ESc201A Home Assignment 6 Sept. 23, 2019.

Solutions of the HA#6 will be on Brihaspati on 30/09/19.

Consider all voltage and current sources to be ideal. Consider all transistors in forward active mode have V_{BE}=0.7V.

1. In fig. 6.1, the transistor Q_1 is under best biasing condition ($V_{cc}/3$) having $\beta_F=100$, $C_\pi=0.5$pF, $C_\mu=0.1$pF, and $V_A=\infty$. If $C_{in}=1\mu$F, $C_{out}=0.5\mu$F, $C_E=3.31\mu$F.

(a) Find the mid frequency small signal voltage gain v_o/v_i.

(b) Using the dominant pole approximation, find the lower cutoff frequency by the **Short Circuit Time Constant (SCTC)** technique.

(c) Using the dominant pole approximation, find the upper cutoff frequency by the **Open Circuit Time Constant (OCTC)** technique.

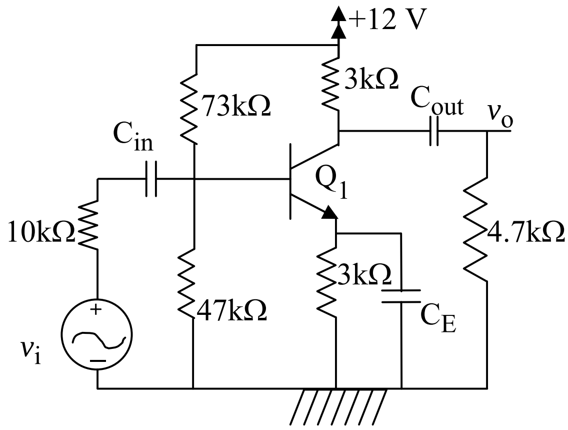

Figure 6.1

2. In fig. 6.2, the MOSFET M_1 is biased at a Q-point of (I_D=0.2mA and V_{DS}=5V) having $V_{GS}-V_{TN}$=1V for Body-Source (BS) shorted.

(a) Find K_N and the threshold voltage (V_{TN}).

(b) Find the ac equivalent circuit at mid band frequencies.

(c) If $C_{in}=0.5\mu$F, $C_{out}=0.5\mu$F, and $C_S=10\mu$F, then calculate the approximate lower cutoff frequency, assuming dominant pole approximation.

(d) If C_{gs}=100pF and C_{gd}=10pF, which of these would give you the dominant pole for the high frequency cutoff?

