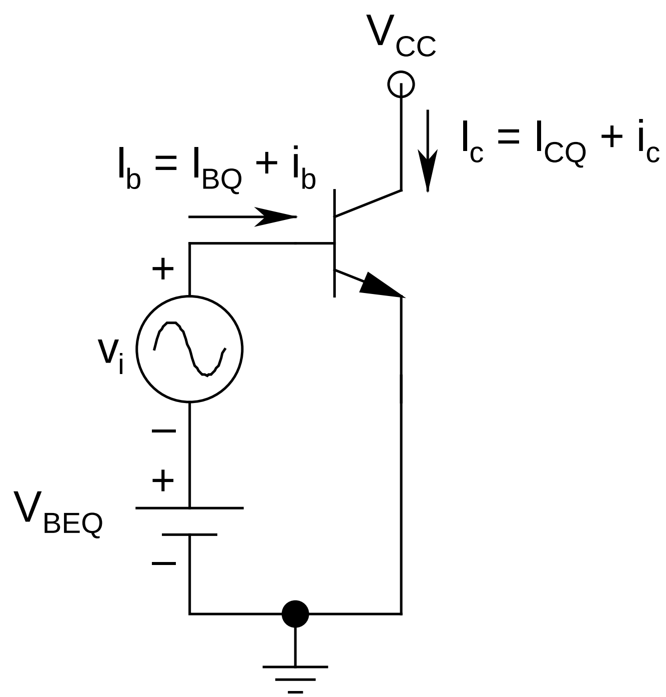
Validity of the Small-Signal Model

- Basically ***linearization*** of the ***operating region*** around the ***Q-point***
- ***This linearization should not contain any higher-order terms***



- To start with, assume $V_A \rightarrow \infty$
  - $I_{CQ} = I_S \exp(V_{BEQ}/V_T)$
- Thus:

$$I_c = I_S \exp\left(\frac{V_{be}}{V_T}\right) = I_S \exp\left(\frac{V_{BEQ} + v_i}{V_T}\right)$$

$$= I_S \exp\left(\frac{V_{BEQ}}{V_T}\right) \exp\left(\frac{v_i}{V_T}\right) = I_{CQ} \exp\left(\frac{v_i}{V_T}\right)$$

- Expand the ***exponential term*** in series:

➢ $I_c = I_{CQ}\left[1+\frac{v_i}{V_T}+\frac{1}{2!}\left(\frac{v_i}{V_T}\right)^2+\frac{1}{3!}\left(\frac{v_i}{V_T}\right)^3+\cdots\right]$

- Thus:

➢ $i_c = I_c - I_{CQ} = I_{CQ}\left[\frac{v_i}{V_T}+\frac{1}{2!}\left(\frac{v_i}{V_T}\right)^2+\frac{1}{3!}\left(\frac{v_i}{V_T}\right)^3+\cdots\right]$

- ***True linearization*** of i_c-v_i relation ***can be achieved*** only if ***all higher-order terms*** can be ***neglected*** $\Rightarrow$ v_i should be $<< V_T$

Small-Signal Model Parameters

- ***Incremental Emitter Resistance*** (r_E):

$$r_E = \frac{v_i}{i_e} = \frac{\Delta V_{BE}}{\Delta I_E} \equiv \frac{dV_{BE}}{dI_E} = \frac{V_T}{I_E}$$

- ***Transconductance*** (g_m):

$$g_m = \frac{i_c}{v_i} = \frac{\Delta I_C}{\Delta V_{BE}} \equiv \left.\frac{dI_C}{dV_{BE}}\right|_{V_{CE}\ \text{constant}} = \frac{I_C}{V_T}$$

- Thus, $g_m r_E = I_C/I_E = \alpha \approx 1$
- ***A frequently used approximation***:
  - $g_m = 1/r_E$
- For $I_C = 1$ mA:
  - $r_E = 26\ \Omega$ and $g_m = 1/26$ A/V
- As $I_C\uparrow$:
  - $g_m\uparrow$ and $r_E\downarrow$
  - Also $P_D\uparrow$
- Gain = $f(g_m)$

  $\Rightarrow$ For ***higher gain***, the circuit has to be fed ***more power***