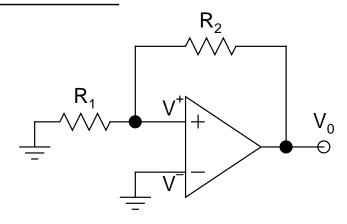
• Closed-Loop: Positive Feedback:

- Connection between output and non-inverting terminal of the OA
- Analysis technique for such circuits needs special care and attention:
 - Concept of Virtual Ground and Summing Point Constraint can't be applied anymore
- Applications:
 - Schmitt Trigger
 - Waveform Generation (Square- and Triangular-Wave)
 - Oscillators (Wein-Bridge and Phase-Shift)

Concept of Positive Feedback: Metastability:

* Circuit looks remarkably similar to an inverting amplifier, but it's not, since the output is fed back to the *non-inverting* terminal



- * Also, the circuit doesn't have any input signal source
- * The output V_0 should be *zero*, if the OA were assumed to be *ideal* (i.e., *without any offset voltage*)
- * Now, suppose that the output V_0 picks up some *positive* noise signal

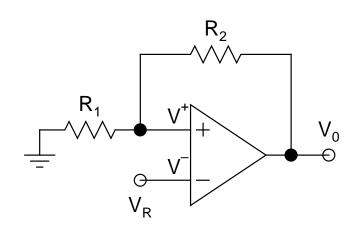
- * This would be fed back to the *non-inverting* terminal of the OA through the voltage divider action of R₁ and R₂
- * With the *inverting terminal grounded*, this would *increase the differential input voltage* of the OA, which would result in a *greater output*
- * This would cause *more fraction* of the output fed back to the input, causing an *even greater output*
- * This process *repeats*, and the output *grows* to be eventually *limited* by the *saturation voltage* V_{SAT^+}

- * The process described is called *signal regeneration*
- * Similarly, if V_0 picked up a *negative noise signal*, then the output would eventually *saturate* to V_{SAT}
- * Once this happens, it is *impossible* to bring the system out of it, i.e., V_0 would get stuck at $\pm V_{SAT}$, unless the power supply to the OA is turned off!
- * Thus, the *presumption* that the output of the circuit would be *zero* is **wrong**, since the chance of the output *picking up noise signals*, and thus the output eventually *saturating* to $\pm V_{SAT}$ is almost a cinch

- * Hence, such a state (i.e., $V_0 = 0$) is known as a *metastable state* for the circuit
- * Metastable: A state of precarious stability
- * The circuit is known as a **bistable** circuit, since the output can stay at either of $\pm V_{SAT}$ for an indefinite time, until and unless it's *forced* to switch state
- * This forced transition is known as external triggering
- * The circuit shown has no such provision
- * A circuit, known as **Schmitt Trigger**, has this *triggering mechanism* built-in

Schmitt Trigger:

- * Named after the inventor *Otto H*. *Schmitt* (in 1934)
- * Note the signal V_R applied at the *inverting* input and the *positive*feedback (output fed back to the non-inverting terminal)

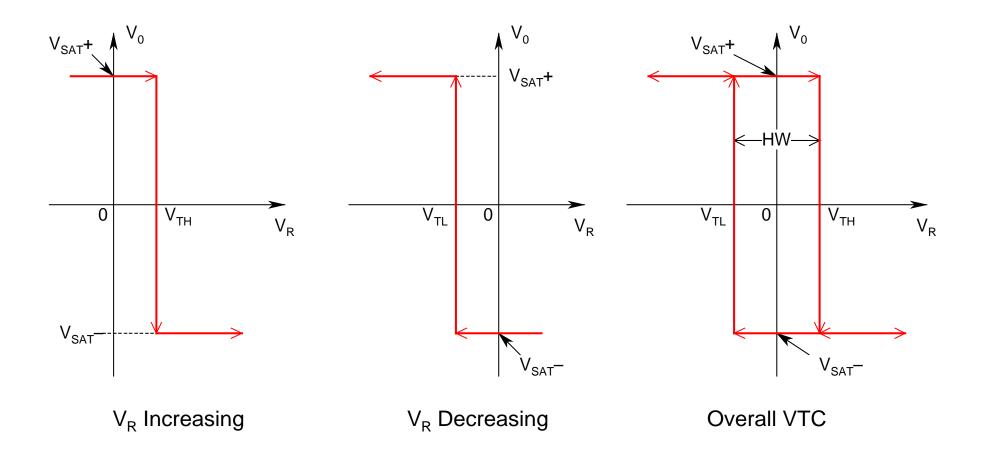


- \Rightarrow V_R is the *trigger signal* that creates change of state
- * Assume *negative* $V_R \Rightarrow V_0$ would *saturate* to V_{SAT^+}

$$\Rightarrow$$
 V⁺ = f'V_{SAT}, where f'= feedback factor = $\frac{R_1}{R_1 + R_2}$