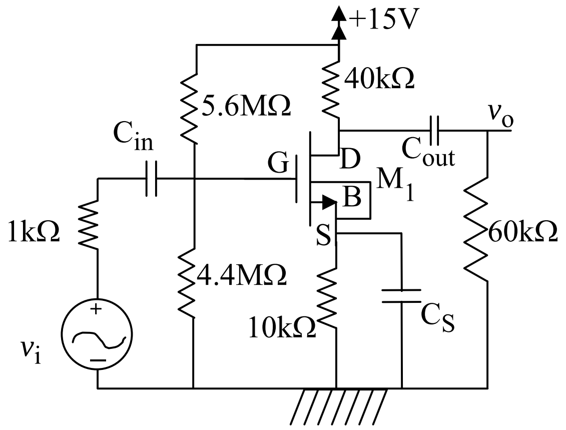

Figure 6.2

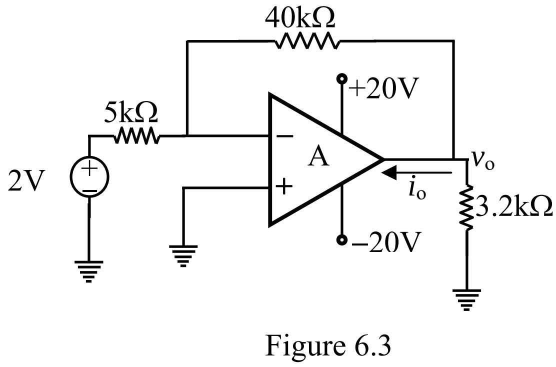

Figure 6.3

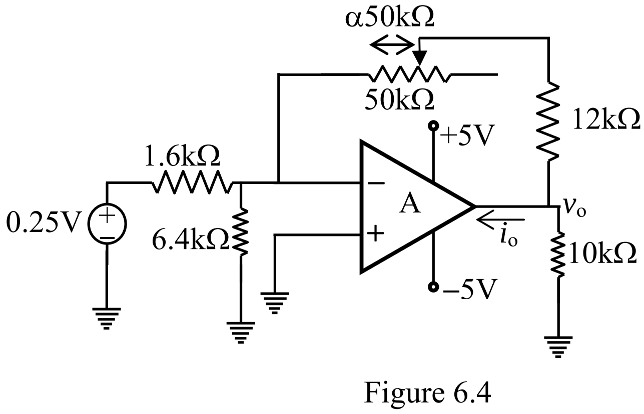

Figure 6.4

3. The OpAmp in fig. 6.3 is considered to be ideal ($R_i\to\infty$, $A\to\infty$, and $R_o\to 0$). Find the output voltage v_o and i_o.

4. Considering the OpAmp in fig. 6.4 to be ideal, find $0 < \alpha < 1$ for which linear amplification is obtained (i.e. v_o does not saturate). Find i_o for which $\alpha = 0.272$.

5. The non-ideal OpAmp of fig. 6.5 has R_i=0.5MΩ, A=5x10^4, and R_o=8kΩ and is working in the linear region. Find (a) The voltage gain v_o/v_i, (b) The differential input voltage (v_+ - v_-) for v_i=1V, (c) Current through the 240kΩ resistance.

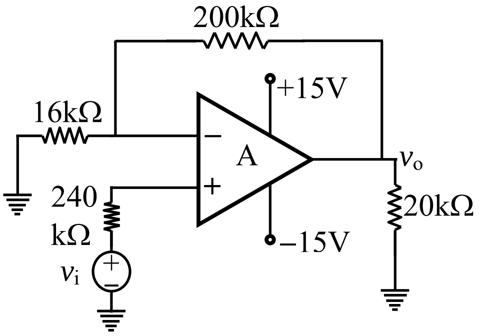

Figure 6.5

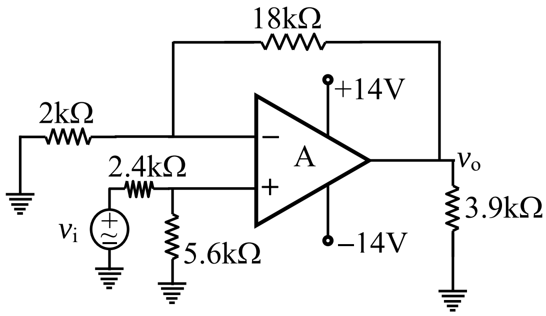

Figure 6.6

6. The input of the non-inverting amplifier of fig. 6.6 is v_i = 0V for t≤ 0, and v_i = 4sin(5π/3)t V for $0 \leq t \leq \infty$. Assuming that the OpAmp is ideal, sketch v_o as a function of time, t.

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1) a) Biasing: As per $(V_{cc}/3)$ rule of thumb $v_{R_C} = \frac{V_{cc}}{3} = 4V$

$V_{R_E} = \frac{V_{cc}}{3} = 4V$. $\therefore v_{CE_Q} = 4V$. $I_{C_Q} \simeq I_{E_Q} = \frac{4}{3k} = \frac{4}{3} mA$

$\therefore V_E = 4V$. $\therefore V_B \geq 4 + 0.7 = 4.7V. = \frac{V_{cc} R_2}{R_1 + R_2} = \frac{12 \times 47}{73 + 47}$

$= 4.7V.$

Check biasing stability $\left\{ I_B \Big|_{\beta=100} = \frac{4}{300} mA \right.$ $I_{R_1} = \frac{12}{120K} = \frac{1}{10} mA. \gg \frac{4}{300} mA.$ Satisfied

$R_B = R_1 \| R_2 = 73k \| 47k = 28.6 k\Omega$, $R_L = 3k \| 4.7k = 1.83 k\Omega$

