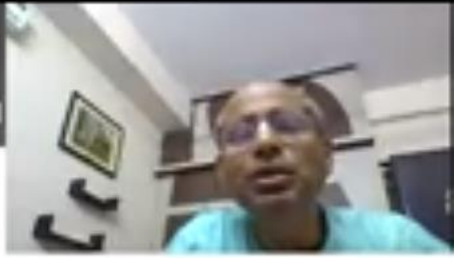


Laplace Transform

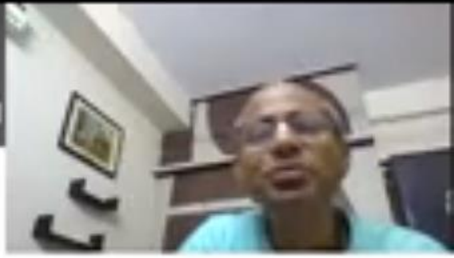
$$F(s) = \mathcal{L} f(t) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\mathcal{L}\, \delta(t) = 1 \quad \text{(Impulse)}$$

$$\mathcal{L}\, u(t) = 1/s \quad \text{(unity step)}$$

$$\mathcal{L}\, t\, u(t) = \frac{1}{s^2} \quad \text{(ramp fn)}$$

$$\mathcal{L}\, e^{-at} u(t) = \frac{1}{s+a}$$

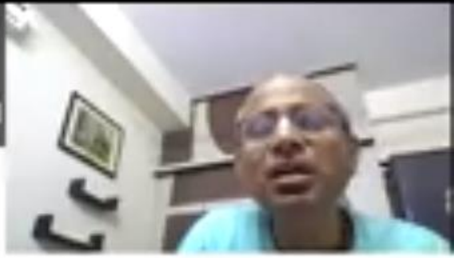


$$\mathcal{L}\, t e^{-at} = -\frac{d}{ds}\frac{1}{s+a}$$

$$= \frac{1}{(s+a)^2}$$

$$\mathcal{L}\, \sin \omega t = \frac{\omega}{s^2+\omega^2}$$

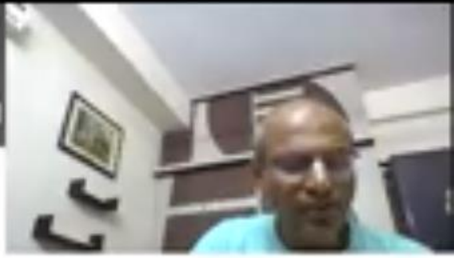
$$\mathcal{L}\, \cos \omega t = \frac{s}{s^2+\omega^2}$$



$$\mathcal{L}\frac{dy}{dt} = sY(s) - y(0)$$

$$\mathcal{L}\,y(t) = Y(s)$$

$$\mathcal{L}\frac{d^2y}{dt^2} = s^2Y(s) - s\,y(0) - \dot{y}(0)$$



Final value Theorem

Let $\mathcal{L}\, f(t) = F(s)$

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\,F(s)$$

Example $\dfrac{d^2 y(t)}{dt^2} + 3\dfrac{dy}{dt} + 2y(t) = 5u(t)$

Initial conditions are

$$y(0) = -1, \quad \frac{dy}{dt}(0) = 0$$

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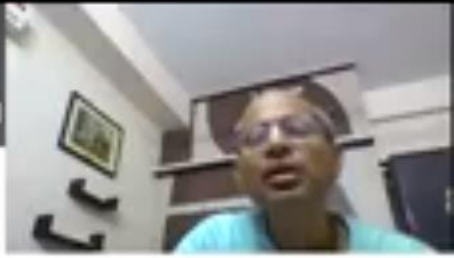
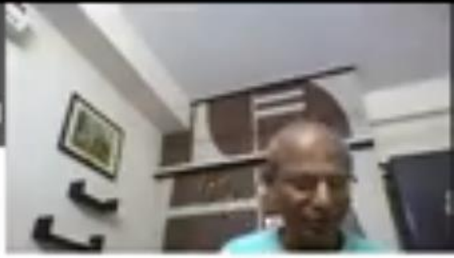


$$s^2 Y(s) - s\,\overset{\uparrow -1}{y(0)} - \overset{\uparrow 0}{\frac{dy}{dt}(0)} + 3\big(sY(s) - Y(0)\big) + 2Y(s) = 5\,\frac{1}{s}$$

$$Y(s) = \frac{-s^2 - 3s + 5}{s(s+1)(s+2)} = \frac{5}{2}\,\frac{1}{s} - \frac{7}{s+1} + \frac{7}{2}\,\frac{1}{s+2}$$

$$y(t) = \mathcal{L}^{-1}\,Y(s) = \frac{5}{2}\,u(t) - 7e^{-t} + \frac{7}{2}\,e^{-2t}$$

$$\lim_{t\to\infty} y(t) = \frac{5}{2}, \quad \lim_{s\to 0} sY(s) = \frac{\cancel{s}(-s^2 - 3s + 5)}{\cancel{s}(s+1)(s+2)} = 5/2$$



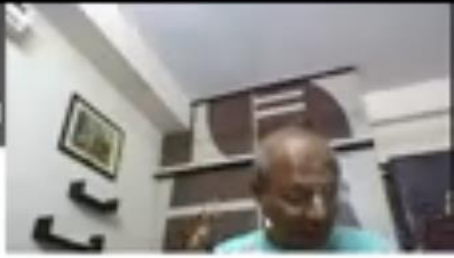
Theorem 2 Real Convolution

$$f_1(t) \leftrightarrow F_1(s)$$

$$f_2(t) \leftrightarrow F_2(s)$$

$$\mathcal{L}\int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau = \mathcal{L}\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau$$

$$= F_1(s)\, F_2(s)$$



Transfer function of a system

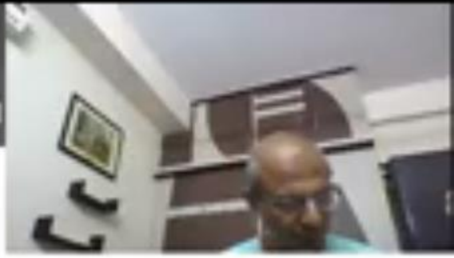
$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = \int_0^\infty h(t-\tau)\, u(\tau)\, d\tau$$

$$Y(s) = H(s)\, U(s)$$

$$\frac{Y(s)}{U(s)} = H(s)$$

, TF is the Laplace Transform of $h(t)$, impulse response



$$\frac{dy}{dt} + 2y = u$$

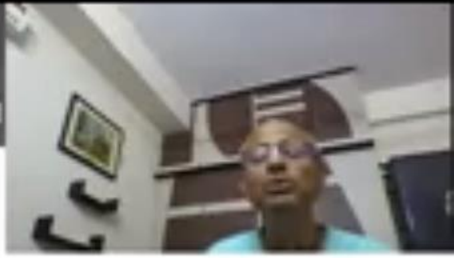
Impulse response

$$h(t) = e^{-2t}$$

$$H(s) = \frac{1}{s+2}$$

$$\frac{Y(s)}{U(s)} = H(s) = \frac{1}{s+2}$$

Laplace Transform of the impulse response of a system is indeed its own TF



Incorrect use of final value thm

$$Y(s) = \frac{3}{s(s+2)}$$

$$\lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{3}{s+2} = 3/2$$

$$Y(s) = \frac{3}{s(s-2)}$$

wrong.

$$\lim_{s \to 0} sY(s) = -3/2$$

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DC gain

$$= \lim_{s \to 0} G(s)$$

$$G(s) = \frac{s+1}{s^2+2s+3}$$

$$G(0) = 1/3$$