- * Now, if V_R is *increased*, i.e., it's made *less negative*, then *zero*, and then *positive*, output can't change state till V^- becomes *greater* than V^+ , which is at $f'V_{SAT^+}$
- * Once this happens, the output will immediately *toggle* to V_{SAT^-} , and remain there till V_R is reduced *below* $f'V_{SAT^-}$ (note that $V^+ = f'V_{SAT^-}$ under this condition)
- * Thus, the circuit has *two stable states* at $\pm V_{SAT}$, and is hence known as a **bistable circuit**
- * For a *change of state* at the output, *appropriate trigger* $signal(V_R)$ needs to be applied at the input

- * The value of V_R at which this change of state takes place is known as the *threshold voltage* of the **Schmitt Trigger**
- * The *upper* one is known as **threshold high** (V_{TH}) and is given by $f'V_{SAT^+}$, while the *lower* one is known as **threshold low** (V_{TL}) and is given by $f'V_{SAT^-}$
- * Note that for values of V_R in between V_{TL} and V_{TH} , the output is indeterminate (: it can be either of $\pm V_{SAT}$), and depends on direction of change of V_R
- * $(V_{TH} V_{TL})$ is known as the **Hysteresis Width** (HW)



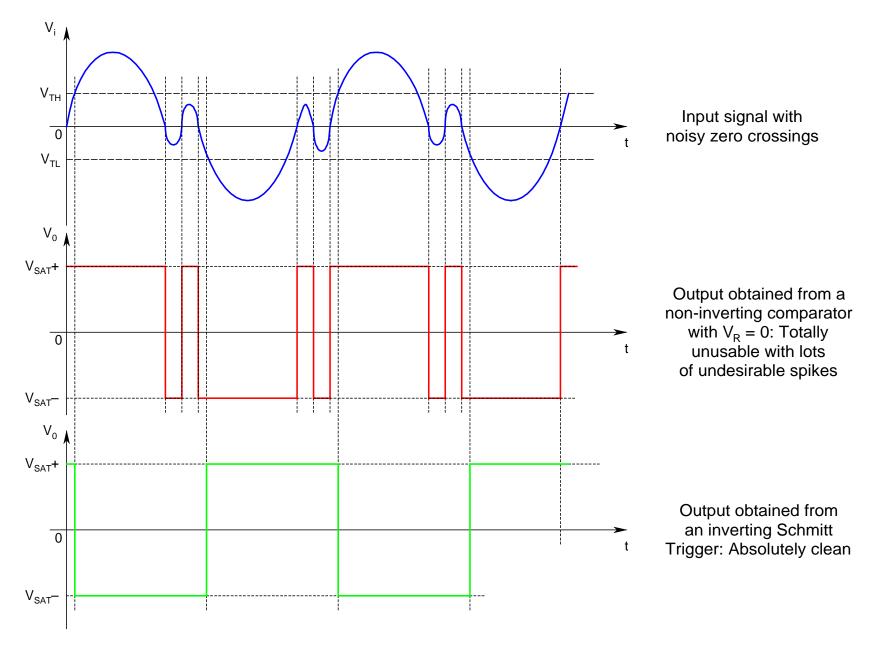
- * The region bound by HW is known as the **dead zone** or **forbidden zone**
- * Note for $V_R > V_{TH}$, the output remains at V_{SAT^-} , while for $V_R < V_{TL}$, the output remains at V_{SAT^+}
 - ⇒ Circuit known as **Inverting Schmitt Trigger**
- * Note that the *hysteresis characteristic* is absolutely *symmetric around zero*, i.e., $|V_{TL}| = V_{TH}$, since $|V_{SAT}| = V_{SAT}$, and f' is a constant
- * However, it is **not necessary** for a Schmitt Trigger to be always symmetric: there are ways to make it *asymmetric*

- * Since the *direction of change* of V_R has a profound impact on the outcome of the circuit, hence, this type of circuits is also known as *directional circuit*
- * Note that once the circuit has reached one of the *stable states*, V_R can be safely *grounded*, since it has *no impact* on the circuit any more
- * For example, if V_R becomes greater than V_{TH} , thus causing a change of state at the output from V_{SAT^+} to V_{SAT^-} , then the only way the output can be switched back to V_{SAT^+} is by reducing V_R below V_{TL}