$r_e = \frac{V_T}{I_C} = \frac{0.026}{4/3 m} = \frac{26 \times 3}{4} = 19.5\Omega$, $r_\pi = (\beta+1) r_e = 101 \times 19.5 = 1.97 k\Omega$

$g_m = \frac{I_C}{V_T} = \frac{1}{r_e} = 51.3 mS$ $\therefore A_v = \frac{v_o}{v_i} = -g_m R_L \frac{R_B \| r_\pi}{R_i + R_B \| r_\pi}$

$= -\frac{51.3 \times 1.83 \times 28.6k \| 1.97k}{10k + 28.6k \| 1.97k}$

or $A_v = -\frac{51.3 \times 1.83 \times 1.84k}{10 + 1.84}$

$= -14.6$

(b) $\omega_L = \left[\frac{1}{R_{C_{in_S}} C_{in}} + \frac{1}{R_{C_{out_S}} C_{out}} + \frac{1}{R_{C_{E_S}} C_E}\right]$ with the voltage source v_i shorted for τ_1 & C_{μ} & C_π open calculation.

Short C_{out} & C_E $\longrightarrow$ $R_{in_S} = R_i + R_B \| r_\pi = 10k + 28.6k \| 1.97k$

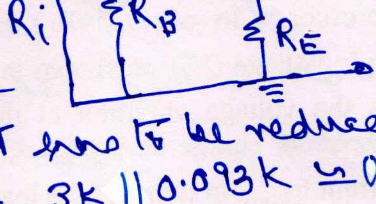
$\tau_{C_{in}} = 1 \times 10^{-6} \times 11.84 k\Omega$ $= 11.84 k\Omega$

$\simeq 12 ms$ $\Longrightarrow$ $f_{C_{in_L}} = \frac{1}{2\pi \times 12m} = 13.26 Hz$

Short C_{in} & C_E & v_i $\longrightarrow$ $R_{out_S} = R_C + R_3 = 3k + 4.7k = 7.7k\Omega$

$\tau_{cout} = 0.5 \times 10^{-6} \times 7.7k = 3.85 ms \longrightarrow f_{C_{out_L}} = \frac{1}{2\pi \times 3.85m} = 41.3 Hz$

short C_{in} & C_{out} & v_i shorted. $\longrightarrow R_{E_S} =$



$= R_E \| \frac{r_\pi + R_i \| R_B}{\beta + 1}$

$= 3k \| \frac{1.97k + 10k \| 28.6k}{101}$

Remember when resistances from the base side is brought over to the emitter side it has to be reduced $(\beta+1)$ times.

$\therefore R_{C_{E_S}} = 3k \| \frac{1.97K + 7.41K}{101} = 3k \| 0.093k \simeq 0.09 k\Omega$

$\therefore f_{C_{E_{S_L}}} = \frac{1}{0.09K \times 3.31\mu F \times 2\pi} = 533 Hz$. $\therefore f_L \cong 533 Hz$ due to Dominant pole or 588 Hz to be more precise.

Here the dominant pole technique fails, as the $\omega_1, \omega_2, \omega_E$ are not widely separate.

(c) $C_\pi = 0.5 pF$, $C_\mu = 0.1 pF$ $\therefore C_{\mu\pi} = 0.1(1 + g_m R_L) pF$ $= 0.1(1 + 93.9) pF = 9.5 pF$

(This is rather larger than usual), $\therefore C_T = 0.5 + 9.5 pF$ and neglect $C_{\mu C} \cong 0.1 pF$

$C_T = 10 pF$. $\Longrightarrow \omega_H = \frac{1}{R_{C_{T_o}} C_T} = \frac{1}{10 pF \times (r_\pi \| R_B \| R_i)}$

$\therefore f_H = \frac{10^6}{2\pi \times 10\mu \times 1.55k} = 10.27 MHz$

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2) (a) $I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$ or $K_n = \frac{2 I_D}{(V_{GS} - V_{TN})^2} = \frac{2 \times 0.2\,mA}{(1V)^2}$

