EE 250: Control Systems Analysis
Module III: s-plane analysis
Lecture 19: Other Applications of Routh Array

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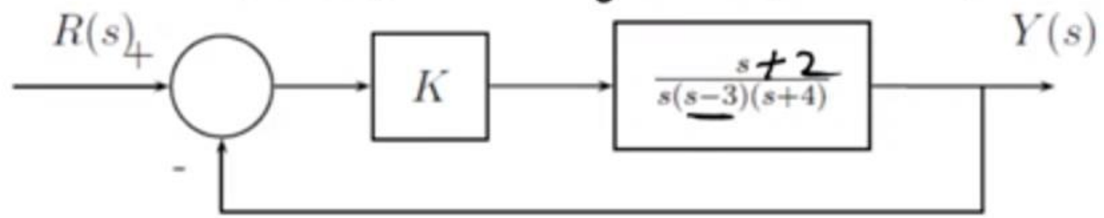
Direct Application

→ Stable

- If all poles are in LHS plane → If 1^{st} column elements are all positive
- How many poles are there in RHS plane

Find range of k for which the system is stable



Find the range of K for which the closed loop system is stable.

$$\frac{Y(s)}{R(s)} = \frac{K(s+2)}{s(s-3)(s+4)+K(s+2)} = \frac{n(s)}{d(s)}$$

$$d(s) = s^3 + s^2 + (K-12)s + 2K = 0$$

Routh array

s^3	1	K-12
s^2	1	2K
s^1	-(K+12)	0
s^0	2K > 0	

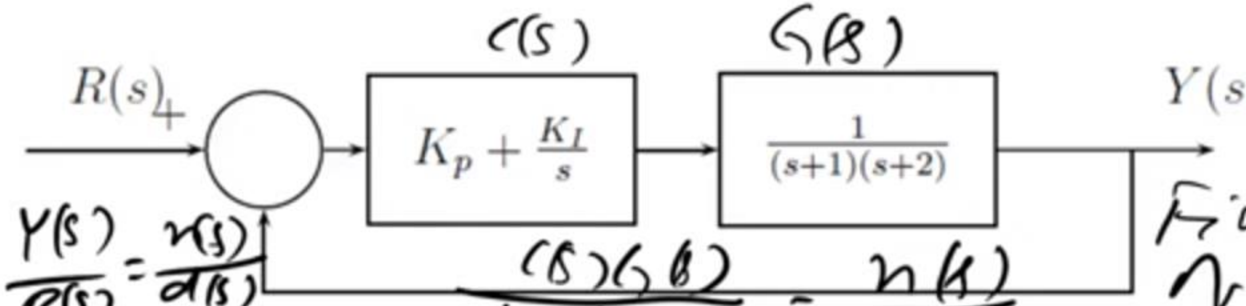
$-(K+12) > 0$

$$-K - 12 > 0$$
$$K + 12 < 0$$
$$K < -12$$

$2k > 0 \Rightarrow k > 0$

System is always unstable

$k > 0$ $\quad$ $k < -12$



$$\frac{Y(s)}{R(s)} = \frac{n(s)}{d(s)} \qquad \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{n(s)}{d(s)}$$

Find value of K_p & K_I

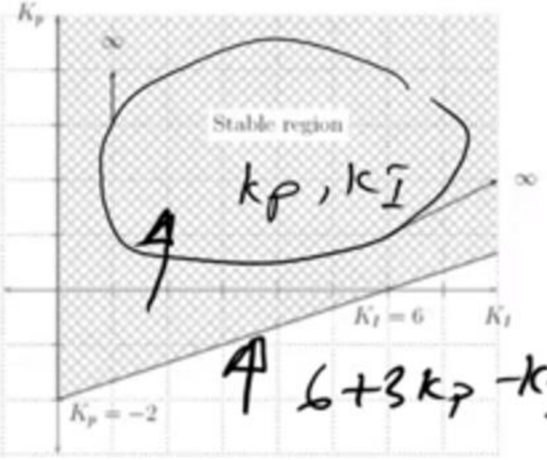
$$d(s) = s^3 + 3s^2 + (2+K_p)s + K_I = 0$$

s^3	1	$2+K_p$
s^2	3	K_I
s^1	$\frac{6+3K_p-K_I}{3} > 0$	0
s^0	$K_I > 0$	

$K_I = 10$

$3K_p > K_I - 6$ $\quad K_p = 5$

or, $K_p > \frac{K_I}{3} - 2$



$6 + 3K_p - K_I = 0$

$$\boxed{\begin{aligned} &K_p > \frac{K_I}{3} \\ &K_I > 0 \end{aligned}}$$

Routh test can find if roots are on $j\omega$ axis. This happens when an entire row becomes zero.

Example 9 (C). onsider the following characteristic polynomial

$$d(s) = s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12$$

The corresponding Routh array

s^5	1	11	28
s^4	5	23	12
s^3	6.4	25.6	0
s^2	3	12	0
s^1	0	0	0
new s^1	6	0	
s^0	12		

All zero row

$3s^2+12=0$

The auxiliary equation

$$d_1(s) = 3s^2 + 12$$
$$\frac{d}{ds}d_1(s) = 6s$$

Since, all the elements in column 1 are positive, no roots are in the right half of s-plane. There exist a pair of conjugate poles on the $j\omega$ axis which can be obtained from the following equation

$$3s^2 + 12 = 0$$
$$s^2 + 4 = 0$$
$$s_{1,2} = \pm j2$$

$s^2 = -4$