- * Thus, grounding V_R would have absolutely no effect on the circuit once a transition has been achieved
- * Therefore, these circuits are considered to have a *memory effect*, i.e., by observing the *output* at any given instant of time, the *last input* applied to the circuit can be easily assessed
- * Also, for values of V_R between V_{TL} and V_{TH} , the output could either be V_{SAT^+} or V_{SAT^-} , which would depend on the last input signal V_R that caused the transition again it is a memory action

Schmitt Trigger as an Effective Noise Suppressor:

- * Recall: A comparator with $V_R = 0$ acts as a zerocrossing detector
 - ⇒ As soon as the input *crosses zero*, the output *changes state*
 - ⇒ Thus, if there is *noise* present during the *zero* crossings of the input, then an absolutely unusable output results
- * Now, if the same signal is applied in place of V_R to a *Schmitt Trigger*, then the output will *change* state only when the signal *crosses* V_{TH} or V_{TL}

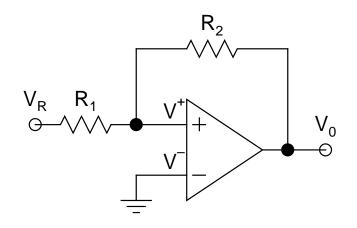


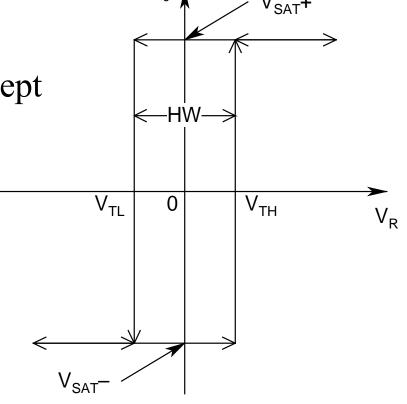
Non-Inverting Schmitt Trigger:

* Using *superposition*:

$$V^{+} = \frac{R_{2}}{R_{1} + R_{2}} V_{R} + \frac{R_{1}}{R_{1} + R_{2}} V_{0} \quad (1)$$

- * Note that $V^- = 0$ (ground)
- * Caution: Virtual Ground concept cannot be used



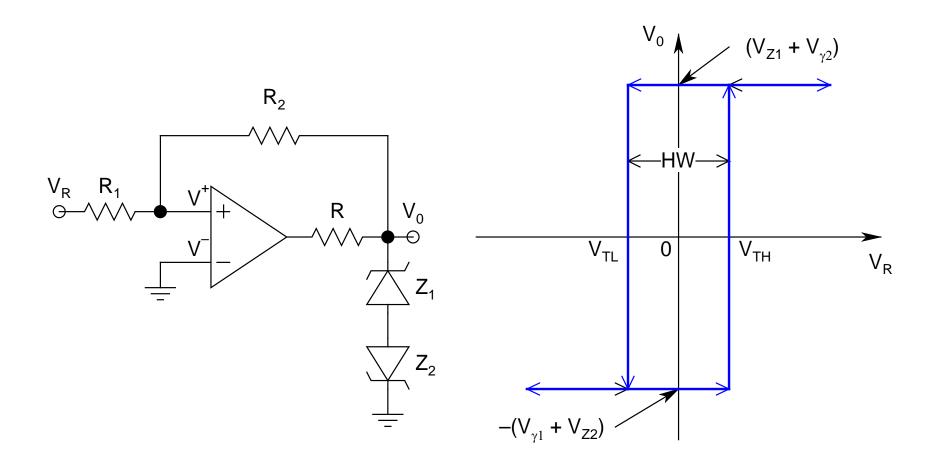


- * Due to *positive feedback*, the output V_0 will be at V_{SAT^+} for $V^+ > V^-$, and at V_{SAT^-} for $V^- > V^+$
- * Now, with $V^- = 0$, if V_0 is at V_{SAT^+} , then *no positive* value of V_R can change the state of the output
 - ⇒ To switch V_0 to V_{SAT^-} , V^+ must be *pulled below* ground (which is the potential V^-), and from (1), it would be possible only if V_R goes below $-(R_1/R_2)V_{SAT^+}$
 - \Rightarrow This corresponds to the *threshold low* (V_{TL}) of the circuit

- * Similarly, when V_0 is at V_{SAT^-} , no negative value of V_R can change the state of the output
 - ⇒ To cause a change in the state of the output under this condition, V_R must be *more positive* than $-(R_1/R_2)V_{SAT^-}$ (*Note*: V_{SAT^-} is *negative*)
 - \Rightarrow This corresponds to the *threshold high* (V_{TH}) of the circuit
- * Thus, $V_{TH} = -f'V_{SAT^-}$, $V_{TL} = -f'V_{SAT^+}$, $f' = R_1/R_2$, and $HW = V_{TH} V_{TL}$