$V_G = \frac{15V \times 4.4M}{(5.6+4.4)M} = 6.6V$

$= \underline{0.4\,mA/V^2}$

$V_S = I_S \times 10k = I_D \times 10k$

$= 0.2m \times 10k = 2V$

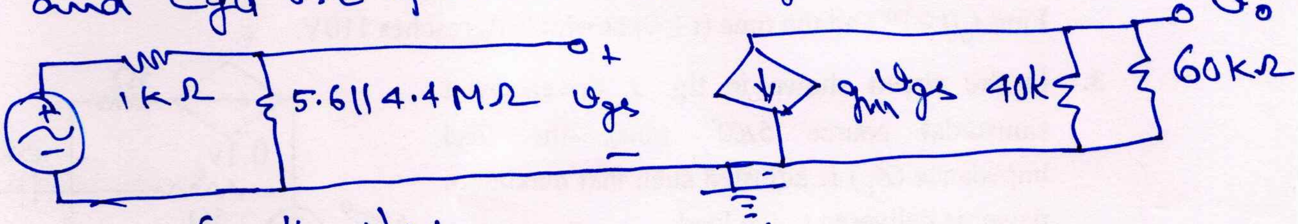
$\therefore V_{GS} = 6.6 - 2 = 4.6V.$

$V_{GS} - V_{TN} = 1V$ or $\underline{V_{TN} = 4.6 - 1 = 3.6}$

(b) $g_m = \sqrt{2 K_n I_D} = \sqrt{0.8m \times 0.2m} = \sqrt{0.16}\,m = 0.4\,mS$

(Here R_D & R_S are given, but in any problem if it is not given then one can find from $V_S \rightarrow V_D$.)

At midband C_{in}, C_{out}, and C_S are shorted and C_{gs} and C_{gd} are open. $\therefore$ A.C. equivalent at midband is



$\therefore \frac{v_{gs}}{v_i} = \frac{(5.6||4.4)M}{1k + (5.6||4.4)M} = \frac{2.46M}{1k + 2.46M} \simeq 1.$

$\frac{v_o}{v_{gs}} = -g_m R_L = -g_m(40k||60k) = -g_m \times 24k = -9.6$

$A_{mid} = -1 \times 9.6 = \underline{-9.6}$

(c) Do exactly as problem 1 but here r_π equivalent is not there $\rightarrow \infty$

$\therefore \tau_{cin} = R_{cin_S} C_{in} = [1k + (5.6||4.4)M] \times 0.5\mu F \cong 2.46M \times 0.5\mu$

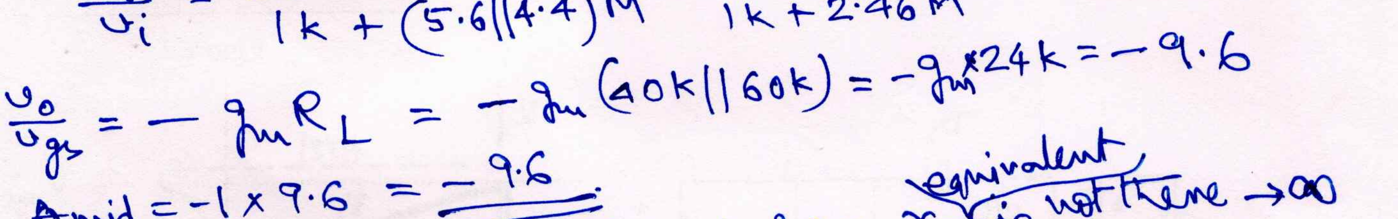
$= 1.23\,s \Longrightarrow f_{cin_L} = \frac{1}{2\pi \times 1.23} = 0.13\,Hz$

$\tau_{cout} = R_{cout_S} C_{out} = 100k \times 0.5\mu = 50\,ms$, $f_{cout_L} = \frac{1}{2\pi \times 50m} = 3.18\,Hz$

$\tau_{C_S} = R_S C_S = 10k \times 10\mu = 100\,ms$, $f_{C_{S_L}} = \frac{1}{2\pi \times 100m} = 1.59\,Hz$.

