 + r_{o2}} v_t = \dfrac{g_{o2}}{g_{o1} + g_{o2} + g_{m2} + g_{mb2}} v_t$.

Thus, $i_t = \dfrac{v_t - v_2}{r_{o2}} - (g_{m2} + g_{mb2}) v_2 = g_{o2} v_t - (g_{o2} + g_{m2} + g_{mb2}) \dfrac{g_{o2}}{g_{o1} + g_{o2} + g_{m2} + g_{mb2}} v_t$

$= \dfrac{g_{o1} g_{o2}}{g_{o1} + g_{o2} + g_{mb2} + g_{m2}} v_t \Rightarrow R_o = \dfrac{v_t}{i_t} = r_{o1} + r_{o2} + (g_{m2} + g_{mb2}) r_{o1} r_{o2}$.

Note that if either of r_{o1} or r_{o2} or both $\to \infty$, $R_o \to \infty$

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6 $I_{EE} = 20\,\mu A \Rightarrow I_{C1} = I_{C2} = I_{EE}/2 = \underline{10\,\mu A}$. $g_{m1} = g_{m2} = g_m = \dfrac{I_{C1}}{V_T} = \dfrac{1}{2.6\,k\Omega}$.

$\Rightarrow A_{dm} = -g_m R_C = \underline{-38.46}$, $A_{cm} = -\dfrac{g_m R_C}{1 + 2 g_m R_{EE}} = \underline{-5 \times 10^{-3}}$, CMRR $= \left|\dfrac{A_{dm}}{A_{cm}}\right| = 7692$

$\Rightarrow$ 77.7 dB (reasonably high). $r_\pi = \beta / g_m = 520\,k\Omega \Rightarrow R_{id} = 2 r_\pi = \underline{1.04\,M\Omega}$

& $R_{ic} = r_\pi + (\beta + 1) 2 R_{EE} = \underline{4.02\,G\Omega}$ (amazingly high)!!

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7 Case of extending the linear range by $\pm I_{EE} R_E = \pm 20\,\mu A \times 4\,k\Omega = \underline{\pm 80\,mV}$.

I_{C1} & I_{C2} remain same at $I_{EE}/2$ ($= 10\,\mu A$). $A_{dm} = -\dfrac{g_m R_C}{1 + g_m R_E} = \underline{-15.15}$

(less than half of previous case), $A_{cm} = -\dfrac{g_m R_C}{1 + g_m (R_E + 2R_{EE})} \simeq \underline{-5 \times 10^{-3}}$ (unchanged,

$\because R_{EE} \gg R_E$), CMRR $= \left|\dfrac{A_{dm}}{A_{cm}}\right| = 3030$ (69.63 dB) $\Rightarrow$ deterioration.

$R_{id} = [r_\pi + (\beta + 1) R_E] \times 2 = 2\,[520\,k\Omega + 201 \times 4\,k\Omega] = \underline{2.65\,M\Omega}$ (enhanced)

& $R_{ic} = r_\pi + (\beta + 1)(R_E + 2R_{EE}) \simeq 4.02\,G\Omega$ (again unchanged, $\because R_{EE} \gg R_E$).

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8. I/p to Q_2 base, o/p from Q_2 emitter ⇒ Q_2 in CC mode. For Q_1, i/p to emitter, & o/p from collector ⇒ Q_1 in CB mode ⇒ Configuration ⇒ CC–CB. (4)

* The absence of R_C in Q_2 collector does not break the symmetry of the base-emitter loop ⇒ the entire analysis done in class holds for this case too.

DC Analysis: $I_{EE} = \frac{-0.7 - V_{EE}}{R_{EE}} = 1.43\ mA \Rightarrow I_{C_1} = I_{C_2} = I_{EE}/2 = 715\ \mu A$.

$g_{m1} = g_{m2} = g_m = 27.5\ m℧$, & $r_{\pi 1} = r_{\pi 2} = r_\pi = 7.27\ k\Omega$.

$\Rightarrow A_{dm} = -g_m R_C = -275$, $A_{cm} = -\frac{g_m R_C}{1 + 2 g_m R_{EE}} = -0.5$.

Now, note that $v_{id} = v_{i1} - v_{i2} = 0 - v_i = -v_i$.

$v_{ic} = \frac{v_{i1} + v_{i2}}{2} = \frac{v_i}{2}$.

The o/p is taken from v_{01} terminal.

$\Rightarrow v_0 = v_{01} = \frac{A_{dm}}{2} v_{id} + A_{cm} v_{ic} = -\frac{A_{dm}}{2} v_i + \frac{A_{cm}}{2} v_i = \left(\frac{A_{cm} - A_{dm}}{2}\right) v_i$

$\Rightarrow$ Voltage Gain $\frac{v_0}{v_i} = \frac{A_{cm} - A_{dm}}{2} = 137.25$ (no phase shift betn i/p & o/p).

* Single-ended gain is half of the double-ended gain.

Overall i/p resistance $R_i = R_{id} \| R_{ic} \simeq R_{id} = 2 r_\pi = 14.54\ k\Omega$ ($\because R_{ic} \gg R_{id}$).

& $R_0 = R_C = 10\ k\Omega$ (by inspection).

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