- * From the VTC, the *non-inverting* behavior is obvious
- * Also, the hysteresis characteristic is *symmetric* around zero, however, as mentioned before, **it need not be**
- * Simplest way to make the hyesteresis characteristic *asymmetric* is to attach a *reference voltage* V_I to the inverting terminal of the OA
 - \Rightarrow By properly choosing the *sign* and *magnitude* of V_I , the entire hystersis characteristic can be *shifted* either to *first and fourth* or to *second and third* quadrants

Non-Inverting Schmitt Trigger With Output Clamp:



- * By putting a **Zener diode clamp** (identify Z_1 - Z_2 as a **double-anode Zener**) at the output, V_0 can be made to swing between extremes **other than the saturation voltages**
- * $V_{0,max} = V_{Z1} + V_{\gamma 2}$ (Z_1 under *breakdown*, Z_2 under *forward bias*)
- * $V_{0,min} = -(V_{Z2} + V_{\gamma 1})$ (Z_2 under *breakdown*, Z_1 under *forward bias*)
- * Quite useful circuit, with the output *adjustable* by a proper choice of the *Zener breakdown voltages* (it can even be made *asymmetric*)

Waveform Generation:

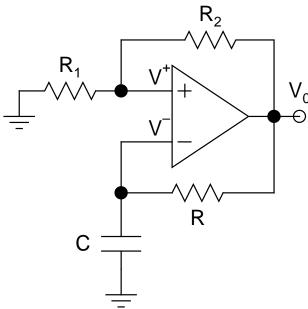
- * A very important instrument in the lab is the *Function Generator*
- * Capable of producing *sinuoidal*, *triangular*, and *square* waveforms
- * Sinusoidal waveforms are generated by Sinusoidal Oscillators
- * Triangular and Square waves can be generated effectively using Schmitt Triggers

Square-Wave Generation:

- * Recall: A Schmitt trigger circuit is essentially bistable, with both output states ($V_{SAT^{+}}$ and $V_{SAT^{-}}$) being stable states, and a triggering mechanism is needed for the change of state to take place
- * Now, if somehow this *triggering mechanism* can be made *automatic*, then the output waveform would keep on *switching* between its two extremes *periodically*, without any further intervention

- * Thus, under such a condition, none of the output states are *stable states*, since they keep on *changing* with respect to time
- * Such circuits are known as *astable circuits*, since they do have *two output states*, however, **none of them are stable states**
- * An extremely popular *square-wave generator* circuit uses only three resistors and a capacitor, along with an OA

- * *Note*: The circuit has *both* positive and negative feedback
- * Due to the *positive feedback* present in the circuit, irrespective of the *negative feedback*, V_0 would be *saturated* at either V_{SAT^+} or V_{SAT^-} ,



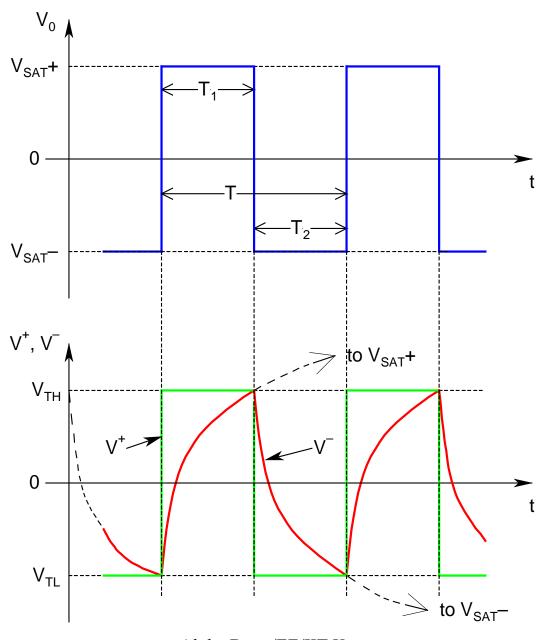
depending on whether V^+ is greater than or less than V^- , respectively

* Assume that at time t = 0, the capacitor is completely discharged, i.e., $V^- = 0$