Two poles are now very close, hence dominant pole cannot be applied. $\therefore f_L \cong f_{cin_L} + f_{cout_L} + f_{C_{S_L}} = 0.13 + 3.18 + 1.59 = \underline{4.9\,Hz}$

(d) Redraws a.c. equivalent.



$C_T = C_{gs} + C_{gd}(1+g_m R_L) = 100pF + 10pF(1+9.6) = 206\,pF \gg C_{gd_{DS}}$

$\therefore f_H = \frac{1}{2\pi \times (2.46M||1k\Omega) \times 206pF} \cong \frac{1}{2\pi(1k) \times 206p} = \underline{772.6\,MHz}$

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3) $v_+ = 0$ Since the OpAmp is ideal $v_d \to 0, (v_+ - v_-) = 0$

$\therefore v_- = 0 \quad \therefore I_{5k\Omega} = \frac{0-2}{5k} = -\frac{2}{5}\,mA$. $I_{40k\Omega} = \frac{0-v_0}{40k}$

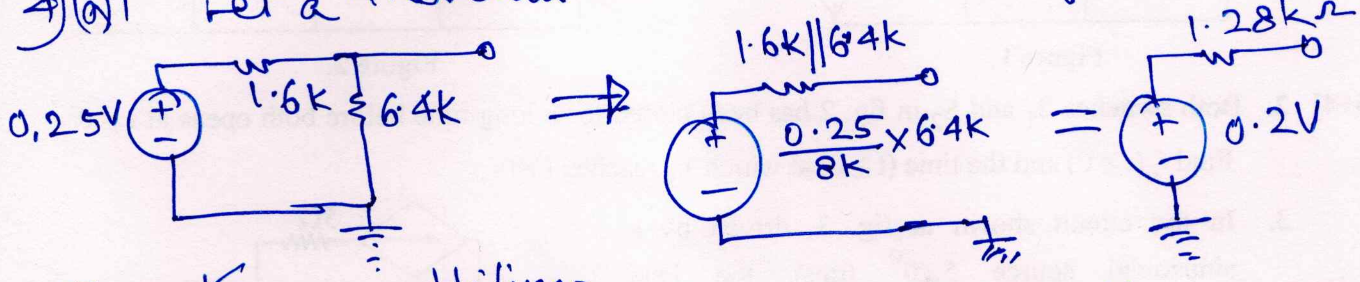
Since ideal current into -ve input $i_- = 0$

$\therefore \frac{0-2}{5}m + \frac{0-v_0}{40}m = 0$ or $v_0 = -\frac{2}{5}\times 40 = \underline{\underline{-16\,V}}$

KCL at the output node: $i_0 + \frac{v_0 - 0}{40k} + \frac{v_0}{3.2k} = 0$

or $i_0 = -v_0\left(\frac{1}{40} + \frac{1}{3.2}\right)\,mA = 16\left(\frac{1}{40} + \frac{1}{3.2}\right) = \underline{\underline{5.4\,mA}}$

4)(a) 1st Let a thevenin be done at left of v_- port.



Inverting amplifier

$\therefore \frac{v_0}{v_i} = -\frac{R_F}{R} = -\frac{12k + \alpha 50k}{1.28k}$

or $v_0 = -\frac{12 + \alpha 50}{1.28}\times 0.2 \cong -5V$ when at saturation

$\therefore \alpha = \frac{1}{50}\left(\frac{5\times 1.28}{0.2} - 12\right) = \frac{32-12}{50} = \frac{20}{50} = \underline{\underline{0.4}}$

(b) For $\alpha = 0.272$ $\quad v_0 = \frac{-12 + 0.272\times 50}{1.28}\times 0.2$

$= -4V$ (Has not saturated)

KCL at output:

$-\frac{4}{10k} + \frac{-4 - (v_- = 0)}{(0.272\times 50 + 12)k} + i_0 = 0.$

or $i_0 = \frac{4}{10k} + \frac{4}{25.6k} = 0.55625\,mA = \underline{\underline{556.25\,\mu A}}$

