

Bivariate Normal Distribution: We say that (X, Y) follows a bivariate normal distribution if its pdf is given by

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-Q/2} \text{ for } -\infty < x < \infty, \quad -\infty < y < \infty,$$

where

$$Q = \frac{1}{1-\rho^2} \left[\left(\frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{x-\mu_1}{\sigma_1} \right) \left(\frac{y-\mu_2}{\sigma_2} \right) + \left(\frac{y-\mu_2}{\sigma_2} \right)^2 \right]$$

with $-\infty < \mu_i < \infty, \sigma_i > 0$ for $i = 1, 2$, and ρ satisfies $\rho^2 < 1$. Clearly, this function is positive everywhere in \mathbb{R}^2 .

Pdf of kth order statistic

The result is

$$g_k(y_k) = \begin{cases} \frac{n!}{(k-1)!(n-k)!} [F(y_k)]^{k-1} [1-F(y_k)]^{n-k} f(y_k) & a < y_k < b \\ 0 & \text{elsewhere.} \end{cases}$$

Joint pdf of (i,j)th order statistics

it is found that

$$g_{ij}(y_i, y_j) = \begin{cases} \frac{n!}{(i-1)!(j-i-1)!(n-j)!} [F(y_i)]^{i-1} [F(y_j) - F(y_i)]^{j-i-1} [1-F(y_j)]^{n-j} f(y_i) f(y_j) & a < y_i < y_j < b \\ 0 & \text{elsewhere.} \end{cases}$$