- * Thus, for the result to be *consistent*, V_0 must be at $V_{SAT^+} \Rightarrow V^+ = f'V_{SAT^+} \ (>0)$, with $f' = R_1/(R_1 + R_2)$
- * Now, for t > 0, C would start to *charge* through R, V^- would *rise*, attempting to reach its *final* maximum value of V_{SAT^+}
- * However, *this won't happen*, : as soon as V^- *rises* above V^+ , which is at a constant potential of $f'V_{SAT^+}$, the output would *change state* to V_{SAT^-}
- * Note: $f'V_{SAT^+}$ is the threshold high (V_{TH}) of this circuit
- * Thus, V⁺ would now become equal to f'V_{SAT}-

- * The capacitor also would now start to discharge towards V_{SAT}
- * Similar to the previous situation, it won't be able to *discharge* all the way down to V_{SAT^-} , since as soon as V^- would *fall* below $f'V_{SAT^-}$, the output would again *toggle* to V_{SAT^+}
- * Note: $f'V_{SAT}$ is the threshold low (V_{TL}) of the circuit
- * These two processes of capacitor *charging* and *discharging* would continue *ad infinitum* and a *square wave* will be produced at the output, ranging between $\pm V_{SAT}$

- * Note that the circuit does not need *any source*: as soon as the *power supply* to the circuit is turned on, the *oscillations* would start, and a *square wave* will appear at the output
- * Refer to the figure on the next page:
 - T_1 : *On* (or *High*) *Time*
 - T₂: *Off* (or *Low*) *Time*
 - $T (= T_1 + T_2)$: *Time Period*
 - f (= 1/T): *Frequency*
 - $\delta (= T_1/T)$: *Duty Cycle*



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Evaluation of T_1 and T_2 :

* Use the familiar expression for *capacitor charging* / *discharging*:

$$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)] exp(-t/\tau)$$

* Note that the *time constant* $\tau = RC$

* **For**
$$T_I$$
: $v_c(0) = V_{TL}$, $v_c(\infty) = V_{SAT^+}$, and $v_c(T_1) = V_{TH}$

$$\Rightarrow T_{1} = \tau \ln \left[\frac{1 - f' V_{SAT^{-}} / V_{SAT^{+}}}{1 - f'} \right] = \tau \ln \left[\frac{1 + f'}{1 - f'} \right]$$

where we have assumed that $V_{SAT^{+}} = -V_{SAT^{-}}$

* For
$$T_2$$
: $v_c(0) = V_{TH}$, $v_c(\infty) = V_{SAT}$, and $v_c(T_2) = V_{TL}$

$$\Rightarrow T_2 = \tau \ln \left[\frac{1 - f' V_{SAT^+} / V_{SAT^-}}{1 - f'} \right] = \tau \ln \left[\frac{1 + f'}{1 - f'} \right]$$

where again we have assumed that $V_{SAT^{+}} = -V_{SAT^{-}}$

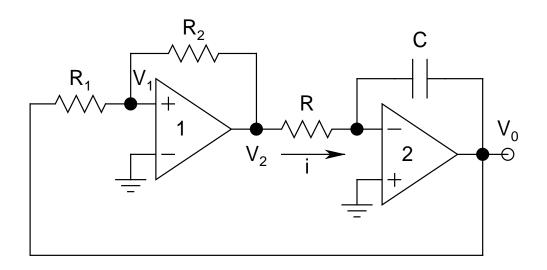
- * Note that for *symmetric* saturation voltages, the on and off times are same, resulting in a *50%* duty cycle square wave
- * Frequency:

$$f = \frac{1}{T} = \frac{1}{T_1 + T_2} = \frac{1}{2T_1} = \left(2\tau \ln \left[\frac{1+f'}{1-f'}\right]\right)^{-1}$$

- * Note that the output swings between $\pm V_{SAT}$
- * In order to obtain other swings, a *double-anode*Zener can be connected at the output
- * Also, the *breakdown voltages* of the Zeners can be chosen to be *unequal*, which would produce different values of *positive and negative amplitudes*, as well as *duty cycle* other than 50%
 - ⇒ Significant waveshaping
- * *Exercise*: Assume that the positive and negative saturation voltages are V_1 and $-V_2$, find T_1 and T_2

Triangular-Wave Generation:

- * Note that with regard to the *square-wave* generator circuit, if τ is made *very large*, and/or f' is made *very small*, then the voltage waveform across the capacitor will be approximately *linear*, since it will traverse only a *very small part* of the exponential characteristic ⇒ *Triangular-Wave Generation*
- * However, there is a much simpler way to generate triangular waves, which is basically a *non-inverting Schmitt Trigger* (with *symmetric* values of V_{TL} and V_{TH}) followed by an *integrator*