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5) a) KCL at inverting input port:

$$\frac{v_-}{16k} + \frac{v_- - v_i}{\underbrace{500k + 240k}_{R_i}} + \frac{v_- - v_o}{200k} = 0 \text{ or } v_-\left(\frac{1}{16} + \frac{1}{740} + \frac{1}{200}\right)m = \frac{v_i}{740}m + \frac{v_o}{200}m$$

or $v_-(0.06885) = v_i(0.00135) + v_o(0.005)$

or $v_i = 51 v_- - 3.7 v_o$ ---- (1)

KCL at the output port:

$$\frac{v_o}{20k} + \frac{v_o - v_-}{200k} + \underbrace{\frac{v_o - A(v_+ - v_-)}{R_o}} = 0.$$

or $$\frac{v_o}{20k} + \frac{v_o - v_-}{200k} + \frac{v_o - 5\times10^4(v_+ - v_-)}{8k} = 0.$$

or $v_o\left(\frac{1}{20} + \frac{1}{200} + \frac{1}{8}\right) + v_-\left(-\frac{1}{200} + \frac{5\times10^4}{8}\right) = \frac{5\times10^4}{8} v_+$

or $v_+ = v_o(2.88\times10^{-5}) + v_-(0.995)$

$= v_o(2.88\times10^{-5}) + \frac{v_i + 3.7v_o}{51} \times 0.995$ using (1)

$= \frac{v_i}{51} + 0.0722 v_o$ ---- (2)

Equating the currents at inverting and non-inverting inputs $\frac{v_- - v_+}{500k} = \frac{v_+ - v_i}{240k}$ or $v_- = v_+\left(1 + \frac{500}{240}\right) - v_i\frac{500}{240}$ ---- (3)

or $v_- = v_+ \times 3.083 - v_i \times 2.083$

$= \frac{v_i \times 3.083}{51} + v_o \times 0.0722 \times 3.083 - v_i \times 2.083$

$= 0.2226 v_o - 2.02255 v_i$ ---- (4)

substituting (3) in (1)

$v_i = 51 \times 0.2226 v_o - 51 \times 2.02255 v_i - 3.7 v_o$

$104.15 v_i = 7.653 v_o$ or $\frac{v_o}{v_i} = \underline{\underline{A_v = 13.6}}$

(b) using (2) $v_+ = \frac{1}{51} + 0.0722 \times 13.6 \times 1 = 1.0015$ V

using (3) $v_- = 1.0015 \times 3.083 - 1 \times 2.083 = 1.00471$ V

$\therefore (v_+ - v_-) = v_d = -3.21$ mV.

(c) $i_{240k} = \frac{v_+ - v_i}{240k} = \frac{1.0015 - 1}{240K} = \underline{\underline{6.25 \text{ nA}}}$

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6) $v_+ = \dfrac{v_i \times 5.6}{2.4+5.6} = \dfrac{5.6}{8} v_i$ considering no input current (ideal).

$= 0.7 v_i$ since ideal $v_+ \simeq v_-$

$\therefore$ KCL at v_- node:

$\dfrac{0.7 v_i}{2k} + \dfrac{0.7 v_i - v_o}{18k} = 0$ or $0.38888 \times 18 v_i = v_o$

or $v_o = 7 v_i = 7 \times 4 \sin\left(\frac{5\pi}{3}\right)t$, $t \geq 0$. ......(1)

$v_o = 0$ $t \leq 0$. ......(2)

At saturation $28 \sin\left(\frac{5\pi}{3}t\right) \simeq \pm 14$ V. or $\sin\left(\frac{5\pi}{3}t\right) = \pm 0.5$

or $\sin\left(\frac{5\pi}{3}t\right) = \pm \sin\left(\frac{\pi}{6}\right)$ $\therefore$ $t = \pm 0.1s, \pm 0.5s, \pm 0.7s, \pm 1.1s$ etc.

$\sin\left(\frac{5\pi}{6}\right), \sin\left(\frac{7\pi}{6}\right), \sin\left(\frac{11\pi}{6}\right), \ldots$ $-$ve t is not to be considered due to condition (2)