



$$\frac{Y(s)}{R(s)} = \frac{n(s)}{d(s)}$$

Routh Array

$$d(s) = s^n + a_1 s^{n-1} + \dots + a_n = 0$$

$$\forall i=1, n \quad a_i > 0 \quad (\text{necessary condition})$$

Sufficient  
condition

$s^n$	<u>1</u>	$a_2$	$a_4$	$\dots$
$s^{n-1}$	$a_1$	$a_3$	$a_5$	$\dots$
$s^{n-2}$				
$\vdots$				
$s^0$				



## Special Cases

$$d(s) = s^4 + 2s^3 + 2s^2 + 4s + 3$$

$$s^4 \quad 1 \quad 2 \quad 3$$

$$s^3 \quad 2 \quad 4 \quad 0$$

$$s^2 \quad 0(\epsilon) \quad 3 \quad \rightarrow \text{negative}$$

$$s^1 \quad \left( \frac{4\epsilon - 6}{\epsilon} \right) \quad 0$$

$$s^0 \quad 3$$

There are two poles  
in the RHS plane



$$d(s) = s^4 + 2s^3 + 2s^2 + 4s + 3$$

$$s \rightarrow s+1$$

$$s = 1/x$$

$$d(x) = \frac{1}{x^4} + \frac{2}{x^3} + \frac{2}{x^2} + \frac{4}{x} + 3 = 0$$

$$= 3x^4 + 4x^3 + 2x^2 + 2x + 1 = 0$$

$x^4$	3	2	1
$x^3$	4	2	
$x^2$	0.5	1	
$x^1$	-6	0	
$x^0$	1		

2 sign change  
 $\Rightarrow$  Two poles in  
RHS plane



## Special case 2

$$d(s) = s^4 + 3s^3 + 6s^2 + 12s + 8 = 0$$

$s^4$	1	6	8
$s^3$	3	12	0
$s^2$	2	8	✓
$s^1$	0(4)	0	
$s^0$	8		

Entire row = 0

$2s^2 + 8 \leftarrow$  Aux eqn

$$\frac{d}{ds} (2s^2 + 8)$$

$$4s + 0$$

Auxiliary eqn is a factor of  $d(s)$



$$2\tilde{s}^2 + 8 = 0$$

$$\tilde{s}^2 + 4 = 0$$

$$s_{1,2} = \pm j2 \quad \text{of } d(s)$$

$$\begin{array}{r} \tilde{s}^2 + 4 \overline{) s^4 + 3s^3 + 6\tilde{s} + 12s + 8} \\ \underline{s^4 \phantom{+ 3s^3} + 4s^2} \phantom{+ 12s + 8} \\ 3s^3 + 2s^2 + 12s \phantom{+ 8} \\ \underline{3s^3} \phantom{+ 2s^2} \phantom{+ 12s} \phantom{+ 8} \\ 2s^2 + 8 \phantom{+ 12s} \\ \underline{2s^2} \phantom{+ 8} \\ 8 \phantom{+ 12s} \end{array}$$

$$\begin{aligned} d(s) &= (\tilde{s}^2 + 4)(\tilde{s}^2 + 3\tilde{s} + 2) \\ &= (\tilde{s}^2 + 4)(s+1)(s+2) \\ &\quad \downarrow \\ &\quad \pm j2, -1, -2 \end{aligned}$$