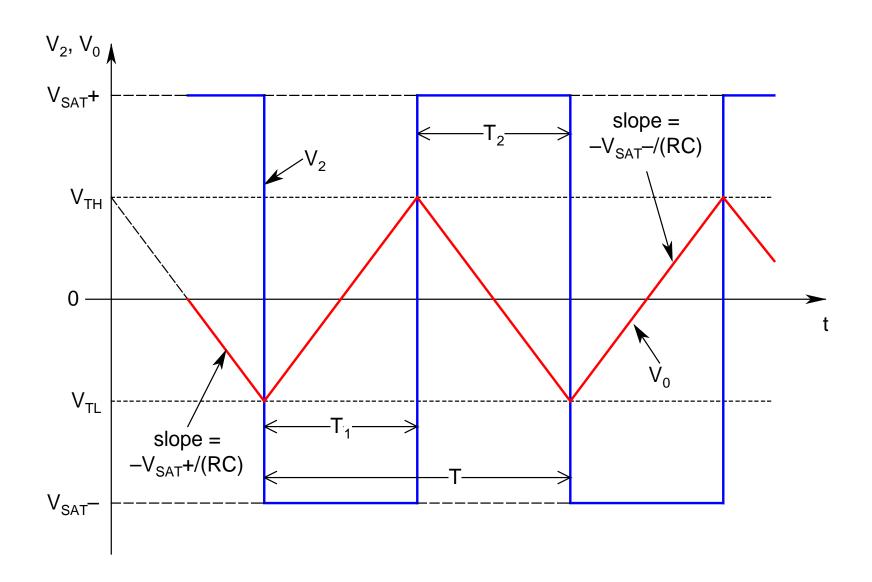


- * Identify OA1 as a *non-inverting Schmitt Trigger* with symmetric threshold, and OA2 as an *integrator*
- * Due to *positive feedback* present in OA1, V_2 will always be either at V_{SAT^+} or V_{SAT^-} , depending on whether V_1 is greater than or less than zero, respectively

- * Note also that OA2 is under *negative feedback*
 - ⇒ Concept of *virtual ground* can be applied, and the *inverting terminal* of OA2 is effectively *ground*
- * Assume that at some arbitrary time, the capacitor was completely discharged (i.e., $V_0 = 0$), and V_2 is at V_{SAT^+}
- * This would push a *constant current* $i = V_{SAT^+}/R$ through R, which would flow through the capacitor as well
- * Note that it is a case of *constant current charging* of a capacitor, which will lead to a *linear* change in voltage across it

- * Thus, $dV_0/dt = -i/C = -V_{SAT^+}/(RC)$
 - \Rightarrow V₀ decreases linearly with time, producing a ramp waveform, with a slope of $-V_{SAT^+}/(RC)$
- * Note that V_0 is fed back to the *Schmitt trigger* as its input, which has a *threshold low* $V_{TL} = -f'V_{SAT^+}$, with $f' = R_1/R_2$
- * : As soon as V_0 falls below V_{TL} , the *Schmitt Trigger* would get *triggered*, and V_2 would *toggle* to V_{SAT^-}
- * This would immediately *change the sign* of the current i, however, if the saturation voltages are *symmetric*, then its *magnitude* would remain the *same*

- * Now, C would start to *discharge* with this constant current, and V_0 would increase *linearly* with time, with a slope of $-V_{SAT^-}/(RC)$
- * It would rise all the way to the *threshold high* V_{TH} $(= -f'V_{SAT^-})$, and again a *change of state* would take place, with V_2 *swinging back* to V_{SAT^+}
- * This sequence of events would keep on *repeating* itself, and at the output, we would get a clear *triangular* waveform, swinging between V_{TL} and V_{TH}



- * Note: When $V_2 = V_{SAT^+}$, V_0 decreases linearly from V_{TH} to V_{TL} with a slope of $-V_{SAT^+}/(RC)$
- * On the other hand, when $V_2 = V_{SAT^-}$, V_0 increases linearly from V_{TL} to V_{TH} with a slope of $-V_{SAT^-}/(RC)$
- * Thus, positive ramp time duration:

$$\frac{V_{TH} - V_{TL}}{T_1} = -\frac{V_{SAT^-}}{RC} \Rightarrow T_1 = RC \times \left(\frac{V_{TH} - V_{TL}}{-V_{SAT^-}}\right)$$

* Similarly, *negative ramp time duration*:

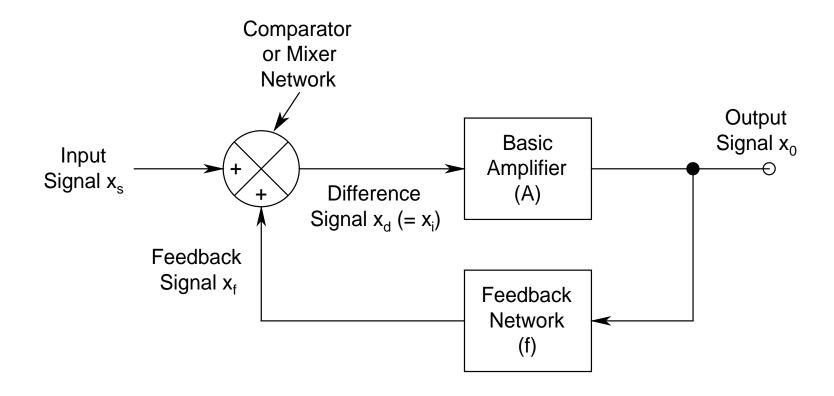
$$\frac{V_{TL} - V_{TH}}{T_2} = -\frac{V_{SAT^+}}{RC} \implies T_2 = RC \times \left(\frac{V_{TH} - V_{TL}}{V_{SAT^+}}\right)$$

- * *Note*: For symmetric saturation voltages, a *50%* duty cycle triangular waveform results
- * Time Period $T = T_1 + T_2$
- * Oscillation Frequency f = 1/T
- * Note that this circuit also does not need any source
 - As soon as the *power supply* is turned on, the *oscillations* would start, and would continue *ad infinitum* till the *power supply* is turned off

Sinusoidal Oscillators:

- * Note: The square- and triangular-wave generator circuits discussed earlier do not need any source
 - As soon as the *power supply* to the circuit is turned on, the *oscillations* start, and continue till the *power supply* is turned off
 - These circuits are also known as *oscillators*
- * However, generating *sinusoidal oscillations* is a totally different ball game
- * Sinusoidal Oscillators achieve this task

Concept of Oscillation: Positive Feedback:



A: Gain of the Basic Amplifier f: Feedback Factor of the Feedback Network $x_0 = Ax_i$, $x_f = fx_0$, $x_i = x_d = x_s + x_f$ * Gain with feedback:

$$A_{f} = \frac{X_{0}}{X_{s}} = \frac{X_{0}}{X_{i}} \frac{X_{i}}{X_{s}} = A \frac{X_{s} + X_{f}}{X_{s}} = A \left(1 + \frac{X_{f}}{X_{s}}\right)$$

$$= A \left(1 + \frac{X_{f}}{X_{0}} \frac{X_{0}}{X_{s}}\right) = A \left(1 + fA_{f}\right)$$

$$\Rightarrow A_{f} = \frac{A}{1 - fA}$$

- * An extremely interesting relation: If $fA \rightarrow 1$,
 - $A_f \rightarrow \infty$, and output appears without any input!
- * This is what *oscillators* are all about: Make *fA* [known as the *Loop Gain* (L)] approach *unity*!

Conditions for Oscillation: Barkhausen's Criteria:

- * Note that both f and A are *functions of frequency* (\omega)
- * Therefore, there may be a *certain frequency* (ω_0) , at which: $L(j\omega_0) = f(j\omega_0)A(j\omega_0) = 1$
- * Note that at this frequency (ω_0) , the *gain* of the system would be *infinite*, and the signal will *self -regenerate*, without the need for any external input
- * Thus, ω_0 becomes the *oscillation frequency*
- * Also, since this oscillation happens only for a particular frequency (ω_0) , hence, a pure sinusoidal waveform results

* German physicist *Heinrich Georg Barkhausen* summarized this condition into two criteria, which came to be known as **Barkhausen's Criteria**:

•
$$|L(j\omega_0)| = 1$$
 and $\angle L(j\omega_0) = 0^\circ$

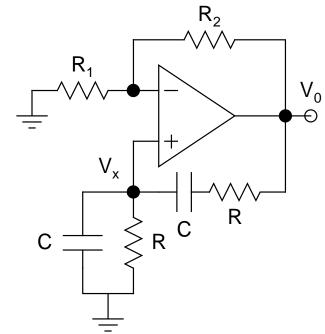
- * Barkhausen's Crietria in words:
 - For a positive feedback system to oscillate, the magnitude of the loop gain must at least be unity, and the total phase shift around the loop should be 0° (or 360°)

- * Note that once the oscillations start, it cannot be stopped without turning the power supply off
- * At ω_0 , if |L| < 1, then with each pass around the loop, the *amplitude* of the output signal would keep on *decreasing*, and eventually, it will *die down* on its own \Rightarrow Under this condition, sustained sinusoidal oscillation is impossible to achieve
- * On the other hand, at ω_0 , if |L| > 1, then with each pass around the loop, the output signal would keep on *increasing*, and will eventually get *limited* by the power supply voltages \Rightarrow Creates distorted output

- * |L| exactly equal to unity is a risky proposition, since component values drift with age and temperature
- * Practical oscillator circuits operate with |L| slightly greater than unity, which lets the oscillations build up, and then control it using automatic gain control (AGC) circuit
- * Two very popular OA-based *sinusoidal oscillators*:
 - Wien-Bridge Oscillator
 - Phase-Shift Oscillator

Wien-Brige Oscillator:

- * Identify that the circuit has both positive and negative feedback
- * Identify that the *non-inverting* terminal with a potential of V_x is the positive feedback node



* In the absence of the positive feedback, the circuit is simply a *non-inverting amplifier*, with the gain (A) given by:

$$A = \frac{V_0}{V_x} = 1 + \frac{R_2}{R_1}$$

* Impedance of series RC circuit:

$$Z_1 = R + 1/(j\omega C)$$

* Impedance of parallel RC circuit:

$$Z_2 = R \parallel \left(\frac{1}{j\omega C}\right) = \frac{R}{1 + j\omega RC}$$

* Thus, the *feedback factor*:

$$f = \frac{V_x}{V_0} = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{3 + j \left[\omega RC - 1/(\omega RC)\right]}$$

* Hence, the *loop gain*:

$$L = fA = \frac{1 + R_2/R_1}{3 + j \left[\omega RC - 1/(\omega RC)\right]}$$

- * For sustained sinusoidal oscillations, Barkhausen's criteria must be satisfied
 - \Rightarrow L must be a real number with zero phase angle
 - \Rightarrow The imaginary part of L must vanish
 - \Rightarrow Gives the oscillation frequency $\omega_0 = 1/(RC)$
- * Also, |L| must equal unity

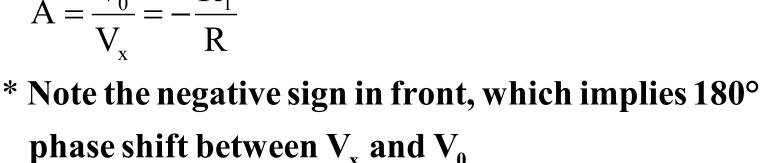
$$\Rightarrow R_2/R_1 = 2$$

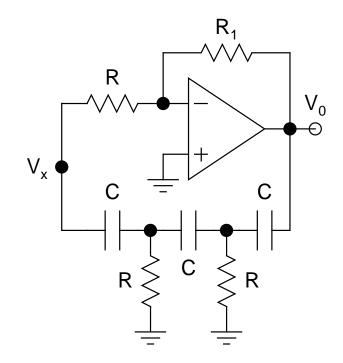
* Actually, R_2/R_1 is made slightly larger than 2 to take care of the drift of component values with temperature and age \Rightarrow Amplitude builds up and eventually gets controlled by the AGC circuit

Phase-Shift Oscillator:

- * Interesting to note that the circuit uses *negative feedback* with the non-inverting terminal grounded
- * Thus, the circuit is simply an inverting amplifier, with the gain (A) of the *basic amplifier* given by:

$$A = \frac{V_0}{V_x} = -\frac{R_1}{R}$$





- * Note also that the feedback path consists of three RC-sections
- * If each of these sections contribute a phase shift of 60°, then the total phase shift of the feedback path will be 180°, which would result in 0° phase shift for the loop gain (L)
- * The feedback factor (f):

$$f = \frac{V_x}{V_0} = \frac{(\omega RC)^3}{\omega RC \left[(\omega RC)^2 - 5\right] + j \left[1 - 6(\omega RC)^2\right]}$$

* Now, for L to be real, the imaginary part of f must vanish at the oscillation frequency ω_0

$$\Rightarrow \omega_0 = \frac{1}{RC\sqrt{6}}$$

- * At ω_0 , f = -1/29
- * This negative sign, implying a phase shift of 180°, coupled with the phase shift of 180° of A, makes the total phase of L to be 0°
 - ⇒ One part of Barkhausen's criteria satisfied

* L at ω_0 :

$$L = fA = \left(-\frac{1}{29}\right) \times \left(-\frac{R_1}{R}\right) = \frac{R_1}{29R}$$

which should be slightly larger than unity

- * Thus, R₁ should be made slightly larger than 29R
- * This would initiate the oscillations, which will tend to blow up
 - ⇒ Amplitude needs to be controlled by AGC circuit
- * Typical oscillation frequency for both these oscillators ~ tens of Hz to hundreds of kHz